

Real Options, Product Market Competition, and Asset Returns

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ABSTRACT

We study how competition in the product market affects the link between firms' real investment decisions and their asset return dynamics. In our model, assets in place and growth options have different sensitivities to market wide uncertainty. The strategic behavior of market participants influences the relative importance of these components of firm value. We show that the relationship between the degree of competition and assets' expected rates of return varies with product market demand. When demand is low, firms in more competitive industries earn higher returns, whereas when demand is high firms in more concentrated industries earn higher returns.

FIRMS' INVESTMENT AND OPERATING DECISIONS CAN explain conditional dynamics in expected asset returns. For example, consider a firm that has an opportunity to expand its current production capacity. Since this growth option has leverage arising from its investment costs, its systematic risk may be higher than that of the assets already in place. Consequently, the firm's exposure to systematic risk increases as the investment in additional capacity is more attractive because the growth opportunity accounts for a larger fraction of firm value.

Most of the recent theoretical work that links firms' optimal investment behavior to their expected returns does not take into account the effect of competition in the product market when analyzing the risk dynamics of the firms in a given industry. However, investment decisions in an oligopolistic industry may involve strategic preemptive motives. Therefore, competitive pressures may affect the value of the growth opportunities of individual firms and, consequently, their exposure to systematic risk.

A recent empirical study on industry concentration and stock returns by Hou and Robinson (2006) finds that firms in less concentrated industries earn higher returns, even after controlling for size, book-to-market, momentum, and

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other known return predictors. They suggest an innovation risk interpretation for their findings. Specifically, firms in less concentrated industries are riskier because they engage in more innovation, thus commanding higher expected returns. While this explanation seems plausible, it does not provide the precise mechanisms through which market structure affects the risk dynamics of the firms in a given industry. Hence the need for asset pricing models that explicitly incorporate features of product markets as determinants of asset returns.

This article studies the effects of competitive interactions among firms on asset returns in a real options framework. We model an industry in which all the firms compete to supply a homogenous product in a market where demand is stochastic. The firms' production decisions are constrained by their installed capacity, but they have the opportunity to invest to expand their capacity when demand is favorable. In formulating their investment strategies, the firms must take into consideration how the investment strategies of their competitors impact their own payoff.

We find that the relationship between the degree of competition and the firms' exposure to systematic risk varies with product market demand. Firms in competitive industries are riskier when demand is low, while firms in concentrated industries are riskier when demand is high. The intuition for this result follows from the effect of competition on the value of growth options, and from the relation between the level of demand and the relative riskiness of assets in place and growth options.

The value of the growth options derives from the firms' ability to decide when to invest in additional capacity. Because the future value of additional capacity is uncertain, there is an opportunity cost of investing today. Thus, the optimal investment rule is to invest when the value of the additional capacity exceeds the investment cost by an amount that corresponds to the value of the option to wait. However, increasing competition leads firms to exercise their options sooner, as the fear of preemption diminishes the value of their options to wait. Therefore, the value of the growth options decreases with more firms in the market. In the limiting case of a perfectly competitive industry, the value of the growth options is zero.

The value of assets in place is the present value of the future cash flows generated by the firm's installed capacity. In our model, production costs introduce operating leverage, which means that the risk of the firms' cash flows increases as their revenues decline due to lower demand. This makes assets in place riskier than growth options when demand is low. But when demand is high, firms are more likely to invest. The anticipated supply from new capital attenuates the effect of demand shocks on the value of assets in place, hence reducing their risk. Thus, assets in place are less risky than growth options when demand is high.

The fact that the value of the growth options decreases with more firms in the market implies that the value of the firms in a competitive industry is mostly due to the value of their assets in place, while growth options account for a larger fraction of firm value in a concentrated industry. Therefore, firms in more competitive industries are riskier when demand is low because assets in

place are riskier than growth options in bad times, whereas firms in more concentrated industries are riskier when demand is high because growth options are riskier than assets in place in good times.

This article contributes to a growing literature pioneered by Berk, Green, and Naik (1999) that links firms' real investment decisions and asset return dynamics. This literature includes Gomes, Kogan, and Zhang (2003), Kogan (2004), Carlson, Fisher, and Giammarino (2004), Cooper (2006), and Zhang (2005). These papers provide models that relate risk and return dynamics to firm-specific characteristics such as size and book-to-market. Specifically, if assets in place and growth options have different sensitivities to changing economic conditions, their systematic risks will be different. The relative importance of assets in place and growth options changes over time in response to optimal investment decisions. Hence, the firm's true conditional systematic risk or beta can vary. By endogenizing expected returns through firm-level decisions, the papers in this emerging literature show how firm characteristics such as size and book-to-market can proxy for conditional beta.¹ We contribute to this literature by showing how competitive interactions among firms in a given industry affect the dynamics of systematic risk.

Our framework also allows for a better understanding of the roles of operating leverage and irreversibility in explaining the risk dynamics of individual firms. In a monopolistic setting, Carlson, Fisher, and Giammarino (2004) and Cooper (2006) rely on operating leverage to explain the positive correlation between the book-to-market ratio and stock returns observed in the data. Intuitively, when firms' revenues fall due to a reduction in their output price, equity values fall relative to installed capital, which can be proxied by book value. If the fixed operating costs are proportional to installed capital, the risk of the firm increases because of higher operating leverage.

In contrast, without relying on operating leverage, Kogan (2004) shows how irreversible investment leads to a positive relation between the book-to-market ratio and the conditional moments of returns in a perfectly competitive industry. Intuitively, as firm value increases, investment in new capital becomes more attractive. The new supply offsets the effect of demand shocks on the value of the firm, reducing the volatility and systematic risk of stock returns. Therefore, the expected return decreases as the value of the firm increases relative to its installed capital.

Kogan's model does not need production costs to obtain a positive relation between the book-to-market ratio and systematic risk because in a perfectly competitive industry the value of the growth options is zero as competition erodes all the value of waiting to invest, and consequently, the value of the firms consists entirely of assets in place. However, growth options can account for a large fraction of firm value for a monopolist. This tends to make low book-to-market firms riskier because of the leverage arising from investment costs in the large growth option component of firm value. Hence, to account for

¹ Fama and French (1992, 1993) provide empirical evidence on the ability of size and book-to-market to explain the cross-section of stock returns.

the positive correlation between book-to-market and risk observed in the data, Carlson, Fisher, and Giammarino (2004) and Cooper (2006) use fixed operating costs that are proportional to installed capital. These costs introduce operating leverage, which makes assets in place riskier than growth options.

Operating leverage and irreversibility are both important in generating our results on the effects of competition on risk dynamics. Firms in competitive industries are riskier when demand is low because operating leverage makes assets in place riskier than growth options. Furthermore, in Section III we show that without production costs increased competition always reduces risk. Hence, our model suggests that an unconditional competition premium must be associated with high production costs, while low production costs lead to an unconditional concentration premium. Regarding irreversibility, in Section III we demonstrate that when investment is totally reversible the degree of competition does not affect the risk exposure of individual firms.

Our work is also related to Aguerrevere (2003), who shows how operating flexibility affects the behavior of equilibrium output prices in a model of strategic capacity expansion. He demonstrates that when firms have the ability to vary their capacity utilization in response to a shock in demand, output price volatility is increasing in the number of firms in the industry.

This article is organized as follows. Section I presents our model of capacity choice and operating flexibility. Section II derives the value of firms in the market. Section III examines the effect of competition on expected returns. Section IV concludes.

I. The Model

Our model extends the model of Grenadier (2002), who derives the equilibrium investment strategies of firms in a Cournot–Nash framework, by introducing an operating option that allows firms to vary their capacity utilization in response to changes in demand.

Consider an industry composed of n firms producing a single non-storable good. At time t , each firm i produces $q_i(t)$ units of output. The output price is a function of the industry output and a stochastic demand shock. Specifically, we assume the following simple form for the inverse demand curve:

$$P(t) = Y(t) \times Q(t)^{-1/\gamma}, \quad (1)$$

where $P(t)$ is the output price, $Y(t)$ is an exogenous shock to demand, $Q(t) = \sum_{i=1}^n q_i(t)$ is the industry output, and the constant $\gamma > 1$ is the elasticity of demand. With this assumed functional form, changes in the variable Y will be reflected in parallel shifts to the demand curve. Thus, Y can be thought of as the relative strength of the demand side of the market. Conditions affecting the strength of demand include the level of industrial production, household income, etc. The demand shock evolves as a geometric Brownian motion

$$dY(t) = \mu Y(t) dt + \sigma Y(t) dZ(t), \quad (2)$$

where μ is the instantaneous proportional change in Y per unit time, σ is the instantaneous standard deviation per unit time, and Z is a standard Wiener process. Both μ and σ are constant.

Firms operate a simple production technology. Each unit of installed capacity can produce one unit of output per unit time at a cost c , where c is the constant marginal cost, assumed to be the same for all firms. At any time t the firms play a static Cournot game. Each firm chooses its output to maximize its profit. This choice depends on current demand and is constrained by the firm's current production capacity, which is denoted by $K_i(t)$. In addition, each firm must condition its output choice on the output choices of the other firms, which are also constrained by their capacity levels.

At any time t , each firm can invest in additional capital to increase its production capacity by an infinitesimal increment $dK_i(t)$. The price of a new unit of capacity is a constant I .² Investment is irreversible; technically this means that the process for the firm's capacity, $\{K_i(t)\}$, is non-decreasing.³

Each firm chooses its production capacity $K_i(t)$ to maximize its value, conditional on the capacity choices of its competitors. Thus, the optimal investment decision is an endogenous Nash equilibrium solution in investment strategies. Production capacity is the strategic variable and each firm must condition its capacity choice on the strategies of its competitors. For each firm i , let $K_{-i} = (K_1, \dots, K_{i-1}, K_{i+1}, \dots, K_n)$ denote the strategies of firm i 's competitors. An n -tuple of strategies $(K_1^*, \dots, K_n^*)$ is a Nash industry equilibrium if

$$K_i^* = K_i(Y, K_{-i}^*), \quad i = 1, \dots, n. \quad (3)$$

To simplify the analysis we assume that the industry is composed of n identical firms. That is, all the firms start with the same initial capacity and, thus, they all have the same size at any time. Our analysis therefore focuses on a symmetric Nash equilibrium.

Let $K(t)$ denote the total industry installed capacity. Since all firms are identical it follows that $K_i(t) = K(t)/n$ and $K_{-i}(t) = (n-1)K(t)/n$. Thus, by focusing on a symmetric equilibrium, the state space is substantially reduced and the firms condition their investment and production decision on the level of the demand shock Y and the total industry capacity K . For example, the instantaneous profit that firm i earns at time t when the industry capacity is $K(t)$ and the demand parameter is $Y(t)$ is given by

$$\pi_i(K(t), Y(t)) = \max_{0 \leq q_i(t) \leq K(t)/n} [(Y(t)Q(t)^{-1/\gamma})q_i(t) - cq_i(t)]. \quad (4)$$

² If $I = 1$, then firm i 's capital K_i can be interpreted as the book value of the firm's assets.

³ The assumption of total irreversibility is done for tractability. The model can be extended to allow the firm to sell part of its capacity. However, none of the basic results of the model are altered under partial irreversibility. Furthermore, assuming that the installed capacity depreciates over time at a given rate does not affect the model results qualitatively.

From the symmetric equilibrium assumption the solution to (4) is

$$\pi_i(K, Y) = \begin{cases} \frac{1}{n} \left(\frac{c}{n\gamma - 1} \right) \left(\frac{n\gamma - 1}{n\gamma c} Y \right)^\gamma & \text{for } Y < \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \\ \frac{YK^{\frac{\gamma-1}{\gamma}}}{n} - \frac{cK}{n} & \text{for } Y > \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}. \end{cases} \quad (5)$$

The interpretation of the expression for the instantaneous profit flow in (5) is as follows. When the industry capacity is K , each firm i has K/n units of installed capacity. The optimal output is less than K/n when $Y < n\gamma c K^{1/\gamma}/(n\gamma - 1)$. Hence, when capacity is not fully used the profit flow depends only on the level of demand. When $Y > n\gamma c K^{1/\gamma}/(n\gamma - 1)$, each firm produces at maximum capacity selling K/n units of output, and the profit per unit produced is $YK^{-1/\gamma} - c$.

II. Valuation

Following Carlson, Fisher, and Giammarino (2004) we assume the existence of traded assets that can be used to hedge demand uncertainty. Specifically, let M be the price of a riskless asset with dynamics $dM(t) = rM(t)dt$, where r is the (constant) risk-free rate of interest, and let X be the price of a risky asset, which evolves as geometric Brownian motion

$$dX(t) = \eta X(t)dt + \sigma X(t)dZ(t). \quad (6)$$

The risky asset and the demand shock are driven by the same Brownian motion Z and have the same instantaneous standard deviation σ of relative changes. Thus, they are perfectly correlated. The difference in their drifts is $\delta = \eta - \mu$. To ensure that the value of the firm is finite we must have $\delta > 0$. Since the relative changes in X and Y are perfectly correlated we can construct a portfolio with X and M that exactly replicates the dynamics of the firm's value. To find the value of the firm we use the traded assets X and M to define a new probability measure Q under which the process $Z^*(t) = Z(t) + \frac{\eta-r}{\sigma}t$ is a standard Brownian motion. Under this risk neutral measure, the demand shock follows the process

$$dY(t) = (r - \delta)Y(t)dt + \sigma Y(t)dZ^*(t). \quad (7)$$

Let $V_i(K, Y)$ be the value of firm i when industry capacity is K and the level of the demand parameter is Y . The firm's problem is to choose the path of capacity expansion that maximizes the present value of its future cash flows. Thus, each firm solves the optimal control problem

$$V_i(K(t), Y(t)) = \max_{\{K_i(s); s \geq 0\}} E^Q \left\{ \int_0^\infty e^{-rt} [\pi_i(K(t), Y(t))dt - I dK_i(t)] \right\}. \quad (8)$$

As in Pindyck (1988) and He and Pindyck (1992), we approach the solution to this problem by examining the firm's incremental investment decision.⁴

The opportunity to invest in an additional unit of capacity is analogous to a perpetual American call option. The underlying asset is the value of an extra unit of capacity and the exercise price is the cost of investing in this unit.

Therefore, the solution to the firm's capacity choice problem involves two steps. First, the value of an extra unit of capacity must be determined. Second, the value of the option to invest in this unit must be determined together with the decision rule for exercising this option. This decision rule is the solution to the optimal capacity problem.

The value of a marginal unit of capacity is the present value of the expected flow of profits from this unit. Given the current capacity K and demand Y , $\Delta F_i(K, Y)$ denotes the value of a marginal unit of capacity. An expression for ΔF_i is derived in the Appendix.

After obtaining the value of the marginal unit of capacity, we can value the option to invest in this unit. Let $\Delta G_i(K, Y)$ denote the value of this option when the current capacity level is K . The exercise price of this option is equal to the investment cost I . Therefore, for any level of committed capacity K , $\Delta G_i(K, Y)$ is a perpetual American call option whose value depends on Y . Hence, there will be a threshold value at which it will be optimal to exercise this option. Specifically, for any K and n there exists a threshold $Y^n(K)$ such that the option to build an additional unit of capacity will be exercised the first time that Y equals or exceeds $Y^n(K)$.

The Appendix shows that the solution to the investment threshold in an industry composed of n identical firms is of the form

$$Y^n(K) = v_n K^{1/\gamma}, \quad (9)$$

where the expression for v_n is given in the Appendix.

In the Appendix we prove that for any given level of industry capacity K , the investment trigger $Y^n(K)$ is decreasing in n . Thus, increasing competition leads firms to exercise their expansion options earlier. Grenadier (2002) obtains a similar result in a model without operating flexibility. The reason is that with more competition each firm has to expand its capacity sooner to avoid losing the investment opportunity to its competitors.

The result that the investment threshold for additional capacity $Y^n(K)$ is decreasing in the number of firms in the industry is based on a given level of industry capacity that is independent of the number of firms in the market. Equivalently, we can show how the degree of competition affects the industry capacity. Specifically, let K^n be the total capacity for an n -firm industry. The endogenous industry capacity is increasing in the number of firms. Specifically, in the Appendix we show that for any number of firms n ,

⁴ The value of the firm can be derived as the solution to an optimal instantaneous control problem. He and Pindyck (1992) show that the solution to this type of capacity choice problem can be obtained by examining the firm's incremental investment decision.

$$K^n = \left[\frac{n\gamma - 1}{n(\gamma - 1)} \right]^\gamma K^1. \quad (10)$$

Thus, the capacity of an n -firm industry is equal to the monopoly capacity times a factor that is greater than one, increases in n , and converges to $[\gamma/(\gamma - 1)]^\gamma$ as n approaches infinity.

However, the investment threshold for new capacity, $Y^n(K^n)$, is the same for any number of firms in the market. To prove this result it suffices to show that $Y^n(K^n) = Y^1(K^1)$ for all n . Substituting equation (10) into equation (A5) in the Appendix proves this result. As the next section illustrates, letting the industry capacity be dependent on the number of firms in the market as in (10) allows us to compare the values and returns resulting from different numbers of firms in the market over the same interval for the demand values, that is, for Y between zero and the investment threshold $Y^n(K^n)$.

Once we solve for the firms' optimal investment rule we can derive an expression for their value. Given the industry capacity K and the current value of Y , we can write the value of firm i , V_i , as the sum of two parts:

$$V_i(K, Y) = F_i(K, Y) + G_i(K, Y), \quad (11)$$

where $F_i(K, Y)$ is the value of the firm's assets in place and $G_i(K, Y)$ is the value of the firm's growth options. Therefore, to get the value of each firm, we need to obtain the value of its assets in place and the value of its growth options.

The value of the assets in place is the value of the firm's installed capacity. When industry capacity is K , each firm i has K/n units of capacity. This capacity provides each firm a cash flow stream of $\pi_i(K(t), Y(t))$, which is the profit given by (5). Also notice that in valuing the assets in place we have to take into consideration the effect of future investment on the value of the firm's installed capacity. Specifically, the fact that firms can increase their capacity when the demand factor Y hits $Y^n(K)$ cuts off some of the upside potential for prices and profits. Therefore, to determine the value of the assets in place we first find the present value of the profit flow $\pi_i(K(t), Y(t))$ for fixed K and then adjust this value for the impact of future increases in industry capacity on the value of the firm's current capacity.

The Appendix shows that the present value of the profit flow $\pi_i(K(t), Y(t))$ is

$$\frac{K}{n} J(K, Y),$$

where $J(K, Y)$ is the value of one unit of capacity for fixed K , and is given by

$$J(K, Y) = \begin{cases} \frac{\left(\frac{c}{n\gamma - 1} \right) \left(\frac{n\gamma - 1}{n\gamma c} Y K^{-1/\gamma} \right)^\gamma}{r - \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2/2} + B(K)Y^\lambda & \text{for } Y < \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \\ \frac{Y K^{-1/\gamma}}{\delta} - \frac{c}{r} + A(K)Y^\alpha & \text{for } Y > \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}. \end{cases} \quad (12)$$

The expressions for the functions $A(K)$ and $B(K)$ and the constants α and λ are given in the Appendix.

To facilitate the interpretation of the results on the effect of competition on asset returns, presented in Section III below, we provide an explanation for the expression for the value of one unit of capacity in (12). Equation (5) gives the profit function for a firm with K/n units of capacity. Thus, the profit per unit is

$$\hat{\pi}_i(K, Y) = \frac{\pi_i(K, Y)}{K/n} = \begin{cases} \left(\frac{c}{n\gamma - 1} \right) \left(\frac{n\gamma - 1}{n\gamma c} YK^{-1/\gamma} \right)^\gamma & \text{for } Y < \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \\ YK^{-1/\gamma} - c & \text{for } Y > \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}. \end{cases} \quad (13)$$

If the installed capacity is always used, the value of one unit of capacity is

$$\frac{YK^{-1/\gamma}}{\delta} - \frac{c}{r}.$$

Thus, when $Y > n\gamma c K^{1/\gamma}/(n\gamma - 1)$, the term $A(K)Y^\alpha$ in (12) is the value of the option to reduce output should Y decrease. This option is valuable because firms can reduce their output when $Y < n\gamma c K^{1/\gamma}/(n\gamma - 1)$ and earn a profit of $(\frac{c}{n\gamma - 1})(\frac{n\gamma - 1}{n\gamma c} YK^{-1/\gamma})^\gamma$ per unit of capacity. This profit is greater than $YK^{-1/\gamma} - c$, which is the profit per unit if installed capacity is always used. Therefore, $A(K)$ is positive for all n .

When the installed capacity is not fully used, that is, when $Y < n\gamma c K^{1/\gamma}/(n\gamma - 1)$, the present value of the profits per unit of capacity is

$$\frac{\left(\frac{c}{n\gamma - 1} \right) \left(\frac{n\gamma - 1}{n\gamma c} YK^{-1/\gamma} \right)^\gamma}{r - \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2/2}.$$

The output of each firm and its capacity utilization increase as Y increases. However, when the industry capacity is K , each firm can produce up to K/n units of output. Therefore, the term $B(K)Y^\lambda$ in (12) represents the impact of the constraint imposed by installed capacity on the value of the firms' assets. This impact can be negative or positive depending on the degree of competition, which means that the sign of $B(K)$ depends on the number of firms in the industry. For a monopolist, the capacity constraint reduces the profit that would otherwise be earned if the capacity was larger. That is, for $n = 1$, $(\frac{c}{\gamma - 1})(\frac{\gamma - 1}{\gamma c} YK^{-1/\gamma})^\gamma > YK^{-1/\gamma} - c$ for all Y . Hence, when capacity is not completely utilized, that is, when $Y < \gamma c K^{1/\gamma}/(\gamma - 1)$, the term $B(K)Y^\lambda$ represents the reduction in the value of the firm's assets in place if Y rises over $\gamma c K^{1/\gamma}/(\gamma - 1)$ and capacity binds. Thus, for $n = 1$, $B(K)$ is negative. As the number of firms in the industry increases, the profit per unit produced when capacity is not fully used declines because increasing competition leads firms to produce more in response to increases in demand. But when demand is high, and the

capacity is fully utilized, the constraint imposed by installed capacity is good for competitive firms because it acts as a commitment device that prevents them from increasing production in response to increases in demand. Therefore, $B(K)$ is increasing in the number of firms in the market, n . In the case of perfect competition, which in the model is obtained as the limit as n approaches infinity, the profit when capacity is not fully used is zero because the output price is equal to the marginal cost c . But when capacity is fully utilized, that is, when $Y > cK^{1/\gamma}$, the profit per unit is $YK^{-1/\gamma} - c > 0$. Therefore, $B(K)$ is positive and $B(K)Y^\lambda$ represents the value of the option to earn a positive profit if demand increases and firms produce at full capacity.

The value of one unit of capacity for fixed K , $J(K, Y)$ in equation (12), does not take into consideration the effect of future investment on the value of the installed capacity. However, the fact that firms can increase their capacity when the demand factor Y hits $Y^n(K)$ removes some of the upside potential for prices and profits, so the value of one unit of capacity must be less than $J(K, Y)$. Denote by $H(K, Y)$ the value of one unit of capacity. The Appendix shows that

$$H(K, Y) = J(K, Y) + E(K)Y^\lambda, \quad (14)$$

where $E(K)Y^\lambda$ represents the impact of future increases in industry capacity on the value of the firm current capacity. Since increased supply has a negative effect on output prices, and therefore on future cash flows, the sign of $E(K)$ is negative.

When the industry capacity is K , each firm i has K/n units of capacity. Therefore, the value of firm i 's assets in place is

$$F_i(K, Y) = \frac{K}{n}H(K, Y). \quad (15)$$

The value of the growth options derives from the firms' ability to decide when to invest in additional capacity. Because the future value of any additional unit of capacity is uncertain, there is an opportunity cost of investing today, rather than waiting and keeping open the possibility of not investing should demand conditions for the firm's output change adversely. Thus, the optimal investment rule is to invest when the value of the additional capacity exceeds the investment cost by an amount that corresponds to the value of the option to wait.

The value of the growth options is

$$G_i(K, Y) = C(K)Y^\lambda. \quad (16)$$

The expression for the function $C(K)$ is given in the Appendix.

The value of the growth option decreases with more firms in the market because, as shown above, increasing competition leads firms to exercise their options sooner, as the fear of preemption diminishes the value of their options to wait.

III. Expected Returns

This section analyzes the effect of product market competition on asset returns by examining how the strategic behavior of firms affects their betas. The demand factor Y is the source of risk in our model, and the beta of a firm measures the sensitivity of relative changes in the firm's value to relative changes in the demand factor. Thus, a firm's beta is the elasticity of its market value with respect to the demand factor. Formally, if the demand factor has a beta of one, then the beta of firm i is⁵

$$\beta_i(K, Y) = \frac{Y}{V_i(K, Y)} \frac{\partial V_i(K, Y)}{\partial Y}. \quad (17)$$

Our model assumes that investment is completely irreversible. To highlight the importance of irreversible investment in generating our results the effect of product market competition on asset returns below, we first compute the beta of the firm when investment is completely reversible. When investment is unconstrained the industry capacity K is not a state variable, and the value of the firm is

$$V_i(Y) = \frac{\frac{1}{n} \left(\frac{c + I}{n\gamma - 1} \right) \left(\frac{n\gamma - 1}{n\gamma c} Y \right)^\gamma}{r - \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2/2}.$$

It follows that beta is $\beta_i(Y) = \gamma$, which is the elasticity of demand. Therefore, when investment is totally reversible, the beta of the firm is a constant independent of the number of firms in the market. Hence, irreversible investment is essential to generate our results on the effects of competition on asset returns.⁶

We study the effect of competition on asset returns by analyzing the betas of the assets in place and growth options. The beta of assets in place is

$$\beta_F(K, Y) = \frac{Y}{F_i(K, Y)} \frac{\partial F_i(K, Y)}{\partial Y} = \frac{Y}{H(K, Y)} \frac{\partial H(K, Y)}{\partial Y}, \quad (18)$$

where the last equality in (18) follows from equation (15). From (18) and (14) we can get the following expression for the beta of assets in place:

$$\beta_F(K, Y) = \beta_J(K, Y) + \frac{E(K)Y^\lambda}{H(K, Y)}(\lambda - \beta_J(K, Y)), \quad (19)$$

where $\beta_J(K, Y)$ is the beta of one unit of capacity for fixed K , which is given by

$$\beta_J(K, Y) = \begin{cases} \gamma + (\lambda - \gamma) \frac{B(K)Y^\lambda}{J(K, Y)} & \text{for } Y < \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \\ 1 + \frac{c/r}{J(K, Y)} + (\alpha - 1) \frac{A(K)Y^\alpha}{J(K, Y)} & \text{for } Y > \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \end{cases}. \quad (20)$$

⁵ For a formal derivation of equation (17) see the proof of Proposition 2 in Carlson, Fisher, and Giammarino (2004).

⁶ We thank an anonymous referee for suggesting this emphasis on the role of irreversibility in generating our results.

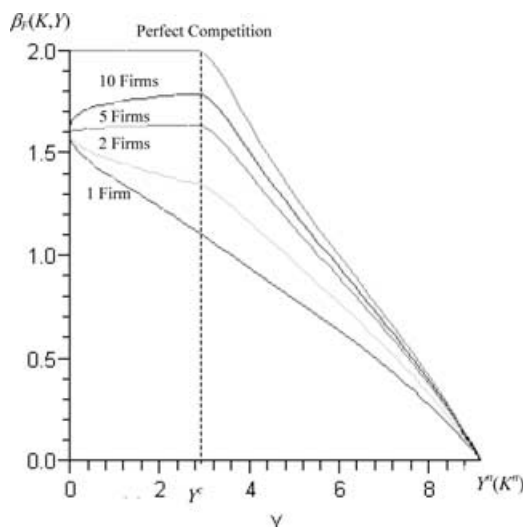


Figure 1. The beta of assets in place as a function of Y for different numbers of firms in the industry. This figure shows the beta of the firm's assets in place as a function of Y when K depends on the number of firms in the market for 1 firm, 2 firms, 5 firms, 10 firms, and perfect competition. The assumed parameter values are $I = 1$, $c = 0.06$, $\gamma = 1.6$, $r = 0.06$, $\delta = 0.05$, and $\sigma = 0.2$.

The interpretation of (19) is as follows. In the previous section we show that the value of one unit of capacity, $H(K, Y)$, is equal to the value of one unit of capacity for fixed K , $J(K, Y)$, adjusted for the impact of future increases in industry capacity on the value of the firm's current capacity. New investment affects the beta of assets in place because the additional supply from the added capacity buffers the effect of changes in demand on the firm's value. Thus, the first term in equation (19) is the beta of the value of the profit stream provided by the firm's current capacity, and the second term takes into account the effect of future investment on the beta of assets in place.

Figure 1 illustrates the relationship between the beta of assets in place and the level of demand for a monopoly, a duopoly, a 5-firm oligopoly, a 10-firm oligopoly, and perfect competition. The beta of assets in place is increasing in the number of firms in the industry for any level of Y between zero and the investment threshold, $Y^n(K^n)$, where K^n is the total capacity of an industry with n firms. To obtain Figure 1 we assume that the relationship between industry capacity K^n and the number of firms n is given by (10), which means, as seen in the previous section, that the investment threshold $Y^n(K^n)$ is independent of the number of firms in the market. Thus, we can compare the betas resulting from different numbers of firms in the market over the same interval for the demand values. Figure 1 also shows the level of Y above which firms will produce at full capacity, which is denoted by Y^c . This level is the same for all n when the relationship between industry capacity and the number of firms is given by (10) because $n\gamma c(K^n)^{1/\gamma}/(n\gamma - 1)$ is the same for all n .

The intuition for the result that the beta of assets in place increases with more competitors is as follows. When capacity is partially used, increased competition leads firms to produce more in response to changes in demand. Thus, assets in place in more competitive markets are riskier because their cash flows are more sensitive to demand shocks. When a firm produces at full capacity, its total production cost is constant because its output is fixed. The constant cost introduces leverage, which increases the risk of the firm's cash flows when revenue falls due to a negative shock to demand. On the other hand, the option to reduce output in response to a fall in demand attenuates the effect of leverage on risk. However, the value of this option decreases with the number of firms in the industry. The reason is that firms in more competitive markets optimally delay their output reductions because the benefits of lower production accrue most to industry rivals. This causes competitive firms to have higher operating leverage as demand falls. Therefore, when capacity is fully used the beta of the firms' assets in place is increasing in the number of firms in the industry.

We can expand our explanation of the effect of competition on the beta of assets in place by examining the expression for the beta of one unit of capacity for fixed K in equation (20). When $Y < n\gamma cK^{1/\gamma}/(n\gamma - 1)$, the expression for the beta of the firm's capacity in equation (20) has two terms. When capacity is not fully used, each firm varies its output in response to changes in demand. The first term is the beta of the firm's profits. This is also the beta of a firm that operates without a capacity constraint. Therefore, the second term captures the impact of the capacity constraint on the risk of the firm's assets. As explained in the previous section, this term is increasing in the number of firms in the market because the constraint imposed by installed capacity is good for competitive firms. Therefore, when capacity is partially used, the beta of the firm's capacity increases with more competition.

When $Y > n\gamma cK^{1/\gamma}/(n\gamma - 1)$, the expression for the beta of the firm's capacity in equation (20) has three terms. The first term is the firm's revenue beta. When a firm produces at full capacity its output is constant and its revenue varies linearly with Y . This can be seen in equation (5) when $Y > n\gamma cK^{1/\gamma}/(n\gamma - 1)$. Thus, the revenue beta is equal to the beta of demand, which is assumed to be one. The second term captures the leverage effect from production costs. This leverage effect is more pronounced in more competitive markets because the value of one unit of capacity, $J(K, Y)$, decreases with the number of firms in the market. The third term derives from the option to reduce output in response to a fall in demand. This option diminishes the effect of leverage on risk, and consequently reduces beta.⁷ However, as explained above, the value of this option decreases with the number of firms in the industry. Therefore, when capacity is fully used, the beta of the firms' current capacity is increasing in the number of firms in the market.

Figure 1 also shows that the beta of assets in place is zero at the investment threshold $Y^n(K^n)$ for any number of competitors. The reason is that production from new capacity completely absorbs the effect of demand shocks on the value

⁷ The sign of this term is negative because $\alpha < 0$ and $A(K) > 0$.

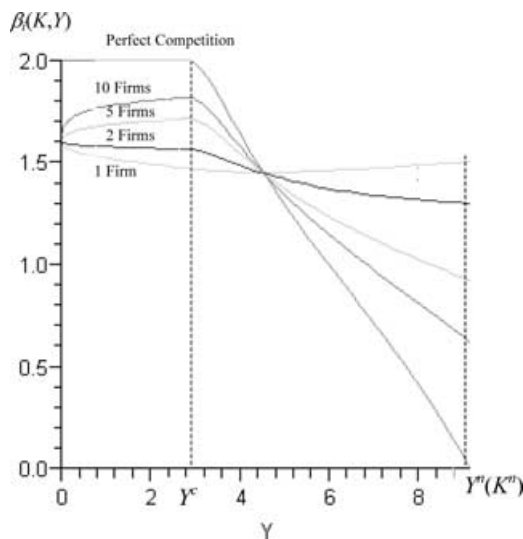


Figure 2. The beta of the firm as a function of Y for different numbers of firms in the industry. This figure shows the beta of the firm as a function of Y when K depends on the number of firms in the market for 1 firm, 2 firms, 5 firms, 10 firms, and perfect competition. The assumed parameter values are $I = 1$, $c = 0.06$, $\gamma = 1.6$, $r = 0.06$, $\delta = 0.05$, and $\sigma = 0.2$.

of assets in place. Also notice that, except for perfect competition, the beta of the assets in place is the same for any number of firms in the industry when $Y = 0$. It follows from equations (19) and (20) that $\beta_F(0, Y) = \gamma$. Thus, for any number of competitors the beta of the assets in place is equal to the demand elasticity when $Y = 0$.

The beta of the growth options is

$$\beta_G(K, Y) = \frac{Y}{G_i(K, Y)} \frac{\partial G_i(K, Y)}{\partial Y} = \lambda, \quad (21)$$

where the last equality follows from equation (16). Equation (21) shows that the beta of the growth options is a constant independent of industry capacity, the demand factor, and the number of firms in the industry.⁸ The reason is that the option to invest in additional capacity is a perpetual American call option. While the beta of the growth options is the same constant irrespective of the number of firms in the industry, the level of demand and the degree of competition affect the value of these options and thus the firm beta.

Figure 2 illustrates the effect of competition on the beta of an individual firm using the same parameters of Figure 1. As in Figure 1, Y^c denotes the level of demand above which firms will produce at full capacity, and $Y^n(K^n)$ denotes the investment threshold. The relationship between the degree of competition

⁸ The Appendix shows that we must have $\lambda > \gamma$, where γ is the elasticity of demand, to ensure that the value of the firm is finite.

and the firms' beta depends on the level of demand. Firms in more competitive industries are riskier when demand is low, while firms in more concentrated industries are riskier when demand is high. The explanation for this result is as follows. The beta of the firm is the value-weighted average of the betas of its assets in place and growth options. The proportion of the growth options in the total value of the firm increases with the level of demand because investment in new capacity becomes more attractive when demand is favorable. Hence, it follows that assets in place amount to a larger fraction of the total value of the firm when demand is low. This means that firms in more competitive markets are riskier because, as seen above, the beta of assets in place increases with the number of competitors. When demand is high, firms in more concentrated markets are riskier because growth options, which are riskier than assets in place, account for a larger fraction of firm value in a concentrated industry.

Notice that in Figure 2 there is a level of demand at which beta is the same for any number of competitors. For levels of demand below this critical level the beta of individual firms is increasing in the degree of competition, while for any level of demand above this critical level firms in more competitive industries have a lower beta. Capacity is fully used at this critical level. As explained above, when firms produce at full capacity the difference among the betas of assets in place for different numbers of competitors declines to zero as demand increases to the level that triggers investment. At the same time, the increase in the value of the growth options is more pronounced for a less competitive industry. Thus, for any two different levels of competition there must be a level of demand at which the betas of individual firms should be the same. Note that the reason there is one level of demand at which beta is the same for any number of competitors in the example of Figure 2 is that this example is generated using the relationship between industry capacity and the number of firms given in (10). This relationship implies that the investment threshold is the same for any number of firms in the industry.

The main result of this paper is that the effect of competition on individual firms' exposure to systematic risk is conditional on the level of demand for the industry output. As illustrated in Figure 2, there is a region of "low" demand values in which increasing competition leads to higher risk and higher returns, and a region of "high" demand values in which increasing competition leads to lower risk and lower returns. The size of these two regions depends on the production cost c . Specifically, the size of the low region increases with production costs. This implies that high production costs are required to generate an unconditional competition premium, while an unconditional concentration premium requires low costs. To clarify this claim, and to obtain further insights into the role of operating leverage and irreversibility in explaining the risk dynamics of individual firms, it is useful to consider a special case in which the production cost is zero.

The base case in Grenadier (2002) is a special case of our model with $c = 0$. Since the variable production cost is zero, the firms always produce at full capacity. In this case, we can get closed-form solutions for the investment threshold and the value of the firm:

$$v_n = \left(\frac{\lambda}{\lambda - 1} \right) \left(\frac{n\gamma}{n\gamma - 1} \right) \delta I. \quad (22)$$

The value of assets in place is $F_i(K, Y) = \frac{K}{n} H(K, Y)$, where

$$H(K, Y) = J(K, Y) + E(K)Y^\lambda$$

with

$$J(K, Y) = \frac{YK^{-1/\gamma}}{\delta} \quad (23)$$

and

$$E(K) = -\frac{v_n^{1-\lambda}}{\lambda\delta} K^{-\lambda/\gamma}. \quad (24)$$

The value of the growth options is $G_i(K, Y) = C(K)Y^\lambda$, with

$$C(K) = \frac{\gamma}{\lambda - \gamma} \frac{I}{n\gamma - 1} v_n^{-\lambda} \frac{K^{\frac{\gamma-\lambda}{\gamma}}}{n} = \frac{K}{n} \frac{\gamma}{\lambda - \gamma} c(K), \quad (25)$$

where $c(K) = \frac{I}{n\gamma-1} v_n^{-\lambda} K^{-\lambda/\gamma}$ and $c(K)Y^\lambda$ is the value of the option to invest in one unit of capacity.

The beta of assets in place is

$$\beta_F(K, Y) = 1 + (\lambda - 1) \frac{E(K)Y^\lambda}{H(K, Y)}. \quad (26)$$

The first term in Equation (26) is the revenue beta, which is the same as the beta of demand because the firms always use all their installed capacity, which means that their revenue varies linearly with Y . The second term captures the effect of new investment. Expansion is more likely as Y increases, and the additional output from the new capacity offsets the effect of demand changes on the value of the assets in place. Thus, the beta of assets in place is decreasing in Y .

When the industry capacity depends on the degree of competition, we can show, by substituting (10) into (23), (24), and (26), that for all n

$$\beta_F(K^n, Y) = \beta_F(K^1, Y). \quad (27)$$

Thus, when the relationship between industry capacity and the number of firms is given by equation (20), the beta of assets in place is independent of the number of firms in the industry.

The beta of the firm is

$$\beta_i(K, Y) = \beta_F(K, Y) + \frac{C(K)Y^\lambda}{V_i(K, Y)} (\lambda - \beta_F(K, Y)). \quad (28)$$

To get (28), notice that the beta of the firm is the value-weighted average of the betas of assets in place and growth options. The second term in (28) is

positive because the beta of the growth options, λ , is greater than the beta of the assets in place, $\beta_F(K, Y)$. To see this note that $\lambda > 1$, and from (24) and (26) we have $\beta_F(K, Y) < 1$. The value of the growth options, $C(K)Y^\lambda$, decreases with more firms in the industry since competition erodes the value of waiting to invest. Combining this result with the result of equation (27), we find that the beta of individual firms decreases with more competition for all levels of demand. This implies that firms in more competitive industries earn lower returns unconditionally. This contrasts with the empirical findings in Hou and Robinson (2006) that firms in more competitive industries earn higher returns. However, as explained above, with production costs and the option to vary capacity utilization in response to changes in demand, firms in more competitive markets earn higher returns when demand is low.

This special case is also useful in understanding the roles of irreversibility of investment and operating leverage in explaining the relationship between conditional beta and firm characteristics such as the book-to-market ratio. For this purpose, we use the following alternative expression for the beta of the firm:⁹

$$\beta_i(K, Y) = 1 + (\lambda - 1) \frac{\frac{K}{n} \left(E(K) + \frac{\gamma}{\lambda - \gamma} c(K) \right) Y^\lambda}{V_i(K, Y)}. \quad (29)$$

Given the industry capacity K , the book value of each firm is IK/n , and the market value of each firm, $V_i(K, Y)$, increases as Y increases. Thus, the book-to-market ratio declines as demand increases. We can get the relationship between the firm's beta and its book-to-market ratio using equation (29) by looking at the sign of the derivative of $\beta_i(K, Y)$ with respect to Y . It is straightforward to show that this sign is the same as the sign of $E(K) + \frac{\gamma}{\lambda - \gamma} c(K)$. In a monopoly, that is, $n = 1$, $E(K) + \frac{\gamma}{\lambda - \gamma} c(K) > 0$, so beta is increasing in Y and thus decreasing in the book-to-market ratio. This does not confirm the positive relation between book-to-market and expected returns observed in the data. However, the sign of $E(K) + \frac{\gamma}{\lambda - \gamma} c(K)$ will become negative as n increases because the value of the option to invest in one unit of capacity, $c(K)Y^\lambda$, is decreasing in n . When $E(K) + \frac{\gamma}{\lambda - \gamma} c(K) < 0$, beta is increasing in the book-to-market ratio.

Intuitively, as demand increases and investment in new capital becomes more attractive, there are two offsetting effects on expected returns. On the one hand, the new supply absorbs the effect of changes in demand on firm value and reduces its beta. On the other hand, the value of the risky growth options increases with high demand offsetting the reduction in risk resulting from new investment. In monopolistic markets, the latter effect dominates and beta increases as demand increases or book-to-market declines. But with more firms in the market the value of the growth options decreases and the first effect can dominate in more competitive markets, causing beta to be decreasing in demand or increasing in book-to-market. In a perfectly competitive industry,

⁹ This expression follows from (16), (23), (24), and (25).

the correlation between book-to-market and beta is positive because the value of the growth options is zero.

Kogan (2004) shows how irreversibility of investment leads to a positive correlation between the book-to-market ratio and risk. His analysis does not rely on operating costs because he investigates the effect of investment frictions on asset prices in a perfectly competitive industry. The rationale for Kogan's result is similar to our intuition for the perfectly competitive case above. In sum, it is possible to obtain a positive relation between book-to-market and beta in a model that does not rely on production costs if the number of firms in the industry is sufficiently large. However, as seen above, such a model implies that more competition always reduces risk, which does not confirm the empirical findings of Hou and Robinson (2006).

We show above that without operating costs, the beta of a monopolistic firm is negatively correlated with the book-to-market ratio, which contrasts with the positive correlation observed in the data. Carlson, Fisher, and Giammarino (2004) and Cooper (2006) study the effects of firm investment decisions on their risk dynamics in a monopolistic industry. These papers use fixed operating costs that are proportional to the level of capacity as a plausible explanation for the observed relationship between the book-to-market ratio and risk. These costs introduce operating leverage, which increases the risk of the firm as demand falls. Therefore, beta is increasing in book-to-market if the leverage effect overwhelms the contribution of the growth options to the risk of the firm.

A major difference between our model and the models of Carlson, Fisher, and Giammarino (2004) and Cooper (2006) is in the nature of the production costs. These two papers assume that production costs are fixed and proportional to capital, while our model assumes that costs are variable. Operating leverage is lower with variable costs because firms can lower costs by reducing output when demand falls. Recent analysis by Xing and Zhang (2004) fails to find much support for the operating leverage hypothesis of Carlson, Fisher, and Giammarino (2004) and Cooper (2006). By measuring operating leverage as the elasticity of operating profits with respect to sales, Xing and Zhang (2004) find that value firms have slightly lower operating leverage than growth firms. Our model is consistent with this evidence because the option to reduce capacity utilization when demand falls reduces operating leverage when firm value is low. This implies that high book-to-market firms have lower operating leverage.

Finally, in a recent empirical study that examines real and financial outcomes of industry booms and busts, Hoberg and Phillips (2007) find support for the key prediction of our model, that firms in competitive industries are riskier when industry demand is low (industry busts) and firms in concentrated industries are riskier when industry demand is high (industry booms). Hoberg and Phillips (2007) use the Herfindahl index, a common measure of market concentration, to classify industries by their competitiveness. In order to identify the conditions that likely surround industry booms and busts they construct proxies of relative industry valuation that reflect beliefs about future industry prospects. Consistent with the main prediction of our model, Hoberg and Phillips (2007) find that during industry booms, systematic risk decreases more

for firms in competitive industries than in concentrated industries. Following decreases in demand (industry busts), systematic risk increases more for firms in competitive industries than in concentrated industries.

IV. Concluding Remarks

This article studies the effects of competitive interactions among firms on asset returns in a real options framework. We analyze the asset pricing implications of product market competition by examining how the strategic behavior of market participants affects their equilibrium investment and production decisions. Our model extends the approach of Grenadier (2002), who derives the equilibrium investment strategies of firms in a Cournot–Nash framework by introducing an operating option that allows firms to vary their capacity utilization in response to changes in demand. Specifically, capital is the only factor of production in our model, and in any given period each firm chooses its output to maximize its current profit. This choice depends on current demand and is constrained by the firm's current production capacity. Each firm must also condition its output choice on the output choices of the other firms, which are also constrained by their capacity levels. All of the firms can expand their production capacity by investing in additional capacity and they must determine when to exercise their investment opportunities. Each firm's investment strategy is conditional on its competitors' investment strategies. Therefore, investment and operating decisions arise from equilibrium in the product market, reflecting strategic interactions among market participants.

We find that the relationship between the degree of competition and the firms' exposure to systematic risk varies with product market demand. Firms in competitive industries are riskier when demand is low while firms in concentrated industries are riskier when demand is high. This result follows from the effect of competition on the value of growth options, and from the relation between the level of demand and the relative riskiness of assets in place and growth options.

Although our model assumes identical firms, the economic mechanism driving our main result is fairly general. The model could be extended to allow for firms of different sizes and cost structures. This would provide greater realism to the model, but the loss of the simplifying feature of a symmetric equilibrium would greatly diminish the tractability of the model.¹⁰

Other issues remain that we leave for further research. In particular, our model assumes that the number of firms in the industry is exogenous. It is important to study the implications of firms' entry and/or exit. Additionally, it would be interesting to consider the risk dynamics in an industry in which firms can merge.

¹⁰ Carlson et al. (2007) analyze risk dynamics in a duopolistic output market with asymmetric firms. They find that increased competition results in risk reduction. Novy-Marx (2008) studies the value premium in a model that allows for firms of different sizes and cost structures.

Appendix

A. Capacity Choice

The solution to the firm's capacity choice problem involves two steps. First, the value of an extra unit of capacity must be determined. Second, the value of the option to invest in this unit must be determined together with the decision rule for exercising this option. This decision rule is the solution to the optimal capacity problem.

B. The Value of a Marginal Unit of Capacity

Given the current capacity K and demand Y , $\Delta F_i(K, Y)$ denotes the value of a marginal unit of capacity. The value of a marginal unit of capacity is the present value of the expected flow of profits from this unit. It follows from (4) that the profit from the marginal unit of capacity at any time t is

$$\Delta \pi_i(K, Y(t)) = \max \left[\frac{n\gamma - 1}{n\gamma} Y K^{-1/\gamma} - c, 0 \right]. \quad (\text{A1})$$

Thus, after it is completed, each incremental unit of capacity will be used only when the additional profit it generates is positive, that is, when $Y > n\gamma c K^{1/\gamma} / (n\gamma - 1)$.

Following standard arguments, the value of the marginal unit of capacity satisfies the differential equation

$$\frac{\sigma^2}{2} Y^2 \frac{\partial^2 \Delta F_i}{\partial Y^2}(K, Y) + (r - \delta) Y \frac{\partial \Delta F_i}{\partial Y}(K, Y) - r \Delta F_i(K, Y) + \Delta \pi_i(K, Y) = 0,$$

where $\Delta \pi_i$ is given by (A1). In the region in which $Y < n\gamma c K^{1/\gamma} / (n\gamma - 1)$, ΔF_i satisfies the equation

$$\frac{\sigma^2}{2} Y^2 \frac{\partial^2 \Delta F_i}{\partial Y^2}(K, Y) + (r - \delta) Y \frac{\partial \Delta F_i}{\partial Y}(K, Y) - r \Delta F_i(K, Y) = 0,$$

subject to the boundary condition

$$\Delta F_i(K, 0) = 0.$$

Therefore, the solution is of the form

$$\Delta F^1(K, Y) = \Delta B(K) Y^\lambda,$$

where λ is the positive root of the characteristic equation

$$\frac{\sigma^2}{2} \xi(\xi - 1) + (r - \delta)\xi - r = 0. \quad (\text{A2})$$

In the region in which $Y > n\gamma cK^{1/\gamma}/(n\gamma - 1)$, ΔF_i satisfies the equation

$$\begin{aligned} \frac{\sigma^2}{2} Y^2 \frac{\partial^2 \Delta F_i}{\partial Y^2}(K, Y) + (r - \delta) Y \frac{\partial \Delta F_i}{\partial Y}(K, Y) - r \Delta F_i(K, Y) \\ + \frac{n\gamma - 1}{n\gamma} Y K^{-1/\gamma} - c = 0, \end{aligned}$$

subject to the boundary condition

$$\lim_{Y \rightarrow \infty} \Delta F_i(K, Y) = \frac{n\gamma - 1}{n\gamma} \frac{Y K^{-1/\gamma}}{\delta} - \frac{c}{r},$$

which implies a solution of the form

$$\Delta F^2(K, Y) = \Delta A(K) Y^\alpha + \frac{n\gamma - 1}{n\gamma} \frac{Y K^{-1/\gamma}}{\delta} - \frac{c}{r}, \quad (\text{A3})$$

where α is the negative root of equation (A2). To solve for $\Delta A(K)$ and $\Delta B(K)$ we consider the point $Y = n\gamma cK^{1/\gamma}/(n\gamma - 1)$, where the two regions meet. $\Delta F(K, Y)$ must be continually differentiable across $Y = n\gamma cK^{1/\gamma}/(n\gamma - 1)$, therefore $\Delta A(K)$ and $\Delta B(K)$ are the solutions to the system of equations

$$\begin{aligned} \Delta F^1\left(K, \frac{n\gamma cK^{1/\gamma}}{n\gamma - 1}\right) &= \Delta F^2\left(K, \frac{n\gamma cK^{1/\gamma}}{n\gamma - 1}\right) \\ \frac{\partial \Delta F^1}{\partial Y}\left(K, \frac{n\gamma cK^{1/\gamma}}{n\gamma - 1}\right) &= \frac{\partial \Delta F^2}{\partial Y}\left(K, \frac{n\gamma cK^{1/\gamma}}{n\gamma - 1}\right). \end{aligned}$$

Solving this system we get

$$\begin{aligned} \Delta A(K) &= \frac{c \left(\frac{n\gamma cK^{1/\gamma}}{n\gamma - 1} \right)^{-\alpha} \left(\frac{\lambda}{r} - \frac{\lambda - 1}{\delta} \right)}{(\lambda - \alpha)} \\ \Delta B(K) &= \frac{c \left(\frac{n\gamma cK^{1/\gamma}}{n\gamma - 1} \right)^{-\lambda} \left(\frac{\alpha}{r} - \frac{\alpha - 1}{\delta} \right)}{(\lambda - \alpha)}. \end{aligned} \quad (\text{A4})$$

C. The Decision to Invest in the Marginal Unit

Having valued the marginal unit of capacity, we can now value the option to invest in this unit. Let $\Delta G_i(K, Y)$ denote the value of this option when the current capacity level is K . The exercise price of this option is equal to the cost of construction. Therefore, for any level of committed capacity K , $\Delta G_i(K, Y)$ is a perpetual American call option whose value depends on Y . Hence, there will be a threshold value at which it will be optimal to exercise this option. Specifically, for any K and n there will exist a threshold, $Y^n(K)$, such that the option to build

an additional unit of capacity will be exercised the first time that Y equals or exceeds $Y^n(K)$.

Following standard arguments ΔG_i satisfies the equation

$$\frac{\sigma^2}{2} Y^2 \frac{\partial^2 \Delta G_i}{\partial Y^2}(K, Y) + (r - \delta) Y \frac{\partial \Delta G_i}{\partial Y}(K, Y) - r \Delta G_i(K, Y) = 0.$$

The solution is subject to the following boundary conditions:

$$\Delta G_i(K, 0) = 0,$$

$$\Delta G_i(K, Y^n(K)) = \Delta F_i(K, Y^n(K)) - I,$$

$$\frac{\partial \Delta G_i}{\partial Y}(K, Y^n(K)) = \frac{\partial \Delta F_i}{\partial Y}(K, Y^n(K)).$$

The first boundary condition arises because $Y = 0$ is an absorbing barrier for the process described in (2), and therefore the option to invest has no value at that point. This implies the following functional form for the option to invest in a marginal unit of capacity:

$$\Delta G_i(K, Y) = \Delta D(K) Y^\lambda \quad \text{for } Y < Y^n(K),$$

where λ is the positive root of (A2). The last two boundary conditions form the system of equations that must be solved to get the values of $Y^n(K)$ and $D(K)$. They are the value-matching condition and the smooth-pasting condition, respectively, and they imply that $Y^n(K)$ is the value of Y that maximizes the value of the option to invest. In solving this system of equations we notice that the option to expand capacity will not be exercised when the current capacity is not fully used, that is, when $Y < \gamma c K^{1/\gamma}/(\gamma - 1)$; there is no reason to incur the investment cost to keep the additional capacity idle for some time. Therefore, using the expression for $\Delta F_i(K, Y)$ for $Y > n\gamma c K^{1/\gamma}/(n\gamma - 1)$ in equations (A3) and (A4), the system becomes

$$\begin{aligned} \Delta D(K) Y^n(K)^\lambda &= \Delta A(K) Y^n(K)^\alpha + \frac{n\gamma - 1}{n\gamma} \frac{Y(K)^n K^{-1/\gamma}}{\delta} - \frac{c}{r} - I \\ \lambda \Delta D(K) Y^n(K)^{\lambda-1} &= \alpha \Delta A(K) Y^n(K)^{\alpha-1} + \frac{n\gamma - 1}{n\gamma} \frac{K^{-1/\gamma}}{\delta}. \end{aligned}$$

Eliminating $\Delta D(K)$ we are left with the following equation for the investment threshold:

$$(\lambda - \alpha) \Delta A(K) Y^n(K)^\alpha + (\lambda - 1) \frac{n\gamma - 1}{n\gamma} \frac{Y^n(K) K^{-1/\gamma}}{\delta} - \lambda \left(\frac{c}{r} + I \right) = 0. \quad (\text{A5})$$

Using the solution for $\Delta A(K)$ in (A4) we can write this equation as

$$c \left(\frac{n\gamma c}{n\gamma - 1} \right)^{-\alpha} \left(\frac{\lambda}{r} - \frac{\lambda - 1}{\delta} \right) v_n^\alpha + (\lambda - 1) \frac{n\gamma - 1}{n\gamma} \frac{v_n}{\delta} - \lambda \left(\frac{c}{r} + I \right) = 0, \quad (\text{A6})$$

where $v_n = Y^n(K) K^{-1/\gamma}$ is the output price at which firms expand their capacity. It follows that for a given number of firms n in the industry, v_n is a constant that is independent of the industry capacity K . This last equation cannot be solved analytically to get an expression for v_n ; however, it is easily solved numerically. Once we get v_n then the investment threshold is

$$Y^n(K) = v_n K^{1/\lambda}. \quad (\text{A7})$$

It follows that the equilibrium investment strategy is for each firm to invest in an additional unit of capacity whenever the demand factor Y rises to the trigger $Y^n(K)$. Given the number of firms n in the industry, the trigger function is an increasing function of K . We can also evaluate how the degree of competition affects the equilibrium investment strategies of firms. By totally differentiating equation (A5) we get

$$\frac{\partial Y^n(K)}{\partial n} = -\frac{Y^n(K)}{n(n\gamma - 1)} < 0. \quad (\text{A8})$$

Thus, increasing competition leads each firm to expand its capacity sooner to avoid losing the investment opportunity to its competitors.

As seen above, the function $Y^n(K)$ is the firms' optimal investment rule. If Y and K are such that $Y > Y^n(K)$, firms should add capacity, increasing K until $Y = Y^n(K)$. Equivalently, given the current level of the demand shock Y , we can determine the industry's optimal capacity by rewriting equation (A5) in terms of $K(Y)$:

$$(\lambda - \alpha)\Delta A(K(Y))Y^\alpha + (\lambda - 1)\frac{n\gamma - 1}{n\gamma}\frac{YK(Y)^{-1/\gamma}}{\delta} - \lambda\left(\frac{c}{r} + I\right) = 0. \quad (\text{A9})$$

When the current level of demand is Y , the solution to equation (A9) gives the optimal capacity, $K(Y)$, for an industry that has no capacity. Alternatively, since investment is irreversible, at any time t , $K(Y(t))$ is the level of "desired capacity." Thus, the irreversibility constraint implies that $K(t) \geq K(Y(t))$ for all t , and $K(t) = K(Y(t))$ if the industry is expanding its capacity at time t .

Equation (A8) shows that the investment threshold for additional capacity is decreasing in the number of firms in the industry. This result is based on a given level of industry capacity that is independent of the number of firms in the market. Equivalently, we can show how the degree of competition affects the industry optimal capacity. Let $K^n(Y)$ be the optimal capacity for an n -firm industry. Comparing equation (A1) for $n = 1$ to the same equation for any n it is easily verified that

$$K^n(Y) = \left[\frac{n\gamma - 1}{n(\gamma - 1)} \right]^\gamma K^1(Y). \quad (\text{A10})$$

D. The Value of Assets in Place

First we derive the value of one unit of capacity for fixed K . Following standard arguments, the value of one unit of capacity $J(K, Y)$ satisfies the following differential equation:

$$\frac{\sigma^2}{2} Y^2 \frac{\partial^2 J}{\partial Y^2}(K, Y) + (r - \delta) Y \frac{\partial J}{\partial Y}(K, Y) - r J(K, Y) + \hat{\pi}_i(K, Y) = 0,$$

where $\hat{\pi}_i$ is the profit per unit of installed capacity given by (13). In the region where $Y < n\gamma c K^{1/\gamma}/(n\gamma - 1)$, J satisfies the equation

$$\begin{aligned} \frac{\sigma^2}{2} Y^2 \frac{\partial^2 J}{\partial Y^2}(K, Y) + (r - \delta) Y \frac{\partial J}{\partial Y}(K, Y) - r J(K, Y) \\ + \left(\frac{c}{n\gamma - 1} \right) \left(\frac{n\gamma - 1}{n\gamma c} Y K^{-1/\gamma} \right)^\gamma = 0 \end{aligned}$$

subject to the boundary condition

$$J(K, 0) = 0.$$

Therefore, the solution is of the form

$$J^1(K, Y) = B(K) Y^\lambda + \frac{\left(\frac{c}{n\gamma - 1} \right)^{1-\gamma} \left(\frac{Y K^{-1/\gamma}}{n\gamma} \right)^\gamma}{r - \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2/2}.$$

We require $r > \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2/2$ to ensure that $J^1(K, Y)$ is well defined. Since $\gamma > 1$ and λ is the positive root of (A2), we must have $\gamma < \lambda$.

In the region where $Y > n\gamma c K^{1/\gamma}/(n\gamma - 1)$, J satisfies the equation

$$\frac{\sigma^2}{2} Y^2 \frac{\partial^2 J}{\partial Y^2}(K, Y) + (r - \delta) Y \frac{\partial J}{\partial Y}(K, Y) - r J(K, Y) + Y K^{-1/\gamma} - c = 0.$$

subject to the boundary condition

$$\lim_{Y \rightarrow \infty} J(K, Y) = \frac{Y K^{-1/\gamma}}{\delta} - \frac{c}{r}.$$

which implies a solution of the form

$$J^2(K, Y) = A(K) Y^\alpha + \frac{Y K^{-1/\gamma}}{\delta} - \frac{c}{r}.$$

To solve for $A(K)$ and $B(K)$ we consider the point $Y = n\gamma c K^{1/\gamma}/(n\gamma - 1)$, where the two regions meet. $J(K, Y)$ must be continually differentiable across $Y = n\gamma c K^{1/\gamma}/(n\gamma - 1)$, therefore $A(K)$ and $B(K)$ are the solutions to the system of equations

$$J^1\left(K, \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}\right) = J^2\left(K, \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1}\right)$$

$$\frac{\partial J^1}{\partial Y} \left(K, \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \right) = \frac{\partial J^2}{\partial Y} \left(K, \frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \right).$$

Solving this system we get

$$\begin{aligned} A(K) \\ = \frac{c \left(\frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \right)^{-\alpha} \left(\frac{\lambda - \gamma}{(n\gamma - 1)(r - \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2/2)} + \frac{\lambda}{r} - \frac{\lambda - 1}{\delta} \frac{n\gamma}{n\gamma - 1} \right)}{(\lambda - \alpha)} \end{aligned} \quad (A11)$$

$$\begin{aligned} B(K) \\ = \frac{c \left(\frac{n\gamma c K^{1/\gamma}}{n\gamma - 1} \right)^{-\lambda} \left(\frac{\alpha - \gamma}{(n\gamma - 1)(r - \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2/2)} + \frac{\alpha}{r} - \frac{\alpha - 1}{\delta} \frac{n\gamma}{n\gamma - 1} \right)}{(\lambda - \alpha)}. \end{aligned}$$

The value of one unit of capacity for fixed K , $J(K, Y)$, derived above does not take into consideration the effect of future investment on the value of the installed capacity. However, the fact that firms can increase their capacity when the demand factor Y hits $Y^n(K)$ cuts off some of the upside potential for prices and profits, so the value of one unit of capacity must be less than $J(K, Y)$. Since $Y = 0$ is an absorbing barrier for the demand shock process the value of one unit of installed capital is of the form

$$H(K, Y) = E(K)Y^\lambda + J(K, Y), \quad (A12)$$

where $E(K)Y^\lambda$ represents the impact of future increases in industry capacity on the value of the firm current capacity. To find $E(K)$ we use the following boundary condition for the value of the K th unit of capacity $H(K, Y)$:

$$\frac{\partial H}{\partial K}(K, Y^n(K)) = 0. \quad (A13)$$

This condition ensures that when the trigger $Y^n(K)$ is reached, K increases by an infinitesimal amount dK and J changes from $J(K, Y)$ to $J(K + dK, Y)$.

Combining (A12) and (A13) and the fact that firms invest when capacity is fully used we have

$$E'(K) = - \left[A'(K)Y(K)^\alpha - \frac{1}{\gamma} \frac{Y^n(K)K^{-1/\gamma-1}}{\delta} \right] Y^n(K)^{-\lambda},$$

where $A'(K)$ and $E'(K)$ are the derivatives of the functions $A(K)$ and $E(K)$ with respect to K .

Using the expressions for $Y^n(K)$ and $A(K)$ in (A7) and (A11), we get

$$E'(K) = \left[\frac{\alpha}{\gamma} \theta_n v_n^\alpha + \frac{1}{\gamma} \frac{v_n}{\delta} \right] v_n^\lambda K^{-\lambda/\gamma-1},$$

where

$$\theta_n = \frac{c \left(\frac{n\gamma c}{n\gamma - 1} \right)^{-\alpha} \left(\frac{\lambda - \gamma}{(n\gamma - 1)(r - \gamma(r - \delta) - \gamma(\gamma - 1)\sigma^2/2)} + \frac{\lambda}{r} - \frac{\lambda - 1}{\delta} \frac{n\gamma}{n\gamma - 1} \right)}{(\lambda - \alpha)}.$$

Therefore,

$$E(K) = -\frac{1}{\lambda} \left[\alpha \theta_n v_n^{\alpha-\lambda} + \frac{v_n^{1-\lambda}}{\delta} \right] K^{-\lambda/\gamma}.$$

E. The Value of the Growth Options

When the industry capacity is K the value of the option to invest in one unit of capacity $g(K, Y)$ satisfies the boundary conditions

$$g(K, 0) = 0 \quad (\text{A14})$$

$$g(K, Y^n(K)) = H(K, Y^n(K)) - I. \quad (\text{A15})$$

Condition (A14) implies that the value of the option has the functional form $g(K, Y) = c(K)Y^\lambda$.

From boundary condition (A15),

$$\begin{aligned} c(K) &= [H(K, Y^n(K)) - I]Y^n(K)^{-\lambda} \\ &= \left[\frac{\lambda - \alpha}{\lambda} \theta_n v_n^\alpha + \frac{\lambda - 1}{\lambda} \frac{v_n}{\delta} - \frac{c}{r} - I \right] v_n^{-\lambda} K^{-\lambda/\gamma}. \end{aligned}$$

To get the value of firm i 's growth options we sum the value of these unit options by integrating to get

$$G_i(K, Y) = C(K)Y^\lambda,$$

where

$$C(K) = \frac{1}{n} \int_K^\infty c(z) dz = \frac{\gamma}{\lambda - \gamma} \left[\frac{\lambda - \alpha}{\lambda} \theta_n v_n^\alpha + \frac{\lambda - 1}{\lambda} \frac{v_n}{\delta} - \frac{c}{r} - I \right] v_n^{-\lambda} \frac{K^{\frac{\gamma-\lambda}{\gamma}}}{n}.$$

REFERENCES

- Aguerrevere, Felipe L., 2003, Equilibrium investment strategies and output price behavior: A real-options approach, *Review of Financial Studies* 16, 1239–1272.
- Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1153–1607.
- Carlson, Murray, Engelbert Dockner, Adlai Fisher, and Ron Giammarino, 2007, Leaders, followers, and risk dynamics in industry equilibrium, Working paper, University of British Columbia.
- Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2004, Corporate investment and asset price dynamics: Implications for the cross-section of returns, *Journal of Finance* 59, 2577–2603.

- Cooper, Ilan, 2006, Asset pricing implications of non-convex adjustment costs and irreversibility of investment, *Journal of Finance* 61, 139–170.
- Fama, Eugene F., and Kenneth R. French, 1992, The cross section of expected returns, *Journal of Finance* 47, 427–465.
- Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.
- Gomes, Joao, Leonid Kogan, and Lu Zhang, 2003, Equilibrium cross section of returns, *Journal of Political Economy* 111, 693–732.
- Grenadier, Steven R., 2002, Option exercise games: An application to the equilibrium investment strategies of firms, *Review of Financial Studies* 15, 691–721.
- He, Hua, and Robert S. Pindyck, 1992, Investments in flexible production capacity, *Journal of Economic Dynamics and Control* 16, 575–599.
- Hoberg, Gerard, and Gordon Phillips, 2007, Real and financial industry booms and busts, Working paper, University of Maryland.
- Hou, Kewei, and David Robinson, 2006, Industry concentration and average stock returns, *Journal of Finance* 61, 1927–1956.
- Kogan, Leonid, 2004, Asset prices and real investment, *Journal of Financial Economics* 73, 411–431.
- Novy-Marx, Robert, 2008, Competition, productivity and the cross section of expected returns, Working paper, University of Chicago.
- Pindyck, Robert S., 1988, Irreversible investment, capacity choice, and the value of the firm, *American Economic Review* 79, 969–985.
- Xing, Yuhang, and Lu Zhang, 2004, Value versus growth: Movement in economic fundamentals, Working paper, University of Rochester.
- Zhang, Lu, 2005, The value premium, *Journal of Finance* 60, 67–103.