

Optimal Decentralized Investment Management

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ABSTRACT

We study an institutional investment problem in which a centralized decision maker, the Chief Investment Officer (CIO), for example, employs multiple asset managers to implement investment strategies in separate asset classes. The CIO allocates capital to the managers who, in turn, allocate these funds to the assets in their asset class. This two-step investment process causes several misalignments of objectives between the CIO and his managers and can lead to large utility costs for the CIO. We focus on (1) loss of diversification, (2) unobservable managerial appetite for risk, and (3) different investment horizons. We derive an optimal unconditional linear performance benchmark and show that this benchmark can be used to better align incentives within the firm. We find that the CIO's uncertainty about the managers' risk appetites increases both the costs of decentralized investment management and the value of an optimally designed benchmark.

THE INVESTMENT MANAGEMENT DIVISIONS OF BANKS, mutual funds, and pension funds are predominantly structured around asset classes such as equities, fixed income, and alternative investments. To achieve superior returns, either through asset selection or market timing, gathering information about specific assets and capitalizing on the acquired informational advantage requires a high level of specialization. This induces the centralized decision maker of the firm, the Chief Investment Officer (CIO), for example, to pick asset managers who are specialized in a single asset class and to delegate portfolio decisions to these specialists. As a consequence, asset allocation decisions are made in at least two stages. In the first stage, the CIO allocates capital to the different asset classes, each managed by a different asset manager. In the second stage, each manager decides how to allocate the funds made available to him, that is, to the assets within his class. This two-stage process can generate several misalignments of incentives that may lead to large utility costs on the part of the CIO. We show that designing appropriate return benchmarks can substantially reduce these costs.

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We focus on the following important, although not exhaustive, list of misalignments of incentives. First, the two-stage process can lead to severe diversification losses. The unconstrained (single-step) solution to the mean-variance (MV) optimization problem is likely different from the optimal linear combination of MV efficient portfolios in each asset class, as pointed out by Sharpe (1981) and Elton and Gruber (2004). Second, there may be considerable, but unobservable, differences in appetites for risk between the CIO and each of the asset managers. When the CIO only knows the cross-sectional distribution of risk appetites of investment managers, but does not know where in this distribution a given manager falls, delegating portfolio decisions to multiple managers can be very costly. Third, the investment horizons of the asset managers and of the CIO may be different. Since the managers are usually compensated on an annual basis, their investment horizon is generally relatively short. The CIO, in contrast, may have a much longer investment horizon.

In practice, the performance of each asset manager is measured against a benchmark comprised of a large number of assets within his class. In the literature, the main purpose of these benchmarks has been to disentangle the effort and achievements of the asset manager from the investment opportunity set available to him. In this paper we show that an optimally designed unconditional benchmark can also serve to improve the alignment of incentives within the firm and to substantially mitigate the utility costs of decentralized investment management.

Our results provide a different perspective on the use of performance benchmarks. Admati and Pfleiderer (1997) take a realistic benchmark as given and show that when an investment manager uses the conditional return distribution in his investment decisions, restricting him by an unconditional benchmark distorts incentives.¹ In their framework, this distortion can only be prevented by setting the benchmark equal to the minimum-variance portfolio. We show that the negative aspect of unconditional benchmarks can be offset, at least in part, by the role of unconditional benchmarks in aligning other incentives, such as diversification, risk preferences, and investment horizons.

We use a stylized representation of an investment management firm to quantify the costs of the misalignments for both constant and time-varying investment opportunities. We assume that the CIO acts in the best interest of a large group of beneficiaries of the assets under management, whereas the investment managers only wish to maximize their personal compensation. Using two asset classes (bonds and stocks) and three assets per class (government bonds, Baa-rated corporate bonds, and Aaa-rated corporate bonds in the fixed income class, and growth stocks, intermediate, and value stocks in the equities class) the utility costs can range from 50 to 300 basis points per year. We therefore argue that decentralization has a first-order effect on the performance of investment management firms.

We demonstrate that when the investment opportunity set is constant and risk attitudes are observable, the CIO can fully align incentives through an

¹ See also Basak, Shapiro, and Teplá (2006).

unconditional benchmark consisting only of assets in each manager's asset class. In other words, cross-benchmarking is not required. Furthermore, we derive the perhaps counterintuitive result that the risk aversion levels of the asset managers for which the utility costs of the CIO are minimized can substantially differ from the risk aversion of the CIO. We then consider the case of time-varying investment opportunities and show that an unconditional (passive) benchmark can still substantially, though not fully, mitigate the utility costs of decentralized investment management.

Next we generalize our model by relaxing the assumption that the CIO knows the asset managers' risk appetite. Specifically, we derive the optimal benchmark assuming that the CIO only knows the cross-sectional distribution of investment managers' risk aversion levels, but does not know where in this distribution a given manager falls. We find that the qualitative results on the benefits of optimal benchmarking derived for a known risk aversion level apply to this more general case. In fact, we find that uncertainty about the managers' risk appetites increases both the costs of decentralized investment management and the value of an optimally designed benchmark.

The negative impact of decentralized investment management on diversification was first noted by Sharpe (1981), who shows that if the CIO has rational expectations about the portfolio choices of the investment managers, he can choose his investment weights such that diversification is at least partially restored. However, this optimal linear combination of MV efficient portfolios within each asset class usually still differs from the optimally diversified portfolio over all assets. To restore diversification further, Sharpe (1981) suggests that the CIO imposes investment rules on one or both of the investment managers to solve an optimization problem that includes the covariances between assets in different asset classes. Elton and Gruber (2004) show that it is possible to overcome the loss of diversification by providing the asset managers with investment rules that they are required to implement. The asset managers can then implement the CIO's optimal strategy without giving up their private information.

Both investment rules described above interfere with the asset manager's desire to maximize his individual performance, on which his compensation depends. Furthermore, when the investment choices of the managers are not always fully observable, these ad hoc rules are not enforceable. In contrast, we propose to change managers' incentives by introducing a return benchmark against which the managers are evaluated for the purpose of their compensation. When this benchmark is implemented in the right way, it is in the managers' own interest to follow investment strategies that are (more) in line with the objectives of the CIO. In Section I, we assume that investment opportunities are constant. This allows us to focus on the loss of diversification and on differences in preferences in a parsimonious framework. We then add market-timing skill and horizon effects in Section II. Both sections assume that the CIO knows the managers' risk attitudes. This assumption is relaxed in Section III.

Perhaps one of the most interesting questions is why the CIO should hire multiple asset managers to begin with. Sharpe (1981) argues that the decision

to employ multiple managers may be motivated by the desire to exploit their specialization or to diversify among asset managers. Alternatively, Barry and Starks (1984) argue that risk-sharing considerations may be a motivation to employ more than one manager. In Section II, investment opportunities are time-varying, consistent with the empirical evidence that equity and bond returns are to some extent predictable.² This allows skilled managers to implement active strategies that generate alphas, when compared to unconditional (passive) return benchmarks. This specific interpretation of alpha may seem unconventional, but it avoids the question of whether asset managers do or do not have private information. Treynor and Black (1973), Admati and Pfleiderer (1997), and Elton and Gruber (2004) assume that managers can generate alpha, but do not explicitly model how managers do so. Cvitanic, Lazrak, Martellini, and Zapatero (2006) assume that the investor is uncertain about the alpha of the manager and derive the optimal policy in that case. We explicitly model the time-variation in investment opportunities and assume that the resulting predictability can be exploited by skilled managers to generate value.

Apart from the tactical aspect of return predictability, time-variation in risk premia can also have important strategic consequences. After all, when asset returns are predictable, the optimal portfolio choice of the CIO depends on his investment horizon.³ This then requires dynamic optimization to find the optimal composition of the CIO's portfolio. The resulting portfolio choice is referred to as strategic as opposed to myopic (or tactical). The differences between the strategic and myopic portfolio weights are called hedging demands as they hedge against future changes in the investment opportunity set. These hedging demands are usually more pronounced for longer investment horizons of the CIO. As the remuneration schemes of investment managers are generally based on a relatively short period, their portfolio weights will be virtually myopic. The CIO, in contrast, usually has a long-term investment horizon. This leads to a third misalignment of incentives.

When unconditional benchmarks are used to overcome costs induced by differences in investment horizons, a key question is whether (1) the benchmark and/or (2) the strategic allocation to the different asset classes exhibit horizon effects. Most strategic asset allocation papers take a centralized perspective and assume that the tactical and strategic aspects are in perfect harmony.⁴ Once investment management is decentralized, tactical and strategic motives are split between the managers and the CIO, respectively. We show that both the strategic allocation, that is, the allocation to the various asset classes, and

² See, for example, Ang and Bekaert (2005), Lewellen (2004), Campbell and Yogo (2006), Torous, Valkanov, and Yan (2005), van Binsbergen and Koijen (2007), and Lettau and Van Nieuwerburgh (2007) for stock return predictability, and Dai and Singleton (2002) and Cochrane and Piazzesi (2005) for bond return predictability.

³ See, for instance, Kim and Omberg (1996), Brennan, Schwartz, Lagnado (1997), Campbell and Viceira (1999), Brandt (1999, 2005), Ait-Sahalia and Brandt (2001), Campbell, Chan, and Viceira (2003), Jurek and Viceira (2006), and Sangvinatsos and Wachter (2005).

⁴ Consider, for instance, Brennan, Schwartz, Lagnado (1997), Campbell, Chan, and Viceira (2003), and Jurek and Viceira (2006).

the optimal benchmarks exhibit strong horizon effects. When investment managers are not constrained by a benchmark, the horizon effects in the strategic allocation are less pronounced, implying that the strategic allocation and optimal benchmarks should be designed jointly.

Our paper also relates to the standard principal-agent literature in which the agent's effort is unobservable. In the delegated portfolio management context, the agent should exert effort to gather the information needed to make the right portfolio decisions, as explored by Ou-Yang (2003).⁵ We abstract from explicitly modeling the effort choices of the asset managers. Instead, the managers add value by timing the market, which we assume the CIO cannot do. The agency problem arises because the investment managers, whose actions are not always fully observable, wish to maximize their annual compensation, whereas the CIO acts in the best interest of the beneficiaries of the firm. When designing the benchmarks, the CIO faces a trade-off between (1) allowing the investment managers to realize the gains from market timing and (2) correcting the misalignments of incentives described above. As a result, the investment problem we solve is nontrivially more difficult than the problem with a CIO and a single investment manager. The strategic allocation of the CIO results from a joint optimization over the benchmark and the strategic allocation to the asset managers.

In the principal-agent literature above, it is common practice to assume that the preferences of the agents (the investment managers) are known to the principal (the CIO). We extend this literature by also considering the realistic case in which the principal has limited knowledge about the agents' preferences. As mentioned before, we assume that the CIO knows the cross-sectional distribution of investment managers' risk appetites, but does not know where in this distribution a given manager falls. We derive (approximate) closed-form solutions for the strategic allocation to the asset classes. In particular, we show that uncertainty about the managers' risk attitudes propagates as a form of background risk (Gollier and Pratt (1996)), which effectively increases the risk aversion of the CIO. Alternatively, limited knowledge of the managers' risk attitudes can be interpreted as a form of Bayesian parameter uncertainty (see, for example, Barberis (2000) and Brennan and Xia (2001)). For ease of exposition, we confine attention to a tractable constant relative risk aversion preference structure and a realistic linear class of performance benchmarks that are assumed to satisfy the participation constraint of the asset managers.

Finally, our work relates to the organizational literature of Dessein, Garicano, and Gertner (2005), who investigate a general manager (the CIO) who attempts to achieve a common goal while providing strong performance-linked compensation schemes to specialists (the investment managers) to overcome the moral hazard problem. They show that to achieve the common goal, individual incentives may have to be weakened. A common way to align incentives is to give the managers a share in each other's output. Our results indicate that in the

⁵ Stracca (2005) provides a recent survey of the theoretical literature on delegated portfolio management.

portfolio management setting, cross-benchmarking, where the benchmark of an asset manager includes assets from other classes, is not required.⁶

The paper proceeds as follows. In Section I, we present the model in a financial market with constant investment opportunities. Section II extends the financial market by allowing for time-variation in expected returns. In Section III, we generalize our framework by considering the problem of a CIO who is uncertain about the managers' risk attitudes. Section IV concludes.

I. Constant Investment Opportunities

A. Financial Market and Preferences

We assume that the financial market contains $2k + 1$ assets with prices denoted by $S_i, i = 0, \dots, 2k$. The first asset, S_0 , is a riskless cash account, that evolves according to:

$$\frac{dS_{0t}}{S_{0t}} = r dt, \quad (1)$$

where r denotes the (constant) instantaneous short rate. The remaining $2k$ assets are risky. We assume that the dynamics of the risky assets are given by geometric Brownian motions. For $i = 1, \dots, 2k$, we have

$$\frac{dS_{it}}{S_{it}} = (r + \sigma'_i \Lambda) dt + \sigma'_i dZ_t, \quad (2)$$

where Λ denotes a $2k$ -dimensional vector of, for now, constant prices of risk and Z is a $2k$ -dimensional vector of independent standard Brownian shocks. All correlations between asset returns are captured by the volatility vectors σ_i . The volatility matrix of the first k assets is given by $\Sigma_1 = (\sigma_1, \dots, \sigma_k)'$ and for the second k assets by $\Sigma_2 = (\sigma_{k+1}, \dots, \sigma_{2k})'$.

The CIO, who acts in the best interest of the beneficiaries of the firm, employs two asset managers. The managers independently decide on the optimal composition of their portfolios using a subset of the available assets. The first asset manager has the mandate to manage the first k assets and the second manager has the mandate to invest in the remaining k assets.

We explicitly model the preferences of both the CIO and the investment managers. Initially, the preference structures are assumed to be common knowledge. We assume that the preferences of the CIO and of the two asset managers can be represented by a CRRA utility function, so that each solves the problem

$$\max_{(x_{is})_{s \in [t, T_i]}} \mathbb{E}_t \left(\frac{1}{1 - \gamma_i} W_{T_i}^{1 - \gamma_i} \right), \quad (3)$$

where γ_i denotes the coefficient of relative risk aversion, T_i denotes the investment horizon, and $i = 1, 2, C$ refers to the two asset managers and the CIO,

⁶ For a treatment of decentralized information processing within the firm, see Vayanos (2003).

respectively. The vector x_i denotes the optimal portfolio weights in the different assets available to agent i . According to equation (3), the preferences of the CIO and the investment managers may be conflicting along two dimensions. First, the risk attitudes are likely to be mismatched. Second, the investment horizon used in determining the optimal portfolio choices are potentially different. The remuneration schemes of asset managers usually induce short, say annual, investment horizons. This form of managerial myopia tends to be at odds with the more long-term perspective of the CIO. The difference in horizons is particularly important for CIOs with long-term mandates from pension funds and life insurers.

For now, we assume that investment opportunities are constant. Section I.B solves for the optimal portfolio choice when investment management is centralized, implying that the CIO optimizes over the complete asset menu. Obviously, in this case, all misalignments of incentives mentioned before are absent. However, when the investment management firm has a rich investment opportunity set and a substantial amount of funds under management, centralized investment management becomes infeasible. In Section I.C, we introduce asset managers for each asset class assuming that the asset managers are not constrained by a benchmark. In Section I.D, the asset managers are then evaluated relative to a performance benchmark, and we show how to design this benchmark optimally. The proofs of the main results are provided in Appendices A to C.

B. Centralized Problem

As a point of reference, we consider first the centralized problem in which the CIO decides on the optimal weights in all $2k + 1$ assets. The instantaneous volatility matrix of the risky assets is given by $\Sigma = (\Sigma'_1, \Sigma'_2)'$. The corresponding optimal portfolio is given by

$$x_C = \frac{1}{\gamma_C} (\Sigma \Sigma')^{-1} \Sigma \Lambda, \quad (4)$$

with the remainder, $1 - x'_C \iota$, invested in the cash account. The utility derived by the CIO from implementing this optimal allocation is

$$J_1(W, \tau_C) = \frac{1}{1 - \gamma_C} W^{1 - \gamma_C} \exp(a_1 \tau_C), \quad (5)$$

where $\tau_C = T_C - t$ and $a_1 = (1 - \gamma_C)r + \frac{1 - \gamma_C}{2\gamma_C} \Lambda' \Sigma' (\Sigma \Sigma')^{-1} \Sigma \Lambda$. When investment opportunities are constant, the CIO's optimal allocation is independent of the investment horizon, as shown by Merton (1969, 1971).

Suppose that the asset set contains six risky assets. The first three risky assets are fixed income portfolios, namely, a government bond index and two Lehman corporate bond indices with Aaa and Baa ratings, respectively. The remaining three risky assets are equity portfolios made up of firms sorted into value, intermediate, and growth categories based on their book-to-market ratio.

Table I
Constant Investment Opportunities

This table gives the estimation results of the financial market in Section I over the period January 1973 through November 2004 using monthly data. The model is estimated by maximum likelihood. The asset set contains government bonds ("Gov. bonds"), corporate bonds with credit ratings Baa ("Corp. bonds, Baa") and Aaa ("Corp. bonds, Aaa"), and three equity portfolio ranked on their book-to-market ratio (growth/intermediate ("Int.")/value). Panel A provides the model parameters and Panel B portrays the implied instantaneous expected returns ($r + \Sigma\Lambda$) and correlations. In determining Λ , we assume that the instantaneous nominal short rate equals $r = 5\%$.

Panel A: Model Parameters							
Source of Risk	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	
Λ	0.331	0.419	−0.0291	0.126	0.477	0.305	
Σ							
Gov. bonds	13.5%	0	0	0	0	0	
Corp. bonds, Baa	8.2%	5.6%	0	0	0	0	
Corp. bonds, Aaa	9.1%	2.7%	2.4%	0	0	0	
Growth stocks	3.7%	6.3%	0.3%	16.5%	0	0	
Int. stocks	3.6%	6.8%	0.3%	11.7%	7.3%	0	
Value stocks	3.6%	7.7%	0.1%	10.4%	6.8%	5.9%	
Panel B: Implied Parameters							
	Expected return			Correlation			
Gov. bonds	9.5%	100%	82%	93%	20%	23%	22%
Corp. bonds, Baa	10.1%	82%	100%	92%	37%	43%	45%
Corp. bonds, Aaa	9.1%	93%	92%	100%	29%	34%	34%
Growth stocks	10.9%	20%	37%	29%	100%	88%	80%
Int. stocks	14.0%	23%	43%	34%	88%	100%	93%
Value stocks	15.7%	22%	45%	34%	80%	93%	100%

The model is estimated by maximum likelihood using data from January 1973 through November 2004. The nominal short rate is set to 5% per annum. Finally, to ensure statistical identification of the elements of the volatility matrix, we assume that Σ is lower triangular.

The estimation results are provided in Table I. Panel A shows estimates of the parameters Λ and Σ . Panel B shows the implied instantaneous expected return and correlations between the assets. In the fixed income asset class, we find an expected return spread of 1% between corporate bonds with a Baa versus Aaa rating. In the equities asset class, we estimate a high value premium of 4.8%. The correlations within asset classes are high, between 80% and 90%. Furthermore, there is clear dependence between asset classes, which, as we show more formally later, implies that the two-stage investment process leads to inefficiencies.

C. Decentralized Problem without a Benchmark

We now solve the decentralized problem in which the first asset manager has the mandate to decide on the first k assets and the second asset manager

manages the remaining k assets. Neither of the asset managers has access to a cash account. If they did, they could hold highly leveraged positions or large cash balances, which is undesirable from the CIO's perspective.⁷ The CIO allocates capital to the two asset managers and invests the remainder, if any, in the cash account.

The optimal portfolio of asset manager i when he is not constrained by a benchmark is

$$x_i^{NB} = \frac{1}{\gamma_i} x_i + \left(1 - \frac{x_i' \ell}{\gamma_i}\right) x_i^{MV}, \quad (6)$$

where

$$x_i = (\Sigma_i \Sigma_i')^{-1} \Sigma_i \Lambda \quad \text{and} \quad x_i^{MV} = \frac{(\Sigma_i \Sigma_i')^{-1} \ell}{\ell' (\Sigma_i \Sigma_i')^{-1} \ell}. \quad (7)$$

The optimal portfolio of the asset managers can be decomposed into two components. The first component, x_i , is the standard myopic demand that optimally exploits the risk–return trade-off. The second component, x_i^{MV} , minimizes the instantaneous return variance and is therefore labeled the minimum-variance portfolio. The minimum-variance portfolio substitutes for the riskless asset in the optimal portfolio of the asset manager. The two components are then weighted by the risk attitude of the asset manager to arrive at the optimal portfolio.

The CIO has to decide how to allocate capital to the two asset managers as well as to the cash account. We call this decision the strategic asset allocation. The investment problem of the CIO is of the same form as in the centralized problem, but with a reduced asset set. In the centralized setting the CIO has access to $2k + 1$ assets. In the decentralized case, each asset manager combines the k assets in his class to form his preferred portfolio. The CIO can then only choose between these two portfolios and the cash account. The instantaneous volatility matrix of the two risky portfolios available to the CIO is given by $\bar{\Sigma} = (\Sigma_1' x_1^{NB}, \Sigma_2' x_2^{NB})'$. Thus, the optimal strategic allocation of the CIO to the two asset managers is

$$x_C = \frac{1}{\gamma_C} (\bar{\Sigma} \bar{\Sigma}')^{-1} \bar{\Sigma} \Lambda, \quad (8)$$

with the remainder, $1 - x_C' \ell$, invested in the cash account. Note that in this case x_C is a two-dimensional vector, containing the strategic allocation to both managers, as opposed to a $2k$ -dimensional vector with the weights allocated to each of the assets as in equation (4).

Throughout the paper, utility costs of decentralized investment management are calculated at the centralized level. In other words, we use the value function of the CIO (the principal) to measure utility losses.

⁷ A similar cash constraint has been imposed in investment problems with a CIO and a single investment manager (e.g., Brennan (1993) and Gómez and Zapatero (2003)).

The value function of the CIO with decentralization is given by

$$J_2(W, \tau_C) = \frac{1}{1 - \gamma_C} W^{1 - \gamma_C} \exp(a_2 \tau_C), \quad (9)$$

where $\tau_C = T_C - t$ and $a_2 = (1 - \gamma_C)r + \frac{1 - \gamma_C}{2\gamma_C} \Lambda' \bar{\Sigma}' (\bar{\Sigma} \bar{\Sigma}')^{-1} \bar{\Sigma} \Lambda$. It is straightforward to show that the value function in equation (5) (the centralized problem) is larger than or equal to the value function in equation (9) (the decentralized problem). This follows from the fact that the two-stage asset allocation procedure reduces the asset set of the CIO. The CIO can only allocate funds between the two managers, which does not provide sufficient flexibility to always achieve the first-best solution.

The two-stage asset allocation results in the first-best outcome only when the asset managers already happen to implement the proper relative weights within their asset classes. In this case, the CIO can use the strategic allocation to scale up the asset managers' weights to the optimal firm-level allocation. A set of sufficient conditions for this to hold is given by

$$\Sigma_1 \Sigma_2' = 0_{k \times k}, \quad (10)$$

$$x_i' \iota = \gamma_i, \quad (11)$$

with $i = 1, 2$. Note that even when asset classes are independent, that is, Condition (10) holds, the first-best allocation is generally not attainable. If asset classes are independent and when managers do not have access to a cash account, managers allocate their funds to the efficient tangency portfolio and the inefficient minimum-variance portfolio of their asset classes. Condition (11) ensures that the investment in the minimum-variance portfolio equals zero. If both conditions are satisfied, the CIO's optimal strategic allocation to the managers is given by γ_i / γ_C , $i = 1, 2$.

Figure 1 illustrates the solution of the decentralized portfolio problem for a CIO who hires two investment managers with equal risk aversion of 10. Panel A shows the MV frontier of the bond manager, the MV frontier of the stock manager, and the CIO's optimal linear combination of these two frontiers. The decentralized MV frontier crosses the MV frontier for stocks at the preferred portfolio of the stock manager, and it crosses the MV for bonds at the portfolio chosen by the bond manager. Panel B compares the decentralized MV frontier with the centralized MV frontier. As argued above, the decentralized MV frontier lies within the centralized MV frontier. The welfare loss due to decentralized investment management can be inferred from the difference in Sharpe ratios (i.e., the slope of the lines in MV space through the point $(0, r)$ and tangent to the centralized and decentralized MV frontier, respectively). Finally, panel B also displays the portfolio choices of the CIO for both the centralized and decentralized scenarios. The results clearly show that the CIO invests more conservatively in the decentralized case. In fact, it can be shown in general that the optimal decentralized portfolio is more conservative than the optimal centralized portfolio.

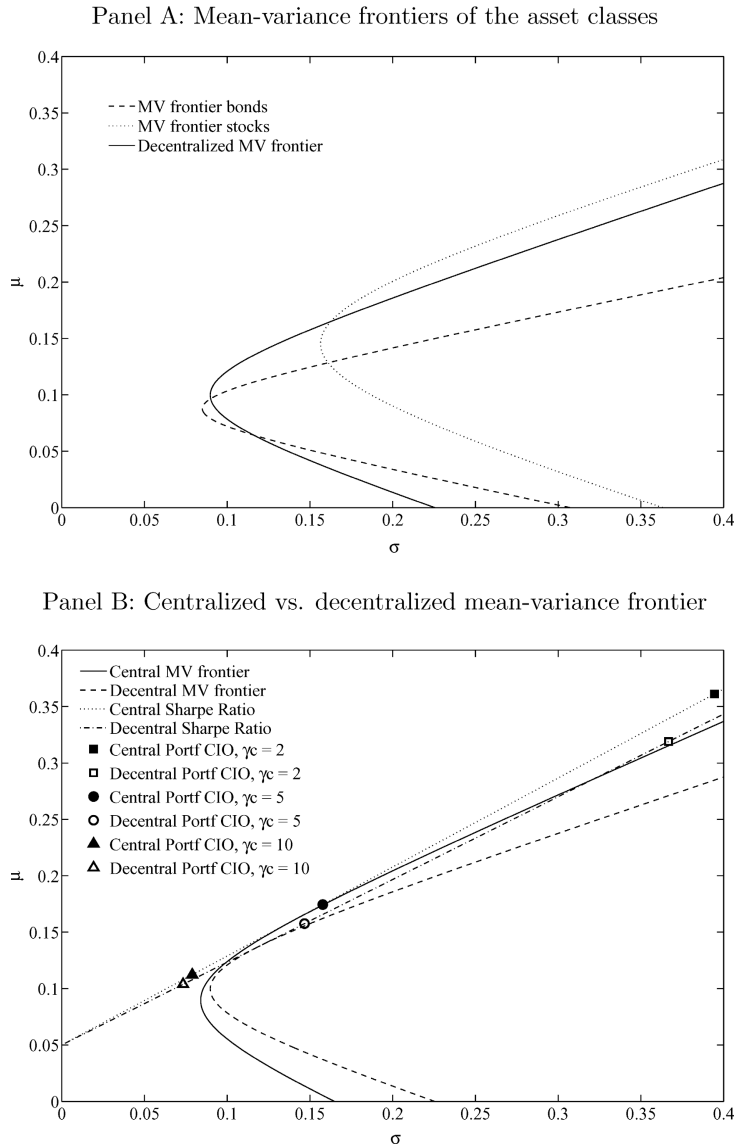
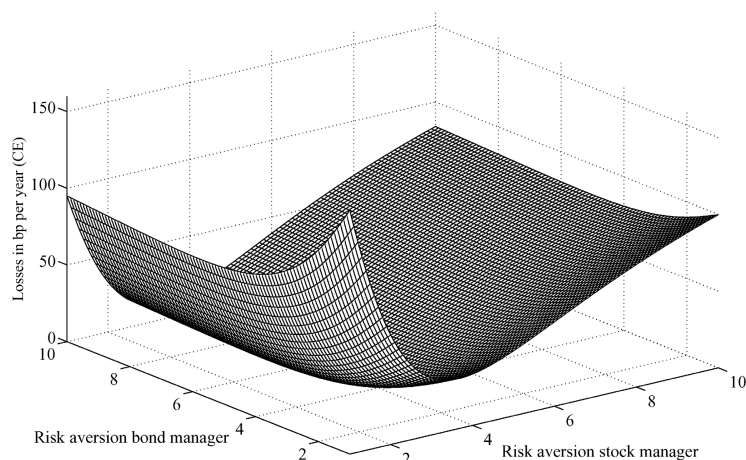


Figure 1. Decentralized investment management problem. This figure shows a decentralized asset allocation problem in which a CIO delegates portfolio decisions to a stock and a bond manager. Both asset managers have a risk aversion coefficient of $\gamma_1 = \gamma_2 = 10$. The bond manager invests in government bonds and corporate bonds with Aaa and Baa ratings. The stock manager invests in growth, intermediate, and value stocks. Panel A shows the mean-variance frontier for stocks and for bonds. The decentralized mean-variance frontier intersects the stock and bond mean-variance frontiers at the preferred portfolios of the bond and the stock manager. The CIO allocates money to the two managers and a riskless asset that pays 5% per year. Panel B compares the mean-variance frontier of the decentralized investment problem with that of the centralized investment problem and depicts the optimal portfolio choices of the CIO for the CIO's risk aversion level γ_C , equal to 2, 5, and 10.

Panel A: The coefficient of relative risk aversion of the CIO equals $\gamma_C = 5$



Panel B: The coefficient of relative risk aversion of the CIO equals $\gamma_C = 10$

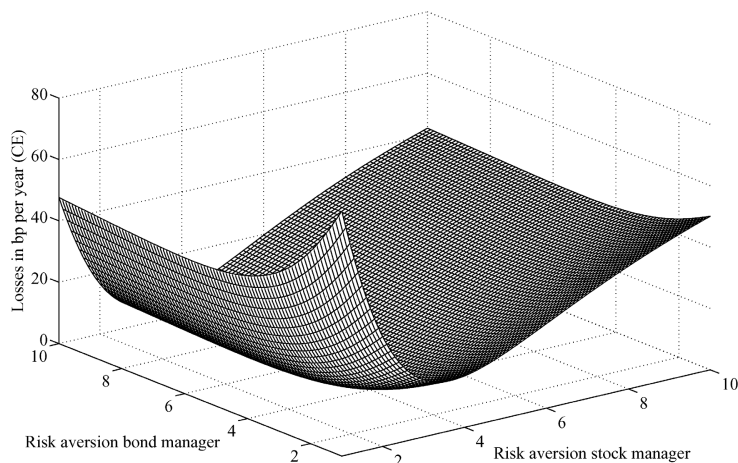


Figure 2. Losses from decentralized investment management. This figure depicts the diversification losses due to decentralized investment management as a function of the risk aversion of the investment managers. The CIO has a risk aversion coefficient $\gamma_C = 5$ in Panel A and $\gamma_C = 10$ in Panel B. The horizontal axes depict the risk appetites of the asset managers. The losses are computed by taking the ratio of the annualized certainty equivalents achieved under decentralized and centralized investment management after which we subtract one and multiply by $-10,000$ to express the losses in basis points per year. For example, 160 basis points implies a loss in terms of certainty equivalents of 1.6% of wealth per year.

In Figure 2, we show the utility losses induced by decentralized investment management for various combinations of managerial risk attitudes. The coefficient of relative risk aversion for the CIO equals $\gamma_C = 5$ in Panel A and $\gamma_C = 10$ in Panel B. We define the utility loss as the decrease in the annualized

certainty-equivalent return at the firm level. Interestingly, this loss is not minimized when the risk aversion of the asset managers is equal to that of the CIO. In fact, the cost of decentralized investment management is minimized for a risk aversion of 3.3 for the stock manager and 5.7 for the bond manager, regardless of the risk aversion of the CIO. Even though the location of the minimum is not dependent on the risk aversion of the CIO, the utility loss incurred obviously is. When the risk aversion of the CIO equals five, the minimum diversification losses are eight basis points per year in terms of certainty equivalents. This number drops to four basis points when the risk aversion of the CIO equals 10 because he moves out of risky assets and into the riskless asset. The utility loss can increase to 80–100 basis points even in this simple example for different risk attitudes of the investment managers. Finally, note that when the CIO is forced to hire a bond manager who does not have the optimal risk aversion level, this may influence the CIO's preferred choice of stock manager and vice versa.

Figure 3 displays the portfolio compositions of the bond manager in Panel A and of the stock manager in Panel B as functions of their risk aversion. Recall that the managers do not have access to a riskless asset. Figure 4 shows the fraction of total risky assets that is allocated to the stock manager as a function of his (and the bond manager's) risk aversion. The bond manager receives one minus this allocation. The allocation of capital between the riskless and the risky assets depends on the risk aversion of the CIO and is not shown.

D. Decentralized Problem with a Benchmark

We now consider the decentralized investment problem in which the CIO designs a performance benchmark for each of the investment managers in an attempt to align incentives. We restrict attention to benchmarks in the form of portfolios that can be replicated by the asset managers. This restriction implies that only the assets of the particular asset class are used and that the benchmark contains no cash position. There is no possibility and, as we show later, no need for cross-benchmarking. We denote the value of the benchmark of manager i at time t by B_{it} and the weights in the *benchmark* portfolio for asset class i by β_i . The evolution of benchmark i is given by

$$\frac{dB_{it}}{B_{it}} = (r + \beta'_i \Sigma_i \Lambda) dt + \beta'_i \Sigma_i dZ_t, \quad (12)$$

where $\beta'_i \iota = 1$, for $i = 1, 2$.

We assume that the asset managers derive utility from the ratio of the value of assets under their control to the value of the benchmark. They face the problem

$$\max_{(x_{is})_{s \in [t, T_i]}} \mathbb{E}_t \left(\frac{1}{1 - \gamma_i} \left(\frac{W_{iT_i}}{B_{iT_i}} \right)^{1 - \gamma_i} \right). \quad (13)$$

This preference structure can be motivated in several ways. First, the remuneration schemes of asset managers usually contain a component that depends

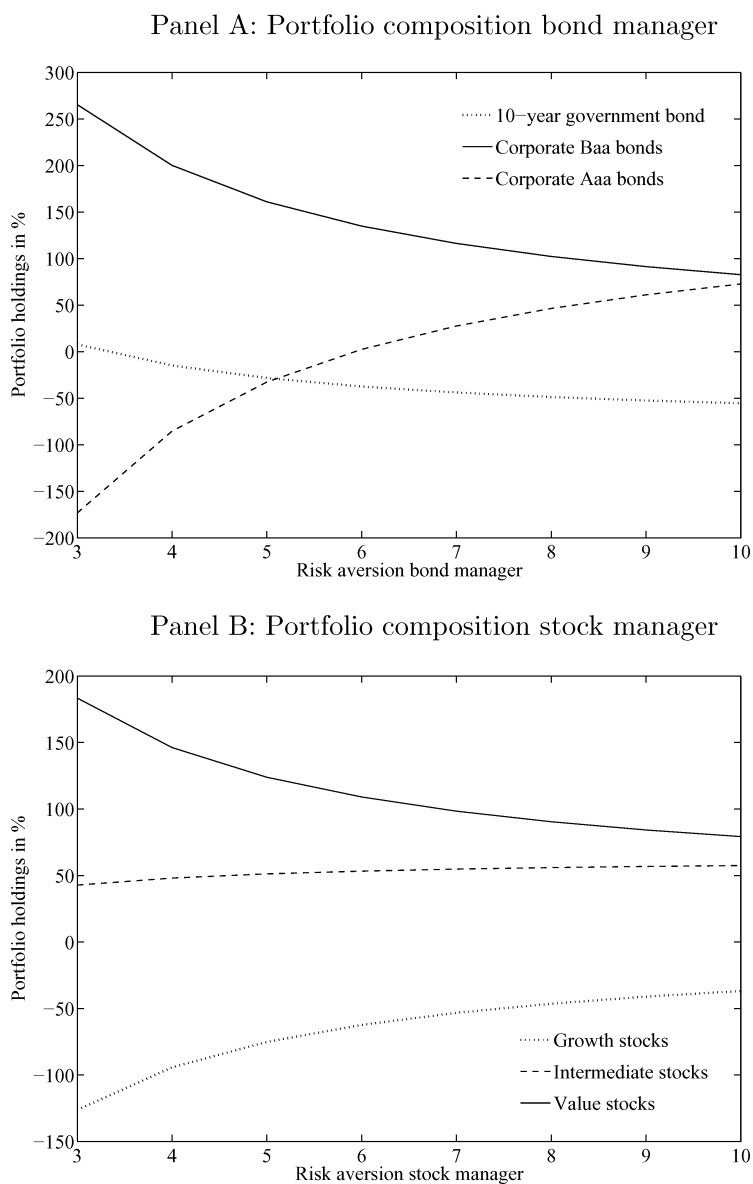


Figure 3. Portfolio compositions without a benchmark. This figure displays the portfolio composition of the bond manager in Panel A and the stock manager in Panel B as functions of their coefficients of relative risk aversion when they are not restricted by a benchmark. The asset managers do not have access to a riskless asset.

on their performance relative to a benchmark. This is captured in our model by specifying preferences over the ratio of funds under management to the value of the benchmark, in line with Browne (1999, 2000). Second, investment managers often operate under risk constraints. An important way to measure risk

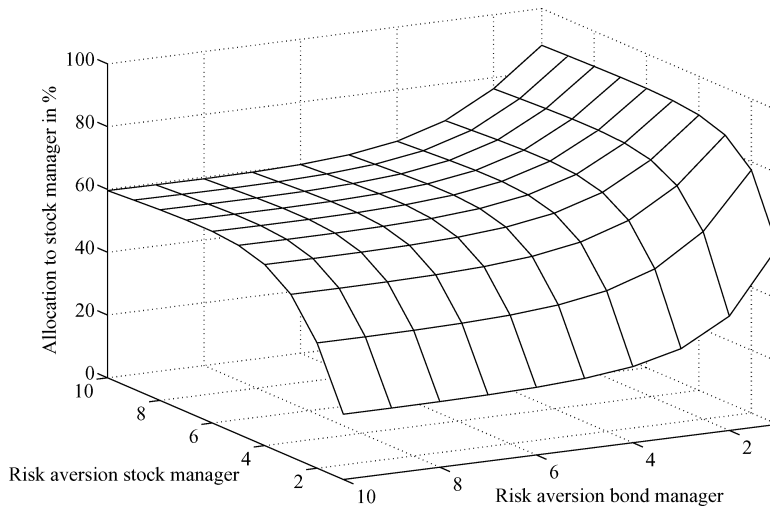


Figure 4. Fraction of risky funds allocated to equities without a benchmark. This figure displays the percentage of total investment in risky assets that is under control of the stock manager as a function of the risk aversion of the bond and the stock manager.

attributable to manager i is to employ tracking error volatility. The tracking error is usually defined as the return differential of the funds under management and the benchmark. Taking logs of the ratio of wealth to the benchmark provides the tracking error in log returns. Third, for investment management firms that need to account for liabilities, such as pension funds and life insurers, supervisory bodies often summarize the financial position by the ratio of assets to liabilities, the so-called funding ratio as further described in Sharpe (2002) and van Binsbergen and Brandt (2007). Hence, the ratio of wealth to the benchmark (liabilities) can be interpreted as a reasonable summary statistic of relative performance.⁸

When the performance of asset manager i is measured relative to the benchmark, his optimal portfolio is given by

$$x_i^B = \frac{1}{\gamma_i} x_i + \left(1 - \frac{1}{\gamma_i}\right) \beta_i + \frac{1}{\gamma_i} (1 - x_i^l) x_i^{MV}, \quad (14)$$

where x_i and x_i^{MV} are given in equation (7). This portfolio differs from the optimal portfolio in the absence of a benchmark in two important respects.

⁸ In addition, Stutzer (2003a) and Foster and Stutzer (2003) show that when the optimal portfolio is chosen so that the probability of underperformance tends to zero as the investment horizon goes to infinity, the portfolio that maximizes the probability decay rate solves an objective similar to power utility with two main modifications. First, the investor's preferences involve the ratio of wealth over the benchmark. Second, the investor's coefficient of relative risk aversion depends on the investment opportunity set. This provides an alternative interpretation of preferences over the ratio of wealth to the benchmark as well as different coefficients of relative risk aversion for the various asset classes.

First, the optimal portfolio contains a component that replicates the composition of the benchmark portfolio. It is exactly this response of the investment manager that allows the CIO to optimally design a benchmark to align incentives. Note that the benchmark weights enter the optimal portfolio linearly. Second, when the coefficient of relative risk aversion, γ_i , tends to infinity, the asset manager tracks the benchmark exactly. Hence, the benchmark is considered to be the riskless asset from the perspective of the asset manager.

The CIO has to optimally design the two benchmark portfolios and has to determine the allocation to the two asset managers as well as to the cash account. It is important to note that $x_i^B = x_i^{NB}$ when $\beta_i = x_i^{MV}$. That is, the optimal portfolios with and without a performance benchmark coincide when the benchmark portfolio equals the minimum-variance portfolio. This implies that when designing a benchmark, the no-benchmark case is in the choice set of the CIO. As a consequence, the optimal benchmark will reduce the utility costs of decentralized investment management. More importantly, when investment opportunities are constant, the benchmark can be designed so that all inefficiencies are eliminated. The composition of the optimal benchmark that leads to the optimal allocation of the centralized investment problem is given by

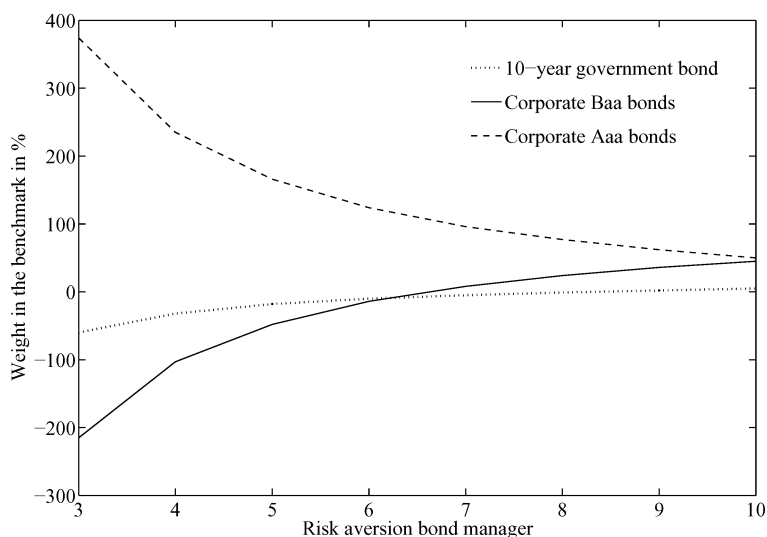
$$\beta_i = x_i^{MV} + \frac{\gamma_i}{\gamma_i - 1} \left(\frac{x_i^C}{x_i^{C'l}} - x_i^{NB} \right), \quad (15)$$

where x_i^C are the optimal weights for the assets under management by manager i when the CIO controls all assets as given in equation (4) and x_i^{NB} is given in equation (6). The benchmark weights sum to one because of the restriction that the benchmark cannot contain a cash position.

The two components of the optimal benchmark portfolio have a natural interpretation. The first component is the minimum-variance portfolio. As we point out above, once the benchmark portfolio coincides with the minimum-variance portfolio, the benchmark does not affect the manager's optimal portfolio. The second component, however, corrects the manager's portfolio choice to align incentives. If the relative weights of the CIO and the portfolio of the manager without a benchmark (i.e., x_i^{NB}) coincide, there is no need to influence the manager's portfolio and the second term is zero. However, when the CIO optimally allocates a larger share of capital to a particular asset in class i , the optimal benchmark will contain a positive position in this asset when $\gamma_i > 1$. The ratio before the second component accounts for the manager's preferences. If the manager is more aggressive (i.e., $\gamma_i \rightarrow 1$), the benchmark weights are more extreme as the manager is less sensitive to benchmark deviations. If the investor becomes more conservative (i.e., $\gamma_i \rightarrow \infty$), we get $x_i^{NB} = x_i^{MV}$ and the benchmark coincides with the relative weights of the CIO.

Finally, the CIO uses the strategic allocation to the two asset managers to implement the optimal firm-level allocation. The optimal weight allocated to each manager is given by $x_i^{C'l}$, with $i = 1, 2$, and the remainder, $1 - x_1^{C'l} - x_2^{C'l}$, is invested in the cash account.

Panel A: Composition of the optimal bond benchmark



Panel B: Composition of the optimal stock benchmark

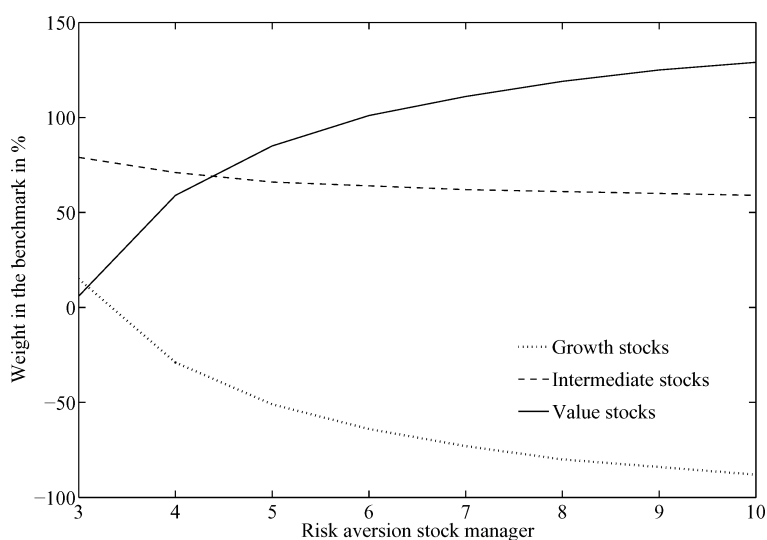


Figure 5. Composition of the optimal performance benchmarks. Composition of the optimal bond benchmark in Panel A and stock benchmark in Panel B as a function of the risk aversion of the asset managers.

Figure 5 shows the composition of the optimal benchmarks for the bond manager in Panel A and for the stock manager in Panel B as functions of their risk aversion. The mechanism through which the benchmark aligns incentives is particularly clear for the fixed income asset class. Without a benchmark, the

bond manager invests too aggressively in corporate bonds with a Baa rating. The optimal benchmark therefore contains a large short position in the same asset that reduces the manager's allocation to Baa-rated bonds. For Aaa-rated bonds, the benchmark provides exactly the opposite incentive.

II. Time-Varying Investment Opportunities

A. Financial Market

In Section I, investment opportunities are constant through time and there are only two inefficiencies caused by decentralized investment management, namely, loss of diversification between asset classes and misalignments in risk attitudes. However, the role of asset managers is rather limited in that they add no value in the form of stock selection or market timing. In this section, we allow investment opportunities, and in particular expected returns, to be time-varying and driven by a set of common forecasting variables. This setting allows asset managers to implement active strategies that optimally exploit changes in investment opportunities in their respective asset classes. These active strategies can generate alphas when compared to an unconditional (passive) performance benchmark. Thus, active asset management can be value-enhancing.

This extension of the problem adds several new interesting dimensions to the decentralized investment management problem. First, differences in investment horizons create another misalignment of incentives. The CIO generally acts in the long-term interest of the investment management firm, while asset managers tend to be more shortsighted, possibly induced by their remuneration schemes. When the predictor variables are correlated with returns, it is optimal to hedge future time-variation in investment opportunities.⁹ As a consequence, the myopic portfolios held by the asset managers will generally not coincide with the CIO's optimal portfolio that incorporates long-term hedging demands. Second, when a common set of predictor variables affects the investment opportunities in both asset classes, active strategies are potentially correlated. This implies that even if instantaneous returns are uncorrelated, long-term returns can be correlated, which aggravates the loss of diversification due to decentralization. Third, the role of benchmarks is markedly different compared to the case of constant investment opportunities. For the sake of realism, we restrict attention to passive (unconditional) strategies as return benchmarks. As we discussed earlier, Admati and Pfleiderer (1997) show that when the asset manager has private information, an unconditional benchmark can be very costly. After all, the asset managers base their decision on the conditional return distribution, whereas the CIO designs the benchmark using the unconditional return distribution.¹⁰ In their framework, it follows therefore, that unless the benchmark is set equal to the minimum-variance portfolio, it

⁹ See, for instance, Kim and Omberg (1996), Campbell and Viceira (1999), Brandt (1999), and Liu (2007).

¹⁰ Although the predictors are publicly observed, we assume that the CIO is time-constrained or not sufficiently specialized to exploit this information. As such, the conditional return distribution

induces a potentially large efficiency loss. In our model, in contrast, the benchmark is used to align incentives in a decentralized investment management firm.

We now consider a more general financial market in which the prices of risk, Λ , can vary over time. More explicitly, we model

$$\Lambda(X) = \Lambda_0 + \Lambda_1 X, \quad (16)$$

where X denotes an m -dimensional vector of de-measured state variables that capture time-variation in expected returns. Although the state variables are time varying, we drop the subscript t for notational convenience. All portfolios in this section are indexed with either the state realization, X , or the investment horizon, τ , in order to emphasize the conditioning information used to construct the portfolio policies.

Most predictor variables used in the literature, such as term structure variables and financial ratios, are highly persistent. In order to accommodate first-order autocorrelation in predictors, we model their dynamics as Ornstein–Uhlenbeck processes:

$$dX_{it} = -\kappa_i X_{it} dt + \sigma'_{Xi} dZ_t, \quad (17)$$

where Z now denotes a $(2k + m)$ -dimensional Brownian motion. The volatility matrix of the m predictors is given by $\Sigma_X = (\sigma_{X1}, \dots, \sigma_{Xm})'$. We assume again that only the CIO has access to a cash account. Finally, we postulate the same preference structures for the CIO and the asset managers as in Section I.A.

We estimate the return dynamics using three predictor variables: the short rate, the yield on a 10-year nominal government bond, and the log dividend yield of the equity index. These predictors have been used in strategic asset allocation problems to capture the time-variation in expected returns (see the references in footnote 3). The model is estimated by maximum likelihood using data from January 1973 through November 2004. The estimation results are presented in Table II.

The estimates of the unconditional instantaneous prices of risk, Λ_0 , are similar to the results in Table I. The second part of Table II describes the responses of the expected returns of the individual assets to changes in the state variables, $\Sigma \Lambda_1$. We find that the short rate has a negative impact on the expected returns of all assets except for government bonds. Furthermore, the expected returns of assets in the fixed income class are positively related to the long-term yield, while the expected returns of assets in the equity class are negatively related to this predictor. The dividend yield is positively related to the expected returns of all assets. The estimates of the autoregressive parameters, κ_i , reflect the high persistence of the predictor variables. Finally, the last part of Table II provides the joint volatility matrix of the assets and the predictor variables.

remains unknown to the CIO and the conditioning information exploited by the asset managers is equivalent to private information.

Table II
Time-Varying Investment Opportunities

This table shows the estimation results of the financial market in Section II over the period January 1973 through November 2004 using monthly data. The model is estimated by maximum likelihood. The asset set contains government bonds (“Gov. bonds”), corporate bonds with credit ratings Baa (“Corp. bonds, Baa”) and Aaa (“Corp. bonds, Aaa”), and three equity portfolio ranked on their book-to-market ratio (growth/intermediate (“Int.”)/value). In determining Λ_0 , we assume that the instantaneous nominal short rate equals $r = 5\%$. We report $\Sigma\Lambda_1$ rather than Λ_1 as the former expression is easier to interpret. The short rate, the yield on a 10Y nominal government bond, and the dividend yield are used to predict returns.

Source of Risk	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9
Λ_0	0.306	0.409	−0.020	0.089	0.498	0.310	0	0	0
	$\Sigma\Lambda_1$								
	Gov.	Baa	Aaa	Growth	Int.	Value		κ_i	
Short rate	0.227	−0.964	−0.209	−0.270	−0.249	−0.012		0.36	
10Y yield	1.269	1.225	0.893	−0.778	−1.086	−1.010		0.12	
DP	0.020	0.071	0.038	0.132	0.121	0.130		0.052	
Σ	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9
Gov. bonds	13.2%	0	0	0	0	0	0	0	0
Corp. bonds, Baa	7.7%	5.4%	0	0	0	0	0	0	0
Corp. bonds, Aaa	8.7%	2.6%	2.4%	0	0	0	0	0	0
Growth stocks	3.1%	5.8%	0.2%	16.5%	0	0	0	0	0
Int. stocks	2.9%	6.2%	0.1%	11.7%	7.2%	0	0	0	0
Value stocks	2.8%	7.1%	−0.2%	10.4%	6.7%	5.8%	0	0	0
Short rate	−1.1%	−0.1%	0.0%	0.3%	−0.1%	−0.1%	2.3%	0	0
10Y yield	0.0%	0.0%	0.0%	0.1%	0.1%	0.0%	0.0%	1.3%	0
DP	−3.0%	−6.7%	0.1%	−14.0%	−2.5%	−0.9%	0.0%	0.6%	4.7%

B. Centralized Problem

We first solve again the centralized investment problem in which the CIO manages all assets. This solution serves as a point of reference for the case in which investment management is decentralized. The centralized investment problem with affine prices of risk has been solved by, among others, Liu (2007) and Sangvinatsos and Wachter (2005). We denote the CIO’s investment horizon by τ_C . The optimal allocation to the different assets is given by

$$\begin{aligned}
 x_C(X, \tau_C) = & \frac{1}{\gamma_C}(\Sigma \Sigma')^{-1} \Sigma \Lambda(X) + \dots \\
 & \frac{1}{\gamma_C}(\Sigma \Sigma')^{-1} \Sigma \Sigma'_X \left(B(\tau_C) + \frac{1}{2}(C(\tau_C) + C(\tau_C)')X \right), \quad (18)
 \end{aligned}$$

where expressions for $B(\tau_C)$ and $C(\tau_C)$, as well as the derivations of the results in this section are provided in Appendix B. The optimal portfolio contains two components. The first component is the conditional myopic demand that optimally exploits the risk–return trade-off provided by the assets. The second

component represents the hedging demands that emerge from the CIO's desire to hedge future changes in the investment opportunity set. This second term reflects the long-term perspective of the CIO. The corresponding value function is given by

$$J_1(W, X, \tau_C) = \frac{1}{1 - \gamma_C} W^{1-\gamma_C} \exp \left\{ A(\tau_C) + B(\tau_C)' X + \frac{1}{2} X' C(\tau_C) X \right\}, \quad (19)$$

with the coefficients A, B, and C provided in Appendix B.

In Figure 6, we illustrate the composition of the optimal portfolio for different investment horizons when the coefficient of relative risk aversion of the CIO equals either $\gamma_C = 5$ in Panel A or $\gamma_C = 10$ in Panel B. Focusing first on the fixed income asset class, we find substantial horizon effects for corporate bonds. At short horizons, the CIO optimally tilts the portfolio towards Baa-rated corporate bonds and shorts Aaa-rated corporate bonds to take advantage of the credit spread. At longer horizons, the fraction invested in Baa-rated bonds increases even further, while the allocation to Aaa-rated corporate bonds decreases. Switching to the results for the equities asset class, we detect a strong value tilt at short horizons due to the high-value premium. The optimal portfolio contains a large long position in value stocks and large short position in growth stocks. However, as the investment horizon increases, the value tilt drops, consistent with the results of Jurek and Viceira (2006).¹¹

C. Decentralized Problem without a Benchmark

We now solve the decentralized problem when the CIO cannot use the benchmark to align incentives. In general, the optimal portfolios of the asset managers depend on both the investment horizon and the state of the economy. However, to make the problem more tractable and realistic, we assume that the investment managers are able to time the market and exploit the time-variation in risk premia, but ignore long-term considerations. That is, asset managers implement the conditional myopic strategy

$$x_i^{NB}(X) = \frac{1}{\gamma_i} x_i(X) + \left(1 - \frac{x_i(X)' \iota}{\gamma_i} \right) x_i^{MV}, \quad (20)$$

where

$$x_i(X) = (\Sigma_i \Sigma_i')^{-1} \Sigma_i \Lambda(X) \quad \text{and} \quad x_i^{MV} = \frac{(\Sigma_i \Sigma_i')^{-1} \iota}{\iota' (\Sigma_i \Sigma_i')^{-1} \iota}. \quad (21)$$

¹¹ This result is also in line with the findings of Campbell and Vuolteenaho (2004), who explain the value premium by decomposing the CAPM beta into a cash flow beta and a discount rate beta. The cash flow component is highly priced, but largely unpredictable. The discount rate component demands a lower price of risk but is to some extent predictable. Campbell and Vuolteenaho (2004) show that growth stocks have a large discount rate beta, whereas value stocks have a large cash flow beta. This implies that from a myopic perspective, value stocks are more attractive than growth stocks. However, the predictability of growth stock returns implies that long-term returns on these assets are less risky, making them relatively more attractive.

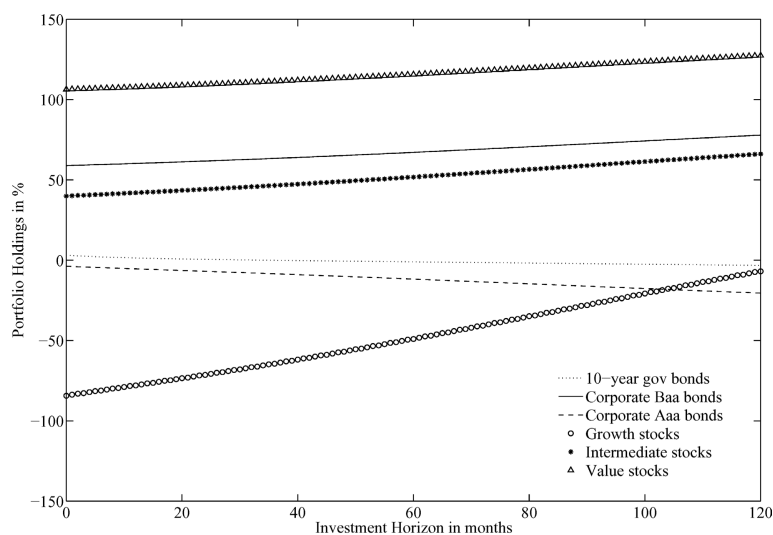
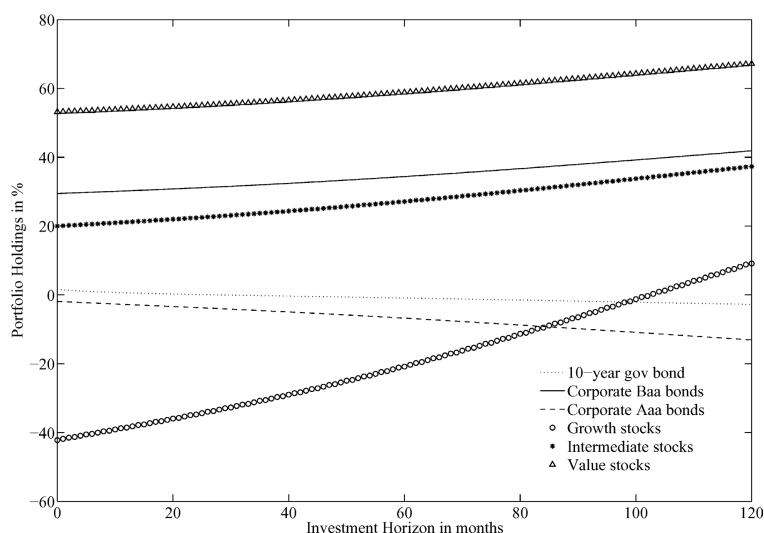
Panel A: The coefficient of relative risk aversion of the CIO equals $\gamma_C = 5$ Panel B: The coefficient of relative risk aversion of the CIO equals $\gamma_C = 10$ 

Figure 6. Optimal portfolio choice in the centralized problem. This figure depicts the optimal allocation to government bonds, corporate bonds with ratings Baa and Aaa, and three stock portfolios ranked based upon their book-to-market ratios (growth, intermediate, and value). The horizontal axis depicts the investment horizon of the CIO in months. The coefficient of relative risk aversion of the CIO equals $\gamma_C = 5$ in Panel A and $\gamma_C = 10$ in Panel B.

This particular form of myopia can be motivated by the relatively short-sighted compensation schemes of asset managers. Since the average hedging demands for 1-year horizons are negligible, we abstract from the managers' hedging motives in this part of the problem.

The CIO does account for the long-term perspective of the firm through the strategic allocation. However, we assume that the CIO implements a strategic allocation that is unconditional, that is, independent of the current state. At each point in time, the allocation to the different asset classes is reset towards a constant-proportions strategic allocation, as opposed to constantly changing the strategic allocation depending on the state. In order to decide on the strategic allocation, the CIO maximizes the unconditional value function

$$\max_{x_C(\tau_C)} \mathbb{E}(J_2(W, X, \tau_C) | W), \quad (22)$$

where J_2 denotes the conditional value function in the decentralized problem above. Obviously, the CIO's horizon, τ_C , influences the choice of the strategic allocation.

To review the setup of this decentralized problem, the asset managers implement active strategies in their asset classes using conditioning information but ignore any long-term considerations. The CIO, in contrast, allocates capital unconditionally to the asset classes, but accounts for the firm's long-term perspective.

In order to determine the unconditional value function, we evaluate first the conditional value function of the CIO, J_2 , for any choice of the strategic allocation. In Appendix B, we show that the conditional value function is exponentially quadratic in the state variables:

$$J_2(W, X, \tau_C) = \frac{W^{1-\gamma_C}}{1-\gamma_C} \exp \left\{ (A(\tau_C, x_C) + B(\tau_C, x_C)'X + \frac{1}{2}X'C(\tau_C, x_C)X) \right\}. \quad (23)$$

One aspect of the CIO's problem is particularly interesting. The active strategy implemented by the asset managers, x_i^{NB} , is affine in the predictor variables: $x_i^{NB}(X) = \zeta_{0i}^{NB} + \zeta_{1i}^{NB}X$. As a consequence, the implied wealth dynamics faced by the CIO are given by

$$\frac{dW_t}{W_t} = (r + \sigma_W(X)' \Lambda(X)) dt + \sigma_W(X)' dZ_t, \quad (24)$$

where $\sigma_W(X)' = x_{1C}(\zeta_{01}^{NB} + \zeta_{11}^{NB}X)' \Sigma_1 + x_{2C}(\zeta_{02}^{NB} + \zeta_{12}^{NB}X)' \Sigma_2$. Since the asset managers condition their portfolios on the state variables, the CIO has to allocate capital to two assets that exhibit a very particular form of heteroskedasticity. Hence, despite the homoskedastic nature of the financial market, the CIO is confronted with heteroskedastic asset returns in the decentralized investment management problem.

We solve for the optimal strategic asset allocation numerically (see Appendix B for details). In Figure 7, we present the strategic allocation to the fixed income and equities classes for different investment horizons. The preference parameters are set to $\gamma_C = 10$ and $\gamma_1 = \gamma_2 = 5$. The strategic allocation to the asset classes exhibits substantial horizon effects and marginally overweights equities. Recall that the strategic allocation to the asset classes is independent of the state variables, by construction, because it is unconditional.

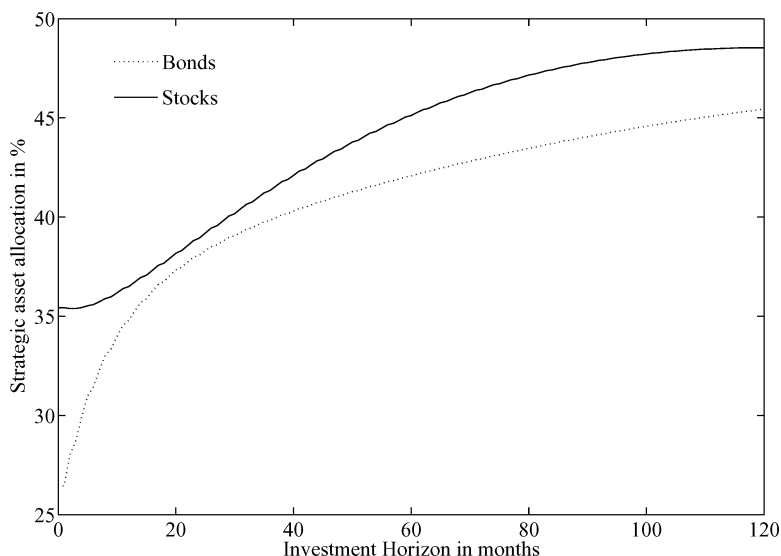


Figure 7. Optimal strategic allocation in the decentralized problem without a benchmark. This figure displays the optimal allocation to the fixed income and equity asset classes in the absence of a benchmark. The horizontal axis depicts the investment horizon of the CIO in months. The preference parameters have been set to $\gamma_C = 10$ and $\gamma_i = 5$, with $i = 1, 2$.

Figure 8 provides the annualized utility costs from decentralized asset management for different risk attitudes of the investment managers. The investment horizon equals either $T = 1$ year in Panel A or $T = 10$ years in Panel B. The utility costs are large and increasing in the horizon of the CIO. For relatively short investment horizons, the costs closely resemble the case with constant investment opportunities, with an order of about 40 to 80 basis points per annum. In contrast, for longer investment horizons, the utility costs are substantially higher, around 200 to 300 basis points per annum. Note that the risk attitudes of the managers, for which the costs of decentralized investment management are minimized, depend on the CIO's investment horizon.

D. Decentralized Problem with a Benchmark

We show in Section I.D that when investment opportunities are constant, a performance benchmark can be designed to eliminate all inefficiencies induced by decentralized asset management. This section reexamines this issue for the case of time-varying investment opportunities. We restrict attention to unconditional benchmarks, meaning the benchmark portfolio weights are not allowed to depend on the state variables.¹² Unconditional benchmarks have the advantage that they are easy to implement. Moreover, investment managers following an unconditional benchmark do not have to trade excessively, which

¹² See also Cornell and Roll (2005).

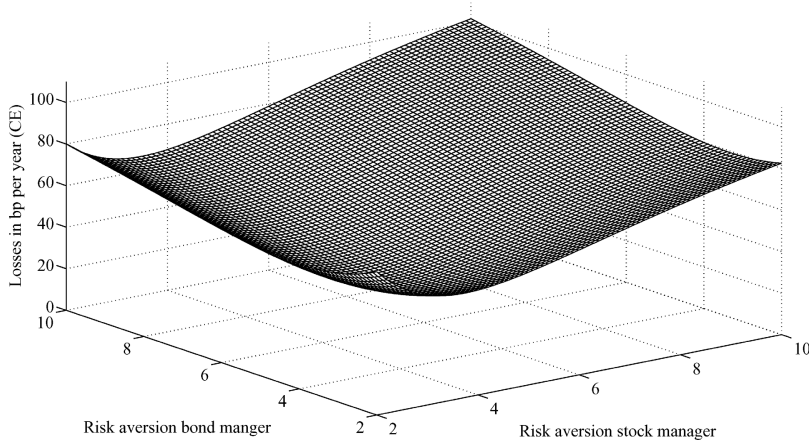
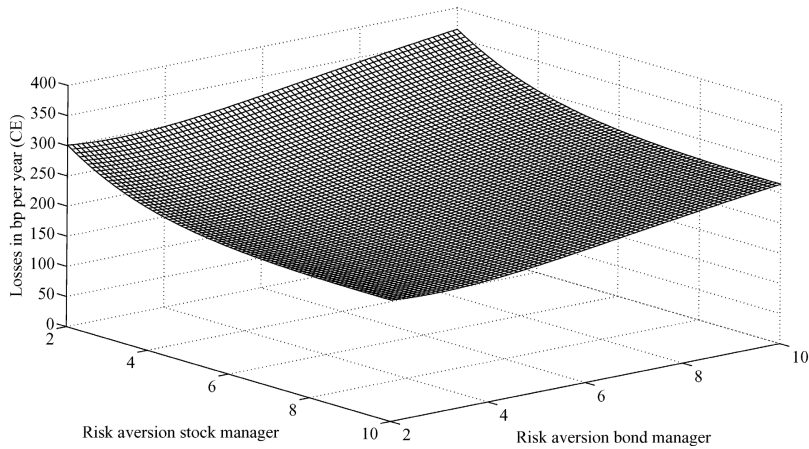
Panel A: The investment horizon of the CIO equals $T = 1$ Panel B: The investment horizon of the CIO equals $T = 10$ 

Figure 8. Utility costs of decentralized investment management without a benchmark. This figure gives a comparison of certainty equivalents following from the centralized and decentralized investment management problem when there is no benchmark and the investment horizon is 1 year in Panel A and 10 years in Panel B. The horizontal axes depict the risk appetites of the asset managers. The coefficient of relative risk aversion of the CIO equals 10. The losses are computed by taking the ratio of the annualized certainty equivalents achieved under decentralized and centralized investment management after which we subtract one and multiply by $-10,000$ to express the losses in basis points per year.

could be the case with a conditional benchmark. Conditional benchmarks are more flexible and may therefore reduce further or even eliminate the costs of decentralization.

The performance benchmark of asset manager i is given by a k -dimensional vector of unconditional portfolio weights, β_i , with $\beta_i' \iota = 1$. Since the benchmark

is chosen unconditionally, asset managers can outperform their benchmark (i.e., generate alpha) by properly incorporating the conditioning information. The benchmark dynamics are

$$\frac{dB_{it}}{B_{it}} = (r + \beta'_i \Sigma_i \Lambda(X)) dt + \beta'_i \Sigma_i dZ_t. \quad (25)$$

To solve for the optimal benchmark, we first determine the optimal response of the asset managers to their benchmarks. The optimal conditional myopic strategy of the investment managers with a benchmark is given by

$$x_i^B(X) = \frac{1}{\gamma_i} x_i(X) + \left(1 - \frac{1}{\gamma_i}\right) \beta_i + \frac{1}{\gamma_i} (1 - x_i(X)' \gamma_t) x_i^{MV}, \quad (26)$$

where $x_i(X)$ and x_i^{MV} are given in equation (21). The CIO chooses the (unconditional) benchmarks and determines the (unconditional) strategic allocation to the asset classes by maximizing the unconditional expectation of the conditional value function,

$$\max_{x_C(\tau_C), \beta_1(\tau_C), \beta_2(\tau_C)} \mathbb{E}(J_3(W, X, \tau_C) \mid W). \quad (27)$$

The conditional value function, J_3 , is again exponentially quadratic in the state variables and the coefficients are provided in Appendix B. Note that both the strategic allocation and the benchmarks are allowed to depend on the CIO's horizon.

We use numerical methods to solve for the optimal benchmarks and allocations to the two asset classes (see Appendix B for details). Panel A of Figure 9 shows the optimal performance benchmarks for different investment horizons of the CIO. The CIO's risk aversion equals 10 and the managers' risk aversion is set to 5. At short horizons, or if the CIO behaves myopically, the optimal benchmarks are similar to when investment opportunities are constant. However, the benchmark portfolios exhibit strong horizon effects. For instance, in the equities asset class, the myopic benchmark reinforces the value tilt already present in the equity manager's (myopic) portfolio. The long-run benchmark, in contrast, anticipates the lower risk of growth stocks and provides an incentive to reduce the value tilt. This illustrates how performance benchmarks can be used to incorporate the CIO's long-term perspective in the short-term portfolio choices of the asset managers.

Panel B of Figure 9 provides the corresponding strategic allocation to both asset classes for different investment horizons. Recall that when investment opportunities are constant, the centralized allocation is always more risky than the decentralized allocation without a benchmark. When investment opportunities are time varying, we find the initial allocation with a benchmark to be similar to (and even somewhat more conservative than) the allocation without a benchmark. However, for longer investment horizons of the CIO, the optimal strategic allocation of the CIO is tilted substantially towards equities.

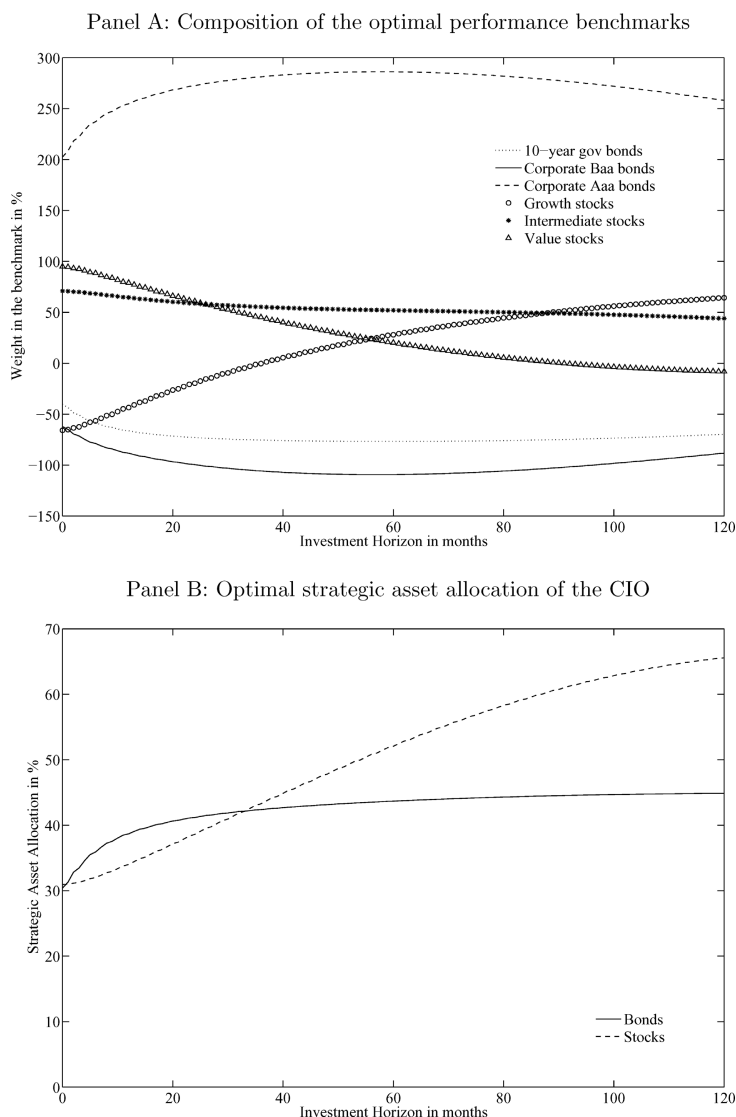


Figure 9. Optimal performance benchmarks and strategic allocation. Panel A portrays the composition of the optimal performance benchmarks for different investment horizons of the CIO. Panel B presents the corresponding optimal strategic asset allocation to the asset classes. We plot the benchmark for the stock and bond manager in the same graph, but there is still no cross-benchmarking. That is, the benchmark weights in both asset classes each sum up to 100%. The horizontal axis depicts the investment horizon of the CIO in months. The preference parameters are $\gamma_C = 10$ and $\gamma_i = 5$, with $i = 1, 2$.

Figure 10 presents the utility gains generated by an optimally chosen benchmark. The CIO's coefficient of risk aversion equals 10 and the horizon is set to $T = 1$ year in Panel A and $T = 10$ years in Panel B. For the 1-year horizon, the value added by the benchmark is limited to approximately 20 basis points.

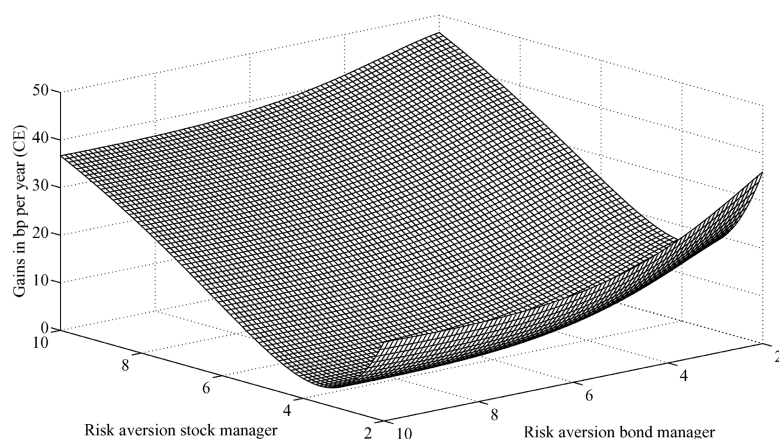
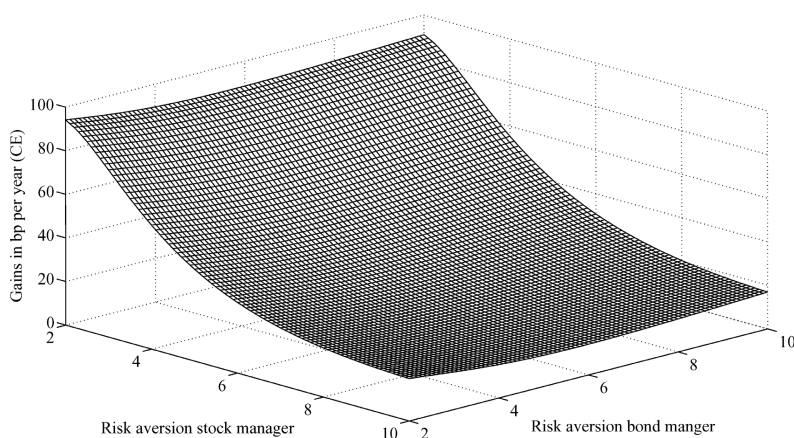
Panel A: The investment horizon of the CIO equals $T = 1$ Panel B: The investment horizon of the CIO equals $T = 10$ 

Figure 10. Value generated by an optimally chosen benchmark. This figure gives a comparison of certainty equivalents following from the decentralized problem with and without an optimally chosen benchmark. We present the annualized gains in basis points from using the benchmark optimally. The investment horizon of the CIO equals 1 year in Panel A and 10 years in Panel B. The horizontal axes depict different risk appetites of the asset managers. The coefficient of relative risk aversion of the CIO equals 10.

However, when the investment horizon increases to 10 years, the benefit of an optimally chosen benchmark increases as the asset managers become less conservative.

We conclude that unconditional performance benchmarks are significantly value enhancing. This extends the results of Admati and Pfleiderer (1997) concerning the role of performance benchmarks in delegated portfolio management problems. In case of multiple asset managers, performance benchmarks can be

useful in aligning incentives along at least three dimensions, namely, diversification, preferences, and investment horizons.

III. Unknown Risk Appetites of the Managers

In the previous sections, we assume that the CIO is able to observe the managers' risk aversion levels in deciding on the strategic allocation and in constructing the performance benchmarks. In reality, the CIO usually has relatively limited information about the managers' preferences. Even though past performance or current portfolio holdings can be informative about the managers' risk attitude, exact inference is often infeasible.

In this section therefore, we generalize our framework by explicitly modeling the CIO's uncertainty about the managers' preferences. Specifically, we focus on the impact of the unknown risk aversion levels of the asset managers on (1) the strategic allocation to each of the asset classes, (2) the utility costs of decentralization, and (3) the value of optimally designed performance benchmarks. We model the CIO's uncertainty with respect to the managers' risk attitudes by assuming that the CIO has a prior distribution over the risk attitudes of the managers. It is important to note that even when the CIO does not wish to implement optimally designed benchmarks, the CIO needs this prior distribution to decide the strategic allocation to each of the asset classes. We then examine the extent to which the implementation of optimal benchmarks is effective in aligning incentives when the CIO can use no more information than his prior beliefs to design the benchmarks.

We assume that the CIO's prior over the managers' coefficients of relative risk aversion is given by a normal distribution truncated between 1 and 10.¹³ More formally, the prior is given by

$$\gamma \sim f(\gamma) = \frac{\exp\left[-\frac{1}{2}(\gamma - \mu_\gamma)' \Sigma_\gamma^{-1}(\gamma - \mu_\gamma)\right]}{\int_1^{10} \int_1^{10} \exp\left[-\frac{1}{2}(\gamma - \mu_\gamma)' \Sigma_\gamma^{-1}(\gamma - \mu_\gamma)\right] d\gamma_1 d\gamma_2}, \gamma \in (1, 10) \times (1, 10), \quad (28)$$

with $\gamma = (\gamma_1, \gamma_2)$. The parameters μ_γ and Σ_γ allow us to vary the average risk appetites of the asset managers as well as the precision.¹⁴ The off-diagonal elements of Σ_γ allow for correlations between the risk attitudes of the managers. Note that when $\Sigma_{\gamma(1,1)}$ and $\Sigma_{\gamma(2,2)}$ tend to infinity, the prior converges to an uninformative uniform prior on the interval (1, 10). Note further that within our

¹³ Increasing the upper bound of this truncated normal distribution to, for example, 15 or 20 does not affect our qualitative results.

¹⁴ Note that the truncated normal distribution is skewed if μ_γ does not equal the average of the upper and lower truncation points. In this case, changing μ_γ affects the precision and, likewise, changing Σ_γ has an impact on the average risk attitude. To analyze the impact of uncertainty about the managers' preferences by varying Σ_γ , we focus our discussion predominantly on a symmetric prior with $\mu_\gamma = 5.5$. The results for alternative, skewed prior distributions are reported for completeness and are qualitatively similar.

model, the CIO could potentially learn about managerial preferences through the volatility matrix of the managers' portfolio returns (Merton (1980)). We consider learning about the managers' preferences to be beyond the scope of this paper, however, and we therefore assume that the uncertainty about the managers' preferences is not alleviated or resolved during the course of the investment period.

In order to determine the optimal strategy of the CIO, we integrate out the uncertainty about the managers' risk aversion levels. This results in a strategic asset allocation and performance benchmarks that are robust to a range of preferences of the asset managers. In Section III.A, we determine the optimal strategic allocation and the costs of decentralization for different priors over the managers' preferences. Next, we examine in Section III.B the extent to which optimal performance benchmarks are useful in reducing the utility costs induced by decentralization. Finally, Section III.C introduces tracking error volatility constraints, which are often observed in the investment management industry to constrain asset managers.

A. Decentralized Problem without a Benchmark

We first consider the case in which the asset managers are not remunerated relative to a benchmark. These managers adopt the strategies given in equation (6). The CIO determines the strategic allocation by maximizing

$$\max_{x_C} \mathbb{E}_t \left(\frac{1}{1 - \gamma_C} W_{T_C}^{1 - \gamma_C} \right), \quad (29)$$

where the expectation is taken with respect to both the uncertainty in the financial market and the risk appetites of the asset managers. We can simplify the problem by first conditioning on the managers' risk aversion levels (γ) and then applying the law of iterated expectations:

$$\max_{x_C} \mathbb{E} \left(\mathbb{E}_t \left(\frac{1}{1 - \gamma_C} W_{T_C}^{1 - \gamma_C} \middle| \gamma \right) \right). \quad (30)$$

The inside expectation, conditional on the managers' preferences and possibly the state variables at time t , can be determined in closed-form for any strategic allocation x_C using the arguments in Sections I and II. To develop the main intuition, we focus initially on the case of constant investment opportunities. The conditional expectation is then given by

$$\mathbb{E}_t \left(\frac{1}{1 - \gamma_C} W_{T_C}^{1 - \gamma_C} \middle| \gamma \right) = \frac{1}{1 - \gamma_C} W_t^{1 - \gamma_C} \exp(a(x_C, \gamma)\tau_C), \quad (31)$$

where $a(x_C, \gamma) = (1 - \gamma_C)(x_C \bar{\Sigma}(\gamma)\Lambda + r) - \frac{\gamma_C(1 - \gamma_C)}{2} x_C' \bar{\Sigma}(\gamma) \bar{\Sigma}(\gamma)' x_C$ and $\tau_C = T_C - t$. Given the prior over the managers' risk appetites, it is straightforward to optimize (numerically) over the strategic allocation. Along these lines we can determine (1) the optimal strategic allocation to both asset classes and (2) the

utility costs induced by decentralization for various prior distributions over the managers' risk aversion levels.

Even though the results in the remainder of this section are determined numerically, we can illustrate the impact of not knowing the managers' preference parameters using an accurate approximation. The CIO's first-order condition with respect to the strategic allocation, x_C , is given by

$$\mathbb{E} \left(\frac{1}{1 - \gamma_C} W_t^{1-\gamma_C} \exp(a(x_C, \gamma) \tau_C) \frac{\partial a(x_C, \gamma)}{\partial x_C} \right) = 0_{2 \times 1}. \quad (32)$$

If the term $\exp(a(x_C, \gamma))$ in equation (32) were constant,¹⁵ the optimal strategic allocation would be given by¹⁶

$$x_C^{\text{approx}} = \frac{1}{\gamma_C} (\mathbb{E}(\bar{\Sigma} \bar{\Sigma}'))^{-1} \mathbb{E}(\bar{\Sigma}) \Lambda. \quad (33)$$

It is straightforward to show that when the risk appetites of the managers are independent, it follows that

$$\mathbb{E}(\bar{\Sigma} \bar{\Sigma}') = \mathbb{E}(\bar{\Sigma}) \mathbb{E}(\bar{\Sigma}') + \begin{bmatrix} \text{Var} \left(\frac{1}{\gamma_1} \right) & 0 \\ 0 & \text{Var} \left(\frac{1}{\gamma_2} \right) \end{bmatrix} \begin{bmatrix} b_1' \Sigma_1 \Sigma_1' b_1 & 0 \\ 0 & b_2' \Sigma_2 \Sigma_2' b_2 \end{bmatrix}, \quad (34)$$

where $b_i = x_i - (x_i' \iota) x_i^{MV}$, $i = 1, 2$. In other words, b_i is a long-short portfolio that is long the speculative portfolio and short the minimum-variance portfolio. We now discuss the last two matrices on the right-hand side of equation (34) in turn. The first matrix shows that the covariance matrix of managed portfolio returns increases as a result of the uncertainty about the managers' preferences. This induces the CIO to reduce the strategic allocation to each of the asset classes. If the uncertainty about the managers' risk attitudes is equal across managers, this effect is symmetric across asset classes. However, the second matrix depends on the properties of the asset class, which implies that even if the CIO has the same information about the managers' risk attitudes, the relative allocations to the asset classes change as the uncertainty about the managers' risk attitudes increases.

Using the approximation in equation (33), we can approximate the value function as

$$\exp(a(x_C, \gamma)) \simeq \exp(a(x_C^{\text{approx}}, \gamma)) \equiv \exp(\tilde{a}(\gamma)). \quad (35)$$

This approximation allows us to solve the first-order condition (32) in closed form:

$$x_C^* \simeq \frac{1}{\gamma_C} (\mathbb{E}(\exp(\tilde{a}(\gamma)) \bar{\Sigma} \bar{\Sigma}'))^{-1} \mathbb{E}(\exp(\tilde{a}(\gamma)) \bar{\Sigma}) \Lambda, \quad (36)$$

¹⁵ This is the case, for instance, if we consider a 0th order expansion in $\gamma = \mathbb{E}(\gamma)$.

¹⁶ We normalize $\tau_C = 1$.

Table III
Strategic Allocation without Benchmarks when Risk Attitudes
Are Unknown

This table gives the strategic allocation of the CIO to the asset classes when the risk attitudes of the managers are unknown and there are no benchmarks. The prior of the CIO over the risk aversion level of each of the managers is a truncated normal distribution with parameters μ_γ and σ_γ , truncated below at 1 and truncated above at 10.

	$\mu_\gamma = 3.1$		$\mu_\gamma = 5.5$		$\mu_\gamma = 7.3$	
	Bonds	Stocks	Bonds	Stocks	Bonds	Stocks
Panel A: Constant Investment Opportunities						
$\sigma_\gamma = 0$	19%	30%	22% (22%)	36% (36%)	24%	37%
$\sigma_\gamma = 1$	18%	27%	22% (22%)	36% (36%)	23%	37%
$\sigma_\gamma = 2$	18%	26%	21% (21%)	33% (33%)	23%	36%
$\sigma_\gamma = 3$	18%	27%	21% (21%)	31% (31%)	22%	33%
$\sigma_\gamma = 25$ (uniform)	20%	28%	20% (20%)	28% (29%)	20%	28%
Panel B: Time-Varying Investment Opportunities ($T = 1$)						
$\sigma_\gamma = 0$	27%	31%	36%	37%	39%	38%
$\sigma_\gamma = 1$	23%	27%	35%	37%	38%	38%
$\sigma_\gamma = 2$	22%	26%	29%	34%	35%	37%
$\sigma_\gamma = 3$	22%	27%	27%	31%	30%	35%
$\sigma_\gamma = 25$ (uniform)	24%	29%	24%	29%	24%	29%
Panel C: Time-Varying Investment Opportunities ($T = 10$)						
$\sigma_\gamma = 0$	33%	35%	47%	51%	51%	57%
$\sigma_\gamma = 1$	23%	26%	26%	42%	51%	56%
$\sigma_\gamma = 2$	23%	25%	25%	30%	26%	36%
$\sigma_\gamma = 3$	23%	25%	25%	28%	25%	31%
$\sigma_\gamma = 25$ (uniform)	24%	26%	24%	26%	24%	26%

which is similar to before except that the covariance matrix and expected returns are weighted by the (scaled) value function of the CIO, $\exp(\tilde{a}(\gamma))$.

In the empirical application, we treat the uncertainty about the risk aversion levels of both managers symmetrically and assume independence: $\mu_{\gamma(1)} = \mu_{\gamma(2)}$ and $\Sigma_\gamma = \sigma_\gamma^2 I$, with I denoting a 2×2 identity matrix. We consider prior distributions with mean parameters $\mu_\gamma = 3.1, 5.5$, and 7.3 and uncertainty parameters $\sigma_\gamma = 0, 1, 2, 3$, and 25 . Note that when $\mu_\gamma = 5.5$ the distribution is symmetric as 5.5 is the average of the truncation points 1 and 10 . When $\sigma_\gamma = 25$, the CIO effectively has a uniform prior over γ , and the parameter μ_γ has no further impact.

The results are summarized in Tables III and IV. In Table III we compute the optimal strategic allocations without benchmarks. In Table IV we report the corresponding costs of decentralized investment management. Each table has three panels, one for constant investment opportunities (Panel A) and two panels for time-varying investment opportunities, with the CIO's investment horizon equal to either $T = 1$ (Panel B) or $T = 10$ (Panel C).

Table IV
Costs of Decentralized Investment Management if Risk Attitudes
Are Unknown

This table gives the costs of decentralized investment management when the risk attitudes of the managers are unknown and there are no benchmarks. The prior of the CIO over the risk aversion levels of each of the managers is a truncated normal distribution with parameters μ_γ and σ_γ , truncated below at 1 and truncated above at 10. The losses are computed by taking the ratio of the annualized certainty equivalents achieved under decentralized and centralized investment management after which we subtract 1 and multiply by $-10,000$ to express the losses in basis points per year.

	$\mu_\gamma = 3.1$		$\mu_\gamma = 5.5$		$\mu_\gamma = 7.3$	
	$\gamma_C = 5$	$\gamma_C = 10$	$\gamma_C = 5$	$\gamma_C = 10$	$\gamma_C = 5$	$\gamma_C = 10$
Panel A: Constant Investment Opportunities						
$\sigma_\gamma = 0$	13.8	6.9	29.8	14.9	54.6	27.2
$\sigma_\gamma = 1$	51.5	25.7	31.7	15.9	53.8	26.9
$\sigma_\gamma = 2$	73.8	36.9	51.4	25.7	53.0	26.5
$\sigma_\gamma = 3$	80.5	40.2	68.3	34.2	62.2	31.1
$\sigma_\gamma = 25$ (uniform)	86.8	43.4	86.8	43.4	86.8	43.4
Panel B: Time-Varying Investment Opportunities ($T = 1$)						
$\sigma_\gamma = 0$	94.4	47.4	119.7	58.9	158.6	78.0
$\sigma_\gamma = 1$	152.5	76.7	124.1	61.2	157.9	77.7
$\sigma_\gamma = 2$	188.6	94.7	161.4	80.4	159.8	78.9
$\sigma_\gamma = 3$	201.9	101.2	189.6	94.7	179.8	89.4
$\sigma_\gamma = 25$ (uniform)	217.6	108.8	217.6	108.8	217.6	108.8
Panel C: Time-Varying Investment Opportunities ($T = 10$)						
$\sigma_\gamma = 0$	434.0	261.1	401.0	234.3	434.6	245.1
$\sigma_\gamma = 1$	586.2	341.2	503.0	295.6	439.3	248.2
$\sigma_\gamma = 2$	650.7	372.6	633.0	363.5	611.5	351.8
$\sigma_\gamma = 3$	679.4	386.4	679.0	386.0	669.9	381.3
$\sigma_\gamma = 25$ (uniform)	717.4	404.6	717.4	404.6	717.4	404.6

We focus our discussion on the prior distribution with $\mu_\gamma = 5.5$, since this distribution is symmetric. The results in Table III indicate that an increase in the uncertainty about the managers' risk aversion leads to a decrease in the optimal allocation to both asset classes. This implies that uncertainty about the managers' preferences effectively increases the risk aversion of the CIO. Not knowing the managers' preferences constitutes a form of background risk, which reduces the investor's appetite for financial risk.¹⁷ The results can also be interpreted as a form of Bayesian parameter uncertainty. This intuition can easily be derived from equations (33) to (36). The effect is quantitatively strong, especially for the equity class. If the prior changes from known preferences (no uncertainty) to a uniform prior between 1 and 10, the CIO reduces the allocation to the equity asset class by 25%–50% of the total allocation. Finally, to verify the

¹⁷ See, for instance, Gollier and Pratt (1996).

accuracy of our approximation, we also present in Panel A in parentheses the approximate optimal strategic allocation using equation (36). We conclude that our approximation has a very high level of accuracy, lending further credibility to the intuitive insights it offers.

We report in Table IV the utility costs incurred by the CIO as a result of decentralization for risk aversion parameters of the CIO equal to $\gamma_C = 5$ and $\gamma_C = 10$. The utility costs are annualized and measured in basis points. The costs of decentralized investment management are generally increasing in the uncertainty about the managers' preferences. The impact of this uncertainty on the utility costs is economically significant. In most cases, the costs double when we move from known levels of risk aversion to a uniform prior distribution over the levels of risk aversion. For instance, in Panel B with $\mu_\gamma = 5.5$ and $\gamma_C = 10$, the utility costs increase from 59 to 109 basis points per annum. These results imply that the common, yet unrealistic, assumption that the preferences of the manager (the agent) are known to the CIO (the principal) can grossly understate the problem and have serious consequences for optimal policies, particularly in the case of time-varying investment opportunities and a long investment horizon for the CIO (see Panel C of Table III).

Note that there are exceptional cases in which the costs of decentralization are slightly decreasing in the uncertainty about the preferences of the managers. If the CIO assigns a high prior probability to high-cost managers to begin with, which is the case when $\mu_\gamma = 7.3$ and σ_γ is low (see, for instance, Figure 8), increasing σ_γ will increase the probability of allocating capital to lower-cost managers. This in turn can lead to a decreasing relationship between the costs of decentralization and the uncertainty about the managers' preferences. However, this effect is quantitatively negligible and up to only one basis point per year.

B. Decentralized Problem with a Benchmark

We now examine how effective benchmarks are in aligning incentives if the CIO does not know the risk aversion levels of the managers. Table V presents the optimal strategic allocation when the asset managers are remunerated relative to optimal performance benchmarks. The main effects are in line with Table III. The optimal strategic allocation to both asset classes decreases as the uncertainty about the managers' risk appetites increases. We also find that the implementation of optimal benchmarks can lead to either an increase or decrease in the strategic allocation relative to the problem without benchmarks, depending on the CIO's prior beliefs.

In the previous subsection, we argue that the inefficiencies caused by decentralization are generally aggravated when the risk appetites of the managers are unknown (Table IV). The value of an optimally designed benchmark (Table VI) depends on the following two effects. First, compared to the case of known risk appetites, the amount of information that can be used to design the optimal benchmarks is lower because risk appetites are now unknown. This suggests that the value of an optimal benchmark diminishes. Second, the inefficiencies

Table V
Strategic Allocation with Benchmarks when Risk Attitudes
Are Unknown

This table gives the strategic allocation of the CIO to the asset classes when the risk attitudes of the managers are unknown and the optimal benchmarks are implemented. The prior of the CIO over the risk aversion levels of each of the managers is a truncated normal distribution with parameters μ_γ and σ_γ , truncated below at 1 and truncated above at 10.

	$\mu_\gamma = 3.1$		$\mu_\gamma = 5.5$		$\mu_\gamma = 7.3$	
	Bonds	Stocks	Bonds	Stocks	Bonds	Stocks
Panel A: Constant Investment Opportunities						
$\sigma_\gamma = 0$	24%	32%	24%	32%	24%	32%
$\sigma_\gamma = 1$	21%	26%	24%	31%	24%	32%
$\sigma_\gamma = 2$	19%	24%	23%	29%	24%	31%
$\sigma_\gamma = 3$	19%	24%	21%	27%	23%	29%
$\sigma_\gamma = 25$ (uniform)	20%	25%	20%	25%	20%	25%
Panel B: Time-Varying Investment Opportunities ($T = 1$)						
$\sigma_\gamma = 0$	31%	35%	40%	34%	41%	33%
$\sigma_\gamma = 1$	25%	28%	38%	33%	41%	33%
$\sigma_\gamma = 2$	23%	25%	31%	30%	37%	33%
$\sigma_\gamma = 3$	23%	25%	27%	28%	31%	31%
$\sigma_\gamma = 25$ (uniform)	24%	26%	24%	26%	24%	26%
Panel C: Time-Varying Investment Opportunities ($T = 10$)						
$\sigma_\gamma = 0$	34%	61%	47%	66%	50%	67%
$\sigma_\gamma = 1$	23%	28%	26%	43%	50%	66%
$\sigma_\gamma = 2$	23%	25%	25%	30%	26%	36%
$\sigma_\gamma = 3$	23%	25%	25%	27%	25%	30%
$\sigma_\gamma = 25$ (uniform)	24%	25%	24%	25%	24%	25%

that can potentially be mitigated by the benchmarks are also much larger. Therefore, there is more scope for the benchmarks to have value-added. This explains why, for low levels of uncertainty, there is a (small) negative relation between the value of benchmarks and the level of uncertainty about the risk appetites. In these cases the first effect dominates. However, as the uncertainty about the risk aversion levels increases, the value of the benchmarks also generally increases and exceeds the value for known preferences because then the second effect dominates.

As explained before, there are exceptional cases in which the costs of decentralization are slightly decreasing in the uncertainty about the managers' preferences (e.g., when $\mu_\gamma = 7.3$). In such cases, increasing the uncertainty about the managers' preferences does not sufficiently enlarge the scope for improvement by optimally designed benchmarks. As a result, the fact that the benchmarks are based on less information dominates and the value of an optimally designed benchmark decreases in the uncertainty about the managers' risk aversion levels.

Table VI
Value of Optimal Benchmarks when Risk Attitudes Are Unknown

This table gives a comparison of certainty equivalents following from the decentralized problem with and without an optimally chosen benchmark. We present the annualized gains in basis points from using the benchmark optimally. The prior of the CIO over the risk aversion levels of each of the managers is a truncated normal distribution with parameters μ_γ and σ_γ , truncated below at 1 and truncated above at 10.

	$\mu_\gamma = 3.1$		$\mu_\gamma = 5.5$		$\mu_\gamma = 7.3$	
	$\gamma_C = 5$	$\gamma_C = 10$	$\gamma_C = 5$	$\gamma_C = 10$	$\gamma_C = 5$	$\gamma_C = 10$
Panel A: Constant Investment Opportunities						
$\sigma_\gamma = 0$	13.8	6.9	29.8	14.9	54.6	27.2
$\sigma_\gamma = 1$	9.1	4.5	28.5	14.3	53.1	26.5
$\sigma_\gamma = 2$	13.4	6.7	29.3	14.6	46.7	23.3
$\sigma_\gamma = 3$	19.8	9.9	31.2	15.6	41.3	20.6
$\sigma_\gamma = 25$ (uniform)	33.7	16.9	33.7	16.9	33.7	16.9
Panel B: Time-Varying Investment Opportunities ($T = 1$)						
$\sigma_\gamma = 0$	28.2	14.5	29.4	13.9	48.6	23.2
$\sigma_\gamma = 1$	12.3	6.0	28.0	13.3	47.3	22.6
$\sigma_\gamma = 2$	13.5	6.5	26.5	12.7	41.6	19.8
$\sigma_\gamma = 3$	18.7	9.0	28.2	13.6	36.7	17.6
$\sigma_\gamma = 25$ (uniform)	31.2	15.1	31.2	15.1	31.2	15.1
Panel C: Time-Varying Investment Opportunities ($T = 10$)						
$\sigma_\gamma = 0$	110.3	71.5	40.4	31.5	26.7	20.4
$\sigma_\gamma = 1$	14.3	7.6	21.4	11.2	26.8	20.4
$\sigma_\gamma = 2$	11.5	6.0	18.2	9.3	23.7	12.2
$\sigma_\gamma = 3$	14.7	7.4	20.3	10.2	24.6	12.3
$\sigma_\gamma = 25$ (uniform)	22.8	11.3	22.8	11.3	22.8	11.3

When the CIO's investment horizon is longer, for example $T = 10$, the results in Panel C may give the impression that benchmarks become less effective in aligning incentives when risk appetites are unknown. It is important to emphasize, however, that the results presented for this case constitute a conservative lower bound on the value of benchmarks. It is common practice in the investment management industry to have the opportunity to revise the benchmark annually. We consider a single, unconditional benchmark that is held constant for 10 years, which is the absolute minimum of what optimally designed benchmarks can actually achieve. In case of annual rebalancing, or an effective 1-year horizon, Panel B shows that the benchmarks are indeed more effective the more uncertain the CIO is about the managers' risk preferences.

To summarize, we find that uncertainty about the managers' risk preferences has a strong effect on the optimal strategic allocation to the different asset classes. We show that this uncertainty increases the costs of decentralized investment management even further. We also show that optimally designed performance benchmarks become more effective to overcome these costs.

C. Risk Constraints

Apart from designing optimal return benchmarks, the CIO can also employ risk constraints in order to change or to restrict the behavior of asset managers. These risk constraints can be formulated either in terms of absolute risk in absence of a benchmark or in terms of relative risk when the asset manager is remunerated relative to a benchmark. Absolute risk constraints restrict the total volatility of the portfolio return. Relative risk constraints limit the volatility of the portfolio return in excess of the benchmark return, as in Roll (1992) and Jorion (2003). We assume that the volatility constraints have to be satisfied at every point in time. In modern investment management firms, risk management systems monitor the risk exposures of portfolio holdings frequently, which makes it plausible to presume that risk constraints have to be satisfied continuously. For ease of exposition, we focus initially on the financial market of Section I in which investment opportunities are constant.

The instantaneous volatility of the portfolio return is given by $\sigma^A(x_i) = \sqrt{x_i' \Sigma_i \Sigma_i' x_i}$, which is the portfolio's absolute risk. The instantaneous volatility of the portfolio return in excess of the benchmark (relative risk) is given by $\sigma^R(x_i) = \sqrt{(x_i - \beta_i)' \Sigma_i \Sigma_i' (x_i - \beta_i)}$, which is also called the tracking error volatility. Using these definitions for absolute and relative risk, we impose risk limits of the form

$$\sigma^j(x_i) \leq \phi_{ij}, \quad (37)$$

with $j = A, R$. To ensure that the optimization problem of the asset managers is well defined, we assume that $\sigma^A(x_i^{MV}) \leq \phi_{iA}$, which states that the limit on absolute risk must exceed the volatility of the minimum variance portfolio. In the case of relative risk constraints, we require that $\phi_{iR} \geq 0$, since we restrict attention to benchmarks that can be replicated by the managers. A relative risk limit of $\phi_{iR} = 0$ implies that the asset manager must exactly implement the benchmark portfolio. We focus on the effect of imposing either one of these constraints, but not both.¹⁸

Whenever the unconstrained portfolio choice in absence of a benchmark does not violate the absolute risk constraint, this portfolio remains optimal for manager i . However, once the absolute risk constraint is violated, Appendix C shows that the optimal portfolio equals

$$x_i^{NB}(\xi_i) = \frac{1}{\gamma_i(1 + \xi_i)} x_i + \left(1 - \frac{x_i' l}{\gamma_i(1 + \xi_i)}\right) x_i^{MV}, \quad (38)$$

where x_i and x_i^{MV} are given by equation (7) and $\xi_i > 0$ satisfies $\sigma^A(x_i^{NB}(\xi_i)) = \phi_{Ai}$. This solution shows that the absolute risk constraint induces an effective increase in risk aversion. The results in Figure 2 then imply that absolute risk constraints can mitigate inefficiencies whenever the investment manager is

¹⁸ Jorion (2003) infers in addition the effect of implementing both absolute and relative risk constraints.

too aggressive. In contrast, when the investment manager is too conservative, absolute risk constraints can actually aggravate the inefficiencies.

We also show in Appendix C that the optimal portfolio in the presence of a performance benchmark and binding relative risk constraint is given by

$$x_i^B(\xi_i) = \frac{1}{\gamma_i(1 + \xi_i)}x_i + \left(1 - \frac{1}{\gamma_i(1 + \xi_i)}\right)\beta_i + \frac{1 - x_i^t}{\gamma_i(1 + \xi_i)}x_i^{MV}, \quad (39)$$

where x_i and x^{MV} are given in equation (7) and $\xi_i > 0$ satisfies $\sigma^R(x_i^{NB}(\xi_i)) = \phi_{Ri}$. In addition, Appendix C shows that the relative risk constraint binds for an investment manager with risk aversion γ_i once the benchmark is designed on the basis of a higher risk aversion $\tilde{\gamma}_i$, with $\tilde{\gamma}_i > \gamma_i$. This implies that the CIO does not require specific knowledge of the manager's risk attitude, more than knowing an upper bound. If the benchmark and relative risk constraint are designed on the basis of this conservative upper bound, the relative risk constraint binds for more aggressive managers. The binding constraint induces an effective increase in the manager's risk aversion to the level for which the benchmark is designed.

Combining these results with our discussion of unknown risk appetites, risk constraints essentially shift the lower truncation point of the CIO's prior over the managers' risk aversion levels upwards. All managers, who are more aggressive than the risk constraint allows, will behave as an asset manager for which the constraint binds on the margin. Hence, risk constraints effectively reduce the CIO's uncertainty about the manager's preferences.

In case of constant investment opportunities, there is no disadvantage from selecting tight risk constraints. However, in the more realistic case of time-varying investment opportunities, the same derivation is valid, albeit ξ_i becomes time dependent and the constraint will bind only at certain points in time. In that case, tight risk constraints will reduce the timing ability of the asset managers. Therefore, the CIO can optimally determine the strategic allocation to both asset classes, the benchmarks for each manager, and the risk constraints for a given prior over the managers' risk tolerances. Tight risk constraints indicate that it is valuable for the CIO to reduce uncertainty about the managers' risk attitude, while wide risk constraints indicate that the CIO prefers to exploit the timing expertise of the managers rather than reducing the uncertainty about their preferences.

IV. Conclusions

We address several misalignments of incentives generated by decentralized investment management. These misalignments between a CIO and the asset managers he employs can lead to large utility costs. One straightforward solution is to implement centralized investment strategies whereby the CIO attempts to manage all assets himself. However, from an organizational point of view, decentralized investment management is an inevitable and stylized fact of the investment industry. We show in this paper that the optimal design of

an unconditional linear benchmark can be very effective in mitigating the costs of decentralized investment management. This is even more pronounced when we generalize our model by relaxing the assumption that the CIO knows the risk aversion levels of the asset managers. The optimal benchmark is derived assuming that the CIO only knows the cross-sectional distribution of investment managers' risk appetites, but does not know where in this distribution a given manager falls.

For ease of exposition, we confine attention to CRRA preferences and linear performance benchmarks. Future work could focus on a more complicated preference structure and/or nonstandard contracts. For example, it seems reasonable that the utility function of the CIO is kinked as in van Binsbergen and Brandt (2007). The compensation scheme for the asset managers may also be nonlinear and/or asymmetric, as in Browne (1999, 2000), Carpenter (2000), and Basak, Pavlova, and Shapiro (2007), for example. Another interesting extension would be to assess the asset pricing implications of decentralized investment management. In delegated portfolio choice problems, Brennan (1993), Gómez and Zapatero (2003), Cuoco and Kaniel (2006), and Cornell and Roll (2005) illustrate the impact of delegation and benchmarking on equilibrium asset prices. Stutzer (2003b) shows that multiple benchmarks imply a factor model with these benchmarks returns as possibly priced factors. Finally, we show that not knowing the risk preferences of the managers to which the CIO delegates the available capital effectively increases the CIO's risk aversion. Since the amount of capital managed institutionally has increased dramatically during recent decades, it is important to further understand the asset pricing implications of unknown risk preferences.

Appendix A: Constant Investment Opportunities

A. Decentralized Problem with a Benchmark

We solve the decentralized problem with the optimally designed benchmark of Section I.D. We derive first the optimal allocations of the asset managers in the presence of a benchmark. Define normalized wealth as $w_{it} = W_{it} B_{it}^{-1}$. Recall that the benchmark comprises only positions in the assets available to the investment managers and no cash. The asset managers are therefore able to replicate the benchmark. The dynamics of the benchmark are given in equation (12). Using Ito's lemma, the dynamics of normalized wealth are

$$\frac{dw_t}{w_t} = (x_i^{B'} \Sigma_i \Lambda - \beta_i' \Sigma_i \Lambda + \beta_i' \Sigma_i \Sigma_i \beta_i - \beta_i' \Sigma_i \Sigma_i' x_i^B) dt + (x_i^{B'} \Sigma_i - \beta_i' \Sigma_i) dZ_t. \quad (A1)$$

The corresponding Hamilton–Jacobi–Bellman (HJB) equation is

$$\max_{x_i^B : x_i^{B'} = 1} \left(J_w w (x_i^{B'} \Sigma_i \Lambda - \beta_i' \Sigma_i \Lambda + \beta_i' \Sigma_i \Sigma_i \beta_i - \beta_i' \Sigma_i \Sigma_i' x_i^B) + \frac{1}{2} w^2 J_{ww} (x_i^{B'} \Sigma_i - \beta_i' \Sigma_i) (x_i^{B'} \Sigma_i - \beta_i' \Sigma_i)' + J_t \right) = 0. \quad (A2)$$

The first-order conditions (FOC) are

$$0 = J_w w (\Sigma_i \Lambda - \Sigma_i \Sigma'_i \beta_i) + J_{ww} w^2 \Sigma_i (\Sigma'_i x_i^B - \Sigma'_i \beta_i) - \xi_t, \text{ and } 1 = x_i^{B'} \iota, \quad (\text{A3})$$

with ξ denoting the Lagrange multiplier. The value function is of the form

$$J_3(W/B, \tau_i) = \frac{1}{1 - \gamma_i} \left(\frac{W}{B} \right)^{1 - \gamma_i} \exp(c \tau_i), \quad (\text{A4})$$

with $\tau_i = T_i - t$. The solution of the FOCs is given by equation (14).

The CIO has to design the benchmarks, that is, $\beta_i, i = 1, 2$, and decide on the strategic allocation to the managers and to the cash account. Since the managers' optimal portfolios are affine in the benchmark weights, (see equation (14)), the benchmark can be designed to solve for the optimal relative fractions invested in the different assets present in the asset classes. The strategic allocation, $x_C \in \mathbb{R}^2$, can subsequently be used to optimally manage the absolute fractions allocated to the different assets. More formally, the optimal portfolio is given by

$$x_C = \begin{bmatrix} x_{1C} \\ x_{2C} \end{bmatrix} = \frac{1}{\gamma_C} (\Sigma \Sigma')^{-1} \Sigma \Lambda, \quad (\text{A5})$$

where x_{iC} denotes the allocation to the assets managed by manager i . We use β_i to solve for the optimal relative fractions invested within the asset class:

$$x_i^B = x_{iC} (x'_{iC} \iota)^{-1}. \quad (\text{A6})$$

The optimal benchmark weights are given by

$$\beta_i = \frac{\gamma_i}{\gamma_i - 1} \left[x_i^C (x'_{iC} \iota)^{-1} - \left(\frac{1}{\gamma_i} x_i + \frac{1}{\gamma_i} (1 - x'_{iC} \iota) x_i^{MV} \right) \right], \quad (\text{A7})$$

and the optimal allocation of the CIO's wealth to the managers is given by $x'_{iC} \iota$.

Appendix B: Time-Varying Investment Opportunities

B. Centralized Problem

The centralized problem in Section II.B relates to the portfolio choice problems in Sangvinatsos and Wachter (2005) and Liu (2007). The problem is solved using standard dynamic programming techniques. The HJB equation reads

$$\max_{x_C} \left(J_W W (r + x'_C \Sigma \Lambda(X)) + \frac{1}{2} J_{WW} W^2 x'_C \Sigma \Sigma' x_C + J_t \right. \\ \left. - J'_X KX + \frac{1}{2} \text{tr}(\Sigma'_X J_{XX} \Sigma_X) + W x'_C \Sigma \Sigma'_X J_{WX} \right) = 0, \quad (\text{B1})$$

where we omit the indices of $x_C(X, \tau_C)$ for notational convenience and $K = \text{diag}(\kappa_1, \dots, \kappa_m)$. The affine structure of the financial market implies that the value function is exponentially quadratic in the state variables:

$$J(W, X, \tau_C) = \frac{W^{1-\gamma_C}}{1-\gamma_C} \exp \left\{ A(\tau_C) + B(\tau_C)'X + \frac{1}{2}X'C(\tau_C)X \right\}. \quad (\text{B2})$$

Solving for the FOC of problem (B1) and using equation (B2) to determine the partial derivatives, we obtain

$$x_C(X, \tau_C) = \frac{1}{\gamma_C}(\Sigma\Sigma')^{-1}\Sigma \left[\Lambda(X) + \Sigma'_X \left(B(\tau_C) + \frac{1}{2}(C(\tau_C) + C(\tau_C)')X \right) \right], \quad (\text{B3})$$

which we can rewrite as $x_C(X, \tau_C) = \zeta_0^C(\tau_C) + \zeta_1^C(\tau_C)X$, with

$$\zeta_0^C(\tau_C) = \frac{1}{\gamma_C}(\Sigma\Sigma')^{-1}\Sigma [\Lambda_0 + \Sigma'_X B(\tau_C)], \quad (\text{B4})$$

$$\zeta_1^C(\tau_C) = \frac{1}{\gamma_C}(\Sigma\Sigma')^{-1}\Sigma \left[\Lambda_1 + \frac{1}{2}\Sigma'_X (C(\tau_C) + C(\tau_C)') \right]. \quad (\text{B5})$$

To find the coefficients A , B , and C , we substitute the optimal portfolio into the HJB equation (B1) and match the constant, the terms linear in X , and the terms quadratic in X . In what follows, we derive the value function for any affine policy, $x(X, \tau) = \zeta_0(\tau) + \zeta_1(\tau)X$, which turns out to be useful in subsequent derivations. The value function for this particular problem is obtained for $\zeta_0(\tau) = \zeta_0^C(\tau)$ and $\zeta_1(\tau) = \zeta_1^C(\tau)$. The resulting ODEs are

$$\begin{aligned} \dot{A} &= (1-\gamma_C)(r + \zeta_0'\Sigma\Lambda_0) - \frac{1}{2}\gamma_C(1-\gamma_C)\zeta_0'\Sigma\Sigma'\zeta_0 \\ &\quad + \frac{1}{4}\text{tr}(\Sigma'_X(C+C')\Sigma_X) + \frac{1}{2}B'\Sigma_X\Sigma'_X B + (1-\gamma)\zeta_0'\Sigma\Sigma'_X B, \\ \dot{B}' &= (1-\gamma_C)[\zeta_0'\Sigma\Lambda_1 + \Lambda_0'\Sigma'\zeta_1] - \gamma_C(1-\gamma_C)\zeta_0'\Sigma\Sigma'\zeta_1 - B'K \\ &\quad + \frac{1}{2}B'\Sigma_X\Sigma'_X(C+C') + \frac{1}{2}(1-\gamma_C)\zeta_0'\Sigma\Sigma'_X(C+C') + (1-\gamma_C)B'\Sigma_X\Sigma'\zeta_1, \\ \dot{C} &= 2(1-\gamma_C)\zeta_1'\Sigma\Lambda_1 - \gamma_C(1-\gamma_C)\zeta_1'\Sigma\Sigma'\zeta_1 - (C+C')K \\ &\quad + \frac{1}{4}(C+C')\Sigma_X\Sigma'_X(C+C') + (1-\gamma_C)\zeta_1'\Sigma\Sigma'_X(C+C'), \end{aligned} \quad (\text{B6})$$

subject to the boundary conditions $A(0) = 0$, $B(0) = 0_{m \times 1}$, and $C(0) = 0_{m \times m}$.

C. Decentralized Problem without a Benchmark

In the decentralized problem without a benchmark in Section II.C, we first solve for the myopic, cash-constrained policy of the managers. The optimization problem of the (myopic) managers can be simplified to

$$\max_{x_i^{NB}: x_i^{NB'} = 1} \mathbb{E}_t \left(x_i^{NB'}(X) \Sigma_i \Lambda(X) - \frac{\gamma_i}{2} x_i^{NB'}(X) \Sigma_i \Sigma_i' x_i^{NB}(X) \right). \quad (\text{B7})$$

As a result, the optimal strategy of the myopic, cash-constrained investment managers is

$$x_i^{NB}(X) = \frac{1}{\gamma_i} x_i(X) + \left(1 - \frac{x_i(X)' \iota}{\gamma_i} \right) x_i^{MV} = \zeta_{0i}^{NB} + \zeta_{1i}^{NB} X, \quad (\text{B8})$$

where $x_i(X)$ and x_i^{MV} are given in equation (21) and

$$\zeta_{0i}^{NB} = \frac{1}{\gamma_i} (\Sigma_i \Sigma_i')^{-1} \Sigma_i \Lambda_0 + x_i^{MV} \left(1 - \frac{\iota' (\Sigma_i \Sigma_i')^{-1} \Sigma_i \Lambda_0}{\gamma_i} \right), \quad (\text{B9})$$

$$\zeta_{1i}^{NB} = \frac{1}{\gamma_i} (\Sigma_i \Sigma_i')^{-1} \Sigma_i \Lambda_1 - x_i^{MV} \left(\frac{\iota' (\Sigma_i \Sigma_i')^{-1} \Sigma_i \Lambda_1}{\gamma_i} \right). \quad (\text{B10})$$

Anticipating the allocations of the asset managers, the CIO has to decide on the strategic allocation. We consider strategic allocations that are independent of the current state of the economy, but that do account for the investment horizon of the CIO. We optimize the unconditional value function

$$\mathbb{E}(J_2(W, X, \tau_C) | W), \quad (\text{B11})$$

with $J_2(W, X, \tau_C)$ denoting the conditional value function, which is exponentially quadratic in the state variables. After all, if we denote the allocation to the i th asset manager by x_{iC} , then the resulting portfolio of the CIO is affine in the state variables:

$$x_C^{\text{Implied}} = \begin{bmatrix} x_{1C} (\zeta_{01}^{NB} + \zeta_{11}^{NB} X) \\ x_{2C} (\zeta_{02}^{NB} + \zeta_{12}^{NB} X) \end{bmatrix} = \zeta_0^{\text{Implied}} + \zeta_1^{\text{Implied}} X, \quad (\text{B12})$$

and the results of Appendix B apply. To determine the unconditional value function, we use Lemma 1.

LEMMA 1: *Let $Y \in \mathbb{R}^{m \times 1}$, $Y \sim N(0, \Sigma)$, $a \in \mathbb{R}^{m \times 1}$, and $B \in \mathbb{R}^{m \times m}$. If $(\Sigma^{-1} - 2B)$ is strictly positive definite, then we have*

$$\mathbb{E}(\exp(a'Y + Y'BY)) = \exp \left(-\frac{1}{2} \ln \det(I - 2\Sigma B) + \frac{1}{2} a'(\Sigma^{-1} - 2B)^{-1} a \right). \quad (\text{B13})$$

Solving for the optimal strategic asset allocation is then reduced to a static optimization of the unconditional value function, which we perform numerically.

D. Decentralized Problem with a Benchmark

The performance benchmark of manager i in Section II.D is parameterized by a vector of constant portfolio weights, β_i , with the corresponding dynamics

specified in equation (25). The asset manager is concerned with wealth relative to the value of the benchmark. The dynamics of normalized wealth, $w_{it} = W_{it}B_{it}^{-1}$, are given by

$$\frac{dw_t}{w_t} = (x_i^{B'}(X)\Sigma_i\Lambda(X) + \beta_i'\Sigma_i[\Sigma_i'\beta_i - \Lambda(X) - \Sigma_i'x_i^B(X)])dt + (x_i^{B'}(X)\Sigma_i - \beta_i'\Sigma_i)dZ_t,$$

where $x_i^B(X)$ denotes the myopic conditional portfolio choice of investment manager i . We first optimize the managers' portfolios when they have no access to a cash account, that is, $x_i^{B'\iota} = 1$. The optimal strategy of the managers is given by

$$x_i^B(X) = \frac{1}{\gamma_i}x_i(X) + \left(1 - \frac{1}{\gamma_i}\right)\beta_i + \frac{1}{\gamma_i}(1 - x_i(X)'\iota)x_i^{MV} = \zeta_{0i}^B + \zeta_{1i}^BX, \quad (\text{B14})$$

where $x_i(X)$ and x_i^{MV} as in equation (21) and

$$\zeta_{0i}^B = \frac{1}{\gamma_i}(\Sigma_i\Sigma_i')^{-1}\Sigma_i\Lambda_0 + \left(1 - \frac{1}{\gamma_i}\right)\beta_i + \frac{1}{\gamma_i}x_i^{MV}\left(1 - \iota'(\Sigma_i\Sigma_i')^{-1}\Sigma_i\Lambda_0\right), \quad (\text{B15})$$

$$\zeta_{1i}^B = \frac{1}{\gamma_i}(\Sigma_i\Sigma_i')^{-1}\Sigma_i\Lambda_1 - \frac{1}{\gamma_i}x_i^{MV}\left(\iota'(\Sigma_i\Sigma_i')^{-1}\Sigma_i\Lambda_1\right). \quad (\text{B16})$$

The implication of equation (B14) is that the optimal portfolio of the managers is again affine in the state variables. The CIO selects the optimal constant proportions strategy and the constant benchmarks, β_1 and β_2 , to optimize the unconditional value function, that is, equation (B11). This yields

$$x_C^{\text{Implied}} = \begin{bmatrix} x_{1C}(\zeta_{01}^B + \zeta_{11}^BX) \\ x_{2C}(\zeta_{02}^B + \zeta_{12}^BX) \end{bmatrix} = \zeta_0^{\text{Implied}} + \zeta_1^{\text{Implied}}X, \quad (\text{B17})$$

where ζ_{0i}^B and ζ_{1i}^B obviously depend on the choice of the benchmark. The conditional value function is exponentially quadratic as in equation (B2), with $\zeta_0(\tau) = \zeta_0^{\text{Implied}}$ and $\zeta_1(\tau) = \zeta_1^{\text{Implied}}$. The coefficients satisfy the ODEs given in equation (B6). To solve for the strategic allocation and the performance benchmark, we evaluate the unconditional expectation of the conditional value function using Lemma 1. We then optimize numerically.

Appendix C: Risk Constraints

We derive in this section the optimal allocations of the asset managers in the presence of either relative or absolute risk constraints as defined in Section III.C. We assume that investment opportunities are constant.

For the case with absolute risk constraints, the optimization problem of asset manager i can be simplified to

$$\max_{x_i^{NB} \in \mathcal{A}_i} \left(x_i^{NB'} \Sigma_i \Lambda + r - \frac{\gamma}{2} x_i^{NB'} \Sigma_i \Sigma_i' x_i^{NB} \right). \quad (C1)$$

and the set $\mathcal{A}_i$ is given by $\mathcal{A}_i = (x \mid x' \iota = 1, \sqrt{x' \Sigma_i \Sigma_i' x} \leq \phi_{Ai})$. Consequently, the Kuhn–Tucker FOCs are

$$0 = \Sigma_i \Lambda - \gamma(1 + \xi_1) \Sigma_i \Sigma_i' x_i^{NB} - \xi_1 \iota \quad (C2)$$

$$1 = x_i^{NB'} \iota, \phi_{Ai}^2 \geq x_i^{NB'} \Sigma_i \Sigma_i' x_i^{NB}, \xi_2 \geq 0 \quad (C3)$$

$$0 = \xi_2 (\phi_{Ai}^2 - x_i^{NB'} \Sigma_i \Sigma_i' x_i^{NB}), \quad (C4)$$

with ξ_1 and ξ_2 denoting the Kuhn–Tucker multipliers. In fact, ξ_2 is the multiplier for the risk constraint scaled by a factor $\gamma/2$ to simplify the interpretation. If the risk constraint is not binding, the managers' optimal portfolio is as derived in Section I.C. Otherwise, the absolute risk constraint binds and the optimal portfolio is given by the solution to equation (C2) for $\xi_2 > 0$ so that the risk constraint holds with equality. This results immediately in the optimal portfolio given in equation (38).

When the asset managers have to satisfy relative risk constraints, their objective is

$$\max_{x_i^B \in \mathcal{B}_i} \left(x_i^{B'} \Sigma_i \Lambda + \beta_i' \Sigma_i [\Sigma_i \beta_i - \Lambda - \Sigma_i' x_i^B] - \frac{\gamma}{2} (x_i^{B'} \Sigma_i - \beta_i' \Sigma_i) (x_i^{B'} \Sigma_i - \beta_i' \Sigma_i)' \right), \quad (C5)$$

where the set $\mathcal{B}_i$ is given by $\mathcal{B}_i = (x \mid x' \iota = 1, \sqrt{(x - \beta_i)' \Sigma_i \Sigma_i' (x - \beta_i)} \leq \phi_{Ri})$.

The FOCs are given by

$$0 = \Sigma_i (\Lambda - \Sigma_i' \beta_i) - \gamma(1 + \xi_1) \Sigma_i \Sigma_i' (x_i^B - \beta_i) - \xi_1 \iota, \quad (C6)$$

$$1 = x_i^{B'} \iota, \phi_{Ri}^2 \geq (x_i^B - \beta_i)' \Sigma_i \Sigma_i' (x_i^B - \beta_i), \xi_2 \geq 0, \quad (C7)$$

$$0 = \xi_2 \left(\phi_{Ri}^2 - (x_i^B - \beta_i)' \Sigma_i \Sigma_i' (x_i^B - \beta_i) \right), \quad (C8)$$

where ξ_1 and ξ_2 indicate the Kuhn–Tucker multipliers. Again, if the relative risk constraint is not binding, the optimal portfolio of Section I.D. prevails. Otherwise, the optimal strategy of manager i is given by the solution to equation (C6) with $\xi_2 > 0$ so that the relative risk constraint is satisfied with equality. This implies the strategy given in equation (39).

Finally, suppose that the benchmark is designed on the basis of a higher risk aversion level, say $\tilde{\gamma}$, than the manager's risk aversion, denoted by γ . In this case, the (relative) risk of the manager's portfolio will exceed the (relative) risk that would correspond to a manager with risk aversion level $\tilde{\gamma}$. If the risk limit is constructed for a manager with risk aversion $\tilde{\gamma}$, then the relative risk constraint will bind for the manager with risk aversion γ . This induces an effective increase in the manager's risk aversion from γ to $\tilde{\gamma}$. To show this, note that the difference between the optimal portfolio of the manager, who has a

risk aversion γ , and the benchmark weights, which are designed for a manager with risk aversion $\tilde{\gamma}$, is given by

$$x^B(\gamma, \beta(\tilde{\gamma})) - \beta(\tilde{\gamma}) = \frac{\tilde{\gamma}}{\tilde{\gamma} - 1} \frac{1}{\gamma} \left\{ x_i - x_i^C (x_i^{C'})^{-1} + (1 - \iota' x_i) x_i^{MV} \right\}. \quad (C9)$$

In this expression, $x^B(\gamma, \beta(\tilde{\gamma}))$ denotes the optimal portfolio choice when the investor has a coefficient of relative risk aversion γ , but is evaluated relative to a benchmark, $\beta(\tilde{\gamma})$, which is based on $\tilde{\gamma}$. This immediately implies for the relative risk of the manager's portfolio:

$$\begin{aligned} & (x^B(\gamma, \beta(\tilde{\gamma})) - \beta(\tilde{\gamma}))' (\Sigma_i \Sigma_i') (x^B(\gamma, \beta(\tilde{\gamma})) - \beta(\tilde{\gamma})) \\ &= \left(\frac{\tilde{\gamma}}{\gamma} \right)^2 (x^B(\tilde{\gamma}, \beta(\tilde{\gamma})) - \beta(\tilde{\gamma}))' (\Sigma_i \Sigma_i') (x^B(\tilde{\gamma}, \beta(\tilde{\gamma})) - \beta(\tilde{\gamma})), \end{aligned} \quad (C10)$$

that is, the relative risk of a more aggressive manager under a benchmark designed for a more conservative manager is larger than when the more conservative manager implements the strategy, since $\tilde{\gamma} > \gamma$. This implies that when the risk constraint is satisfied with equality for a manager with risk aversion $\tilde{\gamma}$, an unconstrained manager with risk aversion γ will implement a strategy that exceeds the relative risk limit. Consequently, the risk constraint on the basis of which the benchmark is designed will be binding and induces an effective increase in the manager's risk aversion from γ to $\tilde{\gamma}$.

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