Convertible Bond Prices and Inherent Biases

PETER CARAYANNOPOULOS AND MADHU KALIMIPALLI

PETER CARAYANNOPOULOS is an associate professor of finance in the School of Business and Economics, Wilfrid Laurier University, in Waterloo, Ontario.
pcarayan@wlu.ca

MADHU KALIMIPALLI is an assistant professor of finance in the School of Business and Economics, Wilfrid Laurier University, in Waterloo, Ontario.
mkalimip@wlu.ca

Our objective is to examine the pricing behavior of corporate convertible bonds that are currently traded in the U.S. secondary convertible bond market. No study to date has used recent data from the U.S. convertible bond market, so our work contributes significantly to an understanding of this market.

The model follows the Duffie and Singleton [1999] credit risk model. This approach lets us calibrate default probabilities in the model to the market prices of both risky debt and equity.

Our results indicate that convertible bonds with relatively low conversion value are often underpriced to the extent that their prices regularly violate pricing bounds implied by the presence of the conversion option. For such bonds, we find that the boundary condition associated with the bond's straight debt value is often violated, and negative option prices are implied. The differences between actual and model prices for bonds with low conversion value are not the result of biases inherent in the pricing model used but are instead attributable to a systematic underpricing of these bonds in the marketplace.

This finding should be of importance to corporate debt issuers and investors alike, in the wake of the tremendous growth in the convertible bond market.[1]

I. LITERATURE REVIEW

The convertible bond is a hybrid security that combines straight bond characteristics with a conversion feature. This feature allows the owner to exchange the convertible bond for another security of different characteristics, usually the issuing firm's common stock. Therefore, while it retains most of the characteristics of straight debt, the convertible bond also offers an upside potential associated with the underlying common stock. Given its hybrid nature, valuation of a convertible bond requires a model that captures both its exposure to credit risk and the upside potential from its equity-like behavior.

Credit risk models are generally classified in two categories: structural models and reduced-form models.[2] Structural models assume the value of a firm is continuous over time and, given the dynamics of firm value through time and appropriate terminal and boundary conditions, derive the value of the firm's debt. Merton [1974] developed one of the first models, which assumes that default is allowed only at the maturity of the debt.

Subsequent structural models relax some of the unrealistic assumptions of the Merton [1974] model.[3] Default can instead occur any time during the life of the bond, and default is triggered when the value of the firm reaches a certain threshold level. Problems arising from unobservable variables and complex capital structure still limit the practical application of such models.

Structural models for convertible bonds were initially developed by Ingersoll [1977a, 1977b] and Brennan and Schwartz [1977]. They follow the same principles as the structural models for the valuation of regular bonds, and allow for the possibility of equity conversion through a set of appropriate terminal and boundary conditions. Brennan and Schwartz [1980] extend their work to allow for the uncertainty inherent in interest rates by introducing the short-term risk-free interest rate as an additional stochastic variable.

Empirical investigations of structural convertible bond valuation models are limited. King [1986] examines a sample of 103 American convertible bonds, and concludes that when market prices are compared with model valuations, the means are not significantly different. Carayannopoulos [1996b], using a structural model that allows for the stochastic nature of interest rates, in a study of monthly data for 30 U.S. convertible bonds finds that market prices are significantly lower than model prices when the conversion option is deep out of the money, i.e., when the conversion value of the convertible bond is low relative to the straight bond value of the security. He does not provide any insight as to whether the observed results are due to model-specific biases or general biases inherent in the observed market prices of these instruments. Furthermore, the simplifying assumptions behind structural models may be restrictive enough to distort some of the empirical results obtained.

Most of the problems associated with the practical application of structural models are mitigated with the use of reduced-form models. Unlike structural ones, reduced-form models do not condition default exclusively on firm value, and parameters associated with firm value need not be estimated for model implementation.

These models also view risky debt as paying off a fraction of each promised dollar if bankruptcy occurs. The time of bankruptcy is treated as an exogenous process, however, and does not depend explicitly on firm value. A typical reduced-form model assumes that an exogenous variable drives default, and the conditional probability of default (also called hazard or intensity rate) during any time interval is non-zero. Furthermore, it is assumed that, upon default, bondholders receive a fraction of the bond's face value, known as the recovery rate that is known a priori.

In general, the value of a corporate bond is equal to the present value of its future cash flows discounted at a risky rate. The risky rate has two components: the risk-free short-term rate and a credit risk premium, while one or both components may vary through time. The credit risk premium is assumed to be a function of the (risk-neutral) probability of default and the recovery rate, if default occurs.

One set of reduced-form models employs a credit rating-based approach that depicts default through a gradual change in ratings driven by a Markov transition matrix.[4] Others depict the default process through the evolution of default spreads or, equivalently, the joint evolution of the conditional probability of default and recovery rate.[5]

Given the equity-like behavior of convertible bonds, reduced-form pricing models need to incorporate both the stock price and credit spread behavior over time. Since the convertible bond behaves more like equity than debt as the equity value of the bond issuer increases, practitioners often guess at an appropriate adjustment to the credit spread to account for the hybrid nature of the convertible bond. The adjustment is usually ad hoc, and depends on the probability of conversion. That is, as the probability of conversion approaches one, the credit spread approaches zero, while as the probability of conversion approaches zero, the credit spread approaches that of an equivalent non-convertible bond.

Tsiveriotis and Fernandez [1998] propose a valuation model that decomposes the convertible bond value into two components. One component is relevant when the bond ends up ultimately as equity, and the other when the bond ends up as debt. The first component is discounted at the risk-free rate, while the second component is discounted at the risk-free rate plus the (unadjusted) credit spread.

Ammann, Kind, and Wilde [2003] test a form of the Tsiveriotis and Fernandez [1998] model using a sample of daily prices of 21 French convertible bonds observed from February 19, 1999, through September 5, 2000. Findings suggest that the model generally overprices convertible bonds for which the embedded option is out of the money, although the degree of overpricing is considerably lower than that observed by Carayannopoulos [1996b] in the U.S. market. Again, however, the study does not clarify whether the model is responsible for the observed overpricing or whether there are biases in the observed market prices of the convertible bonds in the sample.

Takahashi, Kobayashi, and Nakagawa [2001] develop a reduced-form convertible bond valuation model in a Duffie and Singleton [1999] framework. They demonstrate the model using four Japanese convertible bonds. This limited sample, however, does not allow for any meaningful conclusions with respect to the accuracy of the model.

The model we examine is developed following the Duffie and Singleton [1999] framework and along the lines described by Takahashi, Kobayashi, and Nakagawa [2001]. The approach extracts the hazard rate as a function of equity prices, so once the stochastic nature of equity prices is determined, the stochastic process for the hazard rate is also defined. Like other authors, we find the model provides prices that are significantly higher than observed market prices when the conversion option is out of the money, while it underprices convertible bonds when the conversion option is in the money.

We also provide evidence that at least in the case of deep out-of-the-money convertible bonds, differences between actual and model prices are not the result of biases inherent in the pricing model used but rather are due to a systematic underpricing of these bonds in the marketplace when the conversion value of the bond is low relative to the instruments' straight bond value.

II. MODEL

Our model decomposes the time t value of a convertible bond with a maturity T, $B_{conv}(t, T)$, into the value of an equivalent straight bond (i.e., a bond with the same coupon, maturity, and credit risk characteristics, but without the conversion option), $B_{str}(t, T)$, and its embedded option to convert, $O(t,T)$. The conversion option is an option to exchange an asset (the straight bond) for another (the shares the bond can be converted into). Thus:

$$B_{conv}(t,T) = B_{str}(t,T) + O(t,T) \tag{1}$$

In the Duffie and Singleton [1999] framework, the value at time t of a defaultable security with a final payoff X at maturity T and a cumulative payment process $\{D_\tau: t \leq \tau \geq T\}$ is equal to:

$$B_{str}(t,T) = E_t^Q\left[e^{-\int_t^T R(u)du}X + \int_t^T e^{-\int_t^s R(u)du}dD_s\right] \tag{2}$$

where $E_t^Q[.]$ denotes the conditional expectation under a risk-neutral measure Q.

$R(t)$ denotes the default-adjusted discount rate defined by

$$R(t) = r(t) + L(t)h(t) \tag{3}$$

where $r(t)$ is the risk-free short-term interest rate, $L(t)$ is the fractional loss rate of market value if default were to occur at time t, and $h(t)$ is the hazard rate of default at time t.

Noting that the stock price itself is subject to default risk and assuming a zero recovery rate in case of default (or, equivalently, a loss rate of one) and a deterministic risk-free rate, the stock price, S, follows the process:

$$dS = \left[\alpha(S,t) + h(t)\right]Sdt + \sigma Sdz \tag{4}$$

where $\alpha(S, t)$ is the dividend-adjusted (risk-neutral) drift rate per unit of time; i.e., $\alpha(S, t) = r(t) - \gamma(S, t)$; $\gamma(S, t)$ is the dividend yield paid to common shareholders; σ is the volatility of S; and dz is a standard Wiener process with mean zero and variance dt.

In this framework, Equation (1) for a convertible bond with a European conversion option (the option can be exercised only at the maturity of the bond) can be written as:

$$B_{conv} = E_t^Q\left\{e^{-\int_t^T R(U)du}F + \sum_i e^{-\int_t^{\tau_i} R(u)du} cpn_i\right\} + E_t^Q\left[e^{-\int_t^T \lambda(u)du}\max[qS_T - F, 0]\right] \tag{5}$$

where F is the face value paid at the maturity of the bond if the bond is not converted; τ_i is the time at which the i-th coupon is paid; cpn_i is the dollar value of the bond's coupon payment; and q is the conversion ratio of the bond, i.e., the number of shares each bond can be converted into. $\lambda(t) = r(t) + h(t)$ is the default-adjusted discount rate for the payoff from the option (it is assumed that in the case of default the fractional loss, $L(t)$, for the option in (3) is equal to one).

III. IMPLEMENTATION OF MODEL AND DATA DESCRIPTION

The value of a convertible bond with an American type of conversion option (the option can be exercised any time during the life of the bond) can be derived numerically using a trinomial lattice that simulates the evolution of the stock price as denoted by Equation (4). As noted in Equation (1), valuation of the convertible bond involves the decomposition of its value into two parts: the value of an equivalent straight bond, and the value of an option to exchange one asset for another.

The value of the convertible bond at maturity T equals:

$$B_{conv}(T,T) = B_{str}(T,T) + O(T,T) \tag{6}$$

where

$$B_{str}(T,T) = F$$
$$O(T,T) = Max(qS_T - F, 0)$$

Equation (6) suggests that at maturity T, the value of the straight bond, $B_{str}(T, T)$, if the firm is not in default, is equal to its face value, F. The value of the embedded option at the same time, $O(T, T)$, is equal to the maximum of the conversion value *less* the bond's face value or zero. At any point τ ($\tau < T$) in the time dimension of the trinomial lattice, the value of the convertible bond, $B_{conv}(\tau, T)$, is equal to:

$$B_{conv}(\tau,T) = B_{str}(\tau,T) + O(\tau,T) \tag{7}$$

where

$$B_{str}(\tau,T) = e^{-R(\tau)dt}\left\{E_\tau\left[B_{str}(\tau+dt,T)\right]\right\}$$
$$O(\tau,T) = e^{-\lambda(\tau)dt}E_\tau\left[O(\tau+dt,T)\right]$$

and dt is the size of each time step in the trinomial lattice.

If the convertible bond can be converted immediately, an additional condition is needed:

$$O(\tau, T) = Max\left(0,\ qS_\tau - B_{str}(\tau,\ T)\right) \tag{8}$$

Equation (8) ensures that the value of the option cannot fall below its intrinsic value.

In most cases, a convertible bond can be called by the issuer at certain times at predetermined call prices. When a convertible is called, the bondholder will convert if the conversion value exceeds the call price. Therefore, the convertible bond value once the bond is called can be expressed as:

$$B_{conv}(\tau,T) = \max\left(CP(\tau), qS_\tau\right) \tag{9}$$

where $CP(\tau)$ is the call price of the bond at time τ.

Given the convertible bond value described in (7) and callability condition (9), the implicit option value with both callability and convertibility features can be written as:

$$O(\tau,T) = Max\left[0, \max\left(CP(\tau), qS_\tau\right) - \min\left(B_{str}(\tau,T), CP(\tau)\right)\right] \tag{10}$$

where B_{str} is the straight bond value as defined in Equation (7).

It is intuitively attractive to assume that the hazard rate, $h(t)$, is a function not only of time but also of the stock price itself, $h(S,t)$.[6] We assume the functional form for the hazard rate as follows:

$$h(t,S) = \frac{1}{e^{\beta S}} \tag{11}$$

This function has some desirable properties regarding the likelihood of default. As the stock price increases, and thus the likelihood of default diminishes, the default hazard rate approaches zero. As the stock price approaches zero, however, and default becomes more and more likely, the default hazard rate approaches one.

The empirical application of the model requires the estimation of the stock price volatility, σ, the estimation of the parameter β in (11), the fractional loss rate $L(t)$ in (3), and specification of the short-term interest rate r. An estimate of volatility is obtained using a time series of weekly stock prices observed for the 52 weeks prior to the valuation date.

The fractional loss of market value, $L(t)$, however, is very difficult to obtain. Moody's and other rating agencies frequently publish historical recovery rates across different bond seniority (the recovery rate is equivalent to one minus the fractional loss). These recovery rates are expressed as a fraction of face rather than market value. But as Duffie and Singleton [1999] show, differences between the recovery of face value and recovery of market value formulations are generally small even without calibrating one formulation to the other. Thus, recovery rates subject to rating and seniority status as published by Moody's are used to obtain estimates of $L(t)$.

An estimate of β in the expression for the hazard rate given by (11) is obtained by calibrating the model using a non-convertible bond of the same issuing firm, which has the same or similar likelihood of default as the convertible bond under consideration. Thus, the application concentrates on convertible bonds whose issuer has at least one non-convertible debt issue outstanding. For a given issuer, the intensity of default is the same for all its bond issues (the fractional loss may vary due to differences in the seniority status of each instrument, and

EXHIBIT 1

Comparison of Model to Actual Convertible Bond Prices—by Moneyness

CV/SB	Observations	Mean Difference	75%	Median	25%
(0, 0.3]	98	7.44	10.44	6.78	4.02
(0.3, 0.7]	78	9.42	13.96	9.09	3.77
(0.7, 0.9]	33	1.39	10.39	5.07	–5.82
(0.9, 1.1]	30	–0.59	10.58	5.21	–15.38
(1.1, 1.3]	19	–6.87	–3.33	–8.54	–12.72
>1.3	176	–9.17	–6.76	–9.94	–13.15
Total	434	–0.58	8.15	0.76	–10.22

these differences are captured in the different ratings that may apply to various issues of the same issuer).

The implied value of coefficient β can be deduced by choosing its value so that the proposed model fits the price of the non-convertible issue. Thus, the model is calibrated using another non-convertible debt issue that the firm already has in its structure, and then this information is used to price the corresponding convertible bond. The process is similar to the widely used practice of calculating implied volatility from the observed market price of an option. In turn, this volatility can be used to price other instruments with the same underlying asset.

The value of the short-term interest rate, $r(\tau)$, at any time τ in the lattice ($t \leq \tau \geq T$) is given by the forward rates implied by the observed term structure at time t. A cross-section of U.S. Treasury STRIPS is used to establish the observed term structure of interest rates at each valuation date. Treasury STRIPS are zero-coupon instruments created from either the principal (principal STRIPS) or individual coupons (coupon STRIPS) of U.S. Treasury notes or bonds. STRIPS maturities span a period of more than 25 years.[7]

Stock, bond (straight bond as well as convertible issues outstanding), and STRIPS prices are obtained from Datastream. The pricing data refer to the last transaction price of each instrument on a monthly basis. We use prices for instruments that traded on the last trading day for each month from January 2001 through September 2002.[8]

To mitigate potential liquidity problems, we limit the sample to firms whose convertible debt was traded frequently during the sample period. The selection process minimizes non-synchronous trading problems in the sense that the (convertible and non-convertible) bonds and the common shares of a given firm all trade on the same day. We have no information, however, on the timing of the last trade for each security.

Under the liquidity constraint and the calibration requirement that a firm has to have at least one debt issue outstanding other than the convertible, our sample consists of 434 convertible bond observations overall from 25 non-financial firms.

IV. RESULTS

We compare model and observed convertible bond prices in Exhibit 1. The pricing difference is calculated as model price minus actual bond price, and differences are reported as a percent of par value. Overall, in a sample of 434 observations, the mean and median pricing differences are –0.58 and 0.76, while 50% of the differences (those lying in the 25% to 75% range) fall between 8.15 and –10.22.

While the overall results suggest the model is fairly accurate in pricing these instruments, a more detailed analysis of the sample reveals biases associated with the degree of moneyness of the conversion option embedded in the convertible bond. The degree of moneyness is measured by the ratio of the bond's conversion value to the equivalent straight bond value (in terms of maturity, coupon, and call features), $B_{str}(t, T)$, obtained during the numerical process that derives the value of the convertible. The ratio is denoted as CV/SB.

Our sample is partitioned into subsamples as follows: out-of-the-money bonds (three subsamples: ratios below 0.3, over 0.3 but lower than 0.7, and between 0.7 and 0.9); around-the-money bonds (ratios between 0.9 and 1.1); and in-the-money bonds (two subsamples: ratios between 1.1 and 1.3, and ratios higher than 1.3).

Our results indicate that model prices are consider-

EXHIBIT 2

Comparison of Model to Actual Convertible Bond Prices—by Moneyness and Market Prices

CV/SB	Observations	Mean Difference	75%	Median	25%
		Panel A: Discount Bonds			
(0, 0.3]	98	7.44	10.44	6.78	4.03
(0.3, 0.7]	78	9.42	13.96	9.09	3.77
(0.7, 0.9]	28	0.71	12.28	4.83	−11.46
(0.9, 1.1]	22	−0.29	12.35	8.25	−23.96
(1.1, 1.3]	16	−5.54	−0.64	−8.53	−10.72
> 1.3	164	−9.02	−6.67	−9.90	−13.05
Total	406	−0.22	8.41	1.39	−9.99
		Panel B: Premium Bonds			
(0, 0.3]	-	-	-	-	-
(0.3, 0.7]	-	-	-	-	-
(0.7, 0.9]	5	5.21	6.05	5.70	4.92
(0.9, 1.1]	8	−1.42	6.63	4.25	−12.86
(1.1, 1.3]	3	−13.94	−8.49	−16.33	−17.01
> 1.3	12	−11.25	−7.67	−12.46	−15.03
Total	28	−5.79	6.57	−8.28	−14.72

ably lower for in-the-money bonds and higher for out-of-the-money bonds than the respective market prices. For in-the-money bonds, for conversion to straight bond value ratios between 1.1 and 1.3, the mean difference is −6.87, the median −8.54, and the 50% range (−3.33, −12.72). For ratios higher than 1.3, the corresponding numbers are −9.17, −9.94, and (−6.67, −13.15), respectively.

For out-of-the-money bonds, the bias is reversed. When the bond's conversion value is less than 30% of the equivalent straight bond value, the average pricing difference is 7.44, the median is 6.78, and the 50% range is (10.44, 4.02). For conversion to straight bond value ratios between 0.3 and 0.7, the corresponding numbers are 9.42, 9.09, and (13.96, 3.77). For conversion to straight bond value ratios between 0.7 and 0.9, the corresponding numbers are 1.39, 5.07, and (10.39, −5.82), respectively. These numbers clearly indicate the presence of a bias in our results, as the model bond prices are considerably higher than the market prices when the conversion option is out of the money.

When the sample is separated further into discount and premium bonds, the biases with respect to the degree of moneyness persist, as shown in Exhibit 2.[9] When the sample is partitioned with respect to the time to maturity variable, the biases with respect to the moneyness of the bond's embedded option persist within the three maturity groups (under five years, between five and ten years, greater than ten years), as shown in Exhibit 3.

Explaining the In-the-Money Bias

The model's in-the-money bias can be partly explained by the way the model handles the call condition. The vast majority of convertible bonds, in general and in our sample, are also callable. The theoretical model we use assumes the issuing firm's optimal call behavior, i.e., that the convertible is called once its conversion value reaches its call price. There is ample evidence, however, that firms do not follow an optimal call policy and do not usually call a convertible until its conversion value significantly exceeds its call price (see Ingersoll [1977b]).

Thus, a theoretical model that assumes an optimal call policy would be expected to underestimate the embedded conversion option. This result should be more pronounced, the deeper the embedded option is in the money.

EXHIBIT 3

Comparison of Model to Actual Convertible Bond Prices—by Moneyness and Maturity

CV/SB	Observations	Mean Difference	75%	Median	25%
Panel A: Bonds with Maturities Under 5 Years					
< 0.7	76	9.57	14.08	9.20	3.80
(0.7, 0.9]	17	10.00	15.85	9.64	5.70
(0.9, 1.1]	10	7.06	7.96	6.19	3.63
> 1.1	9	−6.11	−3.37	−8.07	−9.67
Total	112	8.15	14.01	8.08	3.60
Panel B: Bonds with Maturities Between 5 and 10 Years					
< 0.7	87	7.47	10.48	6.37	3.47
(0.7, 0.9]	9	2.47	9.67	−1.56	−5.94
(0.9, 1.1]	12	7.87	12.86	10.37	−0.63
> 1.1	26	−1.37	5.41	−0.43	−3.17
Total	134	5.46	9.95	5.79	1.49
Panel C: Bonds with Maturities More Than 10 Years					
< 0.7	13	6.67	9.00	7.30	4.03
(0.7, 0.9]	7	−20.93	−17.91	−21.68	−23.20
(0.9, 1.1]	8	−22.84	−20.21	−24.75	−25.46
> 1.1	160	−10.33	−8.23	−10.33	−13.26
Total	188	−10.09	−7.40	−10.37	−13.53

Explaining the Out-of-the-Money Bias

Explaining the overpricing for convertible bonds with an out-of-the-money embedded option is more difficult. When it is out of the money, the embedded conversion option has relatively low value, and the convertible bond behaves more like a regular non-convertible. Thus, before making any inferences, it is essential that we assess the ability of the credit risk model to correctly price regular bonds in general. Then we can analyze the convertible bond valuation results in the context of any biases found in the valuation of non-convertible bonds.

As noted earlier, our firms have at least one non-convertible debt issue outstanding in addition to the convertible one. The additional non-convertible debt issue is required for calibration purposes. In the case of multiple non-convertible bond issues, the bond with maturity closest to the maturity of the convertible is used in the calibration process.

The presence of multiple non-convertible issues issued by the same firm provides an opportunity to identify any model biases, by comparing model to actual prices for the non-convertible issues not already been used in the calibration process. Overall, we have 866 observations for such non-convertible bonds issued by the same 25 firms that issued the convertibles examined.

Model prices for these bond observations are obtained using the same trinomial lattice that is used to solve Equation (5), assuming a conversion factor, q, equal to zero. Again pricing differences are calculated as model price minus actual price, so a positive difference suggests the model overprices relative to actual prices. Differences are reported as percent of par value.

The results are presented in Exhibit 4. The mean pricing difference is −0.04, and the median is −0.54, while 50% of the pricing differences fall within the interval (2.94, −3.95). Differences are also examined with respect to the degree of discount or premium of each bond in the sample. For the 600 cases when a bond sells at a discount, the mean difference is 0.52, and the median is −0.07, while 50% of the differences fall within the interval (6.13, −5.09).

For deep discount bonds (actual price less than 80% of face value), although the mean pricing difference suggests a model overpricing (2.79), the median is still negative at −0.85. For discount bonds (actual price between

Exhibit 4

Comparison of Model to Actual Non-Convertible Bond Prices—by Moneyness

CV/SB	Observations	Mean Difference	75%	Median	25%
≤ 0.8	198	2.79	18.42	–0.85	–8.94
(0.8, 1]	402	–0.60	3.40	0.10	–4.25
Total Discount	600	0.52	6.13	–0.07	–5.09
(1, 1.2]	265	–1.29	0.84	–0.74	–2.29
> 1.2	1	0.38	-	-	-
Total Premium	266	–1.28	0.84	–0.73	–2.29
Total	866	–0.04	2.94	–0.54	–3.95

Exhibit 5

Comparison of Model to Actual Non-Convertible Bond Prices—by Maturity

Maturity	Observations	Mean Difference	75%	Median	25%
≤ 5 Years	257	2.15	2.72	0.03	–1.19
(5 Yrs, 10 Yrs)	433	1.13	3.94	–0.88	–4.01
>10 Years	176	–6.11	2.80	–2.42	–11.86

80% and 100% of face value), the mean difference is –0.60, the median 0.10, and the 50% range 3.40 to –4.25.

For the 266 cases when the bond sells at a premium, the average pricing difference is –1.28, the median –0.73, and the 50% range (0.84, –2.29). Overall, the analysis suggests that, once calibrated to one of the firm's current debt issues, the model is fairly accurate in pricing the firms' remaining non-convertible debt issues.

The presence of any biases with respect to time to maturity is also investigated. Results are presented in Exhibit 5. In general, the model tends to slightly overprice non-convertible bonds with maturity less than five years (mean 2.15, median 0.03, 50% range 2.72 to –1.19). For bonds with maturity between five and ten years, the average difference is 1.13, the median –0.88, and the 50% range (3.94, –4.01). The model seems to underprice non-convertible bonds with maturities greater than ten years. In this case, the observed average difference is –6.11, the median –2.42, and the 50% range (2.80, –11.86).

A direct comparison of convertible to non-convertible results suggests that actual market prices of out-of-the-money convertible bonds (where the convertible bond has a non-convertible rather than an equity-like behavior) are considerably lower than model prices. These differences far exceed the ones observed in our non-convertible sample for bonds issued by the same firms. The conclusion we draw from this analysis is that convertible bonds with out-of-the-money embedded conversion options trade at prices considerably lower than their straight debt counterparts.

To strengthen this argument, an analysis of the value of the option embedded in the observed convertible bond market prices is presented in Exhibit 6. The value of the embedded option is calculated by subtracting the value of the equivalent straight bond from the observed market price of the convertible bond. A $100 face value is assumed for each bond.

Given the non-negativity of option values, a negative result would indicate that the convertible bond is underpriced. This in turn implies a negative option value and a violation of the boundary condition that states that the value of the convertible bond cannot fall below its straight debt values.

The findings in Exhibit 6 clearly suggest that the market systematically underprices convertible bonds with a deep out-of-the money embedded conversion option. For conversion to straight bond value ratios lower than 0.3, the average option value is –$4.65 and the median –$2.89.

EXHIBIT 6
Implied Conversion Option Values

CV/SB	Observations	Negative Implied Option Values	Mean Option Value	75%	Median	25%
(0, 0.3]	98	78 (79.6%)	–4.65	–0.90	–2.89	–8.73
(0.3, 0.7]	78	52 (66.7%)	–2.69	0.23	–1.64	–6.28
(0.7, 0.9]	33	6 (18.2%)	8.86	13.87	6.32	0.70
(0.9, 1.1]	30	9 (30.0%)	15.26	30.02	16.58	–1.09
(1.1, 1.3]	19	-	22.64	28.74	21.71	15.93
> 1.3	176	-	37.79	45.21	35.23	28.52
Total	434	145.00	16.51	33.04	13.88	–1.77

50% of the option values are between –\$0.90 and –\$8.73, while 79.6% of the observations are negative. For ratios between 0.3 and 0.7, 66.7% of these observations are negative, with a mean option value of –\$2.69 and a median value of –\$1.64. 50% of the observations fall within the interval of \$0.23 to –\$6.28.

Overall, our results strongly suggest that convertible bonds with relatively low conversion values are often underpriced in the market to the extent that a boundary condition associated with the bond's straight debt value is often violated and negative option prices are implied.

V. SUMMARY

We examine the performance of a convertible bond valuation model developed in a Duffie and Singleton [1999] reduced-form credit risk valuation framework. The model produces prices that are lower than the observed market prices of convertible bonds whose embedded conversion option is in the money. It also systematically overprices convertible bonds with an out-of-the-money embedded conversion option.

While the in-the-money bias is expected, to a certain extent, as the result of the optimal firm call policy assumed in the theoretical model, a closer examination of the out-of-the-money bias indicates a systematic underpricing of convertible bonds with low conversion values by the market. The extent of this underpricing is such that conversion option values implied by the observed bond market prices are often negative. These results ought to be of interest to both investors and firm managers, and help them better formulate their investment and financing decisions.

ENDNOTES

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[1]U.S. convertible issuance reached \$104 billion in 2001, up 70% from the previous year's record total, according to "Convertibles as an Asset Class—2001 Update, United States" [2002].

[2]See Nandi [1998] for a review of both types of models.

[3]See, for example, Black and Cox [1976], Chance [1990,] and Shimko, Tejima, and Van Deventer [1993].

[4]See, for example, Jarrow and Turnbull [1995], Das and Tufano [1996], Jarrow, Lando, and Turnbull [1997], and Lando [1998].

[5]See, for example, Duffie and Singleton [1999], Madan and Unal [1998], and Das and Sundaram [2000].

[6]A similar approach to modeling the default hazard rate is followed by Takahashi, Kobayashi, and Nakagawa [2001].

[7]Only STRIPS derived from coupons are used in this exercise. STRIPS derived from principal incorporate embedded options associated with the reconstitution of the stripped bonds that influence their prices.

In general, when STRIPS are used to replicate the cash flows of other Treasury issues, the resulting pricing differences are relatively small. Carayannopoulos (1996a), using monthly observations from July 1989 through March 1994, reports average pricing differences between seasoned bonds and equivalent STRIPS portfolios of \$0.08 per \$100 face value of a Treasury bond.

[8]Monthly data are chosen to minimize the non-trading problems arising out of lack of liquidity in the corporate bond market. Datastream provides very limited bond data prior to January 2001.

[9]Note that the number of premium bonds in the sample is rather limited, but there is an indication that the degree of a

bond's moneyness influences the results in a similar manner, even within the limited number of premium bonds.

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