Multiple Defaults and Merton's Model

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Black and Scholes [1973] and Merton [1974] pioneered the so-called structural approach to pricing defaultable bonds and assessing the creditworthiness of firms. Among others, Black and Cox [1976], Geske [1977], Longstaff and Schwartz [1995], Leland and Toft [1996] and Collin-Dufresne and Goldstein [2001] have extended the Merton [1974] model. They view company equity as a call option on the assets of the firm and company total liabilities as the strike price.

The structural approach has been very popular in the industry. According to Ferry [2003, p. 22], "Merton-type models for estimating credit default probabilities are now so common they are driving pricing in the credit market." The attraction of this type of approach lies in its analytical tractability and the economic interpretation of its inputs, namely, the current value and the volatility of the company's assets, the outstanding debt, and the debt maturity.

Structural models have been criticized for failing to reproduce the level of spreads that are observed in practice. The empirical literature thus far indicates that these models underpredict spreads on corporate bonds; see, for example, Huang and Huang [2000] and Eom, Helwege, and Huang [2003]. Hull, Nelken, and White [2003] note that the performance of the Merton's model could be improved by calculating spreads using the implied volatility of two equity options instead of estimating the instantaneous equity volatility and the debt outstanding. Gemmill [2002] also shows that Merton's model works well when zero-coupon bonds are used for funding.

Campbell and Taksler [2003] find the level of idiosyncratic firm volatility explains well the cross-sectional variation in corporate bond yields. Huang and Kong [2003] conclude that equity market variables have an important role in explaining credit spread changes. These findings confirm that information from the equity markets is a key input in default models, which strengthens the appeal of the structural framework.

The structural approach is mainly used to estimate the relative probabilities of default and credit default swap spreads. Very few attempts have been made to extend the approach to calculate the joint probabilities of default for many firms. Default probabilities for a basket of firms are important for credit analysis, the valuation of various credit derivatives, and risk management. Credit derivatives are among the fastest growing financial contracts in the derivatives market, and default-triggered credit derivatives such as default swaps and default baskets are the most popular.[1]

These products usually involve more than one firm, so we need a model that can accommodate multiple defaults. Although single default probabilities are easily derived within the structural framework, derivation of joint default probability curves is more complex as correlations between firms need to be taken into account.

Zhou [2001] has extended the structural framework to include many firms. He derives in closed form the default correlation of two firms and allows default to occur prior to maturity. Cathcart and El-Jahel [2003] also calculate default correlation in a stochastic interest rate hybrid structural framework, where default depends on the value of the assets of the firm and on a hazard rate process.

These attempts to generalize the structural framework require a trade-off between realistic assumptions and analytical tractability. Although they overcome some limitations of Merton's model, they require computationally intensive numerical solutions that increase in difficulty with the size of the problem.

In a multifirm framework, it is important to have a tractable benchmark that is easy and fast to calculate and against which to compare results from more realistic generalizations. Merton's model remains popular because of its tractability and simplicity. Practitioners can overcome some of its limitations by assuming, for example, that a bond can default only at the time horizon they consider but not at any other time and not necessarily at the maturity of the bond.

We extend the Merton [1974] model to many firms and demonstrate how to calculate various default and survival probabilities in a tractable form.

I. MERTON'S FRAMEWORK

In the Merton [1974] model, the value of the firm's assets is assumed to obey a lognormal diffusion process with a constant volatility. Default occurs at maturity if the value of the firm's assets is less than a promised payment. We extend the Merton [1974] framework to N firms. In the following, for every firm i, $i = 1, \ldots, N$, the asset value dynamics are described by the stochastic process:

$$dV_i = \mu_i V_i dt + \sigma_i V_i dB_{Vi} \tag{1}$$

For each firm, μ_i and σ_i denote, respectively, the asset value drift rate and volatility. For any two firms i, j, $i = 1, \ldots, N$ and $j = 1, \ldots, N$, dB_{Vi} and dB_{Vj} are Wiener processes such that $dB_{Vi}dB_{Vj} = \rho_{ij}dt$. ρ_{ij} denotes the correlation of the asset value of the two firms i and j.

Let X_i denote the promised payment of firm i at maturity T. Firm i defaults at time T if $V_i \leq X_i$ or if $y_i \leq 1$, where y_i is defined such that $y_i = V_i / X_i$. y_i is considered an indicator of the credit quality of firm i.

We also consider D_i and S_i, $i = 1, \ldots, N$, random variables that describe the default and survival status of the N firms at time T. $D_i = 1$ if firm i defaults at time T, and 0 otherwise. The probability of default of firm i is $P(D_i = 1) = P(V_i \leq X_i) = P(y_i \leq 1)$. $S_i = 1$ if firm i survives at time T, and 0 otherwise. The probability of survival of firm i is $P(S_i = 1) = P(V_i > X_i) = P(y_i > 1)$. For ease of notation, we denote $P(D_i = 1) = P(D_i)$ and $P(S_i = 1) = P(S_i)$.

For a single firm, the probability of default in Merton [1974], $P(D_i) = P(V_i \leq X_i) = P(y_i \leq 1)$, is given by:

$$\begin{aligned} P(y_i \leq 1) &= \phi(d_2(y_i, \sigma_i^2)) \\ d_2(y_i, \sigma_i^2) &= -\frac{ln(y_i) + (\mu_i - \sigma_i^2/2)T}{\sigma_i\sqrt{T}} \end{aligned} \tag{2}$$

where ϕ is the cumulative density function of the standard normal distribution.[2]

We use results from probability theory to calculate default and survival probabilities for two firms and two or more firms.

Two Issuers

The case of two issuers illustrates the flexibility of the Merton [1974] approach and the various probabilities that can be derived in closed form within this framework.

The probability of joint default at time T is simply:

$$\begin{aligned} P(D_1 \cap D_2) &= P(y_1 \leq 1, y_2 \leq 1) \\ &= \phi_2(d_2(y_1, \sigma_1^2), d_2(y_2, \sigma_2^2), \rho_{12}) \end{aligned} \tag{3}$$

where $\cap$ stands for "and"; ϕ_2 is the bivariate standard cumulative normal; $d_2(y_i, \sigma_i^2)$ for $i = 1, 2$, is given by Equation (2); and ρ_{12} is the pairwise correlation between the assets of firms 1 and 2.

A host of other useful probabilities can also be derived. For example, for $i = 1, 2$, $j = 1, 2$, and $i \neq j$, $P(D_i | D_j)$, the probability of default of firm i conditional on the default of firm j at time T is:

$$P(D_i|D_j) = \frac{P(D_i \cap D_j)}{P(D_j)} = \frac{\phi_2(d_2(y_i, \sigma_i^2), d_2(y_j, \sigma_j^2), \rho_{ij})}{\phi(d_2(y_j, \sigma_i^2))} \tag{4}$$

The probability of at least one default $P(D_1 \cup D_2)$ at time T, where $\cup$ stands for "or," is:

$$\begin{aligned} P(D_1 \cup D_2) &= P(D_1) + P(D_2) - P(D_1 \cap D_2) && (5) \\ &= \phi(d_2(y_1, \sigma_1^2)) + \phi(d_2(y_2, \sigma_2^2)) - \\ &\quad \phi_2(d_2(y_1, \sigma_1^2), d_2(y_2, \sigma_2^2), \rho_{12}) && (6) \end{aligned}$$

One can also easily deduce probabilities of survival from the probabilities of default. For $i = 1, 2$, the probability of survival of firm i at time T is:

$$P(S_i) = 1 - P(D_i) \tag{7}$$

The probability of at least one survival at time T is:

$$P(S_1 \cup S_2) = 1 - P(D_1 \cap D_2) \tag{8}$$

The probability of joint survival at time T is:

$$P(S_1 \cap S_2) = P(S_1) + P(S_2) - P(S_1 \cup S_2) \tag{9}$$

For $i = 1, 2$, $j = 1, 2$, and $i \neq j$, the probability of survival of firm i conditional on the survival of firm j at time T is:

$$P(S_i|S_j) = \frac{P(S_i \cap S_j)}{P(S_j)} \tag{10}$$

The probability of default of firm i and survival of firm j at time T is:

$$P(D_i \cap S_j) = P(D_i) - P(D_i \cap D_j) \tag{11}$$

The probability of survival of firm i and default of firm j at time T is:

$$P(S_i \cap D_j) = P(S_i) - P(S_i \cap S_j) \tag{12}$$

Finally, one can also compute the default or survival correlation denoted by ρ_D or ρ_S of firm 1 and 2:

$$\begin{aligned}\rho_D = \rho_S &= Corr(D_1, D_2) \\ &= \frac{P(D_1 \cap D_2) - P(D_1).P(D_2)}{(P(D_1)(1-P(D_1)))^{1/2}(P(D_2)(1-P(D_2)))^{1/2}} \\ &= \frac{P(S_1 \cap S_2) - P(S_1).P(S_2)}{(P(S_1)(1-P(S_1)))^{1/2}(P(S_2)(1-P(S_2)))^{1/2}}\end{aligned} \tag{13}$$

Substituting for the relevant probabilities, one gets the solution required.

Many Issuers

The case of more than two firms is more challenging, as one has to take into account the correlations between all the firms. The calculations are slightly more complex but involve no more than the use of the multivariate normal distribution.

Probability of Joint Default

Let the probability of joint N defaults in a basket of N firms at time T be denoted by:

$$\begin{aligned}&P(D_1 \cap D_2 \cap D_3 \cap \ldots \cap D_N) = \\ &\quad P(y_1 \leq 1, y_2 \leq 1, y_3 \leq 1, \ldots, y_N \leq 1)\end{aligned} \tag{14}$$

This probability expression is given by

$$\begin{aligned}&P(y_1 \leq 1, y_2 \leq 1, y_3 \leq 1, \ldots, y_N \leq 1) = \\ &\quad \phi_N(d_2(y_1, \sigma_1^2), d_2(y_2, \sigma_2^2), \ldots, d_2(y_N, \sigma_N^2), \rho_{12}, \rho_{13}, \ldots)\end{aligned} \tag{15}$$

where ϕ is the N-th variate standard cumulative normal. Expressions similar to Equation (15) are used in the option pricing literature; see, for example, Geske and Johnson [1984] and Johnson [1987].

One can also calculate the probability of at least one firm surviving from among a basket of N firms:

$$\begin{aligned}P(S_1 \cup S_2 \cup S_3 \cup \ldots \cup S_N) &= \\ 1 - P(D_1 \cap D_2 \cap D_3 \cap \ldots \cap D_N)\end{aligned} \tag{16}$$

Probability of at Least One Default in a Basket

To calculate the probability of at least one default in a basket of N firms, it is not sufficient to know the probability of an individual default. We also need to give complete information concerning all possible overlaps. That is, we have to derive the probability of joint default of firm i and firm j, and the probability of joint default of firm i, firm j, and firm k, and so on, where $i = 1, \ldots, N$, $j = 1, \ldots, N$, $k = 1, \ldots, N$, and so on. Let

$$\begin{aligned}&F_1 = \Sigma P(D_i) \\ &F_2 = \Sigma P(D_i \cap D_j) \\ &F_3 = \Sigma P(D_i \cap D_j \cap D_k), \ldots\end{aligned} \tag{17}$$

where $P(D_i)$ is given by Equation (2), $P(D_i \cap D_j)$ by Equation (3), and $P(D_i \cap D_j \cap D_k)$ up to $P(D_i \cap D_j \cap \ldots \cap D_N)$ by Equation (15). In Equation (17), two subscripts are never equal, and $i < j < k < \ldots < N$, so that in the sums each combination appears once and only once. Each F_m has $\binom{N}{m}$terms where $\binom{N}{m} = N!/[m!(N-m)]!$ represents the number of different subpopulations of size $m \leq N$. The last sum would be F_N, which reduces to the single term $P(D_1 \cap D_2 \cap D_3 \cap \ldots \cap D_N)$, the probability of N joint defaults.

The probability of at least one default denoted by P_1 is then given by:

$$P_1 = F_1 - F_2 + F_3 - F_4 + - \ldots \pm F_N \quad (18)$$

For $N = 2$, we have only the first two terms F_1 and F_2, the two-firm result we have seen.

From this result we can also automatically derive the probability of joint survival of N firms. This is given by $P(S_1 \cap S_2 \cap S_3 \cap \ldots \cap S_N)$ where

$$\begin{aligned} P(S_1 \cap S_2 \cap S_3 \cap \ldots \cap S_N) &= 1 - P_1 \\ &= 1 - F_1 + F_2 - F_3 + F_4 - + \ldots \pm F_N \quad (19) \end{aligned}$$

Probability of at Least or Exactly *m* Defaults in a Basket

The probability of m exact defaults in a basket of N firms, with $1 \leq m \leq N$, is given by:

$$P_{(m)} = F_m - \binom{m+1}{m} F_{m+1} + \binom{m+2}{m} F_{m+2} - + \ldots \pm \binom{N}{m} F_N \quad (20)$$

and the probability of at least m defaults in a basket of N firms by:

$$P_m = F_m - \binom{m}{m-1} F_{m+1} + \binom{m+1}{m-1} F_{m+2} - + \ldots \pm \binom{N-1}{m-1} F_N \quad (21)$$

Substituting for the relevant probabilities, one gets the solutions required. For proof of these probability results, see Feller [1950].

II. IMPLEMENTATION, RESULTS, AND IMPLICATIONS

The challenge in using these formulas for default or survival probabilities is to evaluate the cumulative multivariate normal probability ϕ_N. There are many routines available for this purpose. We use the function "MULTI-NORMAL-PROB" from Numerical Algorithm Group's (NAG) Excel add-in statistics package. This function can evaluate the multivariate normal distribution up to ten dimensions in less than a fraction of a second. For other numerical methods designed to evaluate the multivariate normal, see, for example, Geske [1977], Geske and Johnson [1984], and Schervish [1985].

To illustrate our results, we construct a basket of $N = 5$ firms, and compare the probabilities of:

1. Joint default.
2. At least one default in the basket.
3. At least two defaults in the basket.
4. Exactly one default in the basket.
5. Exactly two defaults in the basket.

There are two main base case scenarios:

- Scenario 1: $\mu_i = 0.05$, $\sigma_i^2 = 0.16$, for all $i = 1, \ldots, 5$ in the basket (*Exhibits 1–5*):

  For scenario 1, we report the results for two values of pairwise asset correlations $\rho_{ij} = 0.4$ and $\rho_{ij} = 0.1$. For each pairwise asset correlation value, we consider three different credit qualities: $\gamma_i = 3$ high quality, $\gamma_i = 2$ medium quality, and $\gamma_i = 1.2$ low quality. Once the levels of pairwise asset correlation and credit quality have been selected, these values will apply to all firms in the basket. That is, we assume that all firms within a basket have the same credit quality and the same pairwise asset correlation.[3]
- Scenario 2: $\mu_i = 0.05$, $\gamma_i = 2$, for all $i = 1, \ldots, 5$ in the basket (*Exhibits 6-10*):

  For scenario 2, we report the results for two values of pairwise asset correlations $\rho_{ij} = 0.4$ and $\rho_{ij} = 0.1$. For each pairwise asset correlation value, we consider three different volatility levels: $\sigma_i^2 = 0.25$, $\sigma_i^2 = 0.16$, and $\sigma_i^2 = 0.09$. Once again, when selected, these parameter values will apply to all the firms in the basket.[4]

Results

For scenario 1, we have two main observations. First, credit quality is an important factor. In Exhibits 1, 2, and 3, the higher the credit quality, the lower the joint and the at-least probabilities of default.

In Exhibits 4 and 5, the relationship between exact default probabilities and credit quality varies with time to maturity. In the short term, the higher the quality of the basket, the lower the probability of an exact number of defaults. In the long term, this relation is inverted. The crossing points across different-quality baskets depend on the exact number of defaults we are considering. The fewer the number of exact defaults, the sooner the relation will be inverted.

The reason is that in the short term the probability of a certain number of default is high for low-quality firms but over time that exact number could be exceeded. For high- or medium-quality baskets in the short term, the probability of a certain number of default is low, but over time this exact number of default could be reached.

Second, asset correlation also plays a significant role. In Exhibit 1, we note that higher asset correlation implies higher joint default probabilities across all credit qualities. In Exhibit 2, higher asset correlation implies lower probabilities of at least one default, and this result is consistent across maturities and credit qualities.

In Exhibit 3, the probability of at least two defaults varies with time to maturity and the basket credit quality. For short maturities, the higher the asset correlation, the higher the default probability. For longer maturities, the relationship is inverted.

The timing of the crossing points across different asset correlation depends on the basket credit quality. The poorer the credit quality, the sooner this crossing occurs. For medium- and high-credit quality baskets, for example, the probability of at least two defaults derived using a level of asset correlation of 0.1 becomes higher than the probability of at least two defaults derived using a level of asset correlation of 0.4, for maturities longer than 9.5 and 18.0 years, respectively.

In Exhibit 4, the shape of the probability of exactly one default varies with the basket credit quality and time to maturity. High-credit quality baskets typically have an increasing probability of one default. Medium-credit quality baskets present a humped shape. For low-credit quality baskets, this probability increases for short maturities but declines thereafter.

We also observe, for all credit quality baskets, that the probability of exactly one default derived using higher asset correlation will cross the probability of exactly one default using lower asset correlation. Typically high asset correlation probabilities will be lower before the crossing point. The crossing point depends on the credit quality. The poorer the quality, the sooner the crossing occurs.

The probability of exactly two defaults in Exhibit 5 exhibits the same characteristics as the probability of exactly one default. Note, however, first, the probabilities of exactly two defaults are lower. Second, the humped shape is more pronounced at longer maturities.

For scenario 2, Exhibits 6-10, the impact of asset

EXHIBIT 1
Probability of Joint Default (Scenario 1)

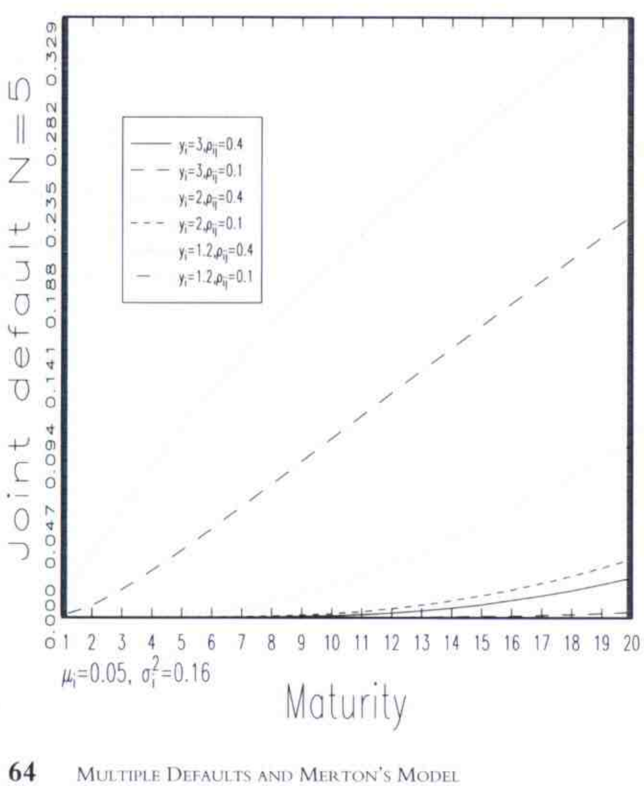


EXHIBIT 2
Probability of at Least One Default (Scenario 1)

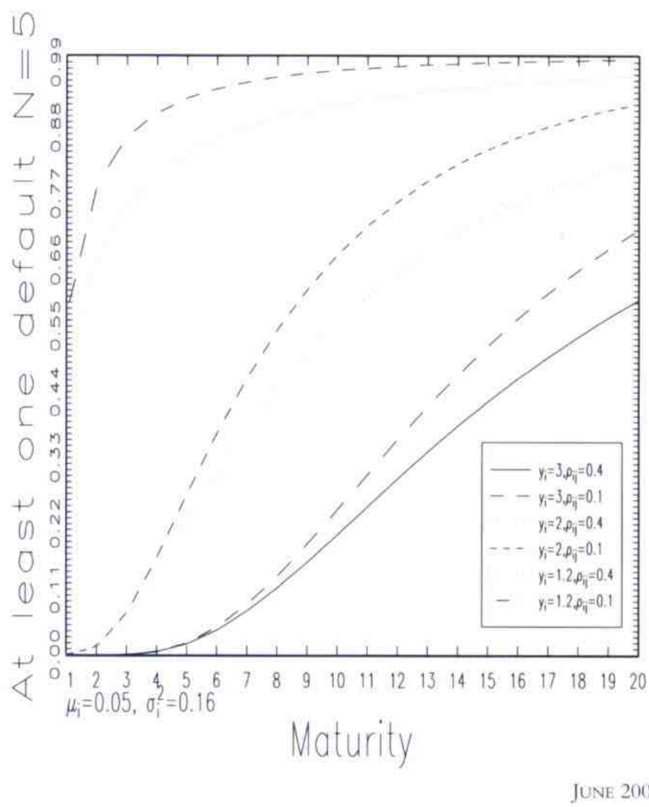


Exhibit 3
Probability of at Least Two Defaults (Scenario 1)

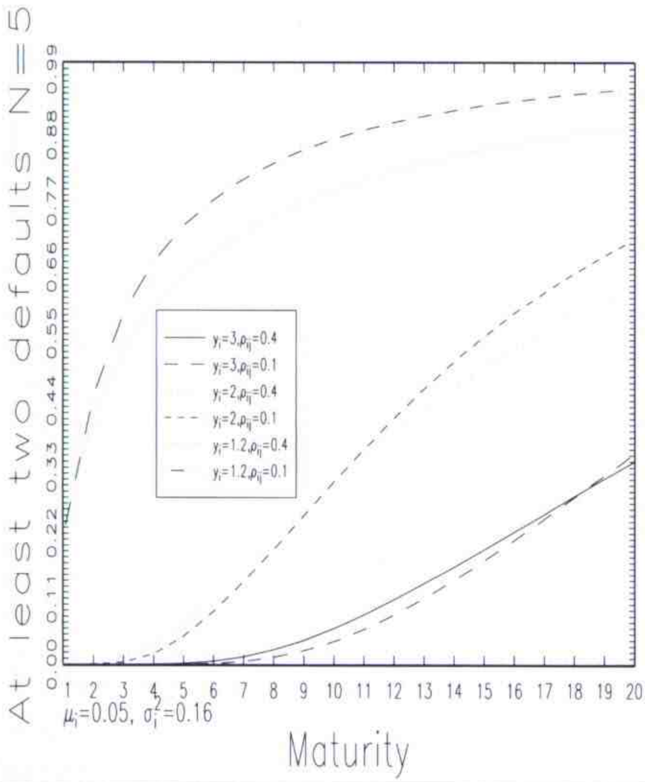


Exhibit 4
Probability of Exactly One Default (Scenario 1)

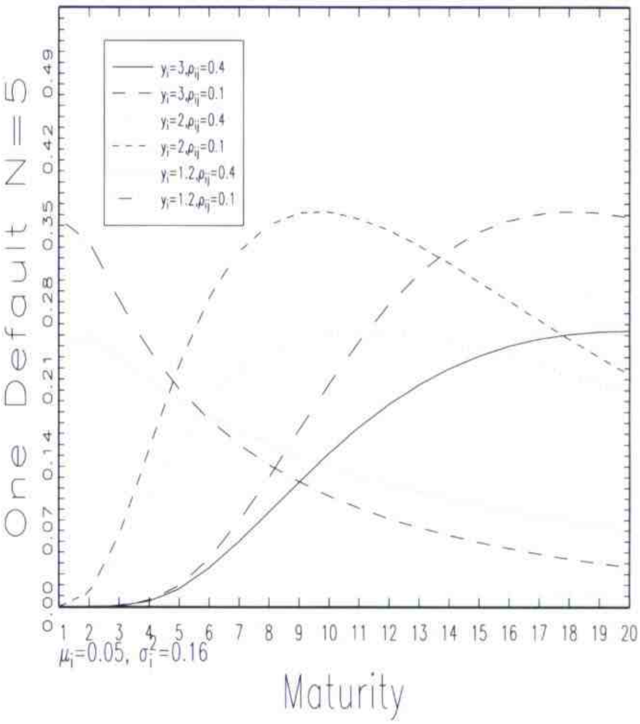
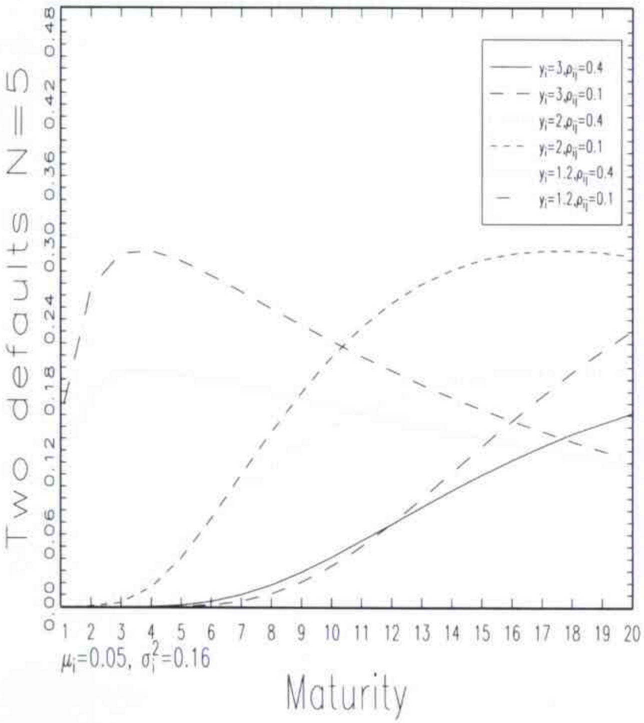


Exhibit 5
Probability of Exactly Two Defaults (Scenario 1)



volatility parameters is comparable to the impact of credit quality. High volatility generates default probability patterns comparable to those generated by low-quality firms, and vice versa. For example, in Exhibit 6, the higher the asset volatility parameter, the higher the joint probability of default.

This observation is consistent for all the other probabilities we consider and also across different correlation levels. In Exhibit 7, for example, higher asset correlation implies lower probabilities of at least one default, and this result holds for all maturities and asset volatilities.

Implications

The Merton [1974] framework allows the calculation of a host of default and survival probabilities for N firms in closed form. This facilitates the study of the impact of various key parameters such as asset correlation, credit quality, time to maturity, and asset volatility on these probabilities.

Asset correlation plays a prime role in determining probabilities of defaults within a basket. A lower asset correlation implies lower joint probabilities of default. Therefore, diversification across regions and industries could

EXHIBIT 6
Probability of Joint Defaults (Scenario 2)

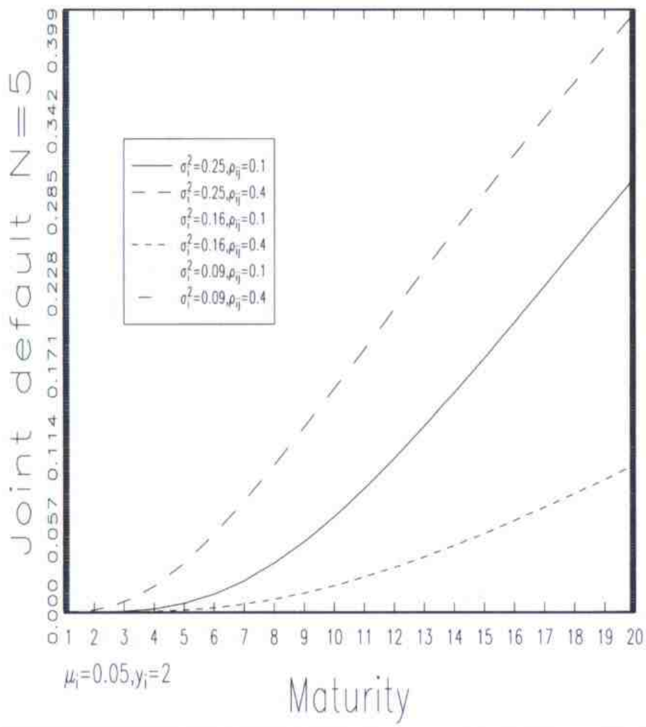
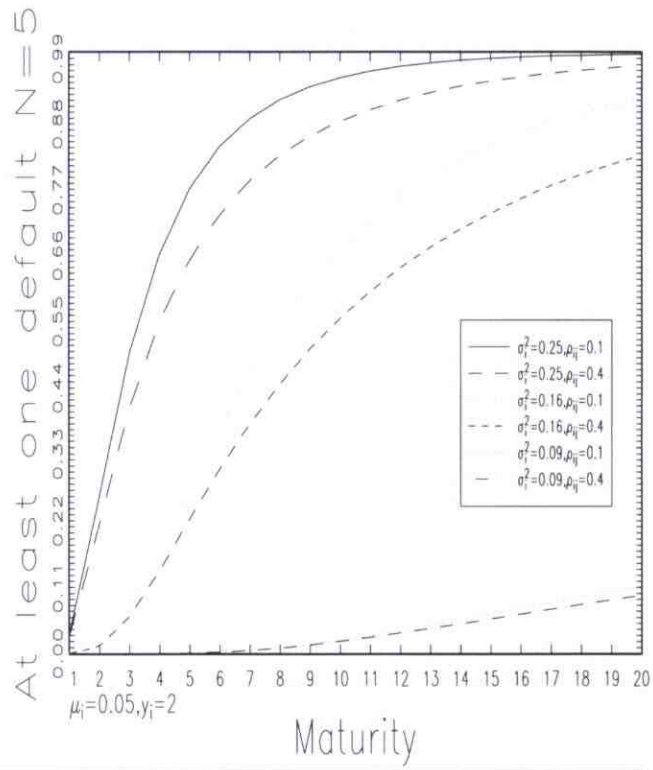


EXHIBIT 7
Probability of at Least One Default (Scenario 2)



significantly reduce the probabilities of joint defaults within a basket. In the case of the probabilities of an exact or at-least number of defaults, this relationship is not always maintained, as lower asset correlation in certain cases would imply higher probabilities.

Credit quality of firms is also an important factor. Joint and at-least probabilities of default are low for high-quality firms and high for poor-quality firms. For the probability of an exact number of defaults, this pattern is not always observed, however. The relationship between the exact probabilities and credit quality changes with the parameters of the model.

Default probabilities vary with maturity. Joint and at-least probabilities usually start low over short horizons and then increase over time. The exact probability of default displays a humped shape. The time of the peak hump depends mainly on the number of defaults considered, the pairwise asset correlation, the credit quality, and asset volatility.

Asset volatilities have a great impact on the various probabilities of defaults. Joint and at-least probabilities of default are low for low asset volatilities and high for high asset volatilities. This pattern for the probability of an exact number of defaults could, however, be inverted for

EXHIBIT 8
Probability of at Least Two Defaults (Scenario 2)

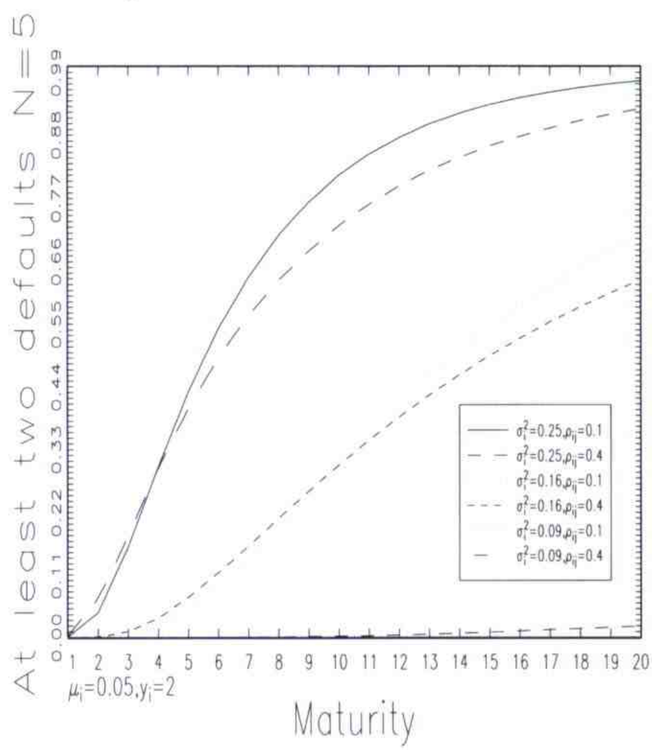


EXHIBIT 9
Probability of Exactly One Default (Scenario 2)

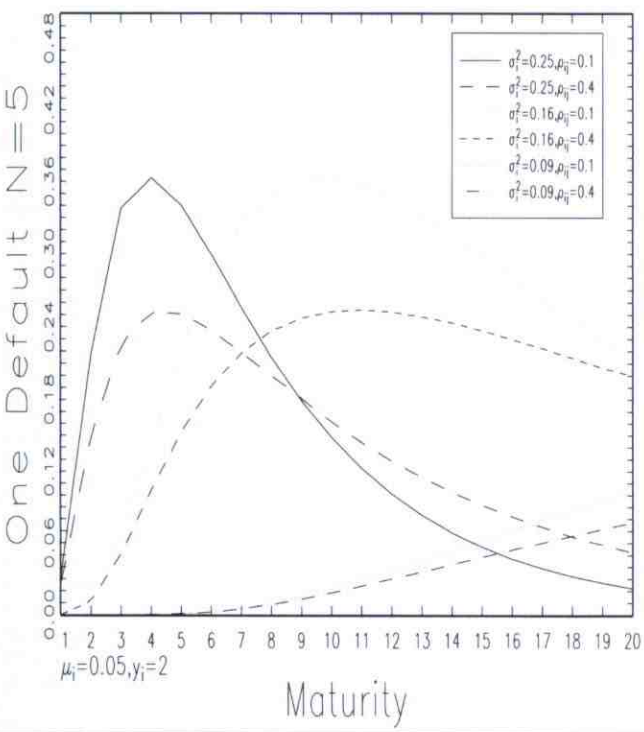
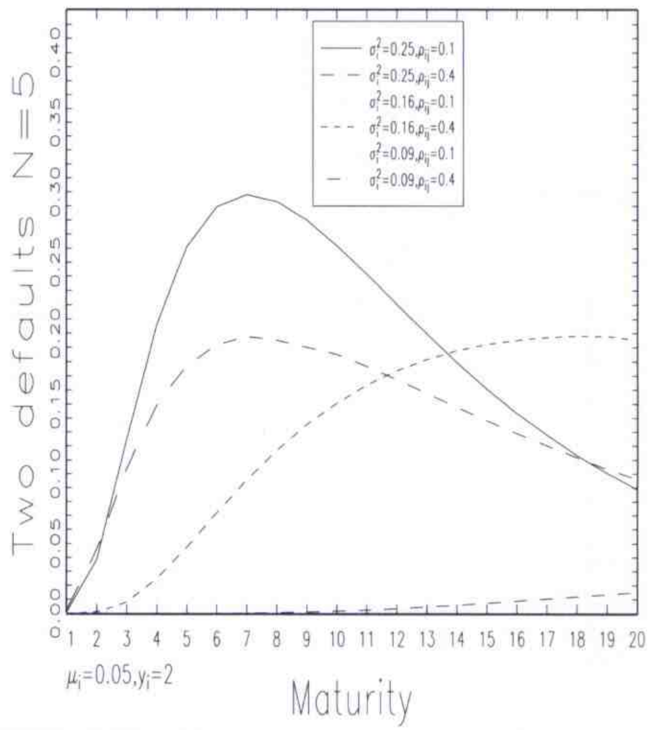


EXHIBIT 10
Probability of Exactly Two Defaults (Scenario 2)



particular maturity, credit quality, and pairwise asset correlation parameter values.

III. CONCLUSION

Recent empirical investigations by Campbell and Taksler [2003] and Huang and Kong [2003] highlight the importance of equity market variables in explaining credit spread movements. This makes the argument for the structural approach even more compelling.

We have presented a straightforward extension of Merton [1974] to accommodate multiple defaults. This extension has several advantages.

First, it presents a unified framework for the calculation of single and joint default probabilities. Second, all results are in closed form and can serve as a comparison benchmark with more complex generalizations. Third, joint default probability curves can be easily built along with various others, such as the at-least and exact default probability curves. Fourth, as credit quality changes with time, the derived default probabilities will be dynamic, and this gives a better understanding of the risk of a basket of firms.

Finally, default probabilities have direct implications for risk management, credit analysis, and the pricing of various credit derivatives such as n-th to default swaps.

ENDNOTES

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[1]These products are considered cash products that can be priced using default probability curves. Default probability curves describe default probabilities for various future times.

[2]For details see Merton [1974].

[3]This assumption could easily be relaxed.

[4]We could easily allow asset volatility to vary among firms within a basket.

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