Effects of Credit Quality on Tax-Exempt and Taxable Yields

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We develop a reduced-form term structure model to estimate the implicit tax rates and the default risk of municipal bonds. The model is a generalization of Green's [1993] municipal bond model without default. It considers the impact of default risk on municipal bond pricing by accommodating both the risk-free arbitrage restricted by the tax code and the interactive effect of taxes and default.

Estimates of tax rates and the default risk implied in municipal bond prices are important for investors, traders, and portfolio managers. First, managers of tax-exempt bond funds and investment advisors need to know the equilibrium marginal income tax rates implied by municipal bond yields in order to provide advice about the best strategies for investing in fixed-income securities. Clearly, municipals are attractive only to investors with a tax bracket higher than the implicit tax rate.

Second, traders and corporate arbitrageurs need to know the marginal tax rate implied by bond yields to perform tax arbitrage effectively so they can take advantage of any discrepancies in returns across assets caused by tax differences. The type of tax arbitrage may range from a simple strategy of borrow-to-invest in municipals to a more sophisticated ex-dividend arbitrage of municipal bond funds (see Elton, Gruber, and Blake [2002]).

Third, fixed-income asset analysts must know the impact of taxes on bond pricing. Changes in taxes affect the after-tax returns on taxable bonds and the relative pricing between tax-exempt and taxable bonds. A good example is the recent tax proposal by the Bush administration, which has already increased the ratio of municipal to Treasury yields.

Market players must also understand risk differences in tax-exempt bonds. For example, a perceived increase in default risk imposes a non-tax cost for financial arbitrage (see Erikson, Goolsbee, and Maydew [2002]). An increase in default risk not only raises the probability of capital loss but also reduces liquidity. Institutions and firms are typically not permitted to hold more than a small fraction of high-risk bonds (Das, Fan, and Geng [2002]). These considerations can significantly affect the ability of firms or traders to arbitrage.

Default risk also affects an individual's willingness to invest. Investors tend to hold undiversified portfolios that are concentrated in municipal bonds from their own state so they can receive a full state tax exemption. Because investors are risk-averse, a higher default risk will reduce the demand for municipal bonds, especially low-rated bonds, and hence narrow the difference between Treasury and municipal yields. Because investors differ in their ability to offset capital losses, and face different tax rates, an increase in default risk has differential impacts on different investors.

Assessing the implicit tax rates and default of municipal bonds has become an increasingly important issue. A variety of events have attracted investor attention, mainly because municipal bond investors are concerned with the potential effects of changes in taxes and risk on the performance of their portfolios.

First, the municipal bond market has experienced considerable growth recently. According to Federal Reserve estimates, outstanding state and local government debt totaled $1.7 trillion at the end of 2001.[1] Individual participation in tax-exempt municipal bonds has expanded significantly through investments in unit investment trusts and mutual and money market funds.[2] Tax-exempt municipal bonds are among the most popular types of investments available today due to the poor performance of the equity markets. Investors need to understand clearly the tax benefits and risks of municipal bonds in order to make wise investment decisions.

Second, President Bush has proposed an aggressive tax cut package intended to stimulate the economy. A significant cut in taxes will cause the municipal market to underperform on a relative basis. The effect of the tax cut will ultimately depend on the risk characteristics of municipal bonds and changes in interest rates and monetary policy.

Third, the recent economic slowdown and uncertainty due to the war in Iraq have substantially increased the risk and the default premium of fixed-income securities. State and local governments are running large deficits, which have substantially eroded the quality of municipal bonds.

Researchers use a variety of models to explain and estimate the implicit tax rates implied by relative taxable and tax-exempt yields. These include the institutional demand model (Galper and Peterson [1971]; Fortune [1973]; Fama [1977]; Kimbal [1977]); the clientele model (Mussa and Kormendi [1979]; Kidwell and Koch [1983]); and the tax-timing model (Constantinides and Ingersoll [1984]; Poterba [1989]). While these models have merits, they cannot fully explain the anomalous behavior of municipal yield curves, which exhibit excessive term premiums compared to Treasury yields.

Green [1993] proposes an equilibrium model of the municipal term structure to explain the puzzling behavior of taxable versus tax-exempt yields. He argues that high-tax investors may shield coupon income from taxation by constructing portfolios of taxable bonds that generate offsetting losses or investment interest expenses. Green uses this argument to obtain investors' implicit valuation of the pre-tax cash flows from par taxable bonds. Assuming the arbitrage activities of dealers and tax-exempt institutions, he derives an equilibrium municipal term structure model to explain the relative yields of taxable and tax-exempt bonds. In this model, investors holding both taxables and municipals may not regard the coupon income of Treasuries as fully taxable at the margin because of offsetting investment interest elsewhere in their portfolios. These implicit tax benefits tend to increase with maturity, thus pulling down the yield curve of Treasury bonds at the long end.

While Green's study provides important insights into municipal bond pricing, his model ignores the effect of default. Municipal bonds are not risk-free, and in fact may even be riskier than corporate bonds due to the unique features of municipal assets and less predictable political processes (see Hempel [1972]; Zimmerman [1977]; Trzcinka [1982]).[3] While ignoring default risk may have only a modest effect on estimation of the parameters for the prime municipal bonds in Green's study, the effect would be much greater for lower-rated municipals.

Empirical evidence has shown that default risk can seriously affect the estimation of implicit tax rates and municipal yield spreads (see Gordon and Malkiel [1981]; Skelton [1983]; Ang, Peterson, and Peterson [1985]; Yawitz, Maloney, and Ederington [1985]; Kim, Ramaswamy, and Sundaresan [1993]; Stock [1994]; Wu and Yu [1996]). These findings suggest that default risk should be accounted for in the estimation of implicit tax rates.[4]

We generalize Green's [1993] model to incorporate the effect of default risk on the relative yields of taxable and tax-exempt bonds. Including the effect of default risk greatly enhances the explanatory power of the municipal term structure model. Ignoring the default risk effect results in an underestimation of municipal bond yields and implicit marginal income tax rates, and more serious underestimation for lower-rated and longer-term bonds.

After default risk is properly accounted for, our reduced-form municipal term structure model can explain the behavior of tax-exempt bond yields over different tax regimes very well. The estimates of implicit tax rates from the defaultable municipal bond model are much more accurate and commensurate with statutory tax rates.

I. YIELD CURVE PUZZLE AND THE GREEN MODEL

Traditional analyses of the yield relationship between taxable and tax-exempt bonds presume that investors are at the margin on all bonds. They can trade freely and without friction, and the taxation of long and short positions is completely symmetric. Investors apply the same discount factors to the after-tax cash flows from both taxable and tax-exempt bonds.

Assume that bonds are default-free and priced at par, and investors hold them to maturity. It follows that the yield on the tax-exempt bond (M_T) is simply the yield on the taxable bond (C_T) times one minus the marginal investor's income tax rate τ:

$$M_T = C_T(1 - \tau) \tag{1-A}$$

This yield relationship should hold for all taxable and tax-exempt bonds maturing at any time T. That is, the yield curve for tax-exempt bonds should be a constant multiple of the yield curve for the taxable bonds of all maturities.

Empirical data, however, do not conform well to the yield relationship in Equation (1-A). First, the implicit marginal tax rate calculated from observed bond yields frequently deviates from the plausible marginal rate for potential investors. In particular, the implicit tax rate estimated from long-term bonds seems far too low to be the marginal tax rate of any investor who could reasonably be expected to hold municipal bonds. Second, historically the yield curves of Treasury bonds are often inverted, but the yield curves of municipal bonds are almost always upward-sloping. Consequently, long-term municipal bonds often have yields that are too high compared to Treasury bonds at the same maturity.

Trzcinka [1982] suggests including a random intercept term in the yield relationship of Equation (1-A) to control for the time-varying risk premium. Specifically, for taxable and tax-exempt bonds in a given rating and maturity class, the yield relationship is proposed:

$$M_t = \Lambda_t + \beta C_t \tag{1-B}$$

where M_t and C_t are the municipal and taxable bond yields, Λ_t is the time-varying risk premium, and $\beta = 1 - \tau$ is constant. Note that in order for β to stay constant, the intercept term Λ_t must increase with maturity to absorb higher municipal yields at longer maturities. This implies that the risk premium must increase with bond maturity.

Green [1993] proposes an alternative model to explain the yield curve anomaly. He argues that the tax code limits investors' ability to be marginal on all bonds simultaneously. Instead, investors can be at the margin only across municipals and some tax-advantaged portfolios of taxable bonds.

A tax-advantaged portfolio can be formed to replicate the cash flow of zero-coupon bonds to avoid taxes before maturity as follows. Suppose there is a par bond with coupon rate C and maturity date T, and another bond with coupon rate $C/2$ and the same maturity date but traded at a discount. A long position with two $C/2$ coupon bonds and a short with one C coupon bond will generate no coupon income and hence no net tax liability prior to maturity. This strategy can be applied to any bonds with different coupons to form a tax-advantaged portfolio.

While individual investors may be precluded from holding this type of tax-advantaged portfolio due to limitations on interest deductions from borrowings and short positions, bond dealers are not. From a dealer's point of view, cash flows from par bonds and a sequence of pure principal payments are equivalent. Any difference in price between a pair of bonds or bond portfolios with an identical income stream creates an arbitrage opportunity for dealers.

Under Green's no-arbitrage condition and the assumption that bonds are priced at par (= 1), taxable bonds will be priced as:

$$1 = \sum_{t=1}^{T} C_T P_t + P_T \tag{2}$$

where C_T is the coupon yield for the taxable bond with maturity T; $P_t = [(1 - \tau)d_t]/[1 - \tau d_t]$; and $t = 1, 2, \ldots, T$, are the pre-tax discount factors that discount the pre-tax cash flow of the taxable bond to its par value. At the same time, investors with these pre-tax discount factors will price a tax-exempt municipal bond as follows:

$$1 = \sum_{t=1}^{T} M_T d_t + d_T \tag{3}$$

where M_T is the yield for the tax-exempt bond with maturity T, and d_t is the after-tax discount factor that equates the cash flow of the bond to its par value.

Solving (3) recursively from par municipal bonds of every maturity, one can obtain the after-tax discount factor at each time t:

$$d_1 = \frac{1}{1 + M_1} \tag{3-A}$$

and

$$d_T = \frac{1 - M_T\left(\sum_{t=1}^{T-1} d_t\right)}{1 + M_T} \tag{3-B}$$

where d_t is the discount factor for the after-tax cash flow at $t = 1, 2, \ldots, T$. From (2) and (3) and rearranging, we can further obtain the relationship between the yield curves of municipals and Treasuries:

$$M_T = C_T(1-\tau)\frac{\sum_{t=1}^{T} \frac{d_t}{1-\tau d_t}}{\sum_{t=1}^{T} \frac{d_t}{1-\tau d_T}} \tag{4}$$

Equation (4) implies that the ratio of the tax-exempt yield to the taxable yield M_t/C_t declines with maturity, which is consistent with observed yield data. When $T = 1$, the ratio multiplying the after-tax coupon on the right side of (4) is equal to one, and $1 - M_t/C_t$ gives the tax rate of the marginal investor. For longer maturities, the ratio multiplying the after-tax coupon is greater than one if forward rates are positive ($d_T < d_t$ for $t < T$). The longer the maturity, the higher this ratio, making M_T increasingly large relative to C_T. Thus, if one uses the formula $1 - M_t/C_t$ to estimate the implicit tax rate from the yields of taxable and tax-exempt bonds, the resulting estimate will be biased downward.

Green [1993] finds that his model explains the relative shapes of the taxable and tax-exempt yield curves very well. While the estimated implicit tax rates are reasonable for short-term bonds, however, the model continues to underestimate the tax rates of long-term bonds. One potential cause for this underestimation is the absence of default risk in his yield model.

II. MUNICIPAL TERM STRUCTURE MODEL WITH DEFAULT RISK

Unlike Treasury securities, municipal bonds are subject to default. Although default risk may not always have a dominating effect on the shape of the municipal yield curve, especially for high-quality bonds, ignoring default risk would almost certainly result in a biased estimation of the implicit marginal income tax rate. We generalize Green's model to account for the effects of default risk of municipal bonds on the relative yield relationship.

Model with Default

The default risk premium is an important component of risky bond spreads. Investors require a higher promised payment to compensate for the expected loss from default. Thus, to successfully explain the yield relationship between risky tax-exempts and riskless taxables, one needs to consider the effects of both taxes and differences in default (see Elton et al. [2001]).

Trzcinka [1982] includes an intercept term in Equation (1-B) to capture the differential risk premium between tax-exempt and taxable bonds. While it is appealing for its simple empirical structure, this specification is not well founded in valuation theory. For instance, if the relative risks of two bonds change over time, both the intercept and slope coefficients in (1-B) will vary. Restricting the slope coefficient to be constant and exclusively related to the marginal tax rate would result in a model misspecification. Therefore, we need a theoretical model that incorporates both effects of taxes and default risk in a consistent manner. A viable approach is to generalize Green's model to account for the effect of default.

In generalizing the municipal term structure model, we impose assumptions as follows:

1. Municipal and Treasury bonds for each maturity are priced at par and both are non-callable.
2. Municipal bonds are subject to default with a default probability λ_t at time t and a constant recovery rate δ, specified as a proportion of the face value.
3. Investors buy and hold the bonds until the date of maturity or default, whichever comes first. The options for tax timing to realize losses earlier are ignored. [5]
4. Taxation of long and short positions is asymmetric for individuals. The interest deductions from borrowings and short positions cannot exceed investment income from the positions these borrowings support. Consequently, investors cannot be marginal on all bonds simultaneously, but a single representative investor can still be at the margin across municipals and portfolios of taxable bonds.
5. The tax rate and the tax regime remain unchanged over the bond life.
6. There is no transaction cost.

All these assumptions except (2) and (6) are similar to Green [1993]. Assumption (2) allows us to introduce the default risk in bond pricing, while assumption

(6) is embedded implicitly in Green's model. The assumption of buy-and-hold is maintained in (3) with a minor revision that investors may be forced to liquidate their positions before maturity if default occurs. We adopt the reduced-form approach to price defaultable bonds (see Jarrow and Turnbull [1995]; Jarrow, Lando, and Turnbull [1997]; Duffie and Singleton [1999]). Using this approach, bonds are priced contingent on the exogenous default process and its payoff discounted by a risk-adjusted rate.

The payoff of each municipal bond is conditional on whether there is a default or not. The present value of the expected cash flow when there is no default is

$$\left(\sum_{t=1}^{T} M_T d_t + d_T\right)\prod_{t=1}^{T}(1-\lambda_t)$$

In the event of default, the bond investor receives a residual (recovery) value δ. The present value of the expected cash flow if default occurs at time i, is given by

$$\left(\sum_{t=1}^{i-1} M_T d_t + \delta\right)\lambda_i \prod_{k=1}^{i-1}(1-\lambda_k)$$

If the bond has not defaulted by time t, its value at t should be equal to the expected present value of its future cash flows with no default plus the expected present value of receiving the net after-tax residual value upon default.

Assuming that the bond is priced at par, the pricing equation that holds for the municipal bond is:

$$1 = \left(\sum_{t=1}^{T} M_T d_t + d_T\right)\prod_{t=1}^{T}(1-\lambda_t) + \sum_{i=1}^{T}\left[\left(\sum_{t=1}^{i-1} M_T d_t + \delta\right)\lambda_i \prod_{k=1}^{i-1}(1-\lambda_k)\right] \quad (5)$$

We can solve for the after-tax discount factor of the par municipal bond recursively, conditional on the default probability and recovery rate.[6] For example, at $t = 1$, we have:

$$d_1 = \frac{1-\lambda_1\delta}{(1-\lambda_1)(1+M_1)} = \frac{1}{(1-\lambda_1)(1+M_1)} - \frac{\lambda_1\delta}{(1-\lambda_1)(1+M_1)} \quad (6)$$

Compared to (3-A), including a positive default probability in the model increases the value of the after-tax discount factor d_1 since $\delta < 1$. The after-tax discount factor is now risk-adjusted. Its value is positively related to the default probability and negatively related to the recovery rate.

Similarly, we can obtain the after-tax discount factors at maturity T:

$$d_T = \frac{1}{(1+M_T)} \times \left[\frac{1-\sum_{i=1}^{T}\left[\left(\sum_{t=1}^{i-1} M_T d_t + \delta\right)\lambda_i \prod_{k=1}^{i-1}(1-\lambda_k)\right]}{\prod_{t=1}^{T}(1-\lambda_t)} - M_T\sum_{t=1}^{T-1} d_t\right] \quad (7)$$

Next consider the pricing of Treasury bonds. Since there is no default, the price of a pure discount Treasury bond, or a tax-advantaged portfolio of taxable bonds with zero coupon, is simply equal to its discounted after-tax value (see also Green [1993]):

$$P_T = \left[1 - \tau(1 - P_T)\right]d_T \quad (8)$$

or

$$P_T = \frac{(1-\tau)d_T}{1-\tau d_T} \quad (9)$$

The value in brackets in Equation (8) is the after-tax cash flow of the zero-coupon bond or portfolio at maturity, which is discounted by the after-tax discount factor d_T. Rearranging (8) gives the pre-tax discount factor P_T. The value of this discount factor is identical to that in Green's model since there is no default risk for Treasury bonds.

We can replicate a par Treasury bond with coupon C_T by a sequence of zero-coupon Treasury bonds, each with a pure principal payment of $C_t = C_T$, at $t = 1, 2, \ldots, T-1$, and $1 + C_T$ at time T. Since these two packages have identical cash flows, they should have the same price. That is:

$$1 = \sum_{t=1}^{T} C_T P_t + P_T \quad (10)$$

where the discount factors P_t apply to the pre-tax payments.

Using (9), we can rewrite (10) as

$$\frac{1 - d_T}{1 - \tau d_T} = \sum_{t=1}^{T} C_T \frac{(1 - \tau)d_t}{1 - \tau d_t} \tag{11}$$

On the other hand, from the valuation formula for municipal bonds in (5), we have

$$\begin{aligned} &1 - d_T = \\ &M_T \left\{ \sum_{t=1}^{T} d_t \prod_{t=1}^{T}(1 - \lambda_t) + \sum_{i=1}^{T}\left[\left(\sum_{t=1}^{i-1} d_t\right)\lambda_i \prod_{k=1}^{i-1}(1 - \lambda_k)\right]\right\} \\ &-d_T\left[1 - \prod_{t=1}^{T}(1 - \lambda_t)\right] + \delta \sum_{i=1}^{T}\left[\lambda_i \prod_{k=1}^{i-1}(1 - \lambda_k)\right] \end{aligned} \tag{12}$$

Equations (11) and (12) can be used to derive the municipal yield curve from the Treasury yield curve. Combining (11) and (12) and rearranging gives:

$$M_T = \frac{C_T \sum_{t=1}^{T} \frac{(1 - \tau)d_t}{1 - \tau d_t} + \frac{d_T\left[1 - \prod_{t=1}^{T}(1 - \lambda_t)\right] - \delta \sum_{i=1}^{T}\left[\lambda_i \prod_{k=1}^{i-1}(1 - \lambda_k)\right]}{1 - \tau d_T}}{\frac{1}{1 - \tau d_T}\left\{\sum_{t=1}^{T} d_t \prod_{t=1}^{T}(1 - \lambda_t) + \sum_{i=1}^{T}\left[\left(\sum_{t=1}^{i-1} d_t\right)\lambda_i \prod_{k=1}^{i-1}(1 - \lambda_k)\right]\right\}} \tag{13}$$

The yield function above applies to municipal bonds of different maturities. For example, for bonds with one period to maturity, we have:

$$M_1 = \frac{C_1(1 - \tau)d_1 + \lambda_1(d_1 - \delta)}{d_1(1 - \lambda_1)} \tag{14}$$

or

$$(1 - \tau) = \frac{M_1 d_1(1 - \lambda_1) - \lambda_1(d_1 - \delta)}{C_1 d_1} \tag{15}$$

For bonds with two periods to maturity, we have:

$$M_2 = \frac{C_2\left[\frac{(1 - \tau)d_1}{1 - \tau d_1} + \frac{(1 - \tau)d_2}{1 - \tau d_2}\right] + \frac{d_2\left[1 - (1 - \lambda_1)(1 - \lambda_2)\right] - \delta(1 - \lambda_1)\lambda_2}{1 - \tau d_2}}{\frac{1}{1 - \tau d_2}\left[(d_1 + d_2)(1 - \lambda_1)(1 - \lambda_2) + d_1(1 - \lambda_1)\lambda_2\right]} \tag{16}$$

When there is default risk ($\lambda > 0$), the yield of municipal bonds must increase to compensate for the potential default [see, for example, Equation (14)]. If default risk were ignored, the estimated marginal tax rate would be forced lower to make the after-tax yield high enough to be consistent with the yield of the risky tax-exempt bond. This leads to underestimation of the marginal income tax rate.

The bias in estimation of the tax rate depends on the probability of default and the recovery rate. If default risk increases with maturity, the estimated tax rate will decline with maturity. In general, the relationship becomes more complicated when bond maturity is lengthened. As Equation (16) shows, the effect of default risk interacts with discount factors and tax rates.

Numerical Simulations

To assess the potential bias in inferring the marginal tax rate from municipal and Treasury yields, we conduct simulation analysis. If default risk is ignored, the implicit tax rate estimated from the yield relationship between tax-exempt and taxable bonds would be biased downward. Since default risk generally increases with maturity for investment-grade bonds, it is expected that the downward bias in the implied tax rate estimate increases with bond maturity. If the default probability is known, we can directly assess the effect of default on municipal bond yields and the estimate of the marginal tax rate.

Standard & Poor's has provided estimates of default probabilities for corporate bonds of different ratings and a transition probability matrix indicating how a bond with a given rating may fare one year later. Unfortunately, similar transition matrix and default probabilities are not available for municipal bonds. We hence use the default transition probability matrix of corporate bonds in our simulation analysis. Although the probabilities of default may be different for municipal bonds, the numerical results nevertheless allow us to assess the potential effect of ignoring default, given the set of default probabilities provided by Standard & Poor's.

S&P's 1995 transition matrix is presented in Panel A of Exhibit 1. Each entry in a row shows the probability in percentage that a bond with a rating shown in the exhibit stub ends up one year later in the rating shown in the column headings.[7] Marginal probabilities of default can be estimated from the transition matrix under the assumption that the transition process is stationary and Markovian. The evolution of default probability can be computed from S&P's transition matrix in Panel A.

Panel B of Exhibit 1 shows the marginal default probabilities by age and the initial rating. In our simulation analysis, we focus on AA, A, and BBB bonds.[8]

The simulation procedure is as follows. In the first step, we calculate the after-tax discount factors d_t from (7), given the municipal yield (M_T), the default probability, and the recovery rate. Given a tax rate, the Treasury yield (C_T) consistent with the yield of the municipal bond with default can be calculated from (13).

In the second step, we restrict the default probability to zero and recalculate the after-tax discount factors d_t from (7). Substituting the after-tax discount factors with no default into (13) and at the same time forcing the default probability to be zero, we reestimate the tax rate from the municipal and Treasury yields obtained from the first step that correctly incorporates the effect of default in the yield model.

That is, we estimate the marginal tax rate from the incomplete model that ignores default using the yields of municipals and Treasuries generated from a model that includes default risk. This allows us to assess the impact of ignoring default on the estimation of the tax rate.

Exhibit 2 reports the numerical estimates of tax rates, which are biased because the default probabilities of municipals are ignored. We use S&P's estimates of default probabilities for AA, A, and BBB bonds at different maturities (from Panel B of Exhibit 1) in these simulations. We set the value of the underlying true recovery rate $\delta = 0.4$, tax rate $\tau = 0.4$, and municipal bond yield $M = 0.04$. When default is ignored, the tax rate is generally underestimated.

There are two interesting findings in Exhibit 2. First, as the default probability increases (e.g., from the AA to BBB rating), the bias in tax rate estimates increases. Second, the bias increases with maturity. For bonds with 30-year maturity, the bias in tax rate estimation is about 2.6% for AA bonds, 5.7% for A bonds, and 12.2% for BBB bonds. For bonds with short maturities, the bias is much smaller: 0.2% for AA bonds, 1% for A bonds, and 2.5% for BBB bonds with two-year maturity.[9]

This explains why Green's model provides better estimates of implicit tax rates for short-term municipal bonds.

III. DATA AND EMPIRICAL RESULTS

Data are collected from three sources: Salomon Brothers' *Analytical Records of Yield and Yield Spread*, Moody's bond yields, and Warga tapes. Salomon Brothers' *Analytical Records of Yield and Yield Spread* provide monthly estimates of yields for current-issue Treasury and municipal bonds at various maturities and for municipal bonds in different default risk categories. This is the same type of data used by Green [1993].

These data are available only for the period from 1973 through 1994. Additional data from 1994 through 2001 are obtained from *Mergent Municipal & Government Manual* and the Bloomberg on-line database, which includes Moody's bond yield data. Although we have data through the end of 2001, we choose to stop at the end of August because of the major market disruption after September 11, 2001. To the extent that Moody's indexes may be based on a different portfolio of bonds, the yield data may differ from Salomon Brothers, but the average difference should be slight because both indexes are constructed from a broad sample to represent each bond class.

Yields of bond indexes provided by Salomon Brothers and Moody's are used to estimate implicit tax rates and default probability. As explained by Salomon Brothers, the data for municipal bonds are derived from new issues coming out at approximately the report dates. Also, the Treasury yield curves follow the yields of higher-coupon issues in the longer maturities whenever there is a choice of coupons.

Our sample period varies slightly for different bond types. For Treasury and prime municipal bonds, we have yields for the whole sample period (1973-2001) with maturities of 1 year, 2 years, 5 years, 10 years, 20 years, and 30 years. For good municipal bonds, we have all yield data for bonds with maturities of 1, 10, 20, and 30 years for the whole sample period (1973-2001), but only up to 1992 for maturities of 2 and 5 years. For medium municipals, we have all yield data for bonds with maturities of 1, 20, and 30 years for the whole sample period, but only up to 1992 for maturities of 2, 5, and 10 years.[10]

We also need data for individual Treasury bonds of different maturities to estimate spot rates or the pre-tax discount factors P_t. Individual Treasury bond data are obtained from the Lehman Brothers Fixed Income

EXHIBIT 1
Default Probability

Panel A. Transition Probability Matrix

	AAA (%)	AA (%)	A (%)	BBB (%)	BB (%)	B (%)	CCC (%)	Default (%)
AAA	90.788	8.291	0.716	0.102	0.102	0.000	0.000	0.000
AA	0.103	91.219	7.851	0.620	0.103	0.103	0.000	0.000
A	0.924	2.361	90.041	5.441	0.719	0.308	0.103	0.103
BBB	0.000	0.318	5.938	86.947	5.302	1.166	0.117	0.212
BB	0.000	0.110	0.659	7.692	80.549	8.791	0.989	1.209
B	0.000	0.114	0.227	0.454	6.470	82.747	4.086	5.902
CCC	0.228	0.000	0.228	1.251	2.275	12.856	60.637	22.526
Default	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000

Panel B. Evolution of Default Probability

Year	AA (%)	A (%)	BBB (%)
1	0.000	0.103	0.212
2	0.017	0.154	0.350
3	0.037	0.204	0.493
4	0.061	0.254	0.632
5	0.087	0.305	0.761
6	0.115	0.355	0.879
7	0.145	0.406	0.983
8	0.177	0.457	1.075
9	0.210	0.506	1.153
10	0.243	0.554	1.221
11	0.278	0.600	1.277
12	0.313	0.644	1.325
13	0.348	0.686	1.363
14	0.383	0.726	1.395
15	0.419	0.763	1.419
16	0.453	0.797	1.439
17	0.488	0.830	1.453
18	0.521	0.860	1.464
19	0.554	0.888	1.471
20	0.586	0.913	1.475
21	0.618	0.937	1.476
22	0.648	0.959	1.476
23	0.677	0.980	1.473
24	0.705	0.998	1.470
25	0.732	1.015	1.465
26	0.758	1.031	1.459
27	0.783	1.045	1.453
28	0.806	1.058	1.446
29	0.829	1.070	1.439
30	0.850	1.081	1.431

EXHIBIT 2

Estimates of Tax Rate When Default Is Ignored

Maturity	λ Based on AA Bond Estimated Values τ	λ Based on A Bond Estimated Values τ	λ Based on BBB Bond Estimated Values τ
2	0.3984	0.3904	0.3755
4	0.3975	0.3867	0.3657
6	0.3964	0.3822	0.3535
8	0.3949	0.3773	0.3404
10	0.3932	0.3723	0.3276
12	0.3911	0.3674	0.3159
14	0.3889	0.3627	0.3058
16	0.3865	0.3583	0.2973
18	0.3841	0.3544	0.2906
20	0.3818	0.3510	0.2855
22	0.3796	0.3481	0.2818
24	0.3777	0.3459	0.2793
26	0.3762	0.3443	0.2780
28	0.3751	0.3434	0.2777
30	0.3744	0.3431	0.2783

$\delta = 0.4$, $M = 0.04$, and true tax rate $\tau = 0.4$.

Database distributed by Warga [1998]. This database provides monthly price, accrued interest, and return data on Treasuries and corporate bonds up to March 1998. Remaining Treasury bond data are collected from the Fixed Investment Securities Database (FISD) through August 2001.

We use only a subset of the Treasury bond data in this study. First, Treasury bonds that are matrix priced rather than trader priced are eliminated from the sample. Eliminating matrix-priced bonds leaves us with a set of Treasury bond prices based exclusively on dealer quotes. This is the same type of data in the CRSP government bond file. Second, we eliminate all bonds with special features that would make them priced differently. Specifically, we exclude flower bonds, inflation-indexed bonds, and callable bonds.[11] Finally, we select only bonds that have maturities shorter than 30 years and longer than 60 days. The final sample for individual bonds consists of a total of 67,281 Treasury bond months.

Significant income tax reforms took place in 1981, 1986, and 1993 (see Jurinski [2000]; Elton, Gruber, and Blake [2002]). To control for the effects of changes in the tax regime, we divide the entire sample period into four subperiods: 1977-1980, 1982-1985, 1987-1992, and 1994-2001. This allows us to compare estimates under different tax regimes.

Descriptive Statistics

Exhibit 3 provides descriptive statistics for the yields of municipal and Treasury bonds for the entire sample period, and for four subperiods. Consistent with Green's [1993] findings, the increase in yields with respect to maturity is smaller for Treasury bonds than municipal bonds, and in some years Treasury yield curves are inverted. The volatility of Treasury yields tends to be higher than municipal yields.

Exhibit 4 reports the term premiums for both the taxable and tax-exempt bonds.[12] These premiums depict the relative steepness of the yield curves for Treasury and municipal bonds. The average term premiums are higher for municipals than for Treasuries. Also, the average term premiums increase as the quality of municipal bonds declines, reflecting the effect of default risk.

The term premiums are generally positive for all three rating classes of municipal bonds, but it is not uncommon for the term premiums of Treasury bonds to be negative, as shown in the column of minimum values. In the period 1977-1980, even the average term premiums are negative. The yields are rarely inverted for municipal bonds.

Exhibit 5 reports the quality spreads of municipal bonds with different ratings and maturities. As expected, the average yields for prime municipal bonds are lower

EXHIBIT 3

Descriptive Statistics for Yields of Treasuries and Municipals—1973-2001

Period	Maturity	Mean				Standard Deviation			
		Treasury	Municipals			Treasury	Municipals		
			Prime	Good	Medium		Prime	Good	Medium
1973–2001	1	7.51	4.44	4.59	4.92	2.76	1.46	1.48	1.42
	2	8.15	5.12	5.44	5.74	2.66	1.31	1.45	1.49
	5	8.13	5.65	6.04	6.37	2.44	1.40	1.59	1.69
	10	8.31	6.02	6.14	7.08	2.34	1.59	1.63	1.97
	20	8.93	6.74	6.86	7.13	2.13	1.73	1.77	1.92
	30	8.54	6.90	7.05	7.34	2.12	1.74	1.79	1.93
1977–1980	1	9.19	4.63	4.74	4.94	2.75	1.36	1.35	1.41
	2	9.07	4.78	4.89	5.15	2.35	1.25	1.25	1.27
	5	8.99	5.07	5.19	5.41	1.86	1.06	1.07	1.15
	10	9.10	5.40	5.51	5.81	1.63	1.02	1.04	1.15
	20	9.14	6.11	6.23	6.54	1.48	1.14	1.16	1.30
	30	9.12	6.38	6.50	6.81	1.43	1.16	1.19	1.33
1982–1985	1	10.32	5.89	6.11	6.44	1.89	1.02	1.05	1.09
	2	11.02	6.41	6.62	6.97	1.75	0.98	1.02	1.03
	5	11.62	7.49	7.71	8.16	1.49	0.98	0.99	1.10
	10	11.84	8.64	8.87	9.35	1.34	1.01	1.02	1.13
	20	11.97	9.64	9.87	10.35	1.18	1.11	1.11	1.24
	30	11.89	9.80	10.07	10.53	1.11	1.14	1.14	1.27
1987–1992	1	6.76	4.88	5.02	5.27	1.63	1.10	1.07	0.87
	2	7.24	5.17	5.30	5.49	1.41	0.78	0.56	0.59
	5	7.80	5.78	5.98	6.16	0.98	0.65	0.61	0.61
	10	8.18	6.34	6.44	6.99	0.76	0.52	0.51	0.49
	20	8.39	6.94	7.05	7.22	0.68	0.51	0.54	0.62
	30	8.39	7.05	7.19	7.39	0.57	0.53	0.56	0.65
1994–2001	1	5.58	3.21	3.32	3.80	0.59	0.40	0.37	0.34
	2	5.97	4.27	—	—	0.73	0.35	—	—
	5	6.03	4.73	—	—	0.74	0.38	—	—
	10	6.18	4.84	4.92	—	0.75	0.43	0.45	—
	20	6.86	5.48	5.56	5.65	0.63	0.44	0.45	0.45
	30	6.51	5.58	5.68	5.83	0.67	0.43	0.45	0.45

Sample period for some good and medium municipal bonds ends in December 1992.

than for good municipal bonds, which in turn are lower than for medium municipal bonds for all periods and maturities. But the spread between good and medium municipals is much higher than the spread between prime and good municipals. As shown in the column of minimum values, the quality spreads are negative in very few cases, suggesting possible but rare mispricing.

Exhibit 6 shows the ratios of the yields of prime municipals to Treasury bonds, which are equal to the traditional measure of $(1 - \tau)$. The ratio increases (or τ declines) as term to maturity increases for all periods. Thus, using the traditional approach, the estimated implicit tax rates decline as the maturity of the bond lengthens. These ratios change over time and are highest for the last two subperiods.

Exhibit 4

Descriptive Statistics for Term Premiums of Treasuries and Municipals—1973-2001

Period	Maturity	Mean				Minimum				Maximum			
		Treasury	Municipal			Treasury	Municipal			Treasury	Municipal		
			Prime	Good	Medium		Prime	Good	Medium		Prime	Good	Medium
1973	2	0.32***	0.44***	0.29***	0.33***	-1.11	-0.20	0.00	0.00	1.26	1.50	1.00	1.00
	5	0.62***	0.97***	0.90***	0.96***	-2.43	-0.05	0.00	0.00	2.59	2.25	2.25	2.25
|	10	0.80***	1.57***	1.56***	1.68***	-3.08	0.00	0.00	0.00	3.31	3.30	3.40	3.50
2001	20	1.10***	2.29***	2.28***	2.21***	-3.28	0.50	0.40	0.30	3.82	4.75	4.40	4.75
	30	1.03***	2.46***	2.46***	2.42***	-3.41	0.60	0.60	0.60	4.33	5.00	4.70	5.00
1977	2	-0.12**	0.15***	0.15***	0.20***	-1.11	0.00	0.00	0.00	0.63	0.40	0.40	0.60
	5	-0.19	0.45***	0.45***	0.47***	-2.43	0.00	0.00	0.00	1.52	1.10	1.15	1.10
|	10	-0.08	0.77***	0.77***	0.86***	-3.08	0.00	0.00	0.00	2.03	1.80	1.80	1.80
1980	20	-0.05	1.48***	1.49***	1.59***	-3.28	0.50	0.50	0.60	2.34	2.80	2.90	3.10
	30	-0.06	1.75***	1.76***	1.87***	-3.41	0.70	0.70	0.80	2.48	3.15	3.25	3.40
1982	2	0.71***	0.51***	0.50***	0.53***	0.12	0.25	0.25	0.00	1.08	1.00	1.00	1.00
	5	1.30***	1.60***	1.59***	1.72***	-0.18	1.05	1.05	1.20	2.01	2.25	2.25	2.25
|	10	1.52***	2.75***	2.76***	2.91***	-0.30	2.10	2.20	2.30	2.50	3.30	3.40	3.50
1985	20	1.65***	3.75***	3.75***	3.91***	-0.43	2.95	3.05	3.20	2.90	4.75	4.40	4.75
	30	1.57***	3.91***	3.95***	4.09***	-0.63	3.05	3.20	3.35	2.94	5.00	4.70	5.00
1987	2	0.49***	0.49***	0.35***	0.38***	-0.10	-0.20	0.20	0.20	1.14	1.50	0.80	0.75
	5	1.05***	0.90***	1.03***	1.05***	-0.21	-0.05	0.55	0.65	2.59	1.95	1.55	1.55
|	10	1.43***	1.46***	1.42***	1.88***	-0.31	0.20	0.15	0.60	3.31	2.75	2.75	2.60
1992	20	1.63***	2.06***	2.03***	1.95***	-0.39	0.50	0.40	0.55	3.82	3.50	3.50	3.30
	30	1.64***	2.17***	2.17***	2.12***	-0.49	0.60	0.60	0.60	4.33	3.55	3.55	3.55
1994	2	0.48***	0.83***	—	—	-0.20	0.63	—	—	1.26	1.20	—	—
	5	0.45***	1.29***	—	—	-1.05	1.00	—	—	1.95	1.92	—	—
|	10	0.60***	1.63***	1.60***	—	-0.98	1.05	1.00	—	2.36	2.40	2.50	—
2001	20	1.36***	2.26***	2.24***	1.85***	0.27	1.70	1.75	0.85	2.54	2.95	3.00	2.60
	30	0.93***	2.36***	2.36***	2.03***	-0.66	1.75	1.85	1.20	2.76	3.25	3.25	2.85

*Sample period for some good and medium municipal bonds ends in December 1992. *, **, and *** represent significance at the 10%, 5%, and 1% level.*

Exhibit 5

Descriptive Statistics for Quality Spreads of Municipal Bonds

Period	Maturity	Mean		Minimum		Maximum	
		ΔPG	ΔGM	ΔPG	ΔGM	ΔPG	ΔGM
1973	1	0.14***	0.33***	0.00	-0.05	0.50	1.50
	2	0.16***	0.30***	-0.10	-0.25	0.50	1.00
	5	0.16***	0.33***	-0.20	-0.15	0.80	1.25
	10	0.13***	0.37***	0.00	0.05	0.60	0.90
2001	20	0.13***	0.26***	0.00	-0.15	0.40	1.00
	30	0.15***	0.29***	0.00	0.00	0.70	0.90
1977	1	0.11***	0.21***	0.05	0.05	0.25	0.60
	2	0.12***	0.26***	0.05	0.05	0.25	0.50
	5	0.11***	0.22***	0.05	0.05	0.25	0.70
	10	0.11***	0.29***	0.05	0.15	0.25	0.85
1980	20	0.12***	0.31***	0.05	0.10	0.25	0.80
	30	0.12***	0.31***	0.10	0.10	0.30	0.70
1982	1	0.22***	0.33***	0.10	-0.05	0.50	0.75
	2	0.21***	0.36***	0.10	-0.25	0.50	1.00
	5	0.21***	0.46***	0.05	0.15	0.30	1.25
	10	0.23***	0.48***	0.10	0.20	0.50	0.90
1985	20	0.23***	0.48***	0.10	0.10	0.40	1.00
	30	0.26***	0.46***	0.10	0.10	0.70	0.90
1987	1	0.14***	0.25***	0.00	0.00	0.40	1.50
	2	0.14***	0.19***	-0.10	-0.10	0.30	0.35
	5	0.18***	0.18***	-0.05	-0.15	0.80	0.50
	10	0.09***	0.32***	0.00	0.15	0.50	0.50
1992	20	0.11***	0.17***	0.00	-0.05	0.35	0.60
	30	0.14***	0.20***	0.00	0.00	0.45	0.65
1994	1	0.11***	0.48***	0.00	0.00	0.40	1.10
	2	—	—	—	—	—	—
	5	—	—	—	—	—	—
	10	0.08***	—	0.00	—	0.60	—
2001	20	0.08***	0.09***	0.00	-0.15	0.25	0.25
	30	0.11***	0.14***	0.00	0.00	0.40	0.45

*Municipal bonds are separated into three rating categories: prime, good, and medium. ΔPG is the yield spread between prime and good municipal bonds; ΔGM is the yield spread between good and medium municipal bonds. *, **, and *** represent significance at the 10%, 5%, and 1% level.*

Estimates of Implicit Tax Rates Using Green's Model with No Default

We first estimate the implicit marginal tax rates using Green's [1993] model with no default. While Green focuses on the yield spreads between Treasury and prime municipal bonds, we include also good and medium municipal bonds. Including these lower-rated bonds allows us to better assess the impact of credit risk on the empirical estimation of municipal yields and implied tax rates.

We first estimate the pre-tax discount factor P_t at time t, which is equivalent to the inverse of one plus the spot yield of a pure discount bond to the power of t. The pre-tax discount factors or spot yields can be obtained from the pure discount Treasury bonds, but there are a very limited number of pure discount bonds in the database. To obtain a sequence of pre-tax discount factors, we use a procedure to strip the coupons from Treasury bonds with different maturities. We then express the after-tax discount factor d_t as a function of the pre-tax discount factor P_t and the marginal tax rate using Equation (9).

By substituting the definition of the after-tax discount factor d_t into Equation (16), we then directly estimate the marginal income tax rate from the model using municipal and Treasury bond yield data. The procedure

EXHIBIT 6
Relative Yields of Prime Municipals to Treasuries

Period	Maturity	Mean	Std. Dev.	Minimum	Maximum
1973–2001	1	0.61	0.10	0.39	0.92
	2	0.65	0.12	0.49	1.03
	5	0.67	0.08	0.52	0.85
	10	0.73	0.08	0.56	0.90
	20	0.78	0.07	0.62	0.91
	30	0.81	0.07	0.65	0.96
1977–1980	1	0.50	0.03	0.45	0.59
	2	0.53	0.02	0.49	0.58
	5	0.56	0.02	0.52	0.62
	10	0.59	0.02	0.56	0.65
	20	0.67	0.03	0.62	0.74
	30	0.70	0.03	0.65	0.78
1982–1985	1	0.57	0.03	0.49	0.64
	2	0.58	0.03	0.51	0.66
	5	0.65	0.04	0.59	0.73
	10	0.73	0.03	0.67	0.79
	20	0.81	0.04	0.74	0.89
	30	0.82	0.04	0.75	0.94
1987–1992	1	0.73	0.06	0.59	0.92
	2	0.75	0.09	0.60	1.03
	5	0.74	0.04	0.67	0.85
	10	0.78	0.03	0.72	0.85
	20	0.83	0.03	0.78	0.89
	30	0.84	0.03	0.79	0.95
1994–2001	1	0.58	0.09	0.39	0.78
	2	0.72	0.05	0.62	0.87
	5	0.74	0.04	0.68	0.82
	10	0.79	0.04	0.69	0.89
	20	0.84	0.04	0.75	0.91
	30	0.86	0.04	0.76	0.95

of stripping coupons and estimating the discount factors is explained in the appendix.

We estimate the implicit marginal tax rates from the non-linear regression model in Equation (4) using the Gauss-Newton method for bonds of different maturities and ratings. Exhibit 7 reports the estimates of the implicit tax rates (in percentage) for the entire sample period and for each subperiod. All marginal tax rate estimates are significant at the 1% level.

The estimates of the marginal income tax rates generally conform to the pattern of the statutory tax rates. Statutory income tax rates declined after 1981, reached their lowest level in the period 1987 to 1992, and then rebounded after 1993.[13]

Exhibit 7 shows that estimated marginal income tax rates for the period 1987-1992 are much lower than for the period 1982-1985, which in turn are lower than the estimates for the period 1977-1980. Consistent with the increase in the statutory tax rates in 1993, the estimate of the marginal income tax rate for one-year bonds in the period 1994-2001 is higher than in the period 1987-1992. These estimates are reasonably close to the statutory tax rates that apply to high-income investors, especially for short-maturity bonds.

Estimates of the implicit marginal income tax rates are much lower still for long-maturity bonds. The estimated marginal tax rates generally decline with maturity for the entire sample period as well as for each subperiod. Although the estimated tax rates for short-maturity bonds appear to be high enough to represent a plausible marginal

EXHIBIT 7

Marginal Tax Rates Inferred from Green [1993]

Panel A. Prime Municipals

Maturity		1973-2001	1977-1980	1982-1985	1987-1992	1994-2001
1	Tax Rate	42.57***	50.29***	43.49***	28.80***	33.18***
	Std. Err	(0.47)	(0.41)	(0.44)	(0.66)	(0.91)
2	Tax Rate	40.33***	48.85***	43.54***	28.93***	31.01***
	Std. Err	(0.55)	(0.36)	(0.43)	(0.70)	(0.81)
5	Tax Rate	38.60***	47.93***	40.21***	28.53***	28.75***
	Std. Err	(0.48)	(0.35)	(0.53)	(0.44)	(0.65)
10	Tax Rate	34.91***	48.38***	34.56***	27.71***	28.58***
	Std. Err	(0.47)	(0.31)	(0.54)	(0.43)	(0.51)
20	Tax Rate	32.05***	43.66***	28.45***	27.04***	28.51***
	Std. Err	(0.49)	(0.53)	(0.76)	(0.40)	(1.00)
30	Tax Rate	30.07***	42.41***	30.72***	26.72***	27.80***
	Std. Err	(0.47)	(0.61)	(0.65)	(0.58)	(1.56)
Average R^2		98.96	99.83	99.74	99.51	99.83

Panel B. Good Municipals

Maturity		1973-2001	1977-1980	1982-1985	1987-1992	1994-2001
1	Tax Rate	40.81***	49.16***	41.37***	27.16***	32.78***
	Std. Err	(0.48)	(0.42)	(0.46)	(0.64)	(0.87)
2	Tax Rate	42.15***	47.65***	41.62***	31.65***	—
	Std. Err	(0.42)	(0.36)	(0.44)	(0.73)	—
5	Tax Rate	39.82***	46.65***	38.29***	29.50***	—
	Std. Err	(0.48)	(0.35)	(0.55)	(0.77)	—
10	Tax Rate	33.14***	47.04***	32.30***	27.30***	29.14***
	Std. Err	(0.49)	(0.33)	(0.57)	(0.43)	(0.51)
20	Tax Rate	29.54***	42.10***	25.87***	25.02***	28.45***
	Std. Err	(0.50)	(0.58)	(0.78)	(0.46)	(0.95)
30	Tax Rate	27.59***	40.71***	24.72***	23.98***	27.02***
	Std. Err	(0.49)	(0.67)	(1.01)	(0.61)	(0.74)
Average R^2		98.74	99.82	99.75	99.87	99.48

EXHIBIT 7 (continued)

Panel C. Medium Municipals

Maturity		1973-2001	1977-1980	1982-1985	1987-1992	1994-2001
1	Tax	37.15***	46.96***	38.24***	25.38***	32.84***
	Std. Err	(0.54)	(0.50)	(0.53)	(0.61)	(0.92)
2	Tax	38.95***	44.95***	38.42***	29.05***	—
	Std. Err	(0.45)	(0.46)	(0.53)	(0.65)	—
5	Tax	36.08***	44.01***	34.03***	27.13***	—
	Std. Err	(0.50)	(0.35)	(0.61)	(0.80)	—
10	Tax	31.57***	43.47***	27.43***	23.48***	—
	Std. Err	(0.66)	(0.44)	(0.73)	(0.81)	—
20	Tax	24.91***	37.92***	26.21***	21.91***	26.71***
	Std. Err	(0.54)	(0.84)	(1.05)	(0.57)	(0.95)
30	Tax	22.34***	36.17***	24.77***	19.86***	24.66***
	Std. Err	(0.55)	(0.84)	(1.28)	(0.77)	(0.89)
Average R^2		98.88	99.69	99.67	99.29	99.26

,, and *** represent significance at the 10%, 5%, and 1% level.*

rate for investors in municipals, the estimated tax rates for long-maturity bonds would seem too low by any reasonable standard.

Furthermore, the downward bias in the estimate of implicit tax rates for long-maturity bonds increases with the default risk of municipal bonds. If we compare the estimates of tax rates for the entire sample in the first column for each rating group, we see a clear pattern of declining implicit marginal tax rates as bond quality worsens. The downward bias in the estimates of tax rates is most serious for low-rated long-maturity municipal bonds. For example, the estimated marginal statutory tax rate is only 19.86% for medium municipals at 30-year maturity in the 1987-1992 period. These results suggest that ignoring the default risk may cause a serious underestimation of implicit marginal tax rates.

Estimation of the Generalized Model with Default

Using the generalized model with default in Equation (13), we estimate the marginal tax rates and default probabilities of municipal bonds with different ratings and maturities. We assume that the default probability is constant for each bond at a given maturity, and set the recovery rate equal to 0.4 for all municipal bonds.[14]

Exhibit 8 reports the results of the non-linear regression estimation. After we account for the default risk, the estimated tax rates become much more stable for bonds over different maturities. All estimated tax rates are statistically significant. Compared to the estimates in Exhibit 7 that ignore default risk, the estimates of marginal tax rates are higher for each maturity class and more so for longer-term bonds. The increase in the estimated tax rates as a result of incorporating default risk is greater for lower-quality bonds.

These results are generally consistent with the findings in the numerical simulations. Results support the contention that the downward bias in the estimation of tax rates due to omission of default risk is positively related to maturity and negatively related to bond quality. The estimated tax rates now become much more reasonable for long-maturity and low-quality bonds.

The estimated tax rates are generally in line with the changes in statutory taxes. The lowest tax rate estimates are obtained for the period 1987-1992. The implicit tax rates in the period 1982-1985 are lower than in the period 1977-1980 and higher than in the period 1987-1992. Not surprisingly, the implicit tax rates are higher in the period 1994-2001 than in the period 1987-1992.

Municipal bonds of different ratings exhibit different probabilities of default. As expected, the estimates of

EXHIBIT 8

Estimates of Marginal Tax Rates and Default Probabilities from Generalized Model

Panel A. Prime Municipals

Maturity		1973-2001		1977-1980		1982-1985		1987-1992		1994-2001	
		Tax Rate	Default Prob	Tax Rate	Default Prob	Tax Rate	Default Prob	Tax Rate	Default Prob	Tax Rate	Default Prob
1	Value	50.50***	0.371***	52.58***	0.139*	49.66***	0.395***	31.61***	0.112***	39.91***	0.133***
	Std. Err	(0.37)	(0.054)	(1.45)	(0.084)	(2.36)	(0.149)	(0.55)	(0.023)	(0.63)	(0.024)
2	Value	51.39***	0.380***	52.42***	0.224***	50.38***	0.527***	33.66***	0.110***	39.41***	0.119***
	Std. Err	(0.34)	(0.021)	(0.36)	(0.023)	(2.69)	(0.206)	(0.62)	(0.032)	(0.57)	(0.023)
5	Value	51.23***	0.394***	52.41***	0.323***	48.55***	0.718**	33.57***	0.114***	39.96***	0.139***
	Std. Err	(0.35)	(0.024)	(1.60)	(0.114)	(4.23)	(0.364)	(0.53)	(0.031)	(0.62)	(0.030)
10	Value	50.92***	0.437***	52.33***	0.307***	48.27***	0.972***	33.66***	0.129	40.42***	0.259***
	Std. Err	(0.37)	(0.027)	(1.69)	(0.127)	(0.49)	(0.043)	(5.46)	(0.339)	(0.42)	(0.022)
20	Value	50.84***	0.505***	50.94***	0.423***	48.10***	1.114***	32.84***	0.460*	40.08***	0.298***
	Std. Err	(0.47)	(0.029)	(0.54)	(0.036)	(0.71)	(0.052)	(3.90)	(0.247)	(0.74)	(0.043)
30	Value	50.32***	0.714***	49.71***	0.450***	48.08***	1.311***	31.55***	0.724***	39.94***	0.313***
	Std. Err	(0.55)	(0.031)	(0.62)	(0.038)	(0.91)	(0.056)	(4.41)	(0.283)	(1.00)	(0.053)
Average R^2		98.86		99.85		99.79		99.65		99.58	

Panel B. Good Municipals

Maturity		1973-2001		1977-1980		1982-1985		1987-1992		1994-2001	
		Tax Rate	Default Prob	Tax Rate	Default	Tax Rate	Default	Tax Rate	Default	Tax Rate	Default Prob
1	Value	51.51***	0.465***	52.64***	0.211***	48.28***	0.442***	31.40***	0.140***	41.37***	0.144***
	Std. Err	(1.07)	(0.053)	(1.42)	(0.082)	(2.50)	(0.157)	(0.90)	(0.039)	(0.87)	(0.029)
2	Value	50.69***	0.575***	51.07***	0.228***	48.53***	0.530***	32.79***	0.104***	—	—
	Std. Err	(1.44)	(0.094)	(0.35)	(0.023)	(0.42)	(0.032)	(0.73)	(0.038)		
5	Value	47.36***	0.557***	51.61***	0.357***	48.35***	0.860**	31.18***	0.129***	—	—
	Std. Err	(1.87)	(0.135)	(1.59)	(0.112)	(4.31)	(0.370)	(0.76)	(0.045)		
10	Value	47.89***	0.679***	51.24***	0.314***	48.03***	1.220***	31.88***	0.203***	41.71***	0.387***
	Std. Err	(1.77)	(0.030)	(0.31)	(0.023)	(0.52)	(0.040)	(0.79)	(0.050)	(0.41)	(0.022)
20	Value	48.55***	0.705***	50.22***	0.456***	47.82***	1.380***	31.56***	0.577***	40.94***	0.537***
	Std. Err	(0.56)	(0.036)	(0.58)	(0.039)	(0.73)	(0.052)	(0.79)	(0.052)	(0.70)	(0.043)
30	Value	48.82***	0.805***	49.22***	0.510***	47.97***	1.471***	31.81***	0.764***	40.55***	0.550***
	Std. Err	(0.61)	(0.034)	(0.68)	(0.042)	(0.96)	(0.058)	(1.11)	(0.079)	(1.01)	(0.046)
Average R^2		98.92		99.78		99.70		99.85		99.64	

EXHIBIT 8 (continued)

Panel C. Medium Municipals

Maturity		1973-2001		1977-1980		1982-1985		1987-1992		1994-2001	
		Tax Rate	Default Prob	Tax Rate	Default	Tax Rate	Default	Tax Rate	Default	Tax Rate	Default Prob
1	Value	52.46***	0.687***	50.97***	0.243***	46.51***	0.528***	30.83***	0.313***	40.27***	0.435***
	Std. Err	(1.21)	(0.020)	(1.71)	(0.099)	(2.83)	(0.178)	(0.83)	(0.035)	(0.86)	(0.028)
2	Value	49.57***	0.712***	50.21***	0.351***	46.29***	0.657***	30.96***	0.205***	—	—
	Std. Err	(1.49)	(0.096)	(1.73)	(0.112)	(0.48)	(0.036)	(0.65)	(0.034)	—	—
5	Value	48.39***	0.782***	49.35***	0.375***	45.73***	0.918***	31.58***	0.265***	—	—
	Std. Err	(0.47)	(0.034)	(0.33)	(0.023)	(0.58)	(0.049)	(10.00)	(0.047)	—	—
10	Value	49.50***	1.071***	48.16***	0.333***	45.66***	1.340***	31.23***	0.477***	—	—
	Std. Err	(0.64)	(0.046)	(0.44)	(0.031)	(0.67)	(0.055)	(0.73)	(0.045)	—	—
20	Value	49.59***	1.100***	46.64***	0.511***	45.71***	1.650***	31.41***	0.770***	42.38***	0.923***
	Std. Err	(0.55)	(0.033)	(0.76)	(0.049)	(0.95)	(0.066)	(0.51)	(0.031)	(0.65)	(0.036)
30	Value	48.45***	1.150***	46.62***	0.516***	45.53***	1.651***	31.36***	0.988***	40.16***	1.060***
	Std. Err	(0.61)	(0.033)	(0.88)	(0.052)	(1.17)	(0.068)	(0.86)	(0.052)	(1.18)	(0.054)
Average R^2		98.72		99.71		99.68		99.65		99.38	

*Standard errors are reported in parentheses. *, **, and *** represent significance at the 10%, 5%, and 1% level.*

default probability are lower for higher-grade and shorter-maturity bonds. Almost all default probability estimates are significantly different from zero.

Ignoring default risk results in a biased estimation of implicit tax rates. When default risk is restricted to zero, the marginal tax rate is forced lower to make after-tax Treasury yields higher, to be consistent with yields of risky municipals. The higher the default risk of municipals, the higher the tax-exempt yields, and the lower the estimated marginal tax rate. To the extent that default risk increases with maturity for investment-grade bonds, the yields of municipal bonds increase with maturity, and the downward bias in the tax rate estimates is more severe for long-maturity bonds when default risk is ignored.

Overall, we find that the generalized model with default offers a much more satisfactory explanation for the anomaly in the relative yield curve of Treasury and municipal yield curves. The goodness of fit is very high for each bond rating and maturity group. After incorporating the effect of default risk, the estimated tax rates become coherent between long- and short-maturity bonds.

IV. CONCLUSIONS

Our generalized model with default risk provides more accurate estimates of implicit tax rates and better explains the relative yield spreads between Treasury and municipal bonds. Long municipal yields are higher than the equivalent after-tax yields of Treasury bonds partly because the probability of default is higher for long-maturity municipals. Failure to account for default risk results in a severe underestimation of the implicit tax rates for long-term municipal bonds. This downward estimation bias tends to be greater for lower-quality municipal bonds.

The results show that credit risk affects the relationship between yields of risky tax-exempt and risk-free taxable bonds. This effect is stronger for lower-quality and longer-maturity municipal bonds. Green [1993] finds that dealer arbitrage activities in the market for taxable bonds substantially reduce the impact of taxes on long-maturity taxable bond prices, and therefore reduce yields of taxable bonds compared to tax-exempt bonds. We demonstrate further that default risk increases the yields of tax-exempt bonds relative to taxable bonds.

Long-maturity tax-exempt bonds are riskier than short-maturity tax-exempt bonds. The higher default risk increases the yields of long-maturity tax-exempt bonds over default-free long-maturity taxable bonds. Our results show that incorporating default risk into the municipal term structure model captures very well this maturity-related risk effect on the relative yields of municipal and Treasury bonds.

Default risk helps explain a considerable portion of the relative yields of tax-exempt and taxable bonds. The estimates of implicit tax rates become quite comparable for short- and long-term bonds. Thus, the anomalous behavior of municipal bond term premiums can be attributed to the interactive effect of taxes and default.

Our findings strongly suggest we should account for credit risk in analyzing the relative yields of tax-exempt and taxable bonds, especially in today's environment, with erosion in municipal bond quality as a result of large deficits in state and local governments.

APPENDIX

Stripping Coupons and Estimation of Discount Factors

Denote T_i as the i-th coupon date, $i = 1, 2, \ldots, T$, where T is the maturity date and t_0 represents the current time, and C is the coupon rate of a Treasury bond. To strip out the zeros, we first estimate the spot rate r_1 of Treasury bonds for the first period (with maturity equal to or less than six months) by the one-period model:

$$V_1 = (1 + C)\, e^{-\frac{r_1}{2}t_1} \tag{A-1}$$

where t_1 is the length of time to maturity, $t_1 = (T - t_0)/180$; V_1 is the current price of the bond adjusted by accrued interest; and C is the coupon payment.

We transform this non-linear function into a log-linear relationship:

$$\ln(V_1) = \ln(1 + C) - (r_1/2)t_1 \tag{A-2}$$

Using (A-2), we estimate the spot rate r_1 for the first (six-month) period by regression. Given r_1, the spot rate for the second (six-month) period r_2 can be determined from a regression for bonds with maturity between six months and one year:

$$V_2 = Ce^{-\frac{r_1}{2}t_1} + (1 + C)e^{-\frac{r_2}{2}t_2} \tag{A-3}$$

where t_1 is the length of time to the first coupon, $t_1 = (T - t_0)/180$; and t_2 is the length of time to maturity, where $t_2 = (T - t_0)/180$.

Similarly, we can transform this non-linear regression into a log-linear regression:

$$\ln(V_2 - Ce^{-\frac{r_1}{2}t_1}) = \ln(1 + C) - \frac{r_2}{2}t_2 \tag{A-4}$$

Using the same procedure, and the formula:

$$V_2 = \sum_{j=1}^{i-1} Ce^{-\frac{r_j}{2}t_j} + (1 + C)e^{-\frac{r_i}{2}t_i} \tag{A-5}$$

all spot rates r_t for $t = 1, 2, \ldots, 60$ can be estimated sequentially for Treasury bonds.

The after-tax discount factors for municipal bonds for each period can be written as:

$$d_t(\tau) = \frac{P_t}{1 - \tau(1 - P_t)} \tag{A-6}$$

where the after-tax discount factor d_t is expressed as a function of the marginal statutory tax rate τ and the pre-tax discount factor P_t, which is the inverse of one plus the spot rate r_t to the power of t.

In empirical investigation, we substitute Equation (A-6) for the after-tax discount factor d_t into Equations (4) and (13). We then directly estimate the marginal statutory tax rate using the yield data of municipal (M_t) and Treasury (C_t) bonds and the pre-tax discount factors (P_t) for models with and with default risk.

ENDNOTES

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[1]See the report in the *Statistical Abstract of the United States*, U.S. Census Bureau, 2002.

[2]See Salomon Smith Barney Bond *Market Monthly*, March 13, 2003.

[3]Municipal assets cannot be seized as easily as corporate assets in bankruptcy proceedings because they may be either physically difficult to seize or legally protected. In addition, the incentive problem or moral hazard is perceived to be more severe for municipalities.

[4]Chalmers [1998] finds indirect evidence that default risk may not explain the municipal bond puzzle, but he does not examine the effect of credit risk on the relative yields of municipal bonds with different maturities and ratings.

[5]As in Green [1993], the tax-timing options involving realization of capital losses before maturity are ignored here.

[6]λ is default intensity as time interval goes to zero.

[7]The non-rated category in the original, which primarily includes bonds that are bought back or issued by companies that merge, has been allocated to the other rating classes so that

each row sums to one.

[8]The procedure of computing the evolution of default probability from the transition matrix is similar to that in Jarrow and Turnbull [1995] and Elton et al. [2001].

[9]Another potential source of estimation errors comes from the assumption of the recovery rate. We also conduct simulation analysis on the sensitivity of implicit tax rate estimates to the assumed recovery rate. We find that the bias caused by the error in the recovery rate is relatively slight. For example, for a ± 0.1 error in the recovery rate, the error in the tax rate is about ± 0.026 for A-rated bonds with 30-year maturity.

[10]The remaining data are missing in Salomon Brothers' *Analytical Records of Yield and Yield Spread*.

[11]We also eliminate bonds with an odd frequency of coupon payments (the normal frequency of coupon payments is twice per year).

[12]Term premiums are calculated by subtracting from the yield for a given month and maturity the corresponding yield for a one-year maturity bond.

[13]Following Ronald Reagan's election, Congress enacted the Economic Recovery Tax Act of 1981 (ERTA). In this tax reform, the top individual tax rates were scheduled to fall 23% over the next three years, and tax rates for both individuals and businesses were substantially reduced. The 1986 Tax Reform Act resulted in another significant change in the income tax code. It cut individual income tax rates from 50% to 28%, while corporate tax rates fell from 46% to 34%. In 1993, President Clinton signed into effect the Revenue Reconciliation Act, which raised income taxes for individuals with the top rate from 31.0% to 39.6%.

[14]We need to set a value for the recovery rate because it cannot be identified. A similar value of δ is used in Yawitz, Maloney, and Ederington [1985]; Fons [1987]; Wu and Yu [1996]; and Duffee [1999]. This value is also very close to that reported by Elton et al. [2001] for AA and A bonds.

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