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Financial Speculators' Underperformance: Learning, Self-Selection, and Endogenous Liquidity

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ABSTRACT

We develop an equilibrium model of learning by rational traders to reconcile several empirical regularities: Cross sectionally, most individual speculators lose money; large speculators outperform small speculators; past performance positively affects subsequent trade intensity; most new traders lose money and cease speculation; and performance shows persistence. Learning from trading generates substantial endogenous liquidity, reducing bid–ask spreads and the impact of exogenous liquidity shocks on asset prices, but amplifying the effects of real shocks. Introducing slightly overconfident traders increases bid–ask spreads, hurting all traders. Finally, behavioral theories cannot reconcile all of these empirical regularities.

WHILE INDIVIDUAL SPECULATORS IN FINANCIAL MARKETS account for a large share of total market activity, studies of different financial markets and time periods find that most speculators realize net trading losses (see Steward (1949), Ross (1973, 1975), Hieronymus (1977), and Barber et al. (2004)). Thus, a natural question to ask is: Why do investors engage in such apparent wealth-destructive activities?

A number of researchers address this question using behavioral- and psychological-based explanations. For instance, Hieronymus (1977), Teweles and Jones (1987), and Hartzmark (1991) propose risk-loving, or utility from gambling. However, Barber et al. (2004) find that losses are too large to be reconciled solely by risk seeking. Other researchers argue that traders mistakenly believe that they can forecast prices and tend to remember profits but forget losses (Teweles and Jones (1987)), or argue that investors may be overconfident about their trading acumen (see Barber et al. (2004) and references therein).

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This paper proposes a rational, learning-based explanation. Rather than approach this empirical puzzle as an isolated fact, we place it within a more complete setting that incorporates the following related findings. First, trading intensities are positively correlated with performance—more active speculators outperform less active speculators (Ross (1973, 1975), Hieronymus (1977), Hartzmark (1987, 1991), Barber et al. (2004)). Second, speculator performance drives future trading intensity—better past performers tend to increase their trade intensities (by more than average), while worse past performers sharply curtail theirs (Barber et al. (2004)). In fact, most new traders trade on a small scale and lose money (Hieronymus (1977), Ross (1973, 1975)), and quickly cease speculation (Ross (1973, 1975)). Finally, there is a strong positive relation between past and future performance, and the most profitable speculators continue to earn positive profits (Barber et al. (2004)).

Taken together, these observations suggest a flow of novice traders in and out of the financial speculation profession. Some members of each cohort continue trading successfully with their trade intensities and profits rising over time, but most cease speculation following disappointing results. We develop an equilibrium, rational, learning-based model in which these predictions feature centrally.

In our overlapping generations environment, individuals do not know their speculative abilities. However, they can trade and obtain information about their trading skills. Only a small fraction of the population is adept at identifying profitable trading opportunities. In equilibrium, each period some novice speculators enter the financial markets. Recognizing that most traders lack financial acumen, the novices first experiment on a small scale. They then use the information contained in their trading profits to decide whether to continue. Those who earn sufficient profits conclude that they are likely to be skilled and expand their speculative activities. However, most speculators do less well, conclude that they are more likely to be inept traders, and leave the financial market. In our equilibrium model, competitive market makers post price quotes, recognizing the speculators' entry–exit process, so that the rewards from speculation are endogenously determined. Our equilibrium model of learning and rational experimentation reconciles each of the following empirical regularities:

1. In a given cross section, most individual speculators lose money.
2. On average, large (more active) speculators outperfrom small (less active) speculators.
3. Performance positively affects subsequent trade intensity. Most new traders start on a small scale, perform poorly, and cease speculation. The few good performers continue to speculate and expand their trading. Past performance is correlated with future performance.

It seems to us that learning must be central to any full explanation of these facts. We do not dismiss overconfidence as part of the explanation; rather we show that (1) a simple rational learning story suffices to reconcile all of these facts, and (2) standard risk seeking or overconfidence behavioral models in which learning is not central cannot do so.

Interestingly, we also prove that *all* speculators are made worse off if some speculators are *slightly* overconfident. Overconfident speculators trade more than is optimal. If they are only slightly overconfident, their expected marginal trading profits on their "excessive trades" are close to the opportunity costs of the value of their work time forgone, implying that they expect to make money on their excessive trading, just not enough to cover their opportunity costs. As a result, market makers post wider quotes, which hurts all traders.

Our model also highlights the role of learning from trading as an additional endogenous source of liquidity to financial markets. In our model, most inexperienced speculators, while learning about their abilities, trade on noise. The *equilibrium* consequence of such learning from trading is to reduce bid–ask spreads sharply. This endogenous liquidity reduces the impact of exogenous liquidity shocks on asset prices, but amplifies the impact of real shocks (in the labor market or in learning costs) on asset prices. Our model thus lays a rational foundation for the presence of agents traditionally identified as "noise traders" in financial markets, and shows that their participation as a group is persistent. Empirical research documents a related phenomenon. For example, Canoles et al. (1998) claim that small-scale futures speculators are a significant source of liquidity. Black's (1986) argument that noise traders may simply confuse the noise they trade on with information is also relevant. Based on this intuition, Wang (2002) and Kanatas and Wang (2003) develop models that feature noise traders who mistakenly believe the contentless signals they receive contain information, and derive the effects on financial markets. In contrast, agents in our model are rational, and "noise trading" is the outcome of a rational learning process. That our comparative statics are driven by the impacts on the equilibrium bid–ask spread highlights the importance of investigating these issues within an equilibrium model.

Bayesian learning has its origins in the statistical decision theory literature (e.g., Raiffa and Schlaifer (1961)). Our model corresponds to a bandit problem in which payoffs are endogenously determined through equilibrium bid and ask prices. Bergemann and Valimaki (1996, 1999) also analyze bandit models with endogenous prizes, but their focus is on oligopoly contexts. The idea of learning as a selection mechanism dates back at least to Jovanovic (1982), who uses a noisy selection mechanism to explain why smaller firms grow faster and are more likely to fail than larger firms. Researchers have also sought to explain asset pricing anomalies with learning (e.g., Veronesi (1999, 2000), Pastor and Veronesi (2003, 2005)). To the best of our knowledge, however, no previous research explains the entry, exit, and performance of speculators in financial markets.[1]

[1] While the goal of our research is to explain the performance of public speculators in financial markets, it can also reconcile related phenomena. Each year, large financial firms hire new finance graduates. The new hires trade on a small "testing" scale. Based on their performance, most are fired, but survivors trade on a larger scale and show better performance on average. So, too, most financial analysts leave the profession soon after their entry, and survivors issue better, more accurate forecasts. For a related model in employment contexts see Banks and Sundaram (1998).

The rest of our paper is organized as follows. In Section I, we present the model and develop the equilibrium conditions. Section II characterizes the unique equilibrium. In this section, we show that the model can match the quantitative features of the data, investigate the qualitative impacts of learning on financial markets, and derive the effect of slight overconfidence on equilibrium outcomes. Section III revisits behavioral explanations of the data, and Section IV concludes. An appendix provides proofs of the key results. Complete proofs are in Mahani (2005).

I. The Model

We consider a stochastic overlapping generations environment over $t = \ldots, -1, 0, 1, \ldots$. Each period, a continuum of measure one of potential speculators (agents) is born. Agents live two periods. They are risk neutral with a common discount factor of one, and are not wealth constrained. Each period, agents allocate one unit of time between financial speculation and working at an alternative occupation that pays w per unit of time. An agent who speculates must spend time gathering financial information, time that is not spent working at the alternative occupation.

Assets. There are N assets in the economy, where N is a large number. Asset values are publicly revealed at the end of each period, and have two components:

$$V_{\gamma,t} = v_{\gamma,t} + Z_{\gamma,t} \quad \text{where} \quad Z_{\gamma,t} = \begin{cases} z & \text{with probability } \dfrac{1}{2}, \\ -z & \text{with probability } \dfrac{1}{2}. \end{cases} \tag{1}$$

Here, $V_{\gamma,t}$ is the value of asset γ at the end of period t, $v_{\gamma,t}$ is public information at the beginning of period t, and $Z_{\gamma,t}$ is an innovation about which speculators can acquire signals. We assume $Z_{\gamma,t}$ is independently distributed from all other innovations. This assumption is made for simplicity and it does not affect our key theoretical predictions.

Speculators. A speculator incurs time cost $\tau_0 > 0$ to enter the financial markets. In addition, she incurs time cost $\tau > 0$ to acquire a signal about an asset's value. Signals are either based on intrinsic information about asset values, in which case the agent is a skilled speculator, or contentless, in which case the agent is an unskilled speculator. Agents differ only in their abilities to acquire and process information. One interpretation of a skilled speculator is that she is better at uncovering "fundamental" values. The interpretation we prefer is that able speculators are "technical" traders with a knack for identifying assets' short-term mispricings.

More specifically, a speculator α who investigates asset γ obtains a signal $\epsilon^{\alpha}_{\gamma,t} \in \{-z, z\}$ related to the assets' end-of-period value, where

$$\mathbf{Pr}\{\epsilon^{\alpha}_{\gamma,t} = Z_{\gamma,t} \mid Z_{\gamma,t}\} = \theta^{\alpha}, \tag{2}$$

and θ^{α} quantifies agent α's skills in gathering and analyzing financial data. An able agent acquires informative signals that are positively correlated with asset values: $\theta^{\alpha} = \bar{\theta} > \frac{1}{2}$. An unskilled agent produces uninformative signals that are uncorrelated with asset values: $\theta^{\alpha} = \frac{1}{2}$. Conditional on an agent's type, θ^{α}, each of her signals is an independent draw.[2] An agent's skill at processing financial data does not vary over her lifetime. Agents do not know their types, but they are born with a prior probability of being skilled. We identify each young agent by her prior probability ρ of skillfulness, and we let F denote the young agents' distribution over ρ.

A prototypical novice speculator in our model is the Japanese hairdresser Kiyoshi Wakino who, according to Barney Jopson (*Financial Post*, September 15, 2004), "day trades in between doing haircuts for 5-6 people in a day . . . times his buys using charting software, cashes in profits once gains reach 10%, while cutting losses after 2%–3% falls." Mr. Wakino has a clear opportunity (time) cost to trading, and while earning a 25% annual return on his trades to date, is still uncertain whether he is an able speculator.

Trade Game. A speculator can submit a buy or sell order for one round lot of any asset. In addition to receiving orders from young and old agents, a market maker receives round lot orders from liquidity traders. The distribution of buy and sell liquidity orders is the same across assets, and the expected measure per asset is $\frac{\mu}{N}$. One can interpret $\frac{\mu}{N}$ as the probability that a liquidity shock forces an agent to trade (buy or sell) any given asset for exogenous reasons. Markets are cleared by competitive, risk-neutral market makers. Market makers announce bid and ask prices at the beginning of each period and execute *all* orders at the corresponding prices.

At the end of each period, traders close positions at the (observed) asset values and realize profits and losses. Agents then use Bayes's rule to update their beliefs about their financial acumen from their trading performance. We assume that agents must trade to "observe" prices and learn whether they are able, that is, paper trading is of limited benefit because it is difficult to ascertain the precise prices at which orders are executed. One can interpret the distribution over priors as reflecting pre-trade learning in the economy due to paper trading, which generates diversity of beliefs across agents.

Timing. The sequence of events each period is as follows: (1) Agents allocate their time to work and speculation. (2) Agents choose which assets to investigate. Market makers simultaneously announce their bid and ask prices.

[2] Our analysis extends almost immediately if we relax the assumption that signals are conditionally independent given a speculator's type, so that correlation could arise due to the speculator's common model of asset values, as long as we maintain the assumption of independence across traders, which preserves our stationary structure.

(3) Agents and liquidity traders submit their (round lot) orders. (4) Asset values are revealed, and traders realize profits and losses.

Strategies and Beliefs. Agent α's actions in period t are fully summarized by $\{y_t^\alpha, \{e_{\gamma,t}^\alpha\}_{\gamma=1}^N, \{x_{\gamma,t}^\alpha\}_{\gamma=1}^N\}$, where $y_t^\alpha = 1$ if she speculates, and $y_t^\alpha = 0$ otherwise; $e_{\gamma,t}^\alpha = 1$ if the agent investigates asset γ at date t, and $e_{\gamma,t}^\alpha = 0$ otherwise; and $x_{\gamma,t}^\alpha = 1$ if she buys one round lot of the asset, $x_{\gamma,t}^\alpha = -1$ if she sells one round lot, and $x_{\gamma,t}^\alpha = 0$ if she does not trade asset γ at time t. Agent α's strategy specifies her actions in each period of her life as functions of her belief (about her skill), and her information.

Market Makers. Each period, market makers set bid and ask prices. Market makers' beliefs assign probabilities to the events that an incoming order originates from liquidity traders, from young agents with different priors, and from old agents with different posteriors.

A. Solving for Equilibrium

We focus on stationary, symmetric Perfect Bayesian Equilibria (PBE, see Mas-Colell, Whinston, and Green (1995)). Stationarity implies that all generations choose the same strategies, so that the distributions of endogenous variables do not vary over time. Because agents' decisions are independent conditional on the price quotes, agents solve a stochastic control problem. Given agents' optimal strategies, market makers confront a one-shot Bayesian game.

We begin by deriving the conditions that a stationary symmetric equilibrium must satisfy.

LEMMA 1: *In any stationary, symmetric PBE, (1) all market makers who handle order flow expect zero profits from quoting bid and ask prices with the same mark-up in each period, and (2) market makers' expected loss due to informed trading is the same across assets.*

This lemma reveals that market makers' strategies are uniquely identified by a bid–ask spread of $2\Delta p$ that is centered around an asset's unconditional expected value. The idea is simple. If a market maker reduces Δp below the level that gives him zero profit, he incurs losses as he cannot differentially attract uninformed trading. Facing such bid–ask spreads, agents optimally investigate each asset with equal probability, which supports the bid–ask spread of $2\Delta p$.

Next, observe that it is not optimal for a speculator to trade an asset that she has not investigated because of the positive bid–ask spread, or to investigate an asset that she does not trade because of the positive time (opportunity) cost. Combining these observations with Lemma 1 reveals that an agent's expected trading profit depends on the number of assets she trades, not the specific assets in her portfolio. Hence, expected payoffs are easy to compute.

LEMMA 2: *The expected trading profit of an agent with belief ρ who trades n assets is*

$$\mathbf{E}\Pi = n(\mathcal{Z}\rho - \Delta p), \quad \mathcal{Z} \equiv z(2\bar{\theta} - 1). \tag{3}$$

An agent's current-period expected payoff as a function of her work–speculate decision is

$$\mathbf{E}\mathcal{P} = (1 - y)w + y\,[w(1 - \tau_0) + n(\mathcal{Z}\rho - \Delta p - \tau w)]. \tag{4}$$

In equation (3), $\mathcal{Z}\rho - \Delta p$ captures the expected profit (or loss) from trading one asset: Δp represents the effective cost of an order, and $\mathcal{Z}\rho$ gives the expected value of the trader's informational advantage. The expected gain from being informed $\mathcal{Z} \equiv z(2\bar{\theta} - 1)$ has two components, size of the shock to an asset's value, z, and the correlation between an informative signal and the random innovation to the asset's value, $(2\bar{\theta} - 1)$. The agent's current-period expected payoff, equation (4), consists of two terms. The first is the income from working full time (when $y = 0$). The second is the expected payoff of a work-and-trade career. Here, $w(1 - \tau_0 - n\tau)$ is the income from working and $n(\mathcal{Z} - \Delta p)$ is the expected profit from trading.

The key dynamic feature of the model is an agent's updating of her belief,

LEMMA 3: *If a trader's prior belief is ρ_0, and she observes m correct trades among n trades, then her updated belief ρ_1 is*

$$\rho_1(\rho_0, m, n) = \left[1 + \left(\frac{1 - \bar{\theta}}{\bar{\theta}}\right)^m \left(\frac{\frac{1}{2}}{1 - \bar{\theta}}\right)^n \left(\frac{1 - \rho_0}{\rho_0}\right)\right]^{-1}. \tag{5}$$

Old Agents. An old agent chooses her actions to maximize her expected period payoff,

$$\max_{y_2, n_2} \mathbf{E_2}\mathcal{P}_2 \quad \text{where} \quad \mathbf{E_2}\mathcal{P}_2 \equiv (1 - y_2)w + y_2[w(1 - \tau_0) + n_2(\mathcal{Z}\rho_1 - \Delta p - \tau w)]\,. \tag{6}$$

The old agent computes her optimal actions using the updated probability, ρ_1, which is computed based on her initial trading performance according to Lemma 3. The maximization problem (6) is straightforward: $\mathbf{E_2}\mathcal{P}_2$ is linear in n_2, with coefficient $(\mathcal{Z}\rho_1 - \Delta p - \tau w)$. If the expected trading profit exceeds the opportunity cost of forgone work, then n_2 has a positive coefficient and $n_2^* = \bar{n}$, where $\bar{n}$ is the maximum number of assets that the agent has time to investigate, $(\tau_0 + \bar{n}\tau) \le 1 < (\tau_0 + (\bar{n} + 1)\tau)$.[3] Otherwise, the agent optimally works full time. In sum, an old agent either trades full time, $n_2^* = \bar{n}$, or works full time, $y_2^* = 0$.

The optimized expected payoff of an old agent,

$$V_2(\rho_1, \Delta p) = \max\{w, \bar{n}(\rho_1\mathcal{Z} - \Delta p)\}\,, \tag{7}$$

is her income from full-time work plus the value of a call option with a strike price of w written on her trading profits. We can write the value function V_2

[3] To simplify presentation, we assume $\tau_0 + \bar{n}\tau = 1$.

in terms of a cut-off probability, $\rho_c \equiv \frac{w/\bar{n}+\Delta p}{\mathcal{Z}}$, such that an old agent trades if $\rho_1 > \rho_c(\Delta p)$ and works if $\rho_1 \leq \rho_c(\Delta p)$:

$$V_2(\rho_1, \Delta p) = w\chi_{\{\rho_1 \leq \rho_c\}} + \bar{n}[\mathcal{Z}\rho_1 - \Delta p]\chi_{\{\rho_1 > \rho_c\}}. \tag{8}$$

The critical probability ρ_c is endogenous: Speculation decisions depend on the bid–ask spread, Δp. Fixing the distribution of ρ_1, increasing Δp reduces the number of young traders who continue to trade when old. The choice of whether to work or to speculate can also be formulated in terms of a critical number of successful trades, m_{1c}, or a critical trade profit, Π_{1c}, such that an old agent trades if and only if $\Pi_1 > \Pi_{1c}$. Effectively, old agents decide whether to trade based on past performance.

Young Agents. A young agent's actions affect her current-period payoffs *and* her future payoffs via the impact on her posterior, ρ_1. The future effect is captured in the value function (8). Taking expectations over future payoffs with respect to first-period information yields

$$\mathbf{E_1}V_2(n_1, \Delta p, \rho) = w(1 - \varrho(n_1, \Delta p, \rho)) + \bar{n}\varrho(n_1, \Delta p, \rho)[\mathcal{Z}\xi(n_1, \Delta p, \rho) - \Delta p], \tag{9}$$

where $\mathbf{E}_1V_2(n_1, \Delta p, \rho)$ is the expected future value (optimized payoff) for a young trader with prior ρ who trades n_1 assets given Δp. In equation (9), $\varrho(n_1, \Delta p, \rho)$ is the probability the young trader continues to trade when old, and $\xi(n_1, \Delta p, \rho)$ is the expected posterior probability conditional on it being optimal to trade when old:

$$\begin{aligned} &\varrho(n_1, \Delta p, \rho) \equiv \mathbf{Pr}\{\rho_1(n_1, \rho) > \rho_c(\Delta p)\} \quad \text{and} \\ &\xi(n_1, \Delta p, \rho) \equiv \mathbf{E}[\rho_1(n_1, \rho) \mid \rho_1(n_1, \rho) > \rho_c(\Delta p)]. \end{aligned} \tag{10}$$

The expected lifetime payoff W_1 of a young trader with prior ρ who trades n_1 assets is

$$\begin{aligned} &W_1(n_1, \Delta p, \rho) \\ &\quad = \overbrace{w(1 - \tau_0) + n_1(\mathcal{Z}\rho - \Delta p - w\tau)}^{\mathbf{E_1}[\mathcal{P}_1 \mid y_1=1](n_1, \Delta p, \rho)} \\ &\quad + \overbrace{w(1 - \varrho(n_1, \Delta p, \rho)) + \bar{n}[\mathcal{Z}\xi(n_1, \Delta p, \rho) - \Delta p]\varrho(n_1, \Delta p, \rho)}^{\mathbf{E_1}[V_2 \mid y_1=1](n_1, \Delta p, \rho)} \\ &\quad = w(2 - \tau_0) + n_1(\mathcal{Z}\rho - \Delta p - \tau w) + \bar{n}\left[\mathcal{Z}\xi(n_1,\ \Delta p,\ \rho) - \Delta p - \frac{w}{\bar{n}}\right]\varrho(n_1, \Delta p, \rho). \end{aligned} \tag{11}$$

Given the Δp set by market makers, a young agent chooses n_1 to maximize W_1,

$$n_1^*(\Delta p, \rho) \equiv \min\left\{\arg\max_{n_1}\{W_1(n_1, \Delta p)\}\right\} \text{ and } \mathcal{W}_1(\Delta p, \rho) \equiv \max_{n_1}\{W_1(n_1, \Delta p)\}. \tag{12}$$

The "min" operator is simply a rule for selecting among possible maximizers. Inspection of (11) reveals that if $n_1 > 0$ then $W_1(n_1, \Delta p, \rho)$ is jointly continuous in ρ and Δp, strictly increasing in ρ, and decreasing in Δp, properties that are inherited by $\mathcal{W}_1(\Delta p, \rho)$.

A young agent with prior ρ has the option to work. Because the bid–ask spread is constant over time, an agent who finds it optimal to work full time when young also finds it optimal to work full time when old. Hence, the payoff of an agent who optimally chooses not to trade is

$$W_c(\Delta p, \rho) = 2w. \tag{13}$$

A young agent compares her expected lifetime payoff if she first speculates, $\mathcal{W}_1(\Delta p, \rho)$, with her payoff of $2w$ if she works full time, testing her speculative abilities if and only if

$$\mathcal{W}_1(\Delta p, \rho) > W_c(\Delta p, \rho) = 2w. \tag{14}$$

An indifferent young agent with $\mathcal{W}_1 = W_c$ always works. If the prior distribution is atomless, this assumption is without loss of generality because this happens on a set of measure zero.

Market Makers. At the beginning of each period, market makers announce prices $v_{\gamma,t} + \Delta p$ for buy orders and $v_{\gamma,t} - \Delta p$ for sell orders. In equilibrium, Δp generates zero expected profits to market makers from both buy and sell orders. Because the trading game is zero sum, a market maker's expected profit is the expected loss of all traders who trade with him.

Liquidity trade buy and sell orders are independent of end-of-period asset values. With probability one-half they earn a profit of $z - \Delta p$, and with probability one half they lose $-z - \Delta p$. Hence, each liquidity trade expects to lose $-\Delta p$, and with an expected liquidity trade of $2\frac{\mu}{N}$, a market maker's expected profit from liquidity trading is

$$\mathbf{E}\Pi^N(\Delta p) = 2\frac{\mu}{N}\Delta p. \tag{15}$$

A representative agent with prior ρ trades when she is young if $\mathcal{W}_1(\Delta p, \rho) > 2w$, in which case she expects a profit (or loss) of $n_1(\mathcal{Z}\rho - \Delta p)$. If trades are spread uniformly across assets, a market maker's expected profit from a young trader with prior ρ is

$$\mathbf{E}\Pi^Y(\Delta p, \rho) = -\frac{n_1}{N}(\mathcal{Z}\rho - \Delta p)\chi_{\{\mathcal{W}_1 > W_c\}}. \tag{16}$$

Moreover, with probability $\varrho(n_1^*, \Delta p, \rho)$ a young trader with prior ρ continues to trade, where $n_1^* = n_1^*(\Delta p, \rho)$ denotes the optimal number of assets to investigate and trade when young. Her expected profit from trading, given she remains a trader, is $\mathcal{Z}\xi(n_1^*, \Delta p, \rho) - \Delta p$. In turn, a market maker's expected profit from an old trader who started with prior ρ is

$$\mathbf{E}\Pi^O(\Delta p, \rho) = -\frac{\bar{n}}{N}\big(\mathcal{Z}\xi\big(n_1^*, \Delta p, \rho\big) - \Delta p\big)\varrho\big(n_1^*, \Delta p, \rho\big)\chi_{\{\mathcal{W}_1 > W_c\}}. \tag{17}$$

A market maker's expected profits (from agents) is the integral over ρ, and therefore

$$N\,\mathbf{E}\Pi(\Delta p) = 2\mu\Delta p - \int_{\mathcal{W}_1 > W_c} \left\{ n_1^*[\mathcal{Z}\rho - \Delta p] + \bar{n}\left[\mathcal{Z}\xi\left(n_1^*, \Delta p, \rho\right) - \Delta p\right]\varrho\left(n_1^*, \Delta p, \rho\right)\right\} dF(\rho). \tag{18}$$

Equilibrium. The equilibrium is summarized by a collection $\{n_1^*(\cdot), \Delta p^*, \rho^*\}$, where:

1. Given Δp^*, a young agent with prior ρ chooses the number of trades $n_1^*(\rho)$ to maximize her expected lifetime payoff, so that
$$W_1\left(n_1^*(\rho), \Delta p^*, \rho\right) \geq W_1\left(n_1, \Delta p^*, \rho\right) \quad \forall n_1 \in \{0, \ldots, \bar{n}\}.$$
2. Young agents optimally choose whether to enter financial markets. There exists a ρ^* such that the young agent ρ speculates if and only if $\rho > \rho^*$, where
$$\mathcal{W}_1(\Delta p^*, \rho) \equiv \max_{n_1}\left\{W_1\left(n_1, \Delta p^*, \rho\right)\right\} \quad \text{and} \quad \mathcal{W}_1(\Delta p^*, \rho^*) = 2w.$$
3. Market makers choose Δp such that they earn zero expected profits:
$$0 = 2\mu\Delta p^* - \int_{\rho^*}^{1} \left\{ n_1^*(\rho)[\mathcal{Z}\rho - \Delta p^*] + \bar{n}\left[\mathcal{Z}\xi\left(n_1^*(\rho), \Delta p^*, p\right) - \Delta p^*\right] \times \varrho\left(n_1^*(\rho), \Delta p^*, \rho\right)\right\} dF(\rho).$$

It is conceptually helpful to separate the optimal participation choices of young agents from the zero expected profit condition of market makers. For a given Δp, we introduce two marginal young agents, (1) the marginal young agent ρ_i, who is indifferent between work and trade, and (2) the marginal young agent ρ_z, such that market makers expect to break even given that all young agents with $\rho > \rho_z$ trade. In equilibrium, the two marginal young agents must coincide, that is, $\rho_i = \rho_z$. The indifferent marginal young agent, ρ_i, is given by

$$\mathcal{W}_1(\Delta p, \rho_i(\Delta p)) \equiv 2w. \tag{19}$$

To define the zero-profit marginal young agent, we introduce the function

$$M(\Delta p, \rho) \equiv 2\mu\Delta p - \int_{\rho}^{1} \left\{ n_1^*(r)[\mathcal{Z}r - \Delta p] + \bar{n}\left[\mathcal{Z}\xi\left(n_1^*(r), \Delta p, r\right) - \Delta p\right]\varrho\left(n_1^*(r), \Delta p, r\right)\right\} dF(r), \tag{20}$$

where $M\,(\Delta p, \rho)$ is the expected profit of market makers given Δp when young agents trade if and only if they are more likely than ρ to be able speculators. Equivalently, $M(\Delta p, \rho)$ is the expected profit of market makers given the "belief" that young agents who are more confident than ρ trade. It then follows that $\rho_z(\Delta p)$ is the "belief" that rationalizes market makers' choice of Δp, that is,

$M\ (\Delta p,\ \rho_z(\Delta p)) \equiv 0$. In equilibrium, Δp^* ensures that the two marginal young agents coincide, so that both the second and the third conditions of equilibrium are satisfied, $\rho_i(\Delta p^*) = \rho_z(\Delta p^*)\ \equiv \rho^*$, and the market makers' belief is consistent with agents' choices.

Clearly, a unique equilibrium exists if $\rho_i(\Delta p)$ is increasing and $\rho_z(\Delta p)$ is decreasing in Δp. The expected lifetime payoff of a young agent, W_1, falls with Δp and rises with ρ, implying that youth speculation, $\rho_i(\Delta p)$, rises in Δp. The argument required to prove that a market maker's profits rise, and $\rho_z(\Delta p)$ falls with Δp, is more subtle. Essentially, we must show that young agents' trading intensities and hence expected trading profits fall with Δp. This is not easy. For some parameterizations, as Δp rises, a young agent may trade more intensively when young, in order to learn more about the profitability of trading when old. Conversely, it may be the case that as Δp rises, an agent trades less when young, but more (in expectations) when old. Hence, tight results do not obtain if we look separately at each stage of an agent's life. The trick is to look at total expected lifetime outcomes.

PROPOSITION 1: *For a speculator with prior ρ, increasing Δp weakly reduces her expected number of lifetime trades, $\eta \equiv n_1 + \bar{n}_{\varrho}$, and strictly reduces her lifetime trading profits, $\mathcal{L} \equiv n_1(\mathcal{Z}\rho - \Delta p) + \bar{n}_{\varrho}(\mathcal{Z}\xi - \Delta p)$. Hence, a market maker's expected profits are strictly increasing in Δp.*

In our stationary setting, a market maker's expected trading losses to speculators (integrating over young and old speculators) also equal the market maker's expect losses from trading with the current generation of young speculators over their entire career. That is,

$$M(\Delta p, \rho) = 2\mu\Delta p - \int_{\rho}^{1} \mathcal{L}(n_1^*, \Delta p, r)\, dF(r). \tag{21}$$

Figure 1 illustrates the equilibrium construction. It shows an out-of-equilibrium value of Δp_0 for which the indifferent marginal young agent is lower than the zero-profit marginal young agent, $\rho_{i,0} < \rho_{z,0}$. Because all young agents with priors larger than $\rho_{i,0}$ trade, the expected profit for market makers is negative, as inspection of the M function reveals. To reach an equilibrium, Δp must be raised. This has two effects. First, it makes trading less profitable for young agents, causing some less confident young traders, who previously entered financial markets and had positive expected lifetime trading profits, to quit trading. That is, $\mathcal{W}_1$ shifts to the right, and as a consequence so does ρ_i. Second, increasing Δp raises the expected profits that market makers earn from each trader. That is, M shifts to the left, and hence so does ρ_z. It is therefore immediate that the process results in a (unique) equilibrium in which the two marginal investors coincide, $\rho_i = \rho_z$.

THEOREM 1 (Existence): *Let the distribution of priors, F, have density f, which is positive on (0,1). Then there exists a unique, symmetric, stationary equilibrium.*[4]

[4] More precisely, there is a unique bid–ask spread, Δp^*, and a unique trading decision by almost all speculators.

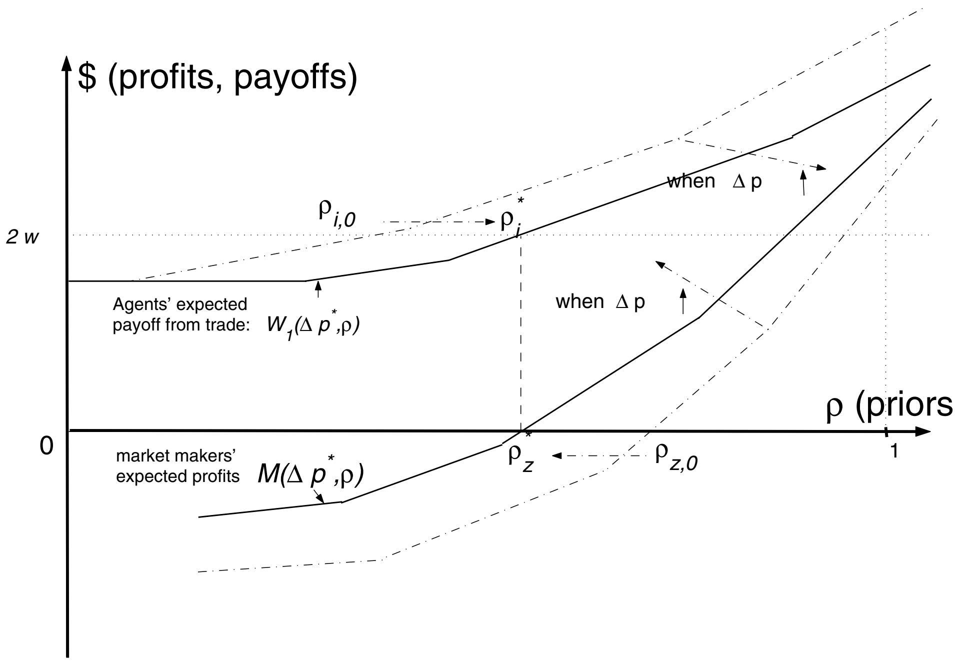

Figure 1. Construction of an equilibrium in the model with heterogeneous priors. Equilibrium is defined as the equality of two marginal young agents: (1) the marginal young agent $\rho_i(\Delta p)$ who is indifferent between work and trade, $\mathcal{W}_1(\rho_i, \Delta p) = 2w$, and (2) the marginal young agent $\rho_z(\Delta p)$ who gives market makers zero expected profit if all young agents above her (i.e., more confident with higher ρs) trade: $M(\rho_z, \Delta p) = 0$. The dashed lines depict the case in which $\rho_{i,0} < \rho_{z,0}$, that is, market makers have negative expected profits and the economy is out of equilibrium. To reach the equilibrium, Δp must increase until $\rho_i = \rho_z = \rho^*$.

Further, a positive measure of young agents speculate if and only if there is a positive information rent: $\mathcal{Z} - \frac{w}{\bar{n}} > 0$.

Above, $\mathcal{Z}$ is a skilled ($\rho = 1$) trader's information gain per asset and $\frac{w}{\bar{n}}$ is a full-time trader's opportunity cost of trading one asset. If $\mathcal{Z} \leq \frac{w}{\bar{n}}$, there is no incentive to trade and all agents work full time. In the economically relevant case of $\mathcal{Z} > \frac{w}{\bar{n}}$, a skilled trader earns a positive information rent, and hence some agents trade in equilibrium.

The proof follows the heuristic existence argument described above. The assumption that F has support [0, 1] has no economic importance and serves only to simplify the existence condition and proofs. One can also prove that an equilibrium exists if F does have mass points, but it takes more work to establish the requisite continuity properties. Section II.A considers the special case of homogeneous priors in which F is degenerate at a prior ρ_0. The proof of existence for a general distribution of priors essentially melds the analyses for the two cases.

II. Equilibrium Characterization

In this section, we document the key qualitative features of the equilibrium. Figure 1 provides the central intuition. Take an equilibrium Δp^* and marginal young investor ρ^*, and consider parameter changes that shift the zero-profit marginal investor ρ_z to the right, shift the indifferent marginal investor ρ_i to the left, or both (changes in the opposite directions are handled similarly). After such changes, the zero-profit marginal investor ρ_z exceeds the indifferent marginal investor ρ_i. As a result, given Δp^*, market makers expect losses. From Proposition 1, market makers set $\Delta p^{**} > \Delta p^*$, reducing ρ_i in the new equilibrium.

The next proposition develops the key comparative statics for a fixed Δp^*. The above argument is then used to derive the consequences for equilibrium when Δp^* responds endogenously to the parameter changes. The seeming obviousness of the comparative statics conceals subtleties. Specifically, while it is easy to derive the effects of parameter changes on the work–speculation decisions by young and old agents, and to derive the consequences for a market maker's profits fixing trading intensity decisions, it is difficult to establish the consequences for a market maker's profits of changes in trading intensities induced by parameter changes. In particular, we must sign the impact for all parameter values. The key is again that market makers' profits are reduced if expected trading intensities rise. In the proposition, $\mathbf{\Delta}(\cdot)$ denotes a change, and $\eta = n_1 + \bar{n}\varrho$ is the expected lifetime trading intensity.

PROPOSITION 2: *If the fixed costs of speculation, τ_0, are sufficiently small, then the effects of increasing $\{\mu, w, z, \rho\}$ are as follows. (Note that $M(\rho, \Delta p) = 2\mu\Delta p - \int_\rho^1 \mathcal{L}(n_1^*(r, \Delta p), r, \Delta p)\, dF(r)$.)*

(i) *Exogenous liquidity trade:* $\mathbf{\Delta}(\mu) > 0 \Rightarrow \mathbf{\Delta}(W_1 - 2w) = 0,\ \mathbf{\Delta}(\eta) = 0,$ and $\mathbf{\Delta}(M) > 0$.
(ii) *Wage:* $\mathbf{\Delta}(w) > 0 \Rightarrow \mathbf{\Delta}(W_1 - 2w) < 0,\ \mathbf{\Delta}(\eta) < 0,$ and $\mathbf{\Delta}(\mathcal{L}) \geq 0$.
(iii) *Trade profitability:* $\mathbf{\Delta}(z) > 0 \Rightarrow \mathbf{\Delta}(W_1 - 2w) > 0, \mathbf{\Delta}(\eta) > 0,$ and $\mathbf{\Delta}(\mathcal{L}) < 0$.
(iv) *Ability of young trader:* $\mathbf{\Delta}(\rho) > 0 \Rightarrow \mathbf{\Delta}(W_1 - 2w) > 0, \mathbf{\Delta}(\eta) > 0,$ and $\mathbf{\Delta}(\mathcal{L}) < 0$.

The following comparative statics results are immediate corollaries of Proposition 2.

- Reducing exogenous liquidity trading, μ, increases the equilibrium bid–ask spread Δp^* and raises ρ^*, decreasing the equilibrium measure of young speculators, $\int_{\rho^*}^1 dF(\rho)$.
- Reducing wages, w, increases both the equilibrium bid–ask spread and the measure of young speculators.
- Increasing trade profitability, z, raises the equilibrium bid–ask spread. The effect on the equilibrium measure of young speculators is ambiguous.

- A first-order stochastic improvement in the distribution of speculator skills, F, widens the equilibrium bid–ask spread, but has ambiguous effects on the measure of young speculators.
- If agents have more diverse priors—a second-order stochastic shift of F around the marginal investor ρ^*—then the equilibrium bid–ask spread widens and fewer young agents speculate.

For example, reducing μ lowers a market maker's expected profits (M shifts to the right). To compensate, market makers raise Δp, which reduces the payoff to speculation by the young ($\mathcal{W}_1$ shifts to the right). In equilibrium, the "new" marginal investor is more likely to be able than the old one, $\rho^{**} > \rho^*$, and fewer young agents speculate.

A. Extreme Cases: Full Information and Homogeneous Priors

We now study equilibrium outcomes for the two extreme formulations of the distribution of priors, namely, that of full information, the case in which young agents know if they are skilled and there is no learning from trading, and that of homogeneous priors, the case in which all young agents have the same prior and no pre-trade learning occurs.

Full Information. If agents know whether they are able, the distribution of priors has two mass points, with a measure $f(0)$ of unskilled agents and a measure $f(1)$ of skilled agents. In equilibrium, only skilled agents (may) speculate, and if they trade, they trade full time. Skilled agents' per-period expected earning is

$$\mathbf{E}\mathcal{P}^S = \max\{w, \bar{n}(\mathcal{Z} - \Delta p)\}. \tag{22}$$

A necessary and sufficient condition for an equilibrium with speculation to exist is the "positive rent" condition of Theorem 1, $\mathcal{Z} > \frac{w}{\bar{n}}$: Wages are not so high that a skilled agent prefers work over speculation. All skilled agents speculate if $\mathcal{Z} > \Delta p + \frac{w}{\bar{n}}$. Market makers' zero-expected profit condition, $\mathbf{E}\Pi^M = 2\mu\Delta p - 2\iota_1\bar{n}(\mathcal{Z} - \Delta p) = 0$, implies that

$$\Delta p^* = \frac{\iota_1\bar{n}\mathcal{Z}}{\mu + \iota_1\bar{n}}. \tag{23}$$

If $\Delta p^* = \mathcal{Z} - \frac{w}{\bar{n}}$, then equation (23) implies that measure $\iota_1 = \frac{\mu}{w}(\mathcal{Z} - \frac{w}{\bar{n}})$ of young agents speculate. If $\Delta p^* < \mathcal{Z} - \frac{w}{\bar{n}}$, then all skilled agents speculate. Hence,

$$\iota_1 = \min\left\{f(1), \frac{\mu}{w}\left(\mathcal{Z} - \frac{w}{\bar{n}}\right)\right\}. \tag{24}$$

Homogeneous Priors. If all young agents share the same prior and some, but not all, young agents speculate, then indifference to speculation implies that $\mathcal{W}_1(\Delta p) = 2w$. If fraction ι_1 of young agents speculate and old agents with the same posterior make the same decisions,[5] the zero-expected profit condition for market makers is

$$\mathbf{E}\Pi^M = 2\mu\Delta p - \iota_1\{n_1[\mathcal{Z}\rho_0 - \Delta p] + \bar{n}[\mathcal{Z}\,\xi(n_1, \Delta p) - \Delta p]\varrho(n_1, \Delta p)\}. \quad (25)$$

Because young agents start with the same prior, a market maker's expected profit is linear in the measure of young agents who participate in financial markets. Given a Δp, the measure of young agents who speculate is given by the zero-expected profit condition

$$0 = 2\mu\Delta p - \iota_1\{n_1[\mathcal{Z}\rho_0 - \Delta p] + \bar{n}[\mathcal{Z}\xi(n_1, \Delta p) - \Delta p]\varrho(n_1, \Delta p)\}. \quad (26)$$

To find an equilibrium, first assume that only some young agents speculate. Indifference of young agents to speculation implies that Δp^* is the solution to $\mathcal{W}_1(\Delta p) \equiv \mathcal{W}_1(\Delta p, \rho_0) = 2w$. For this Δp^*, we solve equation (26) for the measure of young speculators, ι_1^*, that gives market makers zero expected profits. This yields the unique equilibrium.[6]

B. Matching Empirical Observations

We next show that for reasonable parameter choices, the model's equilibrium features the empirical regularities highlighted previously. Specifically,

1. In a given cross section, most individual speculators lose money.
2. Large (more active) traders show better performance than small (less active) traders.
3. Performance positively affects subsequent trade intensity, and most new traders (who start on a small scale and show very poor performance) quickly cease speculation.
4. Performance shows persistence.

For simplicity, we produce the empirical regularities using homogeneous priors. Table I presents representative parameterizations for which the model produces all empirical regularities. The first five columns report the model parameters. In all cases, level of exogenous liquidity trade, μ, is adjusted to produce an equilibrium young agent participation rate of 0.01. We normalize the outside wage w to one and set $\tau_0 = 0.001$.[7] The next six columns detail the equilibrium values of endogenous variables, and show that the model matches the key empirical observations. The ratio of young to old trading intensities, $\frac{n_1^*}{\bar{n}}$, is always less than 0.2, that is, trading intensities rise at least fivefold with experience. The expected profit of a young trader, $\mathbf{E}\Pi^Y$, is negative and

[5] Generically, there is a unique optimal action for an old agent, and therefore, this assumption is without loss of generality.

[6] Mahani (2005) provides the formal proof and analyzes the (economically uninteresting) cases in which no one speculates or all young agents speculate.

[7] If τ_0 is too large, young speculators must expect to make money, to cover the fixed time costs of speculation.

Table I

Equilibrium Quantities with Homogeneous Priors

This table shows the equilibrium quantities from homogeneous prior models for different parameters. In all cases, $w = 1$, $\tau_0 = 0.001$, and μ is adjusted so that 1% of young agents speculate in equilibrium. $\mathbf{E}\Pi^Y = n_1[\mathcal{Z}\rho_0 - \Delta p]$ is the average profit of young speculators. $\mathbf{E}\Pi^O = \bar{n}[\mathcal{Z}\,\xi(n_1, \Delta p) - \Delta p]$ is the average profit of old speculators. $1 - \varrho$ is the probability a young speculator ceases trading when old. Exogenous liquidity is $2\,\mu\,\Delta p$. Endogenous liquidity is the expected total trading losses of unskilled agents. The losses of young agents are: $\iota_1 n_1(1-\rho_0)\Delta p$. For old agents, summing over first-period outcomes yields: $\sum_{\rho_1>\rho_c}(1-\rho_1)\mathbf{Pr}\{m_1\} = \mathbf{Pr}\{\rho_1 > \rho_c\} - \mathbf{E}[\rho_1\chi_{\{\rho_1>\rho_c\}}] = \varrho - \varrho\xi$. Hence, endogenous liquidity $= \iota_1 n_1(1-\rho_0)\Delta p + \iota_1\bar{n}(\varrho - \varrho\xi)\Delta p$.

Model Parameters					Equilibrium Quantities					
No. of Trades ($\bar{n}$)	Prior (ρ_0)	Shocks (z)	Signal Quality ($\bar{\theta}$)	Exog. Liquid. (μ)	Markup Δp^*	Yng./Old Ratio ($\frac{n_1^*}{\bar{n}}$)	Young Profit $\mathbf{E}\Pi^Y$	Old Profit $\mathbf{E}\Pi^O$	Exit Prob. $1-\varrho$	Ratio Endo. to Exog. Liq.
200	0.05	0.2	0.7	0.082	0.010	0.105	−0.135	4.718	0.935	1.673
200	0.05	0.2	0.75	0.042	0.019	0.095	−0.266	6.700	0.936	2.873
200	0.05	0.2	0.8	0.018	0.030	0.06	−0.287	8.515	0.954	4.199
200	0.05	0.2	0.85	0.013	0.043	0.06	−0.436	10.021	0.945	5.627
200	0.05	0.5	0.7	0.025	0.034	0.105	−0.495	10.288	0.935	5.393
200	0.05	0.5	0.75	0.015	0.055	0.095	−0.809	15.237	0.936	8.321
200	0.05	0.5	0.8	0.008	0.082	0.075	−1.011	23.123	0.951	11.588
200	0.05	0.5	0.85	0.005	0.1160	0.06	−1.181	23.540	0.945	15.050
500	0.05	0.2	0.7	0.0284	0.022	0.07	−0.617	14.226	0.948	7.562
500	0.05	0.2	0.75	0.015	0.035	0.052	−0.776	18.258	0.952	10.895
500	0.05	0.2	0.8	0.008	0.051	0.038	−0.851	21.804	0.957	14.052
500	0.05	0.2	0.85	0.005	0.069	0.028	−0.869	24.300	0.962	16.752
500	0.05	0.5	0.7	0.011	0.057	0.07	−1.649	34.0530	0.948	19.962
500	0.05	0.5	0.75	0.006	0.090	0.052	−2.018	44.132	0.952	28.185
500	0.05	0.5	0.8	0.003	0.130	0.038	−2.186	52.992	0.957	35.969
500	0.05	0.5	0.85	0.002	0.176	0.028	−2.215	59.228	0.962	42.617

small, suggesting that inexperienced traders make small average losses. The average profit of old traders, $\mathbf{E}\Pi^O$, is positive and greater than two, suggesting that profits are positively correlated with experience, and that performance is persistent. A very high exit rate, $1-\varrho$, of inexperienced traders also obtains.

The last column of Table I, the equilibrium ratio of endogenous-to-exogenous liquidity, reveals the effect of learning-from-trading on financial markets. Whether there is more endogenous than exogenous liquidity in a given financial market depends on the specific parameters characterizing that market. Nevertheless, an environment with learning affords more informed trading than a full-information setting.

The last three columns of Table II show the probability of a successful trade and the probability of losing money for young traders, old traders, and all traders combined. We match the empirical observation that about two-thirds to three-quarters of speculators lose money.

Table II

Equilibrium Quantities for Homogeneous Priors

This table shows the probability of a successful trade and the probability of making losses (in overall trades) with homogeneous priors and different sets of parameters. In all cases, $w = 1$, $\tau_0 = 0.001$, and μ is adjusted so that 1% of young agents speculate in equilibrium. The following definitions are used:

$$p_{c,Y} \equiv \rho_0 \bar{\theta} + (1 - \rho_0)\frac{1}{2}$$

$$\bar{p}_{c,O} \equiv \sum_{\rho_1 > \rho_c} \left[\rho_1(\rho_0, m_1, n_1)\bar{\theta} + (1 - \rho_1(\rho_0, m_1, n_1))\frac{1}{2}\right] \mathbf{Pr}\{m_1\}$$

$$\bar{p}_{c,T} \equiv \frac{1}{1+\varrho} p_{c,Y} + \frac{\varrho}{1+\varrho} \bar{p}_{c,O}$$

$$p_{L,Y} \equiv \mathbf{Pr}\left\{m_1 < \frac{n_1(z + \Delta p)}{2z}\right\}, \quad \text{where} \quad m_1 \sim \mathbf{Bi}(p_{c,Y}, n_1)$$

$$\bar{p}_{L,O} \equiv \sum_{\rho_1 > \rho_C} \mathbf{Pr}\left\{m_2 < \frac{\bar{n}(z + \Delta p)}{2z} \,\middle|\, m_1\right\} \mathbf{Pr}\{m_1\}, \quad \text{where} \quad m_2 \,|\, m_1 \sim \mathbf{Bi}(\rho_1(m_1), \bar{n})$$

$$\bar{p}_{L,T} \equiv \frac{1}{1+\varrho} p_{L,Y} + \frac{\varrho}{1+\varrho} \bar{p}_{L,O}.$$

Model Parameters					Equilibrium Quantities					
					Prob. of a Successful Trade			Prob. of Loss		
No. of Trades ($\bar{n}$)	Prior (ρ_0)	Shocks (z)	Signal Quality ($\bar{\theta}$)	Exog. Liquid. (μ)	Young ($p_{c,Y}$)	Old ($\bar{p}_{c,O}$)	Overall ($\bar{p}_{c,T}$)	Young ($p_{L,Y}$)	Old ($\bar{p}_{L,O}$)	Overall ($\bar{p}_{L,T}$)
200	0.05	0.2	0.7	0.082	0.510	0.574	0.514	0.634	0.042	0.598
200	0.05	0.2	0.75	0.042	0.513	0.607	0.518	0.636	0.029	0.599
200	0.05	0.2	0.8	0.018	0.515	0.661	0.521	0.572	0.020	0.547
200	0.05	0.2	0.85	0.013	0.518	0.693	0.527	0.771	0.021	0.731
200	0.05	0.5	0.7	0.025	0.510	0.574	0.514	0.634	0.042	0.598
200	0.05	0.5	0.75	0.015	0.513	0.607	0.518	0.636	0.029	0.599
200	0.05	0.5	0.8	0.008	0.515	0.668	0.522	0.654	0.018	0.624
200	0.05	0.5	0.85	0.005	0.518	0.693	0.527	0.771	0.021	0.731
500	0.05	0.2	0.7	0.028	0.510	0.601	0.515	0.711	0.016	0.677
500	0.05	0.2	0.75	0.015	0.513	0.645	0.519	0.803	0.013	0.767
500	0.05	0.2	0.8	0.008	0.515	0.702	0.523	0.784	0.008	0.752
500	0.05	0.2	0.85	0.005	0.518	0.768	0.527	0.887	0.000	0.854
500	0.05	0.5	0.7	0.011	0.510	0.601	0.515	0.711	0.016	0.677
500	0.05	0.5	0.75	0.006	0.513	0.645	0.519	0.803	0.013	0.767
500	0.05	0.5	0.8	0.003	0.515	0.702	0.523	0.784	0.009	0.752
500	0.05	0.5	0.85	0.002	0.518	0.768	0.527	0.887	0.000	0.854

C. Qualitative Effects of Learning from Trading

Table I suggests that learning from trading is a key endogenous source of liquidity. We now investigate this and related aspects of learning. The prior distribution serves to quantify the amount of pre-trade learning (e.g., through education or paper trading) in the economy. In a full-information economy, all learning takes place before trading, whereas in an economy with homogeneous prior all learning occurs via trading.

Consider an equilibrium of the reference model with heterogeneous priors, which is characterized by $(\Delta p^*, \rho^*, n_1^*(\cdot))$. Using ρ^* as a reference point, we can approach a full-information economy via a second-order stochastic shift of the distribution outward around ρ^*, so that in the limiting case, fraction $1 - F(\rho^*)$ of speculators know that they are able, and the remainder know that they are inept. We can approach the homogeneous prior via inward shifts of the ability distribution toward the homogeneous prior in which agents have common beliefs that each is skilled with probability $1 - F(\rho^*)$.[8] Our comparative statics analyses reveal that the outward shifts toward full information raise the equilibrium bid–ask spread and reduce the equilibrium measure of inexperienced speculators. Conversely, moving the probability mass in the prior distribution inward toward the homogeneous prior reduces the equilibrium bid–ask spread and leads to more speculation by young traders. Moreover, with full information, all young traders continue to trade when old, whereas with homogeneous priors, many young traders lose money and cease trading. Therefore, *learning from trading reduces bid–ask spreads and raises the rate of entry and exit,* while *pre-trade learning increases the bid–ask spread and reduces entry and exit.*

This analysis confirms that *learning from trading acts as an endogenous source of liquidity in financial markets.* Recall that increasing exogenous liquidity reduces the bid–ask spread and raises speculation by young agents. In this sense, less diversified priors, which amount to more learning-from-trading, are qualitatively similar to adding exogenous liquidity.

We next explore how the amount of pre-trade learning (captured by the priors) affects the economy's reaction to perturbations in the underlying parameters. To do this, it is important to have comparable equilibria. Accordingly, as we move distribution mass in (toward homogeneous priors) and out (toward full information), we offset each change in the variance of the distribution with a shift in the distribution that leaves the equilibrium marginal investor, and hence the equilibrium bid–ask spread, unchanged. In particular, when moving the distribution mass out (toward full information) we shift the distribution to the left, and when moving the distribution mass in (toward homogeneous priors) we shift the distribution to the right. By construction, the resulting equilibria have the same bid–ask spread and the same marginal investor, but more inexperienced agents speculate in the homogeneous-priors model than in the reference model and in the reference model than in the full-information model. That is, to match an observed average bid–ask spread in real markets, in economies with less diverse beliefs, more participation by novice traders is needed. This is another way of saying that when agents have more homogeneous priors, there is more learning-from-trade, and more endogenous liquidity is provided. It is straightforward to derive the following implications:

[8] It is important to preserve ρ^* as a reference point, as changes to the low end of the distribution of nonspeculators have no impact on equilibrium outcomes.

- Learning from trading reduces the effects of exogenous liquidity shocks on the equilibrium bid–ask spread, but raises their effects on the equilibrium rate of entry in financial markets.
- Learning from trading raises the effects of changes in trading costs on both the equilibrium bid–ask spread and the rate of entry in financial markets.

D. Overconfidence

Having assumed that speculators are rational, we demonstrate above that learning through trading induces behaviors commonly attributed to overconfidence. However, our model offers a natural environment in which to analyze overconfidence. Suppose that for all ρ, a fraction $r(\rho) \in (0, 1)$ of agents with prior ρ incorrectly perceive their probability of being skilled to be $\tilde{\rho} = \rho + d\rho$, where $d\rho$ is positive, but small. Further suppose that market makers are rational: Market makers recognize the presence of overconfident speculators, understand how overconfidence affects decision making, and form expectations based on the actual probabilities. One might think that overconfident traders should earn lower trading profits, ceteris paribus, raising market makers' profits, reducing equilibrium bid–ask spreads, and raising the welfare of rational traders (just as an increase in liquidity trade does). This intuition is wrong.

PROPOSITION 3: *A uniform increase in the measure of (slightly) overconfident speculators, $r(\rho)$, raises equilibrium bid–ask spreads, hurting all traders.*

The full support assumption for overconfident speculators ($r(\rho) \in (0, 1), \forall\rho$) ensures that there is a positive measure of overconfident speculators, with "true" prior ρ and "perceived" prior $\tilde{\rho} = \rho + d\rho$, who modify their trading strategies (to a new strategy different from their "rational" counterparts). Proposition 2 implies that such an overconfident speculator expects to trade more over her lifetime. Because the alternative choice was feasible for a rational agent, it follows immediately that the expected lifetime utility of an overconfident speculator never exceeds her rational counterpart. The key is that for slightly overconfident speculators, their expected marginal trading profits (on their "excessive trades") are close to the opportunity costs of the value of their work time forgone, implying that they expect to earn positive profits on their excessive trading, just not quite enough to cover their opportunity costs. In equilibrium, market makers must quote wider bid–ask spreads, hurting all traders.

III. Competing Behavioral Explanations

In this section, we revisit behavioral models and explore their abilities to reconcile the empirical regularities explained by our model. We first consider bare-bones behavioral models and then augment them with additional features. Importantly, we identify empirical observations that differentiate between competing theories.

Clearly, both overconfidence and risk-seeking behavior can reconcile the simple fact that, on average, individual investors lose money. However, the basic forms of these theories—unadorned with other features such as heterogeneity in ability, wealth/budget constraints, or the learning that we model—are inconsistent with many other empirical findings. For example, in the data, contemporaneous investor performance is positively correlated with the scale of trade and is positively autocorrelated, and exit rates fall with the duration of trade, while the scale of trade rises.

To match the heterogeneity in the scale of trade in the core overconfidence and risk seeking models, investor heterogeneity is required. Investors who are more overconfident or are more risk seeking trade more. But, absent other forms of heterogeneity, more overconfident or more risk-seeking agents who trade more also lose more money on average. That is, contemporaneous performance and scale of trade are negatively correlated and not positively correlated as in the data. Also, absent learning or budget constraints, overconfidence and risk seeking cannot generate the exit patterns observed in the data, as agents have no reason to alter their behavior.

These theories can do a better job of explaining the data if we augment them with wealth constraints that limit the scale of trade. For example, if investors are overconfident, but (initially) poor and risk averse, then they will start on a small scale. If, in addition, we introduce sufficiently high *exogenous* trading costs (endogenous bid–ask spreads would be zero absent both fixed trading costs and adverse selection from able speculators), then because overconfident speculators underperform on average, wealth constraints can generate high initial exit rates (of traders who lose money and then face wealth constraints). In addition, those few traders who initially get lucky and make money face slacker budget constraints, and hence increase their scale of trade. In turn, because they have more money, it takes more trading losses before their wealth constraints bind, so that exit rates fall with experience.

However, even with wealth constraints, overconfidence is inconsistent with the positive correlation between contemporaneous performance and the scale of trade that is in the data. Overconfident investors who do better initially have more money and trade more in the future, and hence lose more. Therefore, overconfidence predicts a zero correlation on a per trade basis, and a negative correlation on an absolute basis. For the same reason, overconfidence in the presence of wealth constraints gives rise to a negative autocorrelation in performance. The expected profit of an overconfident trader who trades n times is $-n\Delta p$, and hence falls with initial performance. In contrast, performance is positively autocorrelated in the data. Furthermore even with wealth constraints, it seems implausible that overconfidence can generate exit rates of greater than 50% within the first year as documented by Ross (1973, 1975).

In addition, to the extent that heterogeneity in (e.g., initial) trading scale is driven by differing degrees of overconfidence rather than different wealth levels, investors who trade more (initially) should lose more money on average, yet should be less likely to exit. In particular, even with learning, given heterogeneity in overconfidence, sufficiently overconfident investors (modeled by a greater difference between "true" and "perceived" priors) trade more, and hence expect to lose more money, but do not revise beliefs by enough to exit as frequently as less overconfident agents who trade on a smaller scale. As a result, exit should be negatively correlated with performance.

The central point that emerges is that to reconcile all facts, investors must learn about their trading abilities through their trading performance. The data are inconsistent with overconfidence or risk-seeking explanations that do not involve such learning. Obviously, the data cannot preclude sufficiently slight overconfidence in agents who learn about their abilities through trade, just as testing statistical hypotheses becomes infeasible when the null and alternative hypotheses are too close. However, the comparative statics in models with and without learning can differ. In presence of learning, slight overconfidence raises participation and bid–ask spreads, because slightly overconfident agents expect positive lifetime trade profits. Absent such learning, to generate the high initial losses and high exit rates, agents must be very overconfident. Added trade from very overconfident agents who expect to lose money causes bid–ask spreads to fall.

With additional structure, it is possible to push our comparative statics results further and derive the cross-sectional effects of heterogeneity across assets. Suppose that there are two groups of assets with different fundamental volatilities, $z^1 > z^2$, that are homogeneous in all other aspects. If the expertise of speculators is transferable across asset classes, then the equilibrium bid–ask spreads will be wider for the more volatile assets, $\Delta p^1 > \Delta p^2$. However, the possible profits from speculation will also be wilder, $z^1 - \Delta p^1 > z^2 - \Delta p^2$, so that more skilled traders will tend to trade the more volatile asset. This would give rise to a positive correlation across three variables, namely, fundamental volatility of assets (as measured by the volatility of earnings or short-term returns, for example), bid–ask spreads, and average performance of speculators. Similar results would obtain if speculator expertise is not transferable (e.g., because some speculators have expertise in mining stocks, while others have expertise in biotech stocks) as long as the distribution of expertise is similar across asset classes. In addition, if expertise is not transferable, we conjecture that (possibly with more structure) (1) the cross-sectional variance in speculator profits will be higher for the more volatile assets, and (2) a greater fraction of speculators trading the more volatile assets will lose money. Deriving the analogous predictions for behavioral models is problematic because such models are partial equilibrium in nature, and their predictions depend on how the model is closed by determining "equilibrium" prices across the asset classes. This suggests that extensions of our model with heterogeneous assets may yield a rich set of panel predictions, predictions that are absent from traditional behavioral models, but that can be used to distinguish between different formulations of learning and expertise formation.

Finally, our model considers risk-neutral agents who live two periods. It is worth considering how predictions would be affected qualitatively in the case of risk-averse agents who have longer trading horizons. Qualitatively, risk aversion reduces the scale of initial trades as well as subsequent trades (even by successful traders who believe that they are probably skilled). With longer trading horizons, over time an agent faces less unresolved uncertainty about his trading ability. The initial uncertainty combined with the convexity induced by the future option(s) induces agents to speculate initially (even though they believe they are likely to lose money when they first trade). As uncertainty is resolved, for any fixed posterior belief, the value of the option is reduced so that over time there should be more exit of marginal performers. As a result, average trading intensities should rise with experience, as should persistence in performance and expected trading profits. This simply reflects that learning continues over time, and that agents with past trading profits correctly believe that they are more likely to be able, expanding their trades as a consequence.

Absent learning, overconfidence or risk seeking predicts very different behavior for individuals. In particular, overconfident or risk-seeking individuals for whom the wealth constraint binds will begin to trade again once they earn enough to relax the wealth constraints: Overconfidence and risk seeking predict temporary exit and repeated re-entry by losers at scales that are limited only by their wealth levels. While we are unaware of explicit data on these predictions—agents who quit trading are not followed—we doubt that this occurs.

IV. Conclusion

We introduce learning from trading to financial speculation to explain several empirical regularities within a consistent framework. Inexperienced traders initiate speculation on a small scale, trading off expected losses against the value of information generated by greater trading. Most inexperienced traders realize losses, conclude that they are unlikely to be skilled, and leave the markets; survivors expand their trades and make more profits. Learning produces the aggressive trading that is traditionally attributed to psychological biases.

Our equilibrium model allows us to derive the effects of changes in parameters on the entry, exit, and performance of financial speculators. Our model also provides insight into the interaction between learning from trading and other elements of financial markets. It develops the intuition that learning by small speculators provides liquidity to financial markets.

Further extensions of our model may explain other anomalies. Introducing correlations across agents' learning processes may drive fluctuations in speculative intensities with corresponding impacts on volume and bid–ask spreads. It also seems worthwhile to connect this model with the literature on "commonality in liquidity" to explain common liquidity movements across assets (e.g., Hasbrouck and Seppi (2001)), and with studies on aggregate liquidity as a priced factor (e.g., Pastor and Stambaugh (2003)). Finally, it would be valuable to extend our model to compute welfare measures. Ironically, many regulations in the financial markets are designed to affect (either limit or enhance) public speculation, without a rigorous understanding of their ultimate welfare effects.

Appendix: Proofs of Results

Proof of Lemma 3: The updated (posterior) probability of $\theta = \bar{\theta}$ is computed via Bayes's rule:

$$\begin{aligned}\mathbf{Pr}\{\theta = \bar{\theta} \mid obs.\} &= \frac{\mathbf{Pr}\{\theta = \bar{\theta} \& obs.\}}{\mathbf{Pr}\{obs.\}} \\ &= \frac{\mathbf{Pr}\{obs. \mid \theta = \bar{\theta}\}\mathbf{Pr}\{\theta = \bar{\theta}\}}{\mathbf{Pr}\{obs. \mid \theta = \bar{\theta}\}\mathbf{Pr}\{\theta = \bar{\theta}\} + \mathbf{Pr}\left\{obs. \mid \theta = \frac{1}{2}\right\}\mathbf{Pr}\left\{\theta = \frac{1}{2}\right\}} \\ &= \frac{\binom{n}{m}(\bar{\theta})^m(1-\bar{\theta})^{N-m}\rho}{\binom{n}{m}(\bar{\theta})^m(1-\bar{\theta})^{N-m}\rho + \binom{n}{m}\left(\frac{1}{2}\right)^m\left(\frac{1}{2}\right)^{N-m}(1-\rho)} \\ &= \frac{1}{1 + \left(\frac{1}{2\bar{\theta}}\right)^m\left(\frac{1}{2(1-\bar{\theta})}\right)^{N-m}\left(\frac{1-\rho}{\rho}\right)}.\end{aligned}$$

Simplifying this expression gives equation (5). Q.E.D.

Proof of Proposition 1: Denote the expected lifetime losses of market makers to a young agent with prior ρ who trades n_1 assets (when young) given Δp by $\mathcal{L}(n_1, \Delta p, \rho)$. That is,

$$\mathcal{L} = n_1(\mathcal{Z}\rho - \Delta p) + \bar{n}\varrho(\mathcal{Z}\xi - \Delta p) \tag{A1}$$

$$= W_1 + \frac{w}{\bar{n}}(n_1 + \bar{n}\varrho) - (2 - \tau_0)w, \tag{A2}$$

which equals the expected liftetime trading profits of the young speculator ρ.

(1) If an agent's trading decisions are unaffected, then

$$\frac{\partial \mathcal{L}}{\partial \Delta p} = -n_1 - \bar{n}\varrho < 0. \tag{A3}$$

(2) If n_1 is not affected by the increase in Δp, but the decision to speculate when old is, then because increasing Δp reduces expected profits from speculation when old, it follows that raising Δp causes the speculator to require $m^* + 1$ successes to speculate rather than m^*. The speculator must have been indifferent to speculation, implying that her expected trading profit following m^* successes was w, so that a market maker's expected losses fall discontinuously by the probability of m^* successes times w. (Equivalently, equation (A2) and continuity of W_1 imply that the jump in $\mathcal{L}$ due to a marginal increase in Δp is $-w\varrho_0$, where $\varrho_0 \equiv \mathbf{Pr}\{\rho_1 = \rho_c\}$.)

(3) When the choice of n_1 is affected by an increase in Δp, then at least two choices of n_1 must leave the agent indifferent, that is, $W_1(\check{n}_1) = W_1(\hat{n}_1)$, for some $\check{n}_1, \hat{n}_1$. Assume that the participation decision when old does not

change (this eases presentation, and only for the complement of a generic set of parameters ρ is both indifferent between trades n_1 and $\hat{n}_1$ when young and indifferent to participation when old). Then, equations (A2) and (A3) imply that the marginal changes in W_1 are $-(\check{n}_1 + \bar{n}\check{\varrho})$ and $-(\hat{n}_1 + \bar{n}\hat{\varrho})$, where $\check{\varrho}$ and $\hat{\varrho}$ are computed from $\check{n}_1$ and $\hat{n}_1$, respectively. The marginal reduction is greater when expected trades, $n_1 + \bar{n}\varrho$, are higher, implying that the increase in Δp causes ρ to reduce lifetime expected trading. Then, equation (A2) and the fact that $\check{n}_1$ and $\hat{n}_1$ must generate the same payoff of W_1 imply that $\check{\mathcal{L}} < \hat{\mathcal{L}}$, that is, a market maker's expected losses are lowered by the reduced trading intensity. Q.E.D.

Proof of Theorem 1: Step 1. If $\mathcal{Z} \geq \frac{w}{\bar{n}}$, there exists a lower bound $\underline{\rho}$ for priors such that $\mathcal{W}_1(0, \underline{\rho}) = 2w$. If $\mathcal{Z} < \frac{w}{\bar{n}}$, then for all $\rho \in [0, 1]$, $\mathcal{W}_1(\Delta p = 0, \rho) < 2w$, and hence no one speculates in equilibrium.

The proof is easy: $\mathcal{W}_1(\Delta p = 0, \rho = 0) = 2w - \tau_0 w < 2w$. If $\mathcal{Z} \geq \frac{w}{\bar{n}}$, $\mathcal{W}_1(\Delta p = 0, \rho = 1) = 2w + 2\bar{n}(\mathcal{Z} - \frac{w}{\bar{n}}) > 2w$. If $\mathcal{Z} < \frac{w}{\bar{n}}$, $\mathcal{W}_1(\Delta p = 0, \rho = 1) < 2w$. We assume $\mathcal{Z} \geq \frac{w}{\bar{n}}$ for the rest of the proof.

Step 2. Define the function $\mathcal{D}_i : [\underline{\rho}, 1] \to [0, \bar{\Delta} p)$ by $\Delta p_0 = \mathcal{D}_i(\rho_0) \Longleftrightarrow \mathcal{W}_1(\Delta p_0, \rho_0) = 2w$.

This function is well defined. For $\rho_0 = \underline{\rho}$, from Step 1$\mathcal{D}_i(\underline{\rho}) = 0$, and since $\mathcal{W}_1$ is continuous in both arguments, nondecreasing in ρ, and nonincreasing in Δp, then for $\rho_0 > \underline{\rho}$, there always exists a Δp_0 such that $\mathcal{W}_1(\Delta p_0, \rho_0) = 2w$. This Δp_0 is unique: For speculating agents, $\mathcal{W}_1$ is strictly increasing in ρ and strictly decreasing in Δp.

Step 3. $\mathcal{D}_i(\rho)$ is continuous and increasing.

This is immediate because $\mathcal{W}_1$ is jointly continuous in ρ and Δp, strictly increasing in ρ , and strictly decreasing in Δp in a neighborhood of $(\mathcal{D}_i(\rho_0), \rho_0)$.

Step 4. Define $\mathcal{M} : [\underline{\rho}, 1] \to \mathbb{R}$ by $\mathcal{M} \equiv M(\mathcal{D}_i(\rho), \rho)$. Then, $\mathcal{M}$ is continuous and increasing on $[\underline{\rho}, 1]$ with $\mathcal{M}(\underline{\rho}) < 0 < \mathcal{M}_1(1) = 2\mu\mathcal{D}_i(1)$.

Obviously, $M(0, \rho) < 0$ because if $\Delta p = 0$, market makers only lose money to skilled speculators. Hence, $\mathcal{M}(\underline{\rho}) < 0$. From the definition of $\mathcal{D}_i(\rho)$, when market makers set $\Delta p = \mathcal{D}_i(1)$, no informed trading occurs and hence a market maker's expected profit is positive. Next note that $M(\Delta p, \rho)$ is jointly continuous in ρ and Δp. (All functions of ρ are almost everywhere continuous and they are inside the integral. A similar argument applies to Δp.) Since $\mathcal{D}_i(\rho)$ is also continuous in ρ, so is $\mathcal{M}$. In Lemma A1 (at the end of this proof) we show that $\mathcal{M}(\rho)$ is increasing in ρ on $[\underline{\rho}, 1]$.

Step 5. Hence, there is a unique Δp^* such that $\mathcal{M}(\Delta p^*) = 0$ (the equilibrium Δp). Q.E.D.

The next lemma shows that M is increasing in ρ for the relevant range of priors, and confirms our construction of equilibrium. Because $\mathcal{D}_i$ is increasing in ρ , its inverse, $\Re$, is well defined.

LEMMA A1: *Define* $\Re : [0, \bar{\Delta}p] \to [\underline{\rho}, 1]$ *by* $\rho_0 = \Re(\Delta p_0) \iff \mathcal{W}_1(\Delta p_0, \rho_0) = 2w$ *, and let*

$$M(\Delta p, \rho) \equiv 2\mu\Delta p - \int_{\rho}^{1} \big\{n_1^*(r)[\mathcal{Z}r - \Delta p] + \bar{n}\big[\mathcal{Z}\xi\big(n_1^*(r), \Delta p, r\big) - \Delta p\big]\varrho\big(n_1^*(r), \Delta p, r\big)\big\}\, dF(r).$$

Then, for fixed $\Delta p_0 \in [0, \mathcal{D}_i(1))$ *, there exists an* $\epsilon > 0$ *such that* $M(\Delta p_0, \rho)$ *is increasing in* ρ *on* $[\Re(\Delta p_0) - \epsilon, 1]$*. Moreover, for fixed* $\rho_0 \in [\underline{\rho}, 1]$*,* $M(\Delta p, \rho_0)$ *is increasing in* Δp *on* $[0, \mathcal{D}_i(1))$*. Therefore,* $\mathcal{M} : [\underline{\rho}, 1] \to \mathbb{R}$ *;* $\bar{\mathcal{M}} \equiv M(\mathcal{D}_i(\rho), \rho)$ *is increasing.*

Proof of Proposition 2: There are three possible channels through which the parameters can affect $W_1 - 2w$ and M: (1) Through the (direct) infinitesimal effect, (2) through the (indirect) effect of some old agents changing their participations decision discontinuously, and (3) through the (indirect) effect of some young agents changing their trade intensity discontinuously. (The third case gives the results for η.) We do not discuss the case that both young, and old, agents' decisions change. (Proof is similar.) To simplify presentation, we assume $\tau_0 = 0$ and $\tau = \frac{1}{\bar{n}}$. All results hold as long as τ_0 is sufficiently small.

A. Continuous Changes. All results follow from simple differentiation:

A.1. $\partial(W_1 - 2w)/\partial\mu = 0,\ \partial M/\partial\mu = 2\Delta p > 0.$

A.2. $\partial(W_1 - 2w)/\partial w = \partial W_1/\partial w - 2 = -\eta/\bar{n} < 0,\ \partial\mathcal{L}/\partial w = 0.$

A.3. $\partial(W_1 - 2w)/\partial z = \partial W_1/\partial z = \eta(2\bar{\theta} - 1)\rho + \bar{n}(2\bar{\theta} - 1)\rho(1 - \rho)(\varrho_{\bar{\theta}} - \varrho_{\frac{1}{2}}) > 0,$

A.4. $\partial(W_1 - 2w)/\partial\rho = \partial W_1/\partial\rho = \eta\mathcal{Z} + +\bar{n}\left[\mathcal{Z}(1 - \rho) - \Delta p - w/\bar{n}\right] > 0$ (the inequality follows because an agent with higher ρ can always adopt the strategy of an agent with lower ρ to earn at least the same payoff.) $\partial\mathcal{L}/\partial\rho = \partial W_1/\partial\rho + w\,\underbrace{\partial\varrho/\partial\rho}_{>0} > 0.$

B. Jumps in Old Participation. Consider an agent who is indifferent between trading and working when old with a positive probability: $\varrho_0 \equiv \mathbf{Pr}\{\rho_1 = \rho_c\} > 0$, where $\rho_c = \frac{\Delta p + w/\bar{n}}{\mathcal{Z}}$. The change in the participation probability is exactly the probability of indifference, $\boldsymbol{\Delta}(\varrho) = \varrho_0$, and it is easy to show that $\boldsymbol{\Delta}(\varrho\xi) = \boldsymbol{\Delta}(\mathbf{E}[\rho_1\chi_{\{\rho_1 > \rho_c\}}]) = \rho_c\varrho_0$.

We already know that W_1 is continuous in terms of changes in old participation ($|\boldsymbol{\Delta}(W_1)| = -w\varrho_0 + \bar{n}\mathcal{Z}\rho_c\varrho_0 - \bar{n}\Delta p\varrho_0 = \bar{n}\varrho_0(\mathcal{Z}\rho_c - \Delta p - w/\bar{n}) = 0$). Therefore, we only need to consider discontinuous changes in $\mathcal{L}$. From $\boldsymbol{\Delta}(\mathcal{L}) = w\boldsymbol{\Delta}(\varrho) = w\varrho_0\mathrm{sign}(\partial(\rho_1 - \rho_c))$, where $\partial(\rho_1 - \rho_c)$ is the change in $\rho_1 - \rho_c$ caused by a change in one of the four parameters and sign $(\cdot)$ is the sign function. The following results are straightforward.

B.1. $\partial(\rho_1 - \rho_c)/\partial\mu = 0 \Rightarrow \dfrac{\mathbf{\Delta}(\mathcal{L})}{\text{sign}(\partial\mu)} = 0.$

B.2. $\partial(\rho_1 - \rho_c)/\partial w < 0 \Rightarrow \dfrac{\mathbf{\Delta}(\mathcal{L})}{\text{sign}(\partial w)} = -w\varrho_0 < 0.$

B.3. $\partial(\rho_1 - \rho_c)/\partial z > 0 \Rightarrow \dfrac{\mathbf{\Delta}(\mathcal{L})}{\text{sign}(\partial z)} = w\varrho_0 > 0.$

B.4. $\partial(\rho_1 - \rho_c)/\partial\rho > 0 \Rightarrow \dfrac{\mathbf{\Delta}(\mathcal{L})}{\text{sign}(\partial\rho)} = w\varrho_0 > 0.$

C. Jumps in Young Trade Intensity. Consider an agent who is indifferent between two trade intensities $\hat{n}_1$ and $\tilde{n}_1$, that is, $\hat{W}_1 = \tilde{W}_1$. This immediately implies that a change from $\hat{n}_1$ to $\tilde{n}_1$ or vice versa does not induce a jump in W_1, so we only need to consider the $\mathcal{L}$ function. From equation (A2), $\mathbf{\Delta}(\mathcal{L}) = w\mathbf{\Delta}(\eta)/\bar{n}$. Using Lemma 5 the following results obtain:

C.1. $\dfrac{\mathbf{\Delta}(\eta)}{\text{sign}(\partial\mu)} = 0 \Rightarrow \dfrac{\mathbf{\Delta}(\mathcal{L})}{\text{sign}(\partial\mu)} = 0.$

C.2. $\dfrac{\mathbf{\Delta}(\eta)}{\text{sign}(\partial w)} < 0 \Rightarrow \dfrac{\mathbf{\Delta}(\mathcal{L})}{\text{sign}(\partial w)} < 0.$

C.3. $\dfrac{\mathbf{\Delta}(\eta)}{\text{sign}(\partial z)} > 0 \Rightarrow \dfrac{\mathbf{\Delta}(\mathcal{L})}{\text{sign}(\partial z)} > 0.$

C.4. $\dfrac{\mathbf{\Delta}(\eta)}{\text{sign}(\partial\rho)} > 0 \Rightarrow \dfrac{\mathbf{\Delta}(\mathcal{L})}{\text{sign}(\partial\rho)} > 0.$

The following lemmas are used in part C of the proof of Proposition 2:

LEMMA A2: *Let* $\varrho_{\bar{\theta}} \equiv \mathbf{Pr}\{\rho_1 > \rho_c \,|\, \theta = \bar{\theta}\}$ and $\varrho_{\frac{1}{2}} \equiv \mathbf{Pr}\{\rho_1 > \rho_c \,|\, \theta = \frac{1}{2}\}$. *Then,*

$$\varrho = \rho\varrho_{\bar{\theta}} + (1-\rho)\varrho_{\frac{1}{2}}, \quad \varrho\xi = \varrho_{\bar{\theta}}\rho. \tag{A4}$$

LEMMA A3: *Assume* $\tau_0 \equiv 0$ *and* $\tau \equiv \frac{1}{\bar{n}}$. *Then,*

$$W_1(n_1, \rho, \Delta p) = 2w + \underbrace{(n_1 + \bar{n}\varrho)}_{\equiv\eta(n_1,\rho,\Delta p)}\left(\mathcal{Z}\rho - \Delta p - \frac{w}{\bar{n}}\right) + \bar{n}\mathcal{Z}\rho(1-\rho)(\varrho_{\bar{\theta}} - \varrho_{\frac{1}{2}}). \tag{A5}$$

Agent ρ's lifetime payoff from trading in excess of her lifetime payoff from working, $W_1 - 2w$, has two parts: (1) Lifetime trade intensity, $n_1 + \bar{n}\varrho$, times net expected profits when young, $\mathcal{Z}\xi - \Delta p - \frac{w}{\bar{n}}$, and (2) $\bar{n}\mathcal{Z}\rho(1-\rho)(\varrho_{\bar{\theta}} - \varrho_{\frac{1}{2}})$.

For an agent who is indifferent between two trades, $\hat{n}_1$ and $\tilde{n}_1$, the second part can be written in terms of the first part, and we directly obtain:

LEMMA A4: *If agent ρ is indifferent between $\hat{n}_1$ and $\tilde{n}_1$, that is, $\hat{W}_1 = \tilde{W}_1$, then*

$$(\tilde{\varrho}_{\bar{\theta}} - \tilde{\varrho}_{\frac{1}{2}}) - (\hat{\varrho}_{\bar{\theta}} - \hat{\varrho}_{\frac{1}{2}}) = \left[\underbrace{(\hat{n}_1 + \bar{n}\hat{\varrho}) - (\tilde{n}_1 + \bar{n}\tilde{\varrho})}_{\hat{\eta}-\tilde{\eta}}\right] \frac{\mathcal{Z}\rho - \Delta p - \dfrac{w}{\bar{n}}}{\bar{n}\mathcal{Z}\rho(1-\rho)}. \tag{A6}$$

Starting from such an indifferent trader for changes in certain parameters we can identify the difference in the slopes of two payoff functions with the expected trade intensity. Hence:

LEMMA A5: *Consider an agent who is indifferent between $\hat{n}_1$ and $\tilde{n}_1$. Then,*

$$\begin{aligned}
&1.\ \partial W_1\hat{/}\partial\Delta p - \partial W_1\tilde{/}\partial\Delta p = -(\hat{\eta} - \tilde{\eta})\\
&2.\ \partial \hat{W_1}/\partial w - \partial \tilde{W_1}/\partial w = -(\hat{\eta} - \tilde{\eta})/\bar{n}\\
&3.\ \partial \hat{W_1}/\partial z - \partial \tilde{W_1}/\partial z = \frac{\Delta p + w/\bar{n}}{z}(\hat{\eta} - \tilde{\eta})\\
&4.\ \partial \hat{W_1}/\partial \rho - \partial \tilde{W_1}/\partial \rho = \underbrace{\frac{(\mathcal{Z} - \Delta p - w/\bar{n})(\Delta p + w/\bar{n})}{\mathcal{Z}\rho(1-\rho)}}_{>0\ when\ \Delta p < \mathcal{Z} - w/\bar{n}}(\hat{\eta} - \tilde{\eta}).
\end{aligned} \tag{A7}$$

Therefore, if an agent is indifferent between $\hat{n}_1$ and $\tilde{\eta}$, then by equation (A2) we have $\hat{W} = \tilde{W} \Rightarrow \hat{\mathcal{L}} - \tilde{\mathcal{L}} = \frac{w}{\bar{n}}(\hat{\eta} - \tilde{\eta})$, and we can sign the direction of changes in a market maker's losses due to an increase in any of the parameters, $\{\Delta p, w, z, \rho\}$, through the impact on n_1. Combining this result with the normal infinitesimal effects yields part C of the proof of Proposition 2.

Proof of Proposition 3: An agent with perceived prior $\rho + d\rho$ uses the same decision rule as one with true prior $\rho + d\rho$. From Proposition 2, an agent with prior $\rho + d\rho$ expects to make at least as many trades over his lifetime as an agent with prior ρ. Furthermore the premise of Proposition 3 implies that agents with sufficiently higher priors expect to trade strictly more than those with lower priors. Because $r(\rho) > 0$ for all $\rho \in [0, 1]$, it then follows that a positive measure of overconfident agents expect to trade more over their lifetime than their rational counterparts. For any such pair $(\rho, \rho + d\rho)$, let $\boldsymbol{\Delta}\eta(\rho, \rho + d\,\rho)$ be the corresponding increase in lifetime trading intensities, resulting in a corresponding increase of $w\tau\boldsymbol{\Delta}\eta(\rho, \rho + d\,\rho)$ in opportunity cost of trading. By continuity, there exists a $d\rho^*(\rho) > 0$ such that for all $d\rho < d\rho^*(\rho)$, the expected lifetime payoff of the overconfident speculator $\rho + d\rho$ is within $w\tau\boldsymbol{\Delta}\eta(\rho, \rho + d\,\rho)$ of that of his rational counterpart, and therefore, her expected lifetime trading profits strictly exceed that of her rational counterpart. But then market makers' expected profits are reduced by the presence of overconfident speculators. To restore market makers' zero expected profits, bid–ask spreads must widen. Q.E.D.

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