Benefits of International Bond Diversification

DELROY M. HUNTER AND DAVID P. SIMON

DELROY M. HUNTER
is an assistant professor of finance in the College of Business Administration at the University of South Florida in Tampa, FL.
dhunter@coba.usf.edu

DAVID P. SIMON
is an associate professor of finance at Bentley College in Waltham, MA.
dsimon@bentley.edu

Fixed-income portfolio managers increasingly are given international mandates on the assumption that business cycles are not perfectly synchronized across countries, and investing internationally provides opportunities both to prospect for higher returns and to reap international diversification benefits. Levy and Lerman [1988], Jorion [1989], and Odier and Solnik [1993] show that adding foreign bonds to domestic bonds or stocks would have reduced portfolio risk without reducing returns over the period from 1960 to 1990.

More recently, Solnik, Boucrelle, and Le Fur [1996], Capiello, Engle, and Sheppard [2002], and Hunter and Simon [2003] have examined whether international diversification benefits have weakened over time and evaporated during high-stress periods, as others typically have found in international equity markets.[1]

Solnik, Boucrelle, and Le Fur [1996] find that rolling correlations between major international bond markets rose from the 1960s through the early 1980s, but have not since then. They also find some evidence that rolling correlations increase when rolling standard deviations of returns in each market rise.

Capiello, Engle, and Sheppard [2002] examine whether bond return correlations are higher when two markets simultaneously experience negative rather than positive returns. They find some evidence of asymmetries, but not to the same extent found in equities.

In Hunter and Simon [2003] we find that bond return correlations between the U.S. and both the U.K. and Germany, but not between the U.S. and Japan, have risen in recent years. This finding is consistent with the observation that business cycle conditions between the U.S. and Europe but not between the U.S. and Japan have increasingly moved together in the past decade. We do not find that international bond return correlations increase during periods of sharp bond market declines or heightened volatility.

Conclusions on diversification gains in all three of these studies are based strictly on correlations, however, and none of us specifically tests whether the addition of foreign bonds to a balanced domestic portfolio significantly expands the portfolio's mean-variance frontier.

We extend our work in Hunter and Simon [2003] to formally examine whether U.K., German, and Japanese government bonds provide U.S. investors with international diversification benefits. Our test differs from Levy and Lerman [1988], Jorion [1989], and Odier and Solnik [1993] in several respects. First, we use formal mean-variance spanning tests, rather than the usual graphic analysis, to show that the addition of hedged, but not unhedged, international bonds to domestic assets provides meaningful international diversification benefits over the period from January 1992 through September 2002.

Second, we do not assume that U.S.

investors hold a narrowly defined portfolio of only stocks or government bonds, but rather test whether foreign bonds provide diversification benefits when investors hold a broad portfolio of U.S. government bonds and bills, corporate bonds, and stocks.

Third, we then use a bivariate GARCH framework to estimate various measures that are useful for international bond investors. Specifically, we test whether foreign and U.S. bond returns, volatilities, and correlations are predictable using information readily available to investors. If so, the predicted moments can be used to construct conditional Sharpe ratios to assess the risk-adjusted performance of international bond markets through time.

To date, most studies examining the benefits of international bond investing have used unconditional (sample) means, volatilities, and correlations rather than time-varying measures. As Christopherson, Ferson, and Turner [1999] point out, unconditional measures may lead to biased results.

Finally, we construct conditional return and yield betas. These are important inputs for managing the interest rate risk and tracking error risk of global bond portfolios.

According to Thomas and Willner [1997], international bond portfolio managers commonly construct overall duration measures for globally diversified portfolios simply by aggregating the durations of foreign and U.S. bonds. They criticize this practice on the grounds that the volatilities of foreign and U.S. interest rates are different and that foreign and U.S. interest rate changes are not perfectly positively correlated. They argue that adjusting the duration of foreign bonds by their yield beta—the sensitivity of a country's interest rate changes to U.S. interest rate changes—is essential before aggregating foreign and U.S. bond durations into overall portfolio risk measures.

Our solution is to construct conditional yield betas from the conditional volatilities and correlations. To the extent that the conditional yield betas are both variable and predictable, they are more useful than static yield betas.

The results indicate that adding foreign government bonds on a currency-hedged basis to a diversified U.S. bond and equity portfolio significantly improves mean-variance efficiency. The findings also demonstrate that the means and volatility of government bond returns are predictable. The most consistent factor driving bond returns is lagged equity returns, while only in the case of Japan is a steeper yield curve associated with higher subsequent bond returns.

We find no evidence that the benefits of international bond diversification on a currency-hedged basis are diminished during high-stress periods. As a consequence of the predictability of bond return moments, yield betas and the trade-off between risk and return move predictably and meaningfully over time.

I. DATA

The sample period runs from January 1992 through September 2002. Weekly Wednesday bond returns are calculated using a linear modified duration approximation (see Campbell, Lo, and MacKinlay [1997]). Bond returns from week t to week $t + 1$ are approximated by multiplying the change in generic U.S., Japanese, German, and U.K. ten-year government rates by their modified duration at week t and then adding accrued interest earned from week t to $t + 1$. Foreign bond returns are expressed both in local currencies and in U.S. dollars, where the latter are obtained from local returns adjusted by the change in the spot exchange rate from week t to $t + 1$.

We also compute currency-hedged returns on the foreign bonds. To hedge the bond returns we assume the investor uses a rolling forward contract and is able to hedge ex post returns perfectly.[2]

The forward premium or discount is approximated by the differential between the dollar and the respective foreign currency one-month LIBOR scaled to a one-week horizon. Because this weekly forward premium or discount is very small, and its volatility is swamped by the volatility of local returns, hedged and local returns are quantitatively indistinguishable. As a result, hedged returns are approximately equal to local returns, and we report only the latter.

The slope of the yield curve in the respective markets is specified as the spread between the ten-year Treasury rate and one-month LIBOR. Weekly equity index returns are calculated as the log differences of the dividend-adjusted index levels. The interest rate and foreign exchange rate data are obtained from Bloomberg, and the equity index data are obtained from Yahoo.com. The foreign exchange rate data are the rates that Merrill Lynch uses to calculate dollar-based returns on non-U.S. bond indexes.

Exhibit 1 provides preliminary information on the bond returns. Panel A shows the means and standard deviations, as well as skewness, excess kurtosis, normality, and autocorrelations. The mean weekly bond returns in local currency terms range from 11 to 19 basis points per week, with standard deviations ranging from 74 to 98 basis points.

EXHIBIT 1

Summary Statistics of Bond and Equity Market Returns—January 1992–September 2002

Panel A. U.S. and Foreign Bond and Stock Returns

	Mean (%) (t-value)	Std Dev	Skewness (p-value)	Excess Kurtosis (p-value)	J-B Stat (p-value)	Autocorrelation $r(1)$	$r(2)$	$r(3)$
US	0.1592	0.939	-0.2996	0.6005	16.675	-0.029	0.050	0.076*
	(4.001)		(0.004)	(0.004)	(0.000)			
Germany	0.1573	0.737	-0.695	0.870	62.344	-0.007	-0.014	0.113**
	(5.030)		(0.000)	(0.000)	(0.000)			
UK	0.1901	0.975	-0.017	1.499	52.067	-0.041	-0.031	0.094**
	(4.598)		(0.868)	(0.000)	(0.000)			
Japan	0.1138	0.862	-0.877	4.000	441.97	0.073*	0.027	0.037
	(3.115)		(0.000)	(0.000)	(0.000)			
Germany $	0.1157	1.601	0.077	0.585	8.483	-0.042	-0.005	0.055
	(1.705)		(0.459)	(0.005)	(0.144)			
UK $	0.1629	1.479	-0.372	1.313	52.83	-0.066	-0.012	0.028
	(2.597)		(0/000)	(0.000)	(0.000)			
Japan $	0.1153	1.558	0.609	3.345	293.6	0.003	0.062	0.074*
	(1.746)		(0.000)	(0.000)	(0.000)			
S&P 500	0.1254	2.159	-0.308	1.647	71.602	-0.092**	0.010	0.020
	(1.369)		(0.003)	(0.000)	(0.000)			
DAX	0.1020	3.022	-0.759	2.049	150.69	-0.051	0.032	0.107**
	(0.796)		(0.000)	(0.000)	(0.000)			
FTSE	0.0688	2.296	0.070	2.453	139.92	-0.094	0.015	0.068
	(.706)		(0.502)	(0.000)	(0.000)			
NIKKEI	-0.154	3.665	-0.340	21.676	10895	-0.174***	-0.321	0.087**
	(-0.99)		(0.001)	(0.000)	(0.000)			

Panel B. Cross-Correlation Between U.S. and Foreign Bond Returns

Lag†	-2	-1	0	1	2
Germany	-0.024	0.058	0.560***	0.023	0.008
U.K.	-0.034	0.006	0.477***	-0.004	-0.017
Japan	-0.016	0.025	0.152***	0.064	0.035
Germany $	0.003	0.043	0.373***	0.052	0.000
U.K. $	0.019	-0.013	0.435***	0.025	-0.047
Japan $	0.036	0.074*	0.078*	0.068	-0.023

, **, * Significant at the 10%, 5%, and 1% level.*

†Lag(x) implies correlation (US_t, $Foreign_{t-x}$).

Weekly returns on the generic ten-year Treasuries in local currency. Germany $, UK $, and Japan $ are returns in U.S. dollars. S&P 500, DAX, FTSE, and Nikkei are local currency returns on the major equity indexes. J-B stat is the Jarque-Bera statistic for a test of the null hypothesis that the variable is normally distributed.

When expressed in dollar terms, German and U.K. bond returns are three to four basis points lower while Japanese returns are about the same, as on average over the sample period the dollar rallied against the deutschemark (or the euro beginning in January 1999) and the pound, and was about unchanged against the yen. The volatility of foreign bond returns about doubles when returns are converted to dollars.

Weekly local currency returns on the S&P 500, the DAX, and the FTSE average between 6.9 and 12.5 basis points, with standard deviations roughly double those of bond returns. Weekly Nikkei returns average −15 basis points, with higher standard deviations than the other equity markets. Thus, all bond markets in local currency terms clearly outperformed their respective equity markets on a risk-adjusted basis over the sample period.

The statistics in Exhibit 1 also show that the hypothesis that the return data are normally distributed is rejected for the most part, and that there is significant autocorrelation in some of the return series.

Panel B shows sample cross-correlations of bond returns. The unconditional contemporaneous correlations of U.S. bond returns with German, U.K., and Japanese bond returns are 0.56, 0.48, and 0.15, respectively. The correlations with the German and Japanese bond returns, respectively, are substantially lower at 0.37 and 0.08 when the returns are expressed in dollar terms.

The lower correlation in dollar terms implies that when U.S. bond returns rise, the DM and the yen weaken, which causes lower correlation than with foreign bond returns measured in local currency terms. Thus, higher U.S. bond returns over the sample period are associated with a stronger dollar against the DM (or euro) and the yen.

The correlation between U.S. and U.K. bond returns, however, is little changed when U.K. bond returns are expressed in dollar terms. Generally, the cross-autocorrelations between the U.S. and foreign bonds in local currency or dollar terms are not statistically significant, suggesting the absence of significant lead-lag relationships between the bond markets.

II. MEAN-VARIANCE SPANNING TESTS

The mean-variance spanning test determines if a statistically significant upward shift occurs in the mean-variance frontier formed from the returns on several domestic assets when augmented by a portfolio of international bonds. Intuitively, if we form an efficient frontier from domestic assets and then add international bonds, the resulting frontier will lie above the original frontier only if the international bonds reduce the portfolio's risk without a commensurate reduction in returns.

The mean-variance spanning test regresses hedged and unhedged dollar returns, respectively, of generic ten-year foreign bonds on a constant and the returns of the components of the domestic portfolio:

$$R_{forbond,t} = b_0 + b_1 R_{USbond,t} + b_2 R_{USBills,t} + b_3 R_{USCorp,t} + b_4 R_{USEquity,t} + e_t \quad (1)$$

The independent variables are the returns on the components of the domestic portfolio and include returns on generic ten-year U.S. Treasury bonds, U.S. Treasury bills, U.S. corporate bonds, and the S&P 500 index. Treasury bill returns are from the Merrill Lynch Treasury bill index, while corporate bond returns are from the Merrill Lynch Master corporate bond index.

We test the null hypothesis that international bonds do not provide diversification benefits by testing whether the constant is equal to zero and the coefficients sum to one: $b_0 = 0$ and $\Sigma_{i=1}^{4} b_i = 1$.[3]

A rejection of the null hypothesis indicates that foreign bonds provide diversification benefits, and hence would be consistent with a statistically significant upward shift of the mean-variance frontier of the augmented portfolio, relative to the frontier formed from the domestic assets only. We estimate the model using a generalized method of moments (GMM) approach, which is robust in the presence of non-normally distributed data and allows for the testing of a conditional model. Details are in Appendix A.

Exhibit 2 presents the mean-variance spanning test results. The first column reports p-values of the test of the null hypothesis when the foreign bond returns are converted to U.S. dollars at the then-current spot exchange rate (i.e., unhedged returns), and the second column reports results for hedged bond returns.

The results in the first column indicate that we cannot reject the null hypothesis that unhedged bond returns do not provide incremental diversification benefits when added to a broadly diversified domestic portfolio. Evidence in Exhibit 1 suggests that the high volatility of unhedged foreign bond returns may account for this finding.

We strongly reject the null hypothesis that hedged foreign bond returns do not provide incremental diversification benefits to the domestic portfolio. Hence, the

EXHIBIT 2

Diversification Benefits of International Bonds in Balanced Domestic Portfolio

International Bonds	Returns Converted to Dollars p-value	Currency-Hedged Returns p-value
Germany	0.357	0.000
Japan	0.299	0.000
U.K.	0.559	0.000

benefits of international bond diversification persist on a currency-hedged basis.

III. BIVARIATE GARCH MODEL AND RESULTS

We use a bivariate conditional correlation GARCH (1, 1) model to estimate three bivariate systems of equations, each system with U.S. bond returns and the bond returns of one of three countries, Germany, Japan, and the U.K. (see Bollerslev [1990]). The model jointly estimates the conditional means and volatilities of returns and the conditional correlation between returns. These results are used to construct conditional Sharpe ratios and conditional yield betas. The models are:

$$R_{i,t} = b_0 + b_1 R_{i,t-1} + b_2 YC_{i,t-1} + b_3 EQRET_{i,t-1} + b_4 R_{j,t-1} + \varepsilon_{i,t} \tag{2}$$

$$h_{i,t} = c + \alpha\varepsilon^2_{i,t-1} + \beta h_{i,t-1} + \gamma\varepsilon^2_{j,t-1} \tag{3}$$

$$h_{ij,t} = (r_0 + r_1 x_{1,t-1} + \dots + r_n x_{n,t-1}) \times [\sqrt{h_{i,t}}\sqrt{h_{j,t}}] \tag{4}$$

Equation (2) is the conditional mean model for both U.S. and foreign bond returns, i and j, respectively, with foreign bond returns expressed both in local currencies and in U.S. dollars. Each conditional mean is modeled as a function of a constant; the first lag of the dependent variable; the first lag of the slope of the own-market yield curve (YC); the first lag of the returns on a broad equity index of the country whose bond returns are the dependent variable (EQRET); and the lagged return on the other dependent variable in the bivariate system. The equity indexes are the S&P 500, the DAX, the Nikkei, and the FTSE for the U.S., Germany, Japan, and the U.K.

The lag of the dependent variable is included to reduce the probability that the residuals of the mean equations will be autocorrelated. The slope of the own-yield curve, as measured by the difference between the own-market ten-year Treasury and one-month LIBOR, is included in the model because several studies of the U.S. and foreign term structures have found that steeper yield curves are associated with higher subsequent returns on longer-maturity bonds (see Campbell and Shiller [1991] and Ilmanen [1996]). The yield curve is thought to steepen primarily because of an increase in the risk premium, which leads to a decline in longer-term rates and higher returns on longer-term instruments.

Lagged equity returns are included in the mean equation to allow for the possibility that in a particular country equity returns lead bond returns. In recent years, economic sluggishness has been associated with equity market weakness and bond market strength. If equity market weakness leads to subsequent bond market strength, the coefficient on lagged equity returns should be significantly negative.[4]

We also include the first lag of the other country's bond returns in each mean equation—the fourth lagged variable on the right-hand side of Equation (2). This is to ascertain if, as has been found in equities markets, there is a lead-lag relationship between U.S. and foreign bond returns (see Lin, Engle, and Ito [1994]).

Equation (3) models the conditional variance of U.S. and foreign bond returns as GARCH (1, 1) processes, where the volatility of each market is dependent on past squared return innovations and past variance. To examine the presence of lead-lag relationships in volatility, each conditional variance is augmented by the lagged squared innovations of the other bond market returns—the third lagged variable on the right-hand side of Equation (3).[5]

The conditional covariance ($h_{ij,t}$) between U.S. and foreign bond returns is modeled in Equation (4). It posits that the correlation between U.S. and foreign bond returns (the expression in brackets) changes over time in a predictable manner. The set of lagged instruments, $x_1, \dots, x_n$, capture this predictability.

In Hunter and Simon [2003] we investigate how the correlation between U.S. and international bond returns changes over time and what variables determine the level of correlation by including a series of instruments, one at a time, in a specification like (4). That is, we include a linear time trend to determine whether correlations have changed over time. Further, to investigate whether correlations change when business cycle conditions across economies are less synchronized, we use absolute differences in the slopes of two particular yield curves—the absolute difference of the spread between ten-year benchmark government bond rates and one-month eurocurrency rates—as well as the absolute difference between these long and short rates. In that work we also examine how, if at all, bond return correlation changes when there is a large shock in either country to equity market returns, bond market returns, and bond market volatilities.

Here we estimate the conditional correlation between U.S. bond returns and the returns on bonds from our three countries by simultaneously including in Equation (4) all the variables we determine in Hunter and Simon [2003] to be significant. For correlations between U.S. and German bonds, the instruments include a linear time trend; the lagged absolute difference between yield curve slopes; a dummy variable for periods when lagged conditional volatility in the U.S. bond market is two standard deviations above its average for the sample period; and a dummy variable for periods when lagged U.S. bond returns are two standard deviations below their average over the sample period.

For the correlations between U.S. and U.K. bonds, the instruments include only a linear time trend and the lagged absolute yield curve differential, as a dummy variable for high-stress periods in financial markets are not significant when entered one at a time.[6]

For the correlations between the U.S. and Japan, the instruments include a linear time trend; the absolute difference between U.S. and Japanese ten-year government bond rates; a dummy variable for periods when lagged U.S. bond returns are two standard deviations below their sample average; and a dummy variable for periods when lagged S&P 500 returns are two standard deviations below their sample average.

The conditional covariance is equal to the estimated r_0 coefficient plus the estimated slope coefficients times the values of predetermined instruments, multiplied by the product of the conditional standard deviations of the two bond returns. If the coefficients in Equation (4) are jointly significantly different from zero, then the correlation between the bond returns is significantly different from zero. Likewise, if the slope coefficients (i.e., those other than r_0) are jointly significantly different from zero, this implies that the correlation between bond returns varies over time in response to changes in the predetermined instruments. The fitted time-varying conditional correlation is equal to $(\hat{r}_0 + \hat{r}_1 * x_{1,t-1} + \dots + \hat{r}_n * x_{n,t-1})$.[7]

We use the quasi-maximum-likelihood (QML) approach of Bollerslev and Wooldridge [1992] to estimate the GARCH models, where the log-likelihood function from the conditional normal specification is maximized, but the estimated standard errors of the parameter estimates are robust to non-normal error distributions. This allows us to make the usual statistical inferences even if the error terms have non-normal distributions.

Foreign Bond Returns in Local Currency

Exhibit 3 reports the results of the bivariate GARCH models specifying foreign bond returns in local currencies. The findings indicate that higher domestic equity market returns are associated with statistically significant lower U.S. and Japanese bond returns in the next week. At the same time, a steeper yield curve is associated with significantly higher subsequent bond returns only for the Japanese bond market.[8]

In the case of Germany, a steeper yield curve is associated with lower subsequent bond returns at the 10% level. The estimates also indicate the presence of some lead-lag relationships in the means. Lagged U.S. bond returns have statistically significant positive effects on current German and U.K. bond returns. The results indicate that a 100 basis point increase in U.S. bond returns leads to roughly 8 basis points higher German and U.K. bond returns in the next week.

Interestingly, the findings also indicate that German and Japanese bond returns have predictive power for subsequent U.S. bond returns. Thus, the findings indicate that Japanese bond returns have predictive power for subsequent U.S. bond returns, but not vice versa. Overall, robust Wald tests indicate that U.S., German, and Japanese government bond returns are statistically significantly predictable at the 1% level.

The conditional variance equation estimates indicate that the conditional volatility of each of the bond returns is significantly mean-reverting, and generally increases in response to lagged squared own-bond return innovations. This evidence of significant variation in volatility for the different bond returns coupled with the variation in the mean noted above supports our examination of conditional Sharpe ratios below.

The results also indicate significant lead-lag relationships between volatilities, as squared U.S. bond return innovations lead to significantly higher German and Japanese conditional bond return volatility. In the other direction, higher squared German bond return innovations lead to

EXHIBIT 3

Results of Bivariate GARCH Model of U.S. and Foreign Bond Returns in Local Currency

Panel A. Conditional Mean Equations

	US	GMY	US	UK	US	JP
Constant	0.1738	0.1920	0.1755	0.1867	0.1832	0.0255
	(3.757)	(17.26)	(3.602)	(5.386)	(10.25)	(1.778)
US Return $_{t-1}$	-0.0791	0.0923	-0.0290	0.0717	-0.0236	0.0082
	(-1.898)	(6.628)	(-0.722)	(1.664)	(-1.770)	(0.635)
FOR Return $_{t-1}$	0.0583	-0.1138	0.0166	-0.0662	0.0571	0.0407
	(1.910)	(-7.640)	(0.429)	(-1.482)	(3.051)	(3.224)
USYC $_{t-1}$ †	0.7906	----	0.5606	----	-1.5816	----
	(0.336)		(0.219)		(-1.658)	
FORYC $_{t-1}$ †	----	-1.7892	----	0.4943	----	5.908
		(-1.893)		(0.220)		(9.937)
S&P500 Return $_{t-1}$	-0.0360	----	-0.0313	----	-0.0428	----
	(-2.657)		(-1.882)		(-7.019)	
Foreign Stock Return $_{t-1}$	----	-0.0057	----	-0.0069	----	-0.0148
		(-1.229)		(-0.463)		(-3.845)
H_0: Constant Mean	15.215	117.20	3.888	4.210	152.06	123.89
($\chi^2 = 4$) (p-value)	(0.004)	(0.000)	(0.421)	(0.378)	(0.000)	(0.000)

Panel B. Conditional Variance Equations

	US	GMY	US	UK	US	JP
Constant	0.2568	0.0203	0.0891	0.0913	0.1822	0.1171
	(17.17)	(1.088)	(2.108)	(1.769)	(29.94)	(18.54)
Own Error Squared $_{t-1}$	0.0360	0.1008	0.0571	0.1075	0.0416	0.1844
	(1.472)	(3.370)	(1.973)	(2.138)	(1.714)	(32.70)
Own Variance $_{t-1}$	0.6021	0.7888	0.8467	0.7884	0.7528	0.6540
	(34.03)	(17.09)	(12.06)	(9.400)	(25.66)	(43.26)
US Error Squared $_{t-1}$	-----	0.0547	-----	0.0071	----	0.0129
		(11.09)		(0.170)		(10.60)
FOR Error Squared $_{t-1}$	0.1111	-----	-0.0022	-----	0.0041	----
	(3.580)		(-0.152)		(0.562)	

Panel C. Time-Varying Conditional Correlations

	US and GMY	US and UK		US and JP
Constant	0.5109	0.3830	Constant	0.3626
	(22.92)	(4.763)		(5.660)
Time trend × 10^3	0.5141	0.6254	Time trend × 10^3	-0.8275
	(10.13)	(3.634)		(-2.776)
Abs (YC differential $_{t-1}$) †	-1.9442	-4.5233	Abs (long rate	2.1941
	(-2.655)	(-1.457)	differential)†	(0.399)
Extreme high US bond	-0.1234	-----	Extreme high lagged US	-0.5707
volatility $_{t-1}$	(-3.499)		bond volatility	(-3.309)
Extreme low US bond	-0.2191	-----	Extreme low lagged US	0.5258
return $_{t-1}$	(-2.431)		stock (SP) return	(3.073)
H_0: Constant correlation	627.87	22.702	H_0: Constant correlation	91.449
(χ^2)(p-value)	(0.000)	(0.000)	(χ^2) (p-value)	(0.000)

Panel D. Standardized Residual Diagnostics

LB (12)	11.457	17.122	10.154	8.356	8.985	13.730
(p-value)	(0.490)	(0.145)	(.6024)	(0.757)	(0.704)	(0.318)
LB^2 (12)	13.364	13.910	9.906	10.363	12.543	32.027
(p-value)	(0.343)	(0.307)	(.6243)	(0.584)	(0.403)	(0.001)
Cross-Product	20.372		4.278		9.232	
LB(12), p-value	(0.060)		(0.978)		(0.683)	
Loglikelihood		-189.835		-393.357		-375.400

Dependent variables are weekly returns on generic ten-year Treasuries in local currency. Independent variables in means are first lags of U.S. and foreign (FOR) bond returns, the lags of the U.S. (USYC) and foreign (FORYC) yield curve slopes, and the lagged returns on the country's equity index. Correlation equation is modeled as a function of several lagged instruments or dummy variables. All t-values are based on quasi-maximum likelihood (QML) estimation robust to non-normality in the residuals. LB(x) [LB^2(x)] is Ljung-Box chi-squared statistic for testing null hypothesis of zero autocorrelation up to lag x in (squared) standardized residuals.

†YC and long rate differentials measured in decimals.

higher U.S. bond return conditional volatility, while U.K. and Japanese bond return innovations do not influence subsequent U.S. bond return volatility. On the whole, the evidence is consistent with the findings of Cappiello, Engle, and Sheppard [2002] that international bond market volatilities exhibit less pervasive and only economically weak linkages compared to equity market volatilities.

While including other country lagged bond returns and lagged squared bond return innovations in the mean and variance equations provides information about the structure of return and volatility interdependencies, market efficiency suggests that the major government bond markets react rapidly to the same new information, and as a result should be contemporaneously correlated. An examination of the time variation in the correlation between U.S. and foreign bond returns provides some evidence on the stability of the benefits from international bond diversification, which should be useful in the management of international bond portfolios. We include in the models only those variables that are statistically significant when entered individually in Hunter and Simon [2003].

For U.S. and German bond return correlations, the intercept term is equal to a statistically significant 0.50, and the linear time trend enters the covariance equation with a significantly positive coefficient, indicating a tendency for the correlation between U.S. and German bond returns to rise over the sample period. The results also indicate that the coefficient on the absolute difference in the slopes of the yield curve is significantly negative, consistent with the notion that greater divergence in business cycle conditions reduces the conditional correlation between U.S. and German bond returns.

The findings also indicate that high-stress periods result in significantly lower, rather than higher, conditional correlations between U.S. and German bond returns. That is, higher conditional U.S. bond return volatility and extreme U.S. bond market sell-offs result in significantly lower correlations. Thus, these results do not suggest that the benefits of diversification evaporate in these markets during high-stress periods. Rather, the evidence for U.S. and German bond markets suggests that the benefits of diversification increase during these periods.

For U.S. and U.K. bond return correlations, the constant is a statistically significant 0.38, and the coefficient on the linear time trend is significantly positive, indicating that the correlation increased over the sample period. The other term in the correlation equation—the absolute difference in the slope of the yield curves—does not enter significantly.[9]

For the U.S. and Japanese bond return correlations, the constant is a statistically significant 0.36, and the linear time trend enters with a significantly negative coefficient, consistent with the view that Japanese bond returns became increasingly decoupled from U.S. bond returns over the 1990s owing to the decade-long recession in Japan. The results also indicate that higher conditional U.S. bond return volatility is associated with subsequently significantly lower, rather than higher, bond return correlations.

Finally, the results also indicate that extreme U.S. equity market weakness is associated with higher conditional correlations between U.S. and Japanese bond returns. Recall, however, that the results in the conditional mean equations demonstrate a tendency for U.S. equity market weakness to be associated with higher subsequent U.S. bond returns. Hence, the higher correlation between U.S. and Japanese bond returns generally occurs when the U.S. bond market is up, and during these periods the higher correlation is innocuous.

Overall, robust Wald tests indicate that bond return correlations in local currency terms are predictable. More important, there is no evidence to suggest that the benefits of international bond diversification dissipate during periods of market stress. To the contrary, the evidence suggests that in the case of U.S. bond return correlations with German and Japanese bond returns, high-stress periods are associated with greater rather than lower diversification benefits. The robustness of our models is supported by several diagnostic tests in Panel D of Exhibit 3.

Foreign Bond Returns in U.S. Dollars

Do these results hold when we perform the same analysis with foreign bond returns in U.S. dollar terms? Exhibit 4 reports the results of the models expressing foreign bond returns in U.S. dollars. German bond returns display significant predictability, as current returns are significantly positively related to lagged U.S. bond returns and significantly negatively related to lagged increases in the slope of the German yield curve and returns on the DAX index.

The constant in the correlation equation is 40% lower when German bond returns are translated into dollars. The other variables in the correlation equation are not statistically significant, with the exception of the positive coefficient on the dummy variable for extremely high conditional U.S. bond volatility. That extremely high U.S. bond return volatility is associated with significantly lower correlation between U.S. and German bond returns when

EXHIBIT 4

Results of Bivariate GARCH Model of U.S. and Foreign Bond Returns in U.S. Dollars

Panel A. Conditional Mean Equations

	US	GMY	US	UK	US	JP
Constant	0.1929	0.0869	0.1726	0.1450	0.1700	0.0022
	(6.792)	(2.991)	(2.979)	(2351)	(9.461)	(0.177)
US Return$_{t-1}$	-0.0414	0.1050	-0.0192	0.0876	-0.0335	0.0469
	(-1.509)	(3.805)	(-0.451)	(1.240)	(-0.426)	(3.861)
FOR Return$_{t-1}$	-0.0036	-0.0767	0.0157	-0.0604	0.0221	-0.0328
	(-0.039)	(-2.185)	(0.555)	(-1.296)	(0.596)	(-1.119)
USYC$_{t-1}$†	-1.2031	----	1.0135	----	-2.1942	----
	(-0.254)		(0.361)		(-1.742)	
FORYC$_{t-1}$†	----	-2.8836	----	5.666	----	4.351
		(-8.795)		(1.562)		(5.588)
S&P500 Return$_{t-1}$	-0.0461	----	-0.0306	----	-0.0551	----
	(-3.710)		(-1.807)		(-3.265)	
Foreign Stock Return$_{t-1}$	----	-0.0313	----	-0.0517	----	0.0047
		(-11.51)		(-2.195)		(1.267)
H_0: Constant Mean	302.41	443.88	(4.533)	9.493	37.451	92.028
($\chi^2 = 4$) (p-value)	(0.000)	(0.000)	(0.338)	(0.049)	(0.000)	(0.000)

Panel B. Conditional Variance Equations

	US	GMY	US	UK	US	JP
Constant	0.1670	0.1041	0.0689	0.0626	0.1523	0.2228
	(13.09)	(25.59)	(2.105)	(1.994)	(10.08)	(14.26)
Own Error Squared$_{t-1}$	0.1292	0.1247	0.0410	0.0279	0.0603	0.1018
	(2.768)	(15.43)	(1.689)	(1.933)	(1.716)	(2.437)
Own Variance$_{t-1}$	0.6776	0.7985	0.8805	0.9551	0.7468	0.7318
	(6.960)	(46.73)	(15.70)	(43.25)	(17.74)	(26.11)
US Error Squared$_{t-1}$	----	0.1847	----	-0.0332	----	0.3633
		(13.42)		(-1.596)		(13.89)
FOR Error Squared$_{t-1}$	0.0130	----	-0.0002	----	0.0059	----
	(0.112)		(-0.026)		(0.423)	

Panel C. Time-Varying Conditional Correlation

	US and GMY	US and UK		US and JP
Constant	0.2852	0.2404	Constant	0.2617
	(12.37)	(2.410)		(22.24)
Time trend $\times 10^3$	0.2204	0.4743	Time trend $\times 10^3$	0.0647
	(1.314)	(2.200)		(1.015)
Abs (YC differential$_{t-1}$)†	2.5143	4.601	Abs (long rate	-6.2028
	(0.634)	(1.363)	differential)†	(-20.75)
Extreme high US bond	0.1337	----	Extreme high lagged US	-0.2257
volatility$_{t-1}$	(2.556)		bond volatility	(-2.555)
Extreme low US bond	-0.1792	----	Extreme low lagged US	0.0382
return$_{t-1}$	(-0.252)		stock (SP) return	(0.396)
H_0: Constant correlation	1344.62	4.848	H_0: Constant correlation	457.80
(χ^2)(p-value)	(0.000)	(0.088)	(χ^2) (p-value)	(0.000)

Panel D. Standardized Residual Diagnostics

	US	GMY	US	UK	US	JP
LB (12)	10.249	22.61	9.914	6.126	8.399	13.21
(p-value)	(0.594)	(0.031)	(0.623)	(0.909)	(0.753)	(0.354)
LB^2 (12)	15.53	16.74	10.132	19.140	12.12	14.24
(p-value)	(0.214)	(0.160)	(0.604)	(0.085)	(0.436)	(0.286)
Cross-Product	5.497		30.974		4.780	
LB(12), p-value	(0.939)		(0.002)		(0.965)	
Loglikelihood		-726.654		-653.58		-803.899

Dependent variables are weekly returns on generic ten-year Treasuries in U.S. dollars. Independent variables in means are first lags of U.S. and foreign (FOR) bond returns, the lags of the U.S. (USYC) and foreign (FORYC) yield curve slopes, and the lagged returns on the country's equity index. Correlation equation is modeled as a function of several lagged instruments or dummy variables. All t-values are based on quasi-maximum likelihood (QML) estimation robust to non-normality in the residuals. LB(x) [LB^2(x)] is Ljung-Box chi-squared statistic for testing null hypothesis of zero autocorrelation up to lag x in (squared) standardized residuals.

†YC and long rate differentials measured in decimals.

the latter are measured in local currency, but significantly higher correlation when measured in U.S. dollars, indicates that dollar strength against the deutschemark (or euro) is associated with periods when U.S. bond volatility is very high (perhaps during high interest rate periods in the U.S.).

The results for U.K. bond returns expressed in U.S. dollar terms are similar to those in local currency terms, with the exception that higher lagged FTSE returns now lead to significantly lower U.K. bond returns. As before, higher U.S. bond returns, are not associated with higher subsequent U.K. bond returns and the slope of the yield curve does not have a significant impact on subsequent U.K. bond returns. In the conditional correlation, while the constant is lower than that in local currency terms, the linear time trend again enters with a significantly positive coefficient, indicating that the correlation between U.S. and U.K. bond returns in dollar terms also rose over the last decade.

For Japanese bond returns in dollars, a steeper Japanese yield curve slope again is associated with higher subsequent Japanese bond returns, and higher U.S. bond volatility leads to a statistically and economically significant increase in Japanese bond volatility in the next week. The constant term in the correlation equation is a statistically significant 0.26. Although the linear time trend is no longer significantly negative, the lagged absolute difference between U.S. and Japanese long-term rates and extremely high U.S. bond volatility now significantly reduce the correlation.

Overall, the results in local and dollar returns are broadly similar. In both models, the most consistent predictor of future bond returns are lagged own-equity returns, while a steeper yield curve slope is associated with higher bond returns only in Japan. One difference in the results is that when foreign bond returns are translated into U.S. dollars, higher equity returns in the U.K. and Germany now lead to significantly lower subsequent own-market bond returns.

The most important difference between the two sets of results is that extremely high conditional U.S. bond return volatility is associated with a higher correlation between U.S. and German bond returns measured in dollars. Because unreported evidence indicates that extremely high U.S. bond volatility is associated with lower subsequent U.S. bond returns, the higher U.S./German correlation when German returns are in dollars implies that the dollar tends to strengthen when U.S. bond returns fall, which provides the higher correlation between U.S. and German bond returns when German bond returns are translated into dollars.

This result suggests that during periods of extremely high U.S. bond return volatility the benefits of diversification for a U.S. investor in German bonds are diminished unless foreign exchange risk is hedged. This finding is consistent with the mean-variance spanning test results.

IV. CONDITIONAL SHARPE RATIOS AND YIELD BETAS

To further assess the diversification benefits offered by foreign bonds, we use the results from the estimated models to compute estimates of the time-varying expected returns and volatilities for each market and the time-varying correlations between U.S. and foreign bond returns. These estimates are used to construct time-varying Sharpe ratios and return and yield betas. Our finding that bond returns and volatility are predictable motivates our use of conditionally expected returns and volatilities to compute time-varying Sharpe ratios. To compute the Sharpe ratios we use one-month LIBOR for each country, scaled to a one-week horizon, as the risk-free rate.

Because investing in international bonds involves a comparison of bonds from countries with different interest rate regimes, conventional risk measures such as duration should be adjusted to account for the differences in volatility and the less-than-perfect return correlations across markets. Thomas and Willner [1997] stress that the duration of foreign bonds held in a portfolio dominated by U.S. bonds should be adjusted by their yield betas—the sensitivity of foreign interest rate changes to U.S. interest rate changes—to reflect the sensitivity of foreign interest rates to U.S. interest rates.

We first compute conditional return betas of the foreign bonds, β_{Ret}, which measure the sensitivity of foreign bond returns to U.S. bond returns, and often are set equal to the ordinary least squares coefficient of a regression of foreign bond returns on U.S. bond returns. In our GARCH framework, we are able to compute a conditional return beta that captures changes in the economic environment, represented by changing conditional volatilities of U.S. and foreign bond returns and their time-varying correlation. This measure is much more useful to active portfolio managers than a static return beta.

The conditional return beta for foreign bond returns with respect to U.S. bond returns is:

$$\beta_{Ret} = ({}^{\sigma_{For}}/_{\sigma_{US}})\rho_{F,U} \tag{5}$$

where σ represents the conditional volatility of returns, *For (U.S.)* represents foreign (U.S.) bonds, and ρ represents the conditional correlation between foreign and U.S. bond returns.

Much of the literature refers to *yield* betas—the sensitivities of foreign yield changes to U.S. yield changes—which are multiplied by the modified durations of foreign bonds to obtain yield beta-adjusted durations. Thomas and Willner [1997] estimate a static yield beta as the slope coefficient in a regression of foreign yield changes on U.S. yield changes. We can obtain yield betas by transforming the time-varying foreign bond return betas that we obtain from our estimates of the variances and correlation of bond returns (see Appendix B for details).

The relation between the return and yield beta is seen in Equation (6), where the return beta is approximately equal to the yield beta multiplied by the ratios of the modified durations of foreign bonds to U.S. bonds:[10]

$$\beta_{Ret} = \left({}^{D_{For}}/_{D_{US}}\right)\beta_{Yld} \tag{6}$$

Intuitively, as the duration of foreign bonds rises relative to U.S. bond duration for a given yield beta, the impact of a change in U.S. interest rates on returns is greater in the foreign bond market, which leads to a higher return beta.[11]

To calculate yield beta-adjusted durations (the duration of foreign bonds, modified to reflect foreign bond sensitivity to U.S. rate changes, D_{YBADJ}), we rewrite Equation (6) in terms of the yield beta, and multiply the yield beta by the foreign bond modified duration, and simplify to get:

$$D_{YBADJ} = \beta_{Ret} D_{US} \tag{7}$$

Thus, yield beta-adjusted durations can be obtained easily from our model estimates and are equal to the return beta times the modified duration of U.S. bonds.

These risk measures are conditional measures unlike the static measures proposed by Thomas and Willner [1997]. If the correlation between U.S. and foreign bond returns changes slowly over time, however, then it may be reasonable to obtain return betas and, in turn, yield betas by setting the conditional correlation of bond returns equal to its unconditional mean or a recent moving average and by forecasting only the return volatilities. Given that bond return volatilities are relatively straightforward to calculate using univariate GARCH models, portfolio managers could get measures of the conditional yield betas without estimating more comprehensive multivariate models of the time-varying correlation of returns.

Exhibit 5 displays summary statistics of the various measures of conditional risk and return for further assessment of the diversification benefits of foreign bonds. The results indicate that the sample means of the conditionally expected returns of U.S., German, and U.K. bonds in local currency terms are not economically different and are clustered between 16 and 18 basis points per week, while the conditional return of Japanese bonds is 12.6 basis points per week.

The sample means of the conditional volatilities of U.S. and U.K. bond returns are higher than those of Japanese and German bond returns. These findings lead to mean conditional Sharpe ratios that average between 0.16 and 0.24, with Japan on the low side and Germany on the high side.

The roughly similar Sharpe ratios for these bond markets imply that the conditional rewards for investing in foreign government bonds on a risk-adjusted basis have not been much different from the rewards from investing in U.S. bonds, abstracting from foreign exchange rate changes. Nevertheless, as evidenced by the range between minimum and maximum values, the conditional Sharpe ratios exhibit a fair degree of variation across time, consistent with considerable fluctuations in expected reward to risk trade-offs for each of these markets.

When foreign bond returns are measured in U.S. dollars, mean conditional returns in the German and Japanese bond markets are considerably lower at 6 and 7 basis points per week, while the mean conditional return in the U.K. bond market is 17 basis points per week. The volatilities of expected returns in U.S. dollars about double in the German and Japanese bond markets, while the volatility of U.K. bond returns increases by about 50%.

As a result, conditional Sharpe ratios fall fairly dramatically in the foreign bond markets when returns are measured in U.S. dollars, ranging from 0.03 and 0.04 in the German and Japanese bond markets to 0.12 in the U.K. bond market. As before, the Sharpe ratios exhibit a substantial degree of variation over time for each market.

From a portfolio perspective, a key factor is the correlation of returns and how correlation varies across time. There is significant variation in the correlation between U.S. and foreign bonds both in local currency terms and in U.S. dollars. The correlation between U.S. and Japanese bonds experiences the widest variation, between –0.99 and 0.84; those for the other markets range from 0.12 to 0.78. Additionally, the sample averages of these time-varying correlations are not very high, ranging from a low of 0.198 for Japan to 0.60 for Germany.

The implication is that even though foreign bonds

EXHIBIT 5
Summary Statistics of Various Measures of Conditional Risk and Return

	Local Currency				U.S. Dollar			
	US*	GMY	UK	JP	US	GMY	UK	JP
Conditionally Expected Return								
Mean	0.1759	0.1684	0.1869	0.1261	0.1645	0.0562	0.1720	0.0731
Standard deviation	0.1016	0.0884	0.0739	0.0755	0.1101	0.1424	0.1751	0.0585
Minimum	-0.1696	-0.1024	-0.1837	-0.3776	-0.2148	-0.3888	-0.7047	-0.1070
Maximum	0.5470	0.4259	0.4569	0.6246	0.5703	0.5968	0.8094	0.2501
Conditional Volatility								
Mean	0.9305	0.7387	0.9533	0.8416	0.9740	1.6612	1.4400	1.7402
Standard deviation	0.0894	0.1719	0.1818	0.2279	0.1316	0.3364	0.2134	0.3767
Minimum	0.8193	0.4217	0.7068	0.6129	0.7590	1.0351	1.0932	1.1119
Maximum	1.3685	1.4547	1.9126	2.2949	1.9149	3.0968	2.2088	3.6507
Conditional Sharpe Ratio								
Mean	0.1893	0.2401	0.2012	0.1584	0.1698	0.0324	0.1212	0.0437
Standard deviation	0.1081	0.1314	0.0804	0.0974	0.1112	0.0828	0.1248	0.0341
Minimum	-0.1873	-0.1001	-0.1162	-0.5200	-0.2187	-0.1887	-0.4071	-0.0702
Maximum	0.5155	0.6769	0.4440	0.7842	0.5772	0.2832	0.6317	0.1578
Conditional Correlation								
Mean		0.6035	0.5018	0.1983		0.4132	0.4336	0.0717
Standard deviation		0.1161	0.1231	0.1715		0.0762	0.0756	0.0770
Minimum		0.1167	0.1656	-0.9942		0.1774	0.3001	-0.3913
Maximum		0.7794	0.6730	0.8427		0.9281	0.6018	0.2038
Conditional Return Beta								
Mean	____	0.4738	0.5008	0.1820	____	0.7058	0.6680	0.1292
Standard deviation	____	0.1124	0.1182	0.1507	____	0.1763	0.1216	0.1592
Minimum	____	0.1163	0.1708	-0.6163	____	0.2634	0.3514	-0.8526
Maximum	____	0.8136	0.8746	0.8606	____	1.5547	1.0601	0.4133
Conditional Yield Beta								
Mean	____	0.4678	0.5125	0.1590	____	0.6993	0.6879	0.1146
Standard deviation	____	0.1059	0.1104	0.1330	____	0.1765	0.1290	0.1406
Minimum	____	0.1140	0.1868	-0.5185	____	0.2538	0.3670	-0.7110
Maximum	____	0.8038	0.8373	0.7615	____	1.6236	1.1521	0.3900

**All means significantly different from zero at the 1% level.*

provide roughly comparable conditional returns as U.S. bonds, these returns are not highly correlated with U.S. bonds and hence provide investors with diversification benefits. Furthermore, because this correlation does not increase significantly when the U.S. bond market is extremely volatile or in a slump, the benefits of diversification are available when they are most needed.

When conditional correlations are estimated measuring foreign bond returns in U.S. dollar terms, mean conditional correlations are lower, ranging from a low of 0.07 with Japan to a high of 0.43 with the U.K. As before, these conditional correlations exhibit a fair degree of variation, consistent with substantial variation in the expected benefits of diversification over the sample period.

This analysis supports the finding from the mean-variance spanning tests that U.S. investors obtain significant diversification benefits from foreign bonds.

To properly assess overall portfolio risk when the portfolio includes foreign bonds requires the estimation of return/yield betas of the foreign bonds. The sample average conditional return betas in local currency terms range from 0.18 for Japanese bonds to 0.50 for U.K. bonds. This implies that over the sample period Japanese bond returns are the least sensitive to changes in U.S. bond returns, experiencing an 18 basis point change in returns (in the same direction), given a 100 basis point change in U.S. returns.

The associated yield betas, representing the change in foreign bond yields, given a change in U.S. bond yields, range from an average of 0.16 for Japanese bonds to 0.51 for U.K. bonds. When dollar returns of foreign bonds are considered, the yield betas increase significantly, except for Japanese bonds where they fall slightly.

To illustrate the time variation of the different measures of risk and return over the sample period, and also to justify our conditional rather than unconditional model, we show in Exhibit 6 a sample graph of the conditional volatilities, conditional correlations and yield beta, and conditional Sharpe ratios derived from our estimation. These represent results from the U.S.-U.K. model in local currency terms.

The first graph indicates that the volatilities (conditional standard deviations) tend to move together, and, although foreign volatility increased sharply for several periods in the early 1990s, on average they are of equal magnitude.

In the second graph, the correlation is both time-varying and trending, moving from a low of 0.19 to about 0.66 over the sample period. The rise in correlation over time corresponds to the increase implied by the model with a time trend in the conditional covariance equation. Similar variation and trend is observed for the yield beta. This might be expected, given that the yield beta is a function of the return beta, which in turn is a function of the correlation and volatilities of U.K. and U.S. bond returns. As indicated by the graphs in the first panel, neither of the volatilities is trending. Similarly, the ratio of modified durations of the U.S. and U.K. bonds, which also affects the yield beta, is not trending.

The last graph displays greater variation in the Sharpe ratios. It indicates that not only are the means equal (as confirmed statistically), but there is also no difference on a period-by-period basis.

V. SUMMARY AND CONCLUSIONS

Our purpose has been to assess whether U.S. investors holding a broadly diversified domestic fixed-income and equity portfolio can obtain incremental diversification benefits from investing in international government bonds. In light of suggestions that the benefits of international diversification may have fallen sharply in recent years, we first use mean-variance spanning tests to determine whether investing in ten-year U.K., German, or Japanese government bonds provides U.S. investors with statistically significant diversification benefits over the period January 1992 through September 2002. We find that these benefits are available, but only if currency risk is hedged.

To examine the factors driving these diversification benefits over time, we use a bivariate GARCH framework that allows us to examine both lead-lag relationships and time-varying correlations between bond returns. Overall, our results suggest that, while there are interdependencies between the major international bond markets, they are weaker than those found between equity markets.

The results also indicate that the correlations between U.S. and both U.K. and German bond market returns rose substantially over the sample period, while the correlations between U.S. and Japanese government bond returns declined over the same period. The evidence presented is consistent with the view that synchronization of business cycles explains some of the change in correlations.

The evidence also suggests that high-stress periods in the U.S. or foreign bond markets, as measured by extremely high conditional volatility or extremely negative lagged returns, lead to lower rather than higher bond return correlations when foreign bond returns are hedged. These findings are consistent with the view that the benefits of international bond diversification on a hedged basis are not diminished during periods of weakness or increased volatility in U.S. or foreign bond markets, and thus are available when needed most—contrary to evidence in international equity markets, which suggests that the benefits of diversification evaporate when U.S. equity return volatility rises sharply or when equity markets weaken substantially.

Our conditional Sharpe ratios provide a perspective on the changing risk-adjusted rewards that international bond investors can expect in the various international bond markets. International bond portfolio managers can use time-varying yield betas instead of static measures to adjust the duration of foreign bonds to reflect the sensitivity of foreign interest rates to U.S. interest rate changes. The benefits of currency-hedged diversification across the major government bond markets are dynamic.

APPENDIX A

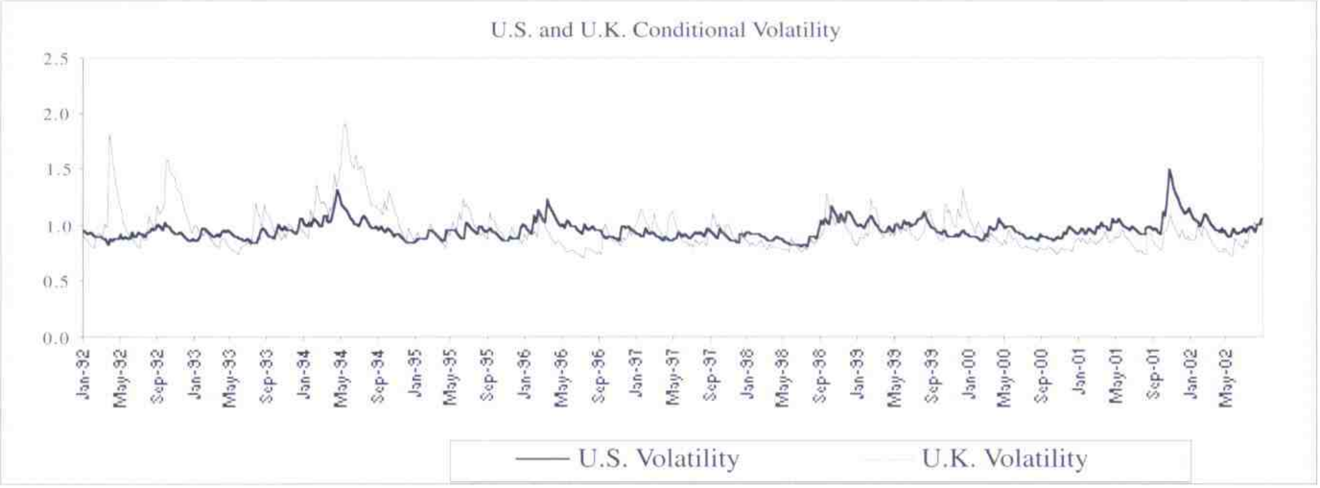
Mean-Variance Spanning Test

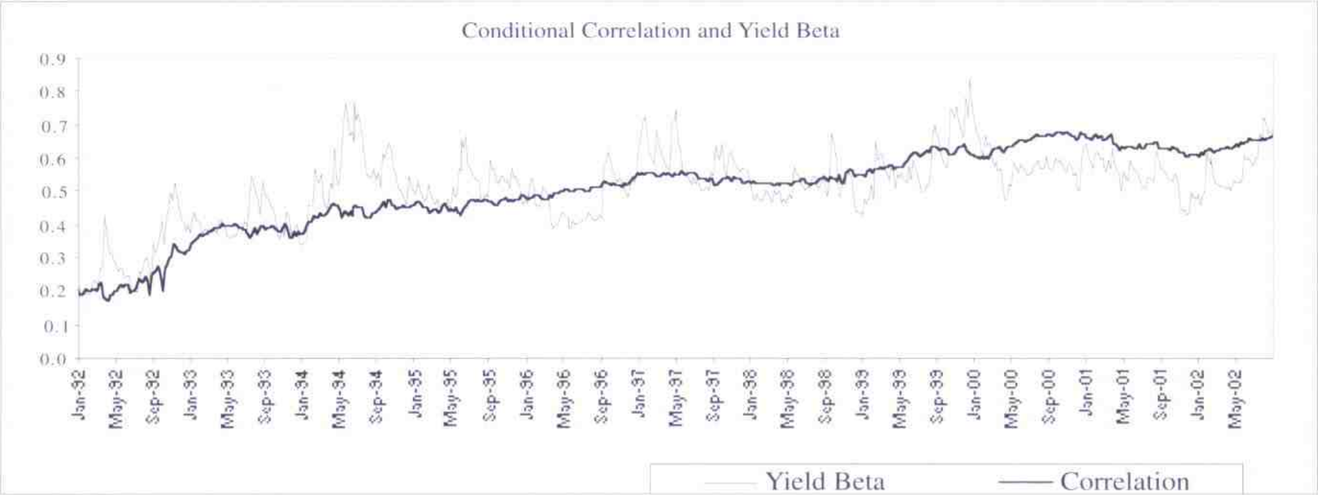
The mean-variance spanning test determines if an upward shift in the efficient frontier of the portfolio of assets that is augmented with international bonds is statistically significant, by regressing the returns of international bonds on a constant and the returns on the domestic assets, and testing if the constant is equal to zero and the coefficients sum to one (see DeRoon and Nijman [2001]). The estimation uses the generalized method of moments technique. The reported p-values are based on the Newey-West heteroscedasticity- and autocorrelation-consistent covariance matrix, with a large enough lag truncation parameter based on the number of observations.

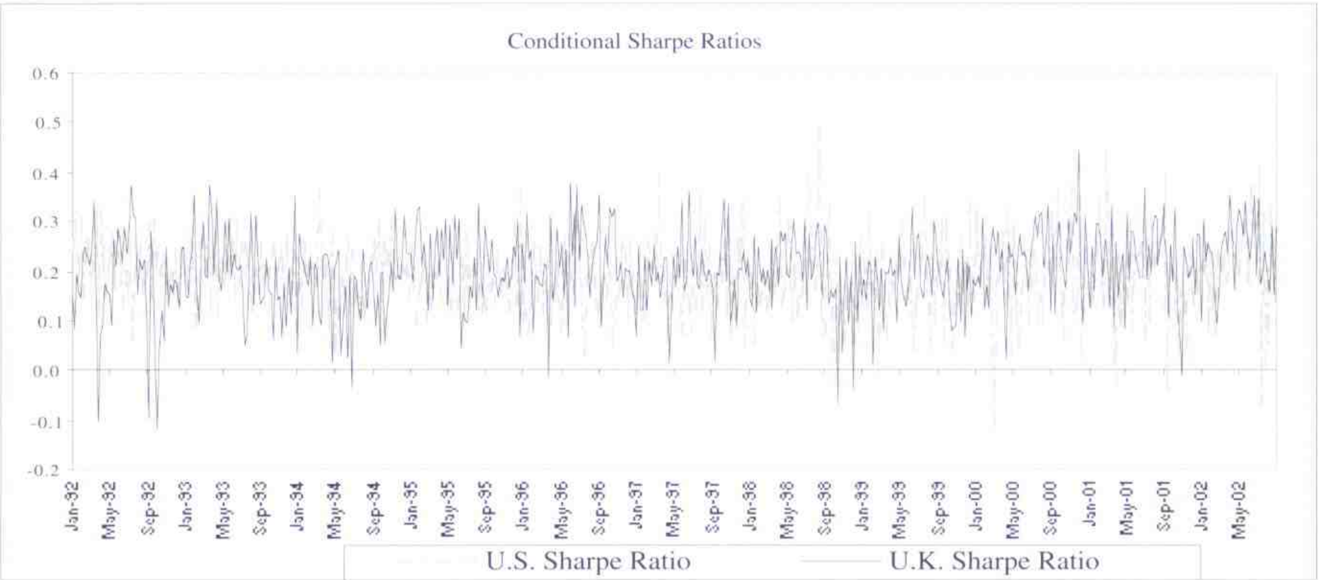
Let $\mathbf{R}$ be an $N \times 1$ vector of the domestic and international bonds, $\mathbf{R}_1$ the $K \times 1$ vector of returns on the domestic (spanning) assets, and $\mathbf{R}_2$ the $(N - K) \times 1$ vector of returns on the international bonds (the remaining $N - K$ test assets). Allowing $\mathbf{Z}_{t-1}$ to be an L vector of lagged instruments, we can write the test as:

EXHIBIT 6

Conditional Risk and Return Measures for U.S. and U.K. Government Bonds in Local Currency







$$E\left[\left(\mathbf{R}_{2,t} - \{\mathbf{a} + \mathbf{B}\mathbf{R}_{1,t}\}\right) \otimes \mathbf{R}_{1t} \otimes \mathbf{Z}_{t-1}\right] = \mathbf{0} \tag{A-1}$$

where $\mathbf{B}$ is an $(N-K) \times K$ matrix of coefficients, $\mathbf{e}_{2,t} = (\cdot)$ is an $N-K$ vector of error terms, and $\otimes$ represents the Kroneker multiplication.

Assuming that the error terms and $\mathbf{R}_{1,t}$ are uncorrelated and that the errors are mean-zero, we test the null hypothesis that international bonds do not provide the benefits of diversification to investors holding the set of domestic assets. This is equivalent to the test that:

$$\mathbf{a} = \mathbf{0} \text{ and } \mathbf{B}\iota_K = \iota_{N-K}$$

or

$$\sum_{j=1}^{K} b_{ij} = 1 \text{ for } i = \mathrm{K}+1, \ldots, \mathrm{N} \tag{A-2}$$

In the GMM estimation, we reject the null hypothesis if the chi-squared value of the J test of overidentifying restrictions is higher than the critical value:

$$J_T = T[\mathbf{h}_T'(b_{ij})\mathbf{W}_T\mathbf{h}_T(b_{ij})] \sim \chi^2_\alpha \tag{A-3}$$

where T is the number of observations; $\mathbf{W}_T$ is a symmetric, positive-definite matrix and is the inverse of a consistent estimate of the covariance matrix of the orthogonality conditions; $\mathbf{h}_T(b_{ij})$ is the sample moment of the orthogonality conditions; and α is the difference between the number of orthogonality conditions $[(N-K) \times K \times L]$ and the number of estimated parameters $[(N-K) \times (K-1)]$. The instruments in the vector L are the lagged U.S. bond returns, the lagged foreign bond returns (in dollars and currency-hedged, respectively), the lagged U.S. yield curve, and the lagged U.S. equity returns.

APPENDIX B

Yield Betas

Yield betas can be approximated from the time-varying return betas that we estimate. Weekly bond returns (R) are set equal to minus the modified duration of the bond (D) at the beginning of the week times the change in interest rates over the week (ΔY) plus weekly accrued interest ($Y/52$):

$$R_{i,t} = -D_{i,t-1} \times \Delta Y_{i,t} + Y_{t-1}/52 \tag{B-1}$$

Because weekly accrued interest is small, and because the difference in weekly accrued interest between U.S. and foreign bond markets has only a minor effect on return and yield betas, we can ignore this term and can rewrite Equation (B-1) as:

$$\Delta Y_{i,t} = R_{i,t}/-D_{i,t-1} \tag{B-2}$$

where the change in ten-year bond rates is equal to the bond return divided by modified duration.

The yield beta can be defined in a manner similar to the return beta as the covariance of U.S. and foreign yield changes divided by the variance of U.S. yield changes:

$$\beta_{Yld} = \frac{\mathrm{Cov}(\Delta Y_{US}, \Delta Y_f)}{\mathrm{Var}(\Delta Y_{US})} \tag{B-3}$$

We can plug Equation (B-2) into Equation (B-3) and obtain:

$$\beta_{Yld} = \frac{\mathrm{Cov}(R_{us}/-D_{us}, R_f/-D_f)}{\mathrm{Var}(R_{us}/-D_{us})} \tag{B-4}$$

Equation (B-4) can be rewritten as

$$\beta_{Yld} = \frac{\mathrm{Cov}(R_{us}, R_f)}{\mathrm{Var}(R_{us})} \frac{D_{us}}{D_f} \tag{B-5}$$

where the yield beta is equal to the return beta times the ratio of the modified duration of U.S. bonds to foreign bonds.

ENDNOTES

The authors thank the Hughey Center for Financial Services at Bentley College for the data.

[1]See Roll [1989], Erb, Harvey, and Viskanta [1994], and Longin and Solnik [2001].

[2]Thus, we abstract from the possibility of being either underhedged or overhedged because actual local currency returns exceed or fall short of expected local currency returns.

[3]This test is well known in the equity literature. See DeRoon and Nijman [2001] for a survey.

[4]McQueen and Roley [1993] demonstrate that the opposite results obtain when market participants are concerned about an overheating economy. Data suggesting a weaker-than-expected economy lead to stronger bond and stock prices, as this makes it less likely that the Federal Reserve is forced to tighten monetary policy aggressively and possibly drive the economy into a recession.

[5]A well-established finding in the equity literature is the tendency of equity return volatility to be more responsive to negative return innovations than to positive return innovations of equal magnitude—the so-called asymmetric volatility effect. Because our preliminary results also show insignificant conditional volatility asymmetries for each bond market we examine, we do not include these terms in the models that we present.

[6]Of course, it is possible that variables that do not enter

the covariance equation significantly by themselves could enter significantly when other variables are included. In experiments with various specifications, those variables do not enter the more general models.

[7]The term in parentheses in Equation (4) (the estimated constant plus the sum of the products of the parameter estimate and the predetermined instrument) is equal to the correlation because the equation sets the conditional covariance equal to this term times the product of the conditional standard deviations of U.S. and foreign bond returns.

[8]The finding that a steeper yield curve slope is not associated with significantly higher bond returns in the U.S., the U.K., and Germany could be attributable to the latter part of the sample period when long-term rates fell sharply after central banks raised short-term rates and their yield curves flattened, although the same results obtain when models are estimated over the first half of the sample period.

[9]While we do not include variables in the correlation equation that reflect high-stress market environments because they did not enter the correlation equation significantly in Hunter and Simon [2003], we included them in preliminary models with the variables above, where they still did not enter significantly.

[10]The transformation results in an approximate yield beta because adjusting the return beta by the ratios of the modified durations does not adjust for the initial yield differences, but these initial yield differences are a small part of the overall variation in returns.

[11]For example, assume that the yield beta is 1.0, that the modified duration of the foreign bond market is greater than that of the U.S. bond market, and that foreign and U.S. yield changes tend to be one-for-one. Then the greater modified duration of the foreign bond market causes the foreign return beta to be greater than 1.0, because equal interest rate changes translate into greater effects on foreign bond returns than U.S. bond returns.

REFERENCES

Bollerslev, T. "Modeling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model." *Review of Economics and Statistics*, 72 (1990), pp. 498-505.

Bollerslev, T., and J. Wooldridge. "Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances." *Econometric Reviews*, 11 (1992), pp. 143-179.

Campbell, J., A. Lo, and C. MacKinlay. *The Econometrics of Financial Markets*. Princeton: Princeton University Press, 1997.

Campbell, J., and R. Shiller. "Yield Spreads and Interest Rate Movements: A Bird's Eye View." *Review of Economic Studies*, 58 (1991), 495-514.

Cappiello, L., R. Engle, and K. Sheppard. "Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns." Working paper, University of California, San Diego, 2002.

Christopherson, J.A., W.E. Ferson, and A.L. Turner. "Performance Evaluation Using Conditional Alphas and Betas." *The Journal of Portfolio Management*, 26 (1999), pp. 59-72.

DeRoon, F.A., and T.E. Nijman. "Testing for Mean-Variance Spanning: A Survey." *Journal of Empirical Finance*, 8 (2001), pp. 111-155.

Erb, C.B., C.R. Harvey, and T.E. Viskanta. "Forecasting International Equity Correlations." *Financial Analysts Journal*, November/December 1994, 32-45.

Hunter, D. M., and D. Simon. "A Conditional Assessment of the Relationships Between the Major World Bond Markets." Working paper, Bentley College and University of South Florida, 2003.

Ilmanen, A. "Market Rate Expectations and Forward Rates." *The Journal of Fixed Income*, Vol. 6, No. 2 (September 1996), pp. 8-22.

Jorion, P. "Asset Allocation with Hedged and Unhedged Foreign Stocks and Bonds." *The Journal of Portfolio Management*, 15 (1989), pp. 49-54.

Levy, H., and Z. Lerman. "The Benefits of International Diversification in Bonds." *Financial Analysts Journal*, 44 (1988), pp. 56-64.

Lin, W., R. Engle, and T. Ito. "Do Bulls and Bears Move Across Borders? International Transmission of Stock Returns and Volatility." *Review of Financial Studies*, 7 (1994), pp. 507-538.

Longin, F., and B. Solnik. "Extreme Correlations in International Equity Markets." *Journal of Finance*, 56 (2001), pp. 649-676.

McQueen, G., and V. Roley. "Stock Prices, News, and Business Conditions." *Review of Financial Studies*, 6 (1993), pp. 683-707.

Odier, P., and B. Solnik. "Lessons from International Asset Allocation." *Financial Analysts Journal*, 49 (March/April 1993), pp. 63-77.

Roll, R. "Price Volatility, International Market Links, and Their Implications for Regulatory Policies." *The Journal of Financial Services Research*, 3 (1989), pp. 211-246.

Solnik, B., C. Boucrelle, and Y. Le Fur. "International Market Correlation and Volatility." *Financial Analysts Journal*, September/October 1996, pp. 17-34.

Thomas, L., and R. Willner. "Measuring the Duration of an Internationally Diversified Bond Portfolio." *The Journal of Portfolio Management*, Fall 1997, pp. 93-99.