Hedging and Replication of Fixed-Income Portfolios

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Recent bond markets have been characterized by unprecedented spread widening and spread volatility. There are substantially more fixed-income derivatives traded, and they are more liquid. Phenomenal growth in the Eurodollar futures and the swaps markets and the introduction of exchange-traded derivative contracts on swaps allow investors to much more easily hedge the risk of a bond portfolio subject to spread risk.

Derivatives have been widely used in financial hedging applications for decades now. Some studies analyze the relationship between certain fixed-income derivatives and specific securities, focusing on the mechanics of hedging operations.[1] Empirical research looks mainly at the closeness of the relationship between equity markets and equity derivatives.[2] We examine the use of derivatives in the more general context of hedging and replication of diversified fixed-income portfolios.

In hedging, derivatives are used to neutralize some or all of the systematic risk exposures of bond portfolio or liability position. Hedging activities can modify the risk profile of an asset or liability position in order to realize a profit from a perceived undervaluation of a portfolio, or to neutralize shocks expected to impact the portfolio in the future.

The goal of index replication is to achieve returns nearly identical to those of a targeted benchmark without actually taking cash positions in the securities that constitute the index. This can be accomplished by using derivatives to match the term structure exposures of the index. Meanwhile, the funds available for investment are placed in short-term instruments. This technique can be used to synthetically create a return profile very similar to that of the benchmark.

Replication is used in a variety of applications. Examples are the management of cash in- and outflows and the initial start-up of a fund.[3] Tax and liquidity issues can motivate some investors to use derivatives in certain markets. "Portable alpha" investors rely upon a close fit of the returns of a replicating derivatives portfolio to the underlying index. In this technique, expertise in outperforming one benchmark may be applied to help outperform another. Derivatives are used to transfer excess returns from one benchmark to another.

Hedging and replication are two closely related uses of derivatives that are nearly opposite in purpose but almost identical in practice. Simply put, derivatives are used in hedging applications to cancel out the risk exposures of securities in a portfolio; in replication, they are used to synthetically reproduce the risk profile of securities that are not held. The same derivatives positions can be used to match the target in either case. The hedger would end up with long positions in the actual securities and a short position in the derivatives portfolio, while the replicating portfolio would have long positions in the derivatives portfolio and in cash.

In designing a derivatives portfolio for hedging or index replication, the same issues must be considered for either application. We focus on index replication, but our results are equally applicable to hedging. The key performance measure in replication is the tracking error—the volatility of the return difference between the benchmark and the replicating portfolio (derivatives and cash).

The short-term interest earned on the cash position in the replicating portfolio (here assumed to follow LIBOR) is much less volatile than the index returns. Therefore, the tracking errors observed in hedging applications (when there is no cash balance, but financing costs may be a consideration) should be almost identical to those observed in index replication. Mean returns may need to be adjusted to reflect short-term rates different from short-term LIBOR.

We evaluate to what degree diversified fixed-income portfolios can be hedged or replicated using several different techniques. We apply various replication strategies to seven Lehman Brothers fixed-income indexes. They are selected to proxy diversified fixed-income portfolios that emphasize different sectors of the U.S. fixed-income market.

The Lehman Brothers Aggregate Index is the dominant institutional benchmark for fixed-income investing in the U.S. We look at the replication of this index, as well as its most important components: the Treasury Index, the Agency Index, the Investment Grade Credit Index, and the Mortgage Index. We also consider the replication of another widely used benchmark, the Government-Credit Index, which consists of all the bonds in the Treasury, the Agency Index, and the Credit Index. To estimate the replication properties of less-diversified portfolios, we also replicate a portfolio of all double-A rated financial bonds that are in the Credit Index. For each index, we investigate the mean of the outperformance of replicating derivatives portfolios over the index returns and their standard deviation or tracking error.

The replication strategies use three different types of derivative products: Treasury futures and Eurodollar futures and swaps. When an index is replicated with a credit component using Treasury futures alone, the spread risk of the index is left totally unhedged, giving rise to large tracking errors. The incorporation of Eurodollar futures and swaps, which include some spread exposure, is intended to improve tracking in such situations.

We also consider hybrid strategies combining different types of derivative products. Hybrid strategies help improve performance in two ways. First, by partitioning the index, one can choose the best hedging instrument for each market segment, rather than using a single type of instrument for the entire index. Second, the replication errors in different market segments tend to be less correlated when different types of hedging instruments are used. This diversification of basis risk reduces the overall tracking error.

We investigate two different approaches to constructing the hedge ratios that determine how much of each hedging instrument should be used to match an index return. In the analytical approach, the durations of the index and the derivative products are used to match the two sets of yield curve sensitivities. In the empirical approach, regression techniques are applied to historical returns to determine the hedge ratios that would have provided the best fit over some recent time period. We discuss the relative advantages of each method and compare the achieved performance.

Several main points emerge from this study. First, particular attention has to be paid to the replication of spread risk, particularly in the current high spread volatility environment. Increased spread volatilities cause much higher tracking errors for both hedging and replication. Second, replication approaches that combine different kinds of derivatives (hybrid replication strategies) are usually preferable from a tracking error perspective.

Both of these observations argue for the inclusion of Eurodollar futures and swaps in replication strategies in addition to Treasury futures. Users of derivatives in hedging and replication might therefore want to consider investment guidelines that allow Eurodollar futures and swaps.

I. HEDGING INSTRUMENTS, INDEX DATA, AND SIMULATION METHODOLOGY

Hedging Instruments

The replication strategies we study use three different types of derivatives: Treasury futures, Eurodollar futures, and swaps. We review these instruments, and compare their relative strengths as hedging vehicles.

Four futures contracts on U.S. Treasury bonds and notes are currently traded on the Chicago Board of Trade: a two-year, a five-year, and a ten-year note contract, as well as a contract on bonds with a remaining maturity of more than 15 years. At expiration, a Treasury security from a basket of acceptable notes or bonds has to be delivered by an investor short the corresponding contract. The contracts are written on a quarterly calendar, with delivery dates in March, June, September, and December. Daily historical closing prices are readily available; we calculate monthly profit and loss for each contract using the closing prices for the last day of each month. Notional amounts on the Treasury futures contracts are $200,000 for the two-year note contract, and $100,000 for the three longer maturities.[4]

Eurodollar futures contracts cover rates on three-month LIBOR deposits that start at some future time. Eurodollar futures contracts expire on the Monday before the third Wednesday of every March, June, September, and December. Each contract controls the interest on a three-month LIBOR time deposit with a notional value of $1 million that settles two business days after the expiration date of the contract. For example, the Eurodollar futures contract that expires on December 16, 2002, cash-settles for one quarter year's simple interest on $1 million, based on the three-month LIBOR on that date.

Since Eurodollar futures contracts are traded out to ten years maturity, they span the forward yield curve on LIBOR deposits out to about ten years.[5] As shown in Appendix A, the contracts beyond five years are largely illiquid, as are the additional contracts that do not follow the quarterly cycle described above. For this reason, many investors and traders use the Eurodollar futures markets only out to five years. We use only quarterly contracts with five years or less to maturity.

The third set of instruments we consider for index replication are fixed-for-floating interest rate swaps in which the investor pays one-month LIBOR and receives a fixed rate.[6] As our replicating portfolio includes a long cash position assumed to earn one-month LIBOR, the floating side is largely cancelled out, and the portfolio essentially consists of long positions in the fixed side of the swaps.[7]

Our motivation for considering swaps-based replication is that the swaps curve implicitly includes a credit spread component that that is generally considered to correspond roughly to double-A rated corporate bonds. This suggests that swaps might provide a better hedge for indexes rich in spread product, especially the credit and related indexes.

While swaps can be written to any desired maturity, we use swaps of four maturities roughly matched to the four Treasury futures contracts: 2, 5, 10, and 30 years. Choice of these maturities facilitates the comparison of results obtained using swaps and Treasury futures; in addition, they are standard market benchmarks, and hence extremely liquid. It is assumed that swaps are traded in units of $1 million.

Investors may have various reasons for preferring one of these instruments to another. As Treasury futures entail various types of optionality, the determination of the precise yield curve exposure of a given contract is not as straightforward as with Eurodollar contracts or swaps.

Treasury futures and Eurodollar futures are exchange-traded, while swaps are over-the-counter contracts usually entered into by either an investor or an issuer and a derivatives dealer as counterparties. Despite the substantial increase in the notional volume of the swaps markets, numerous market participants prefer the convenience and liquidity of standardized exchange-traded derivatives contracts.

Many investors are subject to limits on their notional derivatives exposure as a safeguard against overleveraging. This can present some difficulty in the case of Eurodollar futures contracts. The value of a Treasury futures contract is based on the future value of actual bonds, including principal as well as interest. A Eurodollar futures contract, by contrast, controls only three months of interest, with no principal attached. Treasury futures therefore show much greater price changes for any change in interest rates. As a result, a desired yield curve exposure requires a much higher notional amount of Eurodollar futures than Treasury futures. Eurodollar futures are therefore often excluded from replication applications.[8]

Treasury futures and swaps can be used across the entire maturity spectrum, but Eurodollar futures are most liquid at the short end. We therefore use them only out to five years, in combination with Treasury futures at the long end.

Historical Index Data

We use historical data on the Lehman Brothers U.S. Aggregate index and six component indexes, including monthly data from January 1994 through September 2001, for a total of 93 observations. This period includes the financial crisis of the late summer and fall of 1998, the subsequent spread volatility related to Y2K, and the later economic weakening. To evaluate different market regimes, we present separate results for the full period, the "calm period" from January 1994 through June 1998, and the "volatile period" from July 1998 through September 2001.

Exhibit 1 provides a summary of index performance over the full period and the two subperiods. The mean returns for the full period are quite similar for the seven indexes, with a span of three basis points per month from lowest to highest.

Within each subperiod, however, we see a much greater divergence among the returns of the various

EXHIBIT 1

Summary Statistics of Index Returns, 1/94-9/01, bp/month

	Mean			Standard Deviation		
Index	1/94-9/01	1/94-6/98	7/98-9/01	1/94-9/01	1/94-6/98	7/98-9/01
Aggregate	58.1	57.6	58.8	108.8	120.6	91.4
Gov.-Cred.	57.2	57.1	57.3	120.1	131.2	104.5
Treasury	56.9	55.3	59.2	118.8	125.2	110.7
Agency	58.0	56.3	60.4	110.5	121.9	94.1
Credit	58.4	62.6	52.5	138.6	152.3	118.7
AA-Finance	57.5	54.3	62.0	111.2	117.7	102.9
Mortgage	59.9	59.0	61.1	90.5	101.7	73.4

EXHIBIT 2

Means and Volatility of Monthly Changes of Index Option-Adjusted-Spreads, 1/94-9/01, bp/month

	Mean			Volatilities		
Index	1/94-9/01	1/94-6/98	7/98-9/01	1/94-9/01	1/94-6/98	7/98-9/01
Aggregate	22.2	12.3	36.7	2.3	1.4	3.2
Gov.-Cred.	34.0	17.7	58.0	3.6	2.1	5.1
Treasury	0.1	–0.1	0.4	0.1	0.0	0.1
Agency	26.7	14.9	44.2	5.5	4.5	6.8
Credit	95.3	64.3	140.9	9.6	6.6	13.1
AA-Finance	70.3	45.3	107.0	9.5	6.5	12.8
Mortgage	65.2	54.1	89.6	10.6	7.6	14.8

indexes. The mean return of the Credit Index declined by about 10 bp per month from the calm to the volatile period, while the mean returns of the double-A Finance Index, a component of the Credit Index, increased by about 8 bp per month in the volatile period. These movements in returns reflect spread movements due to changes in risk perceptions by investors in these sectors of the fixed-income market.

The differences in return volatilities among the indexes are more pronounced. These range from a low of 90.5 bp per month for the Mortgage Index to a high of 138.6 bp per month for the Credit Index.

Given the levels of the return volatilities during the two subperiods, one might question the calm and volatile labels. All indexes exhibit higher return volatilities in the earlier subperiod than in the most recent 39 months. This latter period was more volatile not in terms of interest rate movements, but rather in terms of the relationship between Treasuries and all other securities.

Exhibit 2 shows the means and volatilities of the spreads of the seven indexes replicated, expressed as option-adjusted spreads (OAS).[9] Average spreads in the second subperiod are double or triple those in the first. Credit spread volatility increased by about a factor of two from the calm to the volatile period, while the spread volatility of the Agency Index increased by about 50%. This dramatic increase in spread volatility is reflected in increased tracking errors for all the replication methodologies tested.

Simulation Methodology

The same general methodology is followed to simulate the historical performance of each of the replication strategies studied. On December 29, 1993, we set up replicating portfolios of $100 million. Each replicating portfolio is constructed to have the same interest rate sensitivities as an investment of this size in the underlying index. On the last business day of January 1994, we calculate the change in value of the replicating portfolio and divide it by $100 million to obtain the portfolio return.

Since buying futures saves an investor the cash outlay for the underlying securities, futures will underperform those securities by approximately the interest rate that would accrue to a cash deposit due on the expiration date. To bring the index and the replicating portfolio to equal footing, we therefore add one-month LIBOR to the return of the replicating futures portfolio.[10] This way, the derivative portfolios, which by themselves have values of zero at the start of each month, have an investment associated with them, on which returns can be computed.

The replicating portfolio is then rebalanced at the end of the month, based again on a $100 million asset value. The rebalancing allows us to adjust the replication portfolio according to changes in the index or the characteristics of the futures contracts. The index return for the next month is compared with the return of this new portfolio. This procedure is repeated for each month in the data sample, providing a seven and three-quarter year history of monthly performance for each combination of target index and replication strategy. For each historical simulation, we track the mean and standard deviation of three quantities: the index return, the return of the replication derivatives portfolio, and the difference between the two.

We do not include estimates of transaction costs, because they can vary between different accounts, and depend on the services provided to the investor by the futures clearing and execution firm. Transaction costs have historically dropped, and incorporating any estimates into the historical simulations would not provide a forward-looking picture of returns net of transaction costs. Instead, we later compute the approximate transaction costs of each strategy, both at initial start-up and over the long term.

For simplicity, we also do not include a model for the effect of the margin requirements associated with futures accounts. Up to 2% of the notional futures position has to be deposited with the clearing firm in the form of T-bills. In addition, when replication strategies are pursued in practice, the investor will keep a certain amount of cash in overnight deposits to fulfill potential future variation margin requirements. This will usually be another few percent of invested capital. The impact on the total returns of the replication strategies from investing these funds at rates different from one-month LIBOR is restricted to the differential between one-month LIBOR and the returns on the other short-term investments. As these differentials are typically quite small, margin requirements should not have a material effect on overall performance.

II. REPLICATION USING ANALYTICAL DURATIONS

Treasury Futures Replication

To determine the precise Treasury futures positions employed to replicate an index, we use a cell-matching approach. The index is divided into four duration cells, and the yield curve risk of each cell is matched using one contract. The first cell covers all bonds with modified adjusted duration up to three years, and it is replicated using the two-year note futures contract. The next cell covers the part of the index with duration from three to five years, and is replicated with five-year note contracts. The 5.0-7.5 year duration cell uses ten-year note contracts, and the fourth cell, with durations of 7.5 years and up, uses bond contracts.[11]

For the Mortgage Index we use only three cells, due to the dearth of mortgage-backed securities with 7.5+ years' duration. All mortgage bonds with more than five years in modified adjusted duration are aggregated in a 5+ year duration cell and replicated using ten-year note contracts.

In the analytical approach, the number of contracts used in the replication of each index cell is determined by the ratio of durations between the index cell and the appropriate futures contract. The duration of each index cell is the market capitalization-weighted average duration of all its member bonds. This is used to find the dollar sensitivity of the notional index investment to a parallel shift in the yields of all bonds in that duration cell.

The duration of each of the four futures contracts is the modified adjusted duration computed using a multifactor bond futures model.[12] This is an option-adjusted sensitivity to a parallel shift in the yields of all bonds in the deliverable basket. The duration cells are designed to provide a good correspondence between the yield curve sensitivities of the index cells and the associated futures contracts. Dividing the dollar sensitivity of a particular index cell by the dollar sensitivity of the appropriate futures contract gives the number of contracts used in the replication for that cell. A short-term investment in the notional amount of $100 million completes the replication portfolio.

At the end of each month of the simulation, we calculate the return of the replicating portfolio. The profit or loss on each futures position is obtained by multiplying the number of contracts by the notional value per contract by the change in contract price from one month-end to the next. The changes in the values of the four futures posi-

EXHIBIT 3

Treasury Futures Replication, 1/94-9/01, bp/month

Index	Mean Outperformance 1/94-9/01	1/94-6/98	7/98-9/01	Tracking Error 1/94-9/01	1/94-6/98	7/98-9/01	R^2 1/94-9/01	1/94-6/98	7/98-9/01
Aggregate	3.4	2.1	5.3	25.1	12.7	36.0	94.7	98.9	84.5
Gov.-Cred.	3.9	3.2	4.9	25.1	10.4	37.0	95.6	99.4	87.5
Treasury	3.2	4.2	1.9	10.9	8.5	13.6	99.2	99.5	98.5
Agency	2.4	2.6	2.0	23.3	10.7	34.1	95.5	99.2	86.9
Credit	4.8	0.5	10.6	59.5	23.9	87.9	81.6	97.5	45.2
AA-Finance	1.3	1.6	0.8	47.7	16.2	71.7	81.6	98.1	51.4
Mortgage	2.6	−0.4	6.8	33.4	27.2	40.5	86.3	92.8	69.5

tions are totaled, and one-month LIBOR based on a $100 million investment is added. The result, expressed as a percentage of the $100 million notional, is the strategy return for the month. This is compared to the total return of the index for that month. The difference between the two is the replication error (outperformance).[13]

Each historical simulation of a replication strategy results in three time series: index returns, portfolio returns, and outperformance. The means and volatilities of replicating portfolio returns can be found in Appendix B.

Our analysis of strategy performance focuses on outperformance statistics: mean outperformance, volatility of outperformance (tracking error), and R^2, which measures the percentage of index return variability captured by the replication strategy.[14] These quantities are shown in Exhibit 3 for the analytical replication strategy using Treasury futures alone.

As could be expected, the best replication results are achieved for the Treasury Index, with a tracking error of just 10.9 bp per month over the entire period. As we move to indexes with progressively greater amounts of spread exposure (agency, AA-finance, credit), we find that the tracking errors continue to rise, up to 59.5 bp per month for the Credit Index. The resulting tracking error for the Aggregate Index is 25.1 bp per month for the period.

This pattern of tracking errors increasing with credit spread exposures holds within each subperiod as well, but with some striking differences. In the first subperiod, tracking error increases gradually as we take on credit: 8.5 bp per month for the Treasury Index, 10.7 bp per month for the Agency Index, and 23.9 bp per month for the Credit Index. All the replication strategies can be considered to track very well during this period; except for the Mortgage Index, all the R^2 values are over 97%.

In the second subperiod, when spread volatility increased, these differences in tracking errors for different indexes are greatly magnified. The tracking errors for all indexes are greater than in the first subperiod, but by vastly differing amounts. The tracking error for the Credit Index virtually explodes, increasing from 23.9 bp per month in the calm period to 87.9 bp per month in the volatile period. The increase in spread volatilities between the two periods leads to a much more severe increase in tracking error on a relative basis; while Exhibit 2 shows that spread volatilities doubled, we see here that the tracking error more than triples.

Even the tracking error for the Treasury Index increases from 8.5 bp per month to 13.6 bp per month. This demonstrates that the relationship between Treasury futures contracts and the underlying notes and bonds (basis risk) was subject to unusually high volatility during this period as well.

During the volatile period, an R^2 of around 98% is found only for the replication of the Treasury Index. For the other six indexes, the R^2s range from 45% to about 87%. The lowest R^2 values are found for the two indexes that are most heavily exposed to spread risk, implying that only about 50% of the return variability of each of these two indexes is accounted for by their replicating portfolios.

Mean outperformance of the replicating portfolios is almost always positive, except for the Mortgage Index during the first subperiod. For the indexes incorporating substantial amounts of credit product, this outperformance reflects to a large degree the spread widening that occurred over the last eight years, particularly since the middle of 1998. The 11 bp per month outperformance of the replication strategy over the Credit Index is testament to this.

It is less obvious why the strategy outperforms the Treasury Index as well. There are two reasons for this. First, Treasury futures have tended to trade cheap compared to cash. Across a variety of asset markets, the seller of implied volatility has been shown to earn positive returns. This is reflected in our replication results. Second, the short-term rate that we use, one-month LIBOR, exceeds the one-month term repurchase rate at which Treasury bonds can be financed by about 1.5 bp per month over the period of the study.

The Treasury futures portfolios designed to replicate the Mortgage Index exhibit the lowest mean outperformance during the first subperiod. A subsequent increase in mortgage spreads helps the strategy outperform during the volatile period, giving it a positive mean outperformance over the entire time period comparable to the one of the Treasury portfolios replicating the Agency Index.

Hybrid Eurodollar Futures/ Treasury Futures Replication

In our hybrid model using Eurodollar futures and Treasury futures, the index is divided into cells as before, and the two longest cells are replicated using the ten-year note contract and the bond contract as described above. The part of the index with duration under five years is replicated with Eurodollar futures, using a totally different mechanism based on cash flow projection.

Just as a single Eurodollar futures contract can be used to lock in the rate on a future three-month deposit, a sequence of consecutive contracts can be used to hedge the price value of a basis point (PVBP) of a longer-term zero-coupon deposit. The present value of that cash flow depends on the Eurodollar futures rates between the present and its due date. Therefore, the PVBP of a given future cash flow with respect to a particular forward rate can be found by discounting that cash flow to the present, and then determining the sensitivity of the present value with respect to that forward rate. Division of that PVBP by $25 gives the number of contracts on that forward rate required to hedge or replicate that cash flow.[15] Present-valuing future cash flows in order to find rate sensitivities is called *tailing the hedge*.[16]

A continuous set of Eurodollar futures contracts is called a *strip* of Eurodollars, and the approach described here is known as a *stripped hedge*.[17] Each future cash flow is hedged with its own strip of Eurodollar futures. This way, each cash flow is hedged in the closest manner possible. To present-value the future cash flows, discount factors are derived by compounding the Eurodollar futures rates.

If an index has only securities with fixed cash flows that occur exactly on the expiration dates of the futures contracts, this method would specify a near-perfect hedge of the term structure exposures of the index. Index cash flows occur throughout the calendar year, however, and many index securities incorporate embedded options that make their cash flows uncertain. To form the replicating portfolio, we first represent each index security by an equivalent cash flow stream with payments on contract dates only.

We aggregate the cash flows of all bonds with cash flows up to five years within each index into cash flow buckets. Every bucket includes contains three months' worth of future cash flows, corresponding to the three-month term on which the Eurodollar futures are based. The three-month periods are centered on the maturity date of the hypothetical three-month LIBOR deposit underlying each Eurodollar contract, and always cover full three-month periods.

For example, at the end of February 1999, the first Eurodollar contract used in the replication is the March 1999 contract. The hypothetical underlying deposit would mature on June 17, 1999. The first cash flow bucket covers May 1, 1999, through July 31, 1999, for each index.[18]

To account for the credit differential between Eurodollar futures and the index that is replicated, we add a spread (OAS) over the Eurodollar zero curve. The discount factors used in the PVBP calculation described above incorporate those spreads.

Callable bonds with potential cash flows beyond the first five years are aggregated into the third duration cell. The cash flows of callable bonds with maturities of under five years are allocated to the different cash flow buckets using the concept of *cash flow to modified adjusted duration*. This is found by taking a linear combination of the cash flows that would occur if the bond were to be called at the next likely call date and the cash flows that would obtain if the bond survives to maturity. The weight for that linear combination is chosen so that the weighted duration of these two cash flow streams equals the modified adjusted duration derived using an option pricing model.

Cash flow projections for mortgage-backed securities are generated using the zero-volatility model. A single projected path of interest rates corresponding to the forward curve is used to generate the cash flows, which are discounted using a spread over this same curve. Any cash flows that occur after year five are discounted back to the last cash flow bucket.

Exhibit 4

Eurodollar/Treasury Futures Replication, 1/94-9/01, bp/month

Index	Mean Outperformance 1/94-9/01	Mean Outperformance 1/94-6/98	Mean Outperformance 7/98-9/01	Tracking Error 1/94-9/01	Tracking Error 1/94-6/98	Tracking Error 7/98-9/01	R^2 1/94-9/01	R^2 1/94-6/98	R^2 7/98-9/01
Aggregate	2.8	2.0	4.0	22.1	10.1	32.2	95.9	99.3	87.6
Gov.-Cred.	3.8	2.8	5.3	21.4	8.1	31.9	96.8	99.6	90.7
Treasury	3.1	3.5	2.6	9.1	6.1	12.1	99.4	99.8	98.8
Agency	2.3	2.3	2.3	18.4	8.4	26.8	97.2	99.5	91.9
Credit	4.9	0.5	11.0	56.7	23.0	83.5	83.3	97.7	50.6
AA-Finance	1.1	1.0	1.4	43.5	14.7	65.5	84.7	98.4	59.5
Mortgage	−1.9	−4.5	1.6	25.7	25.6	25.7	91.9	93.7	87.7

Exhibit 4 summarizes the results of this hybrid replication approach. The results exhibit many of the same qualitative relationships among the tracking errors for different indexes and time periods that we saw in the Treasury futures replication. We focus on the performance differences between the two replication strategies. Mean outperformance is usually lower than found in Exhibit 3, because of the widening of the spread between the LIBOR and Treasury curves over the period of analysis.

For every index and time period considered, lower tracking errors are obtained using the hybrid strategy than with Treasury futures alone. For indexes whose returns are subject to spread risk, this is not too surprising, since the returns of a Eurodollar futures portfolio incorporate spread returns. It is more surprising in the case of the Treasury Index. Here, it is introduction of an instrument that reflects spread returns that leads to reduced tracking errors. This runs counter to the conventional wisdom that it is best to replicate (hedge) a portfolio of securities with the instruments most closely related in performance characteristics. (More on this phenomenon later.)

Another interesting observation concerns the Mortgage Index. As with the Treasury futures replication, the tracking errors for the volatile period are in most cases much higher than the ones exhibited during the calm period. For the Mortgage Index, though, the tracking errors observed in the two subperiods are virtually identical. This is due in large part to the duration shortening the Mortgage Index experienced during the lower interest rate environment of the volatile period.

Swaps

The methodology for replication with swaps is based on the methodology used for replication with Treasury futures. The bonds in each index are again divided into four duration cells. The 0-3-year duration cell is replicated with the 2-year swap; the 3-5-year cell with the 5-year swap; the 5.0-7.5-year cell with the 10-year swap; and the 7.5+-year cell with the 30-year swap.

Transaction costs in the swaps market are fairly high. It is therefore highly unlikely that any investor would use swaps as part of a monthly rebalancing strategy. In order to make the replication with swaps more realistic, we rebalance the swaps replicating portfolios only once every three months. In between rebalancing, the swaps position is held constant. Returns for the swaps replicating portfolios are still computed on a monthly basis. This is done by marking the full position to market, given the new swaps curve.[19] Given the limited risk inherent in rolling such slightly aged positions into new at-the-money swaps, such positions can usually be rolled at relatively tight spreads.[20]

The results for the replication with swaps are found in Exhibit 5. Comparing these results with those shown in Exhibit 3 for the Treasury futures replication strategy, we find that results are much as one would expect. Swaps do not do as good a job at tracking the Treasury Index, but do much better for spread-based indexes. For the two indexes subject to substantial amounts of spread risk (Credit and Double-A Finance), the use of swaps instead of Treasury futures provides a significant improvement in index replication, especially during the volatile period. For the Credit Index, for example, the tracking error during the volatile period is reduced from 87.9 bp per month using Treasury futures to 68.8 bp per month using swaps, with an accompanying improvement in R^2 from 45.2% to 66.5%.

Although swaps and double-A finance credits are often considered closely related, the reduction in track-

EXHIBIT 5

Swaps Replication, 1/94-9/01, bp/month

Index	Mean Outperformance 1/94-9/01	1/94-6/98	7/98-9/01	Tracking Error 1/94-9/01	1/94-6/98	7/98-9/01	R^2 1/94-9/01	1/94-6/98	7/98-9/01
Aggregate	3.0	0.7	6.3	22.0	12.0	31.0	95.9	99.0	88.5
Gov.-Cred.	3.9	1.8	6.7	27.8	10.5	41.3	94.6	99.4	84.4
Treasury	3.9	2.9	5.1	34.7	11.6	52.1	91.5	99.1	77.8
Agency	2.1	1.5	3.0	16.0	11.8	20.6	97.9	99.1	95.2
Credit	3.8	−1.3	10.8	47.2	20.5	68.8	88.4	98.2	66.5
AA-Finance	0.9	0.1	2.0	34.9	13.9	51.7	90.2	98.6	74.8
Mortgage	1.2	−2.3	6.1	27.9	26.9	29.0	90.5	93.0	84.4

EXHIBIT 6

Hybrid Treasury Futures (Treasuries)**/Swaps** (spread product) **Replication, 1/94-9/01, bp/month**

Index	Mean Outperformance 1/94-9/01	1/94-6/98	7/98-9/01	Tracking Error 1/94-9/01	1/94-6/98	7/98-9/01	R^2 1/94-9/01	1/94-6/98	7/98-9/01
Aggregate	3.2	1.2	5.9	17.6	10.0	24.4	97.4	99.3	92.9
Gov.-Cred.	4.1	2.7	6.1	19.0	7.2	28.2	97.5	99.7	92.7

ing error of the Finance Index from Exhibit 3 to Exhibit 5 is only slightly greater than it is versus the Credit Index. During the volatile period, replicating the Double-A Finance Index with swaps still leaves a tracking error of 51.7 bp per month.

Mean returns for the replication using swaps are of about the same magnitude as those shown in Exhibit 3 using Treasury futures. This might seem surprising, given that swaps were subject to significant spread widening over the period of this study. A contributing factor is that the total notional value of swaps used in the replicating portfolio is often less than the $100 million notional value of the index. The 10- and 30-year swaps, in particular, tend to have much higher durations than the index cells with which they are associated. Therefore, to match the dollar duration of the index, the notional values required for those two swaps in the replication portfolio are lower than the corresponding shares of the index notional.

As a result, the floating payments due on the swaps are lower than the investment income from the cash investment at one-month LIBOR, leading to higher average returns. Essentially, the replication portfolio earns the total return from the fixed side of the swaps portfolio, plus some additional return from the unmatched portion of the principal invested at LIBOR.

As with Treasury futures, we modified our swaps-based replication strategy to form a hybrid strategy incorporating Eurodollar futures at the short end with swaps at the long end. The results based on this hybrid replication strategy are very similar to the results of the pure swaps replication.[21] We do not report these results separately.

Hybrid Replication with Treasury Futures and Swaps

The single-instrument replication of the Treasury Index with the lowest tracking error is attained with Treasury futures, while the lowest tracking error of the Credit Index is obtained using swaps. This motivates a different type of hybrid strategy, in which we determine the parts of the index to be hedged by different instruments by sector rather than by duration. For indexes that include both a Treasury and a credit component, the Treasury component is replicated using Treasury futures, and the component subject to spread risk is replicated using swaps. Each component is divided into four duration cells as before to determine the precise composition of the replicating portfolio.

Exhibit 6 reports the results of this approach applied to the Aggregate and the Government/Credit Indexes.

The tracking errors are lower than those achieved by any of the other three methods used, for the entire time period and each subperiod. Over the entire time period, this hybrid strategy using Treasury futures and swaps produces a tracking error of 17.6 bp per month, down from 25.1 bp per month for the strategy using Treasury futures alone.

Performance Comparison

While we would expect hybrid replication portfolios to show lower tracking errors for indexes whose returns are subject to spread risk, why does this hold for replication of the Treasury Index? During the calm subperiod, for example, the tracking error goes from 8.5 bp per month to 6.1 bp per month when the Treasury futures replicating portfolio is replaced with the hybrid Eurodollar/Treasury futures replicating portfolio. Usually, the returns of two portfolios whose elements are most similar in their risk profiles are expected to track each other better. This tenet of hedging seems to be violated here.

The reason lies in correlation of the outperformance of the subportfolios. The set of bonds that we replicate with Eurodollar futures can be seen as one portfolio. The set of the remaining bonds can be viewed as another portfolio. Total index outperformance is the capitalization-weighted sum of each of the two portfolios. To compute the tracking error for the full index, the correlation of the outperformance of the two subindexes is critical.

We report correlations between the outperformance of the two different short-end replication portfolios (Eurodollar futures and two- and five-year Treasury futures) with the outperformance of the long-end replication portfolio (ten-year note and bond futures contracts) in Exhibit 7. For the Treasury Index, we find a correlation coefficient for the pure Treasury futures replicating portfolio of 0.52 compared to –0.27 for the hybrid Eurodollar/Treasury futures portfolio. The negative correlation between spreads and Treasury rates plays an important role here.

For the other six indexes, the difference in correlation is rarely that great, but there is always a considerable reduction in the correlation. How this plays out over a full replication history is seen in Exhibit 8. It depicts the outperformance of the pure Treasury replications well as the outperformance of the hybrid replication of the Aggregate Index. One sees that in many periods the outperformance numbers from the hybrid replication are closer to zero than the outperformance achieved by the pure Treasury futures replication.

EXHIBIT 7

Correlation of Outperformance Between Replication of Short and Long End of Indexes, 1/94-9/01 (%)

Index	Short End: Treasury Futures Long End: Treasury Futures 1/94-9/01	Short End: Eurodollar Futures Long End: Treasury Futures 1/94-9/01
Aggregate	86.5	10.7
Gov.-Cred.	82.4	–2.8
Treasury	52.4	–26.8
Agency	78.6	10.9
Credit	91.8	66.9
AA-Finance	87.8	65.5
Mortgage	77.5	39.2

EXHIBIT 8

Outperformance of Treasury Futures versus Hybrid Eurodollar/Treasury Futures Replication of Aggregate Index

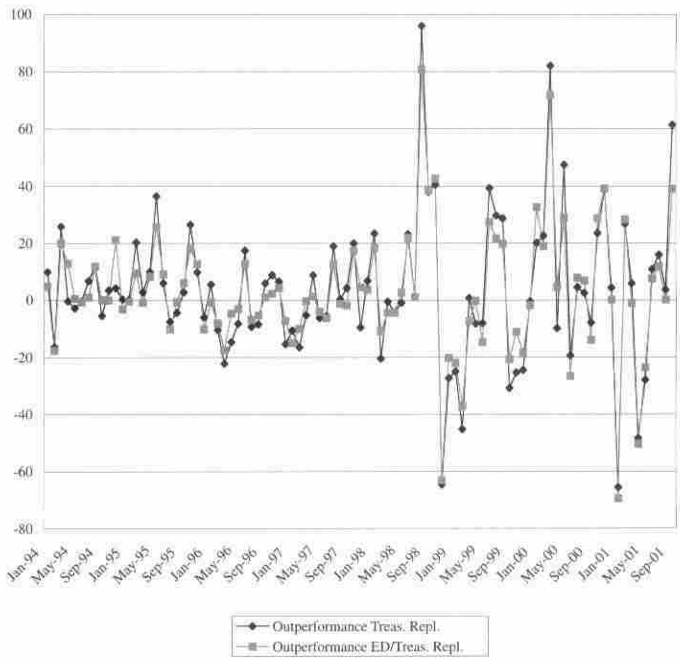


Exhibit 8 also shows how August 1998 signifies a structural break in the sample period. The volatility of both sets of outperformance numbers rises substantially, and it has not so far returned to the levels seen before the summer of 1998.

III. REPLICATION USING EMPIRICAL HEDGE RATIOS

On many occasions, investors use empirical hedge ratios for replication and hedging purposes. Empirical hedge ratios are usually regression-based. Historical returns, rate movements, or spread changes form the foundation for finding the hedge ratios. For example, a hedge ratio for hedging the Lehman Brothers Treasury Index with the ten-year Treasury note contract can be found by regressing past index returns on historical returns of the ten-year note contract over the same period. The estimated coefficient represents the market value of the futures contract that has to be shorted for each dollar invested in the Index.

Alternatively, yield changes could be used as the regression variables. The regression coefficient is then multiplied by the ratio of the duration of the instrument to be hedged to the duration of the hedging instrument to arrive at the proper hedge ratio.

What advantages do empirical durations offer over analytical ones? The underlying assumption of duration analysis is a parallel shift in yields across the curve. If this parallel shift is the only factor determining the change in bond prices, modified duration times the change in yield is a first-order approximation that predicts the price changes of bonds very well. This is clearly the case for Treasury bonds that are close in maturity and coupon.

For the bonds that are far apart on the curve, though, using durations can lead to very rough approximations of hedge ratios. Although parallel movements of the yield curve are the predominant ones, movements in the yield spreads between different bonds can also be important. These spreads, in turn, can move with the general level of yields.

Empirical hedge ratios effectively use historical correlations and volatilities. If, for example, the spread between two Treasury bonds tightens when yields increase, empirical durations will reflect this. If the historical relationships in the data that are embedded in empirical durations hold going forward, empirical hedge ratios should do at least as well as hedge ratios built upon analytical durations.

The reasons to use empirical hedge ratios are even more compelling in the case of hedging spreads. To compute analytical durations, spreads are usually kept constant. In reality, spreads are correlated with movements of the underlying Treasury curve. The actual price movements often diverge from the analytically predicted ones. Similarly, if futures contracts are used for hedging, the futures basis is equivalent to a spread. Changes in the yield curve can change the basis.

Empirical durations are not guaranteed to perform better than analytical ones. First, the future might differ from the past. Second, any estimation necessitates a choice of both regression variables and a data period over which the regressions are estimated.

Hedge ratios could be derived from regressions of returns on returns or rate changes on rate changes. One, two, or more hedging instruments can be used in the regression.[22] Rolling regressions using only the past 12 or so months are often used so as not to include history that is deemed irrelevant for the future. With each passing month, the most recent month is added to the regression, and the first month in the previous month's regression is dropped. In this way, potential non-stationarities or unknown time-varying dependencies of the data on other variables are remedied to some extent.

We investigate empirical replication strategies based on linear regressions of index cell returns on futures returns. We use a rolling window of 12 months of monthly return data; the hedge ratios used in any given month are based on the most recent 12 return observations available.[23] Regressions using 18 and 24 months' worth of data do not lead to significantly different results.

The regressions are performed on returns of 2-, 5-, and 10-year futures contracts and the bond contract, and on 2-, 5-, 10-, and 30-year swap returns in the case of swaps.

For either Treasury futures contracts or swaps, we use the same division of an index into four duration cells as for the analytical replication strategies. Each cell is again replicated with the same single instrument; the only difference is that the hedge ratios are based not on duration, but rather on a regression of past index returns on past futures returns.[24]

The regression includes a constant: if r_t^I and r_t^F are the index return and futures return at time *t*, we find the α and β that minimize the sum of squared residuals ε_t in the equation: $r_t^I = \alpha + \beta r_t^F + \varepsilon_t$. The resulting α is not used in the replication methodology. The slope coefficient β—the hedge ratio—is then converted into the number of futures contracts or swaps.[25]

Instead of four separate regressions, the index returns could be regressed on the returns of all four futures contracts or swaps in one simultaneous regression. Given the usually high correlations among the returns of the instruments across the curve, this would lead to multicollinearity between the regressors, and consequently produce very erratic coefficients. This would make the regression results unusable for practical replication applications. Our methodology avoids these problems and achieves very good results.

EXHIBIT 9

Empirical Treasury Futures Replication, 1/94-9/01, bp/month

Index	Mean Outperformance 1/94-9/01	Mean Outperformance 1/94-6/98	Mean Outperformance 7/98-9/01	Tracking Error 1/94-9/01	Tracking Error 1/94-6/98	Tracking Error 7/98-9/01	R^2 1/94-9/01	R^2 1/94-6/98	R^2 7/98-9/01
Aggregate	2.5	2.3	2.9	23.3	12.3	33.6	95.4	98.9	86.7
Gov.-Cred.	2.6	2.7	2.4	23.0	9.2	34.6	96.3	99.5	89.3
Treasury	2.9	3.3	2.4	8.8	7.4	10.5	99.5	99.6	99.1
Agency	1.7	2.0	1.2	22.3	10.6	32.8	95.9	99.2	88.1
Credit	2.2	0.9	4.1	59.3	23.2	89.2	81.7	97.6	44.6
AA-Finance	−0.2	1.9	−3.3	48.4	15.5	73.9	81.1	98.2	49.6
Mortgage	3.4	1.5	6.2	35.6	32.3	40.1	84.5	89.7	71.0

EXHIBIT 10

Eurodollar/Empirical Treasury Futures Replication, 1/94-9/01, bp/month

Index	Mean Outperformance 1/94-9/01	Mean Outperformance 1/94-6/98	Mean Outperformance 7/98-9/01	Tracking Error 1/94-9/01	Tracking Error 1/94-6/98	Tracking Error 7/98-9/01	R^2 1/94-9/01	R^2 1/94-6/98	R^2 7/98-9/01
Aggregate	2.1	2.1	2.0	21.7	10.9	31.2	96.0	99.2	88.3
Gov.-Cred.	2.9	2.2	3.7	20.1	7.7	29.8	97.2	99.7	91.9
Treasury	3.2	2.8	3.6	8.7	6.9	10.7	99.5	99.7	99.1
Agency	1.6	1.7	1.5	17.6	8.4	25.6	97.5	99.5	92.6
Credit	2.8	0.6	5.9	56.2	22.4	83.3	83.5	97.8	50.8
AA-Finance	−0.1	1.3	−2.0	43.9	14.5	66.1	84.4	98.5	58.8
Mortgage	−1.5	−3.7	1.5	27.3	27.4	27.3	90.9	92.7	86.2

Treasury Futures Replication

The results of the empirical Treasury futures replication are found in Exhibit 9. Comparing the tracking errors with the ones in Exhibit 3, we find mixed results. For replication of the Treasury Index with Treasury futures, empirical hedging produces better results than analytical hedging, reducing the overall tracking error from 10.9 bp per month to 8.8 bp per month. Similar results are obtained for the Government/Credit and the Aggregate Indexes, and to a smaller degree the Agency Index.

For indexes subject to a lot of spread risk, however, tracking errors using Treasury futures can become greater than they are with the analytical approach, particularly during the volatile period. Mean outperformance during the volatile period is lower in most cases than for replication based on analytical durations.

Hybrid Eurodollar/Empirical Treasury Futures Replication

Exhibit 10 provides the results for replicating the short end with Eurodollar futures, and the long end with the ten-year futures and the bond contract. Only the replication with the two Treasury futures contracts is based on empirical hedge ratios. Positions in Eurodollar contracts are calculated as before.

As a result, the differences between the analytical and empirical methods are largely as described for the straight Treasury futures replication. Tracking of the Treasury Index improves in the volatile period, but most other performance differences appear too slight to be significant.

Empirical Replication with Swaps

Exhibit 11 presents the results for the empirical replication with swaps. Comparing them with the analytical results shown in Exhibit 5, we find that empirical hedging reduces tracking errors during the volatile period, but increases them during the calm period. The results for the overall time period are mixed.

In the case of the Mortgage Index replication, the empirical replication gives rise to greater tracking errors in all time periods. Mean outperformance of the portfolio replicating the Mortgage Index improves substantially, though.

EXHIBIT 11

Empirical Swaps Replication, 1/94-9/01, bp/month

Index	Mean Outperformance 1/94-9/01	1/94-6/98	7/98-9/01	Tracking Error 1/94-9/01	1/94-6/98	7/98-9/01	R^2 1/94-9/01	1/94-6/98	7/98-9/01
Aggregate	2.8	2.5	3.3	22.3	19.9	25.4	95.8	97.3	92.3
Gov.-Cred.	2.5	3.3	1.5	25.2	16.1	34.3	95.6	98.5	89.3
Treasury	2.2	4.1	−0.5	32.4	16.2	46.5	92.6	98.3	82.3
Agency	2.6	2.2	3.1	17.4	15.7	19.6	97.5	98.3	95.6
Credit	3.1	1.3	5.6	46.0	26.8	64.1	89.0	96.9	70.9
AA-Finance	1.5	2.4	0.1	35.4	19.2	50.1	89.9	97.3	76.3
Mortgage	4.1	0.7	8.8	35.3	38.2	30.6	84.8	85.9	82.6

EXHIBIT 12

Hybrid Treasury Futures (Treasuries)**/Swaps** (spread product) **Replication, 1/94-9/01, bp/month**

Index	Mean Outperformance 1/94-9/01	1/94-6/98	7/98-9/01	Tracking Error 1/94-9/01	1/94-6/98	7/98-9/01	R^2 1/94-9/01	1/94-6/98	7/98-9/01
Aggregate	3.0	2.8	3.1	13.4	11.4	16.0	98.6	99.1	97.3
Gov.-Cred.	2.8	3.3	2.1	11.4	7.8	15.2	99.1	99.6	98.0

Hybrid Replication with Treasury Futures and Swaps

Another alternative is to replicate the Aggregate and the Government/Credit Index each with both Treasury futures and swaps. Again, the Treasury component of each index is replicated with Treasury futures, and the component subject to spread risk is replicated with swaps. The replication of each component is based on cell-based regressions.

The results in Exhibit 12 show a substantial reduction in tracking errors for both indexes for the volatile and the full period. For the calm period, the tracking errors here are slightly higher than the ones in Exhibit 6.

Performance Comparison

Performance comparisons across the three methods incorporating empirical hedge ratios are largely similar to findings for the replication based on analytical hedge ratios. The hybrid Eurodollar/empirical Treasury futures replication produces lower tracking errors than the pure empirical Treasury futures replication. Swaps lead to lower tracking errors for the Agency, Credit, and the Double-A Index during the volatile period, but for the calm period show higher tracking errors than the empirical replication with Treasury futures. For the Mortgage Index, all three empirical replication approaches produce higher tracking errors than when analytical hedge ratios form the basis for the replication.

At the same time, empirical replication of the Treasury Index with Treasury futures contracts brings about lower tracking errors. Replication of the Treasury Index with swaps leads to about the same tracking error for either analytical or empirical hedge ratios.

Possible reasons for this divergent performance of empirical hedge ratios can be manifold. The estimated hedge ratios could be generally bad predictors of the future ones. Perhaps a shorter time horizon for the estimation or higher-frequency data would produce improved results. It is beyond the scope of this study to investigate these questions, but we can shed some light on the performance of empirical hedge ratios using regression theory.

If the regression from which the hedge ratios are derived is performed in return space, the regression coefficient between the returns of an index and the returns of a replication portfolio is defined by

$$\beta = Cov(r_I, r_R)/Var(r_R)$$

at any time. Here *Cov* and *Var* denote covariance and variance, and r_I and r_R represent the returns of the index and the replication instrument. Now, let us assume that the index to be replicated and the replication instrument trade with a certain yield spread over the same benchmark Treasury bond.[26]

If all spread volatilities are equal to zero, or if the spread of the replication instrument is constant, and the correlation between the spread of the index and the underlying benchmark yield is zero, then the empirical hedge ratios will equal the analytical ones (which are the "true" hedge ratios in this context, i.e., dur_I/dur_R).[27] If the spread of the replication instrument is constant at all times, the empirical hedge ratios will exceed the analytical ones if the correlation between the spread of the index and the benchmark yield is positive, and vice versa if it is negative.

We have assumed that β is a population parameter that can be represented by other known population parameters, namely, variances and correlations. As the term "empirical hedge" suggests, β is usually estimated from historical data. The variances and correlations in the definition of β have to be replaced by their sample estimates, and the analytical durations will be averages of the durations experienced over the historical period that underlies the estimation of β. For the empirical hedge ratios to lead to better performance than the analytical ones, the estimates of β have to be accurate predictors of the actual β for the period for which the replication portfolio should be in place.

High variability of spread volatilities and of spread correlations and of correlations between spreads and rates will lead to greater variability of β. In the extreme, this can make it an inferior estimator of the true beta desirable for the optimal replication, and analytical hedge ratios can lead to lower tracking errors in practice. The increased spread volatility of the last three and a half years probably has made the regression coefficient a less accurate predictor of the hedge ratio for the coming month, causing the replication to perform less well during that period. For the Treasury futures replication, where credit or mortgage spreads and their volatilities play no role, the empirical hedge ratios lead to lower tracking errors.

The substantial increase in the tracking error from empirical replication compared to analytical replication for the Mortgage Index during the calm period has its source in the fact that 1994 provided an environment of rapidly rising rates, which led to changes in mortgage spreads over Treasuries.

IV. LIQUIDITY AND TRANSACTION COSTS

Market liquidity and transaction costs of any replication strategies are of paramount importance to investors using them. Liquidity in both the exchange-traded and the over-the-counter derivatives markets is very high. Appendix A shows open interest and volume for Eurodollar and Treasury futures on January 14, 2000.

For the amounts of contracts needed to replicate portfolios of up to around \$1 billion, replication with exchange-traded derivatives can be conducted with little or no market impact, if the transactions occur over time. The swaps markets are more susceptible to liquidity concerns, since counterparty credit risk and general credit conditions are of more importance. If large amounts have to be traded in short periods of time, the market makers will adjust the bid-ask spreads in line with the marginal risk exposure of the transaction.

Liquidity-related concerns are not as acute for exchange-traded derivatives. The short settlement periods on these markets, and the role of the clearing corporation in collecting and facilitating variation margin payments, significantly reduce counterparty credit risk. Consequently, liquidity does not dry up in stressful markets as it does in the cash or OTC derivatives markets.

The bid-ask spread in futures markets is more a function of market risk than of changes in liquidity. If market risk is perceived to be high (i.e., before announcement of an important economic statistic), the bid-ask spread will widen to compensate market makers for their risk as they provide liquidity to the market. In general, the ability to trade within a particular time frame during periods of market stress is less of an issue in the futures markets than in the OTC derivatives markets.

All the derivatives markets are undergoing a period of immense change. Futures markets in particular are moving rapidly toward electronic trading. The Eurex and the London International Financial Futures Exchange have already done so, and the Chicago Board of Trade and the Chicago Mercantile Exchange are moving in that direction. Electronic trading has implications for how orders are executed and their costs. The exchanges are also making trading more flexible and anonymous. More and more transactions are allowed to take place outside the confines of the exchange.

The CME recently extended "exchange-for-physical" transactions from the first two Eurodollar contracts to all contracts. In an exchange-for-physical transaction, a broker trades a fixed-income security in exchange for

EXHIBIT 13

Transaction Costs of Index Return Replication, 1/94-9/01, bp/month

	Setup Cost—December 1993			Maintenance Costs—January 1994-August 2001		
Index	Treasury Replication Portfolio	Eurodollar and Treasury Futures	Swaps	Treasury Replication Portfolio	Eurodollar and Treasury Futures	Swaps
Aggregate	2.3	3.6	5.7	1.0	0.7	0.4
Gov.-Cred.	2.4	4.0	6.6	1.0	0.7	0.4
Treasury	2.4	4.4	6.5	1.0	0.7	0.4
Agency	2.4	3.6	6.3	1.0	0.7	0.4
Credit	2.4	3.2	7.3	1.1	0.8	0.5
AA-Finance	2.1	3.6	5.0	1.0	0.6	0.4
Mortgage	2.1	5.2	3.5	1.1	0.6	0.3

a strip of Eurodollar contracts that is designed to hedge that bond. Similar to block trades in the Treasury futures markets, the trades take place off the floor, but are reported to the exchange. All futures contracts traded in this way are counted toward open interest and daily volume.

In a block trade, the bid or offer does not have to be posted with the exchange. Instead, the futures buyer or seller trades directly with a broker who makes a market in block trades. The dealer has an obligation to find the best execution for the customer by contacting other brokers. Block trades also allow for greater trading anonymity. Much like liberalization in the rules for exchange-for-physical transactions in the CME, the LIFFE recently reduced the minimum size for block trades in the Long Gilt futures.

Transaction costs have to be considered in the choice of the replication methodology, too. Exhibit 13 gives the cost of setting up replication portfolios for the seven indexes replicated (setup cost) and the cost of rebalancing them (maintenance cost).

The cost assumptions that underlie the numbers in Exhibit 13 are relatively conservative. They are probably higher than what an investor can expect in today's markets.[28] For clearing and execution fees, we assume a cost of $7 for each trade. Rolling from one contract into another one will therefore cost a total of $14. For bid-ask spreads, we assume ½-1 basis point for each Eurodollar contract, one-half a tick for the three-year, five-year, and ten-year note futures contracts, and one tick for the bond futures contract.[29]

For the setup costs of the swaps replicating portfolios, 1.0 basis point is assumed for the 2-, 5-, and 10-year swaps, and 1.5 basis points for the 30-year swap. Clearly, with such costs, the premise of the replication exercises, namely, monthly rebalancing, becomes unrealistic for the swaps market.

For example, with a bid-ask spread for a 30-year swap of 1.5 basis points, and duration of 13 years, this translates into more than 6 ticks, or about $190,000 per $100 million notional. Although the bid-ask spread for shorter-maturity swaps is narrower, the total cost of rolling at the end of every month into a new set of at-the-market swaps would be prohibitively expensive.

An alternative for investors would be to put on new payor or receiver swaps when the swaps position is adjusted. This would entail a steadily growing swaps book, something that is very undesirable to many investors.

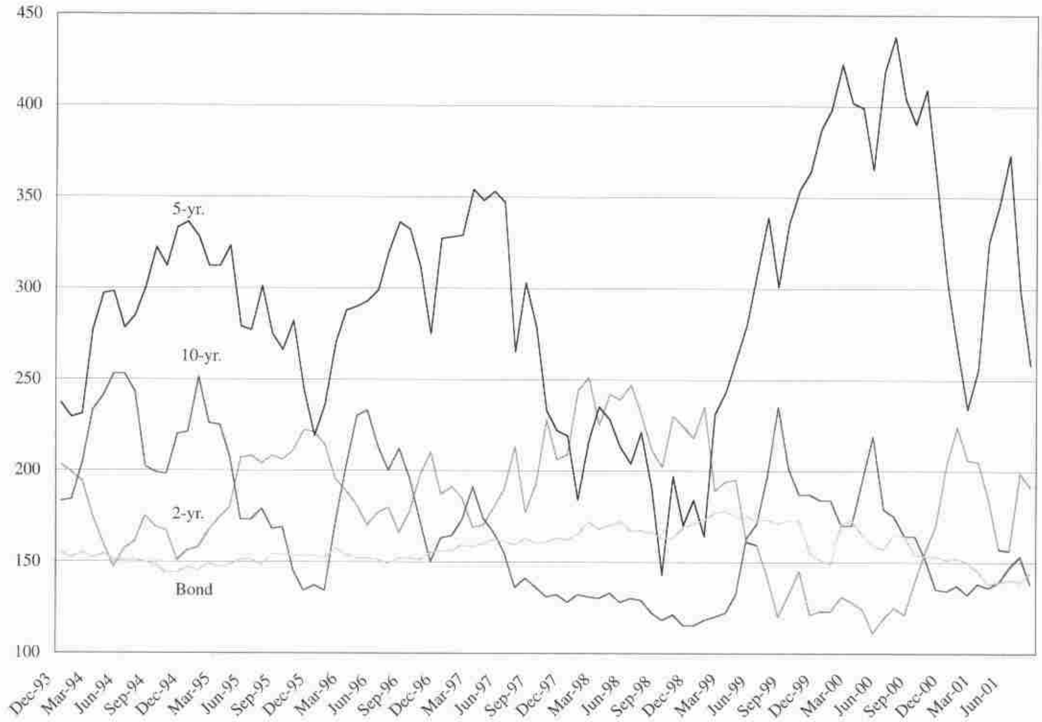
Swaps traders have suggested another alternative to us. For ¼ basis point in bid-ask across the curve, the whole swaps portfolio can be rolled into new at-the-market swaps within the first six months after putting on the original swaps position. For a trader at a large market-making institution, this is very close to a no-risk trade, and the bid-ask can be very tight. We assume a constant ¼ of a basis point bid-ask for the four swaps used in the replication, with quarterly rollover.[30]

For the futures, half the bid-ask is allocated to a one-way trade for computation of the maintenance costs. The setup costs for all replication portfolios and the maintenance costs of the swaps portfolios allocate the full bid-ask spread at the time of purchase.

The numbers in Exhibit 13 are expressed in basis points, and in each case they are computed for $100 million portfolios. For example, the setup costs of replicating the full Aggregate Index using the mixed Eurodollar/Treasury futures method is 3.6 basis points, or 3.6 one-hundreds of 1.0%, which equals $36,000.

EXHIBIT 14

Number of Contracts Used in Treasury Futures Replication of the Aggregate Index



The setup costs are the lowest for the replication portfolios involving solely Treasury futures, largely due to their lower bid-ask spreads, combined with the small number of contracts that have to be purchased in comparison to the hybrid Eurodollar/Treasury futures portfolios. For the full indexes, this advantage amounts to $12,000 for a $100 million portfolio compared to the respective hybrid Eurodollar/Treasury futures replication portfolios.

The relatively high cost of the regular trading of swaps is reflected in the cost of setting up the portfolio. Since this reflects only the one-way cost of a trade, rolling that same portfolio of swaps under our assumed costs for setting up the portfolio would be about twice that amount.

The last three columns of Exhibit 13 summarize the recurring cost of rebalancing the replicating portfolios. Replacing a pure Treasury futures replicating portfolio with a hybrid Eurodollar/Treasury futures portfolio has considerable cost advantages. This can amount to 1/2 basis point a month for an index, or about 6 basis points a year.

Why this substantial cost differential? One reason lies in the lower bid-ask spreads that the Eurodollar futures market is subject to. Another reason lies in the structure of the stripped replication with the Eurodollar futures compared to the replication with individual Treasury futures contracts. For the Treasury replication, rolling contracts involves selling all contracts and buying new ones every quarter. This results in high rollover costs. With the stripped Eurodollar futures replication, at rollover time only the closest contract is completely sold, and new contracts are acquired across the Eurodollar futures curve. Only 100 to 150 contracts are rolled, instead of the whole portfolio of about 600 contracts.

The costs of rebalancing the swaps replicating portfolios come in somewhat below the maintenance costs of the hybrid Eurodollar/Treasury futures replicating portfolios.

Two important elements of the cost of the replication strategies include the number of contracts that have to be rolled on a recurring basis, and the variability in that number across the four duration cells on which our strategies are based. To give an impression of how the replication portfolios evolve through time, Exhibit 14 shows the number of Treasury futures contracts employed in replication of the Aggregate Index. The number of two- and five-year contracts used exhibits the most variability, while the number of bond futures stays around 150. Increases and decreases seen in the

EXHIBIT 15

Ratio of Number of Treasury Futures Used in Aggregate Index Replication—Empirical over Analytical

number of contracts employed largely follow the change in market value shares of the four cells.

Exhibit 15 compares the number of contracts used in replication based on empirical hedge ratios with Treasury futures to the number of contracts used in replication based on analytical hedge ratios. Most of the time, the ratio is below 1.0. Coming out of severe rallies of the Treasury market, the ratios are at their lows, reflecting the lower correlation between spreads and rates during such periods.

Given the fact that the empirical-based replication usually uses fewer contracts than the replication based on analytical hedge ratios, the cost of replication based on empirical hedge ratios should not exceed the cost of analytical replication strategies.

V. SUMMARY AND CONCLUSIONS

We have simulated the performance of portfolios of Treasury futures, Eurodollar futures, and swaps in replicating a number of fixed-income indexes. The results provide us with a quantitative backdrop against which to assess the replication performance of the various sets of instruments.

The extreme spread volatility of the last three and a half years underscores the risk involved in replicating returns of spread indexes with Treasury futures alone. As expected, replication strategies based on instruments with some credit spread exposure, such as Eurodollar futures and swaps, can track this index much more closely. This improvement in tracking errors may be slight in times of normal spread volatility, but can be very significant in times of liquidity crises.

Both Eurodollar contracts and swaps have drawbacks in hedging or replication applications. Eurodollar contracts lack sufficient liquidity beyond five years. The relatively high cost of swaps transactions makes an all-swaps approach impractical for applications that require frequent rebalancing of the full position. We therefore simulate fully rolling the replicating portfolio of swaps into a portfolio of new at-the-money swaps every three months. This is done at a cost of 0.25 basis point per swaps contract.

The insufficient curve coverage of Eurodollar futures leads us to consider hybrid strategies that combine Eurodollar contracts below five years to expiration with Treasury futures. These hybrid strategies show great promise for the replication of all fixed-income indexes. They benefit from several desirable properties of the stripped Eurodollar hedging technique at the short end: low transaction cost, close matching of term structure exposures, and some spread exposure. An additional benefit of the hybrid strategies is that the replication techniques used for the long end and the short end of the indexes exhibit very low or even negative correlation. This serves to reduce the overall tracking error.

As a result of these advantages, the hybrid strategy based on Eurodollar and Treasury futures is able to track index returns more closely than Treasury futures alone, even for the Treasury Index. For the Credit Index, using swaps for the replication provides for a great reduction in tracking errors.

For indexes that have significant Treasury as well as spread components, a different kind of hybrid replication produces the lowest tracking errors. Replication of the Treasury component of such an index with Treasury futures and the spread component with swaps brings about the lowest tracking errors for these indexes.

Replication or hedging based on empirical hedge ratios can improve tracking errors. The best results are achieved for indexes with little or no spread exposure, suggesting that instability in the relationship between different spreads, and between yields and spreads, makes regression-based hedge ratios potentially inaccurate predictors of the actual relationship between index and derivatives returns. Nonetheless, applied to the hybrid replication strategy using Treasury futures and swaps, empirical hedging provides the closest replication of all strategies for the Aggregate Index.

With conservative transaction cost assumptions, the replication strategies can be followed without incurring excessive costs. Eurodollar futures have clear advantages with regard to transaction costs. Replicating portfolios that use Eurodollar futures or swaps exhibit the lowest tracking errors. Users of derivatives in hedging and replication applications might therefore want to consider investment guidelines that allow Eurodollar futures and swaps.

Different investors will need to consider the relative merits of the various replication strategies within the context of their investment goals and constraints. Replication strategies can also be combined with a core position in a given market or augmented with more aggressive deployment of cash. Choices are available to craft an appropriate index replication program to suit any investment style.

ENDNOTES

[1]How to properly hedge (or replicate) the fixed leg of a swap with a strip of Eurodollar futures contracts is described in detail by Kawaller [1994].

[2]See Hill and Naviwala [1999].

[3]Given the relatively larger standard trading sizes in the fixed-income markets, $20 or $30 million is usually insufficient to achieve a portfolio that can be considered well diversified without incurring too much cost.

[4]A detailed description of the Treasury futures market is provided in Burghardt and Belton [1993].

[5]Technically, this statement is not completely accurate, as it neglects the small differences between futures and forward pricing.

[6]The standard fixed-for-floating interest rate swap is set off three-month LIBOR. One-month LIBOR is used here in order to save on the mark-to-market computation of the floating leg between reset dates.

[7]Actually, the swaps position sizes are determined by the need to match the yield curve exposure that would be achieved by investing in the index. The total notional value of swaps may therefore not exactly equal the size of the cash position, in which case the cash position and the floating side do not fully cancel out. This can have an effect on mean returns, as discussed later.

[8]The price value of a basis point (PVBP) of a Eurodollar futures contract is $25, so per $100,000 of notional its PVBP is $2.50. The PVBP of the two-year, five-year, and ten-year note futures contracts and the bond futures contract are $20, $45, $63, and $120 per $100,000 of notional, respectively (assuming futures prices of par and modified durations of 2.0, 4.5, 6.3, and 12.0, respectively).

[9]Since the OAS are computed off an estimated Treasury spline curve based solely on off-the-run Treasury securities, the average of all Treasury bonds will exhibit a slight non-zero OAS. This is caused by deviations of individual Treasury bond yields from this estimated curve.

[10]LIBOR is used because it is a generally accepted money market benchmark rate. We use it as a proxy for other short-term assets used by investors, such as bankers' acceptances or CDs.

[11]We follow here the approach introduced in Dynkin, Hyman, and Wu [1997]. This approach, which uses all four Treasury futures contracts, is shown to be superior to using only a single one of the four contracts. Using all four contracts provides better hedging against non-parallel yield curve changes.

[12]A detailed description can be found in Weissman and Axel [1997].

[13]An in-depth description of the cell-based replication approach, and a detailed numerical example, is provided in Dynkin, Hyman, and Wu [1997].

[14]The R^2 is defined as 1 – Variance(Outperformance)/Variance(Index).

[15]The PVBP for a Eurodollar contract is $25, irrespective of maturity, because the price is based on simple interest on $1 million deposited for 90 days: $1,000,000 × (1/10,000) × (90/360) = $25. Each contract provides this amount of sensitivity to a 1 bp change in a different forward rate.

[16]Building a zero-coupon curve from Eurodollar futures is described in detail in Kawaller [1994]. His application is geared toward the replication and hedging of interest rate swaps with Eurodollar futures. In our application, we have to also aggregate the individual bonds' cash flows into buckets, and we have to account for the credit differential between Eurodollar futures and any specific index through an option-adjusted spread (OAS).

[17]Using only one single Eurodollar futures contract for hedging is called a *stacked hedge*. In practice, the closest contract on the quarterly expiration schedule would be most often used in that approach. While somewhat simpler to implement, the stacked hedge approach leads to much greater tracking errors than the stripped hedge, so we do not include results for it.

[18]In this case, there remains a *stub cash flow* bucket of less than three months, covering cash flows that occur during March and April 1999. These stub cash flow buckets, whose PVBPs are very low, are not replicated.

[19]The rates necessary for discounting the swaps aged one to three months are found by linear interpolation between the rates of the two bracketing quoted swaps.

[20]Following such a methodology eliminates the need for a swaps book that grows increasingly when new at-the-money swaps are added to rebalance the replicating portfolio to stay in line with an index's duration characteristics.

[21]This is not too surprising, given that the Eurodollar markets are used actively by swaps dealers to hedge the net exposures of their books.

[22]To regress rate changes on rate changes, to arrive at the final hedge ratio the regression coefficient would have to be multiplied by the ratio of analytical durations.

[23]Here we follow Appendix 7.D of Duffie [1989]. The resulting coefficients can be transformed into numbers that directly reflect the percentage change in the index value that results from a one percentage point change in the yield underlying the replication instruments.

[24]Futures return is defined in this context as: (Futures Price at time t)/(Futures Price at time t – 1) – 1.

[25]Hedge ratio is defined as the dollar amount of the hedge instrument needed to hedge one dollar invested in the index.

[26]We disregard complications stemming from the aggregation of different bonds into an index. We assume that representation of disparate bonds by the same average yield and duration is appropriate. Treasury futures contracts can be viewed in this context as representing the cheapest to deliver security.

[27]With y and s representing the change in benchmark yield and spread, respectively, from one period to the next, and ρ the correlation coefficient, we have

$$\begin{aligned}\beta &= [\mathrm{dur}_I/\mathrm{dur}_R] \times [\mathrm{Var}(y) + \mathrm{Cov}(y,s_I) + \mathrm{Cov}(y,s_R) + \\ &\quad \mathrm{Cov}\,(s_I,s_R)]/[\mathrm{Var}(y) + \mathrm{Var}(s_R) + 2 \times \mathrm{Cov}(y,s_R)] \\ &= [\mathrm{dur}_I/\mathrm{dur}_R] \times [\mathrm{Var}(y) + \rho_{y,I}\mathrm{Var}(y)^{0.5}\mathrm{Var}(s_I)^{0.5} + \\ &\quad \rho_{y,R}\mathrm{Var}(y)^{0.5}\mathrm{Var}(s_R)0.5 + \rho_{I,R}\mathrm{Var}(s_I)^{0.5}\mathrm{Var}(s_R)^{0.5}]/ \\ &\quad [\mathrm{Var}(y) + \mathrm{Var}(s_R) + 2 \times \rho_{y,R}\mathrm{Var}(y)^{0.5}\mathrm{Var}(s_R)^{0.5}]\end{aligned}$$

where dur_I and dur_R stand for index duration and the duration of the replication instrument, respectively. Interest rate and spread durations are assumed to be the same. The durations here are of an instantaneous nature. Empirical regressions in return space will incur somewhat of an error due to the implicit averaging of past duration ratios that takes place in such a regression.

[28]The advent of electronic trading will probably lead to reductions in fees and bid-ask spreads.

[29]These numbers are conservative, too. For the Eurodollar futures out to two years, ¼ of a basis point is probably more realistic for the bid-ask. For the note and the bond futures contract, the bid-ask can be only half of that indicated in the text. All bid-ask numbers are subject to market conditions.

[30]Moving from a monthly to quarterly roll of the swaps positions, with positions frozen in between, does not lead to any substantive increase in tracking errors.

REFERENCES

Burghardt, Galen D., and T. Belton. *The Treasury Bond Basis*. New York: Mc Graw-Hill Professional Publishing, 1993.

Duffie, Darrell. *Futures Markets*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

Dynkin, Lev, Jay Hyman, and Wei Wu. "Replicating Index returns with Treasury Futures." Lehman Brothers, November 1997.

Hill, Joanne M., and H. Naviwala. "Synthetic and Enhanced Index Strategies Using Futures on U.S. Indexes." *The Journal of Portfolio Management*, Vol. 25 (May 1999), pp. 61-74.

Kawaller, Ira G. "Comparing Eurodollar Strips to Interest Rate Swaps." *The Journal of Derivatives*, Fall 1994, pp. 67-79.

Weissman, Phil, and R. Axel. "The Lehman Brothers' Multi-factor Futures Model." Lehman Brothers, September 1997.

APPENDIX A
The Liquidity of Futures Contracts

The liquidity of instruments used for replication is very important. Exhibits A-1 and A-2 describe the liquidity characteristics of the exchange-traded derivatives used in this study, using open interest and volume data as of January 14, 2000.

EXHIBIT A-1
Open Interest and Total Volume for Eurodollar Contracts Traded on the CME on 1/14/2000

	Open Interest		Total Volume	
	Number of Contracts (Cum.)	Per Cent (Cumulative)	Number of Contracts (Cum.)	Per Cent (Cumulative)
Contracts off the Quarterly Schedule	15,874	0.54	3,030	0.67
March 2000 Contract	506,814	17.79	66,104	15.23
Contracts out to December 2000	1,648,735	56.12	345,347	76.10
Contracts out to December 2002	2,561,350	87.18	434,081	95.65
Contracts out to December 2004	2,841,643	96.72	450,076	99.17
Contracts out to December 2009	2,937,923	100.00	453,828	100.00

EXHIBIT A-2
Open Interest and Total Volume for U.S. Treasury Futures Contracts Traded on the CBOT on 1/14/2000

	Open Interest Number of Contracts	Total Volume Number of Contracts
U.S. 2-Year Treasury Note Contract	32,534	3,343
U.S. 5-Year Treasury Note Contract	386,667	108,813
U.S. 10-Year Treasury Note Contract	576,420	164,550
U.S. Treasury Bond Contract	626,396	362,067

APPENDIX B
Summary Statistics

EXHIBIT B-1
Treasury Replication Portfolio Return Summary Statistics, 1/94-9/01, bp/month

	Mean			Standard Deviation		
Index	1/94-9/01	1/94-6/98	7/98-9/01	1/94-9/01	1/94-6/98	7/98-9/01
Aggregate	61.5	59.7	64.0	118.4	128.0	105.2
Gov.-Cred.	61.1	60.3	62.2	129.6	137.1	120.2
Treasury	60.2	59.4	61.2	125.7	131.3	119.2
Agency	60.4	58.9	62.4	116.2	126.8	101.4
Credit	63.1	63.1	63.1	145.5	156.7	130.4
AA-Finance	58.8	55.9	62.7	113.4	119.2	106.2
Mortgage	62.5	58.6	67.9	99.3	111.0	81.3

APPENDIX B - *continued*

EXHIBIT B-2

Eurodollar/Treasury Futures Replication Portfolio Return Summary Statistics, 1/94-9/01, bp/month

	Mean			Standard Deviation		
Index	**1/94-9/01**	**1/94-6/98**	**7/98-9/01**	**1/94-9/01**	**1/94-6/98**	**7/98-9/01**
Aggregate	60.9	59.6	62.8	115.3	124.7	102.4
Gov.-Cred.	61.0	59.9	62.6	126.9	134.3	117.6
Treasury	60.0	58.8	61.8	122.6	128.0	116.3
Agency	60.3	58.6	62.6	114.0	124.4	99.2
Credit	63.3	63.2	63.5	143.6	154.9	128.2
AA-Finance	58.7	55.3	63.3	110.7	116.5	103.4
Mortgage	58.0	54.6	62.7	80.7	90.3	66.0

EXHIBIT B-3

Swaps Replication Portfolio Return Summary Statistics, 1/94-9/01, bp/month

	Mean			Standard Deviation		
Index	**1/94-9/01**	**1/94-6/98**	**7/98-9/01**	**1/94-9/01**	**1/94-6/98**	**7/98-9/01**
Aggregate	61.1	58.3	65.0	114.5	121.3	105.8
Gov.-Cred.	61.1	58.9	64.0	124.4	129.1	119.2
Treasury	60.8	58.2	64.4	121.4	123.8	119.6
Agency	60.1	57.7	63.4	111.6	119.7	100.9
Credit	62.2	61.3	63.3	138.9	148.0	127.2
AA-Finance	58.4	54.4	64.0	109.1	112.8	104.9
Mortgage	61.1	56.7	67.3	97.5	106.3	85.0

EXHIBIT B-4

Hybrid Treasury Futures (Treasuries)**/Swaps** (spread product)
Replication Portfolio Return Summary Statistics, 1/94-9/01, bp/month

	Mean			Standard Deviation		
Index	**1/94-9/01**	**1/94-6/98**	**7/98-9/01**	**1/94-9/01**	**1/94-6/98**	**7/98-9/01**
Aggregate	61.3	58.8	64.7	115.4	124.0	103.8
Gov.-Cred.	61.3	59.8	63.4	126.5	133.9	117.2