The Cost of Tax Policy Uncertainty: Evidence from the Municipal Swap Market

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As of the end of 2001, there was $1.7 trillion in outstanding municipal debt, representing roughly 9% of the U.S. debt market. In 2001, municipal issuance amounted to $342.60 billion, of which long-term debt totaled $286.5 billion (84%).

There appears to be a clear preference for longer-term debt. One risk of investing in longer-term debt is a reduction in the investor's marginal tax rate. The lower the marginal tax rate, the less beneficial it is to have tax-exempt income. If the marginal tax rate is uncertain, there is similar uncertainty regarding municipal debt's investment performance.

The tax-exempt swap market, although still relatively new, provides a unique forum to study the relationship between taxable and tax-exempt interest rates. The taxable swap market is quite mature, and the bid-offer spread is smaller than that on U.S. Treasury securities at times. Interest rate swaps represent relatively pure exposures to interest rate movements. Collateralization agreements and the absence of principal payment make default risk negligible for interest rate swaps. Also, interest rate swaps are not callable, nor can they be advance- refunded.

Tax-exempt interest rate swaps are not tax-exempt. The cash flows are based on an index of tax-exempt rates, but the resulting cash flows are taxable to the swap counterparty. Our objective here is to develop measures of the difference between the tax-exempt and the taxable swap market, and to explore various interpretations of the empirical results.

It is well established that forward interest rates can be extracted from a series of zero-coupon bonds. Fama [1976, 1984], Brooks, Kim, and Livingston [1993], and others have documented that the implied forward interest rate is higher than the subsequently observed spot interest rate. Hence, forward interest rates are not unbiased predictors of future spot interest rates.

I show that information from both the tax-exempt and the taxable swap market can be used to estimate the implied marginal forward tax rate first developed by Kochin and Parks [1988]. The implied marginal forward tax rate incorporates information regarding market participants' views as to the future course of tax policy and the compensation that municipal investors require for taking the risk of changes in tax policy. For an example of tax policy debate influencing the municipal market, see Slemrod and Greimel [1999].

The tax risk premium is estimated at roughly 60 basis points, assuming the average long-term bond issue maturity is ten years. According to the Bond Market Association, 84% of the $1.7 trillion in municipal debt outstanding at the end of 2001 had been issued as long-term debt over the past 22 years. Hence, municipal governments are estimated to pay an additional $8 billion per year (60 basis points × $1.7 trillion × 84% long-term). As the debate over tax policy heats up, we can expect the tax risk premium to increase.

There are two ways to estimate the implied marginal forward tax rate: the swap approach, and the bond approach. In the swap approach, swap rates from both the taxable and the tax-exempt markets can be used to construct the implied forward rates in each market. The implied forward rates are then used to compute the implied marginal forward tax rate. In the bond approach, the same procedure can be used, starting with yields to maturity (or spot rates) from the taxable and tax-exempt bond markets.

The swap approach is based on the BMA (the Bond Market Association Municipal Swap Index) and LIBOR (London Interbank Offer Rate) swap rates. The floating interest rate used with municipal swaps was formerly called the PSA Municipal Swap Index. We refer to this floating interest rate throughout as the BMA rate (or BMA spot rate), and we refer to swap rates related to the BMA rate as the BMA swap rate. The bond approach is based on U.S. Treasury and general obligation, AAA, option-free municipal bond rates.

The research method adopted here parallels the well-known procedure for extracting forward interest rates from a series of zero-coupon bonds. The focus is on extracting the implied marginal forward tax rates from the taxable and tax-exempt forward curves.

The unbiased expectations hypothesis in the taxable market asserts that the forward interest rates are unbiased forecasts of future spot interest rates. We could pose a similar proposition regarding the implied marginal forward tax rate; that, is the implied marginal forward tax rate is an unbiased predictor of the expected future marginal tax rate. We do not find evidence to support this hypothesis. The implied marginal tax rate is much lower than the subsequent actual marginal tax rate.

The propensity for tax-exempt issuers to prefer long-term bonds appears to reflect a different pattern from that in other debt markets. For example, the average maturity of U.S. Treasury securities is projected to fall to under 5½ years by the end of 2002 (see www.treas.gov/press/releases/po3053.htm). There appears to be a preference for longer-term debt for municipal governments than for the federal government.

We illustrate a different perspective on this decision. According to the research presented here, a case can be made for shorter-term borrowing by municipalities.

This research is important as it sheds light on the task of efficiently managing a municipality risk-adjusted cost of debt. The municipality's objective is to achieve the lowest risk-adjusted cost of debt in the context of the economic duration of its assets, expected debt service costs, and roll-over risk. I evaluate the expected debt service cost by incorporating the impact of tax policy uncertainty.

I. MUNICIPAL SWAP RATES

I first review the theoretical framework for deriving equilibrium swap rates for both the taxable and the tax-exempt market. This theoretical framework is used to derive the implied marginal forward tax rate.

BMA Rates

The BMA rate is technically the BMA Municipal Swap Index or the Bond Market Association Municipal Swap Index. The BMA rate (or BMA index) is an index of rates based on high-grade, seven-day, tax-exempt, variable-rate demand obligations (VRDOs) that is reset each Wednesday. VRDOs are essentially floating-rate bonds. The index is constructed by Municipal Market Data, a Thomson Financial Services Company.

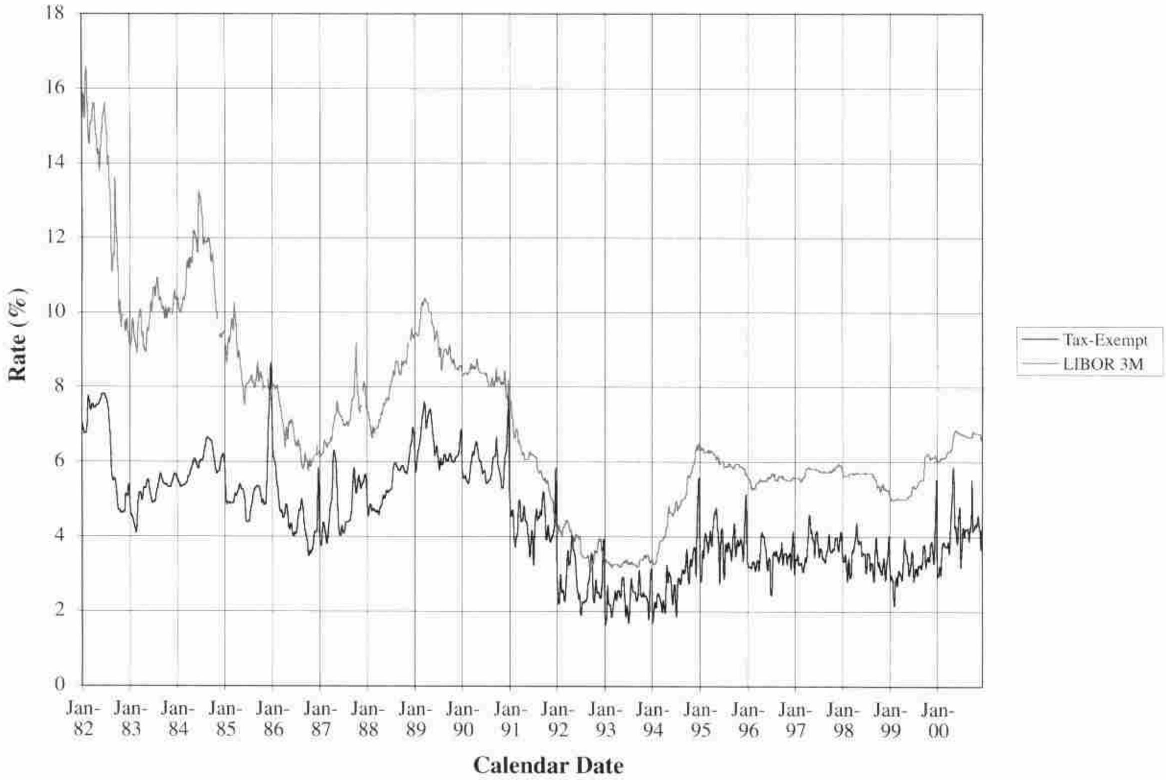
For a VRDO issue to be considered for the index, it must be reset weekly, effective on Wednesday; not be subject to the Alternative Minimum Tax; have at least $10 million outstanding; have the highest short-term rating (VMIG1 by Moody's or A-1+ by S&P); pay interest on a monthly basis; and be calculated on an actual/actual day-count basis. Issues included in the index are screened for outliers, and participating remarketing agents cannot represent more than 15% of the index. The BMA index includes roughly 250 issues in any given week.

Exhibit 1 provides weekly BMA index values and three-month LIBOR from January 7, 1992, through December 13, 2000. The BMA index is quite volatile, and seasonal trends are clearly evident, especially at year-end. The three-month LIBOR is a popular rate on which taxable interest rate swaps are based. Three-month LIBOR exhibits much less short-term variability and is less seasonal.

LIBOR is a very close substitute for the federal funds rate, which is managed by the Federal Reserve Bank. It essentially eliminates seasonal fluctuations in supply and demand for LIBOR. BMA index rates are not close substitutes for the federal funds rate, hence this seasonality. Although other proxies for the taxable rate are possible, such as U.S. Treasury rates, we use LIBOR because of its highly liquid swap market.

EXHIBIT 1

Tax-Exempt and Taxable Spot Interest Rates—January 7, 1982-December 13, 2000



NOTE: JJ Kenny (1/7/82-6/29/89), PSA (renamed BMA, 7/5/89-present), Nearby LIBOR futures (1/7/82-10/16/86), 3-Month LIBOR (10/23/86-present).

We assume that the tax-exempt rate is technically equal to the taxable rate times one minus the tax rate. The implied marginal tax rate ($IMTR_t$) from these two spot interest rates is:

$$IMTR_t = 1.0 - \frac{r_{TE,t}}{r_{T,t}} \tag{1}$$

where $r_{TE,t}$ denotes the tax-exempt spot rate at time t (BMA index), and $r_{T,t}$ denotes the taxable spot rate (LIBOR).

Exhibit 2 plots the *IMTR* based on the 90-day average of the BMA index and the initial three-month LIBOR as well as the highest marginal tax brackets for both individuals and corporations. Although there is variability in the *IMTR*, it does appear related to current tax rates. Notice that both corporate and individual tax rates move together over time. The ratio has also been more stable in the recent past.

The key rate negotiated in an interest rate swap is the fixed rate. In an interest rate swap, one counterparty receives the fixed rate and pays the floating rate, and the other counterparty receives the floating rate and pays the fixed rate. We examine the relationship between the two swaps markets, LIBOR and BMA, focusing on both floating rates and both fixed rates.

Exhibit 3 presents the ratio of the floating BMA index to the floating three-month LIBOR (spot rate ratio) as well as the ratio of ten-year fixed BMA swap rates as a percentage of ten-year fixed LIBOR swap rates. The swap rate data are provided by an anonymous source.

Over the period May 1992 through July 2000 (where swap rate data are available), the average spot rate ratio was 64%, and the average ten-year swap rate ratio was 73%. The spot rate ratio was almost four times as volatile as the swap rate ratio. Specifically, the standard deviation for the spot rate ratio was 2.7% and the standard deviation for the swap rate ratio 10.1%.

EXHIBIT 2
Average BMA Implied Marginal Tax Rate and Actual Tax Rates—January 7, 1982-December 13, 2000

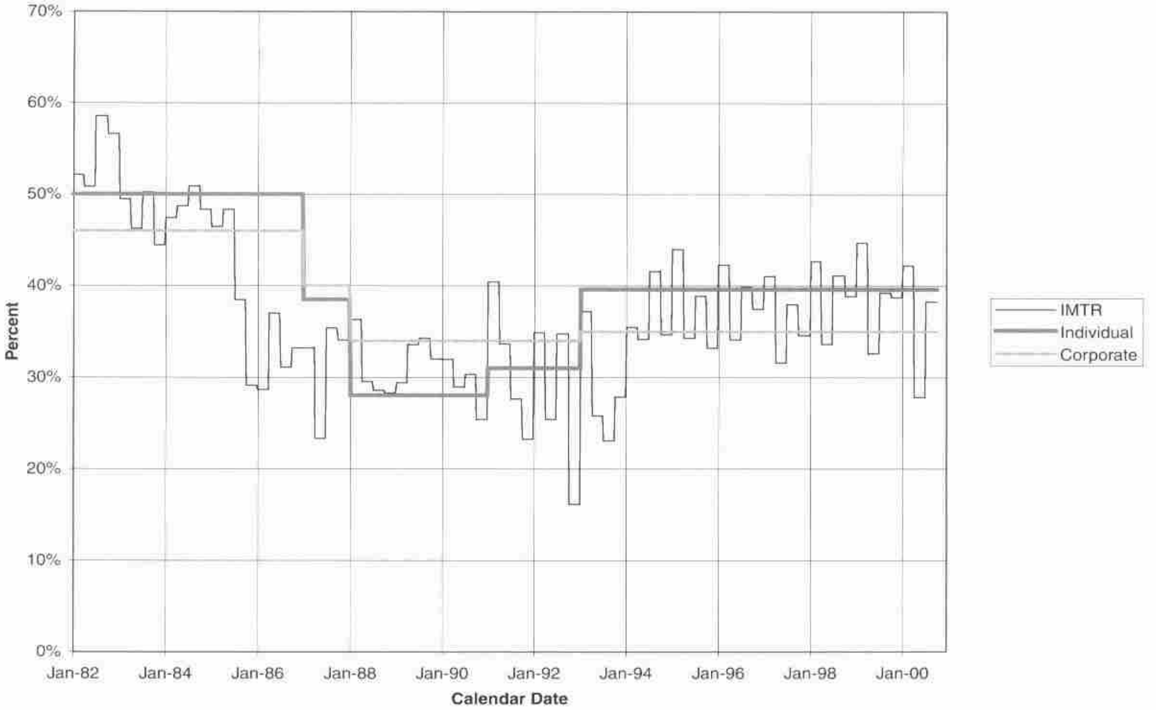


EXHIBIT 3
BMA Rates as Percent of LIBOR—May 15, 1992-July 5, 2000

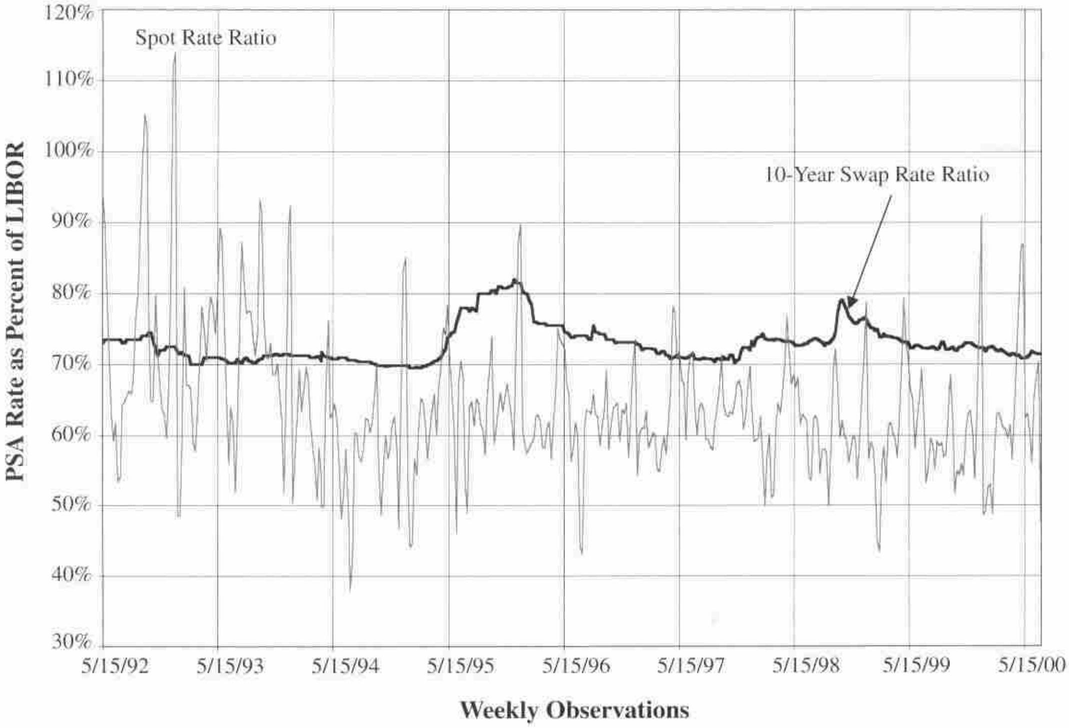


EXHIBIT 4
Swap Curve Based on Historical Average—November 7, 1996-July 7, 2000

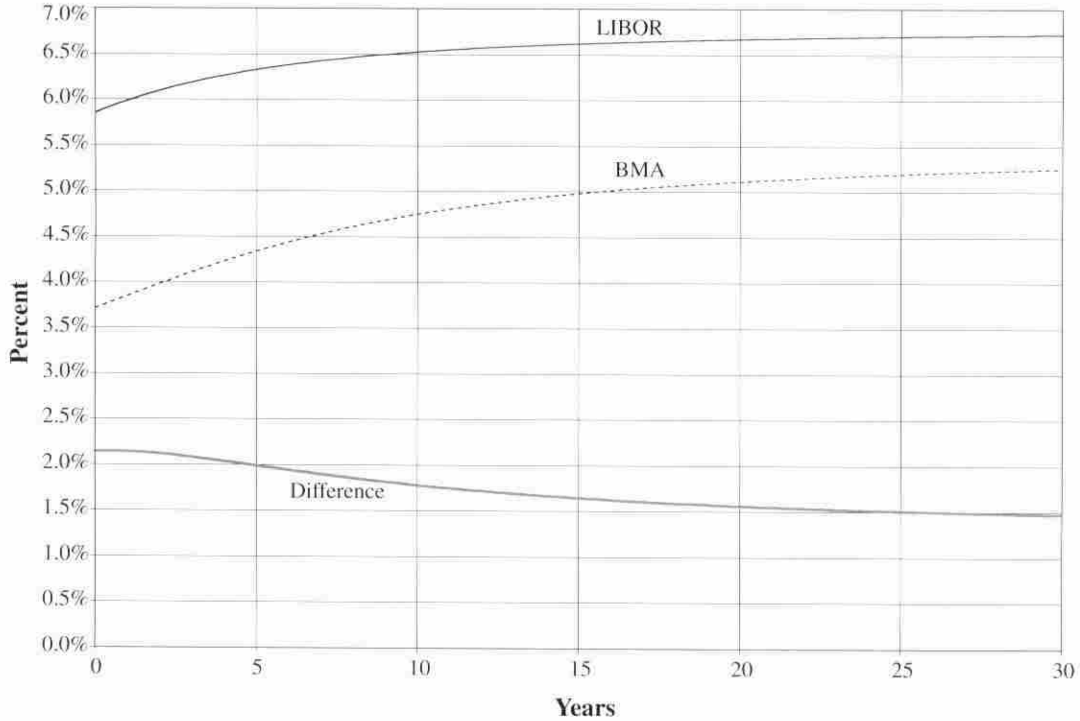


Exhibit 4 illustrates the swap curves for both BMA and LIBOR based on the average shape of each curve over the available data set, from November 1996 through July 7, 2000. Note that the difference between the 30-year LIBOR swap rate and the current LIBOR spot rate is 87 basis points, while the difference between the 30-year BMA swap rate and the current BMA spot rate is 154 basis points. The BMA swap curve is typically steeper than the LIBOR swap curve.

Implied Marginal Forward Tax Rates

To compute the implied marginal forward tax rate, we need the appropriate forward rates for both the taxable and the tax-exempt markets. Appendix A provides the notation. Appendix B reviews basic swap valuation for both taxable and tax-exempt swaps.

Swaps Market. Equilibrium swap rates are based on underlying forward interest rates. We can reverse this process and compute the implied forward rates when we have the swap curve. Appendix C gives the derivation of the forward rates, given the swap rates.

The notional principal for the swap values in Equations (2) and (3) are assumed to be \$1, or the same notional principal used with the discount factor.

Tax-Exempt Forward Rates from BMA Swap Rates:

$$f_{TE,0,n} = \hat{r}_{TE,fx,0,n} - \frac{SV_{TE,0,n-1}\left(\hat{r}_{TE,fx,0,n}, rec\,flt\right)}{DF_{T,0,n}} \tag{2}$$

LIBOR Forward Rates from LIBOR Swap Rates:

$$f_{T,0,n} = \frac{\hat{r}_{T,fx,0,n} DF_{T,0,n-1} - SV_{T,0,n-1}\left(\hat{r}_{T,fx,0,n}, rec\,flt\right)}{DF_{T,0,n-1} + SV_{T,0,n-1}\left(\hat{r}_{T,fx,0,n}, rec\,flt\right)} \tag{3}$$

where

$f_{TE,0,n}$ is the periodic forward tax-exempt interest rate observed at $t = 0$ for period n;

$SV_{TE,0,n-1}(\hat{r}_{TE,fx,0,n}, rec\,flt)$ is the swap value at t of a tax-exempt swap maturing in n periods with fixed swap rate, $\hat{r}_{TE,fx,0,n}$;

$DF_{T,0,n-1}$ is the taxable discount factor observed at $t = 0$ of \$1 received at time $n - 1$;

$f_{T,0,n}$ is the periodic forward taxable interest rate observed at $t = 0$ for period n; and

$SV_{T,0,n-1}(\hat{r}_{T,fx,0,n}, rec\ flt)$ swap value at $t = 0$ of a taxable swap maturing in $n - 1$ periods with fixed swap rate, $\hat{r}_{T,fx,0,n}$.

Note that for both the taxable and the tax-exempt forward rates, if the $n - 1$ swap value is positive (implying that the n-period equilibrium swap rate is lower than the $n - 1$ period equilibrium swap rate), the n-period forward rate is lower than the n-period equilibrium swap rate. Appendix B illustrates the application of Equations (2) and (3).

The ability to compute implied forward rates given the swap curve provides the foundation for computing the *IMFTR*. From Equation (1), we have

$$IMFTR_{t,n} = 1.0 - \frac{f_{TE,t,n}}{f_{T,t,n}} \tag{4}$$

Note that for $n = 0$, we have the implied marginal tax rate.

Bond Market. In the same way that forward rates can be derived from swap rates, forward rates can be derived from bond yields to maturity. Appendix D presents the derivation of the forward rates, given a set of par bond yields to maturity. The main difference here is that coupon payments from tax-exempt bonds are appropriately discounted at the tax-exempt rate.

Tax-Exempt Forward Rates from Tax-Exempt Bonds:

$$f_{TE,0,n} = 2\left[\frac{\left(1 + \frac{y_{TE,0,n}}{2}\right)DF_{TE,0,n-1}}{1 - \frac{y_{TE,0,n}}{2}\sum_{i=1}^{n-1} DF_{TE,0,i}} - 1\right] \tag{5}$$

Taxable Forward Rates from U.S. Treasury Bonds:

$$f_{T,0,n} = 2\left[\frac{\left(1 + \frac{y_{T,0,n}}{2}\right)DF_{T,0,n-1}}{1 - \frac{y_{T,0,n}}{2}\sum_{i=1}^{n-1} DF_{T,0,i}} - 1\right] \tag{6}$$

where the terms are defined in Appendix A.

The *IMFTR*, based on the bond markets, can be computed using Equations (5) and (6) and substituting into Equation (4).

There are several technical considerations in estimating the *IMFTR* in practice. First, plain vanilla swaps in the LIBOR and municipal swap markets have different compounding and day-count conventions. Municipal swaps are typically quoted on a quarterly compounding and quarterly payment frequency, but on an ACT/ACT day-count for the floating rate compared with a 30/360 day-count for the fixed rate. LIBOR swaps are typically quoted on a quarterly compounding and quarterly payment frequency with ACT/360 day-count on the floating side and semiannual compounding and semiannual payment frequency with 30/360 day-count on the fixed side.

Second, the LIBOR market has a highly liquid futures market, the Eurodollar futures market on the Chicago Mercantile Exchange. With the combination of the Eurodollar futures market and the LIBOR swap market, we can estimate a very accurate set of forward rates. The municipal forward market is not well developed, so our estimates are based solely on the municipal swap market.

Third, using swap market data avoids several technical problems with municipal bonds, including advanced refunding, issue-specific default risk, embedded call options, and problems with municipal bonds trading at a premium or a discount.

II. EMPIRICAL EVIDENCE

Although municipal swap data are not generally available, almost four years of both tax-exempt BMA index swap data and three-month LIBOR swap data were acquired. I analyze a complete series of weekly swap rates from November 7, 1996, through July 7, 2000. The objective is to measure the difference in the slopes of the taxable and the tax-exempt swap curves.

There are numerous ways to estimate the term structure. We apply Nelson and Siegel's [1987] parsimonious term structure model (denoted the LSC model for level, slope, and curvature):

EXHIBIT 5

Statistics on LSC Model of Taxable and Tax-Exempt Swap Rates—November 7, 1996-July 7, 2000, Weekly Data

	Level			Slope			Curvature		
	LIBOR	BMA	ITR*	LIBOR	BMA	Diff	LIBOR	BMA	Diff
Average	6.83%	5.54%	19.0%	−0.98%	−1.83%	0.85%	−0.09%	−1.04%	0.95%
Standard Deviation	0.51%	0.38%	1.18%	0.42%	0.30%	0.22%	0.55%	0.36%	0.44%
Maximum	7.79%	6.29%	21.4%	-0.05%	−1.26%	1.54%	1.85%	0.62%	2.00%
Minimum	5.85%	4.81%	15.6%	-2.02%	−2.72%	0.33%	−1.66%	−1.90%	−0.52%

* *Implied Tax Rate = 1 − (BMA/LIBOR).*

$$SC(l,s,c;m) = l + s\left\{\frac{\tau}{m}\left[1-e^{-m/\tau}\right]\right\} + c\left\{\frac{\tau}{m}\left(1-e^{-m/\tau}\left[\frac{m}{\tau}+1\right]\right)\right\} \quad (7)$$

where

- $SC(l, s, c; m)$ denotes the swap curve function of either the tax-exempt or taxable swap rates;
- l denotes level, or the long-term swap rate;
- s denotes slope, the difference between the short-term rate and long-term rate. If the swap curve is upward-sloping, then s is negative;
- c denotes curvature, or the medium-term component;
- m is maturity (in years); and
- τ is the time constant associated with curve hump positioning; it determines the rate of decay toward the long-term rate, and is set to 3 (see Willner [1996]).

The results from application of this curve-fitting model are reported in Exhibit 5. In Exhibit 5, we observe that the average long-term rate (level) suggests an implied tax rate of 19% (= 1 − 5.54/6.83), which appears too low for a long-term expected tax rate. This result is consistent with the empirical literature in the bond market.

The difference in slopes is statistically significant at 85 basis points. Hence, during this period, the BMA swap curve was 85 basis points steeper than the LIBOR swap curve. The steeper curve has implications for the cost of municipal debt relative to maturity. Depending on the interest rate risk profile, municipalities might consider shorter-term debt maturity policies to avoid paying the relatively higher interest rates.

The average R^2 is 0.97 for the LSC model applied to LIBOR and 0.99 for the LSC model applied to BMA, indicating the three-parameter model adequately describes the swap curves.

Applying Equations (2) and (3) to the average parameters reported in Exhibit 5, Exhibit 6 illustrates the forward curves for both LIBOR and BMA. Notice that the relatively steeper BMA swap curve results in even steeper BMA forward curve. Hence the forward difference declines faster than the difference in swap curves.

The BMA forward curve, it is hypothesized, has two components, the interest rate risk premium, and the tax policy risk premium. The BMA forward curve is decomposed into these two risk premiums by applying the slope of the LIBOR forward curve to the BMA forward curve.

The BMA forward curve based on the LIBOR slope is computed as:

$$f_{TE,0,n}(\textit{Taxable Slope}) = \left(\frac{r_{TE,0}}{r_{T,0}}\right) f_{T,0,n} \quad (8)$$

Therefore, the implied forward tax risk premium is the difference between the actual BMA forward curve and the BMA forward curve with LIBOR slope. Exhibit 7 illustrates this analysis based on the data in Exhibit 5.

If the residual difference between the actual BMA forward curve and the BMA forward curve based on the LIBOR slope is attributable to tax risk, then it is clear that uncertain tax policy is very costly to municipalities. Many municipal bonds are callable in ten years. The ten-year maturity forward tax policy risk premium is 125 basis points. If a ten-year maturity is assumed, applying the analysis in Equation (8) to the swap curve implies a swap tax policy

EXHIBIT 6
Forward and Swap Curves Based on Historical Average—November 7, 1996-July 7, 2000

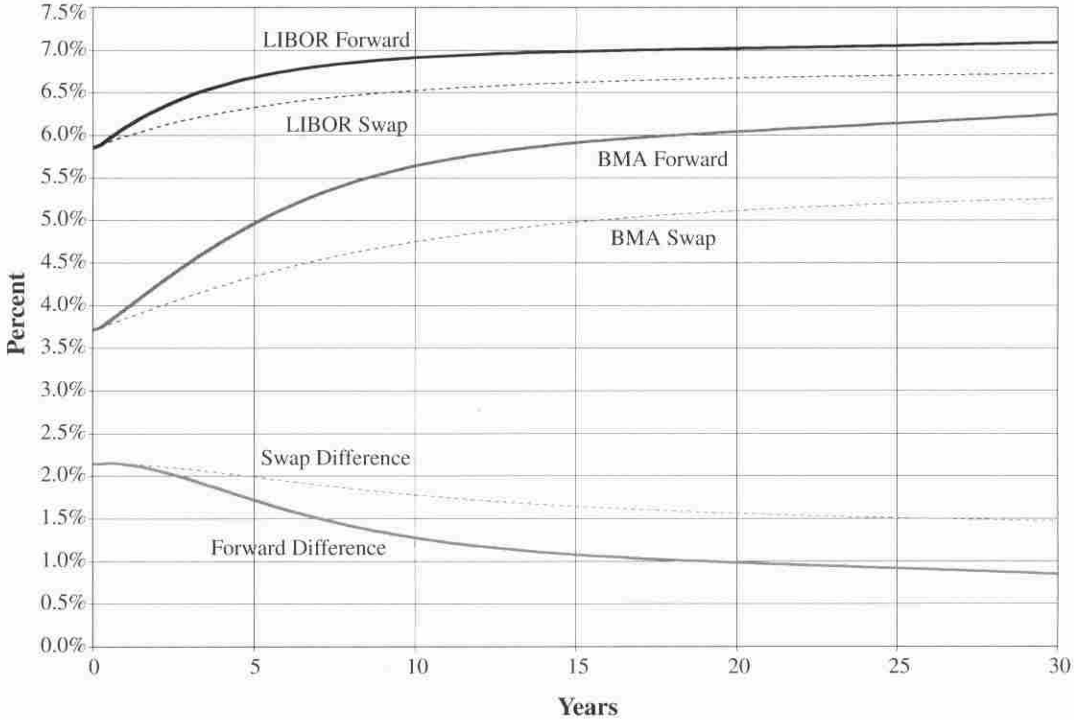


EXHIBIT 7
Average Implied Forward Tax Risk Premium—November 7, 1996-July 7, 2000

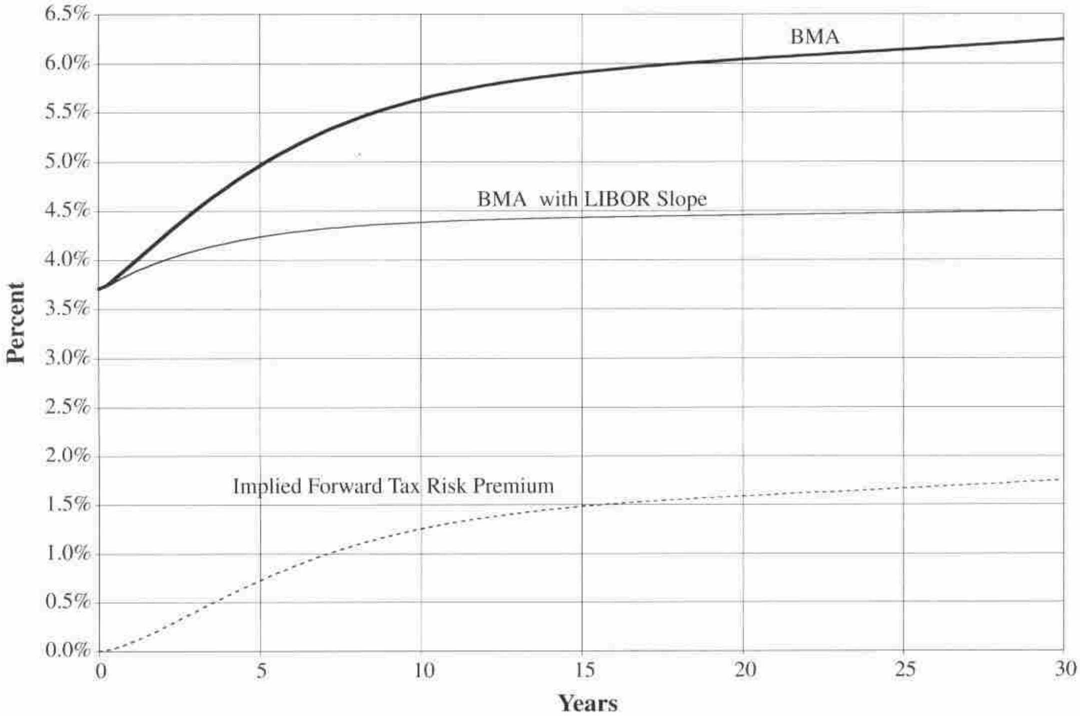
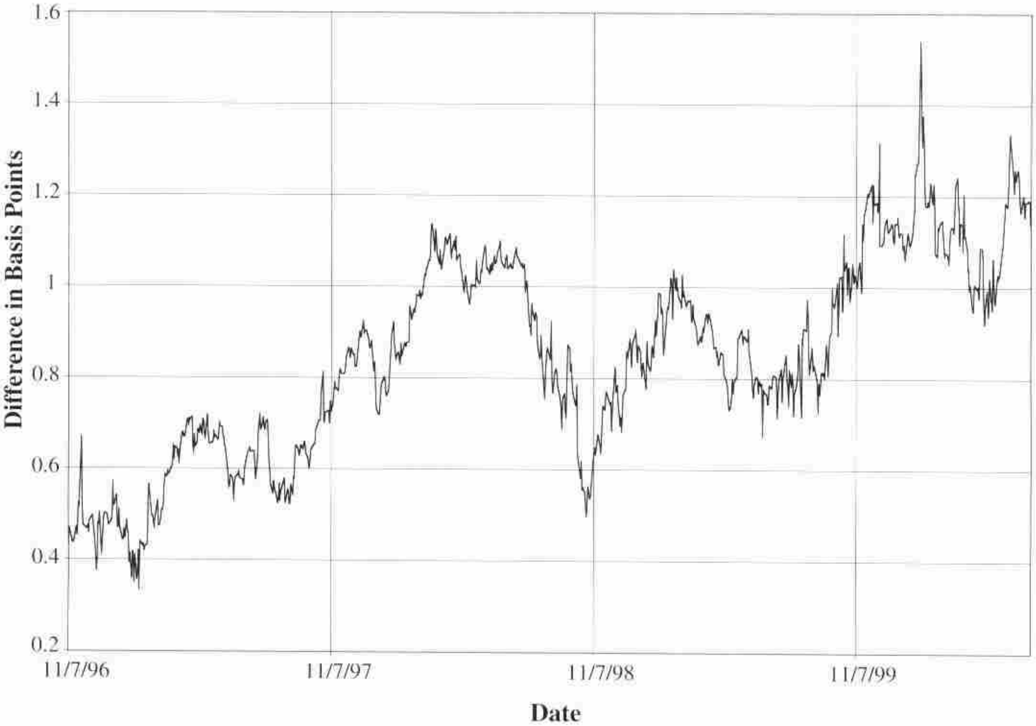


EXHIBIT 8

Difference in Slopes of BMA and LIBOR Swap Curves—LIBOR Slope Minus BMA Slope



risk premium of 61 basis points. Thus, a decision by municipalities to issue ten-year bonds instead of a floating-rate bond implies a payment of 61 additional basis points for subjecting bond investors to tax policy risk.

Exhibit 8 illustrates the difference in the LIBOR and BMA swap slope coefficients over this time period. Although there is variability in the difference in slopes, it is always positive. The general increase in the slope difference may suggest that market participants believe there is now more tax policy uncertainty. Remember that a tax rate reduction is an adverse event in the municipal bond market, as municipal bonds become relatively less attractive.

The difference in slopes in these swap markets should not be based on differences in default risk. Although LIBOR and BMA are subject to differences in default potential, it seems reasonable that these differences are accounted for in the current spot rate. Swaps involve payments of a fixed rate and a floating rate, so the only stochastic variable is the short-term spot rate. The difference in slopes is probably not due to liquidity differences, because again the valuation differences due to liquidity are fully reflected in the current spot rates. Swaps do not have embedded options.

While BMA is more volatile than LIBOR, this volatility appears to be idiosyncratic rather than systematic, and should not result in differences in slopes. For example, the correlation between first differences in the BMA spot rate and percentage changes in the S&P 500 index is –0.04 for this time period.

Green [1993] presents a model for the relationship between taxable and tax-exempt markets based on arbitrage activities within the taxable bond market. He asserts that taxable investors can synthetically create zero-coupon bonds that result in no tax liability until expiration, when there is a taxable capital gain. This arbitrage activity will influence the appropriate tax-exempt yield and hence the tax-exempt-to-taxable yield ratio. Empirical evidence in Green [1993] and Chalmers [1998] finds that Green's model cannot be rejected.

It remains somewhat unclear how high-tax entities can effectively defer taxes on investments. Yet we know that the ability to defer taxes influences the implied tax rate.

Therefore, there appear to be two potential reasons for the difference in slopes in taxable and tax-exempt swap markets: dealer arbitrage, and tax risk. While the ability

EXHIBIT 9
Percentage of Municipal Debt Issued as Short-Term

to defer taxes is a compelling reason for the difference between these two swap curves, we focus on tax risk.

Tax risk is increasingly a negotiated risk in BMA swaps. Here is an excerpt from an actual 1998 swap confirmation where the municipality (the end-user) will receive fixed and pay BMA index (source confidential):

> By reason of enacted legislation, constitutional amendment, judicial decision or decree, or any order, ruling (public or private), regulation or official release of the Department of Treasury or the Internal Revenue Service, or any successors thereto, the 60-day average of the BMA index as a percentage of the 60-day average of the LIBOR index has exceeded 80%, or the 90-day average of the BMA index as a percentage of the 90-day average of the LIBOR Index has exceeded 75%.

If a tax event occurs for this swap, the floating rate will be based on an alternative index of 70% of LIBOR. The tax law language has the effect of reducing the fixed rate on the swap. Conversations with a BMA swap dealer indicate that the cost for including this type of tax law language in a 30-year swap is from 25 to 60 basis points, depending on the actual wording of the events of the taxability section of the swap confirmation. This is consistent with our estimates of the difference in slopes, supporting the position that tax risk is a driving component of the difference in slopes.

III. IMPLICATIONS FOR POLICYMAKERS AND MUNICIPAL ISSUERS

It is generally agreed that forward interest rates are biased upward. This bias results in additional interest expense for issuing fixed-rate debt. For example, examining roughly ten years of daily data between 1986 and 1996, Brooks [1997] estimates the additional interest cost of issuing two-year fixed-rate debt over three-month LIBOR floating-rate debt to be approximately 60 basis points per year ($6,000 per $1 million of debt).

Exhibit 9 shows the percentage of municipal debt issued as short-term debt, according to data from the Bond Market Association. One could argue that municipal governments are behaving rationally by expanding their use of longer-term bonds for fear of potential damaging tax policy changes. Potential tax policy changes that would adversely impact the municipal bond market include a national sales tax or a flat tax. Both these policy changes would reduce the highest marginal tax rate, which would diminish the attractiveness of holding municipal bonds.

Unfortunately, the relative cost of issuing fixed-rate municipal bonds compared to issuing floating-rate municipal bonds is not constant. The ten-year swap rate ratio shown in Exhibit 3 is quite revealing. The period of April 1995 through early 1996 saw the flat tax campaign of Presidential candidate Steve Forbes. Hence Exhibit 3 verifies that tax policy uncertainty influences the relative cost of floating- and fixed-rate debt.

If greater tax policy uncertainty translates into increased cost for fixed-rate municipal debt, we would expect municipal finance directors to steer their entities toward greater use of floating-rate issues. According to Exhibit 9, finance directors are recently going in the right direction. Many municipal governments have tax revenues that are positively correlated with inflation, making the issuance of floating-rate debt both revenue-enhancing (they do not pay the tax risk premium or the interest rate risk premium) as well as risk-reducing (floating-rate debt is a natural hedge against inflation-sensitive revenues).

Finally, increased interest cost due to tax policy uncertainty is also costly to the federal government. As municipal debt servicing provides for more of municipal investors' future consumption needs, there will be reduced demand for purchasing taxable investments, such as U.S. Treasury securities. Hence, tax uncertainty results in an additional tax paid by municipal governments that indirectly is costly to the federal government.

IV. SUMMARY

My purpose has been to introduce the implied marginal forward tax rate that can be derived from information embedded in the taxable and tax-exempt swap markets or by comparing taxable and tax-exempt bond markets. According to the data examined here, issuers pay a premium in tax-exempt, longer-term bonds due to tax policy risk.

Using the implied marginal forward tax rate and limited empirical data, we estimate the tax risk premium at 60 basis points, which translates into a current market price of tax policy uncertainty of approximately $8 billion per year that is paid by municipal debt issuers. That is, the risk of lower marginal tax rates that would adversely impact fixed-rate tax-exempt bonds results in these bonds offering a higher yield to maturity. The additional yield required on longer-maturity tax-exempt bonds has implications for municipal finance directors, government policy analysts, and bond investors.

Computation of the implied marginal forward tax rate will provide interested parties additional information in their assessment of tax-exempt market conditions.

APPENDIX A
Notation

Mathematical notation is defined as follows.

$i = 0, 1, \ldots, n$ is calendar time divided by periods (years here);

T denotes taxable interest rate market;

TE denotes tax-exempt interest rate market;

$r_{T,t}$ is periodic interest rate observed at t, taxable market (annual rate);

$r_{TE,t}$ is periodic interest rate observed at t, tax-exempt market (annual rate);

$f_{T,t,i}$ is periodic forward taxable interest rate observed at t for period i;

$f_{TE,t,i}$ is periodic forward tax-exempt interest rate observed at t for period i;

$DF_{T,t,i}$ is taxable discount factor observed at t of \$1 received at point in time i;

$DF_{TE,t,i}$ is tax-exempt discount factor observed at t of \$1 received at time i;

NP is the notional principal of the swap;

NAD_i is number of accrued days in the i-th swap period;

$NTDY$ is number of total days in the year;

$\tilde{r}_{fl,TE,i}$ is risky future tax-exempt floating rate observed at the beginning of period i;

$r_{T,fx,t,n}$ is fixed rate on a taxable (LIBOR) swap;

$r_{TE,fx,t,n}$ is fixed rate on a tax-exempt (BMA) swap;

$\hat{r}_{T,fx,t,n}$ is equilibrium (zero swap value) fixed rate on a taxable (LIBOR) swap;

$\hat{r}_{TE,fx,t,n}$ is equilibrium (zero swap value) fixed rate on a tax-exempt (BMA) swap;

$SV_{T,t,n}(r_{T,fx,t,n}, \mathit{rec\ flt})$ is swap value at t of a taxable swap maturing in n periods with fixed swap rate, $r_{T,fx,t,n}$;

$SV_{TE,t,n}(r_{TE,fx,t,n}, \mathit{rec\ flt})$ is swap value at t of a tax-exempt swap maturing in n periods with fixed swap rate, $r_{TE,fx,t,n}$;

$\tilde{CF}_{T,i}$ is risky cash flow resulting from a taxable swap at time i;

$\tilde{CF}_{TE,i}$ is risky cash flow resulting from a tax-exempt swap at time i;

$IMFTR_{t,n}$ is implied marginal forward tax rate observed at t for period n;

$IMTR_t$ is the current implied marginal tax rate;

$SC(l, s, c; m)$ denotes the swap curve function of either the tax-exempt or taxable swap rates;

l denotes level, or the long-term swap rate;

s denotes slope, the difference between the short-term rate and long-term rate. If the swap curve is upward-sloping, then s is negative;

c denotes curvature, or the medium-term component;

m is the maturity (in years); and

τ is the time constant associated with curve hump positioning; it determines the rate of decay toward the long-term rate, and is set to 3 (see Willner [1996]).

APPENDIX B
Taxable and Tax-Exempt Swap Valuation

Let us consider a five-year, \$100 million notional principal (NP), annually reset (30/360 day-count) swap. Suppose further that the one-year spot rates and one-year forward rates are as shown in Exhibit B-1.

We select forward rates for BMA to keep a constant ratio between the taxable LIBOR and the tax-exempt BMA rate. This ratio is the focus of this article, and has not been constant across forward rate maturities; rather, it is almost always increasing.

It is important at this juncture to point out that we have made no assumptions regarding future course of interest rates. We have not even stated whether future interest rates follow a particular statistical distribution, such as normal or lognormal. Also, we do not make any restrictive assumptions regarding the behavior of the term structure of interest rates, such as whether the local expectations hypothesis holds. (The local expectations hypothesis states that expected holding-period returns are the same across the term structure.)

We can still precisely value LIBOR-based and BMA-based interest rate swaps using the forward rates in Exhibit B-1. (We have assumed an active forward rate agreement market in both LIBOR and BMA. The BMA market is not that liquid for FRAs.)

Because the swap is annually reset, NAD_i is equal to $NTDY$ for all i. The equilibrium five-year swap rate is 3.9058% for BMA, and 6.1028% for LIBOR, as illustrated below.

BMA Swap Calculation

$$\hat{r}_{TE,fx,0,n} = \frac{\sum_{i=1}^{n} DF_{T,0,i}\left(\frac{NAD_i}{NTDY}\right) f_{TE,0,i}}{\sum_{i=1}^{n} DF_{T,0,i}\left(\frac{NAD_i}{NTDY}\right)} = \frac{\sum_{i=1}^{n} DF_{T,0,i} f_{TE,0,i}}{\sum_{i=1}^{n} DF_{T,0,i}}$$

$$= \frac{0.952381(3.2000) + 0.900597(3.6800) + 0.846823(4.0640) + 0.792906(4.3520) + 0.741588(4.4288)}{0.952381 + 0.900597 + 0.846823 + 0.792906 + 0.741588}$$

$$= 0.224921(3.2) + 0.212691(3.68) + 0.199991(4.064) + 0.187258(4.352) + 0.175138(4.4288)$$

$$= 3.9058\%$$

EXHIBIT B-1
Forward Rates and Related Information

Year*	Taxable Rates (LIBOR)	Tax-Exempt Rates (BMA)	Ratio (BMA/LIBOR)	Discount Factor
Spot ($t = 0$)	5.0000%	3.2000%	64.00%	\$1.000000
1	5.7500%	3.6800%	64.00%	\$0.952381
2	6.3500%	4.0640%	64.00%	\$0.900597
3	6.8000%	4.3520%	64.00%	\$0.846823
4	6.9200%	4.4288%	64.00%**	\$0.792906
5				\$0.741588***

** Rates observed at beginning of the year, and FRA and swap cash flows occur at end of the year. Discount factor based on LIBOR.*

*** 0.64 = 4.4288%/6.9200%.*

**** \$0.741588 = \$0.792906/(1 + 0.0692).*

EXHIBIT B-2
Swap Rates and Related Information

Year	Taxable Rates (LIBOR)*	Tax-Exempt Rates (BMA)**	$SV_{T,0,n-1}(\hat{r}_{T,fx,0,n})$	$SV_{TE,0,n-1}(\hat{r}_{T,fx,0,n})$
1	5.0000%	3.2000%	–0.3472%*	–0.2222%**
2	5.3645%	3.4333%	–0.5728%	–0.3666%
3	5.6736%	3.6311%	–0.6904%	–0.4418%
4	5.9293%	3.7948%	–0.6060%	–0.3878%
5	6.1028%	3.9058%		

* *Based on Equation (C-6). Taxable forward rates and discount factors are given in Exhibit B-1.*
** *Based on Equation (C-1) with tax-exempt forward rates.*

LIBOR Swap Calculation

$$\hat{r}_{T,fx,0,n}=\frac{\sum_{i=1}^{n}DF_{T,0,i}\left(\frac{NAD_i}{NTDY}\right)f_{T,0,i}}{\sum_{i=1}^{n}DF_{T,0,i}\left(\frac{NAD_i}{NTDY}\right)}=\frac{\sum_{i=1}^{n}DF_{T,0,i}f_{T,0,i}}{\sum_{i=1}^{n}DF_{T,0,i}}$$

$$=\frac{0.952381(5.0000)+0.900597(5.7500)+0.846823(6.3500)+0.792906(6.8000)+0.741588(6.9200)}{0.952381+0.900597+0.846823+0.792906+0.741588}$$

$$=0.224921(5.0)+0.212691(5.75)+0.199991(6.35)+0.187258(6.8)+0.175138(6.92)$$

$$=6.1028\%$$

Exhibit B-2 illustrates application of Equations (C-1) and (C-6) from Appendix C to the data in Exhibit B-1. In practice, the swap rates are used as inputs in the model from which we derive implied forward rates.

APPENDIX C
Deriving the Implied Forward Rates from the Swap Market

Implied Forward Rates for Tax-Exempt Market

Our objective is to solve for the n-th forward rate from the tax-exempt swap market. From Appendix E, we have:

$$SV_{TE,0,n}=\sum_{i=1}^{n}DF_{T,0,i}NP\left(\frac{NAD_i}{NTDY}\right)\left(f_{TE,0,i}-r_{TE,fx,0,n}\right) \tag{C-1}$$

Factoring out the notional principal and noting that in equilibrium the value of the swap is zero, we have:

$$NP\sum_{i=1}^{n}DF_{T,0,i}\left(\frac{NAD_i}{NTDY}\right)\left(f_{TE,0,i}-\hat{r}_{TE,fx,0,n}\right)=0 \tag{C-2}$$

Dividing both sides by the notional principal and breaking the summation into two parts (from 1 to $n-1$ and n), we have:

$$DF_{T,0,n}\left(\frac{NAD_n}{NTDY}\right)\left(f_{TE,0,n}-\hat{r}_{TE,fx,0,n}\right)+\sum_{i=1}^{n-1}DF_{T,0,i}\left(\frac{NAD_i}{NTDY}\right)\left(f_{TE,0,i}-\hat{r}_{TE,fx,0,n}\right)=0 \tag{C-3}$$

Noting that the remaining summation is just the value of an $n-1$ period swap, Equation (C-3) can be expressed as

$$DF_{T,0,n}\left(\frac{NAD_n}{NTDY}\right)\left(f_{TE,0,n}-\hat{r}_{TE,fx,0,n}\right)+SV_{TE,0,n-1}\left(\hat{r}_{TE,fx,0,n},rec\,flt\right)=0 \tag{C-4}$$

Solving for the n-period forward rate, we have:

$$f_{TE,0,n} = \hat{r}_{TE,fx,0,n} - \frac{SV_{TE,0,n-1}\left(\hat{r}_{TE,fx,0,n}, rec\,flt\right)}{DF_{T,0,n}\left(\frac{NAD_n}{NTDY}\right)} \qquad \text{(C-5)}$$

Note that in the text we assume that each period is one year; hence NAD_n is equal to $NTDY$.

Implied Forward Rates for Taxable Market

Our objective is to solve for the n-th forward rate from the taxable swap market. From Appendix E, modified for the taxable market, we have

$$SV_{T,0,n} = \sum_{i=1}^{n} DF_{T,0,i} NP\left(\frac{NAD_i}{NTDY}\right)\left(f_{T,0,i} - r_{T,fx,0,n}\right) \qquad \text{(C-6)}$$

Recall that the n-th discount factor is defined as (assuming today is $t = 0$ and adjusting for day-count):

$$DF_{T,0,i} = \frac{DF_{T,0,i-1}}{1+\left(\frac{NAD_i}{NTDY}\right)f_{T,0,i}} \qquad \text{(C-7)}$$

Thus, the n-th period forward rate is also included in the n-th discount factor. Following a similar procedure as in the tax-exempt case, factoring out the notional principal and noting that in equilibrium the value of the swap is zero, we have:

$$NP\sum_{i=1}^{n} DF_{T,0,i}\left(\frac{NAD_i}{NTDY}\right)\left(f_{T,0,i} - \hat{r}_{T,fx,0,n}\right) = 0 \qquad \text{(C-8)}$$

Dividing both sides by the notional principal, and breaking the summation into two parts (from 1 to $n - 1$ and n), we have:

$$DF_{T,0,n}\left(\frac{NAD_n}{NTDY}\right)\left(f_{T,0,n} - \hat{r}_{T,fx,0,n}\right) + \sum_{i=1}^{n-1} DF_{T,0,i}\left(\frac{NAD_i}{NTDY}\right)\left(f_{T,0,i} - \hat{r}_{T,fx,0,n}\right) = 0 \qquad \text{(C-9)}$$

Noting that the remaining summation is just the value of an $n - 1$ period swap, Equation (C-9) can be expressed as:

$$DF_{T,0,n}\left(\frac{NAD_n}{NTDY}\right)\left(f_{T,0,n} - \hat{r}_{T,fx,0,n}\right) + SV_{T,0,n-1}\left(\hat{r}_{T,fx,0,n}, rec\,flt\right) = 0 \qquad \text{(C-10)}$$

Substituting for the n-th discount factor, we have:

$$\frac{DF_{T,0,n-1}}{1+\left(\frac{NAD_n}{NTDY}\right)f_{T,0,n}}\left(\frac{NAD_n}{NTDY}\right)\left(f_{T,0,n} - \hat{r}_{T,fx,0,n}\right) + SV_{T,0,n-1}\left(\hat{r}_{T,fx,0,n}, rec\,flt\right) = 0 \qquad \text{(C-11)}$$

Multiplying both sides by the denominator from the discount factor, we have:

$$DF_{T,0,n-1}\left(\frac{NAD_n}{NTDY}\right)\left(f_{T,0,n} - \hat{r}_{T,fx,0,n}\right) + \left(1+\left(\frac{NAD_n}{NTDY}\right)f_{T,0,n}\right)SV_{T,0,n-1}\left(\hat{r}_{T,fx,0,n}, rec\,flt\right) = 0 \qquad \text{(C-12)}$$

Isolating the n-th forward rate, we have:

$$f_{T,0,n}\left(\frac{NAD_n}{NTDY}\right)\left(DF_{T,0,n-1} + SV_{T,0,n-1}\left(\hat{r}_{T,fx,0,n}, rec\,flt\right)\right) = \hat{r}_{T,fx,0,n}DF_{T,0,n-1}\left(\frac{NAD_n}{NTDY}\right) - SV_{T,0,n-1}\left(\hat{r}_{T,fx,0,n}, rec\,flt\right) \qquad \text{(C-13)}$$

Solving for the forward rate:

$$f_{T,0,n} = \frac{\hat{r}_{T,fx,0,n}DF_{T,0,n-1}\left(\frac{NAD_n}{NTDY}\right) - SV_{T,0,n-1}\left(\hat{r}_{T,fx,0,n}, rec\,flt\right)}{\left(\frac{NAD_n}{NTDY}\right)\left(DF_{T,0,n-1} + SV_{T,0,n-1}\left(\hat{r}_{T,fx,0,n}, rec\,flt\right)\right)} \qquad \text{(C-14)}$$

Again recall that in the text we assume that NAD_n is equal to $NTDY$.

APPENDIX D
Deriving the Implied Forward Rates from the Bond Market

In the bond market, the implied forward rates are derived in a similar fashion in both the taxable and the tax-exempt markets. Cash payments from the municipal bond market are tax-exempt, unlike municipal swaps.

We assume the presence of par bonds across the term structure. Par bonds are bonds that are trading very close to par, so the yield to maturity is approximately the coupon rate. The required par bond yields are approximated by on-the-run U.S. Treasury bonds and municipal GO-Aaa bonds on Bloomberg (option-free). Both series are semiannual compounded on a 30/360 day-count basis.

Semiannual coupon bonds maturing in n years are typically valued as

$$P_n = \sum_{i=1}^{n} \frac{C}{\left(1+\frac{y}{2}\right)^i} + \frac{Par}{\left(1+\frac{y}{2}\right)^n} \tag{D-1}$$

where C is the semiannual coupon, and y is the yield to maturity. Recall our objective is to derive the implied forward rates. On the basis of forward rates, the value of the bonds can be expressed as:

$$P_n = \sum_{i=1}^{n} \frac{C}{\prod_{j=1}^{i}\left(1+\frac{f_j}{2}\right)} + \frac{Par}{\prod_{j=1}^{n}\left(1+\frac{f_j}{2}\right)} \tag{D-2}$$

Defining discount factors as:

$$DF_i = \frac{DF_{i-1}}{1+\frac{f_i}{2}} \tag{D-3}$$

we can express the bond price as:

$$P_n = C\sum_{i=1}^{n} DF_i + ParDF_n \tag{D-4}$$

Assuming \$1 par value and that bonds are trading at par, Equation (D-4) can be expressed as:

$$1 = \frac{y}{2}\sum_{i=1}^{n} DF_i + DF_n \tag{D-5}$$

Factoring out the n-th forward rate, we have:

$$1 = \frac{y}{2}\sum_{i=1}^{n-1} DF_i + \frac{y}{2}\frac{DF_{n-1}}{1+\frac{f_n}{2}} + \frac{DF_{n-1}}{1+\frac{f_n}{2}} \tag{D-6}$$

Solving for f_n, we have:

$$f_n = 2\left[\frac{\left(1+\frac{y}{2}\right)DF_{n-1}}{1-\frac{y}{2}\sum_{i=1}^{n-1} DF_i} - 1\right] \tag{D-7}$$

APPENDIX E
Deriving Equilibrium Swap Rates

Although most swaps have either quarterly or semiannual payment frequencies as well as specific details on day-counting, we illustrate here the basic principles assuming annual resetting of the rates (advanced set, settled in arrears) and whole years (30/360 day-count method).

One important insight in dealing with BMA swaps is that the cash flows from the swap are not tax-exempt to the taxable dealer. Hence, the appropriate discount rate for BMA swaps is based on the taxable interest rate.

Another way to justify this assertion is that BMA index-based derivative valuation models explain observed prices better when one uses taxable interest rates for discounting. The forward rate ($f_{T,t,i}$) is the cost of moving money over period i of the forward contract observed at time t in the taxable market (denoted by T), the discount factor can be expressed as:

$$DF_{T,t,i} = \frac{DF_{T,t,i-1}}{1+f_{T,t,i}} \tag{E-1}$$

where $DF_{T,t,0}$ = \$1.

The value of the cash flows from entering a receive floating, pay fixed, BMA-based swap can be expressed generally as:

$$SV_{TE,0,n}\left(r_{TE,fx,0,n}, rec\,flt\right) \Leftrightarrow \sum_{i=1}^{n} DF_{T,0,i}\tilde{CF}_{TE,i} \tag{E-2}$$

where $SV_{TE,0n}(r_{TE,fx,0n}, recflt)$ is the swap value at $t = 0$ of the tax-exempt (BMA) receive floating, pay fixed, swap maturing in n periods; $DF_{T,0,j}$ is the appropriate discount factor for each i-th cash flow using the taxable rate; $\tilde{CF}_{TE,j}$ is the risky cash flow resulting from the receive floating, pay fixed, BMA swap. The symbol $\Leftrightarrow$, although not an equality, denotes a relationship because the cash flows occur at future periods and are risky.

For BMA swaps, Equation (E-2) can be expressed more precisely as:

$$SV_{TE,0,n}\left(r_{TE,fx,0,n}, recflt\right) \Leftrightarrow \sum_{i=1}^{n} DF_{T,0,i} \times NP\left(\frac{NAD_i}{NTDY}\right)\left(\tilde{r}_{TE,0,i} - r_{TE,fx,0,n}\right) \tag{E-3}$$

where NP is the notional principal of the swap; NAD_i is the number of accrued days in the i-th swap period; $NTDY$ is the number of total days in the year (usually 360 or 365); $r_{TE,fx,0,n}$ is the fixed rate on the swap observed when the swap was transacted ($t = 0$); and $\tilde{r}_{TE,0,i}$ is the risky future tax-exempt floating rate observed at the beginning of period i.

We adopt the common swap convention of advanced set rates that are settled in arrears. Hence the forward rate is observed at the beginning of period i, and the discount factor provides the present value of \$1 from today to the end of period i. Technically, the BMA swap floating rates are an average of the seven-day rates observed over the quarter.

At this point, there are three basic approaches to evaluating the current value of the swap. First, we could adjust the discount factor using some risk premium. Second, we could assume a dynamic hedging strategy to mitigate the floating-rate risk similar to option pricing techniques. Third, we could apply a market comparables approach (MCA). For a detailed analysis of the different approaches to valuing interest rate swaps in general, see Brooks [1998].

MCA does not rely on any assumption about the distribution of interest rates, so it is the method of choice here. Assuming no trading costs or other market frictions and the existence of a forward rate market for LIBOR and BMA, we can costlessly eliminate the risky future BMA floating rate with a forward rate agreement (FRA) on BMA. We also assume that we can borrow and lend via the LIBOR FRA market.

LIBOR, the London Inter-Bank Offer Rate, is a good approximation of swap dealers' marginal cost of funds. Therefore it is a good interest rate to discount cash flows related to interest rate swaps. Because FRAs are costless to enter, we can replace the floating rate in the valuation equation with the FRA market rate. Specifically, the market prices of the floating rates in the BMA swap are the current forward rates. Hence our BMA swap valuation equation for receiving floating is:

$$SV_{TE,0,n}\left(r_{TE,fx,0,n}, recflt\right) = \sum_{i=1}^{n} DF_{T,0,i} NP\left(\frac{NAD_i}{NTDY}\right)\left(f_{TE,0,i} - r_{TE,fx,0,n}\right) \tag{E-4}$$

The fixed rate on the swap that gives it a current value of zero is called the market swap rate. Solving for this fixed rate (denoted with a hat and emphasizing this is an n-th maturity swap), and noting that the notional principal can be cancelled out, we have:

$$\hat{r}_{TE,fx,0,n} = \frac{\sum_{i=1}^{n} DF_{T,0,i}\left(\frac{NAD_i}{NTDY}\right) f_{TE,0,i}}{\sum_{i=1}^{n} DF_{T,0,i}\left(\frac{NAD_i}{NTDY}\right)} = \sum_{i=1}^{n} w_i f_{TE,0,i} \tag{E-5}$$

where

$$w_i = \frac{DF_{T,0,i}\left(\frac{NAD_i}{NTDY}\right)}{\sum_{i=1}^{n} DF_{T,0,i}\left(\frac{NAD_i}{NTDY}\right)} \tag{E-6}$$

Note that the sum of w_i equals one. Hence, the equilibrium swap rate can be viewed as a weighted average of forward rates, where the weights are a complex function of the taxable forward rates. Appendix B provides more details on swap valuation and swap rates.

ENDNOTE

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REFERENCES

Brooks, Robert. "Approaches to Valuation Illustrated with Interest Rate Swaps." *Derivatives Quarterly*, 4 (1998), pp. 51-62.

——. "Interest Rate Modeling and the Risk Premiums in Interest Rate Swaps." Charlottesville, VA: The Research Foundation of the Institute of Chartered Financial Analysts, 1997.

Brooks, Robert, Myung J. Kim, and Miles Livingston. "The Unbiased Expectations Hypothesis and Error-Learning." *Advances in Quantitative Analysis of Finance and Accounting*, 2 (Part A) (1993), pp. 105-114.

Chalmers, John M.R. "Default Risk Cannot Explain the Muni Puzzle: Evidence from Municipal Bonds that are Secured by U.S. Treasury Obligations." *The Review of Financial Studies*, 11 (1998), pp. 281-308.

Fama, Eugene F. "Forward Rates as Predictors of Future Spot Interest Rates." *Journal of Financial Economics*, 3 (1976), pp. 361-377.

——. "The Information in the Term Structure." *Journal of Financial Economics*, 13 (1984), pp. 509-528.

Green, Richard C. "A Simple Model of the Taxable and Tax-Exempt Yield Curves." *The Review of Financial Studies*, 6 (1993), pp. 233-264.

Kochin, Levis A., and Richard W. Parks. "Was the Tax-Exempt Bond Market Inefficient or Were Future Expected Tax Rates Negative?" *Journal of Finance*, 43 (1988), pp. 913-931.

Nelson, Charles R., and Andrew F. Siegel. "Parsimonious Modeling of Yield Curves." *Journal of Business*, 60 (1987), pp. 473-489.

Slemrod, Joel, and Timothy Greimel. "Did Steve Forbes Scare the U. S. Municipal Bond Market?" *Journal of Public Economics*, 74 (1999), pp. 81-96.

Willner, Ram. "A New Tool For Portfolio Managers: Level, Slope, and Curvature Durations." *The Journal of Fixed Income*, Vol. 5, No. 1(1996), pp. 48-59.