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Price Convexity and Skewness

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ABSTRACT

This paper develops a model in which investors who are prohibited from short selling agree to disagree on the precision of a publicly observed signal. The model implies that the equilibrium price is a convex function of the public signal. The model predicts that (1) the stock price reacts more to good news than to bad news; (2) the skewness of stock returns is positively correlated with contemporaneous returns, but negatively correlated with lagged returns; (3) short sale constraints increase rather than decrease skewness; and (4) disagreement about information precision increases skewness. Empirical tests conducted find supportive evidence for all these predictions.

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The following example illustrates the basic idea. Consider a stock market in which short sales are not allowed. Investors initially agree on the value of a stock but disagree on the precision of a publicly observed signal (for example, an earning announcement). In particular, "high precision" investors think that the signal's precision is higher than "low precision" investors do. Accordingly, high precision investors adjust their valuation more in response to this signal. If the signal is positive, high precision investors will value the stock higher than low precision investors. The latter may wish to sell the stock short but end up being crowded out of the market since short selling is not allowed. In this case the market reacts to the positive signal through the reaction of high precision investors. On the other hand, if the signal is negative, high precision investors may be crowded out in which case the market reacts to the negative signal through the reaction of low precision investors. Therefore, the market

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reacts more to a positive signal than to a negative signal, resulting in a price that is a convex function of information.

Central to the model is the assumption that investors agree to disagree on the precision of the public signal. This assumption is motivated by the observation that people may react to the same signal to a different extent. For example, after earnings announcements analysts adjust their estimates by different amounts, which suggests that the analysts disagree on how to interpret this public signal. One way to capture this heterogeneity is to assume that analysts hold different opinions about the precision of the public information. This assumption does not lose generality if we consider other types of analyst heterogeneity. First, analysts may disagree in their preannouncement estimates. But if analysts update their beliefs in a Bayesian way, disagreement in prior means (expectations) cannot explain why they update their estimates by different amounts. Second, analysts' confidence in their own preannouncement beliefs may also differ. Note, however, that modeling heterogeneous prior confidence and modeling heterogeneous confidence in a new public signal are equivalent, as saying that one is more confident in her prior beliefs is equivalent to saying that she is less confident in the new signal. If we were to assume instead that investors have heterogeneous confidence in their priors but they agree on the precision of the new signal, the model leads to the same set of results. Finally, disagreement may also exist with respect to how significant the news is. For example, analysts may disagree about whether the announcement represents changes in long-run growth trends or just seasonal fluctuations. This dimension of disagreement also results in different reactions to the signal and can be captured by assuming heterogeneous precisions.

A second key assumption of the model consists of short sale constraints. This second assumption serves the purpose of crowding potential short sellers out of the market, which can be justified by the cost, risk, and restrictions associated with short selling.[1] Agreement to disagree on a common signal's precision represents a new dimension of heterogeneous beliefs that has not been sufficiently investigated in the literature. By focusing on this new dimension of heterogeneity and imposing short sale constraints, this paper's contributions are twofold.

First, it offers a new result on how heterogeneous beliefs and short sale constraints jointly impact asset prices. It has long been recognized that in the presence of heterogeneous beliefs and short sale constraints, risky assets are held by the most optimistic investors (Miller (1977)). Furthermore, the opportunity to resell an asset at a higher price can create speculative bubbles (Harrison and Kreps (1978), Morris (1996), Scheinkman and Xiong (2003), Hong, Scheinkman, and Xiong (2006)).[2] By examining a different dimension of heterogeneous beliefs, namely, agreement to disagree on the precision of a common signal, we

[1] For justifications of short sale constraints as a modeling assumption, see Scheinkman and Xiong (2003).

[2] Different opinions also exist. Lintner (1969) and Merton (1987) suggest that short sale constraints decrease asset prices because short sale constraints cause market segmentation, which increases risk. Jarrow (1980) shows that short sale constraints can both increase and decrease

obtain the novel result that asset prices react asymmetrically to good and bad news. Thus, price changes are asymmetric on the up and down sides. Empirically, we find that stock prices react to a positive earnings surprise more than to a negative surprise of an equal size.

Second, this paper produces new insights into the determination of skewness. Because the equilibrium price is a convex function of information, and a convex transformation skews a distribution to the right, market returns should be more positively skewed when the price function is more convex. In this model, price convexity arises from disagreement over information precision and short sale constraints, thus skewness should increase in both the intensity of disagreement and the cost of short selling. Empirical testing finds confirmative evidence. In particular, skewness increases with trading volume, which proxies for disagreement,[3] and decreases with stock size, institutional ownership, and ownership breadth, which inversely proxy for short selling costs.[4] We also find that controlling for institutional ownership and ownership breadth removes the effect of size on skewness, suggesting that stock size correlates with skewness though their common correlation with the two ownership variables.

In addition, the model predicts that skewness is positively correlated with contemporaneous returns but negatively correlated with lagged returns. The positive contemporaneous correlation is expected because price convexity simultaneously raises skewness and average returns. The negative lagged correlation is expected because corrections for earlier over- and under-reactions have an inverse effect on price convexity and in turn on skewness. The joint dynamics of returns and skewness also find empirical support in our tests.

Note that these predictions do not contradict the existing literature on negative skewness. The predictions above indicate that skewness should be more positive for some stocks and less positive for other stocks—nothing is asserted about whether skewness should be positive or negative. If the sign of skewness is to be determined, other skewness factors will have to be exclusively taken into consideration,[5] which is beyond the scope of this paper. However, existing evidence on the skewness of individual stock returns is mixed. Although market indices tend to be negatively skewed, the skewness of an individual stock is often positive (Harvey and Siddique (1999, 2000), Chen et al. (2001)).

asset prices, but they can only increase asset prices if people agree on the covariance of the assets. Diamond and Verrecchia (1987) introduce rational expectations into the debate and show that short sale constraints do not necessarily decrease asset prices if pessimistic opinions are already taken into consideration by optimistic investors.

[3] See Varian (1989), Harris and Raviv (1993), Kandel and Pearson (1995), Odean (1998), Chen, Hong, and Stein (2001), and Hong and Stein (2003) for justifications and evidence that volume proxies for disagreement.

[4] See D'Avolio (2002) and Geczy, Musto, and Reed (2002) for detailed discussions on the specialness of stocks.

[5] Proposed skewness factors include the leverage effect (Black (1976), Christie (1982)), volatility feedback (Pindyck (1984), French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992)), stochastic bubbles (Blanchard and Watson (1982)), short sale constraints (Diamond and Verrecchia (1987), Hong and Stein (2003)), negative news threshold (Ekholm and Pasternack (2005)), and state-switching (Veronesi (1999)).

By investigating the joint effects of short sale constraints and heterogeneous beliefs on skewness, this paper joins the renewed interest on skewness in the literature.[6] Diamond and Verrecchia (1987) argue that short sale constraints reduce the speed with which prices adjust private information, especially bad news, leading to more negative skewness of stock returns. Hong and Stein (2003) develop a theory to explain market crashes based on differences of opinions and short sale constraints, for which Chen et al. (2001) find supportive evidence. This paper finds that short sale constraints, when combined with heterogeneous beliefs on the precision of public signals, increase rather than decrease skewness of individual stock returns.[7] The empirical setup of this paper differs from that of Chen et al. (2001) in that their paper tries to predict *future* skewness conditional on currently available information, whereas this paper is concerned with the correlations between *contemporaneous* skewness and contemporaneous and lagged information. Furthermore, this paper produces implications about the joint dynamics of returns and skewness that are also empirically confirmed. Finally, the predictions of this paper help explain why large stocks tend to be more negatively skewed. Earlier research (Harvey and Siddique (1999, 2000) and Chen et al. (2001)) observes that larger stocks tend to be more negatively skewed, but does not provide further discussion on this issue. According to our model, this can be attributed to the fact that larger stocks are easier to sell short, leading to less convex price functions.

The rest of this paper proceeds as follows. Section I sets up the model and Section II solves the equilibrium. Section III explains the crowding out mechanism and its effect on the average confidence of market participants. Section IV shows that asset prices are convex functions of information due to the crowding out effect and that this convexity leads to testable predictions about both price reactions to earnings surprises and skewness of market returns. Section V empirically tests these predictions. Section VI closes the paper with a brief summary and a discussion about directions for future research. All proofs are presented in the Appendix.

I. The Model

Consider a competitive market for two assets, namely, a risky asset (the "stock") and a riskless bond that has infinite supply and demand at the gross interest rate R. There are three periods, 0, 1, and 2. At time 0, investors of exponential utility, $U(w) = -e^{-aw}$, enter the market each with a unit endowment of the risky asset. At time 1, a signal about the payoff of the risky asset is publicly observed. At time 2 the payoff is realized. Consumption happens only at time 2.

[6] See, for example, Diamond and Verrecchia (1987), Veronesi (1999), Chen et al. (2001), Harvey and Siddique (1999, 2000), Cao, Coval, and Hirshleifer (2002), and Hong and Stein (2003).

[7] After writing the first draft of this paper, I become acquainted with the work of Chang and Yu (2004) and Bris, Goetzmann, and Zhu (2004). Both papers find evidence that short sale constraints increase rather than decrease skewness of individual stock returns. These findings support the model of this paper.

The time 1 signal s equals the asset's payoff x plus noise ε:

$$s = x + \varepsilon. \tag{1}$$

The random variables x and ε follow the independent normal distribution $x \sim N(\mu_x, \tau_x^{-1})$ and $\varepsilon \sim N(0, \tau_\varepsilon^{-1})$, where $\tau_x = 1/\sigma_x^2$ and $\tau_\varepsilon = 1/\sigma_\varepsilon^2$ are the precisions of x and ε, respectively. Investors agree over the distribution of x but disagree over the distribution of ε. More precisely, they disagree over its precision τ_ε:

ASSUMPTION 1: *Investors disagree over the precision of* ε.

Accordingly, investors are classified into low precision traders, τ_L, and high precision traders, τ_H, $\tau_L < \tau_H$, where τ_L (τ_H) traders believe the precision is $\tau_L(\tau_H)$. A proportion λ of investors are τ_L traders; the rest are τ_H traders.[8] By definition, τ_L traders associate lower quality with the signal than τ_H traders do.

Disagreement over information precision is a direct application of the idea of "difference of opinions" (Harris and Raviv (1993), Kandel and Pearson (1995)) to information precision. One possible origin of this disagreement is noise. Black (1986) forcefully argues that the world we are attempting to model is full of noise. Sometimes people take information as noise and other times people take noise as information. It is implausible that people always agree on how to differentiate information from noise. Kandel and Pearson (1995) provide evidence that investors interpret public announcements differently. Ekholm (2006 p. 127) finds that different types of investors respond differently to public information and concludes that "differences in investor overconfidence emerge as the strongest candidate [explanation]."

Our second key assumption is that short selling is prohibited. Formally,

ASSUMPTION 2: *Short selling of the risky asset is prohibited.*

This assumption serves the purpose of crowding potential short sellers out of the market, and is justified by the cost, risk, and restrictions on equity short selling. If we were to assume that short selling is possible but incurs higher cost and risk, our results still hold qualitatively. For simplicity, we assume that short selling is prohibited.[9]

It is useful to compare this model to that of Hong et al. (2006). Following Scheinkman and Xiong (2003), Hong et al. (2006) assume that two groups of investors observe private signals and are overconfident in their own private signals. In contrast, we assume that all investors observe a common public

[8] It can be further assumed that $\lambda\tau_L + (1-\lambda)\tau_H = \tau_\varepsilon$, that is, the average belief about information precision is correct. In that case the "average" investor neither overestimates nor underestimates the information precision, and thus the average investor can be considered rational in the sense of Muth (1961). This assumption is not necessary for the result of this paper.

[9] Notice that the expectation of an asset's payoff can be negative if the signal is sufficiently negative. To avoid dealing with conditional distributions, we assume unlimited liability and allow an asset's price to be negative. None of our results depends on this simplifying assumption.

signal but attach different precisions to the public signal. Furthermore, while they design their model to study the effect of short sale constraints on speculative bubbles, we construct our model to study the joint effects of short sale constraints and heterogeneous beliefs about information precision on the skewness of market returns.

II. The Equilibrium

The competitive equilibrium is a set of price and demand functions under which (1) traders maximize their expected utility, and (2) the market clears.

Investors update their beliefs when new information arrives, as summarized in Lemma 1.

LEMMA 1: *The posterior precision and mean of type $\theta (\theta = L, H)$ traders are, respectively,*

$$\hat{\tau}_\theta = \tau_x + \tau_\theta, \tag{2}$$

$$\hat{\mu}_\theta = \frac{\tau_x}{\hat{\tau}_\theta}\mu_x + \frac{\tau_\theta}{\hat{\tau}_\theta}s. \tag{3}$$

Equation (2) says that the posterior precision of low (high) precision investors is also lower (higher). Since posterior precision measures how confident one is in his posterior beliefs, low precision traders are less confident than high precision investors after the signal is observed. In other words, the confidence in the signal transforms into confidence in posterior beliefs. For this reason, we also refer to τ_L (τ_H) investors as less (more) confident investors. Equation (3) says that when updating their beliefs, τ_L traders put less weight on the signal and more weight on their prior beliefs than do τ_H traders. Intuitively, those who think the information is of higher quality weight the information more.

Because investors' current decisions depend on future information, we employ backward induction. The rest of this section derives the equilibrium backward from time 2 to time 0. At time 2, the payoff is realized. Thus, there is no uncertainty about the payoff of the asset. There is no Pareto-improving trading. And the shadow price of the asset is simply the realized payoff.

At time 1, the decision problem is essentially a static one because there are no future chances for trading. Type θ ($\theta = L, H$) traders' optimization problem is

$$J_{1,\theta}(W_{1,\theta}, I_1) = U_{1,\theta}V_{1,\theta}, \tag{4}$$

where

$$U_{1,\theta} \equiv -\exp(-aRW_{1,\theta}), \tag{5}$$

$$V_{1,\theta} \equiv \mathrm{Max} E_{1,\theta}[\exp(-a y_{1,\theta}(x - RP_1))]. \tag{6}$$

In equations (4)–(6), $J_{1,\theta}(W_{1,\theta}, I_1)$ is type θ traders' value function at time 1, $W_{1,\theta}$ is type θ traders' wealth at time 1, taking the equilibrium price as given, and $y_{1,\theta}$ is type θ traders' asset demand when the price is P_1. The value function at time 1 consists of two parts, the risk-free part $U_{1,\theta}$, which is the expected utility if all wealth is invested in the risk-free bond, and the risky part $V_{1,\theta}$, which is the maximum expected gains in utility from investing in the risky asset.

For expositional convenience, we first present the time 1 equilibrium for the case of no short sale constraints.

PROPOSITION 1: *In the absence of short sale constraints, the equilibrium at time 1 is,*

$$P_{1,m} = \frac{\lambda\hat{\tau}_L\hat{\mu}_L + (1-\lambda)\hat{\tau}_H\hat{\mu}_H - a}{R(\lambda\hat{\tau}_L + (1-\lambda)\hat{\tau}_H)}, \tag{7}$$

$$y_{1,L} = \frac{b(1-\lambda)\hat{\tau}_L\hat{\tau}_H(\hat{\mu}_L - \hat{\mu}_H) + \hat{\tau}_L}{\lambda\hat{\tau}_L + (1-\lambda)\hat{\tau}_H}, \tag{8}$$

$$y_{1,H} = \frac{b\lambda\hat{\tau}_L\hat{\tau}_H(\hat{\mu}_H - \hat{\mu}_L) + \hat{\tau}_H}{\lambda\hat{\tau}_L + (1-\lambda)\hat{\tau}_H}, \tag{9}$$

where $b = \frac{1}{a}$.

Proposition 1 says that in the absence of short sale constraints, the equilibrium price $P_{1,m}$ is given by the weighted average of the opinions of both types of traders, and the optimal asset demands, $y_{1,L}$ and $y_{1,H}$, are determined by the difference of opinions. This is consistent with the result in Varian (1989). Notice that $y_{1,L}$ and $y_{1,H}$ in equation (8) and (9) could be negative, if the difference of opinions is sufficiently large. Yet under short sale constraints this could not happen.

Proposition 2 examines the time 1 equilibrium under short sale constraints.

PROPOSITION 2: *Under short sale constraints, the equilibrium at time 1 is,*

(a) If condition (a): $\hat{\mu}_H - \hat{\mu}_L > \frac{a\hat{\sigma}_H^2}{1-\lambda}$ *holds, only* τ_H *traders hold the risky asset;* τ_L *traders sell all their risky assets. The equilibrium price is* $P_{1,H} = \frac{1}{R}(\hat{\mu}_H - \frac{a\hat{\sigma}_H^2}{1-\lambda})$.

(b) If condition (b): $\hat{\mu}_L - \hat{\mu}_H > \frac{a\hat{\sigma}_L^2}{\lambda}$ *holds, only* τ_L *traders hold the risky asset;* τ_H *traders sell all their risky assets. The equilibrium price is* $P_{1,L} = \frac{1}{R}(\hat{\mu}_L - \frac{a\hat{\sigma}_L^2}{\lambda})$.

(c) If conditions (a) and (b) do not hold, the equilibrium in Proposition 1 is still valid.

The conditions (a) and (b) in Proposition 2 can be rewritten as

$$\text{(a}'\text{)} \qquad s_1 > s_1^h \equiv \mu_x + \frac{a\hat{\tau}_L}{(1-\lambda)(\tau_H - \tau_L)\tau_x}, \tag{10}$$

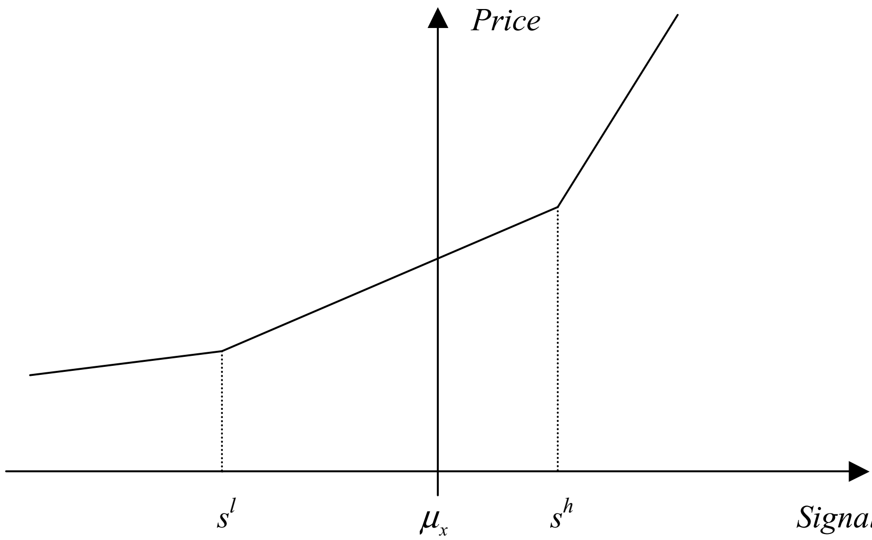

Figure 1. Price convex in signal. The market reacts to signals through the reaction of market participants. When the signal is sufficiently positive (negative), only high (low) precision investors participate in the market. Since high precision investors react more to the signal than low precision investors, price reactions are stronger to positive than to negative signals.

$$\text{(b}'\text{)} \qquad s_1 < s_1^l \equiv \mu_x - \frac{a\hat{\tau}_H}{\lambda(\tau_H - \tau_L)\tau_x}. \tag{11}$$

The two critical values, s^l ands^h, together divide the real line into three parts: (i) $s \leq s^l$, in which case τ_H traders are crowded out; (ii) $s^l < s < s^h$, in which case no traders are crowded out; and (iii) $s \geq s^h$, in which case τ_L traders are crowded out. The equilibrium price functions in these three cases are $P_{1,L}$, $P_{1,m}$, and $P_{1,H}$, respectively. Figure 1 illustrates this equilibrium.

The time 0 equilibrium can be derived from that at time 1 analogously. However, due to the segmented price function at time 1, a closed-form solution does not exist. One way to proceed is to use numerical methods to calculate the equilibrium at time 0. Alternatively, if we assume that investors are myopic, a closed-form solution can be derived. The myopia assumption simplifies analysis by eliminating hedging demands.[10] The equilibrium price at time 0 is a constant that depends on investors' expectations of future dynamics. None of our results depends on this constant, thus we proceed without deriving it.

III. Selective Market Participation and Average Market Confidence

Proposition 2 above establishes a "crowding out" mechanism whereby investors of different confidence levels selectively participate in the market: When

[10] See Hong et al. (2006) for a demonstration of the derivation under the assumption of myopic investors.

the signal is bigger (smaller) than the critical value s^h (s^l), τ_L (τ_H) traders are crowded out. We summarize the properties of this crowding mechanism in the following proposition.

PROPOSITION 3: *(1) The average confidence of market participants increases with the asset's price. (2) For each type of trader, the likelihood of participation increases with their proportion in the investor population. (3) High precision investors are more likely to participate.*

Property (1) claims that the average confidence of market participants is higher after price increases than after price decreases. A positive (negative) signal simultaneously increases (decreases) the asset price and crowds out less (more) confident traders. Therefore, the average confidence of market participants is higher (lower) after price gains (losses).[11] Property (2) amounts to a *majority effect*. It is easier for the majority group to crowd out the *minority* group than vice versa because each member in the majority group bears a smaller share of the gross risk if the minority group is crowded out, whereas each member in the minority group bears a larger share if the majority group is crowded out. Intuitively, opinions shared by more people are more likely to be reflected in the market. Property (3) states that more confident investors are more likely to participate in the market. Due to their higher confidence in the signal and consequently in their posterior beliefs, more confident investors perceive less risk and demand less of a risk premium. They therefore value the asset higher and demand more of the risky asset, and are less likely to be crowded out of the market.[12]

This third property also implies that the average confidence of market participants is higher than the average confidence of the average potential investor. Loosely speaking, those who think they have better knowledge and information are more confident and hence are more likely to participate. This can happen even if each individual has access to exactly the same information set because the investors can agree to disagree over the quality of commonly observed information. Thus market participants do not represent the average potential investor, but rather a self-selected group of people who think they are smart at playing the game. Surveys of investors are therefore likely to find that they tend to be over- rather than under-confident.

[11] I deliberately avoid using the term "market confidence" to avoid possible confusion. In the financial press, market confidence usually refers to investors' belief about the price level, that is, optimism. Thus, to say market confidence is higher means that the market is more optimistic. In this paper, confidence refers to the strength of the investors' belief. Statistically, confidence refers to the second-order moment while optimism refers to the first-order moment.

[12] In Figure 1, the lower critical value s^l is greater than the higher critical value s^h in absolute value, implying that high precision investors are less likely to be crowded out. It is also interesting to note that even if the signal equals the prior expectation μ_x, and thus contains no surprise, high precision investors still demand more of the risky asset than low precision investors. This is because although the signal contains zero surprise, it creates a difference in posterior investor confidence and therefore in the demand for a risk premium.

IV. Price Convexity and Skewness of Market Returns

A. Price Convexity

The crowding out mechanism established by Proposition 2 implies that the price function at time 1 is convex. This is because the market overreacts to good news and underreacts to bad news. Intuitively, the market overreacts to positive signals because the market reactions correspond to those who have more confidence in the signal, as those who have less confidence in the signal are crowded out; similarly, the market underreacts to negative signals because the market reactions correspond to those who have less confidence in the signal. Mathematically, when $s > s^h$ we have $P_{1,H} > P_{1,m}$, and when $s < s^l$ we have $P_{1,L} > P_{1,m}$. Graphically, the price function is steeper when the signal is larger than the upper critical value and flatter when the signal is smaller than the lower critical value (Figure 1).[13]

This price convexity implies that, conditional on an equal amount of surprises in good and bad news, the market's reaction to good news is stronger than its reaction to bad news, and thus, if both the amount of surprises and price reactions can be observed, this asymmetry should obtain in price reactions. Firms' earnings announcements provide a scenario to test this conjecture. The prediction is that for the same amount of surprises in earnings announcements, price reactions should be stronger for positive surprises than for negative surprises.[14] To measure earnings surprises we use analyst forecasts to proxy for preannouncement market opinions. Section V provides empirical evidence for this prediction.

Note that the convex price reaction at time 1 is not the whole story. Given there exist over- or under-reaction, a natural question that arises is: What will happen when the over- or under-reaction is corrected? That is, is the multiple period price function also convex?

Let us start with a close look at what will happen in the model at time 2. Recall that at time 2, the payoff is realized. In this case no trading happens and the shadow price of the asset equals the realized payoff x. Next, the payoff from time 1 to time 2 is simply $x - p_1$;[15] since p_1 is convex and x is normally distributed, $x - p_1$ has to be concave. Finally, the payoff from time 0 to time 2,

[13] The kinks in the price function come from the assumption that the asset's payoff is realized at time 2 and thus there are no more chances for trading. For assets living for more than two periods, new information may keep flowing in before liquidation and investors may have multiple chances to trade. In that case investors will hedge against future price movements. This hedge motivation will smooth the kinks in the price function, making it a smooth convex curve. In addition, investors could be of multiple types, thus the price function may have multiple instead of two kinks. The number of kinks increases with the number of investor types, which also makes the price function smoother.

[14] The author thanks an anonymous referee and Ho-Mou Wu for pointing this out. The author is also grateful to the anonymous referee for the suggestions on the joint dynamics of return and skewness.

[15] Here we are talking about the payoff in dollars. Since it can be transformed into a rate of return if divided by the earlier of two prices, the intuition derived here will go through, especially for high frequency returns such as daily returns, for which the rate of return is usually small: When the rate of return is small, it is insensitive to the base period price function.

$x - p_0$, is normally distributed. It therefore follows that the price concavity at time 2 exactly counteracts the price convexity at time 1, leaving the payoff from time 0 to time 2 symmetrically distributed.

This result may appear uninteresting at first sight. However, it is a useful starting point toward a more interesting observation. Notice that nonconvexity of the overall payoff, $x - p_0$, is imposed instead of derived. In particular, it is imposed by the assumption that at time 2 the payoff is realized, with investors effectively observing a conclusive signal. This conclusive signal contains no noise at all. Consequently, all earlier over- and under-reactions have to be corrected both *completely* and *instantly*. In other words, the correction speed is infinite, which makes the degree of concavity at time 2 exactly equal to the degree of convexity at time 1. In addition, this conclusive signal allows no opportunity for disagreement. This "non-disagreement" property is directly responsible for the no-trading result at time 2.

Stepping back from these two extreme restrictions, it is clear that the multiple period price function is still convex. Usually investors can only observe noisy signals. Such noisy signals may partially and gradually (as opposed to completely and instantly) correct earlier over- and under-reactions. How investors interpret new information in light of old information and previously held opinions is a complex issue about which little is known. The difficulty in interpreting asset pricing information makes such corrections even more difficult and time consuming. Existence of stubborn beliefs suggests that the correction process is probably very slow. In addition, noisy signals allow for disagreement, which creates more convexity when combined with short sale constraints as demonstrated in the time 1 equilibrium. Both partial correction and disagreement over each signal suggest that the multiple period price function is still convex. The discussion above also suggests that *noisiness* of information plays an important role in determining the degree of price convexity. The noisier the signals, the more convex the price function.

B. Implications for Skewness

Price convexity also has direct implications for the distribution of market returns.[16] A convex price function tends to make market returns skew to the right. To see this, assume for now that the signal is symmetrically distributed, so that positive and negative signals of an equal size are equally possible. Also assume for now that there exist no other factors affecting the skewness of market returns. Then, because the price function is steeper on the right-hand side,

[16] Another implication relates to the ex ante stock price level. If it is known ex ante that a signal will be observed and it will be interpreted differently by different investors, the ex ante expectation of the ex post price will be increased due to the well-known property of convex functions, whereby the expectation of a function's value is greater than the function's value at the expectation of its argument. Here, it means that a higher price expectation raises the equilibrium price before the signal actually comes. Therefore, the equilibrium price is higher when the information is foreseen than otherwise. In other words, the news (or even "rumor") that some news is coming increases the equilibrium price by raising the expectation of the future price. This is consistent with Wall Street saying "buy with rumors."

a positive signal increases the asset's price more than a negative signal of equal size decreases the asset's price. The asymmetry reflected in the rate of market returns is that the positive return is larger than the negative return in absolute value. Correspondingly, the right tail of market returns is long and thin while the left tail is short and fat. The skewness of such a distribution will be positive. Furthermore, when the price function is more convex, the right tail is longer and thinner while the left tail is shorter and fatter; again, the skewness will be more positive.

The above argument relates price convexity to skewness in the highly simplified case in which (1) the distribution of the signal is symmetric and (2) no other factors affect the skewness of market returns. Neither assumption is literally true. However, the relationship between price convexity and skewness does not depend on these assumptions. With respect to the first assumption, Ekholm and Pasternack (2005) argue that the distribution of new information might be negatively skewed, while Chen et al. (2001) suggest that signals could be positively skewed due to discretionary behavior of managers. With respect to the second assumption, several hypotheses have been proposed to explain the negative skewness observed in market returns. The "leverage effect" (Black (1976), Christie (1982), Bekaert and Wu (2000)) and "volatility feedback" (Pindyck (1984), French et al. (1987), Campbell and Hentschel (1992)) hypotheses explain negative skewness through asymmetric volatility; the "stochastic bubble" hypothesis (Blanchard and Watson (1982)) explains negative skewness through the popping of bubbles; the "short sale constraints" hypothesis (Diamond and Verrecchia (1987), Hong and Stein (2003)) explains negative skewness through the revelation of bad news hidden by short sale constraints.

Despite the above considerations, the connection between skewness and price convexity is still valid. Asymmetry in signals and other skewness factors affect the overall level of skewness, but not the relative skewness conditional on the degree of price convexity, as long as other skewness factors do not completely counteract the effect of price convexity on skewness. Complete counteraction requires the effect of other skewness factors to be perfectly negatively correlated with the effect of price convexity. This extreme case does not seem plausible unless we have pre-existent reasons to believe so.

C. Testable Hypotheses

The above discussion points to predictions about skewness conditional on current returns, lagged returns, and the degree of price convexity.[17] First, skewness

[17] Besides the predictions that follow, the discussion in IV.A also suggests that skewness of market returns decreases with the speed of correction for over- and under-reactions. Because we do not know how to measure the speed of correction, this conjecture is left for future exploration. However, faster correction implies more informative information and faster learning, which reduces the intensity of disagreement over each signal. Therefore, correction speed is not independent from disagreement intensity. In this sense, a test about disagreement intensity is also a test about correction speed.

should be positively correlated with contemporaneous returns. This is because price convexity simultaneously raises skewness and expected market returns. Price convexity raises expected market returns due to the well-known property of convex functions whereby the expectation of a function's value is greater than the function's value at the expectation of the function's arguments. Second, skewness should be negatively correlated with lagged returns. This should obtain due to the correction for earlier over- and under-reactions. In terms of probability, a higher return in an earlier period can be partly attributed to a more convex price function during that period. This more convex price function implies more space for correction and results in more price concavity in the current period and in turn more negatively skewed returns in the current period if the correction actually happens. Thus, completing this logic chain, current skewness should be negatively correlated with earlier returns. These predictions are summarized in the following hypothesis.

Hypothesis 1: Skewness is positively correlated with contemporaneous return, and negatively correlated with lagged returns.

This hypothesis concerns the joint dynamics of skewness and returns. It has two parts. The first part relates to the over- and under-reactions to new signals. The second part relates to the corrections for earlier over- and under-reactions. If higher returns in the previous period are due to better news, they will not be corrected. Therefore, negative correlations between skewness and lagged returns, if confirmed, can be interpreted as evidence that higher returns in the earlier periods are at least partly due to the convexity of the price function. Our earlier discussion suggests that corrections for earlier over- and under-reactions might be a slow process. Thus, in empirical tests we should consider a long time window.

Turning to the relation between skewness to the degree of price convexity, the more convex the price function is, the more positive the skewness of market returns should be. In our model, price convexity is driven by both disagreement over information quality and short sale constraints: The price function is more convex when disagreement is more intense, or when short sale constraints are more binding. Therefore, skewness should increase with intensity of disagreement and with effectiveness of short sale constraints. Factors other than disagreement and short sale constraints that also interact with the crowding out mechanism to make price convexity more observable should also be positively correlated with skewness.

Existing studies offer fruitful results on the measurement of disagreement. Volume is generally considered to be a good proxy for disagreement. For instance, Varian (1989), Harris and Raviv (1993), Kandel and Pearson (1995), Odean (1998), and Hong and Stein (2003) all suggest that trading volume can be used as a measure of disagreement. Intuitively, when investors disagree more on how to interpret new signals, their posterior valuations differ more, leading to heavier trading volume.[18] We summarize our prediction on the relation between skewness and disagreement-based price convexity as measured by trading volume as follows:

Hypothesis 2: Skewness increases with trading volume.

Existing studies also offer fruitful results on the measurement of the costs of short selling. D'Avolio (2002) finds that larger stocks and stocks with higher institutional ownership are easier to sell short. According to our model, such stocks should have more negatively skewed returns. A third and closely related factor, ownership breadth, should also be negatively correlated with short selling costs because a stock owned by more investors is easy to borrow due to the following considerations. First, the "search" cost (Duffie et al. (2002)) in locating a security lender is lower. Second, broader ownership potentially implies more competition among the lenders and this competition tends to squeeze the lenders' asking price. Furthermore, broader ownership also implies more diversified and continuously distributed opinions. In this case crowding out of some investors may still leave investors of "adjacent" opinions in the market. Prices should therefore be less significantly influenced in this case than in the case in which jumps in opinions exist due to lack of interest in the stock. Thus, ceteris paribus, stocks of broader ownership should have a less convex price function thus less positive skewness. We summarize these predictions on the relation between skewness and the costs of short selling in the following hypothesis:

Hypothesis 3: Skewness decreases with stock size, institutional ownership, and ownership breadth.

Note that the three variables, size, institutional ownership, and ownership breadth, are not independent. Larger stocks are more likely to be owned by institutional investors, larger stocks tend to have more institutional investors, and thus broader ownership and larger institutional ownership can at least be plausibly associated with broader ownership.[19] The correlations among these three variables, together with the existing evidence that larger stocks are more negatively skewed, suggest the possibility that institutional ownership and

[18] Seminar participants have pointed out that trading volume is a noisy proxy for disagreement, especially for disagreement about public information precision, as trading volume is also affected by factors such as private information, portfolio rebalancing, liquidity, and transaction costs. To what extent detrending methodologies help isolate trading volume changes due to disagreement is not clear. Another often used proxy for disagreement, dispersion of analyst forecasts, is even worse for the purpose of this paper. Forecast dispersion measures how analysts disagree about ex ante expectations of earnings, not the extent to which analysts (not even the investors) interpret public information differently. In addition, forecasts are only available at scattered times, making them inappropriate for testing skewness. At present, trading volume remains the best proxy I can find. Thus the evidence with respect to trading volume should be interpreted with caution. The test of the volume–skewness correlation is a joint test of the disagreement–convexity–skewness relationship and the volume–disagreement relationship.

[19] Chen, Hong, and Stein (2002) find that the correlation between stock size and ownership breadth is 0.691. The positive correlation between ownership breadth and institutional ownership is especially plausible when institutional stock holdings are used to calculate ownership breadth, as is the case in our empirical study.

ownership breadth serve as the channel through which size is linked to skewness. That is, the effect of size on skewness may be indirectly obtained from its correlation with institutional ownership and ownership breadth, both of which are directly correlated with short sales costs and thus have direct effects on skewness. If this is true, then the inclusion of institutional ownership and ownership breadth should reduce the explanatory power of size on skewness. Formally,

Hypothesis 4: Controlling for institutional ownership and ownership breadth reduces the negative correlation between skewness and size.

This hypothesis, if confirmed, provides an explanation for the phenomenon recently documented by Harvey and Siddique (2000) and Chen et al. (2001) that larger stocks tend to be more negatively skewed. In these papers, the reason for this result is not clear. This phenomenon is even more puzzling from the perspective that larger stocks are usually considered less risky, while more negative skewness is associated with greater risk. The model in this paper suggests that returns of larger stocks tend to be more negatively skewed because larger stocks are easier to sell short and thus have less convex price functions. In this case, more negative skewness of larger stocks arises due to a less distorted reaction of stock prices to information, that is, more negative skewness is not equivalent to more risks.

V. Empirical Evidence

A. Asymmetric Price Reactions

The model directly implies that the market price reacts more to good than to bad news. This property underlies all the predictions about skewness. To test this implication we need to identify appropriate reaction events and for each event we need to be able to measure (1) the size of the news surprise, good, or bad, and (2) the size of the price reaction. Neither measurement is straightforward. To measure surprises, one needs a measure of investors' prior beliefs, which is by no means readily available. To measure the price reaction, one needs to determine when a piece of information begins to affect the market, which is again not readily identifiable due to possible leakage of information.

Price reactions to earnings announcements probably serve as one of the best experiments for testing this implication. For these events we need to measure earnings announcements surprises and their related price reactions. Earnings surprises can be measured as the difference between average analyst forecasts and actual earnings. Price reactions can be measured as the postannouncement cumulative returns over selected time periods.[20]

[20] There is an extensive literature on postannouncement stock returns, usually referred to as the "post-announcement drift" literature. See Fama (1998) and Kothari (2001) for recent reviews. Most studies focus on long-run cumulative returns after the announcement. Less attention has been paid to short-term price reactions. I am aware of no existing evidence that compares price reactions to positive and negative earnings surprises of equal sizes.

Using the ZACKS historical data set (1982 to 2003) on analysts' earnings forecasts,[21] we calculate earnings surprises as the differences between announced earnings and the average of analyst forecasts. We employ quarterly rather than annual announcements because quarterly announcements convey relatively more novel information compared to annual announcements. In the calculation, we use each analyst's most recent forecast. Forecasts more than 30 days old are considered outdated and are excluded. We also exclude observations with very large positive or negative earnings (absolute value greater than 10 dollars), very large forecasts (absolute value greater than 10 dollars), or large forecast errors (forecasts that deviate from the announcement by more than 100% for earnings outside the [–10, 10] cent interval or by more than 10 cents for earnings in the [–10, 10] cent interval). From the remaining observations, we calculate the percentage deviation of the average analyst forecasts from actual announcements as a measure of earnings surprises. If an announcement event has less than three valid forecasts, it is excluded from the analysis as too few forecasts cast doubt on the accuracy of the average forecasts. The above truncations are conducted for the purpose of correcting data entry errors, rare events, and unreliable forecasts. While these truncation criteria are somewhat arbitrary, they represent a compromise between maintaining sample size and improving data accuracy. Our result is not sensitive to the truncation criteria, as we discuss below.

Price reactions are calculated as postannouncement cumulative returns over selected periods. We obtain daily stock returns from the Center for Research in Security Prices (CRSP) at the University of Chicago, which we use to calculate price reactions. Since we are mainly interested in short time reactions, we restrict our analysis to 5 trading days beginning with the announcement day. We also exclude very large reactions, which we define as a 1-day reaction greater than 10%, a 2-day reaction greater than 15%, or a 3-day reaction greater than 20%, as such large price changes are likely to be rare events that are driven chiefly by other important explanatory factors. Although it is possible that these large price changes can still be traced back to earnings surprises, the relationship could be much more complicated.

To compare price reactions to positive and negative surprises, earnings surprises in the range of [–50%, 50%] are grouped into 10 equal size intervals symmetric around zero. Average forecasts that deviate from the actual earnings by more than 50% cast doubt on the accuracy and reliability of the forecasts. Within each small interval we calculate the average postannouncement cumulative returns of 1–5 trading days. Table I presents the results.

The bottom row of Table I summarizes the data. Our final sample has 34,811 announcement events. The average earnings surprise of 0.6% is slightly positive with a relatively large standard deviation of 16%. Also notice that positive surprises occur more often than negative ones, especially in the neighborhood of zero, consistent with Degeorge, Patel, and Zechhauser (1999) and Durtschi and Easton (2005), who show that firms manage earnings to meet analyst

[21] The author thanks Jennifer Francis for making the data available.

Table I

Price Reactions to Earnings Surprises

Earnings surprises are calculated as the percentage deviations of actual announcements from average analyst forecasts. The data source is the Zacks historical database of analyst forecasts from 1982 to 2003. To be included in the analysis an announcement event must have at least three valid forecasts within 30 days of the announcement. A forecast is valid if it is the analyst's most recent forecast for the event, and if it does not deviate from the announcement by more than 100% for earnings outside the [−10, 10] cent interval or by more than 10 cents for earnings in the [−10, 10] cent interval. Announcement events with absolute earnings or absolute average forecasts greater than $10 are excluded. Extraordinary price reactions, defined as larger than 10% for 1-day reactions, or larger than 15% for 2-day reactions, or larger than 20% for 3-day reactions, are also excluded. Earnings surprises are grouped into 10 intervals of width 10% that are symmetric around zero. Price reactions are calculated as the postannouncement cumulative returns of 1–5 trading days. Daily stock returns are obtained from CRSP.

				Postannouncement Cumulative Returns (%)									
	Earnings Surprises (%)			1 Day		2 Days		3 Days		4 Days		5 Days	
Intervals	No. of Obs.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
[−50, 40)	737	−0.447	0.030	−0.30	3.15	−0.54	4.75	−0.70	5.47	−0.59	6.49	−0.71	7.17
[−40, 30)	988	−0.347	0.029	−0.52	3.34	−0.39	4.71	−0.42	5.47	−0.45	6.28	−0.35	7.02
[−30, 20)	1,534	−0.247	0.029	−0.27	3.21	−0.41	4.76	−0.31	5.64	−0.27	6.23	−0.18	6.74
[−20, 10)	2,952	−0.144	0.028	−0.33	3.12	−0.40	4.54	−0.36	5.40	−0.32	6.11	−0.25	6.65
[−10, 0)	9,776	−0.039	0.027	−0.08	2.93	−0.15	4.25	−0.14	4.92	−0.17	5.50	−0.05	6.00
(0, 10]	11,681	0.042	0.027	0.32	2.85	0.59	4.14	0.66	4.85	0.68	5.47	0.76	5.97
(10, 20]	3,912	0.143	0.029	0.64	3.03	1.03	4.35	1.11	5.16	1.12	5.77	1.22	6.32
(20, 30]	1,779	0.244	0.028	0.67	3.09	0.96	4.49	1.04	5.24	1.06	5.95	1.13	6.64
(30, 40]	876	0.346	0.028	0.80	3.19	1.06	4.64	1.04	5.50	1.05	6.24	1.04	6.75
(40, 50]	576	0.447	0.031	0.96	3.25	1.36	4.64	1.25	5.55	1.15	5.89	1.32	6.61
All	34,811	0.006	0.160	0.16	3.00	0.30	4.36	0.33	5.10	0.34	5.73	0.43	6.26

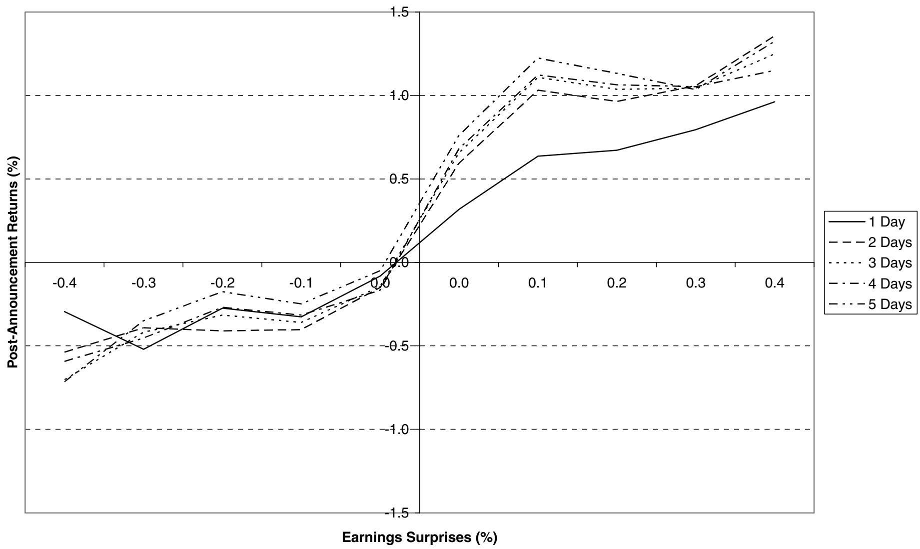

Figure 2. Price reactions to earnings surprises. This figure graphs the data in Table I. With the help of the 0.5% and −0.5% gridlines, it is clear that price reactions are stronger to positive surprises than to negative surprises of equal sizes.

forecasts. The average postannouncement returns for 1–5 days are 0.16%, 0.30%, 0.33%, 0.34%, and 0.43%, respectively. The standard deviations range from 3% to 6.26%. The average postannouncement returns tend to be slightly positive. But since the average earnings surprises also tend to be positive, it is not clear so far whether the positive postannouncement returns are due to "good" news or to asymmetric reactions.

The main body of Table I compares price reactions in corresponding positive and negative surprise intervals. It can be seen that, in absolute value, price reactions are always obviously higher on the positive side than on the negative side for corresponding intervals. This pattern is more readily obvious in Figure 2, which plots the data in Table I. With the help of 0.5% and –0.5% gridlines, Figure 2 illustrates that the right-hand side is always higher than the left-hand side in absolute value. Price reactions to negative earnings surprises seldom reach –0.5%, while reactions to positive surprises almost always exceed 0.5%. For surprises in the [–10%, 10%] interval, into which about two-thirds of the observations fall, the one-day average reaction is three times larger in response to positive surprises than to negative surprises. The average reactions are 0.32% and –0.08%, respectively. Furthermore, if we look at earnings surprises within the [–50%, 20%] range, a convex price pattern can be observed. These results lend support to our model's prediction of asymmetric price reactions.

The asymmetry in the price reactions to earnings surprises is robust to the various data selection criteria in our test. If we use earnings surprises in dollars in place of percentages, the reaction is still asymmetric. If we relax our data truncation criteria, the asymmetric pattern continues to hold, though the standard deviations are larger. For example, if forecasts during the past 90 days, instead of 30 days, are used, the pattern still obtains. We also try to increase the number of small intervals by decreasing the size of each interval. This makes the average surprises in corresponding positive and negative intervals very close. The asymmetry in price reactions continues to hold. We also try calculating returns 1 day before the announcements. Our results are robust to this modification.

The asymmetric price reactions in Table I and Figure 2 are not due to the asymmetry in earnings surprises documented by Degeorge et al. (1999). This is because we are comparing price reactions to positive and negative earnings surprises of equal size, thus the asymmetry of earnings surprises is already controlled for. In addition, although it is reasonable to expect that asymmetries in earnings surprises in particular and in information flows in general can affect the average skewness of market returns, it is not obvious how such asymmetries are related to the correlations between skewness and volume, institutional ownership and ownership breadth (discussed earlier and documented later).

Table I and Figure 2 also suggest that earnings surprises are obviously not the only factor that determines price reactions. Price reactions grouped solely on earnings surprises are widely dispersed as indicated by the large standard errors. If we do not attribute this solely to noise, factors other than earnings surprises that have a significant effect on price reactions must exist. In addition, some other patterns can also be observed. Price reactions are somewhat flat when earnings surprises exceed 20%; the flatness is less obvious on the negative side. Furthermore, the reactions are not only asymmetric but also nonlinear on the positive and negative sides of zero. What effects cause the nonlinearity, and through what channels, is not clear. Finally, although reactions to positive (negative) surprises tend to be positive (negative), this is not always the case. Although one can form conjectures about what factors might cause departures from this pattern, a comprehensive understanding of this relation is, to the best of my knowledge, absent. Since a systematic treatment of how prices react to earnings surprises is beyond the main theme of this paper, we leave this question to future research.[22]

[22] Earnings surprises and price reactions calculated as such may contain measurement error, as several assumptions are implicitly made. First, analyst forecasts are assumed to well represent investor opinions. This is true if analyst forecasts are well respected by investors or analysts do a good job of saying what investors want to hear. Neither is necessarily true. Second, surprises measured as such may not be real surprises. Since there are time lags between forecasts and announcements, market opinions may have already changed by the time announcements are released (even if forecasts well represent market opinions when the forecasts are made). For example, the market can infer earnings from economic statistics or research reports. Although analysts have access to these information sources, they may not update their forecasts even if their opinions are already updated. For example, too-frequent updating of forecasts may contaminate the analyst's reputation. In some cases it is also possible that earnings are leaked out before the announcement is made, in which case surprises calculated as such may contain spurious surprises that are already inferred by or leaked to the market and the market price may already reflect such information. Therefore, price reactions calculated from the announcement day (or the day before the announcement) may also be doubtful.

Nevertheless, the results in Table I and Figure 2 are encouraging. Price reactions are clearly stronger for positive earnings surprises than for negative ones of similar size, and the reactions display a pattern of convexity, although not a clear-cut one. Admittedly, this table is far from conclusive: The standard errors of price reactions are relatively large and the reaction pattern is not only asymmetric but also nonlinear on both sides of zero. We therefore interpret Table I and Figure 2 as an encouraging starting point. We now proceed to test the model's predictions about skewness.

B. Determination of Skewness

B.1. Data

The data for testing the skewness hypotheses come from two sources: CRSP and the CDA/Spectrum Institutional 13(f) Common Stock Holdings database. CRSP contains daily stock returns, prices, trading volume, and shares outstanding for individual stocks from July 3, 1962 to December 31, 2003. We include all NYSE stocks except observations with missing values. Following standard practice, we exclude ADRs, REITs, close-end funds, and other exotica, and we focus on ordinary common shares, for which our model is more applicable. We exclude NASDAQ stocks because the NASDAQ dealership structure makes its trading volume, one of our key analysis variables, incomparable to that of NYSE. Finally, we exclude AMEX stocks because AMEX includes mainly small stocks and trading volume is a noisier measure of disagreement for small stocks than for large stocks.[23]

Using CRSP data, we calculate annual returns, volatility, skewness, volume, and size. Using daily log returns we calculate volatility and skewness using 1 year of daily observations. We use log returns instead of simple returns because simple returns are obviously more positively skewed. Skewness is the third-order decentralized moment calculated as

$$E(x-\mu)^3/\sigma^3 = \sum_{i=1}^{n} \frac{(x_i-\bar{x})^3}{n} \Bigg/ \hat{\sigma}_x^3, \tag{12}$$

where $\bar{x}$ and $\hat{\sigma}_x$ are the sample mean and standard error. Skewness measures the symmetry of a distribution. Positive skewness indicates a longer right tail while negative skewness indicates a longer left tail. One year is selected as a compromise between data accuracy and sample size. To increase the accuracy of high-order moments such as skewness, one needs more observations. For stocks with less than 40 valid observations in a year, we exclude that year from the calculation. This exclusion guarantees all skewness coefficients are calculated from at least 40 observations.

[23] We redo our analysis using both NYSE and AMEX stocks. The result is qualitatively the same. All explanatory variables still have the predicted signs. The most noticeable difference is that the parameter estimates of volume are positive but insignificant, confirming our choice of NYSE stocks only.

We calculate daily turnover (TO) as the ratio of shares traded to shares outstanding and detrend the turnover series using a moving average of 20 trading days.[24] Annual turnover is calculated as the mean detrended daily turnover times 250, which is approximately the number of trading days in 1 year. We also use 250 to annualize returns and volatility. Stock size is calculated as the annual average market capitalization, which is the product of daily prices and shares outstanding. We use the annual average market capitalization instead of market capitalization on a selected day since skewness is measured over the same period instead of on any selected day. The annual average also helps reduce random errors due to date selection and allows for more data accuracy.

The second data source, the Spectrum 13(f) database, contains quarterly stock holdings of 13(f) institutions from 1980 to 2003. We use the institutional 13(f) database because of its broader coverage than the CDA/spectrum S12 database.[25] For each stock in the 13(f) database, we calculate two variables, institutional ownership (IO) and ownership breadth (OB). Institutional ownership is the percentage of total shares outstanding owned by 13(f) institutions. Ownership breadth is the percentage of all reporting institutions that report a positive stock position,[26] where reporting institutions are all institutions that report any positions in that quarter. Ownership breadth defined as such actually represents "institutional ownership breadth," and thus is a little bit mislabeled.

Table II presents the summary statistics for the variables used in the analysis. Because volatilities, capitalization, and institutional ownership are nonnegative, we take natural logarithms of these variables. A visual check confirms that the logarithm transformation makes these variables' distributions more symmetric. We present summary statistics for both original and transformed variables. Because we use lagged returns of up to three periods as control variables, our sample starts as of 1965 instead of 1962. The availability of 13(f) data naturally defines two subsamples, 1965 to 1979 and 1980 to 2003. We present descriptive statistics separately for the whole sample and the two subsamples. For the 13(f) data, we have fewer observations because not all stocks in the CRSP database have institutional holding records in the 13(f) database.

Among the most noticeable patterns in Table II is the positive average skewness of individual stock returns. This is consistent with previous findings that individual stock returns are often positively skewed. Volatility is slightly higher in the second subsample (1980 to 2003) than in the first subsample (1965 to 1979). This is consistent with the finding of Campbell et al. (2001) that firm-level variances display a significantly positive trend between 1962 and 1997.

[24] Sometimes the logarithm of volume is used because volume cannot be negative, thus tends to be positively skewed. Here, because we use the annual average of daily volume, this transformation is not necessary. A visual check finds no obvious asymmetry in annual average turnover.

[25] The 13(f) database covers entire investment companies, including banks, insurance companies, parents of mutual funds, pension funds, university endowments, and numerous other types of professional investment advisors, while the S12 database covers only mutual funds.

[26] Positions less than 10,000 shares or $200,000 need not report. Small institutions with less than $100 million under control are not required to report under 13(f).

Table II

Summary Statistics

Daily price, return, trading volume, and shares outstanding for NYSE stocks (1962.07.03 to 2003.12.31) are collected from CRSP. Only common stocks are included. ADRs, REITs, close-end funds, and other exotica are excluded. Skewness is calculated from 1 year's worth of daily returns. If a stock has less than 40 valid observations in a year, skewness is not calculated and those observations are excluded. Capitalization is the average of the product between price and shares outstanding in a year. Turnover is trading volume divided by shares outstanding, detrended by MA(20). Returns, turnover, and volatility are annualized. Ownership data are calculated from the Spectrum 13(f) data set. Institutional ownership (IO) is the percentage of shares owned by institutional investors. Ownership breadth (OB) is the percentage of institutions that own the stock in the 13(f) database. Institutional ownership and ownership breadth in a year are calculated as the average of the four quarters. Natural logs are taken on capitalization, volatility, and institutional ownership. For the subperiod of 1980 to 2003, skewness, (log) capitalization, return, turnover, and (log) volatility have 30,834 observations. Institutional ownership and ownership breadth have 24,904 observations.

Period No. of Obs.	1965 to 2003 48,701		1965 to 1979 17,867		1980 to 2003 30,834 (24,904)	
	Mean	Std.	Mean	Std.	Mean	Std.
Skewness	0.253	1.156	0.318	0.821	0.216	1.310
Capitalization (billions)	2.261	10.259	0.516	1.848	3.273	12.707
Log capitalization	12.805	1.824	11.912	1.466	13.322	1.812
Turnover (raw) (%)	64.057	66.996	33.283	34.289	81.889	74.441
Turnover (detrended) (%)	0.226	4.445	0.099	2.482	0.299	5.256
Return (%)	0.070	0.522	0.073	0.414	0.069	0.575
Volatility (std.) (%)	0.376	0.237	0.345	0.158	0.394	0.271
Volatility (log std.)	−1.099	0.462	−1.151	0.408	−1.069	0.488
Institutional ownership					0.108	0.451
Institutional ownership (log)					−2.764	1.117
Ownership breadth					0.115	0.224

Turnover is also higher in the second subperiod, which could possibly be attributed to factors such as innovations in trading technology and wider equity market participation. For the time period for which we have ownership data (1980 to 2003), average institutional ownership and ownership breadth are 10.8% and 11.5%, respectively.

Motivated by existing evidence on the correlation between size, institutional ownership, and ownership breadth, Table III cross-tabulates skewness, institutional ownership, and ownership breadth with market capitalization. Consistent with the findings of Harvey and Siddique (2000), Chen et al. (2001, 2002), and D'Avolio (2002), Table III shows that skewness decreases with market capitalization and that institutional ownership and ownership breadth increase with market capitalization. All three variables strongly interact with size, which reminds us of the possibility that the ownership variables may serve as a channel that links skewness and size.

Table IV reports the correlations of the variables used in later regression analyses. These correlations are largely encouraging. All correlations are significant

Table III

Skewness, Institutional Ownership, Ownership Breadth, and Size

This table cross-tabulates the variation of skewness, institutional ownership (IO), and ownership breadth (OB) with stock size (market capitalization). Skewness is daily skewness calculated from 1 year's worth of daily returns. IO is the percentage of shares owned by institutional investors. OB is the percentage of institutions that own the stock in the 13(f) database. Each year, stocks are grouped into 10 deciles according to annual average market capitalization. Mean and standard deviations for skewness, IO, and OB are calculated for each of the subsamples for the time periods indicated.

Period Variable	1965 to 2003 Skewness 48,701		1965 to 1979 Skewness 17,867		1980 to 2003 Skewness 30,834		1980 to 2003 IO 24,904		1980 to 2003 OB 24,904	
No. of Obs.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.	Mean	Std.
Smallest	0.353	1.243	0.462	0.919	0.285	1.402	0.011	0.221	0.006	0.170
2	0.427	1.230	0.475	0.828	0.397	1.422	0.022	0.331	0.010	0.203
3	0.376	1.266	0.477	0.914	0.313	1.441	0.033	0.390	0.013	0.222
4	0.308	1.255	0.421	0.923	0.237	1.420	0.043	0.432	0.015	0.222
5	0.264	1.246	0.351	0.868	0.210	1.432	0.057	0.453	0.019	0.219
6	0.235	1.185	0.314	0.857	0.186	1.346	0.075	0.480	0.024	0.215
7	0.207	1.184	0.249	0.824	0.181	1.361	0.098	0.509	0.030	0.202
8	0.166	1.075	0.211	0.711	0.138	1.244	0.132	0.547	0.038	0.198
9	0.126	0.950	0.171	0.557	0.099	1.124	0.189	0.558	0.056	0.178
Largest	0.086	0.851	0.137	0.488	0.055	1.009	0.351	0.547	0.131	0.153

at the 5% or higher significance levels,[27] except for those between skewness and turnover and between turnover and returns lagged beyond one period. First, notice that skewness is positively correlated with contemporaneous returns and negatively correlated with lagged returns, exactly as predicted by Hypothesis 1. Second, skewness is negatively correlated with size, institutional ownership, and ownership breadth, which is consistent with Hypothesis 3. Third, consistent with Table III, size is positively correlated with institutional ownership and ownership breadth, with correlation coefficients of 0.893 and 0.533, respectively, and the absolute values are very big and highly significant. These represent the largest correlations in Table IV.

In contrast, the correlation between skewness and volume is less encouraging. The correlation coefficient is only 0.006 with a p-value of 0.19. If turnover proxies for disagreement, then according to the model we should expect a positive and significant correlation between volume and skewness. The weak correlation we obtain may cast doubt on the quality of volume as a proxy for disagreement and even on the power of the model itself. But it is also possible that volume interacts with skewness through other channels in counteracting directions. In the finance literature, trading volume is also frequently used as a

[27] Throughout this paper, the term "significant" means significant at the 5% level while "highly significant" means significant at the 1% level.

Table IV

Pearson Correlation

Skew is daily skewness calculated using 1 year's worth of data. Cap is logarithm market capitalization. TO is detrended turnover. Std. is logarithm standard deviation of stock returns. Ret, Ret(–1), Ret(–2), and Ret(–3) are returns in the current and lagged 1, 2, and 3 years. Log IO is logarithm institutional ownership. Institutional ownership is the percentage of shares owned by institutional investors. OB is ownership breadth, the percentage of institutions that own the stock in the 13(f) database. Correlations between skewness, Log Cap, return, lagged returns (Ret(–1), Ret(–2), Ret(–3)), Log Std., and TO are calculated using data from 1965, for a total of 48,701 observations. Correlations with IO (institutional ownership) and OB (ownership breadth) are calculated using data from 1980, for a total of 24,904 observations. All correlations are significant at the 5% or higher level except for that between skewness and turnover and those between turnover and lagged returns beyond 1 year.

	Cap	TO	Std.	Ret	Ret(−1)	Ret(−2)	Ret(−3)	Log IO	OB
Skew	−0.113	0.006	−0.011	0.316	−0.018	−0.033	−0.011	−0.101	−0.124
Cap		0.019	−0.366	0.123	0.204	0.196	0.182	0.893	0.533
TO			0.070	−0.028	0.015	0.002	0.003	0.021	0.056
Std.				−0.274	−0.321	−0.217	−0.143	−0.366	−0.124
Ret					0.126	−0.014	0.043	0.142	0.078
Ret(−1)						0.095	−0.071	0.196	0.085
Ret(−2)							0.070	0.200	0.093
Ret(−3)								0.188	0.075
Log IO									0.556

proxy for liquidity. According to the "liquidity premium" effect documented by Brennan, Chordia, and Subrahmanyam (1998), heavier trading volume is associated with lower returns because more liquid stocks are considered less risky. This argument is supported by the significantly negative correlation between turnover and contemporaneous returns in Table IV. In addition, Hypothesis 1 above states that lower returns are associated with more negative skewness. Therefore, the liquidity premium effect may imply a negative correlation between volume and skewness. This liquidity premium effect connects volume and skewness through returns; thus it can be taken into account by controlling for the level of returns.

The correlation between skewness and volatility is significantly negative. This could be attributed to the "asymmetric volatility" effect. Higher volatility resulting from lower lagged returns tends to decrease skewness, resulting in a negative correlation between skewness and volatility. In the volatility feedback model of Campbell and Hentschel (1992), higher volatility is associated with more negative skewness. This is supported here by the significantly negative correlation of volatility with current and lagged returns. The correlation coefficients between volatility and current and lagged returns of up to 3 years are –0.274, –0.321, –0.217, and –0.143, respectively.[28] However, volatility may

[28] The negative correlation between volatility and contemporaneous returns can possibly be attributed to the long-time window (1 year); thus, the asymmetric volatility effect creeps into contemporaneous returns. Black (1976) documents asymmetric volatility in weekly data.

Table V
Pearson Partial Correlation with Skewness

Cap is logarithm market capitalization. TO is stock turnover detrended by MA(20). Std. is logarithm standard deviation of stock returns. Ret, Ret(–1), Ret(–2), and Ret(–3) are returns in the current and lagged years. Log IO is logarithm institutional ownership. Institutional ownership is the percentage of shares owned by institutional investors. OB is ownership breadth, the percentage of institutions that own the stock in the 13(f) database. In parentheses are p-values testing the null hypothesis of zero correlation.

Correlation Variable	Partial Variables	Partial Correlation	Raw Correlation
Log std.	Ret, Ret(–1), Ret (–2), Ret (–3)	0.060 (<0.0001)	−0.011 (0.012)
TO	Same as above	0.017 (0.0002)	0.006 (0.19)
Cap	Log IO, breadth	0.001 (0.86)	−0.101 (<0.0001)

also correlate with skewness through other channels. For example, since the volatility–volume correlation is usually positive, volatility could be positively correlated with disagreement and thus positively correlated with skewness. This positive correlation could be counteracted by the asymmetric volatility effect. If current and lagged returns are taken into account to at least partially control for the asymmetric volatility effect, it is possible that volatility and skewness are positively correlated.

The above discussion finds support from the partial correlation analysis in Table V. To facilitate comparisons, the raw Pearson correlations are also reported. After controlling for returns and lagged returns, volatility and volume become significantly positively correlated with skewness. Before the control, they are negatively correlated or uncorrelated with skewness, respectively. Thus, Tables IV and V suggest that contemporaneous and lagged returns should be controlled for when examining the correlations among volume, volatility, and skewness, even when we are not interested in the skewness–return correlations per se. Table V also supports the conjecture that institutional ownership and ownership breadth may serve as the channels that link size to skewness. After controlling for institutional ownership and ownership breadth, the correlation between size and skewness becomes insignificantly positive. This is in clear contrast to the significantly negative correlation we obtain before the control.

B.2. Regressions

Table VI reports the regression results. We start with Hypothesis 1 by regressing skewness on contemporaneous and lagged returns. All variables have the predicted sign and are highly significant. Contemporaneous returns have a positive coefficient and lagged returns all have negative coefficients. These results lend strong support to Hypothesis 1, which predicts a positive correlation between skewness and contemporaneous return and a negative correlation

Table VI
Determination of Skewness

The dependent variable is skewness calculated from 1 year's worth of daily returns if there are more than 40 observations in a calendar year. Log cap is logarithm market capitalization. Turnover is average daily turnover in a year detrended using MA(20). Log std. is logarithm standard deviation of daily returns. Ret, Ret($t-1$), Ret($t-2$), Ret($t-3$) are returns of the current year and the 3 previous years. Log IO is logarithm institutional ownership (percentage of shares held by 13(f) institutional investors). OB is ownership breadth, defined as the number of institutions that own the stock as a percentage of all 13(f) institutions filing with SEC in that quarter. Annual IO and OB are the average of the four quarters. All regressions include time dummies (unreported) for each year. In parentheses are t-values. The reported R-squares are adjusted for degrees of freedom.

Reg. ID	1	2	3	4	5	6	7	8	9
Period	1965 to 2003	1965 to 1979	1980 to 2003	1965 to 2003	1965 to 1979	1980 to 2003	1980 to 2003	1980 to 1991	1992 to 2003
Intercept	0.35	0.34	0.27	1.34	1.239	1.373	−0.334	−0.904	−0.746
	(10.30)	(13.62)	(8.01)	(27.28)	(23.74)	(21.73)	(−1.37)	(−2.63)	(−1.86)
Ret	0.78	0.73	0.79	0.82	0.74	0.84	0.89	0.90	0.87
	(75.74)	(41.50)	(61.61)	(78.72)	(42.15)	(62.99)	(55.91)	(37.49)	(40.71)
Ret($t-1$)	−0.23	−0.18	−0.24	−0.14	−0.11	−0.15	−0.18	−0.15	−0.20
	(−17.24)	(−9.50)	(−14.06)	(−9.88)	(−6.01)	(−8.04)	(−8.40)	(−4.45)	(−7.22)
Ret($t-2$)	−0.13	−0.09	−0.14	−0.04	−0.028	−0.053	−0.047	−0.017	−0.067
	(−9.30)	(−4.91)	(−7.82)	(−3.11)	(−1.46)	(−2.80)	(−2.21)	(−0.48)	(−2.43)
Ret(t–3)	−0.13	−0.09	−0.15	−0.04	−0.010	−0.060	−0.071	−0.032	−0.089
	(−9.68)	(−4.79)	(−8.23)	(−3.18)	(−0.56)	(−3.22)	(−3.34)	(−0.96)	(−3.25)
Turnover				0.0057	0.0059	0.0062	0.0084	0.0095	0.0081
				(5.20)	(2.52)	(4.66)	(4.97)	(3.27)	(3.84)
Log std.				0.054	0.173	0.014	0.024	0.258	−0.110
				(3.93)	(9.41)	(0.75)	(1.11)	(7.81)	(−3.71)
Log cap.				−0.084	−0.060	−0.091	0.019	0.072	0.027
				(−23.77)	(−12.56)	(−19.28)	(1.34)	(3.45)	(1.27)
Log IO							−0.15	−0.18	−0.20
							(−6.44)	(−6.05)	(−5.05)
OB							−0.37	−0.22	−0.36
							(−6.87)	(−2.49)	(−5.06)
No. of obs.	48,071	17,867	30,834	48,701	17,867	30,834	24,904	10,906	13,998
Adj.-R^2	0.1254	0.1112	0.1270	0.1395	0.1357	0.1394	0.1487	0.1375	0.1483

between skewness and lagged returns. As a robustness check, we split the whole sample in regression (1) into two subsamples, 1965 to 1979 and 1980 to 2003, and repeat the regression for each of the two subsamples. Regressions (2) and (3) report the subsample results. The coefficient estimations continue to have the predicted sign and are highly significant.

Next, we add trading volume, volatility, and stock size to the regressions. The inclusion of trading volume tests Hypothesis 2, and because size is an inverse proxy for short selling costs, the inclusion of size partially tests Hypothesis 3. We add stock size first and the ownership variables later to identify the change in the size coefficient for the purpose of testing Hypothesis 4. Again, to verify robustness, regression (4) pertains to the whole sample and regressions (5) and (6) pertain to the two subsamples. In all three regressions turnover realizes positive coefficients that are significant or highly significant. This supports Hypothesis 2, which predicts that skewness increases with trading volume. Volatility also records positive coefficients in all three regressions, but it is insignificant in the 1980 to 2003 subsample. This is not surprising if one considers that individual stocks became more volatile during the second subperiod. Higher volatility implies more opportunity for the asymmetric volatility effect, which may decrease the correlation between skewness and volatility. Size records negative coefficients that are highly significant in all three regressions, which is consistent with the evidence in Tables III and IV and the prediction in Hypothesis 3. This finding is also consistent with earlier evidence that larger stocks are more negatively skewed. Adding these three variables slightly increases the coefficients of current and lagged returns in all three samples without changing the signs. Also noteworthy is that the coefficients on lagged returns of two and three periods for the 1965 to 1979 subsample are not significant anymore. But all other coefficients are still significant or highly significant.[29]

Regressions (7) to (9) add institutional ownership and ownership breadth to further test Hypotheses 3 and 4. Regression (7) pertains to the whole period for which we have ownership data, 1980 to 2003. To verify robustness we further split this time period into two subperiods of equal length, 1980 to 1991 and 1992 to 2003. The regression results for the two subperiods are reported in regressions (8) and (9). In all three regressions, institutional ownership and ownership breadth have significantly negative coefficients, and five of the six coefficients on these two ownership variables for the three samples are highly significant. This is consistent with Hypothesis 3, which posits that skewness decreases institutional ownership and ownership breadth, both of which inversely proxy for short selling costs.

The most striking change after adding the ownership variables is found in the coefficient on size. The size coefficient becomes insignificant; it is significantly

[29] If skewness is regressed on volume, volatility, and size without controlling for current and lagged returns, the coefficient on volume is still positive, but reduced by approximately 50%. This can be attributed to the liquidity premium effect, which decreases the correlation between skewness and volume. The large t values for contemporaneous returns are due to the large sample size. We conduct a Belsley et al. (1980) test for multicollinearity. All condition numbers are smaller than 2, mitigating the concern of multicollinearity.

negative above. This change lends strong support to Hypothesis 4, which conjectures that the ownership variables are responsible for the size effect of skewness. Together with the insignificant partial correlation between skewness and size after controlling for institutional ownership and ownership breadth in Table V, this suggests that the effect of size on skewness derives from its correlation with institutional ownership and ownership breadth. Because institutional ownership and ownership breadth inversely proxy for short sale constraints, the effect of size on skewness actually represents the effect of short sale constraints on price convexity.

The size coefficient is positive for the 1980 to 2003 sample and the two subsamples, but it is only significant for the 1980 to 1991 subsample. Why size should be positively correlated with skewness is not clear. Since it is only significant for one subsample, it could be due to reasons that are idiosyncratic to that time period. Another possibility is that size proxies for heterogeneity of opinions among investors. Larger stocks usually have more investors. Furthermore, larger firms are exposed to a more complex set of business factors, which give investors more opportunity to disagree. Both considerations tend to make investor beliefs regarding larger stocks more heterogeneous, which in turn leads to more convex price functions and thus more positive skewness.

Adding these two ownership variables does not change the coefficients of the return variables significantly. Noticeably, the turnover coefficient increases from 0.0062 to between 0.0081 and 0.0084, a 33% increase. This is consistent with the positive correlation of trading volume with IO and OB. When the ownership variables are absent, turnover captures part of their negative effect on skewness; thus, the turnover coefficient is reduced.

VI. Summary

This paper develops a model in which investors who are prohibited from short selling agree to disagree on the precision of a common public signal. In equilibrium, a very positive signal crowds out low precision investors while a very negative signal crowds out high precision investors. The equilibrium asset price is a convex function of the signal, due to the different information sensitivity of high and low precision investors. This price convexity result is robust to our maintained assumptions. For example, we assume that investors share common priors on the asset's payoff. If investors have different expectations, or are heterogeneously confident in their expectations of the asset's payoff, our results are still qualitatively valid as long as investors' prior beliefs about the asset's payoff are sufficiently independent from the precisions they attach to the new signal.

The model produces a set of testable predictions. In particular, it predicts that price reactions are stronger for good news than for bad news, that skewness of market returns is positively correlated with contemporaneous returns and negatively correlated with lagged returns, and that skewness increases with both disagreement intensity and short selling costs. We find evidence in support of all of these predictions. Specifically, we find that skewness of stock returns increases with trading volume, a proxy for disagreement intensity, but decreases with stock size, institutional ownership, and ownership breadth, inverse proxies for short selling costs. We also find that after controlling for institutional ownership and ownership breadth, the effect of size on skewness disappears. This finding suggests that the two ownership variables serve as the channel that links stock size and skewness.

This paper also brings forward unanswered questions that point to future research. First, we cannot answer why market indices are often negatively skewed. This is particularly intriguing when the positive skewness of individual stocks is taken into consideration. This question calls for a multiple asset framework. Second, we only briefly touch upon the determination of investor and market confidence. In our model investor confidence is exogenously assumed; the average market confidence is then determined by whether more or less confident people participate in the market. What if signal quality, information costs, and the distribution of beliefs are taken to be endogenous? Such questions should lead to a better understanding of the determination of investor and market confidence. Third, the implications of investor and market confidence reach beyond those that we discuss. Confidence can be immediately linked to information sensitivity, which in turn can be related to price volatility, trading volume, and securities trading decisions.[30]

Appendix: Proofs

Proof of Proposition 1: Because x is normally distributed, the value function can be rewritten as $V_{1,\theta} = \text{Max} \exp[-a y_{1,\theta}(\hat{\mu}_\theta - RP_1) + \frac{(a y_{1,\theta})^2}{2}\hat{\sigma}_\theta^2]$. Solving this maximization problem gives the optimal demand $y_{1,\theta} = \frac{\hat{\mu}_\theta - RP_1}{a\hat{\sigma}_\theta^2}$. The market clearing condition is $\lambda y_{1,L} + (1-\lambda) y_{1,H} = 1$. Substituting $y_{1,L}$ and $y_{1,H}$ in the market clearing condition gives the equilibrium price, and substituting this price into the demand functions gives the equilibrium demands. Q.E.D.

Proof of Proposition 2: Condition (a) follows from $y_{1,L} < 0$. In this case, τ_L traders wish to hold a short position in the risky asset but the best thing they can do is to sell out. Condition (b) follows from $y_{1,H} < 0$. In this case, τ_H traders wish to hold a short position but the best thing they can do is to sell out. If none of these two conditions hold, the short sale constraint does not bind and the equilibrium in Proposition 1 is still valid. Q.E.D.

Proof of Proposition 3: Claim (1) follows from the fact that less confident traders are crowded out when the signal is sufficiently positive and more confident traders are crowded out when the signal is sufficiently negative. Specifically, τ_L traders are crowded out with the probability

[30] Xu (2005) pursues research in this direction.

$$P(s > s^h) = \Phi\left(-\frac{a\hat{\tau}_L}{(1-\lambda)(\tau_H - \tau_L)\tau_x\sigma_s}\right), \tag{A1}$$

and τ_H traders are crowded out with the probability

$$P(s < s^l) = \Phi\left(-\frac{a\hat{\tau}_H}{\lambda(\tau_H - \tau_L)\tau_x\sigma_s}\right). \tag{A2}$$

It then follows that

$$P(s < s^l) - P(s > s^h) = \Phi\left(\frac{a\hat{\tau}_L}{(1-\lambda)(\tau_H - \tau_L)\tau_x\sigma_s}\right) - \Phi\left(\frac{a\hat{\tau}_H}{\lambda(\tau_H - \tau_L)\tau_x\sigma_s}\right), \tag{A3}$$

where $\sigma_s = \sqrt{\sigma_x^2 + \sigma_\varepsilon^2}$ is the standard deviation of the signal and $\Phi(\cdot)$ is the normal cumulative distribution function. Claim (2) follows from the fact that $\Phi(\cdot)$ is a monotone increasing function. If $\frac{\hat{\tau}_L}{1-\lambda} < \frac{\hat{\tau}_H}{\lambda}$, then $\frac{a\hat{\tau}_L}{(1-\lambda)(\tau_H - \tau_L)\tau_x\sigma_s} < \frac{a\hat{\tau}_H}{\lambda(\tau_H - \tau_L)\tau_x\sigma_s}$, thus $P(s < s^l) < P(s > s^h)$. Claim (3) follows. Q.E.D.

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