THE JOURNAL OF FINANCE • VOL. LXII, NO. 1 • FEBRUARY 2007

The Value of Embedded Real Options: Evidence from Consumer Automobile Lease Contracts

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ABSTRACT

Under the common assumption of constant interest rates, we show that penalties for early termination of a lease are often structured in such a way that the cancellation option embedded in consumer automotive leases has little value. Furthermore, our estimates drawn from a sample of three popular car models over 1990 to 2000 indicate that the stand-alone value of the lease-end purchase option is, on average, about 16% of the market value of underlying used vehicles, or about $1,462 per contract. Finally, we examine the sensitivity of our option value estimates to model parameters and default risk.

RECENT YEARS HAVE WITNESSED A DRAMATIC GROWTH in the leasing of automobiles. For instance, overall industry sales data indicate that about a third of the new vehicles and trucks sold in the United States are leased (Hendel and Lizzeri (2002), and Miller (1995)), and the Federal Reserve Board survey of family finances reports that the use of leased vehicles by individuals (nonbusiness consumers) has risen from 2.9% to 5.8% over the 1992 to 2001 period (Azicorbe, Kennickell, and Moore (2003)). Consumer automobile lease contracts for new cars often include two embedded options that are not associated with a typical debt contract. The first is a cancellation option that allows the lessee to terminate the lease early. The second is a European call option, which gives the lessee the right, but not the obligation, to purchase the leased (used) vehicle at the scheduled termination of the lease at a predetermined exercise price. Both of these options are simple examples of real options because the asset underlying these options is a used car, that is, a real asset; however, these options constitute a special case of real options since they are embedded in the lease contract, which is a financial (credit) instrument.

Theoretical analyses of lease contracts show that the cancellation option is quite valuable (Schallheim and McConnell (1985)). With respect to the purchase option, Hendel and Lizzeri (2002) argue that they can play an important role in

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mitigating moral hazard and asymmetric information-type frictions in markets for durable goods and thereby improve consumer welfare. Since the value of the underlying asset is quite variable over the typical 2- to 4-year term of consumer leases, financial economics suggests that these long-term consumer real options can have significant value. Yet, there exists little empirical evidence on the value of these options.[1] The objective of this paper is to develop stand-alone estimates of the value of these embedded options in leases written on new passenger cars by using used car price data for three popular makes and models over the 1990 to 2000 period.

When a consumer purchases a car, he assumes ownership of it. Under a lease contract, the consumer may use the durable asset for a certain period and then decide whether to purchase it. Thus, a lease contract separates usage rights from ownership rights. In addition, a lease contract often allows the lessee to cancel the agreement to use the asset. We examine whether these economic flexibilities built into the automobile lease contract are material by applying the standard call option pricing model. In doing so, we focus on the value of managerial flexibility in marketing a durable asset. This approach differs from typical studies on real options, which focus on the value of managerial flexibility in the production of goods and services.

The extant literature provides no empirical evidence on the value of the cancellation option and the available evidence on the purchase option is limited to two studies that provide only sketchy estimates. Miller (1995) estimates the value of the call option by comparing the cash flows on a closed-end lease that specifies a predetermined residual value with the cash flows on a contract that offers the lessee the opportunity to purchase the leased vehicle at the fair market price prevailing at the termination of lease contract. Miller's estimates based on lease quotes from one dealer show that the value of the purchase option in a 3-year lease accounts for 5% to 6% (9% to 11%) of the retail sales price (residual value at the end of the lease contract) of a new passenger car. Drawing from a sample of 33 lease advertisements in newspapers in June 1993, he concludes that the standard approaches for evaluating the lease-versus-purchase decision overstate the costs associated with closed-end leases by ignoring the value of the embedded call option. Furthermore, Angel (1997, p. 5) observes that "Third party lessors often reduced the residual factor by three percentage points on (2-year) leases that contained such purchase options; the decrease in the residual factor raised the monthly lease payment to offset the value of the option." This implies that for a car with a Manufacturer's Suggested Retail Price (MSRP) of $20,000, the average value of the 2-year purchase option is $600.

Our analysis begins with an examination of the structure of the cancellation option—the lessee's right to cancel the lease at any time. In theory, this

[1] In contrast, the real estate literature has developed extensive analyses, both theoretical and empirical, of options embedded in mortgage contracts (e.g., Hall (1985), Schwartz and Torous (1989), and Deng, Quigley, and Van Order (2000)).

option derives its value primarily from fluctuations in the market value of the leased vehicle and secondarily from uncertain interest rate movements. In practice, however, lessors levy an early-termination penalty that is determined in such a way that the lessee can never benefit from variations in the market price of the used car. Under the standard assumption of a fixed interest rate, we show that the structure of the penalty clause nullifies the value of the cancellation option. When we allow for variable interest rates, while it is theoretically possible for the cancellation option to be valuable in certain states of nature, we argue that the lessor-imposed conditions effectively neutralize the value of early termination of the lease when interest rates fall. Next, we apply the model of McConnell and Schallheim (1983) to used car price data to estimate the stand-alone value of the purchase option embedded in consumer leases for three popular makes and models of new cars. Assuming frictionless durable goods markets, this model generates benchmarks for the value of the purchase option. Our findings indicate that the call option has considerable value, on average about 16% of the market value of the underlying used vehicles. This amounts to an average of $1,462 per call option embedded in a lease contract. In addition, we examine the sensitivity of our option value estimates to the model's parameters and discuss plausible reasons as to why lessors do not explicitly price the purchase option embedded in automobile leases.

Akin to most structured financial instruments, retail automobile lease contracts are complex instruments that contain implicit options such as default and prepayment options, other than the cancellation and purchase options. We revise the base-case estimates of the call option to account for the possibility of default, but we do not explicitly examine the prepayment option. We concentrate on the cancellation and purchase options because they derive their value largely from fluctuations in the price of the underlying asset, which is by far the most important source of uncertainty that participants in the lease market face. Given the complexity of interactions among the multiple embedded options, we examine the value of the cancellation and purchase options separately, without regard to potential interdependence between them. In addition, we ignore the effects of the other assets and liabilities of the lessors and lessees (i.e., portfolio effects) on the value of these options. Therefore, the evidence we present on the value of the consumer flexibilities should be viewed as a base-case estimate that ignores potential portfolio effects.

The rest of the paper proceeds as follows. In Section I, we review the valuation of a standard operating lease with an option to buy and discuss examples of the embedded cancellation and purchase options. Section II explains how we assemble used car price data and presents estimates of the value of the purchase option under different assumptions about the price of the underlying asset, its volatility, and the strike price of the option. In Section III, we examine the sensitivity of the option value estimates to the key parameters of the model, adjust the call value estimates for the possibility of default by the lessee, and offer plausible explanations as to why the purchase option is rarely explicitly priced in consumer automobile lease contracts. Section IV concludes the paper.

I. Pricing Automobile Leases and Their Embedded Options

In a typical consumer lease arrangement, a financial institution (lessor) buys a new vehicle from a manufacturer's franchised automobile dealer and leases it to a personal user (lessee). Most consumer leases for new automobiles are closed-end leases, that is, the lessee is not responsible for the difference between the market price of the leased asset and its residual value at the end of the lease period if the former falls below the latter. Theoretical models of closed-end leases discuss two important options, namely, a European purchase option at lease expiration, and a compound cancellation option.

A. Valuation of a Standard Operating Lease with the Two Embedded Options

McConnell and Schallheim (1983) model the cancellation option as a compound call option. To illustrate, consider a two-period lease in which the lessee can choose to cancel or renew the contract for the second period. This contract is effectively a single-period lease plus a call option on the use of the asset in the second period, with the exercise price equal to the contractual lease payment. In general, a T-period cancelable lease is a combination of a one-period lease plus a stream of *T-1* one-period call options to renew the lease.

The net advantage of leasing (NAL) a brand new automobile for T periods, with a cancellation option at each point t, is given by the difference between the present value of lease services and the present value of the contractual lease payments:[2]

$$NAL = \sum_{t=0}^{T-1} PV(L_t) N_t(h_t + \sigma\sqrt{t}) - \sum_{t=0}^{T-1} PV(L^*) N_t(h_t), \tag{1}$$

where $PV(L_t)$ is the risk-adjusted present value (as of time $t = 0$) of the expected value of lease services L_t, $PV(L^*)$ is the present value of the contractual lease payment L^* discounted at the riskless rate r, σ^2 is the variance of the rate at which the automobile depreciates over time, $N_t(.)$ is a t-dimensional multivariate normal distribution function defined as $N_t(h_1, \ldots, h_t) = \int_{-\infty}^{h_1} \cdots \int_{-\infty}^{h_t} f(x_1, \ldots, x_t)\, dx_1, \ldots, dx_t$, and $h_t = [\ln(\lambda^t A_0/\bar{A}_t) + (\ln r - \sigma^2/2)t]/\sigma\sqrt{t}$, where $\lambda^t A_0$ is the expected (depreciated) car value at time t discounted to the present period at a risk-adjusted rate, A_0 is the market value of the brand new automobile, and $\bar{A}_t$ is the period-t boundary value of the automobile. If at time t the market value A_t is greater than the critical value, then

[2] A complete development of this model may be found in McConnell and Schallheim (1983), Section 3.

the lessee will make the lease payment and extend the life of the lease for one more period.

If the lessee has the option to buy the car at lease expiration (time T), then the value of the call option C must be added in. The value of the purchase option with a strike price of X is given by (see MS)

$$C = S_0 N_T(h_T + \sigma\sqrt{T}) - PV(X) N_T(h_T), \tag{2}$$

where $S_0(=\lambda^T A_0)$ is the present value of the expected market price of the leased automobile T periods from now, and $PV(X)$ is the present value of the exercise price.

This expression for C resembles the traditional Black and Scholes (1973) call option model on a stock that pays no dividends. No correction for the stochastic dividend generated from the use of the automobile is necessary here because, as equation (1) shows, the lease payments are set to cover this dividend. However, there are two differences between (1) and the classical option pricing model. First, the probability multipliers are based on a multivariate normal distribution. Second, S_0, the underlying asset price, is based on the expectation of the depreciated asset value and hence is not directly observable.

B. Valuation of the Cancellation Option in Frictionless Markets

Note that in equation (1), each periodic lease payment L^* includes both an amount for driving the car and a portion for the option to cancel the lease. A natural question that arises is, in the absence of market frictions such as prepayment penalties, how valuable is the early-termination option held by the lessee? To answer this question, notice that the lessor holds a long position in the underlying asset and a short position in the compound call option. The lessor can hedge the early-termination risk by buying a lease cancellation insurance policy.[3]

Assuming constant interest rates, Schallheim and McConnell (1985) model the fair value of this insurance policy as the difference between the present value of rental payments on a cancelable lease and those on a noncancelable lease of identical maturity, where the two sets of rental payments are discounted at the risk-free rate. Their numerical analysis of 5- and 7-year leases shows that the value of the insurance premium varies from 2% to 32% of the value of the underlying asset across a range of parameter values. Thus, the cancellation option can have significant value, at least in theory. We emphasize that the insurance premium denotes the value of the cancellation option in the absence of prepayment penalties.

[3] Equivalently, the lessor holds a short position in the compound put option. In the simpler case of standard options, as contrasted with compound options, a long position in the underlying asset coupled with a short position in a European call is known as a covered call position and is equivalent to a short position in a European put option. To hedge the risk in a short put, the lessor can buy an insurance policy, which is equivalent to a long put position.

C. Valuation of the Cancellation Option with Cancellation Penalties

The lessee has an incentive to cancel a lease primarily under the following two conditions: (1) The market value of the leased car depreciates more rapidly than the contractual rate of depreciation, and (2) the market rate of interest drops below the rate specified in the lease contract. Thus, the two primary sources of the value of the cancellation option are asset price risk and interest rate risk. McConnell and Schallheim (1983) and Schallheim and McConnell (1985) model the value of the cancellation option due to the underlying asset price uncertainty, but they ignore the interest rate component of the option value by assuming a constant interest rate. We are not aware of a two-factor model of lease pricing that simultaneously accounts for both sources of risk. Note that the lessee would not want to cancel a lease when the market price falls at a slower pace than the contractual rate of depreciation, nor when the market interest rate rises above the rate specified in the lease contract.

Below, we present two examples of real-world lease contracts; each includes the typical early-termination clause and the option to purchase the underlying car. We then show that the structure of the prepayment penalty found in the major American manufacturers' closed-end lease documents is materially different from the structure assumed in theoretical models of the cancellation option.

General Motors Acceptance Corporation Lease (as of August 1998):

> "EARLY END. You may end this lease anytime . . ."
> "WHAT YOU OWE AT EARLY END. In general, you will owe us any unpaid monthly payments. We will give you a credit for any unearned rent charge and a credit if we sell the vehicle for more than the residual value. . . . You will owe us:
> The base monthly payment times the number of payments not yet due,
> – Any unearned rent charge, figured by the actuarial method,
> – Any surplus . . . on the vehicle sale,
> + If there is no surplus, any early excess mileage and wear charge. . .
> = The Total. If the Total is more than zero, you will owe us the Total. If the Total is less than zero, we will not give you a refund or credit."
> "PURCHASE OPTION AT END OF LEASE TERM. You have an option to buy the vehicle at the end of the lease for $______, plus official fees and taxes."

Ford Credit Lease (www.fordcredit.com)

> "VOLUNTARY EARLY TERMINATION AND RETURN THE VEHICLE. You may terminate this lease early, if You are not in default, by returning the Vehicle to Lessor and paying the following: (a) an early termination fee of $200, plus (b) the difference, if any, between the Unpaid Adjusted Capitalized Cost and the Vehicle's Fair Market Wholesale

Value, plus (c) all other amounts then due under this lease. You will never pay more than the sum of the remaining unpaid lease payments, plus any excess wear and use and mileage charges, and all other amounts then due under this lease."

"VOLUNTARY EARLY TERMINATION AND PURCHASE THE VEHICLE. You may purchase the Vehicle from Lessor at any time for the sum of the remaining payments, less any unearned Rent Charges, plus the purchase option price and all other amounts then due under this lease."

"Purchase Option at End of Lease Term. $ ______ plus official fees and taxes is Your lease end purchase option price. You have the option to purchase the Vehicle from Lessor in cash for the purchase option price at the end of this lease term if You are not in default."

From the above two examples, we can specify the cancellation (prepayment) penalty as follows (ignoring other charges such as fees and excess mileage and wear charges):

$$\begin{aligned}
\textit{Penalty} &= \text{Max}\{[\text{Outstanding Lease Payments } (\textit{OLP}) \text{ less Unearned} \\
&\quad \text{Interest Charges } (\textit{UIC}) \text{ less Surplus } (\textit{SUR}, \text{ which is equal to} \\
&\quad \text{the difference between the Fair Market Wholesale Value}(A_t) \\
&\quad \text{and the residual value } (X) \text{ of the car})], 0\} \\
&= \text{Max}\{[\textit{OLP} - \textit{UIC} - \textit{SUR}], 0\}. \qquad (3)
\end{aligned}$$

To examine the implications of this *Penalty* specification, assume for the moment that the market interest rate is the same as the rate set in the lease contract so that we can focus on the asset price risk, which is the primary determinant of the cancellation option. Let *ACC* represent the adjusted capitalized cost[4] of the leased vehicle at the time the contract is signed, $t = 0$, T denote the number of total monthly lease payments, and n indicate the number of remaining lease payments at the cancellation time, t. The lease contract divides the unpaid value of the vehicle under the contract, *ACC*, into two components, specifically, the value of usage rights given by (*ACC* minus X) and the value of ownership rights represented by the contractual residual value X. The lessor sells the usage rights to the lessee and retains both the ownership rights and the residual value risk (i.e., the risk that the market value of the used vehicle deviates from the contractual residual value). At any time t subsequent to the signing of the lease, the market value of usage rights is given by (A_t minus X, which is equal to *SUR*). The corresponding unamortized cost of usage rights at t is equal to $(ACC - X) \times (n/T)$, which is equal to $OLP - UIC$. Thus, the early-termination penalty is equal to either the difference between

[4] The capitalized cost includes the negotiated purchase price of the vehicle with factory and dealer installed options, the cost of dealer sold programs such as an extended warranty, any dealer conveyance fee, or any government fees less any initial cash reduction paid by the lessee (cash or trade-in value of the old car) or by the manufacturer as a rebate.

the unamortized contractual cost of usage rights and its market value or zero, whichever is greater. The lessee has an incentive to terminate the lease early if the actual (i.e., market) rate of depreciation in the value of the leased vehicle is faster than the contractual rate of depreciation. Since the prepayment penalty is set equal to the ex post difference between the two depreciation rates, the lessee can never profit from a decrease in the market value of the leased vehicle. The penalty ensures that any increase in the market value of the leased vehicle goes back to the owner of the asset; thus, the lessor is made whole for any excessive economic depreciation over the term of the lease contract.

Notice that the early-termination payoff shown on the right-hand side of the *Penalty* equation is similar to the exercise value of a put option on usage rights bought by the lessee. The contractual value of unused usage rights, $OLP - UIC$, denotes the strike price of the put, while the market value of usage rights, SUR, captures the value of the underlying asset (in excess of X) at the time the put option is exercised. Since the early-termination penalty is determined and payable only upon termination of the lease contract, and not at the time the lease is signed ($t = 0$), the value of the early-termination option must be zero under the assumption of fixed interest rates. In other words, in most automobile leases the lessee's flexibility with respect to early cancellation of the contract is structured not as a valuable option in the sense of the typical derivative contract, but simply as a penalty.[5]

This result is in sharp contrast to the theoretical model of McConnell and Schallheim (1983), who also assume a constant interest rate. In their model, the lessee purchases the cancellation option by paying a fee in advance as part of each lease payment and has a chance to profit from canceling the existing lease contract if the market price of the leased car drops sharply. However, in practice, the lessee is not allowed to buy the cancellation option in advance, and the penalty imposed upon cancellation completely offsets the gain due to the drop in the market value of the leased automobile. Thus, the "ex post settling up" or "pay-on-termination" clause of the cancellation option is a very important feature of real-world automobile leases, as the penalty renders the cancellation option worthless under the standard assumption of constant interest rates.

For a numerical example, consider a car with an adjusted capitalized cost (ACC) equal to \$30,000. If the residual value X in a 30-month lease is set at \$15,000, then the value of the lessee's usage rights at $t = 0$ is $ACC - X =$ \$15,000. The lessor retains the residual value of \$15,000. Now suppose that in 15 months the market value of the used car A_t drops to \$20,000. This results in a market value of usage rights ($A_t - X$) equal to \$5,000. The unamortized value of the contractual usage rights is \$7,500 = (15/30) $\times$ [\$30,000 – \$15,000]. Since the car has depreciated in market value faster than at the contractual rate, the lessee has an incentive to cancel the lease. If the lessee chooses to cancel the lease, she will face an early-termination penalty equal to \$2,500. Thus, the

[5] There exists a similar penalty in commercial real estate mortgage markets. Quite often commercial mortgages include a feature known as "yield maintenance," which is basically a prepayment penalty triggered when a borrower pays off the mortgage prior to maturity. The yield maintenance is designed such that the lender is entitled to receive the current market value of the loan.

early-termination penalty ensures that the lessor receives the prorated value of usage rights under the contract.

	Summary of Leasing Terminology and Numerical Values	
A_0	Adjusted capitalized cost (ACC): cost of new vehicle adjusted for rebates, installed options, cash or trade-in value, etc.	\$30,000
X	Contract residual value (value of ownership rights): Exercise price of purchase option	\$15,000
$ACC - X$	(Total) Depreciation component of lease payment	\$15,000
$(ACC - X)/T$	Contractual depreciation rate per month	\$500
$ACC - A_t$	Cumulative market depreciation (as of time t)	\$10,000
$A_t - X$	Market value of usage rights at time t	\$5,000
$(ACC - X)(n/T)$	Unamortized contractual value of usage rights	\$7,500

To consider the effects of a slower rate of economic depreciation than the contractual rate, suppose that $A_t = \$25{,}000$ rather than \$20,000 in 15 months. In this case, *Penalty* = 0, but the lessee would not want to cancel the lease (such that he could keep the \$2,500 (the excess of *SUR* over *OLP* − *UIC*) in the form of below-market lease charges). To realize this value of \$2,500, the lessee would need to prepay the lease and purchase the leased vehicle from the lessor, the owner. In other words, to exercise this prepayment option, the lessee would have to incur the cost of a purchase option plus common transaction costs such as registration fees and sales tax.[6] As we point out later in this section, lease prepayments due to slower economic depreciation rates are rare in practice. Observed prepayments are predominantly attributable to the aggressive marketing strategy of using unduly high residual values (X) to lower monthly lease payments (see Astorina and Mrazek (2000), and Kravitt and Raymond (1995)).

Next, we consider the effect of a drop in the market interest rate on *Penalty,* while assuming that the market rate of depreciation is equal to the contractual rate of depreciation. Notice that in equation (3) both *OLP* and *UIC* are computed using the fixed contractual interest rate (i.e., the decrease in the interest rate does not alter *Penalty*). This results in a prepayment penalty equal to zero. Therefore, the lessee can profit from a drop in the market rate by canceling the existing lease with a higher interest rate and replacing it with a new contract struck at a lower rate. In general, if the decline in the market value of the leased car (in excess of the contractual depreciation rate) is less than the gain from the interest rate decrease, the lessee would benefit from early termination. In other words, if *Penalty* is less than the gain from the fall in the market interest rate, the cancellation option would have positive value. This event might occur when used car prices and interest rates are negatively correlated. However, if the two variables are positively correlated and the used car price falls sharply relative to the gain from the decrease in the interest rate, the lessee cannot profit from cancellation.

[6] In sharp contrast to the lease market, in a real estate mortgage the borrower owns the underlying property, and hence there is no need for him to buy the purchase option in order to exercise the prepayment option.

Theoretical modeling of the cancellation option under both price and interest rate uncertainty requires a two-factor model of lease pricing, which, as we note before, does not exist. In the hybrid securities area and the mortgage literature, a number of theoretical and empirical papers including Kau et al. (1992, 1995), Titman and Torous (1989), and Deng, Quigley, and Van Order (2000) stress the importance of jointly modeling the prepayment and default options. However, no closed-form solutions have been developed so far. McConnell and Schallheim (1983) point out the difficulties in modeling both the cancellation and default options within the context of leasing contracts.

In the remainder of this study, we will ignore interest rate risk and focus on the underlying asset price uncertainty. This treatment assumes that the cancellation option has little value, substantially simplifying the estimation of the purchase option value. We believe this is a reasonable assumption because of the widespread practice of offering cut-rate lease contracts. Offering sharply lower-than-market interest rates serves to minimize the interest rate component of the cancellation option. Moreover, lessees stand to forfeit security deposits and any capitalized cost reductions if they terminate the contract early. Whether ignoring interest rate uncertainty is completely satisfactory is an open question that we cannot resolve here. Nevertheless, sensitivity analysis under the one-factor model does permit us to examine the potential impact of errors in holding the interest rate constant.

A survey of the literature on lease securitizations reveals two relevant pieces of empirical evidence. First, lessors suffered average end-of-term (EOT—at lease expiration) losses of \$1,672 and \$2,592 in 1998 and 1999, respectively, per vehicle. Astorina and Mrazek (2000) and Kravitt and Raymond (1995) note that lessors suffered large residual value losses primarily because they used unduly large residual values (X in equation (3)) to lower monthly lease payments (referred to as aggressive subvention). Second, in the 1990s a large percentage of automobile lease contracts was terminated prior to scheduled maturity. For instance, at the industry level, the Association of Consumer Vehicle Lessors (ACVL (2000)), whose members account for an estimated 80% of all consumer vehicle leasing in the United States, reports that from 24% to 59% of outstanding 2- to 4-year lease contracts were terminated early over 1996 to 1999, on average. At the lease portfolio level, Kravitt and Raymond (1995) report that about 86% of lease contracts included in the World Omni Automobile Lease Securitization Trust terminated prior to the scheduled maturity date over the previous 3 years and 33% of 2-year leases and 42% of 3-year leases in the Ford Credit Auto Lease Trust prepaid between 1989 and 1992.

How do we reconcile the puzzling incidence of large early terminations in the face of large EOT residual value losses suffered by lessors? Notice that lessees would have little incentive to terminate their contracts voluntarily before the scheduled maturity date because they could avoid the loss by simply waiting to return the vehicles at the normal end of their lease contracts. However, the story for lessors is quite different; facing the specter of substantial end-of-term losses, manufacturers and dealers often adopted a proactive lease termination program to lower residual value risk (the risk of unexpected depreciation). These programs encouraged lessees to purchase, trade-in, or refinance leased vehicles in the last year of the contract for the sum of remaining principal payments, contractual residual value, and processing fees. Early termination reduces the frequency and severity of the residual value loss because the leased vehicle has had less time to depreciate.[7] Furthermore, when a lessee prepays, there typically is no residual value loss since he is liable to repay the full outstanding contract balance plus applicable fees and taxes. More importantly, we found little empirical evidence to refute our argument that the cancellation option as structured in automobile lease contracts has little value.

D. Noncancelable Lease Valuation with a Purchase Option

The analysis in the previous section suggests that the "pay-on-cancellation" clause has two important effects under the assumption of fixed interest rates: (1) It nullifies the value of the cancellation option, and (2) it allows us to treat the complex lease-end purchase option modeled by equation (2) as a simple European call option.

McConnell and Schallheim (1983) show that for a noncancelable lease with an option to buy at lease expiration, the NAL has the very simple form:

$$NAL = A_0 - S_0 - \sum_{t=0}^{T-1} PV_t(L^*) + C, \tag{4}$$

where C, the value of the purchase option, is given by

$$C = S_0 N(h_T + \sigma\sqrt{T}) - PV(X)N(h_T) \tag{5}$$

and $N(.)$ is the typical univariate standard normal distribution evaluated at $h_T = [\ln(S_0/X) + (\ln r - \sigma^2/2)T]/\sigma\sqrt{T}$. To illustrate the size of the monthly lease payments and the corresponding value of the purchase option, we continue the numerical example from Section C above. Again, we set A_0 equal to the adjusted capitalized cost of \$30,000 and the residual value X to \$15,000, and we assume that the depreciated value of the automobile in 30 months is the same as the residual value. To find S_0, we discount the depreciated car value to the present at the risk-free rate of 6%; thus, we are assuming a beta of zero. Last, we set the variance σ^2 at 0.0025. The value of the purchase option is \$2,048 and the monthly lease payments are \$738 without the option and \$808 with the purchase option (assuming perfectly competitive markets in which case the NAL is zero).

E. Real-World Market Imperfections

The theoretical model above is based on the standard assumption of frictionless capital markets: The underlying asset market is without moral hazard,

[7] For example, Astorina and Mrazek (2000, pp. 14–15) note, "The greater the number of leases that prepay, the less vulnerable a securitization is to residual value risk. This is due mainly to the stipulation that closed-end leases require the lessee to make whole the lessor upon early termination. Therefore, when a lease is prepaid, the risk of not realizing the booked residual value is largely eliminated."

information asymmetry, or other frictions such as default.[8] Furthermore, it views the lease-end purchase option as a stand-alone option. These assumptions imply that the option value estimates generated by this model represent frictionless market benchmarks for the value of the embedded purchase option.

Since automobile leases are typically unsecured legal contracts, there exist states of nature in which default is the optimal strategy for the lessee. However, because the leased vehicle is owned by the lessor, and can generally be recovered more quickly than lessee-owned assets, the default option is less valuable. In case of default, the lessor will attempt to seize the asset for quick disposal in the wholesale market in addition to legally pursuing an early-termination penalty plus legal costs. Lessors have other means to discourage default. It is common practice in the automobile leasing market to require both a security deposit and capitalized cost reductions in the form of cash, the trade-in value of the old car, or a rebate from the manufacturer. Since these will be forfeited upon default, they act as collateral and provide an additional incentive for the lessee to make the required lease payments.

In reality, as Astorina and Mrazek (2000) note, a small fraction of leases do default for a variety of reasons, and lessors use credit enhancements such as overcollateralization, surety, security deposits, reserve accounts, a spread over the riskless rate, subordination, and residual value insurance to minimize credit loss. Empirical evidence at the industry level shows net credit losses (after recoveries) averaged 48 and 49 basis points (bps), respectively, in 1999 and 2000 (see ACVL (2001)). At the firm or portfolio level, American Honda Finance Corporation (1999) disclosed in the Honda Auto Lease Trust 1999-A Prospectus covering about $3.5 billion of outstanding principal in leases of Honda and Acura vehicles that net repossession losses as a percentage of the average value of lease contracts outstanding for fiscal years 1996 to 1999 are 0.36%, 0.38%, 0.49%, and 0.38%, respectively. In contrast, in a sample of 425,000 conventional mortgages originated between 1975 and 1983, Avery et al. (1996) find an overall default rate of 2.16% with an average severity of loss equal to 39.2%. Assuming an average mortgage life of 7 years, we obtain a yearly average loss ratio of 12 bps. Using a similar data set from Freddie Mac, Lekkas, Quigley, and Van Order (1993) report slightly lower loss rates. For corporate bonds, the historical evidence reported by Keenan, Hamilton, and Berhault (2000) from 1970 to 1999 is 6 bps for Baa bonds, 68 bps for Ba bonds, and 333 bps for single B bonds. In sum, lease losses owing to default appear to fall between conventional mortgages and Ba-rated bonds.

However, these credit losses are substantially smaller than the residual value losses, that is, the difference between the resale value of returned vehicles and their contractual residual value, suffered by lessors. This is to be expected because only a small fraction of leases runs into default, but all leases are exposed to residual value risk. ACVL (2001) reports that average end-of-term residual value losses per vehicle are $1,672, $2,592, and $2,212, respectively, for 1998, 1999, and 2000. The Honda Auto Lease Trust 1999-A Prospectus discloses

[8] Quigley and Van Order (1995) examine empirically the importance of market frictions in the context of a contingent claim model of mortgage default.

that residual value losses as a percentage of residual values of returned vehicles sold by American Honda Finance Corp. for fiscal years 1996 to 1999 are 2%, 7%, 12%, and 14%. These huge residual value losses are indicative of the potential strategic errors made by lessors in setting the contractual residual value as well the fact that the primary source of risk in leasing is the uncertainty about the underlying vehicle price at lease termination. Therefore, first we focus on the valuation of the lease-end purchase option in the absence of default by the lessee, which hinges critically on the contractual residual value and the underlying vehicle price uncertainty. Subsequently, we adjust the base-case call value estimates for the possibility of default.

Many studies show that embedded options play an important role in durable goods markets characterized by moral hazard and asymmetric information problems. Used cars represent a classic example of markets subject to adverse selection: The owner of a used car knows more about its quality than does a potential buyer. Akerlof (1970) shows that adverse selection is an important source of inefficiency in trade.[9] Hendel and Lizzeri (2002) model the choice of manufacturers to market their output to consumers with heterogeneous valuations for product quality under two contracts, a sale contract and a lease contract that specifies a rental payment and a strike price at which the leased good can be purchased on termination of the lease. They show that the lease contract can add value because it allows manufacturers to better address information asymmetry problems by properly structuring the exercise price of the purchase option. Moreover, manufacturers can benefit by designing lease contracts to segment the market since different contracts appeal to consumers with different tastes for quality.[10] Smith and Wakeman (1985) suggest that the purchase option serves to mitigate the moral hazard problem by giving the user an incentive to take care of the asset.

The lack of empirical evidence on the value of the implicit options is perhaps due to the difficulty in estimating the key parameters of option value. Theoretical lease models compute the used car value by depreciating the initial cost (denoted by "adjusted capitalized cost" in lease contracts) at a stochastic rate. However, the capitalized cost of a vehicle often varies across lessors. Manufacturers sell their products through franchised independent dealers. While the manufacturer sets an MSRP and a dealer price, each dealer negotiates a net sale price with either the consumer or the independent leasing company

[9] Hendel and Lizzeri (2002) attribute the recent increase in leasing to the observed improvement in the durability of cars, that is, slower depreciation of cars. An increase in durability makes used cars better substitutes for new cars, but it worsens the adverse selection problem in the used car market.

[10] Hendel and Lizzeri (2002) show that under asymmetric information about quality, the lease contract enables lessors to better price the option of keeping the cars by manipulating the strike price. In their model, consumers who put more value on quality do not value the option to purchase used cars very much because they are not likely to keep the used cars. Therefore, these high-valuation consumers choose the lease contract. On the other hand, low-valuation consumers are more likely to keep the car as it ages. They choose to buy the car because they can retain the high realization of the used car's quality and the implicit price of the option of keeping the car is lower under the sale contract.

acting on behalf of the consumer. Thus, the actual car price used in computing monthly lease payments depends on the relative ability of the parties to negotiate. Incentives,[11,12] other charges that appear on a dealer invoice,[13] and other fees that the selling dealer charges to handle the paperwork make it difficult to estimate the original capitalized cost.[14] These extra costs complicate the process of obtaining a reliable estimate of new car prices and, in turn, of the depreciation rate. We circumvent these problems by obtaining three alternative estimates of the used car wholesale values to proxy for the value of the underlying asset.

II. Data and Empirical Analysis

A. Description of ALG and NADA Data

Lessees return three out of every four leased cars at the expiration of the lease (Hendel and Lizzeri (2002)). The lessor, typically a commercial bank or a finance company, is not in the business of retailing used vehicles; most likely, the lessor will dispose of the returned automobile in the wholesale used car market rather than sell it at a retail price. Accordingly, the wholesale value of a used car seems to be the appropriate choice to represent the value of the underlying asset for the purchase option.

Two primary sources of data on used car prices are Automotive Lease Guide (ALG)[15] and the N.A.D.A. Official Used Car Guide (NADA).[16] We obtain base-level wholesale used car prices from the Eastern Edition of NADA. The NADA prices are available monthly from November 1990 to November 2000 and are averages of used car sales prices reported by dealers. In addition, we collect annual used car residual values from the November/December Northern Edition of the ALG for 1995 through 2000. These residual values are published either as a percentage to be applied to the MSRP of a vehicle or in terms of absolute dollars as with the NADA used car prices.

The absolute dollar values are widely used by major leasing companies, both manufacturer subsidiaries and third-party independent lessors, as the best estimate of what the wholesale value is expected to be at the end of k years. The ALG residual values are based on an objective depreciation rate for the given model, based on the historical performance of that model, and subjective expert

[11] From time to time during a model run, the manufacturer may offer various incentives to the dealer and/or the retail consumer. These incentives alter the net discount structure and/or the true consumer cost.

[12] At various times, manufacturers also offer fleet incentives to lessors that either buy a specified number of vehicles during the model year run or own another specified number of vehicles in total. However, this incentive is supposed to be unavailable on a vehicle destined to be leased to a nonfleet-qualified end user such as an individual consumer. The manufacturers have variously enforced this restriction over time. When not enforced, the lessor would have the option of the fleet allowance or the retail rebate, whichever was greater, when available.

[13] Typical examples are a manufacturer's holdback and advertising charges.

[14] Advertising charges are normally excluded on leases made through independent third-party lessors, further clouding the original capitalized cost.

[15] Automotive Lease Guide (ALG), 2034 De La Vina St., Santa Barbara, CA 93105.

[16] N.A.D.A. Official Used Car Guide Company, 8400 Westpark Dr., McLean, VA 22102.

opinion of how the new model will fare relative to its competitors. ALG values do not account for unusual wear and tear and the direct expense of termination, both of which are lease-specific. These raw prices are adjusted for optional equipment and mileage to maintain constant quality over the sample period.

Automakers introduce quality and style improvements at the beginning of each model year. To minimize the effects of quality changes introduced by manufacturers across car vintages over time, we restrict our sample to only three of the best-selling nameplates in the United States during the 1990s: General Motors Saturn, Honda Civic, and Toyota Camry. These car models were dominant in their market class throughout the decade and are relatively comparable from one year to the next. This set also covers the most popular body style within each car model, and allows us to study both domestic and foreign brands.

In estimating the value of the purchase option based on both sources of used car price data, we use the ALG residual values to denote exercise prices. We also collect yields to maturity on Treasury notes from the Federal Reserve Bulletin. Over 1995 to 2000, these rates averaged 5.53%, 5.57%, and 5.59% for 2-, 3-, and 4-year terms, respectively.

B. Constant Quality Adjustments

Manufacturers introduce new car models typically by the month of November of the prior year. We consider a sample of k-year leases on brand new cars signed each November from 1995 to 2000. To price the embedded call, we need to estimate the volatility of (natural log) price changes. We construct a constant quality monthly time series of used car prices holding age fixed at k years. We then follow a process analogous to the formation of a fixed maturity time series of prices for a debt issue. For ease of exposition let us set $k = 2$. Suppose time t represents November 1991; at this point, 1990 model cars (introduced by November 1989) are 2 years old. We gather monthly prices for this 2-year-old car for the next 12 months. By November 1992, this car turns 3 years old and no longer qualifies for the 2-year price series. Therefore, we switch to monthly prices for a 1991 model car in November 1992 to maintain the term of the price series at approximately 2 years. We repeat this switching process every November until the end of the sample period. The price time series for 3- and 4-year-old cars are derived analogously.

We make the following two adjustments to the NADA and ALG price data to ensure constant quality. First, price adjustments are made to ensure that the physical characteristics of automobiles (e.g., automatic transmission, air conditioning) remain constant over the study period. Second, following ALG, we adjust reported prices to ensure that the 2-, 3-, and 4-year-old cars have odometer mileages of 30,000, 45,000, and 60,000, respectively.

C. Volatility Estimates from NADA Prices

Define the term of a lease to be k years, where $k = T/12$. Under a closed-end k-year lease contract on a new vehicle, the lessee has at lease expiration the option to buy a k-year-old car. While the term of consumer automobile leases

Table I

Summary Statistics on Used Car Price

This table reports on the distribution of NADA prices for k-year-old ($k = 2$, 3, and 4) used cars of constant quality for the sample period November 1990 through November 2000. The sample period includes car model years 1990 through 2000. The price series for Saturn begins April 1992, with its 1991 model year. All prices are averages from reporting automobile dealers. For each k, reported statistics are mean and standard deviation in the first row, and (median) and (number of observations) in the second row.

	$k = 2$		$k = 3$		$k = 4$		Total	
Model	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)
All	9,955	1,342	8,806	1,282	7,518	1,166	8,864	1,610
	(9,600)	(315)	(8,425)	(279)	(7,175)	(243)	(8,800)	(837)
Camry	11,459	902	10,248	894	8,824	800	10,286	1,377
	(11,675)	(109)	(10,625)	(97)	(9,025)	(85)	(10,375)	(291)
Civic	9,164	895	8,054	771	6,858	679	8,121	1,227
	(9,350)	(109)	(8,075)	(97)	(6,875)	(85)	(8,075)	(291)
Saturn	9,153	381	8,018	406	6,765	422	8,091	1,048
	(9,175)	(97)	(8,050)	(85)	(6,825)	(73)	(8,200)	(255)

can vary anywhere from 6 to about 66 months, we focus on the more common terms of 2, 3, and 4 years.

Table I presents summary statistics on NADA monthly wholesale constant quality prices for 2-, 3-, and 4-year-old cars for the three models from November 1990 to November 2000. The price series for Saturn begins in April 1992; hence, it has fewer observations than the other two models. Since the popular lease terms during our study period are 2 to 4 years, we limit our analysis to call options on 2-, 3-, and 4-year-old used cars.[17] From the second row of the Total column, we have a total of 837 monthly price observations across the three car models. The mean (median) price and standard deviation of constant quality used cars in our sample vary from \$7,518 (\$7,175) and \$1,166, respectively, for a 4-year-old car to \$9,955 (\$9,600) and \$1,342 for a 2-year-old car. The rest of the table presents price distribution statistics for each of the three car models.

As automobile companies launch their new models in the last quarter of each year, retail prices for new cars are higher near the beginning of the season (i.e., in November) and then decline as the season progresses (Pashigian, Bowen, and Gould (1995) and Pashigian (2001)). Wholesale used car prices exhibit a similar seasonal pattern.[18] The Toyota Camry provides a good example of the typical price pattern; Figure 1 displays the (natural) log of a 2-year-old used car

[17] The Association of Consumer Vehicle Lessors (ACVL, 2001) reports that the average term of all consumer leases has dramatically increased in the last few years. For 2000, the average term was 39.3 months, an increase of over 20% from 1997.

[18] Consistent with the decline in new car prices, the residual factor applied to MSRP tends to decline over the model year. For instance, Angel (1997) reports that party lessors were quoting a residual factor for a 2-year lease of 60% of MSRP for a new 1996 Chrysler Cirrus, but a leftover 1995 Chrysler Cirrus would have a 24-month residual factor of only 52%.

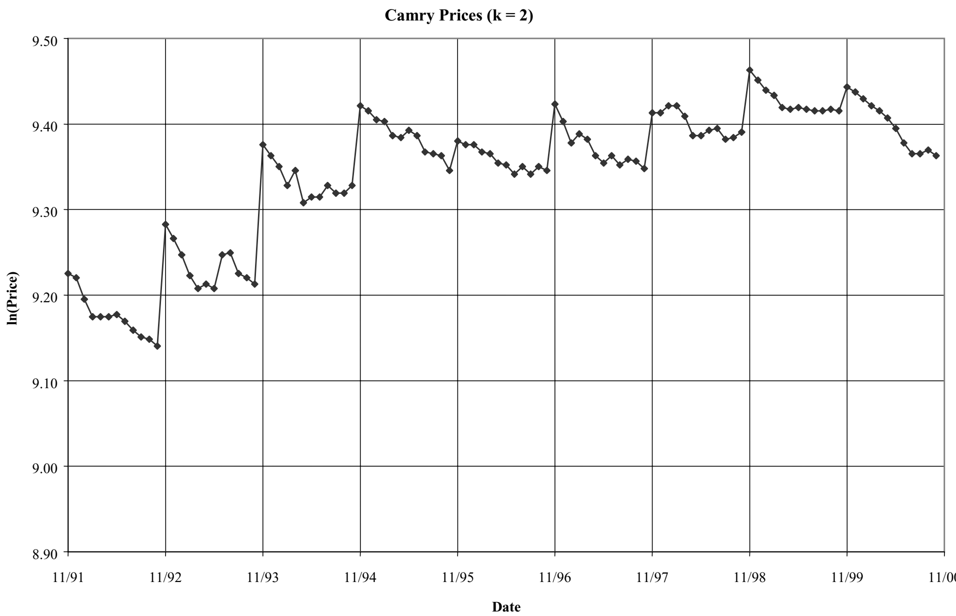


Figure 1. Price behavior of a constant maturity used car. This figure displays (natural) log monthly prices of a 2-year-old Camry from November 1991 through November 2000. The graph shows discontinuity due to pricing of the latest model in November of each year.

price in month t, from November 1991 to November 2000. In November of each year, when a new car model is introduced, the graph shows a discontinuity due to new pricing for the latest model.

The total variance of a used car price appears to consist of two components, namely, a monthly seasonal plus a sharp spike caused by the introduction of a new model. Moreover, while the amount of within-year depreciation seems to be fairly stable throughout the sample period, the November spike appears to be much smaller in the latter part of the sample. To incorporate both sources of variation, we take the differencing interval to be 1 year and calculate the continuously compounded annual percentage price change from overlapping observations. We use the NADA monthly constant quality used car prices from November 1990 to November 1995 to estimate σ, which is used to price a call option at the signing of k-year leases in November 1995. We generate rolling estimates of σ utilizing all price data available up to t.

In Table II, we report estimates of the annual standard deviations of percentage changes in NADA prices for k-year-old cars. For example, a 1990 Camry turns 2 years old in November 1991 and continues through November 2000, yielding 97 monthly differences.[19] The volatility estimates are derived from overlapping annual percentage changes in the NADA monthly price series prior to the signing of a k-year lease contract.

[19] The price series for Saturn begins from April 1992, resulting in 84 price change observations.

Table II

Estimates of Annual Standard Deviations

This table presents standard deviations (*SD*) of percentage price changes in the NADA price series based on overlapping monthly observations for the full data set covering November 1990 through November 2000; k represents the lease term in years, and N denotes the number of monthly observations.

	$k = 2$		$k = 3$		$k = 4$		Total	
Model	N	SD	N	SD	N	SD	N	SD
All	278	0.0407	242	0.0555	206	0.0711	726	0.0556
Camry	97	0.0440	85	0.0654	73	0.0719	255	0.0604
Civic	97	0.0324	85	0.0452	73	0.0695	255	0.0498
Saturn	84	0.0392	72	0.0512	60	0.0560	216	0.0490

Across all models, our estimates increase from an annual standard deviation of 4.07% for a 2-year-old car to 7.11% for a 4-year-old car. Camry has the largest standard deviation of the three models, while Civic has the lowest for the 2- and 3-year leases. For all the three makes, the older the used car, the greater the standard deviation. To put these estimates in perspective, Ibbotson Associates (2000) report the following annual standard deviation estimates of realized returns for different asset classes: small company stocks, 33.6%; large company stocks, 20.1%; long-term corporate bonds, 8.7%; long-term government bonds, 9.3%; 1-year U.S. Treasury bills, 3.2%; and annual inflation rates, 4.5%. Quigg (1993) finds that implied annual standard deviations for individual commercial property range from 18% to 28%. Thus, used car prices in our sample are far less volatile than common stock prices or real estate prices; their volatility seems closer to that of intermediate-term government and corporate bonds.

D. Purchase Option Estimates Based on NADA Used Car Prices

We report the average value of the purchase option across all car models with k years to expiration in Table III. These estimates are based on constant quality NADA prices as proxies for the current used car price S_0 and the corresponding ALG residuals to represent the strike price X in the MS model (equation (5)). As we explain earlier, we construct k-year leases (k = 2, 3, and 4) for each of the three new car models in November of each year t (from 1995 to 2000). For Camry and Civic, we have six observations for each of the k-year maturity calls (corresponding to car model years 1995 through 2000), but we have only five observations on 4-year leases for Saturn. In all we have a total of 53 estimates of call values.

From Panel A, the grand mean (median) call value is $1,462 ($1,493). On average, the mean value of the call option is 15.7% of the underlying asset price, with a median of 15.3%. The overall standard deviation estimate of 5.5% indicates considerable variability in call value estimates across car models and lease terms. Expressed as a ratio of the strike price (ALG residual value), the

Table III

Estimates of Call Option Values Using NADA Used Car Prices

This table reports C, the call option value estimates using the Black–Scholes model; their values as percentages of S, the underlying asset (used car) price, and of X, the residual value; and their decomposition into intrinsic and time value components for November 1995 through November 2000. For each k, the lease term in years, reported statistics are mean and standard deviation in dollars in the first row, and (median, $) and (number of observations) in the second row.

	$k = 2$		$k = 3$		$k = 4$		Total	
Variable	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)
				Panel A: All Car Models				
C	1,158	422	1,639	396	1,597	510	1,462	488
	(1,166)	(18)	(1,621)	(18)	(1,598)	(17)	(1,493)	(53)
C/S	10.9%	3.9%	17.1%	3.5%	19.2%	5.1%	15.7%	5.5%
	(10.8%)	(18)	(16.1%)	(18)	(20.2%)	(17)	(15.3%)	(53)
C/X	11.1%	4.4%	17.6%	4.3%	19.2%	6.0%	15.9%	6.0%
	(10.9%)	(18)	(16.1%)	(18)	(20.0%)	(17)	(15.6%)	(53)
Intrinsic value	210	270	240	279	119	201	190	253
	(88)	(18)	(150)	(18)	0	(18)	(25)	(53)
Time value	948	239	1,399	239	1,470	420	1,269	383
	(1,019)	(18)	(1,354)	(18)	(1,515)	(17)	(1,237)	(53)
				Panel B: Camry				
C	1,108	506	1,848	351	1,948	325	1,635	540
	(1,099)	(6)	(1,699)	(6)	(2,040)	(6)	(1,699)	(18)
C/S	9.0%	4.0%	16.6%	3.1%	20.0%	3.4%	15.2%	5.8%
	(8.6%)	(6)	(15.7%)	(6)	(21.2%)	(6)	(15.6%)	(18)
C/X	9.0%	4.5%	16.8%	3.7%	20.0%	3.9%	15.2%	6.1%
	(8.5%)	(6)	(15.6%)	(6)	(21.2%)	(6)	(15.6%)	(18)
Intrinsic value	121	296	167	275	92	147	126	235
	0	(6)	0	(6)	(25)	(6)	0	(18)
Time value	988	301	1,681	128	1,856	260	1,508	447
	(1,099)	(6)	(1,668)	(6)	(1,924)	(6)	(1,632)	(18)
				Panel C: Civic				
C	1,349	389	1,653	419	1,497	496	1,500	430
	(1,536)	(6)	(1,572)	(6)	(1,348)	(6)	(1,454)	(18)
C/S	13.2%	3.5%	18.3%	3.9%	19.3%	5.3%	16.9%	4.9%
	(14.7%)	(6)	(18.1%)	(6)	(18.4%)	(6)	(15.8%)	(18)
C/X	13.7%	4.0%	19.1%	4.8%	19.4%	6.5%	17.4%	5.6%
	(15.6%)	(6)	(18.7%)	(6)	(17.8%)	(6)	(16.3%)	(18)
Intrinsic value	383	306	354	326	163	252	300	296
	(500)	(6)	(350)	(6)	0	(6)	(350)	(18)
Time value	966	134	1,299	122	1,335	265	1,200	244
	(928)	(6)	(1,310)	(6)	(1,348)	(6)	(1,169)	(18)
				Panel D: Saturn				
C	1,016	357	1,416	350	1,294	536	1,239	426
	(1,116)	(6)	(1,400)	(6)	(1,515)	(5)	(1,195)	(17)
C/S	10.5%	3.6%	16.6%	3.9%	18.1%	7.2%	14.9%	5.8%
	(11.8%)	(6)	(16.4%)	(6)	(20.2%)	(5)	(12.8%)	(17)
C/X	10.6%	3.9%	16.9%	4.7%	18.1%	8.4%	15.0%	6.4%
	(11.9%)	(6)	(16.5%)	(6)	(20.0%)	(5)	(12.4%)	(17)
Intrinsic value	125	111	200	242	104	220	143	192
	(125)	(6)	(125)	(6)	0	(6)	(63)	(18)
Time value	891	283	1,216	124	1,169	412	1,088	308
	(1,019)	(6)	(1,216)	(6)	(1,341)	(5)	(1,071)	(17)

grand mean and median call value estimates are 15.9% and 15.6%, respectively. The untabulated estimates show the option value ranges from 3% to 28% of the value of the underlying used car price in the subsamples for the three brands. In 24 of the 53 estimates, the intrinsic value of the call option is zero (not reported), indicating that the current used car price S_0 is less than or equal to the strike price X. The median intrinsic value of the purchase option, \$25, is small relative to the median time value of \$1,237. This decomposition shows that the value of the option stems primarily from return volatility and the term of the call, rather than the difference between S_0 and X. Furthermore, our overall estimates show that the value of call options increases from 10.9% of the underlying asset price, for a 2-year term, to 19.2% for a 4-year term to expiration.

Panels B through D present call value estimates for the three car makes in our sample. The median estimates of the call option as percent of the underlying used car price are 15.6%, 15.8%, and 12.8%, respectively, for Camry, Civic, and Saturn. Within each car type, our estimates tend to increase with the term to maturity of the k-year lease/call. Clearly, these call value estimates are much larger than those reported by Angel (1997) and Miller (1995).

E. Estimates Based on ALG Data

The previous set of estimates uses the NADA wholesale prices and ALG residuals to proxy for S_0 and X, respectively. In this subsection, we use the ALG constant quality values to proxy for both S_0 and X; this assumption yields estimates of values of at-the-money calls.[20] Unlike monthly NADA data over the 10 years from November 1990, we collect ALG residual values only at annual intervals from November 1995 through November 2000. This yields six annual observations for each $k = 2$, 3, and 4 years for each of the three car makes, resulting in 54 observations. From Panel A of Table IV, the overall mean (median) ALG constant quality used car price in our sample is \$9,466 (\$9,338). Given the small number of observations, we use ex post standard deviations of percentage changes in used car prices. From Panel B, our estimates of annual standard deviations vary from 3.78% to 6.07% across the sample.

From Table V, the grand mean (median) of the value of at-the-money calls is \$1,420 (\$1,364). These estimates vary from a minimum of \$878 to a maximum of \$2,128 (not reported). Expressed as a percent of the underlying used car price (which is set equal to the strike price X), the overall mean and median estimates are 15.4% and 16%, respectively. Furthermore, for all the three car makes, average option value estimates increase monotonically across the term of the option, $k = 2$, 3, and 4 years. The remaining entries show that the estimates across Camry, Civic, and Saturn are similar to those based on the NADA used car prices.

[20] ALG residuals may also be viewed as forecasts of wholesale values k years into the future. In this case, S_0 must be set equal to the present value of ALG. Detailed results are available from the authors.

Table IV

ALG Used Car Prices and Annual Standard Deviation Estimates

Panel A reports summary statistics on residual values obtained from ALG for November 1995 through November 2000. For each k, the lease term in years, reported statistics are mean and standard deviation in dollars the first row and (median, \$) and (number of observations) in the second row. Panel B presents annual standard deviations (*SD*) of percentage price changes based on log differences of the ALG exercise price series; N stands for number of observations.

Panel A: Residual Values

	$k=2$		$k=3$		$k=4$		Total	
Model	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)
All	10,689	1,408	9,393	1,274	8,315	1,158	9,466	1,596
	(10,038)	(18)	(8,825)	(18)	(7,775)	(18)	(9,338)	(54)
Camry	12,538	512	11,063	467	9,813	443	11,138	1,230
	(12,650)	(6)	(11,125)	(6)	(9,850)	(6)	(11,125)	(18)
Civic	9,854	377	8,683	306	7,746	213	8,761	933
	(9,850)	(6)	(8,700)	(6)	(7,738)	(6)	(8,700)	(18)
Saturn	9,675	407	8,433	395	7,388	449	8,499	1,039
	(9,638)	(6)	(8,488)	(6)	(7,600)	(6)	(8,488)	(18)

Panel B: Standard Deviations

	$k=2$		$k=3$		$k=4$		Total	
Model	*N*	*SD*	*N*	*SD*	*N*	*SD*	*N*	*SD*
All	18	0.0523	18	0.0487	18	0.0489	54	0.0499
Camry	6	0.0487	6	0.0481	6	0.0522	18	0.0497
Civic	6	0.0607	6	0.0602	6	0.0544	18	0.0584
Saturn	6	0.0474	6	0.0378	6	0.0400	18	0.0417

F. Estimates Based on a Used Car Price Index

In estimating the embedded call option values reported earlier, we use the average wholesale NADA and ALG values to proxy for the current used car price, S_0. To scrutinize the robustness of these estimates, we now turn to an alternative model of used car prices. This methodology is based on the hedonic pricing model of Griliches (1961) and Rosen (1974). Let S_t be the (natural) log of the constant quality used car price for month t. We use the following regression to obtain a used car price index for each of the three car models:

$$S_t = \alpha UsedAutoCPI_t + \sum_{k=2}^{4} \beta_k DTerm(k)_t + \sum_{m=1}^{12} \gamma_m DMonth(m)_t + \varepsilon_t. \quad (6)$$

The variable *UsedAutoCPI* is the component for Used Auto Prices published by the U.S. Department of Labor Bureau of Labor Statistics' Consumer Price Index (CPI). We use this variable as a proxy for the strength of the overall used car market. The term $DTerm(k)_t$ is a dummy variable for whether the car is a

Table V

Estimates of Call Option Values Using ALG Used Car Prices

This table reports C, the call option value estimates using the Black–Scholes model, and their values as percentages of S, the underlying asset (used car) price, for November 1995 through November 2000. For each k, the lease term in years, reported statistics are mean and standard deviation in dollars in the first row, and (median, \$) and (number of observations) in the second row.

	$k = 2$		$k = 3$		$k = 4$		Total	
Variable	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)
	Panel A: All Car Models							
C	1,142	165	1,453	215	1,666	252	1,420	302
	(1,109)	(18)	(1,400)	(18)	(1,604)	(18)	(1,364)	(54)
C/S	10.7%	0.8%	15.5%	1.2%	20.1%	1.6%	15.4%	4.0%
	(10.9%)	(18)	(16.0%)	(18)	(20.5%)	(18)	(16.0%)	(54)
	Panel B: Camry							
C	1,330	108	1,707	120	1,965	126	1,668	290
	(1,329)	(6)	(1,709)	(6)	(1,974)	(6)	(1,709)	(18)
C/S	10.6%	0.8%	15.5%	1.2%	20.1%	1.7%	15.4%	4.2%
	(10.9%)	(6)	(16.0%)	(6)	(20.6%)	(6)	(16.0%)	(18)
	Panel C: Civic							
C	1,070	97	1,356	117	1,556	130	1,327	232
	(1,087)	(6)	(1,378)	(6)	(1,585)	(6)	(1,321)	(18)
C/S	10.9%	0.8%	15.6%	1.2%	20.1%	1.6%	15.5%	4.1%
	(11.1%)	(6)	(16.1%)	(6)	(20.6%)	(6)	(16.1%)	(18)
	Panel D: Saturn							
C	1,025	76	1,297	103	1,477	137	1,266	217
	(1,039)	(6)	(1,352)	(6)	(1,541)	(6)	(1,306)	(18)
C/S	10.6%	0.9%	15.4%	1.3%	20.0%	1.7%	15.3%	4.1%
	(10.9%)	(6)	(15.9%)	(6)	(20.5%)	(6)	(15.9%)	(18)

2-, 3-, or 4-year-old automobile (an indicator variable for the age of the car in years); these dummy variables serve the purpose of allocating a particular price to the age-appropriate series. Alternatively, one may think of $DTerm(k)_t$ as a hedonic characteristic to account for time in service. The variable $DMonth(m)_t$ is a calendar month dummy equal to one if t falls in month m, and zero otherwise (to capture seasonality in the used car automobile prices). Since we have already adjusted the price S_t for mileage and optional equipment, we do not include any other hedonic characteristics. The variance of the first differences $\hat{\gamma}_m - \hat{\gamma}_{m-1}$ provides an estimate for the volatility of a k-year-old car; we use this variance and the exponential of $\hat{S}_0$ as inputs into the MS purchase option model.

From Panel A of Table VI, the overall mean used car price is \$242 less than the mean ALG price reported in Panel A of Table IV. The results for each of the car makes reported in the remaining panels indicate that the price index-based

Table VI

Summary Statistics on Used Car Prices and Standard Deviations Based on the Used Car Price Index

Panel A reports on the distribution of expected prices for k-year-old ($k = 2, 3, 4$) used cars of constant quality. For each k, the lease term in years, reported statistics are mean and standard deviation in dollars in the first row and (median, \$) and (number of observations) in the second row. Panel B reports annualized standard deviations derived from the price series that incorporates the CPI for used cars, the Used Car Price Index model.

Panel A: Summary Statistics

	$k = 2$		$k = 3$		$k = 4$		Total	
Model	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)
All	10,698	1,361	9,226	1,194	7,747	968	9,224	1,682
	(9,974)	(18)	(8,503)	(18)	(7,194)	(18)	(9,012)	(54)
Camry	12,520	350	10,832	309	9,037	331	10,796	1,496
	(12,577)	(6)	(10,861)	(6)	(9,012)	(6)	(10,861)	(18)
Civic	9,960	302	8,476	292	7,031	260	8,489	1,259
	(9,974)	(6)	(8,454)	(6)	(7,002)	(6)	(8,454)	(18)
Saturn	9,613	195	8,369	126	7,174	64	8,385	1,033
	(9,591)	(6)	(8,373)	(6)	(7,156)	(6)	(8,373)	(18)

Panel B: Standard Deviations

Model	'90–'95	'90–'96	'90–'97	'90–'98	'90–'99	'90–'00
Camry	0.0149	0.0145	0.0141	0.0144	0.0130	0.0133
Civic	0.0136	0.0138	0.0114	0.0139	0.0136	0.0145
Saturn	0.0205	0.0150	0.0110	0.0134	0.0111	0.0096

estimates are lower on average than the corresponding ALG values by a few hundred dollars. From Panel B, the annualized standard deviations of monthly percentage price changes are much lower than those obtained from the NADA and ALG data sources. The lower price volatility is attributable to two sources. First, the model basically averages together all the November prices to obtain $\hat{\gamma}_1$, and so on for the remaining months. As a result, some of the intrayear volatility is lost. Second, the use of the CPI for used cars explains quite a bit of the variation in the NADA price series. While the used car CPI is helpful in predicting the k-year-old car value at the inception of the lease, it takes away variation from the monthly time dummy coefficients.

We re-estimate call option values with $\hat{S}_0$ as a proxy for automobile value while continuing to use the corresponding ALG residual values to represent the strike price X. In Table VII, the overall mean (median) value of the purchase option across all lease terms and car makes is \$1,167 (\$1,182), which amounts to 12.8% (12.8%) of the current used car price. This is roughly 2% to 3% less than the mean estimates based on the NADA and ALG data. The remaining results show that our estimates are lower across all car makes and lease terms. In summary, our estimates vary from about 13% to 16% of the underlying used

Table VII
Call Values Derived from Used Car Prices Based on the Used Car Price Index

This table reports C, the call option value estimates using the Black–Scholes model; their values as percentages of S, the underlying asset (used car) price, and of X, the residual value; and their decomposition into intrinsic and time value components for all three car models based on the price series using the Used Car Price Index. For each k, the least term in years, reported statistics are mean and standard deviation in the first row and (median) and (number of observations) in the second row.

	$k = 2$		$k = 3$		$k = 4$		Total	
Variable	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)
C	1,129	450	1,276	442	1,095	436	1,167	441
	(1,182)	(18)	(1,274)	(18)	(1,081)	(18)	(1,182)	(54)
C/S	10.5%	3.9%	13.8%	4.3%	14.2%	5.4%	12.8%	4.8%
	(10.6%)	(18)	(14.2%)	(18)	(14.3%)	(18)	(12.8%)	(54)
C/X	10.7%	4.3%	13.8%	5.0%	13.5%	6.0%	12.7%	5.2%
	(10.6%)	(18)	(13.9%)	(18)	(13.3%)	(18)	(12.8%)	(54)
Intrinsic value	173	259	87	203	32	136	97	210
	(25)	(18)	0	(18)	0	(18)	0	(54)
Time value	956	275	1,189	340	1,063	391	1,069	346
	(1,010)	(18)	(1,252)	(18)	(1,081)	(18)	(1,051)	(54)

car price under different assumptions about S_0, X, and σ. These estimates are substantially higher than the 3% to 6% reported in previous studies.

III. Sensitivity Analysis

This section begins with a basic sensitivity analysis of the value of the call option to key parameters of the model. We then present revised estimates of the call option for default by the lessee.

A. Basic Sensitivity of Call Values

There are several benefits to examining the effects of the structure of the embedded purchase option on its value. First, sensitivity analysis allows us to perform a what-if analysis to assess the impact of changing the values of the parameters on call prices C. Second, our estimates of the three parameters (current value of the leased automobile (S_0), strike price (X), and volatility of used car price changes (σ)) are subject to error. It is important to scrutinize the sensitivity of the reported purchase option values to potential errors in the parameter estimates. Third, although the structure of the purchase option is irrelevant in the absence of market frictions, lessors can manipulate some of the parameters strategically in the presence of moral hazard and information asymmetry (see, for example, Hendel and Lizzeri (1999, 2002)). Toward this

Table VIII

Sensitivity Analyses of NADA Price-Based Call Values

This table presents average sensitivities of the call price across the three car brands to the five input variables, the underlying asset (used car) price (S), the exercise price (X), the standard deviation, the risk-free rate, and time. The sensitivities of the call price reported are *Delta*, the rate of change with the price of the underlying asset; $\partial C/\partial X$, the rate of change with the exercise price; *Vega,* the rate of change with the volatility; *Rho*, the rate of change with the risk-free rate; and *Theta*, the rate of change with time. For each k, the lease term in years, reported statistics are mean and standard deviation in the first row, and (median) and (number of observations) in the second row.

	$k = 2$		$k = 3$		$k = 4$		Total	
Variable	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)
Delta	0.94	0.12	0.95	0.04	0.92	0.08	0.93	0.09
	(0.98)	(18)	(0.95)	(18)	(0.96)	(17)	(0.9648)	(53)
$\partial C/\partial X$	−0.83	0.12	−0.79	0.04	−0.72	0.07	−0.78	0.09
	(−0.86)	(18)	(−0.79)	(18)	(−0.75)	(17)	(−0.79)	(53)
Vega	1,356	1,760	1,676	1,335	2,026	1,719	1,680	1,606
	(633)	(18)	(1,812)	(18)	(1,726)	(17)	(1,133)	(53)
Rho	17,693	2,854	22,188	2,362	24,002	3,853	21,243	4,026
	(17,425)	(18)	(21,429)	(18)	(23,455)	(17)	(20,708)	(53)
Theta	−504	78	−431	62	−355	62	−431	90
	(−505)	(18)	(−416)	(18)	(−352)	(17)	(−425)	(53)

end, in Table VIII we present the comparative statics results for the purchase option estimates reported earlier in Table III.

From the first two rows, the grand mean (median) of *Delta*, the partial derivative of C with respect to S_0 equal to $N(h_k + \sigma\sqrt{k})$, is 0.93 (0.96). That is, if we underestimate the current price of a used car by a dollar, on average that error will depress the value of the purchase option by \$0.93. Similarly, the grand mean (median) of $\partial C/\partial X$ (given by $-e^{-rk}N(h_k)$) is 0.78 (0.79). Thus, if the strike price is overestimated by a dollar, the value of the call will be reduced by \$0.78 (0.79). In practice, manufacturers view the exercise price of the purchase option as a strategic variable and authorize their financial subsidiaries to tactically vary it from the ALG residual values, promising that the manufacturing division will absorb any future loss. For example, faced with low demand (during a recession and/or for slow-moving models), automobile manufacturers often resort to raising residuals used in lease programs in lieu of cash rebates to sell the product. Manufacturers or their captives occasionally use up to 110% of ALG values, even though this may cause dramatic "losses" for the leasing subsidiary.[21] It is not uncommon for independent lessors to use aggressive

[21] The Association of Consumer Vehicle Lessors (ACVL) reports (August 3, 2001) weighted average losses on leased cars returned at termination of \$1,672, \$2,592, and \$2,212 for 1998, 1999, and 2000, respectively. This reflects both residuals higher than ALG suggested and weak used car markets. Note that the unspecified portion due to higher-than-ALG suggested values is billed back to the manufacturer.

(higher than ALG) residuals to lower monthly lease payments to gain market share.[22]

It is important to note, however, that this tactic generates residual risk in the presence of agency and informational asymmetry about the quality of used cars. That is, it raises the risk of the market values of vehicles falling below the residual values used in lease contracts (i.e., increases the probability that the purchase option expires out-of-the-money) and of incurring expenses associated with vehicle remarketing.[23] In addition, using a strike price that is higher than the expected market value of a used car aggravates the moral hazard (in maintenance) problem faced by the lessee—it distorts the incentives of the lessee to maintain the leased asset in good order (Smith and Wakeman (1985) and Waldman (1997)).[24]

The sensitivity of the purchase option to the volatility of used car price changes, $\partial C/\partial\sigma$, is known as *Vega* (equal to $S_0\sqrt{k} \times N'(h_k + \sigma\sqrt{k})$). The grand mean (median) of *Vega* is 1,680 (1,133). In Table II, our annual price volatility estimates vary from 3.24% to 7.19%. These estimates are comparable to those reported by Pashigian (2001) for his sample of Kelley's Blue Book trade-in values and the used car CPI. Suppose the midpoint value of 5.25% underestimates the level of true price volatility by 1%. This implies the average value of the call is downward biased by \$16.80, holding other things constant. Employing a hedonic model that prices the physical characteristics of used cars, Gordon (1990) finds the average price volatility over 1975 to 1983 is higher, at 8.5%. Although this level of volatility seems to be an overestimate for our sample period, a rough estimate of the impact of such a large increase on the value of calls is on the order of about \$55, on average.

Since automobile manufacturers and dealers often offer cut-rate or below-market financing rates to boost sales, it is also of interest to examine the sensitivity of C with respect to r, known as *Rho* ($Rho = kXe^{-rk}N(h_k)$). This shows that the call value is positively related to the risk-free rate. In the standard stock-based option valuation, the interest rate is an exogenous, market-wide parameter. However, it is endogenous to the lease pricing process since the lessor and lessee can negotiate the rate. By offering a cut rate, instead of charging the market (risk-free) rate, the lessor deliberately underprices the call option to reduce monthly lease payments. From Table VIII, the grand mean estimate of *Rho* is 21,243, indicating that for a 1% decrease in the risk-free rate (say from 6% to 5%), the value of the call decreases by \$212.43 on average. In contrast,

[22] *Vehicle Leasing Today*, Spring 2001: Bankers panel: Judgment day for residuals or opportunity for the strong? Association of Consumer Vehicle Lenders; www.acvl.com.

[23] The Association of Consumer Vehicle Lessors reports that 92% of leased vehicles are returned on average and the average loss per returned vehicle is \$2,212 for 2000 (ACVL.com, press release 8/3/01).

[24] Waldman (1997) provides an alternative explanation for setting a higher residual price in a lease contract. In his model, a monopolist can sometimes maximize long-run profits by eliminating the market for secondhand durable goods so that she can charge higher prices for new units. She can achieve this by selling her output and offering to repurchase old machines at high prices, such that old vintages are removed from the market prior to when consumers would have an incentive to retire them.

as we note in Section I, the value of the cancellation option increases as the market interest rate falls. This means that by offering a cut-rate lease contract, the lessor seeks to reduce the incentive of the lessee to cancel the lease prematurely. Thus, the widespread use of cut-rate leases serves to alleviate the undervaluation of the cancellation option under the one-factor model.

In addition to lowering the financing rate, the lessor and lessee may agree to increase the term of the lease, say from 2 to 3 years, to work out a mutually acceptable lower level of monthly lease payments. Since cars depreciate at a faster rate in market value in initial year, shorter-term leases tend to have higher monthly payments relative to longer-term leases. However, we know that a European call option usually becomes more valuable as the term to expiration increases. Therefore, the lessor must consider the increased cost of the call option while offering a longer-term lease contract. From Table VIII, the grand mean of *Theta*, the sensitivity of the purchase option to its term to expiration, is −431. Thus, if the lessor increases the lease term by a full year so as to lower monthly lease payments, then the value of the embedded purchase option would increase by approximately \$431 (this estimate is accurate only for a small change in k). Alternatively, if the lessee decides to prepay the lease 1 year prior to the scheduled maturity date, the value of the call would decrease by about \$431.

The term of the lease, that is, the level of lease durability, has other important implications. Aghion and Bolton (1987) show that a manufacturer with market power can use a long-term lease coupled with a penalty clause for early return of the leased asset to deter entry by competitors. Waldman (1996) argues that a monopolist may have an incentive to reduce lease durability to remove old units (which serve as potential substitutes for new units) from the market. Clearly, a lessor has to evaluate all these financial and real effects in deciding on the optimal term of a lease contract.

B. *Adjustment of Call Values for Default Risk*

The purchase option valuation model in equation (5) and the corresponding estimates discussed earlier assume thus far that the related lease contract is default-free. However, like most private and public debt contracts, an automobile lease contract is exposed to the risk of default by the lessee. If the lessee commits a default on the outstanding lease payments, the purchase option becomes null and void and its value drops to zero immediately regardless of the rate of recovery on the defaulted instrument. Thus, the essence of default risk in this context is that the lessor ties the value of the call option to the lessee's performance on the lease contract. As the binomial tree in the Appendix illustrates, this tie-up reduces the value of the call to zero in the default states. Evidently, the call option conditional on the lessee's performance on the lease contract is always less valuable than an equivalent unconditional call. Therefore, the standard Black–Scholes option model overestimates the value of a lease-end purchase option.

As the Appendix illustrates, we extend Jarrow and Turnbull (JT, 1995) to account for the possibility of default by the lessee (see also Hull and White

(1995)). JT analyze two types of credit risk, namely, the asset underlying a derivative security may default and the writer of the derivative security may default. They show that the price of a call option on a risky zero-coupon bond written by a risky writer is equal to the price of an option written by a default-free writer discounted by the expected payoff from the risky writer. In contrast, we focus on default by the lessee, who owns the purchase option. Furthermore, in our context, the asset subject to default, the lease, is not the asset underlying the purchase option. To keep the exposition simple, we continue with a two-period three-date lease contract, where $t = 0$, 1, and 2. The lessee makes the initial ($t = 0$) lease payment, but the second payment due at $t = 1$ is subject to a default probability of $\lambda\mu$. As in JT, we assume independence among the lease default process, the underlying used car price process, and default-free spot interest rate process. The binomial tree shows that the call has a zero value in default states at $t = 1$ (which persists in $t = 2$), and its value in nondefault states equals that of a default-free call. The value of the call at $t = 0$ is given by

$$C(0) = \frac{\nu(0, T)}{P(0, T)} C^*(0) = e^{-y(T)\times T} C^*(0), \tag{7}$$

where $C(0)$ denotes the value of a call option subject to the lease default risk, $C^*(0)$ is the value of the default-free call given by equation (5), $\nu(0, T)$ is the price of a T-period zero-coupon bond from a particular default risk class, $P(0, T)$ is the price of an equivalent maturity risk-free zero bond, and $y(T)$ is the credit spread for a T-period bond. In words, the default-prone call value is equal to the discounted value of the default-free call, where the discount rate is given by the credit spread appropriate for the default risk class of the lease.

To apply equation (7), we assume that lessees belong to the same default risk class as issuers of BBB-rated corporate bonds. Annual credit spreads for industrial bonds rated BBB vary from a low of 0.47% for a 2-year bond to 1.60% for a 4-year bond over our sample period. We discount the default-free call value estimates reported in Table III by these credit spreads and present the resulting credit risk-adjusted call values in Table IX. The grand mean and median are \$1,432 and \$1,445 as compared with \$1,462 and \$1,493 for call options without credit risk (see Table III). These revised estimates indicate that the lease-end purchase option accounts for about 15% of the value of the underlying used car even after adjusting for the possibility of default on the lease contract. Next, let us relax the assumption of independence and recognize that the lessee is more likely to default when the underlying used car price is low, that is, when the residual value risk is high. Then default would be more concentrated in states in which the value of the purchase option is low than when the two processes are independent. This is likely to render the above default risk-adjusted call value estimates (under the assumption of independence) conservative.

Despite these significant estimates, it is rare to find the direct pricing of purchase options in the automobile lease market. We offer three explanations for the lack of explicit pricing of this embedded option. First, it is almost certain that the option value is incorporated into the monthly lease payment (and

Table IX

Default Risk-Adjusted Call Value Estimates Using NADA Used Car Prices

This table reports the default risk-adjusted call option value estimates (applying equation (7)), their values as percentages of S, the underlying asset (used car) price, and of X, the residual value, and their decomposition into intrinsic and time value components for November 1995 through November 2000. For each k, the lease term in years, reported statistics are mean and standard deviation in dollars in the first row and (median, $) and (number of observations) in the second row.

	$k = 2$		$k = 3$		$k = 4$		Total	
Variable	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)	Mean (Median)	*SD* (*N*)
C	1,144	418	1,609	401	1,550	496	1,432	479
	(1,162)	(18)	(1,573)	(18)	(1,597)	(17)	(1,445)	(53)
C/S	10.8%	3.9%	16.8%	3.6%	18.6%	4.9%	15.3%	5.3%
	(10.6%)	(18)	(15.8%)	(18)	(19.7%)	(17)	(14.9%)	(53)
C/X	11.0%	4.3%	17.3%	4.3%	18.6%	5.8%	15.6%	5.8%
	(10.6%)	(18)	(15.8%)	(18)	(19.5%)	(17)	(15.3%)	(53)
Intrinsic value	210	270	240	279	119	201	190	253
	(88)	(18)	(150)	(18)	(0)	(18)	(25)	(54)
Time value	935	238	1,369	242	1,423	413	1,239	373
	(999)	(18)	(1,344)	(18)	(1,480)	(17)	(1,199)	(53)

therefore into the contractual interest rate). This situation is similar to one that arises in mortgage markets: The option to prepay a mortgage is not priced explicitly. The real estate literature (e.g., Dunn and McConnell (1981) and Stanton and Wallace (1998)) suggests that a mix of points and a higher mortgage interest rate are used by lenders to capture the value of the prepayment option.

Second, there are a number of mechanisms by which lessors can indirectly recover the value of the embedded call option.[25] These include fairly transparent solutions such as additional fees (acquisition fee, set up charge, etc.) and a manufacturer's subsidy for higher residual values that lessors may offer to lower monthly lease payments instead of cash rebates to promote sales. The more subtle mechanisms are the benefits of leasing in mitigating different types of market frictions (taxes, cost of asset disposal, etc.); see Miller and Upton (1976), Lewellen, Long, and McConnell (1976), Myers, Dill, and Bautista (1976), Barclay and Smith (1995), and Benjamin, de la Torre, and Musumeci (1998). In a world with adverse selection and heterogeneous consumer preferences for quality, Hendel and Lizzeri (2002) show that manufacturers can gain by manipulating the strike price to segment the market. Furthermore, they show

[25] A similar issue arises in mortgage markets: The option to prepay a mortgage is not priced explicitly. The real estate literature (e.g., Dunn and McConnell (1981) and Stanton and Wallace (1998)) suggests that a mix of points and a higher mortgage interest rate are used by lenders to capture the value of the prepayment option.

that the purchase option enables a manufacturer with market power to expand output profitably.

Third, our estimates are likely biased because the application of the Black–Scholes call model values the purchase option as a stand-alone option, whereas both the lessor and the lessee view a lease as a "package." As Moel and Tufano (2002) note, a common shortcoming of real option analyses is that they view managerial flexibilities as stand-alone options, without adequately accounting for the fact that firms regard these options as parts of their broader portfolio. The decision by the lessee to exercise the purchase option and the decision by the lessor to fully or partly price the value of the purchase option in determining the lease payments is a complex firm-level (not project-level) portfolio choice problem that depends on the existence and profitability of other choices. In the context of the value of managerial flexibility in production, Moel and Tufano (2002, p. 62) conclude, "While real option models are good stylized representations of plant-level decisions, they often fail to capture aspects of firm-level decision making. We find evidence reminding us that the decision to close a mine may be a firm-level one rather than a marginal mine-level, choice."[26]

IV. Conclusion

Most automotive lease contracts offer two embedded options that are not commonly seen in standard debt contracts, specifically, a cancellation option that allows the lessee to terminate the contract early and a European call option that gives the lessee the right, but not the obligation, to purchase the vehicle at the termination of the lease at a predetermined exercise price. While theoretical work suggests that these options are quite valuable, we rarely observe explicit pricing of these options in practice. Recent models of sell-versus-lease decisions show that the purchase option can empower the lease contract to mitigate the severity of moral hazard and adverse selection in durable goods markets. If the purchase option can play such a powerful role, we should expect it to command a significant economic value.

This study examines the stand-alone value of these options and presents two interesting results. First, we show that the practice of pricing the penalty at the time of early termination renders the cancellation option worthless under the common assumption of constant interest rates. When the interest rate is allowed to vary over time, the cancellation option may have positive value for the lessee; however, lessors try to minimize this value by offering cut-rate contracts and imposing other penalties for early termination.

Second, assuming no default and prepayment by the lessee, we apply the Black–Scholes methodology to the used car price data to estimate the

[26] Lamont (1997) and Shin and Stulz (1996) find that investment decisions in multidivisional firms are affected by the profitability of other divisions, and in particular, that investments in smaller divisions are reduced when earnings in larger divisions suffer. For a similar point in the context of underpricing of initial public offerings of common stocks, see Loughran and Ritter (2002).

stand-alone value of the call option embedded in consumer leases for several popular makes and models of new cars. Our findings indicate that the call option has considerable value, on average about 16% of the market value of the underlying used vehicles. This amounts to an average of \$1,462 per call option embedded in a lease on a new car. This evidence is robust to the possibility of default by the lessee. We examine the sensitivity of our option value estimates to the model parameters and offer some explanations as to why lessors do not explicitly price the call option. The purchase option premium for leases written on other car models with lower reliability ratings (more speculative properties) is likely to be much larger than the estimates based on the three popular nameplates covered in our sample.

Appendix: Pricing the Lease-End Purchase Option Subject to Default by the Lessee

A lessee may default[27] on the monthly lease payments for many reasons, including job-related events, illness, and changes in family characteristics. If a lease is in default, then the purchase option loses 100% of its value. The purpose of this Appendix is to model the impact on the value of the end-of-lease (European) purchase option in the event of a lessee default. We assume a two period lease with three dates $t = 0$, 1, and 2. Lease payments are made at the beginning of each period. While the lessee makes the first payment (at $t = 0$) for sure, he may default on the time 1 payment with probability $\lambda\mu$. In this case, the lease-end purchase option becomes null and void. In all other respects, the structure of the economy is similar to the one described in Jarrow and Turnbull (JT, 1995) to price derivative securities subject to credit risk. Hence, the economy is frictionless and without arbitrage opportunities. The used car price follows a binomial process with a risk-neutral probability of the up state equal to γ. Moreover, the default (by the lessee) process is independent of both the spot (riskless) interest rate and the automobile price processes.

As the binomial tree below shows, the evolution of the call price in the presence of default by the lessee has four nodes at $t = 1$ and six nodes at $t = 2$. At expiration ($T = 2$),

$$\begin{aligned} C(2) &= \text{Max}[S(T) - K, 0], \text{ if there is no default at t} = 1 \\ &= 0, \qquad\qquad\qquad\quad \text{if default occurs at } t = 1. \end{aligned} \tag{A1}$$

At $t = 1$, the call option has zero value in both up and down states if default occurs on lease payments, that is, $C(1)_{u,b} = C(1)_{d,b} = 0$, where subscripts u and d denote up and down states and b indicates the default state. If there is no default at time 1, then the call values in the up and down states equal the

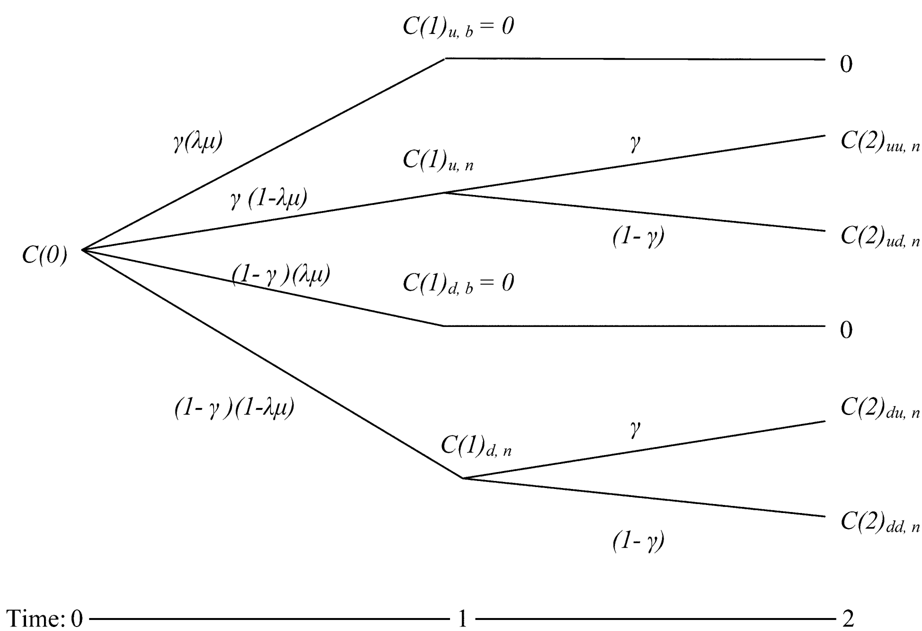
[27] We do not distinguish between delinquency and default. For our purposes, a lease is in default if the lessee fails to make a scheduled lease payment.

corresponding expected values under risk-neutral probabilities of the payoffs at expiration discounted by the respective risk-free interest rates $r(1)$:

$$C(1)_{u,n} = [\gamma C(2)_{uu,n} + (1-\gamma)C(2)_{ud,n}]/r(1)_u \tag{A2}$$

$$C(1)_{d,n} = [\gamma C(2)_{du,n} + (1-\gamma)C(2)_{dd,n}]/r(1)_d, \tag{A3}$$

where n indicates no default.



Therefore, the current value of a defaultable purchase option is

$$C(0) = (1-\gamma\mu)[\gamma C(1)_{u,n} + (1-\gamma)C(1)_{d,n}]/r(0), \tag{A4}$$

where $r(0)$ is current period risk-free interest rate.

Let $C^*(t)$ be the time-t value of a call that is not subject to default risk on lease payments. Then, at expiration $C^*(T) = \text{Max}\,[S(T) - K, 0]$, and its current value may be shown to be

$$C^*(0) = [\gamma C(1)_{u,n} + (1-\gamma)C(1)_{d,n}]/r(0). \tag{A5}$$

Therefore, the option values with and without default risk are related as $C(0) = (1-\lambda\mu)\,C^*(0)$.

Last, assume the lessee belongs to the same credit risk class as the issuer of a BBB-rated corporate bond (similar to the "XYZ" bond described in detail in JT, p. 63). Then, using equations (16c), (17), (26), and (63) of JT, we have the continuous-time expression for the value of the call option subject to default risk:

$$\begin{aligned} C(0) &= \frac{\nu(0,T)}{P(0,T)} C^*(0) \\ &= e^{-y(T)\times T} C^*(0), \end{aligned} \tag{A6}$$

where $\nu(0, T)$ is the price of a T-period zero-coupon bond from the XYZ risk class, $P(0, T)$ is the price of an equivalent maturity risk-free zero bond, and $y(T)$ is the credit spread for a T-period bond. Equation (A6) shows that the value of a defaultable call is proportional to the default-free (Black–Scholes) call value.

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