Interest Rates—Normal or Lognormal?

JEFFREY HO AND LAURIE GOODMAN

JEFFREY HO
is an executive director and a member of the Mortgage Strategy Group at UBS in New York City.
jeffrey.ho@ubs.com

LAURIE GOODMAN
is a managing director and head of the Mortgage Strategy Group at UBS in New York City.
laurie.goodman@ubs.com

The historically low interest rates experienced in 2003 have exposed some serious problems in the typical interest rate models, especially as applied to evaluating mortgage securities. This is serious, as many investors rely heavily on option-adjusted spread models to make asset allocation decisions between mortgages and other instruments, as well as relative value decisions within the mortgage market.

In this article we discuss the distortions in OAS modeling. We show in the first part that it is wrong to assume, as most models do, an interest rate process based on a lognormal interest rate distribution (volatility measured in percentages is constant). Basically, interest rates do not behave as if they follow a lognormal distribution. In fact, they behave with a non-linear rate directionality. Incorporating this directionality allows for a very robust description of the term structure of volatility, as well as for skewness to occur naturally.

We then apply those results, showing that using a typical lognormal distribution means that at low rate levels such as those prevailing in mid-2003, rates will be too high along high rate paths.

The practical implications of this analysis are that lognormal models result in fixed-rate durations that are too long and caps that are valued way too high, making floaters appear to be misleadingly rich in OAS models.

In the second part of the article, we model mortgage current-coupon rates. We show that the correlation between interest rates and volatility and mortgage spreads means that the negative convexity of mortgage instruments, as measured by OAS models, is overstated. Beyond the OAS modeling of mortgages, we discuss incorporation of the directionality of mortgage spreads into our favorite regression-based relative value models to make them even more powerful.

Our conclusion: Investors need to think long and hard about how they use OAS numbers in portfolio management.

I. OAS MODELING

Typically, OAS numbers are calculated using a Monte Carlo simulation that captures a bond's performance over a range of scenarios. In Monte Carlo simulation, a large number of interest rate paths are chosen from an interest rate distribution centered on forward rates. Cash flows for the mortgage securities are determined using a prepayment model, which gives prepayment rates at each node along each path. The OAS is the spread at which the average present value across all paths is equal to the market price of the security.

There are two calibration issues involved here:

- The correct interest rate distribution—Most models assume lognormally distributed rates (i.e., no negative rates), although a few normal models are in use.

The interest rate lattice (distribution of rates at each time) is calibrated to return today's observed yield curve and prices on a few at-the-money caps or swaptions. Even though more serious models make a somewhat rough attempt to account for the observed term structure of at-the-money implied volatilities, there generally is no attempt to account for volatility skews (i.e., changes in volatility for strikes away from the money).

- The current-coupon mortgage rate, which drives the prepayment model—To value mortgage securities on the lattice, it is essential to produce mortgage current-coupon rates for feeding into a prepayment model. Mortgage rates along each Monte Carlo path are typically calculated as a fixed spread over a five- or ten-year maturity swap or Treasury bullet priced using the same Monte Carlo path.

We consider each of these points in turn. First we correct the interest rate distribution, and then we deal with the current-coupon mortgage rate.

II. VOLATILITY CALIBRATION

A few pictures will clearly illustrate the problem with the way volatility is currently handled. In Exhibit 1, we show weekly observations of five-year cap volatility, measured in percentage terms, versus five-year swap yields over January 8, 1988–February 28, 2003.

Note that for rate levels below 5%, volatility, measured in percent, appears to be directional. That is, low rate levels appear to be associated with high volatility levels. This flies in the face of lognormal models, which implicitly assume that volatility, measured in percent, is constant across all rate levels.

In Exhibit 2, we show weekly observations of five-year cap volatility, measured in basis points, versus five-year swap yields for the same period. Above an 8.5% yield, volatility, measured in basis points, appears to be directional with rates, but the correlation is positive; that is, high rate levels are associated with high volatility levels. Although the observed directionality contradicts the assumption of normal rate models (i.e., volatility, measured in basis points, is constant across all rate levels), those who use normal models feel that their models are superior in the current low rate environment.

The problem of directional volatility actually impacts the modeling of mortgage securities in both lognormal and normal models in all rate environments. This is because an OAS calculation simulates a wide range of rate levels, no matter where current rates are, and each approach will get it wrong for both the low- and the high-rate scenarios.

III. GOING INTERNATIONAL—JAPANESE INTEREST RATE EXPERIENCE

If we look at Japan's experience with volatility and interest rates over the last six years, we can glimpse how the problem of normal versus lognormal assumptions worsens as rates approach zero. The solid triangles in Exhibit 3 show weekly five-year yen cap volatility, measured in percent, versus five-year yen swap yields for February 23, 1996–February 28, 2003. The open squares plot five-year USD caps versus five-year USD swap yields since 1988.

The USD low in rates was 3.00%, while the yen high in rates was 2.99%. There appears to be decent continuity as we move from the higher rate regime of the U.S. (across a yield of 3%) to the lower rate regime of Japan. Given this, we include as the solid line in Exhibit 3 a non-linear best fit function that models volatility, measured in percent, as a function of yield.

This model is:

$$\% \text{ Vol} = 124.75 + 12.26 \times \text{Yield} - 72.93 \times \sqrt{(\text{Yield})} \tag{1}$$

where % Vol is volatility measured in percent.

Clearly, as Exhibit 3 illustrates, volatility measured in percent is directional. As rates decline, volatility measured in percent rises, and vice versa. Thus, modeling the distribution as a constant percent volatility understates the true volatility at low rates, and overstates it at high rates.

In Exhibit 4, we replot the data in Exhibit 3, using volatility measured in basis points. Given that volatility measured in basis points has a more complex shape than volatility measured in percent, we favor modeling the directionality of volatility as measured in percent.

What's most interesting about our extrapolated view of volatility as rates fall below 3% is that a constant basis point volatility assumption becomes as misleading as percentage point volatility—but it goes astray in the completely opposite direction. That is, given a five-year swap rate right around 3%, a normal OAS model (constant basis point volatility) will be modeling the lower half of its interest rate lattice with too much volatility.

EXHIBIT 1
Directionality—% Volatility

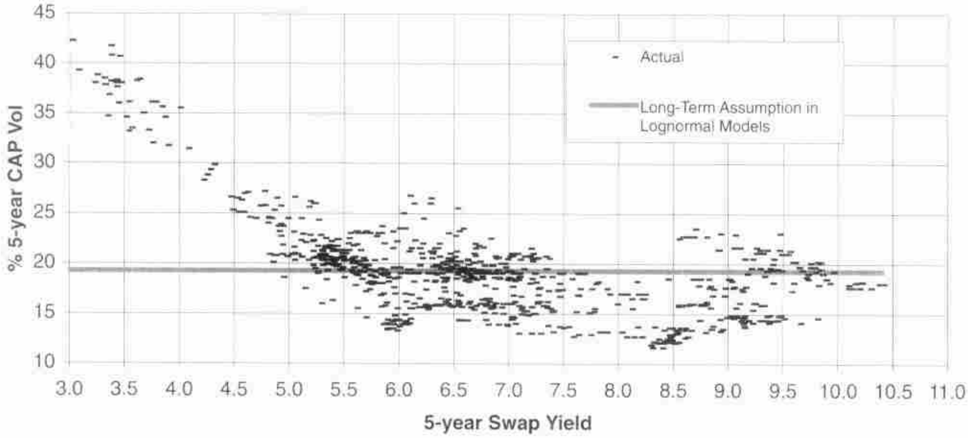


EXHIBIT 2
Directionality—Basis Point Volatility

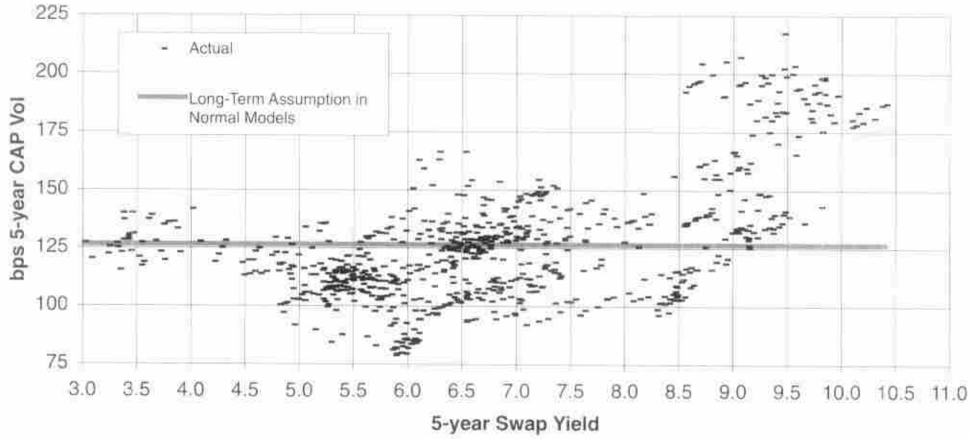


EXHIBIT 3
Combined Directionality—Yen + USD Cap Volatilities

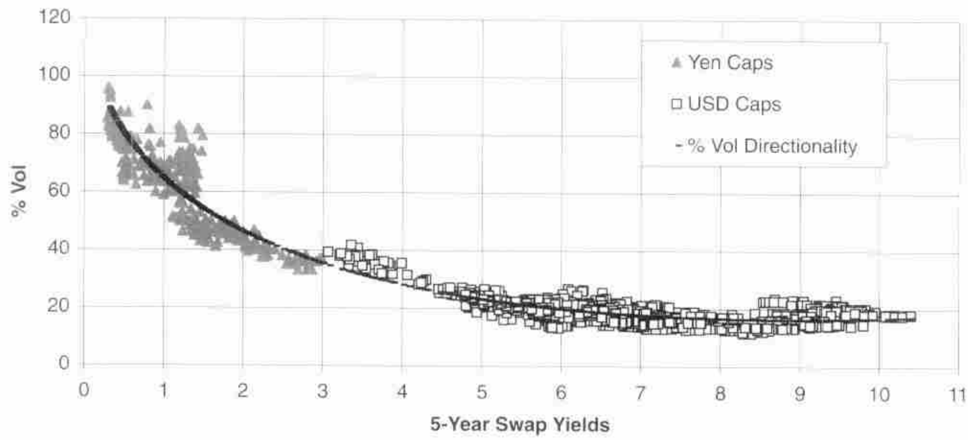
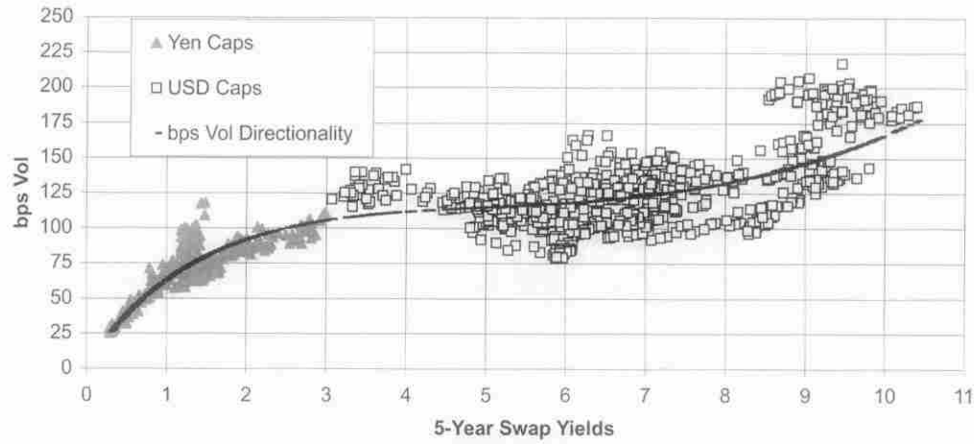


EXHIBIT 4
Combined Basis Point Directionality



IV. ADVANTAGES OF DIRECTIONAL VOLATILITY MODEL

Let's say you buy into the non-linear directionality of volatility as described in Equation (1) for five-year caps, and decide to build your interest rate lattice accordingly. If we assume that short rate volatility, as measured in percent, has the same sort of directionality, the resulting rate model predicts a lot of additional features that are actually observed in the term structure of real-world implied volatilities.

Lower Volatility on Longer Instruments

In an upward-sloping yield curve, longer-dated options will have lower volatility than shorter-dated ones. This is because an upward-sloping yield curve comes with higher forward rates. If volatility declines as rates go up, it follows that longer-dated options (keyed to higher forward rates) will have a lower volatility. This is certainly the case in the real world.

The first panel of Exhibit 5 shows implied volatility, measured in percent, for caps and swaptions as of March 7, 2003. The first column shows cap volatilities. Thus, the cap volatility for the three-year period is 54.2%; for the ten-year period it is 29.10%. Longer-dated options have lower cap volatilities, as measured in percent.

The remaining columns refer to the back end of the swaption. Thus, a 3×7 swaption (a three-year option to exercise into a seven-year instrument at option expiration) has an implied volatility of 25.1%. A 5×10 swaption (a five-year option to exercise into a ten-year underlying at expiration) has a volatility of 18.85%. Note

EXHIBIT 5
Cap/Swaption Volatility Matrix as of 3/7/03

		Swaptions (backend tenor)						
Expiration	Caps	1.00	2.00	3.00	4.00	5.00	7.00	10.00
Actual Implied Vols								
0.08		48.00	53.50	45.50	42.50	39.20	32.80	26.50
0.25		50.00	57.50	49.00	45.00	42.00	34.80	29.00
0.50		60.50	56.25	48.00	44.20	41.00	34.50	29.50
1.00	48.20	61.25	51.40	44.90	40.70	37.80	33.10	28.40
2.00	60.50	46.00	40.10	35.90	33.40	31.50	28.20	25.20
3.00	54.20	35.60	32.90	30.40	28.60	27.30	25.10	22.70
4.00	47.50	30.20	28.30	26.80	25.50	24.60	22.50	20.60
5.00	42.25	26.10	25.20	24.30	23.00	22.25	20.65	18.85
7.00	35.50							
10.00	29.10							
Average Underlying Forward 3-mo LIBOR								
0.08		1.26	1.75	2.27	2.70	3.07	3.63	4.21
0.25		1.30	1.82	2.34	2.77	3.13	3.68	4.25
0.50		1.39	1.94	2.45	2.87	3.22	3.75	4.31
1.00	1.24	1.64	2.20	2.68	3.08	3.40	3.90	4.44
2.00	1.72	2.20	2.70	3.12	3.46	3.74	4.17	4.66
3.00	2.23	2.68	3.12	3.48	3.77	4.02	4.40	4.82
4.00	2.67	3.08	3.46	3.77	4.04	4.25	4.60	4.92
5.00	3.04	3.40	3.74	4.02	4.25	4.45	4.77	4.98
7.00	3.61							
10.00	4.19							
Model Vol								
0.08		58.37	49.70	42.72	37.98	34.62	30.30	26.71
0.25		57.54	48.65	41.90	37.35	34.13	29.97	26.49
0.50		55.78	46.97	40.65	36.39	33.39	29.48	26.16
1.00	58.67	51.45	43.59	38.22	34.55	31.98	28.54	25.53
2.00	50.19	43.59	38.04	34.21	31.54	29.58	26.93	24.47
3.00	43.12	38.22	34.21	31.38	29.34	27.81	25.70	23.74
4.00	38.29	34.55	31.54	29.34	27.72	26.50	24.72	23.30
5.00	34.86	31.98	29.58	27.81	26.50	25.48	23.95	23.07
7.00	30.46							
10.00	26.81							

that volatility is lower, the longer the time to option expiration, and the longer the life of the underlying.

The second panel of Exhibit 5 shows the average underlying forward three-month LIBOR for each option, while the last panel displays the volatility that would be expected according to Equation (1). The average underlying forward three-month LIBOR is our shortcut way to determine what summary yield level to plug into Equation (1).

For example, the underlying summary three-month LIBOR behind a five-year cap is the average of forward rates from time zero to Year 5 (3.04%). For swaptions, we first average forward rates over the length of the tenor. Thus, for a 3 × 7 swaption, we calculate average forward LIBOR for seven-year periods, beginning now, performing the calculation every three months, and ending three years from now. Thus, we have 12 sets of seven-year averages, which in turn are averaged to give a 4.4% average forward LIBOR.

The last section of Exhibit 5 shows volatilities that would be derived from our simple volatility model. They are not a perfect fit, but the general patterns hold. In particular, in an upward-sloping yield curve, options with longer to expiration will have lower volatilities than those with shorter times to expiration. Similarly, options of longer-dated tenors will have lower volatilities than shorter-dated tenors. This is because a longer tenor blends lower front-end short rates with higher forward short rates.

In current OAS models, we build in the fact that longer expirations and longer tenors have lower volatilities than shorter counterparts, by incorporating a term structure of interest rate volatility. Using non-linear directional volatility, we get the correct qualitative conclusions naturally, without using a term structure of volatility. Obviously incorporating one would enable a better calibration.

Skewness

Another advantage of incorporating non-linear directional volatility is that skewness naturally occurs. As we move to higher rate strikes, we naturally adjust our volatilities, measured in percent, down. And as we move to lower rates, we naturally adjust our volatilities, measured in percent, up.

Exhibit 6 shows implied volatilities across various yield strikes for Eurodollar futures as of March 7, 2003. The average column is the average of the [call + put] implied yield volatilities. Thus, for a 3.25% yield strike, average volatility is 56.07%, which is the average of [the 54.6% call volatility + the 59.26% and 54.35% put volatilities].

Notice first that because we are dealing with short-dated expiration, short-tenor options in a historically low USD yield environment, our non-linear directional volatility approach would have braced us for extremely high volatilities, which are greater than any observed in Exhibit 5. Second, using levels as of March 7, 2003, higher volatilities are coincident with lower rate strikes.

The model volatility in Exhibit 6 is calculated by plugging the yield strikes into Equation (1). Notice that the model projects the results we observe in this environment—that higher volatilities will be coincident with lower strike yields.

Exhibit 6

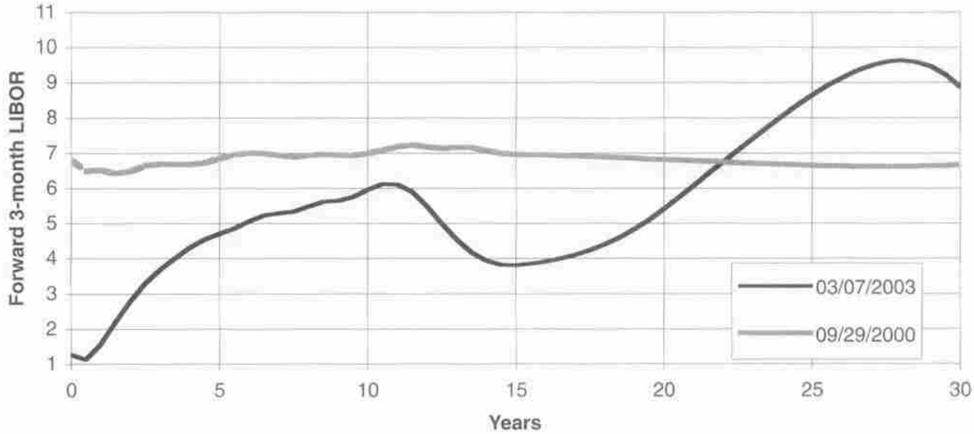
Eurodollar Futures Volatility Skew

Yield			Call Implied Yield Volatilities								Put Implied Yield Volatilities							
Strike	Model	Avg	03/03	06/03	09/03	12/03	03/04	06/04	09/04	12/04	03/03	06/03	09/03	12/03	03/04	06/04	09/04	12/04
3.75	29.50	52.61																52.61
3.50	31.22	53.76																53.76
3.25	33.12	56.07								54.60							59.26	54.35
3.00	35.21	57.71								55.15						60.99	59.62	55.08
2.75	37.52	58.74							59.78	55.95					58.67	62.47	59.68	55.88
2.50	40.09	60.55						63.58	61.57	57.15					59.84	63.45	61.16	57.09
2.25	42.94	59.72					60.59	64.95	62.61	58.54			50.96	52.78	60.53	64.95	62.61	58.69
2.00	46.13	60.41				56.22	61.60	66.39	62.91	60.31			50.73	54.94	61.61	66.22	63.10	60.48
1.75	49.73	58.14		46.85	50.93	57.26	63.28	68.01	64.41	61.69		47.70	50.34	55.91	63.36	67.97		
1.50	53.82	58.69	63.15	41.48	49.69	56.48	66.03	69.18	66.02	64.30	62.76	40.30	49.81	56.92	66.19	69.31		
1.25	58.54	53.44	33.18	47.98	52.76	59.44	66.17	71.52	66.63		33.18	47.78	52.55	56.65				
1.00	64.08	65.80	79.35	58.75	57.66	61.32	69.19	71.84	69.44		79.86	58.92	57.95	59.56				
0.75	70.79	67.72		66.24	62.84	65.75	73.03	77.09						61.38				
0.50	79.31	77.21		85.85	76.83	67.01	79.15											
0.25	91.35	100.50			100.50													

As of 3/7/03.

Exhibit 7

Interest Rate Environment (3/7/03 versus 9/29/00)



More Generally

If we look back for a totally different yield curve environment for, say [a high rate + a flat curve], we find that a non-linear directional volatility model will in many cases prepare us for what the implied volatility surface will look like. Exhibit 7 compares the forward three-month LIBOR of September 29, 2000 (an arbitrary selection, for when rates were high and the curve almost flat), to a low rate, steep curve environment (the close of March 7, 2003), to highlight the implications of a drastic difference in yield environments. Exhibit 8 shows actual implied caps and swaption volatilities from September 29, 2000, along with the model volatilities that Equation (1) would have predicted.

The model works quite well, correctly anticipating that higher yields on September 29, 2000, meant that volatility would be lower on that date than on March 7, 2003 (this can be seen by comparing the results in Exhibit 8 to those in Exhibit 5). Additionally, given an essentially flat yield curve in September 2000, Equation (1) would have anticipated a much flatter volatility term structure on September 29, 2000, than on March 7, 2003.

But our results are even more general. Our yield directional volatility model also generally does a good job across currencies. Exhibit 9 shows two-, five-, and ten-year swap yields, actual cap volatilities, and model cap volatilities in nine currencies. Norway (the highest-rate environment) is predicted to have the lowest cap volatilities, while the U.S., Switzerland, and Japan are predicted to have the highest cap volatilities. This is borne out by the configuration of actual cap volatilities.

Exhibit 8

Cap/Swaption Volatility Matrix as of 9/29/00

Actual Implied Volatilities								
		Swaptions (backend tenor)						
Expiration	Caps	1.00	2.00	3.00	4.00	5.00	7.00	10.00
0.08		9.75	10.90	11.00	11.10	11.20	11.30	11.40
0.25		10.50	11.80	11.90	12.00	12.10	12.20	12.20
0.50		12.25	12.60	12.70	12.80	12.80	12.80	12.80
1.00	9.75	14.00	14.30	14.20	14.10	14.00	13.90	13.75
2.00	13.10	15.75	15.35	15.15	14.90	14.60	14.25	14.00
3.00	14.65	15.85	15.45	15.25	15.00	14.70	14.35	14.10
4.00	15.40	15.75	15.35	15.15	14.85	14.55	14.15	13.70
5.00	15.80	15.60	15.15	14.90	14.55	14.20	13.70	13.25
7.00	15.95							
10.00	15.60							
Average Underlying Forward 3-Mo LIBOR								
0.08		6.59	6.53	6.57	6.60	6.63	6.73	6.79
0.25		6.57	6.52	6.57	6.60	6.63	6.73	6.79
0.50		6.54	6.51	6.57	6.60	6.64	6.73	6.80
1.00	6.61	6.51	6.52	6.58	6.61	6.66	6.74	6.81
2.00	6.54	6.52	6.56	6.61	6.65	6.70	6.77	6.84
3.00	6.57	6.58	6.61	6.65	6.70	6.75	6.80	6.88
4.00	6.60	6.61	6.65	6.70	6.74	6.78	6.83	6.90
5.00	6.63	6.66	6.70	6.75	6.78	6.81	6.86	6.93
7.00	6.73							
10.00	6.79							
Model Vol								
0.08		18.32	18.44	18.36	18.31	18.25	18.07	17.95
0.25		18.37	18.46	18.37	18.31	18.24	18.06	17.95
0.50		18.42	18.48	18.37	18.31	18.23	18.06	17.94
1.00	18.29	18.48	18.46	18.35	18.29	18.20	18.04	17.92
2.00	18.43	18.46	18.38	18.29	18.21	18.11	17.99	17.87
3.00	18.36	18.35	18.29	18.20	18.11	18.03	17.93	17.81
4.00	18.30	18.29	18.21	18.11	18.04	17.97	17.89	17.77
5.00	18.25	18.20	18.11	18.03	17.97	17.92	17.84	17.73
7.00	18.07							
10.00	17.96							

We have used Equation (1) (fitted to five-year cap and five-year swaps data) as our rough model for volatility for all tenors, expirations, and strikes. A model fitted to a wider range of data would obviously do a better job, but we want to show how even an approximate model can predict out-of-sample certain important features of real-world implied interest rate volatilities.

We do not intend in these examples to predict actual observed volatilities with the model, just generally to show our model can predict overall key characteristics of the actual data. Such predictions should be impossible if the assumptions implicit in either normal or lognormal rate modes were true.

V. LOGNORMAL DISTRIBUTION BIASES

If you accept the possibility that volatility is truly directional, you next have to consider how results from your current model are biased. Are your OASs too high or low? Are your durations and convexities too high or low?

There will obviously be a lot of "it depends" answers. We can try to figure it out intuitively, but a better approach would be to calculate actual OAS numbers, given the three approaches in question (lognormal, normal, and our directional volatility approach).

We will work with simple binomial lattices with ten annual time steps, calibrated to return swap yields as of March 10, 2003, and the observed price of a ten-year at-the-money cap. In the lognormal version of the lattice, volatility (measured in percent) will be the same throughout. In the normal version, volatility (measured in basis points) will be constant. In the directional volatility approach, we calibrate the model using the volatility determined by [Equation (1) + a constant term]. We essentially modify the Black-Derman-Toy [1990] approach to implement directional volatility.

Exhibit 10 shows the resulting lattices for the three different approaches. The appendix shows the calibration of the three lattices. There are a few interesting things to note here.

The lognormal lattice has the greatest upward dispersion in rates, and can be expected to attribute greater value to caps. The normal lattice allows negative rates, plus has the greatest downward dispersion in rates. We might expect such a lattice to assign more value to floors or calls. The directional lattice, which incorporates directional volatility, takes the middle ground in terms of dispersion.

The differences among the lattices are quite striking. On the lognormal lattice, rates at the end of Year 10 vary from 0.52% to 53.63%. On the normal lattice, rates vary from –3.99% to 17.21%. On the directional lattice, rates vary from 0.02% to 27.44%.

VI. RESULTS

Let's now turn to some actual valuation results. Exhibit 11 shows the valuation of ten-year caps of various strikes. We graph these same numbers in Exhibit 12.

ATM strike (ten-year swap rate), is 4.23%, and all three lattices are calibrated to produce the same value. The key thing to note here is that for the steep curve, low-rate, high-volatility environment (such as that prevailing on March 10, 2003, to which this lattice is calibrated), the lognormal lattice overvalues out-of-the-money caps and undervalues in-the-money caps. Note in Exhibit 11 that a ten-year cap with an 8% strike has a value of $3.42 using the lognormal lattice, $1.62 using the normal lattice, and

EXHIBIT 9
International Swap Yields and Cap Volatilities (bid side)

Term	US	UK	Euro	Swiss	Norway	Sweden	Australia	Hong Kong	Japan
Swap Yields									
2	1.58	3.37	2.38	0.61	5.22	3.56	4.41	1.80	0.10
5	2.88	3.87	3.13	1.45	5.33	4.04	4.78	3.28	0.27
10	3.97	4.32	3.97	2.38	5.39	4.57	5.19	4.49	0.70
CAP Vols									
2	64.50	22.03	28.30	77.75	12.50	21.50	19.25	49.50	75.00
5	43.00	21.42	23.20	44.20	11.80	18.75	18.25	37.00	78.00
10	29.30	18.31	16.90	25.00	10.80	15.55	15.40	23.60	58.50
Model Vols									
2	52.48	32.22	41.44	75.27	22.12	30.79	25.66	48.97	102.91
5	36.33	28.74	34.13	54.71	21.72	27.69	23.90	32.88	90.16
10	28.13	26.12	28.12	41.42	21.51	24.87	22.23	25.26	72.31

As of 3/10/03.

Exhibit 10

Three Approaches to Building an Interest Rate Lattice

Lognormal Rates Approach

									53.628
								39.168	
							29.059		32.010
						21.927		23.379	
					15.870		17.345		19.107
				11.506		13.088		13.955	
			7.914		9.473		10.353		11.405
		4.996		6.868		7.813		8.330	
	2.536		4.724		5.655		6.180		6.808
1.177		2.982		4.100		4.663		4.972	
	1.513		2.820		3.375		3.689		4.064
		1.780		2.447		2.784		2.968	
			1.683		2.015		2.202		2.426
				1.461		1.662		1.772	
					1.203		1.314		1.448
						0.992		1.057	
							0.785		0.864
								0.631	
									0.516

Normal Rates Approach

									17.210
								15.617	
							14.102		14.855
						12.672		13.262	
					11.027		11.747		12.500
				9.404		10.317		10.907	
			7.590		8.672		9.392		10.145
		5.563		7.049		7.962		8.552	
	3.213		5.235		6.317		7.037		7.790
1.177		3.208		4.694		5.607		6.197	
	0.858		2.880		3.962		4.682		5.435
		0.853		2.339		3.252		3.842	
			0.525		1.607		2.327		3.080
				-0.016		0.897		1.487	
					-0.748		-0.028		0.725
						-1.458		-0.868	
							-2.383		-1.630
								-3.223	
									-3.985

Directional Volatility Approach

									27.433
								22.134	
							15.531		19.876
						15.247		15.987	
					12.235		11.218		14.356
				9.948		11.023		11.552	
			7.911		8.819		8.104		10.377
		5.989		7.135		7.932		8.303	
	3.127		5.334		6.223		5.814		7.467
1.177		3.026		4.621		5.411		5.544	
	0.940		2.649		3.802		5.814		4.905
		0.815		2.138		3.019		3.702	
			0.668		1.566		2.884		2.701
				0.493		1.053		1.793	
					0.319		0.821		1.012
						0.183		0.453	
							0.121		0.199
								0.061	
									0.022

EXHIBIT 11
Valuation of 10-Year Caps

Strike	Lognormal	Normal	Directional
2	20.16	21.25	20.91
3	14.55	15.30	15.04
4	10.74	10.87	10.86
4.23 (ATM)	**10.00**	**10.00**	**10.00**
5	7.85	7.29	7.25
6	5.97	4.66	4.57
7	4.49	2.83	3.01
8	3.42	1.62	1.81
9	2.71	0.90	1.22

$1.81 using the directional volatility lattice.

The normal and directional lattices produce very similar values across strikes. We can thus conclude that the embedded caps in collateralized mortgage obligation floaters are being overvalued by lognormal models in the current environment. Since lognormal interest rate distributions are the most commonly used, OAS levels on CMO floaters are artificially too low. (Intuitively, too high a value is being placed on the cap, which the investor is short.)

In Exhibit 13, we turn to the more important kind of optionality for fixed-rate mortgage investors—callability. We mimic a discount, par, and premium pass-through by giving a 10NC3 structure the price and coupon of a 4.5, a 5.0, and a 5.5 coupon. This analysis treats the securities as if they each had an American par call after three years (i.e., the call can be exercised any time from Year 3 to Year 10).

For all securities, the lognormal approach produces the longest duration. For example, for the par-priced security, the duration (calculated using +/-50 basis point price moves) is 6.22 years using the lognormal lattice, 5.86 years using the normal lattice, and 5.99 years using the directional lattice. This makes sense, as the lognormal lattice reflects higher interest rates after a number of years, suggesting that the price depreciation will, on average, be more severe.

This can clearly be seen in the fact that in the +100 case for each bond, the dollar prices are lowest in the lognormal distribution. It also reflects the fact that in the −100 basis point case, the lognormal distribution has the highest prices (as the distribution does not allow for neg-

EXHIBIT 12
Ten-Year Cap Valuations—Different Approaches

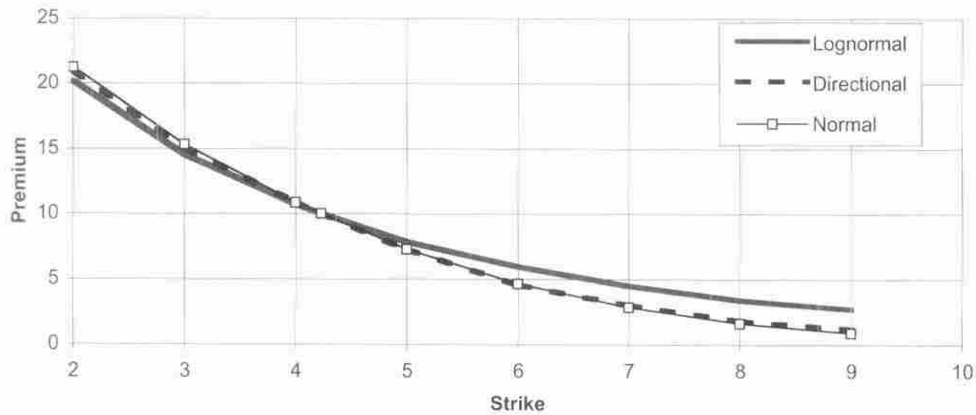
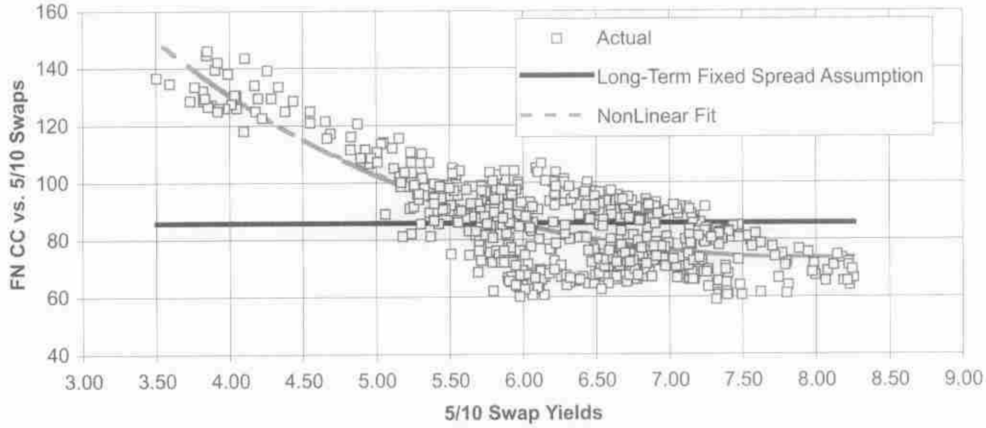


EXHIBIT 13
OAS Results of 10NC3

Cpn Price	5.5 102.75			5.0 100.00			4.5 97.25		
	Lognormal	Normal	Directional	Lognormal	Normal	Directional	Lognormal	Normal	Directional
OAS	50.53	60.10	61.52	54.40	54.84	58.48	49.18	47.82	52.00
-100 Px	108.51	107.72	108.00	105.94	105.47	105.61	103.66	103.18	103.35
-50 Px	105.63	105.28	105.34	102.97	102.93	102.95	100.61	100.22	100.39
Base Px	**102.75**	**102.75**	**102.75**	**100.00**	**100.00**	**100.00**	**97.25**	**97.25**	**97.25**
+50 Px	99.92	99.89	99.84	96.76	97.07	96.96	94.04	94.17	94.05
+100 Px	96.77	97.00	96.89	93.65	94.06	93.84	90.81	91.20	90.94
Dur	5.56	5.25	5.34	6.22	5.86	5.99	6.75	6.22	6.52

EXHIBIT 14
Effect of Constant Spread Assumption



ative rates). This upward bias in the OAS duration using lognormal models helps explain why, during rally periods, market participants find that empirical durations are generally shorter than OAS durations. Note that the normal approach produces the shortest durations, while the directional volatility takes the middle ground.

Note also that both the lognormal approach and the normal approach produce very similar OAS numbers for the par-priced mortgage. For a premium security (the 5.5 coupon), however, the OAS is lower on the lognormal distribution and higher on the normal distribution. Thus, the lognormal distribution is harsher on premium mortgages than either the proposed directional volatility or normal volatility.

We are uncomfortable drawing convexity conclusions from this simple analysis. Our lattice is very coarse, and we have done a less-than-perfect job in capturing the mortgage qualities of the instruments. We have not allowed for extension risk, nor have we taken into account the correlation between mortgage rates and volatility. Once we take the latter into account (as below), it is clear that the convexity on OAS models will be overstated regardless of the distribution used.

Directionality of Mortgage Spreads

As if faulty volatility assumptions aren't bad enough, there is another problem with OAS modeling of mortgage securities, arising from a low rate environment. This is the rate directionality of the spread of mortgage current-coupon rates versus bullet bonds.

Exhibit 14 shows spreads of the FN30 current-coupon versus the average of the five- and ten-year swap yields (5/10 yields) since June 30, 1992. In most OAS models, current-coupon mortgage rates are assumed to be a constant spread to some bullet bond. Historically, this assumption has worked acceptably well for 5/10 yields higher than 5.5%. Like the lognormal or normal interest rate assumptions, the assumption of constant long-term nominal mortgage spread across all rate levels falls apart in low interest rate environments.

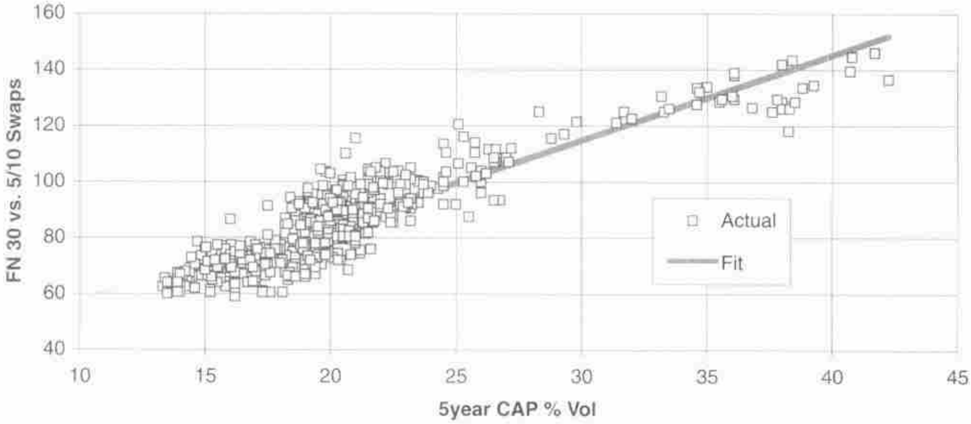
Where does the directionality of nominal current-coupon spreads come from? Straight from the directionality of volatility. The directionality of volatility and mortgage spreads are so similar that we venture to suggest that both essentially represent the same risk factor.

Exhibit 15 shows a scatterplot of mortgage spreads versus five-year cap volatility over June 30, 1992–March 7, 2003. The best fit line has an R-square of 63%.

It is no coincidence that nominal mortgage spreads are directional, as are implied volatilities (measured in percent). More interest rate options are traded in the U.S. mortgage market than anywhere else in the fixed-income world (albeit in implicit form, as prepayment options are embedded in an amortizing fixed-income instrument). But the direction of causality is unclear.

Holding all else constant, higher volatilities result in wider spreads. But there is a technical effect—when rates are low, there is more optionality, and fewer ways to escape that optionality. Thus there is a greater demand on the part of mortgage investors to buy volatility—which drives up implied versus actual. The fact that high volatility causes wider spreads on the current-coupon mortgage and that low rates cause a demand for volatility results in very strong correlation (63%) between mortgage nominal spreads and volatility.

EXHIBIT 15
Directional Volatility and Nominal Mortgage Spreads



We think the relationship demonstrated in Exhibits 14 and 15 suggests that current-coupon mortgage rates in OAS models should be modeled as linear functions of volatility or as non-linear functions of 5/10 swap or Treasury yields.

Directional Mortgage Spreads = Less Negative Convexity

As mortgage securities are path-dependent, we couldn't rely on simple backward induction in our simple lattices to demonstrate the impact of a non-constant mortgage spread. To eventually produce a true mortgage OAS result, we are working on setting up Monte Carlo simulation on those lattices (required because of path-dependence) and integrating a simple prepayment model to generate mortgage cash flows.

In the meantime, we can reasonably guess that modeling non-constant current-coupon mortgage spreads will likely improve the calculated convexity of mortgage securities. That is, allowing mortgage spreads to be directional suggests mortgage rates will be less volatile than their bullet benchmarks.

To be more explicit—as discount rates go down, volatility goes up and spreads widen, resulting in mortgage rates declining less than their bullet benchmark. As discount rates go up, volatility drops and spreads narrow, resulting in mortgage rates rising less than their bullet benchmark. If mortgage rates become less volatile, prepayments and cash flows also become less volatile, thus improving convexity.

Much more can be done to examine the impact on OAS modeling from the drastic changes we have suggested here. Especially if rates stay around current levels for any significant amount of time, the issues we raise must eventually be addressed by anyone who uses an OAS result. We hope at least to start discussions of what we view as the key issues.

VII. IMPROVING REGRESSION-BASED RICH/CHEAP MODELS

OAS modeling aside, there is something else we can do to improve our monitoring of mortgage relative value. For some time, we have evaluated the mortgage basis (FN 30 CC versus 5/10 swaps) with a regression as follows:

$$\text{Spread} = B[0] + B[1]\text{Yield} + B[2]\text{Yield}^2 + B[3]\text{Curve} + B[4]\text{Vol} \quad (2)$$

A variant of this model is introduced in Goodman and Ho [1997]. The fact that we have modeled spread as a function of yield and yield^2 means we have already been correctly accounting for the non-linear directionality of spreads to yield levels. We have also incorporated the yield curve.

For now, we'll leave alone the way these variables enter the regression. Given the correlation between volatility and mortgage spreads, however, a change is called for in how volatility enters the model.

In model-building jargon, volatility needs to be *orthogonal to yields* before we can use it in Equation (2). The way to do this is to substitute volatility (measured in percent) with [the residual of volatility (measured in percent) versus model volatility], where model volatility describes the observed non-linear directionality of volatility (measured in percent) to yields. Rather than using Equa-

EXHIBIT 16
Orthogonalizing Five-Year Cap Volatilities

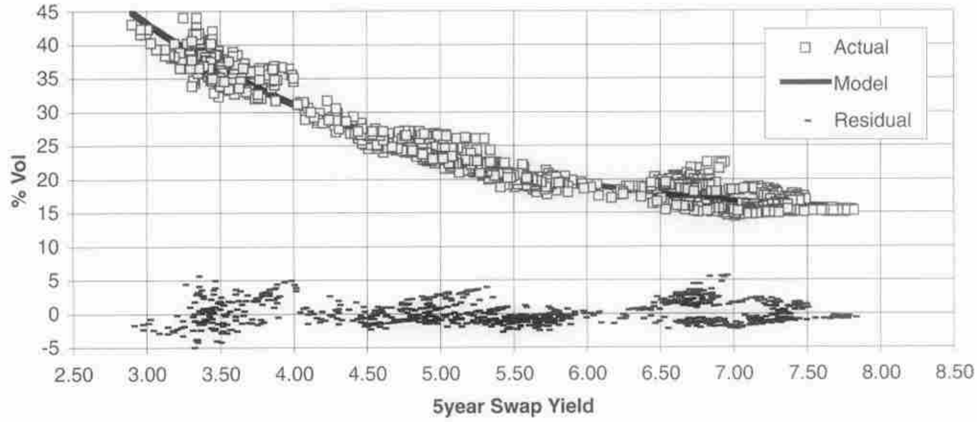
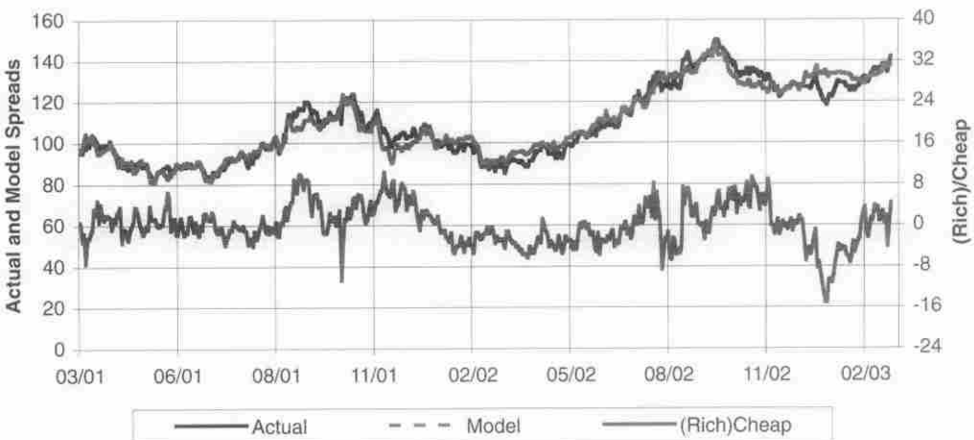


EXHIBIT 17
New and Improved Basis Model



tion (1), we opt for a version that relies on U.S.-only cap volatilities since June 30, 1999.

As Equation (1) is ultimately only suggestive of the directionality of volatility as rates decline further, we will want the actual U.S.-only experience to dominate. We do this in Exhibit 16, which plots actual and model five-year cap volatility (measured in percent) as a function of five-year swap yields and log yields. We prefer using log as the non-linear operator, since the resulting intercept is reasonably close to that of Equation (1).

The fitting equation then becomes:

$$\text{Model Vol} = 96.29 + 8.61(\text{Yield}) - 71.86(\ln \text{Yield}) \tag{3}$$

Substituting for volatility the model residuals of Equation (3) gives us the spread model as of March 10, 2003:

$$\text{Spread} = 184.43 + 5.01(\text{Yield}) - 3.26(\text{Yld}^2) - 0.09(\text{Crv}) + 1.93(\text{Tvol}) \tag{4}$$

The Tvol term is the orthogonalized volatility (true volatility) variable. As of March 10, 2003, this model has the basis cheap by 4.3 basis points, with a 1.05 Z-score significance. The basis cheapened 8 basis points over the then-latest two trading days, and hasn't been this cheap since November 19, 2002. The historical results, using true or orthogonalized volatility, are shown in Exhibit 17.

If we simply use Equation (2) (no volatility correction), we would end up also as cheap, albeit slightly less so. This may seem like a lot of work with no dramatic change in results, but the new regression estimate is more robust, and its coefficients are more reliable, because a multicollinearity problem is eliminated.

VIII. CONCLUSION

We have certainly not solved the issue of incorporating a non-linear rate directional volatility assumption. We have clearly shown the following:

- It is not correct to use either a normal or lognormal rate distribution. The correct approach employs a model that incorporates non-linear directional volatility.
- Using non-linear directional volatility naturally allows longer-tenor instruments, plus options with longer terms to expiration, to have lower volatility in an upward-sloping yield curve environment.
- Using lognormal and normal models can grossly distort relative value decisions. Using lognormal models, OASs on floaters are too low (the instruments appear richer on OAS models than they really are). On fixed-rate instruments, all durations are too long, and OASs on premiums are too low relative to par-priced instruments.
- The directionality of mortgage spreads is not adequately incorporated into most OAS models. This means that the negative convexity in mortgage instruments is overstated.
- We can incorporate the directionality of mortgage spreads into our regression-based current-coupon model, allowing a more robust estimation of the mortgage basis.

APPENDIX

Calibrating the Lattices

For any column in the lognormal and directional lattices, any rate is a function of the rate in the node directly above it and a volatility (measured in percent, labeled % Vol) as follows:

$$\% \text{ Vol} = \ln(\text{Rate Above}/\text{Rate Below})/2$$

such that

$$\text{Rate Below} = \text{Rate Above}/\text{Exp}[2(\text{vol})]$$

For the normal lattice, the relationship between rates and volatility is:

$$\text{Basis Points Vol} = \text{Rate Above} - \text{Rate Below}$$

The top rate in any column is solved for iteratively, so that the lattice produces the currently observed yield curve.

To calibrate these lattices for volatility, we iterate for the constant % volatility, constant basis point volatility, and constant adjustment to directional volatility so that the lognormal, normal, and directional lattices return the currently observed price for a ten-year ATM cap.

Discounting of cash flow in all three lattices is done as:

$$\text{PV} = (\text{Cash Flow Above} + \text{Cash Flow Below}) / [2(1 + \text{rate})]$$

REFERENCES

Black, Fischer, Emanuel Derman, and William Toy. "A One Factor Model of Interest Rates and its Application to Treasury Bond Options." *Financial Analysts Journal*, 46 (1) (January/February 1990), pp. 33-39.

Goodman, Laurie, and Jeffrey Ho. "Modeling the Mortgage-Treasury Spread." *The Journal of Fixed Income*, September 1997, pp. 85-91.