An Empirical Study of Credit Default Swaps

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Recent years have witnessed exponential growth in the credit derivatives market. The British Bankers Association estimates the notional principal amounts outstanding of all types of credit derivatives at nearly \$2 trillion, around 45% in credit default swaps ("2001/2002 Credit Derivatives Survey" [2002]).

Despite the importance of the credit default swap market, there are very few studies that examine this market or the behavior of models that price these securities. We apply the important credit risk models of Duffie and Singleton [1999] and Jarrow and Turnbull [1995] to price credit default swap contracts around the time of the Asian currency crisis.

A long position in a credit default swap insures the buyer of credit protection against losses due to default on the insured bond. In the parlance of the credit default swap market, the insured bond is the reference security. Typical default payments are par less the value of the defaulted reference security.

The buyer of credit protection pays a swap premium quoted as a number of basis points times a notional principal amount of the reference security. The swap premium is quoted at annual rates, but swap premium payments are typically paid quarterly. If the reference security should default, the buyer of credit protection would receive the default payment, and the flow of swap premium payments would cease.

We compare the expected value of the swap premium (the cost of default insurance) with the expected value of payments in default (the amount of insurance) as generated by the Duffie and Singleton [1999] and Jarrow and Turnbull [1995] credit risk models. We find that the cost of default insurance is greater than the amount of insurance for credit default swaps written on bonds subject to the recent Asian currency crisis. Credit default swaps not subject to the Asian currency crisis generally show the opposite result; the cost of insurance is less than the amount of insurance.

We speculate that these results are evidence of the moral hazard problem when payments from the credit default swap may be triggered by bond restructuring as well as default. For discussion of the moral hazard problem, see Bennett [2000] and Cass [2000].

We also find that the Jarrow and Turnbull [1995] model produces lower premium values and default payment values for credit default swaps than those found by Duffie and Singleton [1999]. This happens because Jarrow and Turnbull [1995] obtain higher hazard rates than Duffie and Singleton [1999], given the same information set concerning a particular credit default swap.

Finally, we suggest a solution to an important issue concerning the implementation of term and credit risk structure-consistent credit risk models. That is, we know from Helwege and Turner [1999] that below-investment-grade yield curves are often biased, so a model that agrees with a biased yield curve will itself be biased. We overcome this problem by suggesting a method to calibrate a term- and credit risk-consistent credit risk model so that it agrees with the market value of the underlying reference bond.

Since the candidate credit risk model, once fine-tuned to an estimate of a corporate yield curve, agrees with the market price of the reference bond, we have some assurance that our estimate of the credit-risky yield curve is reasonable. A reasonable estimate of the credit-risky yield curve is important because the credit default swap is priced on this yield curve.

I. THE MODELS

Our objective is to price credit default swaps. Since credit default swaps are American-style options, there is no known closed-form solution. Consequently, we choose a binomial lattice implementation. We pick the Duffie and Singleton [1999] and Jarrow and Turnbull [1995] models because we want to explore the choice between the return of market value (RM) and the return of Treasury (RT) recovery assumptions that the two models (respectively) use.

To obtain a binomial version of Jarrow and Turnbull [1995] and Duffie and Singleton [1999], we suggest the binomial stochastic process for the default-free rate of interest and the hazard (default) rate as follows:

$$r(t, i) = u_{rt} e\left(z_{\sigma t}(2i - t)\sqrt{\Delta T}\right) \quad (1)$$

$$h(t, i) = h(0, 0)e\left(v_t \Delta T + \rho_{h,r} \frac{\sigma_{ht}}{\sigma_{rt}} r(t, i)\, \Delta T\right) \quad (2)$$

The first binomial stochastic process is attributable to Black, Derman, and Toy [1990], where r (t, i) refers to the default-free interest rate that evolves in state i and time t. Meanwhile u_{rt} and $z_{\sigma t}$ are time-dependent parameters that calibrate the interest rate tree by forward induction through use of state prices to the exogenously supplied Treasury zero yield and volatility curves, respectively, and ΔT is the time step. Note that when t = 0, then r (t, i) is defined to be today's known short-term default-free rate of interest r (0, 0).

The second binomial stochastic process describes the evolution of the one-period hazard rate h (t, i) and the joint probability distribution between r (t, i) and h (t, i). Through covariance between the default-free rate of interest and the hazard rate, correlation $\rho_{h,r}$ between these parameters is included. This covariance is scaled by the time-dependent default-free interest rate volatility $\sigma_{r,t}$ leading to a multiplicative term that models the volatility of hazard rates as the responsiveness of hazard rates to the current default-free rate of interest.

In Equation (2), the time-dependent parameter v_t calibrates the hazard rate tree by forward induction through use of corporate state prices to the corporate zero yield curve, and σ_{ht} is the hazard rate volatility parameter. Note that when t = 0, then h (t, i) is defined to be today's one-period hazard rate h (0, 0).

We first calibrate the default-free interest rate process at today's date t = 0 to the Treasury zero yield and the Treasury volatility curves by adjusting the calibration factors u_{rt} and $z_{\sigma t}$, respectively, for all future dates. This obtains the default-free interest rate binomial tree whose values of [r (t, i)] are included in the hazard rate process in Equation (2).[1]

We then calibrate the hazard rate process, which is correlated with the Treasury interest rate process generated by the first calibration, to a credit-risky zero-coupon yield curve by adjusting the calibration factor v_t. Simultaneously this calibration adjusts the structure of defaultable state security prices until the yield implied by the portfolio of all defaultable state securities that mature at a given date agrees with the corresponding yield from our estimate of the credit-risky zero-coupon yield curve. This process continues for all future dates so that, at each date, the yield of the replicating portfolio of defaultable state securities agrees with the corresponding yield from our estimate of the credit-risky term structure.

Substituting (1) and (2) into a general expression for the value of a two-period defaultable zero, we obtain a binomial version of Jarrow and Turnbull [1995].[2] The result is as follows:

$$V_0 = \left\{\left([1 - h(0, 0)] \times [1 - 0.5(h(1, 0) + h(1, 1))]\right)V_2 + \phi_2\right\} \times e^{-r(0,0)} 0.5[e^{-r(1,0)} + e^{-r(1,1)}] \quad (3)$$

where

$$\phi_2 = h(0, 0)\delta\left[0.5\left(e^{r(1,0)} + e^{r(1,1)}\right)\right] + 0.5[h(1, 0) + h(1, 1)]\, \delta[1 - h(0, 0)] \quad (3\text{-A})$$

Equation (3) says that the zero must survive two periods (with probability 1 − h (t, i) for each period) if the final redemption amount V_2 is to be paid. Then again, the zero may default and instead a recovery amount ϕ_2 is paid at scheduled maturity. These expected values are discounted to the present using the binomial structure of default-free interest rates.

Equation (3-A) highlights the RT recovery assumption, where conditional upon no prior default a recovery amount δ is paid at the end of the current period. Should default occur prior to maturity, the recovery amount is reinvested in a Treasury security until promised maturity. These recovery amounts are then included in (3). The recovery amounts are multiplied by Treasury zero prices so they are ultimately expressed as a fraction of the value of a Treasury zero.

This is the RT assumption that underlies Jarrow and Turnbull [1995], for the recovery amount that is paid at the end of the first period is a fraction of a one-period Treasury zero. Similarly, the recovery amount paid at the end of the second period is expressed as a fraction of a two-period Treasury zero.

If we adjust (3) to conform to the return of market recovery assumption, we obtain Duffie and Singleton [1999]. Specifically, the Duffie and Singleton [1999] binomial version of (3) becomes:

$$V_0 = \{1 - (h(0,0) + 0.5[h(1,0) + h(1,1)])\, L_{t+1}\}V_{t+1} \times e^{-r(0,0)}0.5[e^{-r(1,0)} + e^{-r(1,1)}] \qquad (4)$$

This expression says that, in the event of default h(t, i) a loss rate L_{t+1} is experienced at the end of the period. One minus this expected loss rate is then multiplied by the promised value V_{t+1} to find the expected value. Then we find the present value of this expected value.

In (4), L_{t+1} is the end-of-period loss rate that is equal to this expression under the RM recovery assumption:

$$L_{t+1} = 1 - \{1 - (h(0,0) + 0.5[h(1,0) + h(1,1)])\}\omega V_{t+1} \qquad (4\text{-A})$$

In other words, (4-A) says that upon default the investor loses a fraction L. This amount is one minus the recovery amount. In turn, this recovery amount is a fraction ω of the end-of-period survival-contingent {1 – h (t, i)} value of a $1 face value zero.[3] In contrast, the RT assumption in (3-A) models recoveries as a fraction of a Treasury zero.

The advantage of the RM formulation is that if recoveries are fractions of survival-contingent values, then values associated with prior-period defaults are included in (4) as a *multiplicative* term. This facilitates forward induction, allowing us to apply default-free interest rate modeling techniques directly to a corporate rate of interest without the extra computational complexity of adding values associated with prior-period defaults to each corporate state price. Equation (3-A) shows that to implement the RT recovery method, one needs to make a separate *additive* calculation of payoffs in (3) in the event of default for all possible prior periods at each interest rate state.

II. EMPIRICAL PROCEDURES

An investment bank that wishes to remain anonymous allowed us to select a sample of default swap trades. For each trade, we obtain the default swap ticket. This document provides important details of the premium, credit event parameters, and payout structure in the event of default of the reference security.

The premium is specified as the total number of basis points to be paid each year. Payments are made in installments, typically quarterly, and are based on the specified notional principal amount. Applicable credit events are specified, which include failure to pay, bankruptcy, repudiation, and restructuring. Payoffs in the event of default are defined as par value less the value of the defaulted security, where the value of the defaulted bond is established as the average of (typically) five independent dealer quotes. From this document we are able to identify the reference security and the precise date of trade.

Our sample includes 31 straight U.S. dollar credit default swaps for the September 1997 through February 1999 time period. This time period covers the worst of the Asian currency crises, so credit default swap premiums reflect these events.

To obtain information about the effect these events had on the credit default swap market, we subdivide our sample into two categories: *crisis swaps*, which are swaps whose reference entity was domiciled in the Asian countries that were subject to the Asian currency crisis, and *non-crisis swaps*, which are swaps whose reference entity was domiciled in non-Asian countries that evidently did not experience a currency crisis. We then plot the yield of the reference bond, the corresponding-maturity Treasury yield, the resulting credit spread, and the credit swap premium. In general, the reference security will be different at points along the time series plot, but since all are long-term dollar securities the resulting plot is reasonably continuous.

Exhibits 1 and 2 show the time series plot of the non-crisis and crisis subsamples. Exhibit 1 shows that for credit default swaps *not* experiencing a currency crisis the swap premium plots within the credit spread. Exhibit 2 tells another story. Beginning in June 1998, credit swap premiums rise dramatically, now plotting above the credit spread. Later the credit premiums rise even further, now plotting above the Treasury yield.

These data should provide a challenge for Duffie and Singleton [1999] and Jarrow and Turnbull [1995] because they are required to measure the market prices of credit default swaps subject to widely varying credit risk conditions. To implement the models, we need estimates of the Treasury yield curve, the credit-risky yield curve applicable to the reference bond, and the correlation between the Treasury interest rates and the credit-risky interest rates applicable to the reference bond. We also need Treasury interest rate volatility and the volatility of credit risk. Finally, we need the price of the reference bond, the binomial structure of hazard rates, and the recovery rate of the reference bond in the event of default. Note that all of this information must apply on the date of the credit default swap trade.

We estimate the zero-coupon Treasury yield curve on the date of each credit default swap transaction by applying the Nelson and Siegel [1987] yield curve estimation procedure to Treasury transaction prices supplied by the National Association of Insurance Commissioners' (NAIC) database. The NAIC database includes the details of each bond transaction that member insurance companies report to the national body, representing 30% to 40% of all U.S. domestic over-the-counter bond trading activity. Each transaction includes all the important details such as the CUSIP of the bond, transaction date, clean price, accrued interest, and day-count conventions. A separate cross-sectional file provides all the major bond covenants of each issue.

This information can be linked with the transaction file, so we are able to avoid using transactions tainted by optionality. We estimate a ten-year Treasury yield curve, since the maturity of the reference bond in all our default swaps is never longer than ten years.

We also obtain the price and yield of the reference bond and the one-, two-, three-, four-, five-, seven-, and ten-year at-the-money implied interest rate cap volatility from Datastream's 901b program at the same dates the credit default swaps were sold. We linearly interpolate implied cap volatility to form an estimate of the Treasury volatility curve.

One barrier to pricing credit default swaps is the absence of precise information concerning the credit-risky yield curve applicable to a given credit default swap. Helwege and Turner [1999] find that for bonds rated below investment-grade, credit ratings are not precise enough to discriminate among bonds of different credit quality.

Yield curves estimated for below-investment-grade bonds tend to be downward-sloping, not because of any crisis at maturity problem but because below-investment-grade longer-term bonds are more creditworthy than shorter-term bonds of the same credit rating. Once constructed from paired short- and long-term bonds from the same firm, below-investment-grade yield curves tend to be upward rather than downward-sloping. This suggests that applying, say, a generic BB yield curve in an attempt to value a credit default swap written on a BB bond may lead to important bias. Since term structure- and credit risk structure-consistent models are fine-tuned to agree with the underlying term structures, errors in estimating these yield curves will result in errors in pricing the credit default swap.

We overcome this problem as follows. We first calibrate the Black, Derman, and Toy [1990] default-free interest rate process to the Treasury zerocoupon and cap volatility curves on the trade date of the credit default swap. We initially assume that the credit-risky yield curve is the sovereign yield curve plus a credit spread calculated as the difference between the reference bond's yield and the corresponding maturity U.S. Treasury yield. Then we calibrate the candidate credit risk model by adjusting the credit spread until the candidate credit risk model replicates the value of the reference bond using reasonable values for the recovery fraction, credit risk volatility, and correlation between hazard rates and Treasury interest rates. Later we test our initial estimates of key parameters to ensure that these choices do not materially affect our results.

Since the candidate credit risk model, once fine-tuned to an estimate of a corporate yield curve, agrees with the market price of the reference bond, we have some assurance that our estimate of the credit-risky yield curve is reasonable. A reasonable estimate of the credit-risky yield curve is important because the credit default swap is priced relative to this yield curve.

We choose recovery rates that are consistent with the credit rating of the reference bond. This information is

EXHIBIT 1
Non-Crisis Yield Curve versus U.S. Treasury Yield Curve

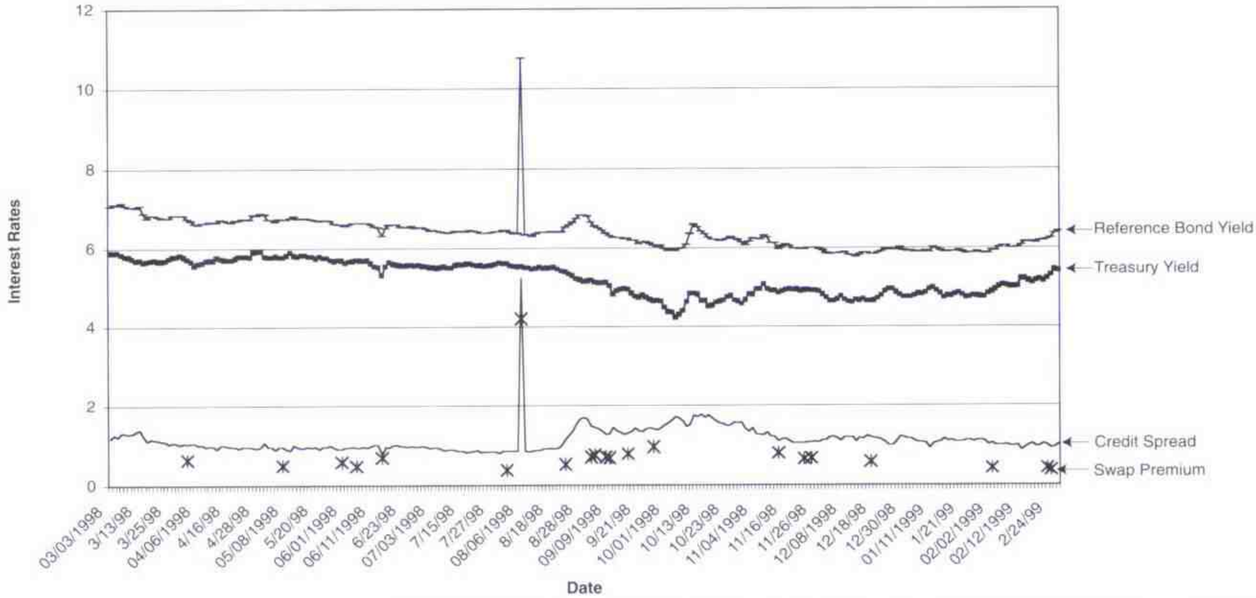
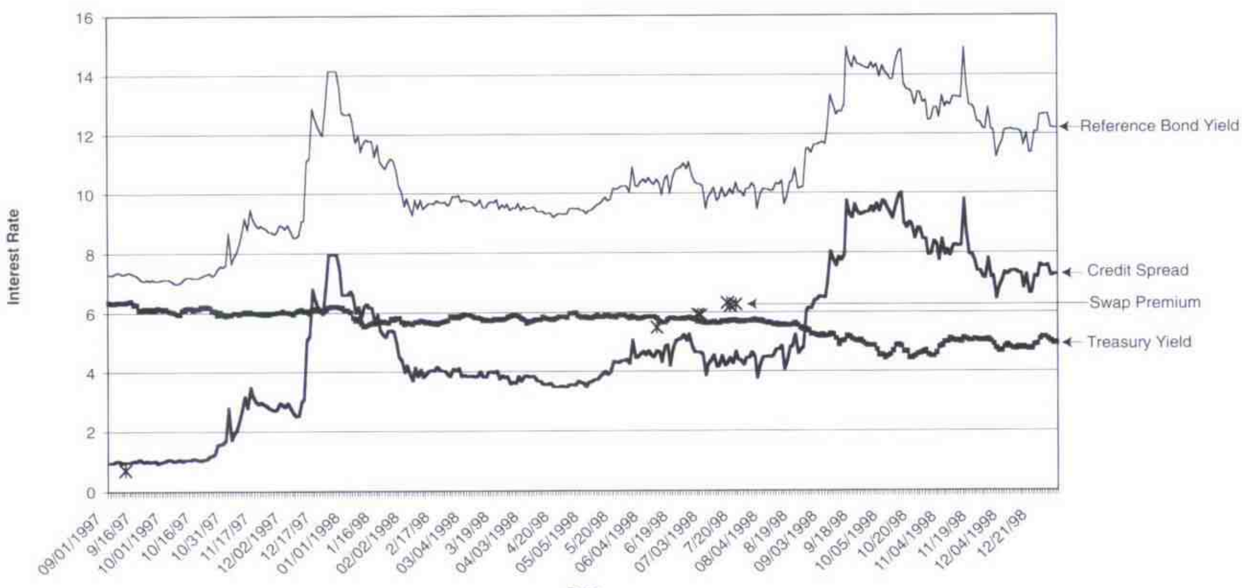


EXHIBIT 2
Crisis Yield Curve versus U.S. Treasury Yield Curve



available from Altman, Cooke, and Kishore [1999], who report the average trade price of defaulted bonds one month after default. These averages are computed over the 1971-1998 time period, and are reported by broad rating category.

We find the historical credit rating of the candidate reference bond at the Standard & Poor's website. We choose credit risk volatility as one-tenth of Treasury interest rate volatility, as we suspect that credit risk volatility is related to but lower than Treasury interest rate volatility.

Finally, we assume that the correlation between Treasury interest rates and hazard rates applicable to the insured bond is −0.1, as Collin-Dufresne, Goldstein, and Martin [2001] and Duffee [1998] suggest that correlations between

Treasury interest rates and the credit spread are negative.

According to these estimates, we run a given candidate model to see if these choices for the candidate credit-risky yield curve, recovery fraction, credit risk volatility, and correlation between Treasury and credit-risky interest rates are able to replicate the known price of the reference bond. The binomial structure of default risk-free interest rates and hazard probabilities is calibrated until it replicates the given Treasury and the initial estimate of the credit-risky yield curve. The binomial structure of Treasury interest rates and hazard probabilities, along with the recovery fraction, also generates a binomial structure of credit-risky interest rates. This binomial structure of credit-risky interest rates is then used to price the reference bond.

Initially, the market price of the reference bond is not replicated. We discover that changes in the credit risk volatility and correlation between Treasury rates and hazard rates do not appreciably change the price of the reference bond that we obtain by the two credit risk models. For example, changing credit risk volatility by a factor of ten results in less than a \$0.01 change in the price of the reference bond.

Similarly, the price of the bond obtained by both models is insensitive to large variations in correlation between Treasury rates and hazard rates. These results are not surprising, as the underlying bond is a cash instrument that should be much less sensitive to volatility and correlation than a derivative security.

Furthermore, we find the price of the underlying bond is not sensitive to changes in the recovery rate. This is what we should expect, because as the recovery rate rises (declines), a renewed calibration of the candidate credit risk model will have higher (lower) hazard probabilities. In other words, the calibration process takes the credit spread as given, so for a given credit risk model a rise in the recovery rate is offset by a rise in the hazard rate. Therefore, the renewed calibration replicates the credit spread with a different combination of hazard and recovery rates that results in the same price of the reference bond.

The reference bond's price obtained by the two candidate models is sensitive to the size of the credit spread, however. Therefore, we generate a new credit-risky yield curve by adjusting the credit spread until we replicate the market price of the reference bond. We find that the Duffie and Singleton [1999] and Jarrow and Turnbull [1995] models agree as to the extent of the spread. The difference is always under 2 basis points, even for below-investment-grade bonds.

III. EMPIRICAL ESTIMATES

Using the calibrated default-free interest volatility, credit spread, and hazard rates, and initial estimates of the recovery rate, credit correlation, and credit volatility, we find the expected default value, the present value of payments from the credit default swap in the event of default, according to Equation (5):

$$\text{Default Value} = E_0\left(\sum_{t=0}^{n}\sum_{i=0}^{n} h(t,i)[100 - B(t,i)\delta]\, e^{-[r(t,i)]}\right) \tag{5}$$

Note that where $i > t$, $h(t, i)$ and $r(t, i)$ are zero. The entire expression is calculated under the risk-neutral expectations operator E_0 since interest rates are stochastic and correlated with factors that influence default.

Equation (5) says that the default value of the swap is the present value of expected payoffs in the event of default. This is the amount of insurance purchased by the buyer of a credit default swap. The payment is determined by the swap contract. In our sample, the payment in the event of default is defined as face value less the value of the defaulted reference bond $B(t, i)\delta$, where δ is the recovery rate.

The two models differ in how they determine the hazard rates and the values received in the event of default. Specifically, Duffie and Singleton [1999] model recovery amounts as a fraction of the survival-contingent value of the reference security (RM), while Jarrow and Turnbull [1995] model recovery amounts as a fraction of a Treasury zero (RT). As we note earlier, because both models are calibrated to the same exogenous credit spread, different recovery amounts result in different estimates of the hazard probabilities.

Next we convert the basis point price of the credit default swap to premium values:

$$\text{Premium Value} = E_0\left(\sum_{t=0}^{n}\sum_{i=0}^{n}[1 - h(t,i)] x S_t x M x\, e^{-[r(t,i)]}\right) \tag{6}$$

Like (5), Equation (6) is calculated under the risk-neutral expectations operator E_0, since interest rates are stochastic and correlated with factors that influence default. The buyer pays the premium value calculated as the swap rate S_t as quoted in basis points times the notional amount of the reference bond M that we set to be 100. Payments are conditional upon no prior default [1 – h (t, i)] at any time t and state i.

The present value of this stream of conditional payments is found by solving backward using the binomial structure of default-free rates of interest. This is the cost of insurance paid by the buyer of the credit default swap.

Subtracting (6) from (5), we find the economic value of the credit default swap from the buyer's perspective. It is tempting to suggest that a credit default swap should have an economic value of zero, that is, (6) = (5), so the cost of default insurance [Equation (5)] is equal to the expected benefit of default insurance [Equation (6)]. In fact, Bomfin [2002] suggests that this is the case for credit default swaps that insure floating-rate reference bonds when markets are complete and frictionless.[4]

As the developing credit default swap market can hardly be characterized as a frictionless market, however, we must consider the impact of common market imperfections. First, consider the likely impact of liquidity. Given exogenous estimates of recovery rates, the structure of hazard rates will be determined by the credit spread. We think the credit spread will be wider than it should be in a frictionless market because U.S. Treasury bonds are more marketable than the corresponding-maturity but more credit-risky reference bond.

Consequently, hazard rates would be overestimated because the candidate credit risk model will be calibrated to a credit spread that reflects differences in liquidity as well as credit risk. This would underestimate the value of payments to the seller of the swap in (6), and overestimate the value of the credit default swap payoff to the buyer in (5).

This suggests that the Duffie and Singleton [1999] and Jarrow and Turnbull [1995] models would typically yield positive economic values for credit default swaps. That is, the amount of insurance (6) will be greater than the cost of insurance (5).

Second, consider the problem of moral hazard. All swaps in our sample include restructuring as a credit event that triggers payments from the default swap. Since the buyer of the credit default swap is often the investor in the reference bond insured by the credit default swap, in essence the insured party may also influence payments on the insurance contract. Therefore we have a moral hazard problem.

For example, if the reference bond is likely to default, the owner of the reference bond may press for an early restructuring, before needing to do so because of actual default. An early restructuring is desirable from the owner's perspective since delays in restructuring may result in more severe defaults later as the issuer may delay facing up to its problems.

The owner of the reference bond could tempt the issuer to restructure early by, say, offering to accept a replacement security with a lower coupon in place of the high-coupon reference security. Since the owner of the reference bond is insured by the credit default swap, losses incurred by the restructuring would be offset by payments on the credit default swap.

Overall, at least one gains, and possibly both the issuer and the owner of the reference security gain, but neither would suffer losses. Instead, the seller of credit protection incurs losses by paying the difference between the value of the reference bond and the replacement bond.

The seller of credit protection now faces an asymmetric information problem. Some buyers of credit protection may know more about the possibility of restructuring than the seller of credit protection. These knowledgeable buyers of credit protection would buy credit default swaps that include restructuring as a credit event if the swap premium underprices the likelihood of restructuring. Therefore, the seller of credit protection is encouraged to require higher credit default swap premiums.

All candidate credit default swap models may not be able to pick up this higher premium, as it would be included in the premium value (6) but maybe not in the default value (5). The latter is a possibility, as some but not necessarily all the investors in the reference bond own the credit default swap written on this bond. Furthermore, some investors will not even know that other investors have bought credit protection since credit default swaps trade over the counter.

Therefore, the likelihood of restructuring due to the moral hazard problem may not be included in the price of the reference bond, and consequently may not be included in the credit spread. Since all candidate credit risk models are fine-tuned to agree with the credit spread, the credit risk model may not include the impact of the moral hazard problem, so Equations (5) and (6) may underestimate the likelihood of restructuring. This would lead to negative economic values, since (6) includes the extra swap premium in S, but (5) and (6) may underestimate the hazard probability. In other words, the cost of insurance (5) may be greater than the amount of insurance (6).

It is an interesting exercise to see whether the candidate models do in fact find positive or negative economic values for market-traded credit default swaps. We find the values of the 31 credit default swaps using the calibrated Treasury and corporate yield curves and the default-free interest rate volatility curve in the manner outlined earlier.

Since the RM assumption that underlies the Duffie and Singleton [1999] model cannot separately identify the recovery rate as a single value (it is survival-contingent), we replace ω with the recovery rate δ in (4-A) when we run the model. As the implied cash value of a fraction of a survival-contingent zero (RM) is lower than the corresponding fraction of a Treasury zero (RT), the value of the RM recovery rate should be higher in order for it to be comparable to the RT recovery rate.

The result of this exercise is reported in Exhibits 3, 4, and 5.

Exhibit 3 shows that for credit default swaps that evidently did not experience problems with the Asian currency crisis, the Duffie and Singleton [1999] model estimates positive economic values from the point of view of those buying credit protection. Exhibit 4 shows that under the Jarrow and Turnbull [1995] model, most have positive economic values but 10 of 23 credit default swaps have negative economic values. Most of these ten, however, are only slightly negative.

Exhibit 5 paints a different picture. For the credit default swaps that evidently did experience problems with the Asian currency crisis, both models agree that all credit default swaps have a high negative economic value.[5]

Since we expect to observe positive economic values due to a liquidity bias, it appears that the consistently negative economic values found in Exhibit 5 are likely due to a moral hazard problem. Evidently, we generally observe the anticipated positive economic values for credit default swaps in the absence of the Asian currency crisis. When the possibility of restructuring is more likely, namely, for the below-investment-grade reference bonds subject to the Asian currency crisis, we consistently observe negative rather than positive economic values.

This means that compensation for bearing default risk as measured in (6) is more than the anticipated cost of default risk as measured in (5), even though the hazard rate is probably overestimated due to a liquidity bias. We suggest that this occurred because the moral hazard problem was so severe in the case of reference bonds subject to the Asian currency crisis that sellers of credit protection demanded (and received) a measurable restructuring risk premium due to the moral hazard problem.

Also notice that the Duffie and Singleton [1999] model consistently reports higher premium prices and default values than Jarrow and Turnbull [1995] in Exhibits 3, 4, and 5. This happens because any given recovery fraction used for both models implies smaller recovery amounts

EXHIBIT 3

Non-Crisis Swaps—Duffie and Singleton [1999]

Date	Bond Life (in months)	Credit Spread (in basis points)	Rating	Swap Life (in months)	Default Value (in $ per 100)	Premium Value (in $ per 100)	Economic Value (default – premium)
03/31/98	119	99	BBB	108	6.89	4.47	2.42
05/06/98	117	104	BBB	60	4.38	2.16	2.22
05/28/98	117	88	BBB	108	7.17	4.11	3.06
06/03/98	116	94	BBB	57	3.77	2.00	1.77
06/16/98	47	76	A	47	2.57	2.33	0.24
07/30/98	115	97	BBB	115	6.98	2.81	4.17
08/06/98	78	504	B	60	21.87	18.26	3.61
08/21/98	114	92	BBB	60	3.84	2.28	1.56
09/01/98	114	156	BBB	114	11.51	5.28	6.24
09/02/98	114	140	BBB	60	5.98	3.31	2.66
09/02/98	114	140	BBB	114	10.36	5.66	4.70
09/07/98	113	122	BBB	60	5.12	3.08	2.04
09/08/98	113	131	BBB	60	5.38	3.00	2.38
09/15/98	113	136	BBB	60	5.66	3.46	2.21
09/24/98	113	126	BBB	113	9.23	7.12	2.11
11/11/98	111	122	BBB	84	6.89	4.75	2.14
11/17/98	111	114	BBB	60	4.74	3.05	1.70
11/20/98	111	103	BBB	60	4.36	2.93	1.43
11/24/98	111	115	BBB	111	8.15	5.00	3.15
12/16/98	110	115	BBB	84	6.44	3.47	2.97
02/04/99	108	75	BBB	108	5.18	3.10	2.08
02/22/99	108	76	BBB	108	8.91	3.05	5.85
02/23/99	108	71	BBB	60	2.97	1.63	1.34

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Exhibit 4
Non-Crisis Swaps—Jarrow and Turnbull [1995]

Date	Bond Life (in months)	Credit Spread (in basis points)	Rating	Swap Life (in months)	Default Value (in $ per 100)	Premium Value (in $ per 100)	Economic Value (default – premium)
03/31/98	119	100	BBB	108	3.86	4.27	-0.41
05/06/98	117	106	BBB	60	2.27	2.09	0.18
05/28/98	117	90	BBB	108	3.99	3.93	0.06
06/03/98	116	95	BBB	57	2.51	1.95	0.57
06/16/98	47	77	A	47	Fail	Fail	Fail
07/30/98	115	98	BBB	115	3.90	2.68	1.22
08/06/98	78	505	B	60	15.77	16.00	-0.23
08/21/98	114	93	BBB	60	2.19	2.22	-0.03
09/01/98	114	157	BBB	114	6.56	4.90	1.66
09/02/98	114	141	BBB	60	3.42	3.19	0.24
09/02/98	114	141	BBB	114	5.86	5.29	0.57
09/07/98	113	123	BBB	60	2.92	2.97	-0.05
09/08/98	113	132	BBB	60	3.12	2.89	0.23
09/15/98	113	137	BBB	60	3.24	3.33	-0.09
09/24/98	113	127	BBB	113	5.19	6.71	-1.52
11/11/98	111	123	BBB	84	3.88	4.54	-0.65
11/17/98	111	115	BBB	60	2.70	2.95	-0.25
11/20/98	111	104	BBB	60	2.43	2.85	-0.42
11/24/98	111	116	BBB	111	4.57	4.74	-0.17
12/16/98	110	116	BBB	84	3.62	3.32	0.30
02/04/99	108	76	BBB	108	4.32	2.99	1.33
02/22/99	108	77	BBB	108	4.96	2.90	2.05
02/23/99	108	72	BBB	60	1.68	1.59	0.09

Exhibit 5
Crisis Swaps

Swap	Date	Bond Life (in months)	Credit Spread (in basis points)	Rating	Swap Life (in months)	Default Value (in $ per 100)	Premium Value (in $ per 100)	Economic Value (default – premium)
DS	09/08/97	61	98	AA	60	4.25	2.99	1.27
JT	09/08/97	61	99	AA	60	Fail	Fail	Fail
DS	06/09/98	54	432	BB	54	17.56	21.66	-4.09
JT	06/09/98	54	433	BB	54	12.06	19.52	-7.47
DS	06/30/98	53	453	BB	53	18.13	22.29	-4.16
JT	06/30/98	53	454	BB	53	12.69	20.00	-7.31
DS	06/30/98	53	453	BB	53	18.13	20.89	-2.76
JT	06/30/98	53	454	BB	53	12.69	18.80	-6.11
DS	07/13/98	39	532	BB	12	4.97	6.55	-1.58
JT	07/13/98	39	533	BB	12	4.09	6.30	-2.21
DS	07/14/98	52	415	BB	12	3.92	6.04	-2.13
JT	07/14/98	52	416	BB	12	3.21	5.82	-2.62
DS	07/15/98	52	419	BB	12	3.96	5.99	-2.03
JT	07/15/98	52	420	BB	12	3.24	5.81	-2.57
DS	07/17/98	52	419	BB	12	3.96	6.04	-2.08
JT	07/17/98	52	420	BB	12	3.24	5.85	-2.61

DS = Duffie and Singleton [1999] and JT = Jarrow and Turnbull [1995].

and lower hazard rates in Duffie and Singleton [1999].

This result is consistent with the analysis of Delianedis and Lagnado [2002], who agree that the RM assumption typically implies lower hazard rates than the RT assumption. These lower hazard probabilities but higher default payoffs in Duffie and Singleton [1999] lead to higher default values in (5).

Similarly, the premium value in (6) is higher for Duffie and Singleton [1999], since by calibration a smaller recovery amount implies a lower hazard rate. This leads to higher probabilities of premium payments and so to higher premium values in (6).[6]

IV. SUMMARY AND CONCLUSIONS

We examine the empirical performance of the Duffie and Singleton [1999] and Jarrow and Turnbull [1995] models on a sample of 31 credit default swaps that traded during 1997-1999. We find that the return of market value assumption that underlies Duffie and Singleton [1999] produces higher premium and default values for credit default swaps than the return of Treasury assumption that underlies Jarrow and Turnbull [1995]. We are able to estimate these models because we can generate a credit-risky yield curve that is applicable to the underlying reference bond.

We generally find that the expected value of the credit default swap premium is lower than the expected value of recoveries in the event of default for non-crisis credit default swaps, which is what would be expected, given that credit spreads reflect liquidity as well as credit risk. Both models consistently find that the expected value of the credit default swap premium is higher than the expected value of recoveries in the event of default for credit default swaps that were subject to the Asian currency crisis.

We suggest this happens because of a moral hazard problem. Sellers of crisis credit default swaps suspected that some buyers were also investors in the reference bond. Sellers then believed that some buyers could influence the conditions of payoff from the credit default swap. Faced with information asymmetry, being unable to distinguish between buyers who had accurate information concerning the likelihood of restructuring and those who did not, the sellers of credit protection demanded, and evidently received, a restructuring premium.

ENDNOTES

The authors thank Nicolas Papageorgiou and participants at the HEC International Credit Risk Conference for valuable comments and Sadiq Merchant for research assistance. Antonio Díaz acknowledges the financial support provided by Ministerio de Ciencia y Tecnología grant MEC 2001-1599.

[1]Details of how to implement Black, Derman, and Toy [1990] can be found in Clewlow and Strickland [1998, Chapter 8].

[2]Since we include correlation between credit-risky and Treasury interest rates, (3) represents a minor extension of Jarrow and Turnbull [1995], who assume zero correlation between these two parameters.

[3]Duffie and Singleton [1999] also propose a return of face value (RF) recovery assumption. In this case, the fractional loss L_t is simply a fraction of next period's promised amount or $(1 - \omega_t V_t + 1)$. They demonstrate that there is little difference between the results obtained using either the RF or the RM recovery assumption, a result that we also find here. For the sake of brevity, we omit mention of this in the main text.

[4]This also assumes that the credit default swap contract is not vulnerable. Specifically, the writer and the buyer of credit protection are not subject to credit risk. A weaker condition is that the writer and the buyer are equally vulnerable, or alternatively the credit arrangements are collateralized. See Bomfin [2002].

[5]Exhibit 2 shows that the very first Asian credit default swap's premium was not extraordinarily high. Furthermore the credit rating reported in Exhibit 4 is investment-grade, and the date of trade was September 8, 1997, prior to the Asian currency crisis. We conclude that the first Asian credit default swap was not subject to the Asian currency crisis. Also note that the Jarrow and Turnbull [1995] model results fail to converge using historical AA recovery rates, as the model did earlier in attempting to price the non-Asian swap of June 16, 1998.

[6]To assure ourselves that these results are not materially affected by our choices for the correlation between default-free and credit-risky interest rates, and for credit risk volatility, we recompute these results for a wide range of possible values for these parameters. We find that the premium value (6) and the default value (5) increase very slightly, less than one-half of a cent, as the hazard rate volatility increases by a factor of ten. Similarly, as hazard and Treasury rate correlation varies from -0.1 to ± 0.5, the default value changes only very slightly. Schönbucher [1999] finds the same results when pricing hypothetical credit default swaps.

REFERENCES

Altman, E., D. Cooke, and V. Kishore. "Defaults and Returns on High Yield Bonds: Analysis through 1998." New York University Salomon Center, 1999.

Bennett, O. "Documentation Dilemmas." *Risk*, 14 (2000), pp. S6-7.

Black, F., E. Derman, and W. Toy. "A One Factor Model of Interest Rates and its Application to Treasury Bond Options." *Financial Analysts Journal*, 46 (1990), pp. 33-39.

Bomfin, A. "Credit Derivatives and their Potential to Synthesize Riskless Assets." *The Journal of Fixed Income*, 12 (December 2002), pp. 6-16.

Cass, D. "Credit Protection Seller's Congress." *Risk*, 14 (2000), pp. S12-20.

Clewlow, L., and C. Strickland. *Implementing Derivatives Models.* New York: Wiley, 1998.

Collin-Dufresne, P., R. Goldstein, and J.S. Martin. "The Determinants of Credit Spread Changes." *Journal of Finance*, 56 (2001), pp. 2177-2207.

Delianedis, G., and R. Lagnado. "Recovery Assumptions in the Valuation of Credit Derivatives." *The Journal of Fixed Income*, 11 (March 2002), pp. 20-30.

Duffee, G. "The Relationship between Treasury Yields and Corporate Bond Yield Spreads." *Journal of Finance*, 53 (1998), pp. 2225-2241.

Duffie, D., and K. Singleton. "Modeling Term Structures of Defaultable Bonds." *Review of Financial Studies*, 12 (1999), pp. 687-720.

Helwege, J., and C.M. Turner. "The Slope of the Credit Yield for Speculative-Grade Issuers." *Journal of Finance*, 54 (1999), pp. 1869-1884.

Jarrow, R., and S. Turnbull. "Pricing Derivatives on Financial Securities Subject to Credit Risk." *Journal of Finance*, 50 (1995), pp. 53-85.

Nelson, C.R., and A.F. Siegel. "Parsimonious Modeling of Yield Curves." *Journal of Business*, 60 (1987), pp. 473-490.

Schönbucher, P. "A Tree Implementation of a Credit Spread Model for Credit Derivatives." Working paper, Department of Statistics, Bonn University, 1999.

"2001/2002 Credit Derivatives Survey." British Bankers Association, 2002.

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