Two-Factor Structural Model of Determinants of Brazilian Sovereign Risk

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The credit spread, defined as the yield difference between a risky and a risk-free bond with similar characteristics, is related to the default probability of the security. When the issuer of a risky bond is a foreign country, or when there are constraints on repayment, this spread is known as sovereign risk.[1]

The credit spread paid by Brazilian bonds abroad is very high, which has significant implications for the country's economic growth and the cost of rolling over its debt. It is important to try to identify the determinants of this spread in order to adjust public policies to reduce it.

In financial theory, credit risk models differ in the way they treat default probability and the recovery rate of capital. There are two broad types of models, structural and reduced-form. Structural models, initially proposed by Merton [1974], adopt a contingent claims approach to option pricing. The risky bond is modeled as a derivative over some measure related to the economic and financial conditions of the debtor that trigger the default event—defined as when the measure crosses a critical barrier. Application of structural models produces the default probability at any future time (stopping time is defined) and also the term structure of the credit spread.

In reduced-form models, proposed by Duffie and Singleton [1999] and others, the spread of a risky bond follows a stochastic process that is not explicitly related to the financial state and the economic conditions of the firm or bond issuer, and the default probability cannot be determined (inaccessible stopping time). In reduced-form models, the default probability or the stochastic intensity of default is taken as a given or estimated from rating agency data.

Hund [2002] points out the difficulty of reconciling the behavior implied by structural credit risk models with the realities observed in the credit spread market data. Structural models applied to sovereign risk analysis face some limitations, such as a lack of specific legislation about sovereign default, coupled with the difficulty of defining a single appropriate measure related to a government's willingness and capacity to pay. Structural models in these circumstances need adaptation, such as using as a default index a measure related to the level of the country's indebtedness.

Some authors recently have applied structural models to the pricing of sovereign risk. Lehrbass [1999] uses an equity index of the borrower country expressed in foreign currency as the default index. Wiggers [2002] adopts the domestic output in foreign currency, and follows the Leland [1994] approach of optimal endogenous default.

One significant advantage of structural models applied to sovereign risk, especially for government policymakers, is that they allow a better understanding of how public policies and market expectations affect sovereign risk.

We propose a two-factor structural

model, applying the contingent claims approach to explain the Brazilian sovereign risk implicit in the Brazilian government bond (C-bond) based on macroeconomic fundamentals and agents' expectations.[2]

This model can explain the evolution of the spread, distinguishing the effect of macroeconomic fundamentals—which summarize past policy and shock effects on the economy—from the effect of the risk premium—which reflects market expectations about the future level of indebtedness and also the political credibility of the country. The model is able to forecast the credit spread and estimate the default probability perceived in the market at any future time.

These features are essential for pricing credit derivatives and therefore enable investors and portfolio risk managers to find the optimal allocation of portfoliowide assessment of country risk and create hedge positions across markets according to a suitable risk measure technique.[3]

I. STRUCTURAL MODEL

Structural credit risk models (SM), initially proposed by Merton [1974] and extended by Black and Cox [1976], Leland [1994], Longstaff and Schwartz [1995], Zhou [1997], and Saá-Requejo and Santa-Clara [1999], among others, focus on explaining a risky bond price by the financial and economic fundamentals of the firm or the bond issuer. These authors apply the contingent claims approach of option theory to pricing default-risky securities, and obtain the term structure of the credit spread required for various bond rating levels. Bond ratings are related to the current indebtedness level of the issuer.

Structural models are defined in a partial equilibrium framework, where the bond price follows the contingent claims approach. The asset value of the issuing firm is the underlying variable that evolves according to a stochastic differential equation. Default can occur at any time before bond maturity when the asset value hits a certain barrier (for example, a minimum ability to honor debts). Such an event involves debt restructuring and causes losses to creditors, who may receive a new risk-free bond, but at a discounted value.[4]

Huang and Huang [2002] show that structural models explain about 60%-80% of the spread on corporate bonds rated by Moody's as speculative-grade (Ba) and roughly 100% of the spread for those rated as high-risk obligations (B).

Let y be the macroeconomic fundamental that triggers default, representing the country indebtedness, such as the debt/GDP ratio. To avoid arbitrage opportunities, we represent the stochastic process y in a martingale equivalent risk measure as a two-factor model:[5]

$$dy/y = \lambda_t dt + \sigma\, dz^* \tag{1}$$

$$\lambda_t = \lambda_{t-1} + \zeta_t \quad \text{for} \quad \zeta_t \sim (0, \varphi) \tag{2}$$

Equation (1) describes a risk-neutral process, where the risk-neutral drift λ_t is the sum of the real drift and the associated risk premium, and dz^* is a Wiener process in a martingale measure.

The fundamental y process described in Equations (1) and (2) is an empirical matter (derived in Appendix A). The random walk hypothesis described by Equation (2) makes the model tractable and suggests smooth changes in agents' expectations over time, which would represent changes in the macroeconomic climate of the country (future behavior of the country's debt and political credibility) or even in international market liquidity.

Such a hypothesis is similar to the one adopted by Bates [1991], which interprets the frequency of the jump parameter in a martingale equivalent risk measure obtained from options market data as an indicator of market expectations and as an anticipation of the 1987 crash.

A default event is triggered the first time the variable y hits a critical value α (default barrier), i.e., when the fundamental reaches a value that makes debt repayment unlikely. Because y evolves stochastically, this moment (first hitting time) is uncertain and has a probability distribution.

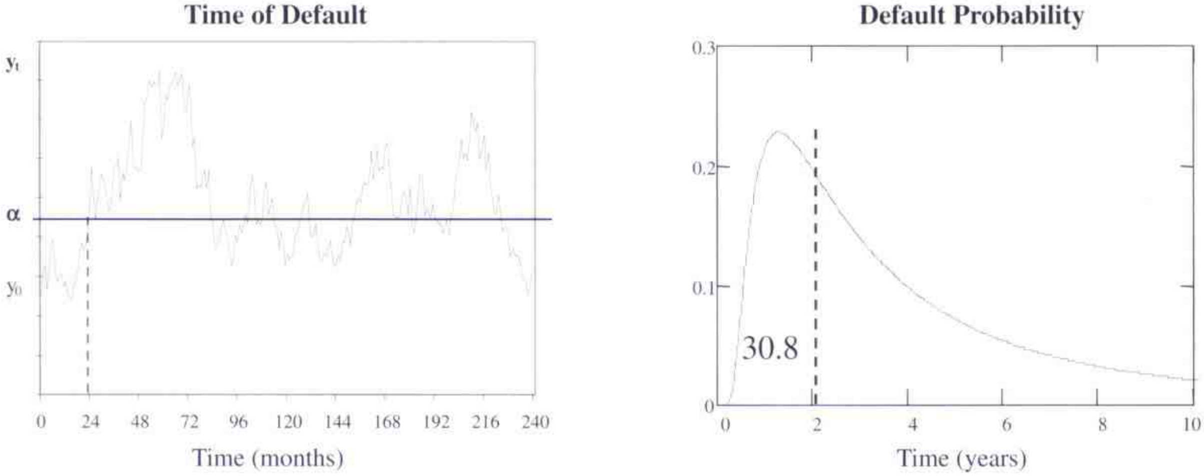
Appendix B shows the first hitting time distribution, which is illustrated in Exhibit 1. In the first graph, according to a realization of y initially set as y_0, default occurs after 24 months and occurs when the variable y crosses the barrier α. The second graph shows the default probability distribution, where the default probability in the first two years corresponds to the area below the curve.

Pricing a risky coupon bond that pays coupon c_i, at every time i and matures at time T implies considering the default probability of each promised coupon payment. In an ex ante default event at time $\tau > t$, the price $P(t, \tau|\beta)$ of a risky coupon bond at time t is given by Equation (3), where all the cash flows c_i for $i < \tau$ are totally paid. After the default event, the creditors receive a fraction β of an identical but risk-free bond:

$$P(t, \tau|\beta) = \sum_{i=t}^{\tau} c_i e^{-r_{t,i} \cdot i} + \beta \sum_{i>\tau}^{T} c_i e^{-r_{t,i} \cdot i} \tag{3}$$

Exhibit 1

Default Event



where r_t is the term structure of the risk-free interest rate at time t.[6] We follow Lehrbass [1999] and adopt Moody's assumption about the recovery rate for high-risk obligations, choosing β equal to 40% as a plausible estimate.[7]

Let $\pi(\tau \mid y_t, \lambda_t, \sigma, \alpha)$ be the default probability in each time period $\tau > t$. The price of a risky coupon bond P_t is given by Equation (4), which represents the expected present value of every coupon's payments during the lifetime of the bond, considering the default probability of those payments, plus a random term that includes all other influences. The model can be estimated, without loss of generality, conditional on λ_t.[8]

$$\begin{aligned} P_t &= \sum_{\tau>t}^{T} \pi(\tau|y_t, \lambda_t, \sigma, \alpha)\, P(t, \tau|\beta) + e_t \\ &= p(t, y_t, \lambda_t|\Psi) + e_t \quad \text{for} \quad e_t \sim (0, V) \end{aligned} \tag{4}$$

Estimation Methodology

In Equation (4), the market data on the C-bond price P_t and the fundamentals y_t are observable.[9] The information set is given by $\Omega_t = \{(P_i, y_i)\ i \le t\}$, and the parameters $(\lambda_t, \sigma, \alpha, \varphi)$ must be estimated.

The term structure of interest rates is obtained from Bloomberg through its time series data in "Fair Market Curve Indexes." Bloomberg calculates the spot rate for maturities of 3 to 6 months, and 1-10, 15, 20, and 30 years using bonds considered fairly priced and issued without options. Appendix A shows the estimation of σ for the real process y_t, while the other parameters are estimated conditional on the expected value of σ.

Equations (2) and (4) describe a non-linear Kalman filter model, i.e., a state space model that can be estimated, following West and Harrison [1997], by locally applying a first-order Taylor expansion in Equation (4):

$$\begin{aligned} p(t, y_t, \lambda_t) \cong\ & p(t, y_t, E(\lambda_t|t-1)) + \\ & \left.\frac{\partial p(t, y_t, \lambda)}{\partial \lambda}\right|_{\lambda = E(\lambda_t|t-1)} (\lambda_t - E(\lambda_t|t-1)) \end{aligned} \tag{5}$$

Hence, conditional on the parameter vector $\Psi = (\alpha, \varphi)$, one can estimate the variance of the error $(V \mid \Psi, \Omega)$ and the state variable $(\lambda_t \mid \Psi, \Omega)$, and calculate the predictive log-likelihood function $LL(\Psi \mid \Omega)$.

The parameter Ψ can be estimated by the maximum log-likelihood $\Psi^* = \text{argmax } LL(\Psi \mid \Omega)$ or using the Markov chain Monte Carlo (MCMC) method, presented in Gamerman [1997], which constructs a sample of the estimator distribution $(\Psi \mid \Omega)$. Appendix C shows a summary of the estimation methodology.

The model described in Equations (2) and (4) explains the risky coupon bond price, P_t, by the fundamental y_t, a risk premium λ_t, and an error term e_t. The usefulness of this description of the bond price depends on: 1) the amount of information in the fundamental y; 2) the accuracy of Equation (1) in describing it; and 3) the hypothesis of Equation (2), where the equivalent risk premium follows a smooth trend.

Spread Forecast

To forecast the credit spread, we apply the structural model we have developed. The spread is a deterministic function of the bond price and the term structure of the risk-free interest rate $s(p_t, r_t)$. Since the bond price depends on fundamentals, as shown in Equation (4), assume $S(t, y_t, \lambda_t | \Psi, r_t) = s(p(t, y_t, \lambda_t | \Psi, r_t), r_t)$.[10]

Unlike autoregressive models, this representation is subject to some limitations in forecasting the spread since all the information in the spread of the previous period is not contemplated. To overcome such limitations, consider the marginal structural model (MSM) as a first-order Taylor expansion of $S(t, y_t, \lambda_t | \Psi, r_t)$ around the last period, as shown in Equation (6). The first-order Taylor expansion should consider the effect of fundamental and risk premium variation. According to Equation (2), the expected value of risk premium variation $E(\Delta\lambda_t | t-1)$ equals zero, so Equation (6) determines the expected value of the spread variation based on variations in the fundamental and on the partial derivatives of the proposed structural model:[11]

$$E(S_t | t-1) = S_{t-1} + s_y(y_{t-1}, E(\lambda_t | t-1))\Delta y_t \qquad (6)$$

where: $s_y(y_{t-1}, E(\lambda_t | t-1)) = \left.\dfrac{\partial s(y, \lambda)}{\partial y}\right|_{y=y_{t-1}}$

II. DEFAULT INDEX

The ratio between the stock of debt and the source, direct or indirect, of the revenues to service it is a measure of the degree of effort required of the debtor, a measure that is related to its capacity and willingness to continue servicing the debt. A debtor will default the first time this quantity reaches a critical value (default barrier).[12]

The sources of the revenue to service the external debt depend on the time period considered for macroeconomic adjustment. In the short run, the revenue source is the value of a country's exports.[13] In the medium term, a greater portion of production can be exported, changing the real exchange rate. In this case, we take industrial output as an approximation of domestic revenue sources. The source of revenue to service the public debt is nominal tax collections, approximated by industrial output multiplied by the level of the IGP-DI (General Price Index–Domestic Supply).[14]

The balance of payments and the fiscal balance are two economic fundamentals whose net imbalances are adjusted through variations in net external indebtedness and public sector debt. The degree of effort to service these debts appears to be an indicator of the instability of a country, and thus of the possibility of its default.

Equation (1) assumes a partial equilibrium model of option pricing where the innovation on the fundamental process y should be independent of the bond price or spread, meaning that: 1) the spread should not affect future values of y, which is known as the Granger causality test (see Hamilton [1994]); and 2) given previous period information, the fundamental y should not be related to the spread or to any other variable that determines the spread. The latter assumption imposes constraints on choosing the fundamental measure y (discussed in Appendix A).

The indebtedness measures y we use as a default index include:[15]

A. (External debt – Reserves) / Real value of exports over the past 12 months [(D – R)/Exp];[16]
B. (External debt – Reserves) × Exchange rate lagged 1 period / Index of industrial output over the past 12 months [(D – R) C [–1]/Ind]; and
C. Net internal public sector debt deflated by the IGP-DI / Industrial output over the past 12 months (DI/Ind/ IGP).

Exhibit 2 shows the evolution of the default indexes since 1995.

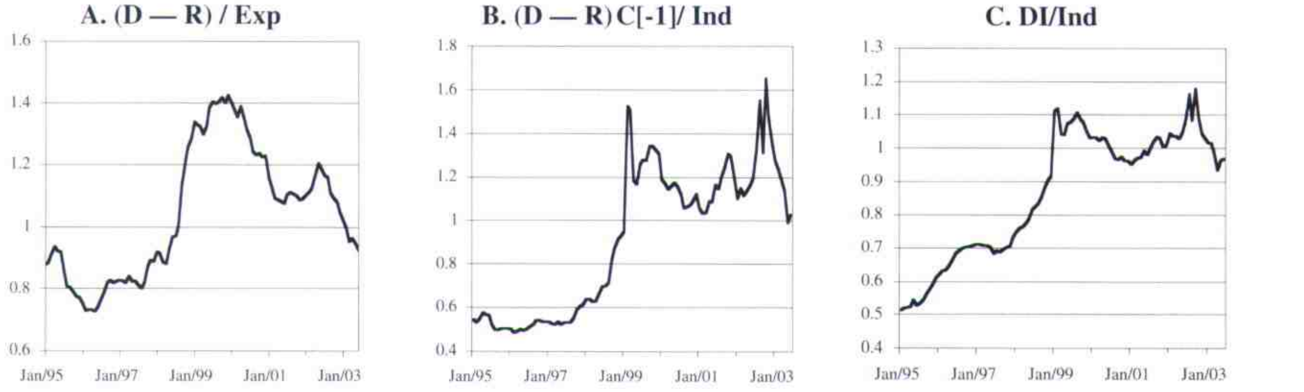
Finally, the structural model relates the risky coupon bond price, a daily market variable, to macroeconomic variables such as indebtedness, observed only on a monthly basis. To preserve the same time units, we measure the bond price by its monthly average.

III. EMPIRICAL RESULTS

Our model determines the default probability at any particular time and also forecasts the Brazilian credit spread. When the goal is to forecast the spread, we must guarantee that the information set in the prior period is independent of the current information set. In the proposed two-factor structural model, we assume the default barrier α and the variance of the risk premium φ parameters are constant over time, and estimations are conditioned to a specific sample datum. The risk premium can be estimated in a recursive procedure conditional on the information set available until the previous time.

To implement the spread forecast $E(s_t | t-1)$, the sample data are segmented into two parts, I = [01/1997; 06/2001] and O = [07/2001; 06/2003], and the $\psi = (\alpha, \varphi)$ parameter is estimated exclusively on the basis of the first sample, while the forecast analysis relies on the second sample.

EXHIBIT 2
Default Indexes



Structural Model

In the second half of 2002, there was considerable uncertainty about future government policy with regard to the Brazilian economy, especially after August 2002, in the face of presidential elections when victory of the opposition candidate (now President Luiz Inácio Lula da Silva) was becoming likely.[17] To reflect the sudden increase in the market risk premium, we allow a high variance on the risk premium estimator at the beginning of this period (August 2002).

Empirical use of the model raises questions concerning both the spread and risk premium forecast. The model assumes that the risk premium changes smoothly over time and can be estimated, contrary to an hypothesis of a constant risk premium over time. To ascertain the time-varying risk premium and estimate it accurately, we compare the proposed two-factor structural model (SM) to a restricted structural model (RSM) with a constant risk premium over time ($\lambda_t = \lambda$).[18]

The non-linear nature of the two-factor model suggests we take care in estimating the Ψ parameters. The predictive log-likelihood function LL(Ψ | I) can present, for example, local modes and is subject to identification problems.[19] We perform the estimation in two steps. First, we define a grid for a set of reasonable values of the parameters and choose a first estimate for the optimum value Ψ^* | I. Then we use the MCMC algorithm, and take this value as the starting point to obtain the empirical distribution of (Ψ | I).

Exhibit 3 shows for both model specifications the maximum log-likelihood estimator (MLE) and an adjustment capacity measure for the spread evolution (AC), defined as the ratio of the sum of the squared errors and the variance of the spread. Exhibit 4 shows the mean and standard deviation estimates of parameter Ψ.

EXHIBIT 3
Performance Statistics

	SM		RSM	
Index	MLE	%AC	MLE	%AC
A. (D – R)/Exp	–2458	45	–2688	249
B. (D – R)C[–1]/Ind	–2460	45	–2690	56
C. DI/Ind/IGP	–2458	39	–2674	67

A. (External debt – Reserves)/Real value of exports over past 12 months.

B. (External debt – Reserves) Exchange rate lagged 1 period/Index of industrial output over past 12 months.

C. Net internal public sector debt deflated by IGP-DI/Industrial output over past 12 months.

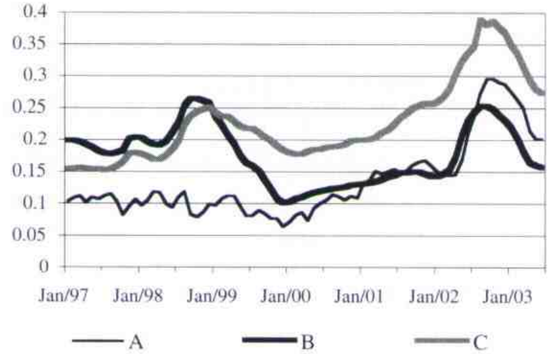
The results show that:

1. The SM does not explain around 40% of the spread variation, which we consider reasonable for a structural model.
2. In the SM, the three default indexes show almost the same performance considering the maximum log-likelihood.
3. The SM is significantly better than RSM, so one cannot reject the time-varying risk premium hypothesis.
4. In the RSM, the last default index C shows the best performance.
5. The default barrier α is estimated accurately in both models as is λ in the RSM.

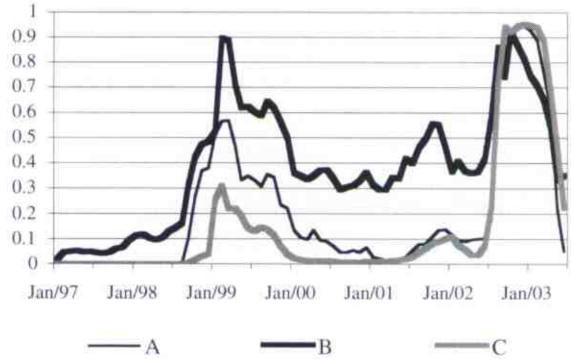
EXHIBIT 4
Model Parameters

	SM				RSM			
Index	$E(\alpha)$	$V(\alpha)^{.5}$	$E(\varphi)$	$V(\varphi)^{.5}$	$E(\alpha)$	$V(\alpha)^{.5}$	$E(\lambda)$	$V(\lambda)^{.5}$
A. (D – R)/Exp	2.09	0.64	0.019	0.0155	2.17	0.087	0.04	0.0050
B. (D – R)C[–1]/Ind	3.1	0.26	0.077	0.0275	2.84	0.021	0.15	0.0143
C. DI/Ind/IGP	2.8	0.05	0.013	0.0205	1.50	0.002	0.02	0.0004

EXHIBIT 5
Risk Premium and Default Probability



Risk Premium (λ^*_t)



Default Probability in 36 Months

6. The risk premium variances φ for default indexes A and C present identification problems.

The proposed two-factor structural model SM provides some interesting results, such as estimations of the risk premium and default probability distribution. Exhibit 5 shows for each default index: 1) the expected value of the risk premium $E(\lambda^*_t | \Theta)$, and 2) the probability of default occurring in the 36 months following each period.[20]

The results show that:

1. The risk premium estimated by the model displays systematically rising risk, even though fundamentals show improvement for the majority of the default indexes. This indicates that although the indebtedness situation has improved, market expectations have been systematically worsening.
2. The default probability and the risk premium are systematically higher following the Russian default (August 1998), reaching their highest levels at the time of the Brazilian presidential election (October 2002). The first resulted in changing the Brazilian exchange rate from a crawling peg to a floating regime, and the second was addressed through an austere fiscal and monetary policy adopted by the new government.
3. The risk premium shows a smooth increase and decline path, according to expectations about future crises and consolidation of a less risky environment.

Exhibit 6 shows the market data and the forecast spread according to the two-factor structural model SM, for each default index. Exhibit 7 shows the estimated Brazilian default probability distribution for the next ten years for each default index in two periods: August 2002, the critical period of Brazilian presidential elections, and as of June 2003.

These graphs show that:

EXHIBIT 6
Market Data and Forecast Spread

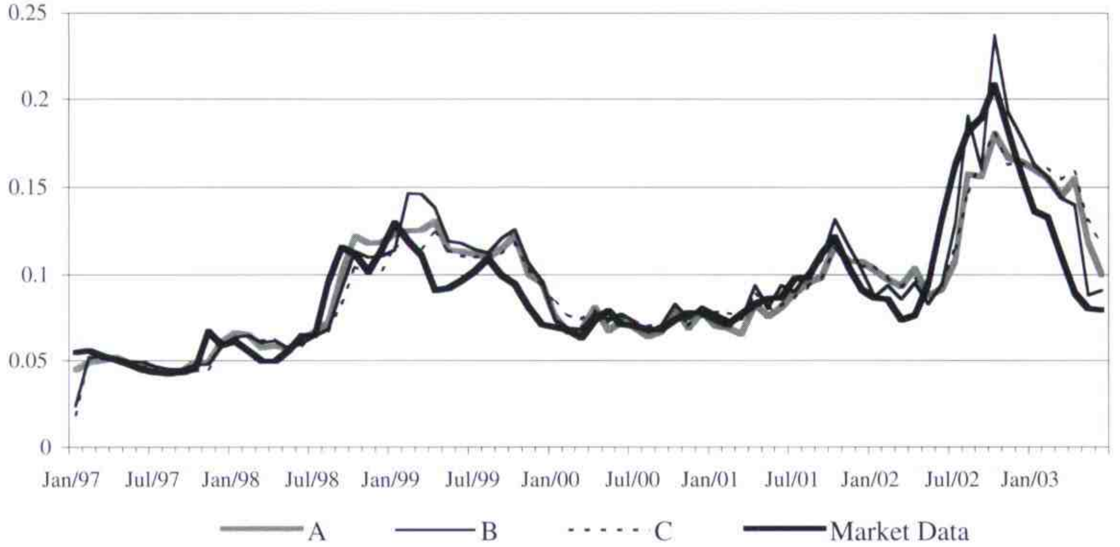
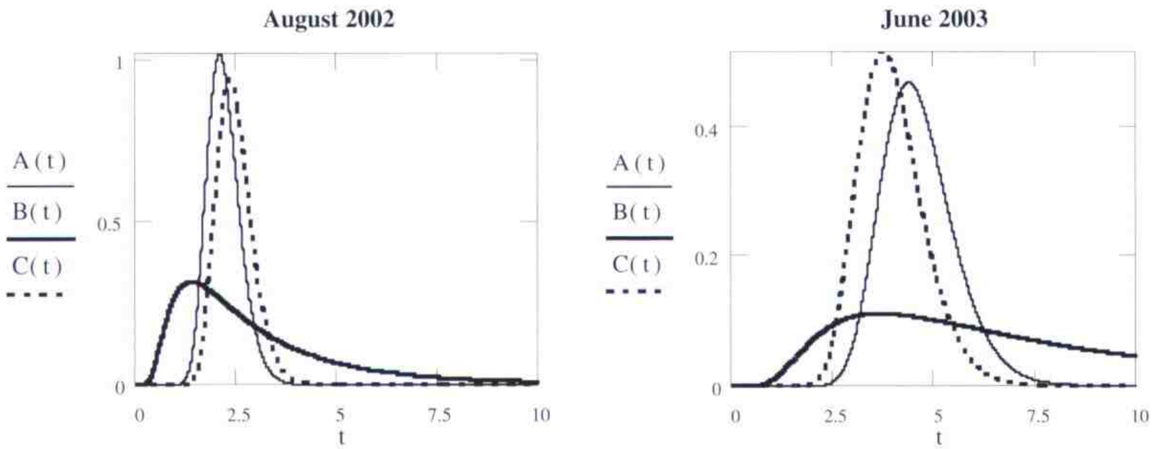


EXHIBIT 7
Brazilian Default Probability Distribution



1. In August 2002, the critical period of the Brazilian elections, default indexes A and C estimate a default probability higher than 99% for the next four years (corresponding to the term of the government in power), while index B has a 78% probability.
2. In June 2003, a year later, after the fiscal and monetary austerity imposed by the new government, the default probability changes for all indexes. The mode moves from around 2.5 years to around 5.0 years.
3. The default probability distribution of index B is less concentrated due to the greater uncertainty of its process.

Spread Forecast

Despite the advantages of the structural model, it has some limitations, including that: 1) it does not consider the last period spread influence; 2) it assumes that default index follows a certain stochastic process; 3) the risk premium, an unobservable variable, needs estimation; and 4) the ill-behaved shape of the predictive log-likelihood function, due to the non-linearity of the model, can lead to identification problems in the estimation process. The marginal structural model overcomes some of these limitations.

EXHIBIT 8
Forecasting Ability

Index	SM	MSM	EM
A. (D – R)/Exp	1.85	0.98	0.95
B. (D – R)C[–1]/Ind	1.52	0.79	0.89
C. DI/Ind/IGP	1.53	1.04	0.98

Econometric models such as Garcia [2001], Muinhos [2002], and Favero [2003] have been proposed to explain the Brazilian credit spread. These models are reduced-form models that can deal with more than one default index, but they have well-known disadvantages. An econometric model can be rationalized as a linear approximation of a non-explicit behavior model.

Considering that the two-factor structural model function $S(y, \lambda_t | \psi)$ can be approximated by a second-order polynomial and that $E(\Delta\lambda_t | t - 1)$ equals zero, it is easy to show that the credit spread variation can be represented by Equation (7), which is an econometric model (EM):[21]

$$E(S_t | t - 1) = S_t + a\,\Delta y_t + b\,y_t\,\Delta y_t \tag{7}$$

The predictive ability of all the models presented so far can be measured by the Theil-U statistic, which compares predictive error with the random walk model error.[22] Exhibit 8 shows that the marginal structural model (MSM) has the best forecasting ability, even compared to the random walk hypothesis. This means that the partial derivatives of the structural model applied in the MSM aggregate more information to forecast the spread than the econometric model estimated coefficients.

Finally, to evaluate the advantage of considering more than one variable to express the default index, we estimate a set of econometric models considering combinations of two and three default indexes leading to a Theil-U of 0.82 (higher than the value obtained by the MSM for index B), for a model considering indexes B and C. This result confirms the forecasting ability of the marginal structural model MSM.

IV. CONCLUSION

We have developed a two-factor structural model for pricing Brazilian sovereign risk. Application of the model to the Brazilian government bond (C-bond) indicates that:

1. The credit spread can be explained by macroeconomic fundamentals that indicate default, such as the level of indebtedness and the corresponding risk premium. That is, it is explained by the result of past policies, and also by the market's expectations of the evolution of future indebtedness related to credibility regarding economic policy.
2. The time-invariant hypothesis for the risk premium is inadequate, meaning that credit spread evolution cannot be explained solely by fundamentals.
3. In the structural model, all default indexes perform similarly in explaining the Brazilian credit spread evolution. The second index—net external debt/industrial output in lagged exchange rate—has more forecasting ability than the others.
4. The risk premium and default probability estimates show a significant increase since the Russian default in August 1998, resulting in the change of the Brazilian exchange rate from a crawling peg to a floating regime at the beginning of 1999.
5. The default probability distribution during the period of the 2002 Brazilian elections—when the victory of the opposition candidate (then elected) was becoming likely—was signaling a 97% probability of a default event in the ensuing four years. Almost a year later, in June 2003, due to the fiscal and monetary austerity of the new government, the default probability for the following three years dropped to less than 1%.
6. The marginal structural model proposed to forecast the Brazilian credit spread shows a Theil-U of 0.8 and better forecasting ability than the econometric models.

In general, expectation is a key issue in a stable economy, but in the Brazilian economy it has a particularly important effect. The internal interest rate is determined partly by the sovereign risk, and indebtedness and market expectations—represented by the risk premium—are crucial in the spread evolution.

Reducing indebtedness implies incurring social costs such as a surplus in public funding or in balance of payments accounts. Yet adjustments in market expectations, which do not have large impacts on social costs, are related to political credibility, sustainability of public policies, and institutional issues. This trade-off indicates how the Brazilian economy is particularly vulnerable to institutional and credibility questions.

APPENDIX A

Model Validation

To validate the model, we demonstrate that: 1) fundamental y, defined as the indebtedness level of the bond issuer, follows the stochastic differential equation (1); and 2) the partial equilibrium model of option pricing where the risky bond price (or spread) is defined as a contingent claim on that indebtedness level is appropriate.

Does Indebtedness Follow Equation (1)?

Variables such as stock of debt or production level measures are usually integrated in the economy. Defining the default index as the level of indebtedness, expressed by the ratio of those quantities, implies that such index is also integrated. This suggests that the default index evolves as Equation (1).

To evaluate this assumption, we transform the scale, $Y_t = \ln(y_t)$, and consider the ADF augmented Dickey-Fuller (ADF) equation:

$$E(\Delta Y_t) = a + \rho\, Y_{t-1} + \Sigma_i \beta_i \Delta Y_{t-i} \qquad \text{(A-1)}$$

Applying Equation (A-1) to each default index, we get the results: 1) the ADF test does not reject the unit root hypothesis; 2) the F-test does not reject that $\beta_i = 0$. These results suggest that the data do not allow us to reject Equation (1).

To estimate the real process for the indebtedness level y that triggers default, we follow Equations (A-2) and (A-3) to estimate the parameters b and σ and the expected drift $E(\mu_t \mid b, \sigma)$ by maximum log-likelihood:

$$dy/y = \mu_t\, dt + \sigma\, dz \qquad \text{(A-2)}$$

$$\mu_t = \mu_{t-1} + \zeta_t \text{ for } \zeta_t \sim (0, b) \qquad \text{(A-3)}$$

The estimates shown in the Appendix A exhibit indicate that we cannot reject Equation (1) as a good specification for the indebtedness level.

Is the Partial Equilibrium Model of Option Pricing Appropriate?

In a partial equilibrium model of option pricing, the security is a contingent claim on the underlying variable y. In this approach, derivative securities are redundant and do not affect the price of assets in the market. Therefore, it is necessary for the bond price, or the spread, not to affect the evolution of the underlying y.

We demonstrate this statement by considering two measures: the lagged effect, and the contemporaneous effect. For the lagged effect, we conduct a Granger causality test between each index and the credit spread. The results are shown in the last column of the exhibit.[23]

APPENDIX A EXHIBIT

Parameter Estimates

Index	b	σ	Granger Causality Test
A. (D-R)/Exp	0.05	0.02	Not rejected
B. (D-R)C[-1]/Ind	0.001	0.083	Rejected
C. DI/Ind/IGP	0.041	0.031	Not rejected

The contemporaneous effect is not a matter of correlation, but rather of identification. If the hypothesis of partial equilibrium is appropriate, we should expect that the underlying y innovation is correlated with the spread, but we need arguments to defend that the underlying y causes the spread and not vice versa.

For instance, the nominal Brazilian exchange rate (R$/US$) and the Brazilian spread are contemporaneously related, especially in stress periods, which means that risk information passes through the exchange rate variable. Therefore an adequate measure of fundamental y, at least, should not include the contemporaneous exchange rate.[24]

This is not a simple question. It is known that in some circumstances options prices can influence their underlying variables, especially when the market is incomplete; primary and derivative asset markets inherently interact. Detemple and Selden [1991] discuss this issue.

The exhibit shows the estimates and the annualized standard error equivalent. Indexes B and C require some comment. To avoid confounding the effect of exchange rate risk on index B, this indicator is defined in terms of lagged exchange rates.

In Brazil, public debt C is the sum of three types of debt—the external portion of public debt denominated in U.S. dollars, the internal portion denominated in Brazilian *reals* but indexed by the exchange rate, and the remaining portions. Despite the lagged effect of the spread on B, and the exchange rate effect on C, we consider these indexes because they are so important.

APPENDIX B

First Hitting Time Distribution

We adopt a stochastic process with dz as a Wiener process:

$$dx = \eta_t\, dt + \sigma\, dz \qquad \text{(B-1)}$$

Following Karatzas and Shreve [1991], the first passage time density of x evaluated at $\tau > t$, i.e., $\tau = \inf\{t \geq 0,\ x(t) > 0\}$, is given by Equation (B-2):

$$\pi(\tau \mid x_t, \eta, \sigma)$$
$$= \frac{|x_t|}{\sigma\sqrt{2\pi(\tau - t)^3}} \exp\left[-\frac{(x_t + \eta_t(\tau - t))^2}{2.\sigma^2(\tau - t)}\right] \quad \text{(B-2)}$$

Through Itô's lemma, we have that if y follows Equation (1), then $x_t = \ln(y_t/\alpha)$ follows the stochastic differential equation:

$$dx = (\lambda_t - 0.5\sigma^2)\,dt + \sigma\,dz \quad \text{(B-3)}$$

Hence, the first passage time density of y evaluated at $\tau > t$, i.e., $\tau = \inf\{t \geq 0, y(t) \geq \alpha\}$, is given by Equation (B-4):

$$\pi(\tau|y_t, \lambda_t, \sigma, \alpha)$$
$$= \frac{|\ln(y_t/\alpha)|}{\sigma\sqrt{2\pi(\tau - t)^3}} \exp\left[-\frac{\left(\ln(y_t/\alpha) + \left(\lambda_t - 0.5.\sigma^2\right)(\tau - t)\right)^2}{2.\sigma^2(\tau - t)}\right] \quad \text{(B-4)}$$

APPENDIX C

Estimation of the Model

The bond model given by Equation (4) may be rewritten as:

$$P_t = p(t, y_t, \lambda_t | \Psi, r_t) + e_t \qquad e_t \sim (0, V) \quad \text{(C-1)}$$

$$\lambda_t = \lambda_{t-1} + \zeta_t \qquad \zeta_t \sim (0, \varphi) \quad \text{(C-2)}$$

Applying a first-order Taylor expansion in Equation (C-1) at the point of the expected value of the λ_t gives Equation (C-3):

$$\begin{aligned} P_t &= p(t, y_t, E(\lambda_t | t-1)) + p_\lambda(t, y_t, E(\lambda_t | t-1)) \times \\ &\quad (\lambda_t - E(\lambda_t | t-1)) + e_t \\ &= \tilde{p} + F_t\lambda_t + e_t \end{aligned} \quad \text{(C-3)}$$

Equation (C-3) can be estimated as in Chapter 8 of West and Harrison [1997], where $F_t = p_\lambda(t, y_t, E(\lambda_t | t-1))$.

Therefore, we have the equations:

Priori:

$$E(\lambda_t | t-1) \qquad V(\lambda_t | t-1) = R_t = C_{t-1} + \varphi \quad \text{(C-4)}$$

Prediction:

$$E(y_t | t-1) = p(y_t, E(\lambda_t | t-1)) \qquad Q_t = R_t + 1 \quad \text{(C-5)}$$
$$e_t = y_t - E(y_t | t-1) \qquad A_t = R_t/Q_t \quad \text{(C-6)}$$

Posteriori:

$$E(\lambda_t | t) = E(\lambda_t | t-1) + A_t e_t$$
$$V(\lambda_t | t) = C_t = R_t - A_t^2 Q_t \quad \text{(C-7)}$$

Variance of the Error:

$$s_t = s_{t-1} + e_t^2 \qquad n_t = n_{t-1} + 1$$
where $n_0 = s_0 = 0$. (C-8)

$$V(\lambda_t | t-1) = R_t s_t / n_t \quad V(y_t | t-1) = Q_t s_t / n_t \quad \text{(C-9)}$$

Predictive Log-Likelihood:

$$LL(\Psi) = -0.5 \sum_t [\ln(Q_t s_t) + t\ln(t + e_t^2/(Q_t s_t)] \quad \text{(C-10)}$$

Using the LL(.) function above, we employ the Markov chain Monte Carlo algorithm (MCMC) of Gamerman [1997] with the rejection criteria to obtain a sample of the Ψ distribution conditional on the information set, and a suitable convergence criterion.

ENDNOTES

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[1]Even if the debtor has the resources to pay, in some cases a lack of foreign exchange can impede payment of foreign debts.

[2]The liquidity and market dimension of the C-bond indicate it is representative of Brazilian sovereign debt in general. The C-bond was issued on April 15, 1994, and has characteristics as follows: maturity: 4/15/14; original value: US$ 7,407,002,000; amortization: 21 equal semiannual payments after 4/15/04 of roughly 5.86% of the principal value; no collateral; six-month coupon payments: first and second years 4% per year; third and fourth years 4.5%; fifth and sixth years 5%; from seventh year on 8%.

Credit rating agencies (Moody's and Standard & Poor's) list several economic, social, and political factors that underlie their sovereign rating criteria. Cantor and Packer [1996] relate these criteria to the current ratings and credit spread.

[3]According to the British Bankers Association "Credit Derivatives Report" [2001/2002], the credit derivatives market is the fastest-growing segment of the over-the-counter derivatives market, especially since the Asian crisis of 1997 and the Russian crisis of 1998. In 1999, the world volume for this market (excluding asset swaps) was US$586 billion, which by the end of 2004 is estimated to stand at US$5 trillion. Estimates are that one-third of this amount involves emerging markets, another third corporations, and the balance private banks. These derivatives, considered exotic and complex instruments, allow the mitigation of default risk and losses.

[4]This approach, known as recovery-of-treasure, follows Jarrow and Turnbull [1995] and Longstaff and Schwartz [1995].

[5]Since the variable y represents a macroeconomic fundamental, which is a non-traded asset, we rely on the martingale equivalent risk measure approach, which transforms the probability distribution of a real stochastic process y into a risk-neutral process, avoiding arbitrage opportunities. For details about application of the martingale measure in pricing derivatives, see Neftci [2000] or Tavella [2002].

[6]For simplicity, we do not consider the stochastic behavior of interest rates.

[7]At the time of the Russian default, the recovery rate of the Russian eurobond was set at 30%. The recovery rate could also be estimated from sample data, but preliminary estimation showed this did not imply an increase of the log-likelihood, which may indicate an identification problem.

[8]The risk premium λ_t^* can be calculated as $\lambda_t^* = \lambda_t - \mu_t$, after the estimation of λ_t, where the real drift process μ_t is estimated from the observed fundamental data.

[9]The observed C-bond price corresponds to the average price (MID) effectively paid in US$$10^4$ after calculating the pro rata accrued interest rate, given the last coupon payment.

[10]The model could be specified to forecast the spread rather than the bond price. In this case, Equation (4) would be replaced by: $s_t = s(t, y_t, \lambda_t | \Psi, r_{jt}) + u_t$, and the estimation would have as a criterion the predictive log-likelihood of the spread. Naturally, the estimators Ψ of the two versions are different. We specify the model for the bond price for ease of calculation. The function p(.) is simpler to calculate than the function s(.), which is particularly important at the estimation stage.

[11]Given ($S_t = s(y_t, a_t) + u_t^*$) as the representation of the structural model that determines the spread, where S_t is the spread market data, a_t is the expected value of the risk premium $E(\lambda_t | t-1)$, and u^* is the error in prediction at this scale, we can introduce a first-order expansion of s(.) indicated below, where (o) corresponds to the other omitted portions: $s(y_t, a_t) = s(y_{t-1}, a_{t-1}) + s_\lambda(y_{t-1}, a_{t-1})(a_t - a_{t-1}) + s_y(y_{t-1}, a_{t-1})(y_t - y_{t-1}) + o$. Defining (u) adequately and noting that according to Equation (2) the second term [the expected value of the risk premium variation $E(\Delta\lambda_t | t-1)$] equals zero, we get Equation (6).

[12]The debtor's degree of effort measure can be interpreted as an approximation of the asset/liability ratio used in corporate models. The assets of the country can be represented by the expected present value of the accrued flow of resources, a quantity equal to the current value multiplied by a constant that depends on the difference between the growth rate of this source and the relevant discount rate, where the growth rate of the revenue source follows a random walk.

[13]To the extent that imports are more flexible than downturns in domestic activity or to increases in the exchange rate.

[14]We use industrial output because it is published monthly and in preliminary tests performed much the same as a measure of gross domestic product, or of tradable GDP calculated on a monthly basis by interpolation.

[15]Seasonal effects of exports and industrial production are removed by considering their 12-month moving average transformation. The indicators were normalized and multiplied by a constant in order to produce one value as of June 2003.

[16]The Brazilian Central Bank publishes the total external debt quarterly. We obtained monthly measurements by interpolation.

[17]Uncertainty evidenced itself in inflation and failure of the market to accept long-term public bonds.

[18]Therefore in SM we have $\psi = (\alpha, \varphi)$ and in RSM, $\psi = (\alpha, \lambda)$.

[19]Different combinations of parameters can correspond to similar predictive log-likelihood values, making the estimation of parameters imprecise.

[20]The expected value of the risk premium is calculated as the difference between the risk premium and the estimated drift of the observed process.

[21]The second-order polynomial is $S(y_t, \lambda_t | \psi) \cong a_0 + a_1 y + a_2 y_t^2 + a_3 \lambda_t + a_4 \lambda_t^2 + a_5 \lambda_t y_t = S(y_t, \lambda_t | a)$, where $S(y_t, \lambda_t | a)$ explains 99% of $S(y_t, \lambda_t | \psi)$ variation.

[22]$TU = \{\sum_{t \in O} (s_t - E(s_t | t-1))^2 / \sum_{t \in O} (s_t - s_{t-1})^2\}^{0.5}$

[23]The Granger causality test is defined as: H0: B(L) = 0 applied to the model $y_t = A(L)y_{t-1} + B(L)s_{t-1} + e_t$.

[24]A VAR model of spread and nominal exchange rate using daily data shows that one cannot reject that each quantity precedes the other and that their residuals are correlated.

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