THE JOURNAL OF FINANCE • VOL. LXI, NO. 6 • DECEMBER 2006

Derivative Pricing 60 Years before Black–Scholes: Evidence from the Johannesburg Stock Exchange

LYNDON MOORE and STEVE JUH*

ABSTRACT

We obtain daily data for warrants traded on the Johannesburg Stock Exchange between 1909 and 1922, and for a broker's call option quotes on stocks from 1908 to 1911. We use this new data set to test how close derivative prices are to Black–Scholes (1973) prices and to compute profits for investors using a simple trading rule for call options. We examine whether investors exercised warrants optimally and how they reacted to extensions of the warrants' durations. We show that long before the development of the formal theory, investors had an intuitive grasp of the determinants of derivative pricing.

WE OBTAIN DAILY PRICES FOR MARKET-TRADED WARRANTS on the Johannesburg Stock Exchange (JSE) from 1909 to 1922 and utilizing the Black–Scholes (1973) model assess whether the market prices of these warrants were "fairly" priced. We compile daily prices of 15 warrant price series from 1909 to 1922 and a broker's daily call option quotes written on 112 companies' stocks between January 1908 and May 1911. We test whether the market prices of these derivatives are close to Black–Scholes model prices and also calculate the profits for investors who follow the simple trading strategy of buying one of every call option offered for sale by the broker. We then compare the early twentieth century data with contemporary JSE and U.S. security market data.

Tests of how close the market prices of derivative securities are to fair prices can be grouped into three categories. The first category encompasses tests of modern exchange-traded derivatives, that is, derivatives for which market participants have access to the derivative pricing theory of Black and Scholes (1973) and Merton (1973). The data necessary for tests of modern exchange-traded

*Moore is from Victoria University of Wellington. Juh is from Lehman Brothers. The paper was written when both authors were at Northwestern University. The authors would like to thank Kathleen Hagerty for many long, detailed explanations of derivative pricing theory and excellent suggestions to improve the paper. Ran Abramitzky, Andrew Hertzberg, Jon Huntley, Robert Korajczyk, Robert McDonald, Joel Mokyr, Warwick Moore, Eugene Orlov, Ronnie Sadka, Ellis Tallman, Ryan White, and an anonymous referee all provided excellent advice and suggestions, as did seminar participants at Northwestern University and at the European Social Science History Conference. Discussions with officials of the Johannesburg Stock Exchange were very useful, as were the data they provided. All errors remain our own. The financial support of the Economic History Association and from a Northwestern University Dissertation Year Fellowship are gratefully acknowledged.

derivatives are available as of 1973, which corresponds to the opening of the Chicago Board Options Exchange (CBOE). Early tests of the Black–Scholes model, such as those of Galai (1977) and Macbeth and Melville (1979), do not find major deviations of CBOE market prices from model prices.

The second category comprise tests of pre-1973, non-exchange-traded derivatives. Tests in this second category rely on data collected from option brokers. Using the diaries of an option broker from 1966 to 1969, Black and Scholes (1972, p. 413) demonstrate that although "in general writers obtain favorable prices," once transaction costs are taken into account, there is no opportunity for traders to profit from any mispricing of market prices relative to Black–Scholes prices. In contrast, Kairys and Valerio (1997, p. 1719), who use price quotes from a single broker between 1873 and 1875, show that the broker's option quotes were "regularly overpriced relative to a theoretical valuation model." Kruizenga (1964) uses nominal put and call quotes, not necessarily the prices actually written into option contracts, reported to the Securities and Exchange Commission from 1946 to 1956 and concludes that buying 6-month call options would have yielded an average annual profit of 35%. Boness (1964), who uses data on the actual transactions of two brokerage houses from 1957 to 1960, concludes that buying either put or call options was unprofitable for the buyer. Clearly, the empirical results using pre-1973 data are mixed. It is therefore difficult to tell if the better post-1973 fit between option prices and theoretical prices is due to the development of a satisfactory derivative pricing model or to the creation of an options exchange with standardized contracts and transparent price information.

The third category consists of tests of pre-1973, exchange-traded derivatives. Kassouf (1969) attempts to derive the true relation between stock and warrant prices by examining the market prices of every warrant that traded on the American Stock Exchange in the months of November between 1945 and 1964. However, Kassouf's study suffers from the use of monthly averages of high and low prices for both stocks and warrants, rather than contemporaneous observations of each. Bachelier (1900) develops a theoretical model of option pricing based upon a Brownian motion model of stock price changes (culminating in the Black–Scholes model; see Sullivan and Weithers (1991, p. 170)) and tests his option pricing model with data collected from the Paris Bourse on 3% Rentes, the standard French government debt instrument of the time. He finds that observed prices on the bourse are close to his model's prices.[1]

Our paper adds to the existing literature by comparing the mispricing of early twentieth century exchange-traded warrants with the mispricing of a broker's call options during the same period in South Africa. That is, we determine whether trading on an exchange affected derivative mispricing. We then compare the mispricing of early twentieth century JSE-traded warrants with modern JSE-traded derivatives, and determine whether the development of

[1] It is difficult to compare Bachelier's findings with ours here. French options had a fixed cost, either 50, 25, or 10 centimes, with the strike price relative to the spot price being negotiated in the marketplace. Bachelier's findings that the average observed market "spread" (defined by him as the strike price plus the cost of the option less the spot price) is within 8.6% of his model's implied spread for 30-day call options, and within 29.7% for 10-day call options.

the modern theory by Black and Scholes (1973) and Merton (1973) has driven exchange-traded derivative prices closer to fair prices. While our approach cannot hold everything else constant, it provides some evidence as to whether post-1973 market prices of derivatives are closer to fair prices due to the development of a satisfactory model for derivative prices or to the development of centralized marketplaces that are available to investors.

In the early twentieth century, books such as Higgins (1906) and Conway (1913) provided a theory of derivative pricing.[2] The theory these books contain is qualitatively correct, although not precisely quantified. Thus, in the early twentieth century the investor had a good idea of what influenced the value of derivatives but did not have an exact formula to use in determining their prices. The closeness of market warrant prices to Black–Scholes (1973) prices that we observe therefore suggests that investors were rational and priced derivatives as well as could be expected given their available knowledge.

The JSE opened in 1887, shortly after the discovery of the Witwatersrand gold fields. While it closed at the outbreak of the Boer War in 1899, it reopened in December 1901. During the early 1900s there was a large increase in stock market activity and trading opportunities in Johannesburg. Several short-lived stock exchanges started up, competing with the main JSE. A few mutual funds and some fairly sophisticated derivative products, both over the counter and exchange traded, also emerged. The primary source of turnover on the JSE was in mining companies' stocks, although a few industrial and financial companies were also traded. Many of the large companies listed on the JSE were also cross-listed on the London Stock Exchange and on the Parisian unofficial market. Many of these companies also had agents in London, Paris, and Berlin.

The Johannesburg market was an active one. Trading was conducted from 9:00 a.m. to 6:00 p.m. on weekdays, with more limited hours on Saturdays. Trading often continued on the street outside the exchange after the exchange closed for the day.

There were various other short-lived exchanges that competed with the JSE for business in the early years: the Rand Stock Exchange (1897 to 1898), the Rapid Call Exchange (1904 to 1905), the Consolidated Exchange (1904 to 1905), the Public Exchange (1905 to 1906), another business also known as the Rand Stock Exchange (1907), and the Public Call Exchange (1907 to 1908). Of these exchanges, the Rapid Call Exchange and the Public Call Exchange traded 30-day call options, although reported prices were very irregular. In effect, these smaller exchanges were doing little more than providing large stockbrokers a public forum for trading and offering quotes on some mining companies, often the smaller ones. These exchanges also provided a market for syndicates, which were small companies that pooled investors' money and prospected for minerals in a given area. The larger syndicates began with tens of thousands of pounds of investors' money, and the successful syndicates could turn into

[2] The work of Bachelier (1900) was not available to market participants in the early twentieth century: "prior to 1960, Bachelier's work was known primarily to a handful of specialists in Brownian motion" (Sullivan and Weithers (1991, p. 170)). Bachelier's work was only translated into English in 1964.

publicly traded companies on the JSE if they found minerals in commercial quantities.

As a further indication of a sophisticated market, large, listed companies such as Consolidated Mines Selection and Rand Mines, which acted somewhat like mutual companies, were also present in the market. These companies did not run mining operations directly but instead kept a portfolio of stocks of other mining companies.

The African Share Agency (ASA) began trading in 1906 using the name The New Call Exchange. The ASA acted purely as a broker. It offered to trade shares and also offered various other financial products such as term deposits, 10-, 15-, 30-, 60-, and 90-day call options, "time bargains" that were similar to put options, £5 and £10 "cover limits," and "double options" (the simultaneous writing of a put and a call).[3] The ASA also sold shares on 12-month installment plans, promoted various funds, and published a financial newsletter, *The Transvaal Financier*. The ASA advertised almost daily for business in a local newspaper, the *Rand Daily Mail*, beginning in 1906 and published current stock prices and fairly regular quotes for call options from January 1908 to September 1909 and also from August 1910 to May 1911.[4]

Options had been used since the very beginning of the Johannesburg gold rush. The initial scramble to secure good mining ground was typically a mixture of outright purchases and leases, with an option to buy later. Until 1908, in addition to a fixed fee, companies often gave warrants as a form of compensation to the underwriters of their new share issues. These warrants could only be exercised by the underwriters, so there was no secondary market for them. Beginning in 1908, but concentrated between 1917 and 1921, some companies that underwent capital restructuring offered bearer warrants to their shareholders. Franklin and Colberg (1958, p. 29) report a similar phenomenon in the United States during the 1920s, "long-term and even unlimited-period option warrants were issued frequently in connection with new stock issues." The South African warrant-issuing companies were developing mines and thus were looking for financing. For instance, the Geduld Proprietary 1919 annual report explained that "the decision . . . to raise new capital for the purpose of sinking and equipping the vertical shaft (was arrived at in October). The method decided upon was to issue to shareholders 180,000 shares . . . the subscriber of each share being entitled to an option to take up a further share."[5]

[3] Cover limits were issued for periods of 10 days, during which time the holder could command the shares. "In the event of a fall in price the account becomes automatically closed should the fall extend to 1 shilling or 2 shillings, as the case may be, and no further liability is incurred by the contract holder, whilst the profits are unlimited in extent." (*Rand Daily Mail*. October 19, 1906)

[4] In October 1909 an Act of Parliament halted all advertising by stockbrokers. The ASA got around this, we believe, by reorganizing their business so that legally they became a stock exchange. By August 1910 they could resume quoting call options in the *Rand Daily Mail*, and they were the only stockbroker advertising to write call options to the public. The ASA's price lists appeared under the title "The New Call Exchange" until 1907, then under the title "The African Share Agency," and then again from August 1910 under the title of "The New Call Exchange."

[5] (*South African Mining Journal*, May 3, 1919). We use the term "warrants," which is the term

The mining companies issued the warrants for free to existing shareholders, intending that the warrants be exercised so that the company would gain further working capital to develop the mine the company possessed. Financing a new mine tended to be a precarious affair. The company would try to raise enough capital to dig the mine shaft and determine the ore quality. It would then count on extracting sufficient gold from the ore to bring to market before its funds ran out. The warrant-issuing companies may have believed that capital raised through warrant exercise would be cheaper than other forms of capital. For instance, because the South African Alkali company reported that "to raise the necessary capital . . . to install such machinery and plant as is required . . . 35,000 new shares are now offered . . ., with the right to subscribers to take up within twelve months a further 17,500 shares . . . the amount realised by this issue will cover the cost of new machinery, . . . besides liquidating all outstanding liabilities of the company" (*South African Mining Journal*, April 24, 1920), they appear to have believed that raising capital through common stock and warrant issuance was cheaper than continuing to rely on debt financing. Further, of the five warrant-issuing companies that also issued debt to raise capital, in all five cases those companies' warrants had expired out-of-the-money or were just about to expire out-of-the-money. Thus, those five companies were in need of additional capital, as they had not received the expected inflow of capital from warrant exercise, and issued debt to obtain the required capital for mining operations.

The JSE shut down at the beginning of the First World War, and reopened again on January 4, 1915. While warrant issuance was not forbidden during the early years of the war, any new capital emissions had to receive authorization from the Imperial Treasury in London. This regulation included the issuing of warrants and therefore partly explains the disappearance of warrants from our data set between 1911 and 1917.

The rest of the paper proceeds as follows. We describe our data in detail in Section I. In Section II we assess whether early twentieth century derivatives were fairly priced in the Black–Scholes (1973) sense. In Section III we examine whether investors exercised their warrants optimally, and in Section IV we analyze how market prices adjusted to changes in warrant specifications. In Section V, to better assess investor efficiency in pricing warrants, we compare the performance of South African investors in the early twentieth century with that of contemporary investors on both the JSE and U.S. exchanges. We summarize and conclude in Section VI.

I. Data

A. Warrants

Of the 15 warrants in our sample, 11 were issued by gold mining companies,

used in the United States for securities that allow the holder to obtain a unit of stock from the warrant-issuing company that the company itself will create and issue to the warrant holder if the warrant is exercised. In South Africa these securities were (and still are) referred to as "options."

2 were issued by silver mining companies, 1 was issued by a diamond mine, and 1 by an alkali extractor. Although we only collect warrant data from 1909 to 1922, the *Rand Daily Mail's* price list of the JSE market contains a few other warrants that were issued in later years. We use data given in the *Rand Daily Mail* as of "High 'Change" each day, where High 'Change was the time in the middle of the morning when the entire official share list was read out on the floor of the JSE and both bid and ask quotes were given by open outcry. Officials noted any transactions made at this time. We use price quotes at High 'Change when available. Occasionally, a security might not have recorded a price at High 'Change, but will have a bid or ask quote, or a transaction recorded at another time during the day. If that was the case, we use quotes for those other times during the day. We ensure that on a given day prices for a stock and that stock's warrant are in the same time period. The data are not time-stamped per se; rather, prices are recorded in intervals of time. The reported time periods vary from day to day. For example, prices one day may be reported as occurring between 1:00 p.m. and 2:30 p.m. and the following day between 1:00 p.m. and 3:00 p.m.

If a transaction were recorded, we use the transaction price as our price. In the absence of a transaction, we use the average of the bid and ask quotes if both were available. If only the bid quote were recorded, we add half the average bid–ask spread to the bid quote. If only the ask quote were recorded, we subtract half the average bid–ask spread from the ask quote. For each stock and warrant, we calculate the average bid–ask spread as the arithmetic average of all spreads for which we observe both the bid and the ask quote for that security.

A little under one-half of our observations are based on transactions. Of the remainder, more than three-quarters are based on both the bid and the ask quote. Quotes usually changed throughout the day and almost always changed from day to day.

Prices were quoted in shillings and pence, with the lowest tick size being one-half of one penny, although it was unusual to find tick sizes of less than a penny. Twelve pennies equal one shilling, and 20 shillings equal one pound, making the typical tick size 1/240th of a pound.

The warrants in our sample are all American style, meaning that they could be exercised at any time up to the expiry date. Details on warrant specifications come from the companies' annual reports, usually published in the *Rand Daily Mail*. Annual reports usually contain balance sheet information and often a description of the proceedings of the annual general meeting (AGM). The balance sheet and the report of the AGM generally state how many warrants had been exercised. Not all companies published dividend notices and balance sheet information in the *Rand Daily Mail*, so we augment our data with various issues of the *South African Mining Journal* and *The Star*, another daily newspaper in Johannesburg.

Table I presents summary statistics of the warrant data we collect.

Table I

JSE Warrant Specifications—1909 to 1922

Obs is the number of warrant price observations in our sample. Duration is the initial time to expiry of the warrant. We measure the mean stock price, mean warrant price, and strike price in shillings. We measure the mean stock price over the duration of the warrant. We define dilution as the number of warrants initially issued divided by the number of issued shares. % "in" is the percentage of warrant observations when the stock price is above the exercise price. The time period is the period during which we observe the warrant price on a daily basis.

Warrant	# Obs	Duration	$\overline{Share}$	$\overline{Warrant}$	Strike	Dilut.	% "in"	Time Period
Bantjes Consolidated	71	2 years	57.02	36.93	22.5	0.47	100	January 1909 to November 1910
Vogelstruis Cons. Deep	123	2 years	24.48	12.20	22.5	0.44	86.2	January 1909 to November 1910
South Randfontein Deep	105	2 years	7.58	1.99	10	0.51	27.6	January 1910 to November 1911
Daggafontein 3	412	3 years	24.32	7.58	20	0.04	97.8	August 1917 to March 1919
Daggafontein 4	149	4 years	18.63	2.72	22.5	0.19	32.9	September 1919 to June 1920
Daggafontein 5	321	5 years	13.95	3.75	25	0.19	0.3	September 1919 to April 1921
Modderfontein East 3	800	3 years	22.60	6.52	21.25	0.54	49.5	August 1917 to June 1920
Modderfontein East 4	1050	4 years	21.45	7.26	22.5	0.54	31.5	August 1917 to April 1921
West Springs	673	2.5 years	19.58	5.62	22.5	0.22	43.1	July 1918 to September 1921
Southern Van Ryn	30	1.5 years	10.41	1.50	20	0.63	0	August 1918 to December 1919
Geduld Proprietary	672	3 years	47.74	12.96	40	0.19	85.7	January 1919 to December 1921
Transvaal Silver	230	1 year	62.80	40.93	25	0.40	100	January 1920 to December 1920
Pretoria Silver	101	1 year	6.44	0.54	25	1.44	0	September 1921 to December 1922
Frank Smith Diamonds	455	3 years	6.89	3.59	7.5	0.18	28.6	March 1920 to December 1922
South African Alkali	209	1 year	20.97	8.19	25	0.21	44.5	July 1920 to November 1921

B. Call Options

During our sample period, the *Rand Daily Mail* printed the quotes of the African Share Agency (ASA), generally listing either a bid and an ask price or the price of the last sale for each stock, as well as 30-day call prices for those stocks from January 1908 to September 1909 and then from August 1910 to May 1911. In January 1908 and February 1908, the ASA concurrently listed 30- and 60-day call prices on 20 companies. The ASA advertised that they were willing to write 10-, 15-, 30-, 60-, and 90-day calls, although they only posted prices for the 30- and 60-day calls. It appears that 30-day calls were the most sought after by investors.[6]

Unfortunately, the exact strike prices for the options were not usually listed. However, we find nine call option quotes in 1907 and 1908 for which the exact strike price is mentioned. Four of the quotes are from the ASA and five are from H. Emdon & Co., a competing broker. Most strike prices we obtain were in the vicinity of the last traded price, ranging from −1.75% below the last traded price to 12.5% above the last traded price. The arithmetic average of the nine quotes are of a strike price 3% above the last traded price. Based on these nine call option quotes, we assume that the strike price of each call option was the average of the stock's bid and ask price for that day, effectively assuming that options were issued at-the-money. We use the average of the bid and ask prices in 30 days as the price the option holder would receive for the stock if he were to exercise his option at expiry.

The cost of a call option generally varied by three pennies, which is quite a large tick size relative to each underlying stock's usual tick size of one penny. The call option quotes were given more frequently for the smaller, non-dividend-paying stocks, although there were a fair number of quotes for the better-quality, dividend-paying stocks such as City Deep, City and Suburban, Consolidated Main Reef, Jumpers, Main Reef West, Rooiberg Minerals, and Zaaiplats Tin. In our sample of 112 stocks, we have a total of 11,181 30-day call option quotes. As was the case in the U.S. market for equity options in the 1870s (see Kairys and Valerio (1997)), the ASA options market was one-sided. The ASA offered the call options for sale; it did not purchase call options at its own quoted prices.

Although the ASA was the preeminent quoter of option prices and loudly proclaimed its market leadership through its advertisements, other stockbrokers also advertised that they were willing to write call options. For instance, R. Norden & Co. advertised that it was willing to "sell options or calls on any of the listed stocks up to 90 days" (*Rand Daily Mail,* January 23, 1907), and H. Emdon and Co. and John MacNamara advertised in the *Rand Daily Mail* that they were willing to write options, though they did not post the price at which they would write call options. There were instances of financial innovation, such as when H. Emdon and Co. offered a "Christmas Box" in 1908. For 12 pounds 10 shillings, the purchaser would receive 30-day call options on 100 African

[6] Kairys and Valerio (1997) note a preference for 30-day options by U.S. investors in the 1870s. Franklin and Colberg (1958) report the same preference by U.S. investors during the 1930s.

Farms, 100 Bush Tins, and 100 Kaalfonteins or three other combinations of call options (*Rand Daily Mail*, December 14, 1908).

II. Pricing and Volatility Measures

A. Warrant Pricing

Black and Scholes (1973) and Galai and Schneller (1978) show that adjustments need to be made to the standard Black–Scholes model when it is used to price warrants in order to account for the diluting effect of warrant exercise on a company's outstanding capital. We use Lauterbach and Schultz's (1990) adjustment to the Black–Scholes model, which although identical to Galai and Schneller, more explicitly represents the model's inputs:

$$W = \left(\frac{N}{N+M}\right)\left[\left(S - \sum_i e^{-rt_i} D_i + \frac{M}{N} W\right) N(d_1) - e^{-rT} X N(d_2)\right], \quad (1)$$

where

$$d_1 = \frac{\ln\left(\dfrac{S - \sum_i e^{-rt_i} D_i + \dfrac{M}{N} W}{X}\right) + rT}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

and

W = the warrant price
S = the stock price
X = the exercise price
N = the number of shares of outstanding stock
M = the number of outstanding warrants
$N(d)$ = the probability that a random draw from a standard normal distribution will be less than d
D_i = the amount (per share) of the i^{th} dividend
t_i = the time until the i^{th} dividend is paid, in years
T = the time to expiry of the warrant, in years
r = the risk-free interest rate, with the same time to expiry as the warrant
σ = the standard deviation of the equity returns, $S + (M/N)W$, on an annualized basis.

Lauterbach and Schultz's model makes three adjustments to the standard Black–Scholes model. First, it substitutes $S + (M/N)W$ for the stock price. Second, the volatility measure used to price warrants is the annualized standard deviation of the equity returns of the firm, not the annualized standard deviation of the returns of the stock price, where equity is defined as $S + (M/N)W$.

Third, the entire formula is multiplied by $N/((N/\gamma)+M)$, where γ is the number of shares that can be purchased with each warrant.[7]

Although we use a warrant pricing formula, the results are similar to those that obtain if we use the standard Black–Scholes (1973) model and treat the warrants as if they were call options.

Only one stock paid dividends in our sample, Geduld Proprietary. We use a binomial tree with 100 steps to calculate the model price of Geduld Proprietary, using the ex post realized dividend yield.[8]

Income taxes in South Africa were very low during this period. They were first imposed in 1904, with the tax rate beginning at 2.5% and rising to 5% on all income over £1000.[9] Most of the government's revenue was raised by customs duties and railroad taxes. The Transvaal Republic, in which Johannesburg was situated, levied a tax on gold mining companies that was paid directly by the company. Brokerage charges were also modest. On October 1, 1906, the JSE changed the official brokerage rate from 1% paid by the seller to 0.5% paid by the buyer and 0.5% paid by the seller (*Rand Daily Mail,* September 26, 1906). Since taxes and brokerage charges were both very small, we have not made any adjustment for them in calculating Black–Scholes (1973) prices.

For the Black–Scholes (1973) pricing equation to hold exactly, the model requires that investors be able to short-sell. Although we find two mentions of the practice of short-selling on the JSE, we do not find any advertisements by institutions offering such a possibility. The lack of easily accessible short-selling possibilities may be one explanation for the mispricing we find.

The Black–Scholes (1973) model uses several inputs that require judgment, namely, the risk-free interest rate and the annualized standard deviation of equity returns, which we refer to in this paper as the volatility.

With respect to the risk-free rate, we use a 5% interest rate to price all warrants in our data set because a Union of South Africa bond with a nominal yield of 5% traded very close to par throughout our study period.

The most important input in any option or warrant pricing procedure is the volatility, σ. The warrants were issued with a time to expiry of up to 5 years; therefore volatility changed over its life. This changing volatility makes valuing warrants more problematic than valuing the shorter-lived options, as Black and Scholes (1973) and others note. Early twentieth-century investors did not have access to the Black–Scholes model; thus they were not explicitly estimating

[7] The number of shares that can be purchased with each warrant is equal to one for every warrant in our sample. Therefore in equation (1) we simplify $(\frac{N}{\frac{N}{\gamma}+M})$ to $(\frac{N}{N+M})$.

[8] Using a binomial tree gives very similar results to using equation (1). The advantage of using a binomial tree method is that it ensures that the possibility of early exercise is taken into account. Geduld Proprietary paid a dividend of 1.25 shillings (March 1919), issued stock at below market prices (worth approximately 1 shilling per share in June 1919), made a bonus issue of 2 per 25 shares (worth approximately 4.5 shillings in January 1920), and paid dividends of 1.5 shillings (July 1920), 2 shillings (January 1921), 1.5 shillings (July 1921), and 2 shillings (January 1922). The books for the last dividend closed on December 31, 1921, the date the warrant expired.

[9] http://encyclopedia.thefreedictionary.com/History%20of%20Cape%20Colony%20from%20the%20Second%20Anglo-Boer%20War

volatility. However, it is likely that they were implicitly estimating volatility. Indeed, the contemporary literature was aware that (what we refer to as) volatility is important in pricing derivatives. For example, Higgins (1906) advises that investors "ascertain the past average fluctuations over a considerable period of time of the stock to be operated in."

Early 20th-century investors faced a difficult job in (implicitly) estimating the volatility to use in pricing warrants. First, several companies were only listed at the same time as their warrants, so that no past history of volatility was available. Second, with the exception of South Randfontein Deep, Bantjes Consolidated, and Vogelstruis Consolidated Deep, these companies issued warrants during a very uncertain time. In particular, World War I was being fought in the earlier part of many of the warrants' lives, and the resolution of the war was bound to be a factor in the movement of gold prices. Furthermore, because most South African gold was exported to Great Britain for minting during this period when Britain went off the gold standard in September 1918, there was an immediate rise in both the price and volatility of the price of gold. All South African diamond mines were also shut down during the war, and decisions about when to restart mining operations were an issue during the first few years after the end of the war. Third, most of the companies that issued warrants (the only clear exception being Geduld Proprietary) were only marginally profitable, and the companies in our sample constantly faced major financing and investment decisions, such as whether or not to shut down their mines.

To provide robustness to our results, we consider two plausible measures of volatility, a perfect foresight measure and a 90-day backward-looking measure. Under perfect foresight we use the actual realized (ex post) annualized standard deviation of equity returns over the life of the warrant, and use that (constant) volatility measure to calculate Black–Scholes (1973) prices throughout each warrant's life. Investors may have expected a change in the volatility of gold mining companies' stocks over the life of a warrant, and been (somewhat) able to predict what the change in volatility would be. For example, investors may have expected volatilities to change as the war ended and gold demand and prices were impacted. Diamond investors may also have expected a change in volatility, as the diamond mines would restart and demand would pick up after the war. Forward-looking behavior by investors suggests the use of an ex post volatility measure.

Under a 90-day backward-looking window, we use data on equity returns for the previous 90 days (when available). We could argue that an investor's best predictor of future volatility is the immediate past volatility, rather than a somewhat arbitrary expectation of what volatility will be in the future. The 90-day backward-looking volatility measure is only possible at the beginning of the warrant's life for the Frank Smith, Geduld Proprietary, South Randfontein Deep, Bantjes Consolidated, Vogelstruis Consolidated Deep, and South African Alkali warrants. The other seven warrants were issued at the same time as their stocks were listed on the JSE. Therefore, for the first 90 days of these warrants' lives, we measure the volatility based on as many data as

Table II
JSE Warrant Mispricing—1909 to 1922

We calculate the numbers in the shillings columns by averaging $|W_{BS} - W_{market}|$ over all observations. We calculate the numbers in the percent column, the average absolute percentage mispricing, by dividing the number in the shillings column by the average warrant price and multiplying by 100. We present the volatility measure we use for each warrant in the σ^2 perfect foresight volatility column, and the low-to-high range of volatility measures we use in the σ^2 previous 90 days' volatility column. We use the annualized standard deviations of equity returns, measured over the lives of each warrant, in the perfect foresight columns. We use the annualized standard deviations of equity returns, measured over the previous 90 days, in the previous 90 days' volatility columns. For those warrants that were extended, we calculate the average mispricing over all observations. We switch from a Black–Scholes price, using the initial expiration date, to the extended date on the date the extension is announced when the extension is free, and the date the extension "payment" is made for the South African Alkali and Southern Van Ryn warrants. Model prices for the Geduld Proprietary warrant are calculated using a binomial tree with 100 steps to allow for the possibility of early exercise, since Geduld Proprietary was the only stock that paid dividends.

	Perfect Foresight Volatility			Previous 90 Days' Volatility		
	Shillings	Percent	σ^2	Shillings	Percent	σ^2 (Range)
Bantjes Consolidated	0.9	2.4	0.39	1.1	3.1	0.19–0.54
Vogelstruis Cons. Deep	1.6	12.7	0.73	1.1	8.7	0.43–0.98
South Randfontein Deep	0.4	20.5	0.63	0.6	30.4	0.47–1.00
Daggafontein 3	1.8	23.3	0.31	1.8	23.2	0.15–0.51
Daggafontein 4	0.9	31.2	0.50	1.1	40.7	0.36–0.71
Daggafontein 5	1.1	28.6	0.62	1.8	46.8	0.36–1.27
Modderfontein East 3	1.0	16.0	0.40	1.2	18.5	0.20–0.68
Modderfontein East 4	1.5	20.0	0.43	1.7	23.9	0.20–0.74
West Springs	1.7	30.5	0.44	2.1	38.0	0.24–0.73
Southern Van Ryn	1.3	85.4	0.46	1.3	85.7	0.37–0.52
Geduld Proprietary	3.3	25.3	0.31	3.4	26.0	0.15–0.50
Transvaal Silver	1.5	3.7	0.88	1.1	2.7	0.54–1.89
Pretoria Silver	0.7	137.3	2.16	0.7	138.2	1.31–3.09
Frank Smith	0.6	17.0	0.90	0.6	17.8	0.50–1.31
South African Alkali	2.2	27.1	0.97	2.8	34.0	0.52–1.41
Average		23.7			27.4	

are available, beginning when we have at least 10 observations of daily equity returns. We annualize the volatility by multiplying the daily standard deviation of equity returns by the square root of 295. The JSE was open for trading on 295 days in 1918, 294 days in 1919, 294 days in 1920, and 293 days in 1921.

Table II presents the measures of volatility that we use in calculating Black–Scholes (1973) model prices, along with the average absolute mispricing in shillings and the average absolute percentage mispricing.

We see a big difference in the accuracy of warrant pricing between the various warrants. Based on the average absolute percentage mispricing measure, investors did a good-to-fair job of pricing the Bantjes Consolidated, South Randfontein Deep, Vogelstruis Consolidated Deep, Daggafontein three-year

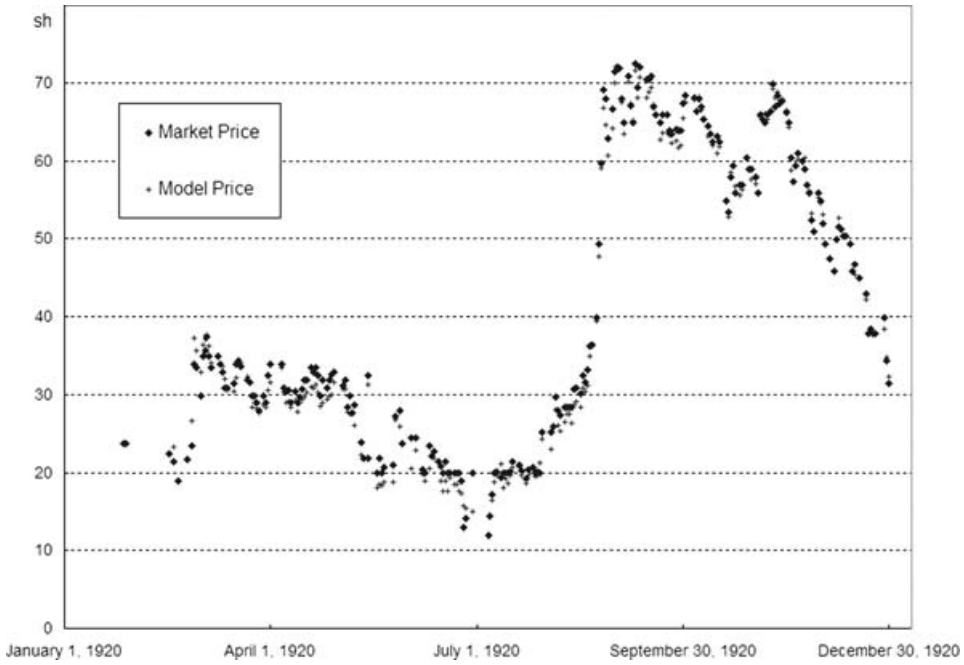

Figure 1. Transvaal Silver. Daily market prices and Black–Scholes model prices of the Transvaal Silver warrant using a 90-day backward-looking window of equity returns to calculate volatility. Both prices are measured in shillings.

warrants, Modderfontein three- and four-year warrants, Frank Smith, and Transvaal Silver warrants. Investors' average absolute percentage mispricings are between 2.4% and 23.3% based on our perfect foresight measure. The Transvaal Silver warrant (Figure 1) was always well in-the-money, and the fit of market prices to Black–Scholes (1973) prices is very close throughout the warrant's life. Investors did not do so well pricing the Daggafontein 4- and 5-year warrants, Geduld Proprietary (adjusting for dividends), South African Alkali, and West Springs warrants, with average absolute percentage mispricings between 25.3% and 31.2%. Although movements in the market price of the West Springs warrant (Figure 2) are in line with the movements of the Black–Scholes model price, there is a noticeable difference in the levels, as the warrant was substantially overpriced. Investors performed very poorly with the Southern Van Ryn and Pretoria Silver warrants with average absolute percentage mispricings of 85.4% and 137.3%, respectively. The Pretoria Silver warrant (Figure 3) was the most poorly priced. Investors did not appear to adjust warrant prices at all for changes in the underlying stock price. The summary mispricing measure we report at the bottom of Table II, 23.7% using perfect foresight volatility and 27.4% for the 90-day backward looking window, is an arithmetic average of the 15 warrants' average absolute percentage mispricings weighted by the number of observations for each warrant.

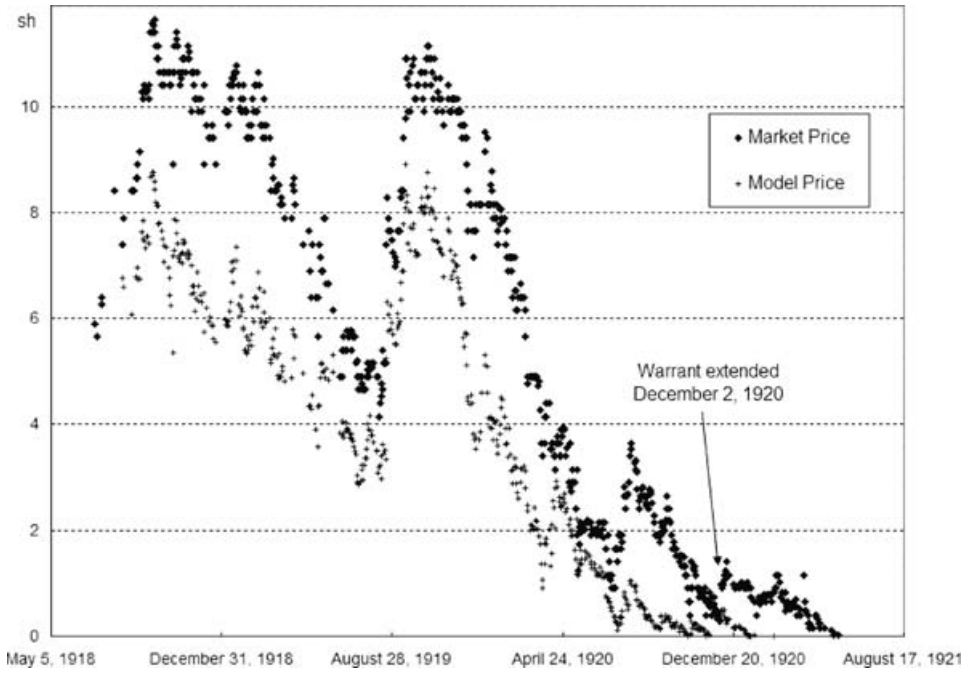

Figure 2. West Springs. Daily market prices and Black–Scholes model prices of the West Springs warrant using a 90-day backward-looking window of equity returns to calculate volatility. Both prices are measured in shillings.

In general, we find that if investors could accurately foresee the volatility of the equity over the life of the warrant, there would be less mispricing than if they used a 90-day backward-looking window of equity returns to calculate volatility. However, for some warrants the 90-day backward-looking window generates a better fit between market prices and Black–Scholes (1973) prices.

The value of a warrant or an option consists of two components, the intrinsic value and the time value. The intrinsic value is simple to calculate, as it is the difference between the underlying stock price and the strike price. The time value is more complex to calculate, as it depends on the underlying stock price, the strike price, the time remaining until expiry, the interest rate, the dividends (if any) to be paid during the derivative's life, and the volatility. The value of deep in-the-money warrants, such as Transvaal Silver, depends mostly on the intrinsic value, thus it is not surprising that deep in-the-money warrants are the easiest for investors to price. The value of out-of-the-money warrants, on the other hand, depends solely on the time value. Given the difficulty in estimating the correct volatility to use and in processing all the relevant information, it is not surprising that out-of-the-money warrants, such as the Southern Van Ryn and the Pretoria Silver warrants, were the most difficult for early twentieth-century investors to price.

Comparing the mispricing between two warrants is problematic, as a 2-shilling mispricing of a 72.5-shilling warrant (the maximum price for the

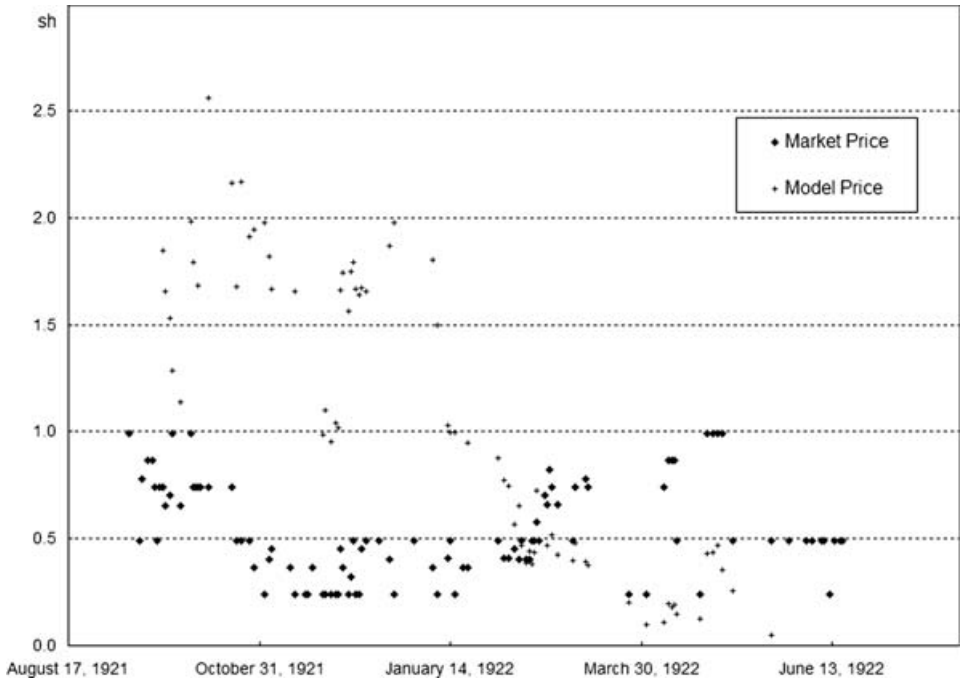

Figure 3. Pretoria Silver. Daily market prices and Black–Scholes model prices of the Pretoria Silver warrant using a 90-day backward-looking window of equity returns to calculate volatility. Both prices are measured in shillings.

Transvaal Silver warrant) is less serious than a 2-shilling mispricing of a 1-shilling warrant. For example, the Pretoria Silver warrant had a market price of 0.75 shillings on October 5, 1921 while the Black–Scholes price (using perfect foresight volatility) is 2.44 shillings. To overcome this challenge, at least in part, we measure the absolute mispricing as a percentage of the average warrant price. Although out-of-the-money warrants tend to be more poorly priced than in-the-money warrants, due to the difficulty of calculating the time value, our use of percentages to measure the mispricing tends to exaggerate the result. Out-of-the-money warrants have a smaller average warrant price (the denominator) than at-the-money or in-the-money warrants, *ceteris paribus*, which boosts the average absolute percentage mispricing.

We emphasize that we cannot discriminate definitively between two alternative explanations for mispricing; either there was mispricing because investors used higher volatilities than actually occurred, or could have been reasonably expected to occur, or because investors accurately allowed for the underlying stocks' volatilities, but could not process this information efficiently to come up with something close to the prices predicted by the Black–Scholes (1973) model. However, if we were to use a single, higher-volatility number for all warrants except Southern Van Ryn and Pretoria Silver, the Black–Scholes and market prices would be much closer over the entire time period during which

those warrants were traded. This result indicates that the use of too high a measure of volatility is more likely to have been a cause of the mispricing than irrationality. That is, the warrants' implied volatilities are higher than their ex post realized volatilities. For example, if we were to use a volatility figure of 0.61 throughout the Modderfontein East 4-year warrant's life rather than the perfect foresight figure of 0.43, the average absolute percentage mispricing falls from 20% to 8.4%. Other qualitative predictions of warrant pricing theory, such as no early exercise, are substantially borne out in our data, further suggesting that investors' use of too high a volatility is the more likely explanation for the mispricing.

B. Call Option Pricing

We consider two different methods to assess the fairness of the 30-day call option quotes. The first is to use the standard Black–Scholes (1973) formula, using the actual ex post volatility over the life of the option (the perfect foresight measure), together with a risk-free rate of 5%. Of the 11,181 30-day call options quotes in our data set, 8,671 or 77.6% of them were overpriced relative to the price implied by the Black–Scholes model.[10] Of the 112 stocks in our data set, more than half of the options quoted on 84 of them were overpriced relative to Black–Scholes model prices (including 18 stocks for which all the options written were overpriced), and more than half of the options quoted on 28 of them were underpriced relative to Black–Scholes model prices (including 10 stocks for which all options written were underpriced). There are a few quoted option prices without an observed stock (and therefore an implied strike) price for that day; these observations do not appear in our data set. In the first period, January 1908 to September 1909, 67.8% of all option quotes were overpriced. In the second period, August 1910 to May 1911, 95% of all option quotes were overpriced.

We define the average absolute percentage mispricing as $\frac{|C_{ASA} - C_{BS}|}{C_{BS}} \times 100$ averaged over all observations, where C_{ASA} is the price at which the ASA offered to write the call option and C_{BS} is the Black–Scholes (1973) model price. In the first period, we find that the average absolute percentage mispricing of the call options is 10.4%, but in the second period it rises to 52.4%. We believe a primary cause of this increase is the reduced competition in the market for writing call options following an Act of Parliament that halted all advertising by stockbrokers in October 1909. By August 1910, the ASA could resume quoting

[10] We exclude 18 West Roodeport Deep options from the 11,199 options we observe because we are not sure how these options, written on West Roodepoort Deep immediately before its takeover by Princess Estate, were treated by the ASA. Given the final price of West Roodeport Deep before the takeover, none of the options finished in-the-money. For companies that underwent restructurings that resulted in a jump in the share price, we make the share price after the restructuring comparable to that prevailing when the option was written. For example, both Aurora West and Bantjes Consolidated restructured by consolidating every four old shares into one new share. For options written over the reorganization period, we divide the stock price at expiry by four, to make it comparable to the strike price of the option.

Table III
African Share Agency 30-Day Call Options—Cost and Payout of a Simple Trading Strategy

We calculate the total cost of purchasing one of each call option offered by the ASA in the option cost column. We calculate the total payoff of exercising all call options that expire in-the-money in the option payoff column. We divide the option payoff by the option cost and multiply by 100 in the payoff/cost % column.

	Option Cost, £	Option Payoff, £	Payoff/Cost %
January 1908–September 1909	570.5	410.2	71.9
August 1910–May 1911	292.7	47.5	16.2
Whole period (1908–1911)	863.1	457.8	53.0

call options in the *Rand Daily Mail* and they were the only stockbroker advertising to write call options to the public. This reduction in competition allowed the ASA to exploit its unique market position at the expense of option buyers by raising the prices of its call options further above Black–Scholes prices. Our results therefore suggest that findings of option mispricing that rely on a broker's quotes are quite sensitive to the competitiveness of the market in which the broker operated. It is possible that differing levels of competition in the option writing market can explain the divergent findings of Boness (1964), Kruizenga (1964), Black and Scholes (1972), and Kairys and Valerio (1997).

Our second method of assessing the mispricing of the options allows us to compare our results with those of Kairys and Valerio (1997). Specifically, we calculate the returns for a hypothetical investor who follows the same simple strategy that they postulate: The investor buys one of each quoted call option; if in 30 days the option finishes in-the-money, he exercises it, otherwise he does not exercise the option. We summarize the cost and payoff of this strategy in Table III.

An investor who followed this strategy would have spent £863.1 between 1908 and 1911, and earned a gross payout of £457.8 from exercising those options that finished in-the-money. Thus, an investor would have recouped 53% of his or her money spent on call options in Johannesburg, which is more than the 27% rate that Kairys and Valerio (1997) find for U.S. call options in the 1870s.

We are not sure how the ASA handled dividend payments in their option contracts. We assume that the strike price was adjusted downward by the amount of any dividend payment for options written over a period in which a dividend was paid. The absence of any downward change in the quoted price of call options in the 30 days preceding an ex-dividend date (an alternative that would make options written over ex-dividend dates roughly as attractive as those written just before or just after a dividend payment) suggests that the ASA made an adjustment to the strike price, although we do not have definitive evidence to this effect. The payoff-to-cost ratio fell from 71.9% in the first period to 16.2% in the second period, partly due to a strong run-up in the market in 1908 and the

Table IV
African Share Agency 30-Day Call Option Data (10 Most Frequently Quoted Options)

Quotes is the total number of call option quotes the African Share Agency made using that company as the underlying asset. We calculate the % Exercised as the total number of option quotes that expire in-the-money divided by the total number of option quotes. We calculate the % Payoff as the monetary sum gained from exercising all options that finished in-the-money divided by the cost of purchasing all options multiplied by 100 (including both those exercised and those unexercised). % Mispricing, the average absolute percentage mispricing, is calculated as $\frac{|C_{ASA} - C_{BS}|}{C_{ASA}} \times 100$ averaged over all observations, where we calculate C_{BS} using the perfect foresight volatility measure, that is, the annualized standard deviation of the returns of that company's common stock over the 30 days of the option's life.

		January 1908 to September 1909			August 1910 to May 1911		
Company	# Quotes	% Exercised	% Payoff	% Mispricing	% Exercised	% Payoff	% Mispricing
Randfontein Estates	409	39.8	85.2	24.2	41.7	27.8	53.3
African Farms	390	55.6	71.7	36.0	32.9	13.0	62.0
Cons. Main Reef	386	55.4	68.4	32.1	33.5	16.5	65.5
South African Land	366	47.1	64.5	33.9	17.9	7.9	51.1
Knight Central	342	46.3	62.0	33.1	14.8	8.7	55.7
Lace Proprietary	289	38.8	105.3	32.0	18.8	5.6	57.3
Lydenburg Farms	285	49.0	54.2	42.9	12.9	3.4	67.7
Jupiter	274	53.8	76.1	26.3	5.9	2.1	43.6
East Rand Central	264	53.7	82.0	32.6	26.1	12.9	48.1
Benoni	251	34.3	14.6	34.0	31.6	11.9	63.6

first half of 1909. This run-up was followed by flat conditions in 1910 and 1911, combined with options that were more overpriced. Table IV presents summary statistics for the 10 most frequently quoted 30-day call options.

We follow an identical method for the 431 quotes for 60-day call options in January and February 1908. Over the 2 months when they were quoted, an investor would have spent £29.2 and earned £9.5 from exercising in-the-money 60-day calls, for a payoff-to-cost ratio of 32.4%. If instead of purchasing 60-day calls, the investor purchased 431 30-day calls on the same dates, an investor would have achieved a payoff-to-cost ratio of 35.3%.

Given that we use the average of the stock's bid and ask price as the assumed strike price of each call option (based on the nine call options for which we do observe the actual strike price), we probably underestimate the true strike price that would have been written into each call contract. We use the average of the stock's bid and ask price in 30 days as the price the option holder would receive if the investor were to exercise his or her option and sell the underlying stock immediately. This method will tend to overestimate the price at which the investor could sell the stock. Thus, our findings are likely to understate the extent of the overpricing.

We can also compare the mispricing of the ASA's call options and the JSE-traded warrants by calculating the implied volatility for each security. We have

concurrent prices for both the options and the warrants for three securities (Bantjes Consolidated, Vogelstruis Consolidated Deep, and South Randfontein Deep). Of the 16 concurrent observations, of which 7 were before the prohibition on advertising by stockbrokers and 9 were after the ASA became the single quoter of call options, the implied volatility of the call option is always higher than the implied volatility of the warrant. In 15 of the 16 observations, the implied volatility of the warrant is closer than the implied volatility of the call option to the 90-day backward-looking actual volatility. Taking an average of these 16 observations, the implied volatility of the call options is 114% higher than the implied volatility of the warrants. These facts indicate that market forces on the JSE tended to drive warrant prices closer to Black–Scholes (1973) prices than was the case for the non-market-traded call options.

III. Warrant Early Exercise

In Table V we present data on the exercise of warrants. These data come from companies' annual reports, published in the *Rand Daily Mail* or in the *South African Mining Journal*. In addition, we collect data from the minutes of the AGMs reported in the *Rand Daily Mail* and from quarterly reports and

Table V
Warrant Exercise

Shares Outstanding is the number of shares of common stock of each of the companies in our sample. The companies in our sample are all small- to medium-sized, based on market capitalization. # Warrants Issued is the number of warrants initially issued by each company. 0 denotes no warrants were exercised that year. "–" denotes the warrant had either not been issued or had expired by that year. We do not find the Frank Smith accounts, but from the minutes of the annual general meetings it appears that no warrants were exercised. Of the Geduld Proprietary warrants, 5,027 were exercised either in 1920 or early 1921. There were no South Randfontein Deep warrants exercised between 1910 and 1911. Vogelstruis Consolidated Deep warrant holders exercised 20,546 warrants (out of 131,000) in 1909, and none in 1910 (the warrant finished out-of-the-money). Bantjes Consolidated warrant holders exercised 22,260 warrants (out of 131,000) in 1909 and the remainder of the warrants in 1910 (the warrant finished well in-the-money).

Warrant	# Shares Outstanding	# Warrants Issued	# Exercised				
			1918	1919	1920	1921	1922
Modder East 3-year	690,459	373,979	0	0	251,855	–	–
Modder East 4-year	690,459	373,980	0	0	129	235	–
Daggafontein 3-year	648,033	250,000	1,732	248,268	–	–	–
Daggafontein 4-year	648,033	125,000	0	0	0	–	–
Daggafontein 5-year	648,033	125,000	0	0	0	0	–
Frank Smith	530,874	98,156	–	–	0	0	0
Geduld Proprietary	970,000	180,000	–	21,229	5,027	151,796	–
South African Alkali	82,500	35,000	–	–	0	7,817	0
Southern Van Ryn	274,000	180,000	16,724	0	–	–	–
Transvaal Silver	175,000	70,000	–	–	69,920	–	–
West Springs	531,978	115,000	0	0	0	0	–
Pretoria Silver	90,030	115,000	–	–	0	0	0

special reports in the *South African Mining Journal*. The table is necessarily incomplete, as these sources did not always mention how many warrants had been exercised.

One of the strongest results from derivative pricing theory is that it is always inefficient to exercise a call option before expiry if the stock does not pay dividends (Black and Scholes (1973)). Warrants should also not be exercised early, except in unusual cases of very narrowly held warrants (Emanuel (1983)).

Based on the data in our study, the exercise decisions of investors are very close to what modern theory recommends. For most stocks that did not pay dividends, warrant holders delayed exercise until the last possible opportunity. When they did exercise early, data from quarterly reports suggests that it was typically only 1 or 2 months ahead of the expiry date. A very small number of Modderfontein East 4-year and Daggafontein 3-year warrants were exercised more than a year before the expiry date.

Early exercise was significant for Geduld Proprietary, for which 11.8% of warrant holders exercised in the first year, and a further 2.8% exercised more than 3 months before the warrants expired.[11] Seventeen percent of Bantjes Consolidated warrant holders exercised more than 11 months early, although the stock did not pay dividends. Despite being inefficient, exercise must have seemed assured as the stock price was almost always at least twice the strike price during those months, although warrant holders did lose the benefit of delaying payment of the exercise price. Some 15.7% of Vogelstruis Consolidated Deep warrant holders exercised inefficiently more than a year early. Although the warrant was deep in-the-money during the year they exercised, it finished well out-of-the-money because the mine was shut down during the course of the warrant's life.

Warrant holders did not price the Geduld Proprietary warrant well. We calculate the model price for Geduld Proprietary warrants using a binomial tree with 100 steps and with the market stock price of Geduld common stock and the ex post realized dividend yield as inputs. Accounting for the dividends that the stock was to pay gives model prices that are substantially lower than market prices, especially at the beginning of the warrant's life (Figure 4). If we were to calculate binomial tree model prices using the market stock price, and a (counterfactual) 0% dividend yield (Figure 5), market prices would be much closer to model prices (the average absolute percentage mispricing using the 90-day backward-looking window falls from 26.0% to 13.0% when the dividend yield of Geduld Proprietary is set to 0%). This suggests that warrant holders did not do a good job of adjusting warrant prices for the dividends that Geduld Proprietary was to pay during the warrant's life.

Although the South African Alkali and Southern Van Ryn companies did not pay any dividends, warrant holders exercised early in significant numbers. This early exercise is explained by the fact that the holders could gain a warrant extension by exercising only a fraction of their warrants.

[11] On 10.7% of the days during which the Geduld Proprietary warrant was listed, the use of a binomial tree valuation model with 100 steps indicates that immediate exercise was optimal.

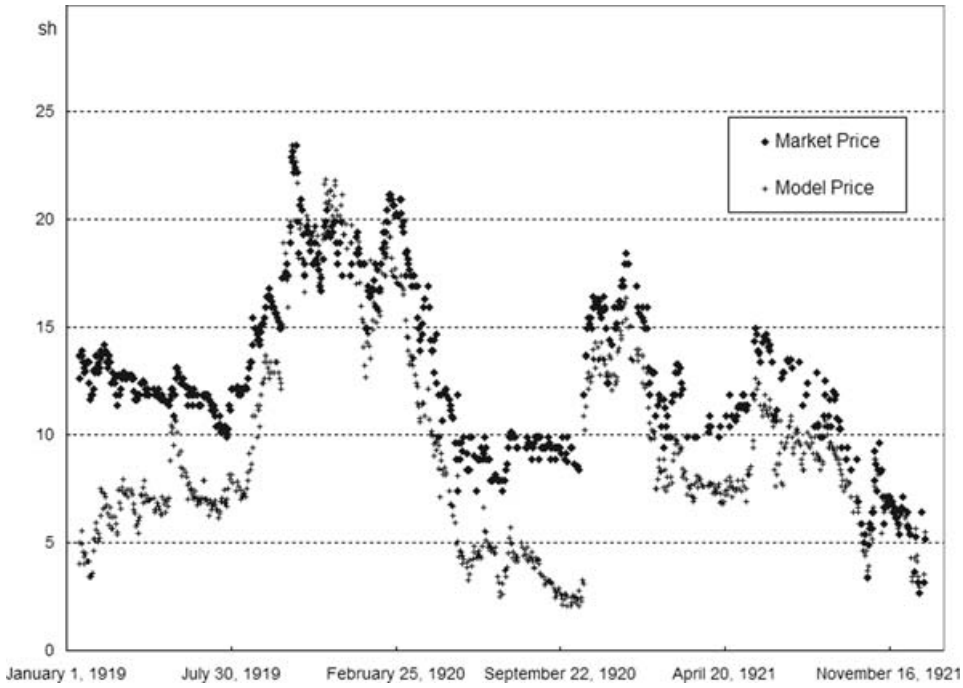

Figure 4. Geduld Proprietary (adjusted). Daily market prices and binomial tree model prices of the Geduld Proprietary warrant using a 90-day backward-looking window of equity returns to calculate volatility. We use the ex post realized dividend yield to calculate the binomial tree model prices using 100 steps. Both prices are measured in shillings.

IV. Warrant Extensions

Of the 15 warrants in our sample, 7 of them had their time to expiry extended. The reason appears to be that the issuing company concluded that the warrants would finish out-of-the-money and not be exercised. However, to obtain the necessary funds to continue development of the mine, the company wanted investors to exercise the warrants. Thus, the company extended the life of the warrants hoping the share price would rise so that further funds would flow into the company.

Company statements such as those of Southern Van Ryn suggest that the company believed there would be some fraction of shareholders who would almost automatically exercise their warrants if only they were given more time to do so: "There is every likelihood that in view of the new policy now adopted shareholders will take up their options as they fall due."[12]

[12] Statement by the Southern Van Ryn company on or about May 17, 1919 (*South African Mining Journal*), after a management reorganization. It appears management believed that the future direction of the company would determine warrant exercise, rather than the fundamentals of the strike price relative to the stock price on the expiry date.

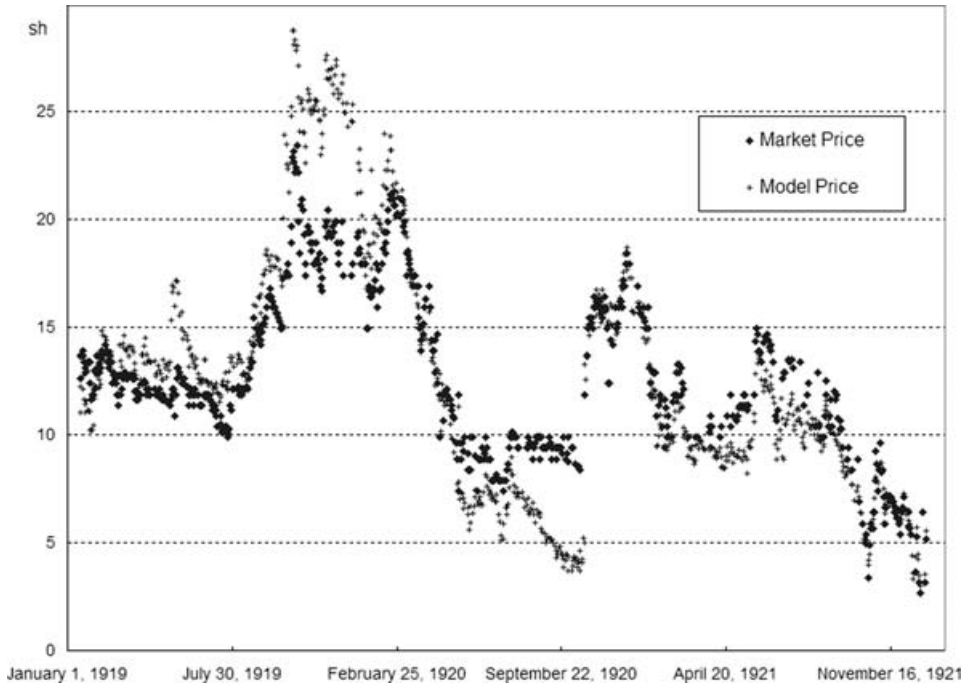

Figure 5. Geduld Proprietary (unadjusted). Daily market prices and binomial tree model prices of the Geduld Proprietary warrant using a 90-day backward-looking window of equity returns to calculate volatility. We use a dividend yield of zero to calculate the binomial tree model prices using 100 steps. Both prices are measured in shillings.

Of the seven warrants that were extended, none of them finished in-the-money at the end of the warrant extension because in each case the mines turned out to be unprofitable. Thus, extending the duration of the warrants only allowed investors to gain more information as to the unprofitability of the mine. Table VI contains details of the warrant extensions along with a brief description of what happened to the companies after the warrants went unexercised for a second time.

We examine whether warrant holders reacted optimally to the news of the warrants' extensions. We separately assess the effects of a free extension (in the case of Daggafontein 4- and 5-year warrants, Frank Smith, and West Springs) against those of an extension with a cost, in the case of the South African Alkali and Southern Van Ryn warrants. South African Alkali and Southern Van Ryn warrant holders could exercise a portion of the warrants early, out-of-the-money, to receive an extension on the remainder. We do not observe prices for the Pretoria Silver warrant before its extension; therefore, this warrant does not appear in Table VII.

For the free extensions, we ask whether the warrant price changed by the amount implied by the Black–Scholes (1973) model on the announcement date

Table VI

Warrant Extension

Initial Expiry is the date on which the warrant was set to expire, when it was originally issued by the company. Extended is the date on which the company officially extended the warrant's duration. Final Expiry is the date on which the warrant expired, after the company had extended the warrant's duration. Company Continuation gives brief details of what happened to each company after their warrants expired. The Frank Smith management extended the company's warrants by a period of 2 years, or until the mine reopened, whichever came first. South African Alkali holders had to exercise one-third of their warrants by March 31, 1921 to receive a 1-year extension of their remaining warrants. Southern Van Ryn holders had to exercise one-fourth of their warrants by December 31, 1918 to receive a 1-year extension of their remaining warrants. The initial expiry date of the West Springs warrants was unknown when they were issued, being 1 year after the ratification of the World War I peace treaty. Warrant price data for Pretoria Silver are only available for the extended warrant.

Panel A: Warrants That Were Extended

Warrant	Initial Expiry	Extended	Final Expiry	Company Continuation
Daggafontein 4-year	March 28, 1920	March 23, 1920	September 28, 1920	Mine shut down in March 1921
Daggafontein 5-year	March 28, 1921	March 23, 1920	September 28, 1921	Mine shut down in March 1921
Frank Smith	February 28, 1923	November 30, 1921	February 28, 1925	Company wound up in 1924
S. African Alkali	May 4, 1921	March 4, 1921	May 4, 1922	£50,000 debt issued in September 1922
Southern Van Ryn	December 31, 1918	n.a.	December 31, 1919	£3,500 debt issued in May 1921
West Springs	January 9, 1921	December 2, 1920	June 9, 1921	£350,000 debt issued in 1923
Pretoria Silver	September 1921	n.a.	July 31, 1922	£30,000 debt issued in June 1922

Panel B: Warrants That Were Not Extended

Modderfontein East	Issued shares to parent company in 1921, issued £465,465 debt in 1924
Geduld Proprietary	Continued as a regular dividend payer
Transvaal Silver	Borrowed money from parent company, issued £160,080 in equity in 1922
Bantjes Consolidated	Began paying dividends in 1912
Vogelstruis C. Deep	Mine shut down in June 1910
S. Randfontein Deep	Taken over by Randfontein Central in 1912

Table VII
Actual and Black–Scholes Implied Price Changes Following Warrant Extensions

We calculate the price change in shillings implied by the Black–Scholes (1973) model from the day before the warrants were extended to the day on which the warrants were extended in the Black–Scholes (previous 90 days' volatility) column. We use a 90-day backward-looking window of equity returns to calculate the volatility to use in the Black–Scholes formula. We also calculate the price change in shillings implied by the Black–Scholes model from the day before the warrants were extended to the day on which the warrants were extended in the Black–Scholes (perfect foresight) column. The perfect foresight column uses the actual realized volatility of equity returns over the life of the warrant. We calculate the change in the market price, in shillings, from the day before the warrants were extended to the day on which the warrants were extended in the market price column.

	Black–Scholes (Previous 90 Days' Volatility)	Black–Scholes (Perfect Foresight)	Market Price
Daggafontein 4	1.0	1.8	0.6
Daggafontein 5	0.8	1.1	0.0
Frank Smith	0.6	0.6	−0.3
West Springs	0.4	0.4	0.7

of the warrant extension. In Table VII we report the amount by which the price should have changed, based on the Black–Scholes model, and compare this to the actual change in the market price of the warrant on the announcement date.

Investors did not seem to be particularly good at calculating the change in the time value of warrants that were extended. They did not value the Daggafontein 4-year warrant extension as highly as the Black–Scholes (1973) model suggests, and they appeared to entirely ignore the extension effect of the 5-year warrant.

The Frank Smith warrant extension is particularly difficult to assess. The extension length was uncertain, being either 2 years or until the mine reopened, whichever came sooner. The warrant was unofficially extended in April 1921 after the company had been approached by several shareholders. This extension was made official at the AGM on November 30, 1921. In addition, the price data are noisy due to two problems, namely, the use of discrete tick sizes that became more important as the price of the warrant approached zero and the calculation of the time value of a warrant that was well out-of-the-money. During April and May, as investors became more convinced that the warrant extension would be approved at the AGM, the investors slowly repriced the warrants upward toward the Black–Scholes (1973) price with the extended expiry date.

The second type of warrant extension involves a cost. Warrant holders could have allowed their warrants to expire out-of-the-money, or they could have exercised the minimum fraction of warrants out-of-the-money (which acts as the extension cost). By doing so, they would obtain "new" warrants with identical specifications as the original ones, except that the new warrants would have an extended expiry date. South African Alkali warrants were extended by a year

until May 4, 1922, as long as warrant holders exercised one-third of them by March 31, 1921. Southern Van Ryn warrants were extended by a year, until December 31, 1919, as long as warrant holders exercised one-fourth of them by December 31, 1918. We value the benefit of the warrant extensions by measuring the increase in the remaining fraction of the warrants' prices implied by the Black–Scholes (1973) pricing formula. We use the perfect foresight volatility measure and extend the warrants' durations by 1 year.

We check the South African Alkali and Southern Van Ryn extensions to determine if investors rationally exercised a fraction of their warrants to obtain an extension on the remaining fraction. South African Alkali warrant holders would have incurred a cost of 2.4 shillings per warrant to prolong their remaining warrants' lives by a year, which would have increased the value of their remaining fraction of warrants by 2.6 shillings per warrant. Some 22.3% of warrant holders took advantage of this warrant extension. Southern Van Ryn warrant holders would have incurred a cost of 1.8 shillings per warrant to extend their warrants' lives by a year, which was only worth 0.3 shillings per warrant to them. Only 9.3% of these warrants were exercised early. In this case early exercise was inefficient, because the cost of early exercise was substantially larger than the benefit.

V. Modern Derivative Pricing

We compare the early twentieth-century JSE mining derivatives with a contemporary data set of mining and industrial companies' derivatives traded on the JSE in 2003. This analysis assists us in determining whether the mispricing we find with respect to the early twentieth-century is due to the lack of a formal derivative pricing model. We then compare South African derivative pricing, past and present, with U.S. mining and industrial option prices to assess whether the exchange on which the products are traded is relevant to their mispricing.

We obtain data on 18 call options and one warrant for mining companies listed on the JSE between 2001 and 2003 from PSG Investment Services. Modern-day options are issued by banks, they do not need the permission of the company on whose stock they are written, and they are usually written for durations of less than 12 months. The options have a coverage ratio of between 10 and 200, which means that an investor requires between 10 and 200 options to exercise into one share. The JSE web site explains that the coverage ratio is in place to facilitate liquidity in the options market.[13] The only warrant in our modern sample that is exercisable one-for-one for shares, Harmony Gold, has a 2-year duration. The issued warrants comprise 4.3% of the ordinary shares outstanding.

Trading of options began in October 1997 on the JSE, with issuance performed by eight large banks including Deutsche Bank, Standard Bank, and UBS Warburg. The JSE market for options exists alongside the South African

[13] www.jse.co.za

Table VIII
Modern JSE Derivative Specifications—2001 to 2003

Obs is the number of option (warrant) observations we observe. Duration is the length of the option's (warrant's) life. We measure the mean share price and mean option (warrant) price in Rand over the life of the option (warrant). The strike price is measured in Rand. % Obs "in" is the percentage of price observations when the share price is above the strike price. The only warrant in this sample, the Harmony warrant, has a dilution of 4.3%.

Call Option	# Obs	Duration	$\overline{Share}$	$\overline{Option}$	Strike	% Obs "in"
Anglo Gold CB	42	6 months	264	26.2	330	0
Durban Roodepoort CA	129	9 months	34	7.7	45	7.8
Durban Roodepoort CB	88	12 months	32	7.3	65	0
Goldfields CA	134	9 months	112	16.3	150	0.7
Goldfields CD	81	6 months	105	10.6	150	0
Goldfields CE−SB	82	6 months	106	17.2	100	69.5
Harmony Warrant	202	2 years	110	68.0	43	96.0
Harmony CA−IB	100	9 months	135	23.8	180	0
Harmony CB	72	6 months	125	27.7	130	54.2
Anglo American CB	60	9 months	127	8.3	180	0
Anglo American CE−IB	67	9 months	127	11.9	160	0
Anglo American CE−SB	61	6 months	122	10.5	130	13.1
Ang.Amer. Platinum CB−IB	67	9 months	331	34.7	400	0
Ang.Amer. Platinum CC	47	7 months	304	38.8	360	0
Ang.Amer. Platinum CD	44	6 months	305	17.5	390	0
BHP Billiton CA	30	6 months	41	2.2	52	0
Impala Platinum CA	76	12 months	544	78.7	590	15.8
Impala Platinum CC	47	6 months	523	45.3	650	0
Kumba Resources C1	33	18 months	35	3.3	85	0

Futures Exchange (SAFEX), which trades options on futures. The SAFEX market, established in 1997, is primarily geared toward institutional investors, whereas the options market is aimed at small retail investors. Officials at the JSE suggest that institutional investors do not generally trade in the JSE options market because they do not have mandates from their clients to do so.

We use exactly the same method for these data as we do for the early twentieth-century warrants to calculate volatilities and Black–Scholes (1973) prices. Table VIII provides summary statistics of these modern securities. Table IX presents data on the mispricing.

The large mispricing of modern options on the JSE is troubling because it appears to indicate large unexploited profit opportunities. Although the options are not extremely liquid, they appear to be liquid enough to make a profit by hedging. The bid–ask spread is generally quite low. The average of each option's bid–ask spread divided by the price of that option is around 4%, and the bid–ask spread is probably not large enough to explain such mispricing. While there are many options on futures for mining companies listed on SAFEX, the market is extremely thin, with most of the trades corresponding to market indexes rather than options on futures of individual stocks.

JSE officials suggest that the mispricing may be due to overly optimistic investors. Since approximately 30% of the options listed are put options and 70%

Table IX

Modern JSE Derivative Pricing—2001 to 2003

We average $|C_{BS} - C_{market}|$ over all observations in the Rand column. We obtain the numbers in the percent column, the average absolute percentage mispricing, by dividing the number in the Rand column by the average price of the derivative and multiplying by 100. We present the volatility measure we use for each derivative in the σ^2 perfect foresight volatility column, and the low to high range of volatility measures we use in the σ^2 previous 90 days' volatility column. We use the annualized standard deviation of common stock returns, calculated over the duration of the derivative's life, as our volatility parameter in the perfect foresight columns. We use the annualized standard deviation of common stock returns, calculated over the previous 90 days, as our volatility parameter in the previous 90 days' volatility columns.

	Perfect Foresight			Previous 90 Days' Volatility		
Call Option/Warrant	Rand	Percent	σ^2	Rand	Percent	σ^2 (Range)
Anglo Gold CB	13.8	52.7	0.39	10.1	38.8	0.38–0.46
Durban Roodepoort CA	3.0	39.5	0.75	2.0	26.6	0.56–1.27
Durban Roodepoort CB	4.2	57.6	0.73	2.9	40.2	0.56–1.18
Goldfields CA	7.1	43.4	0.61	4.5	27.5	0.49–1.00
Goldfields CD	6.6	62.2	0.54	5.8	54.4	0.49–0.64
Goldfields CE–SB	0.9	5.0	0.54	1.0	6.0	0.49–0.64
Harmony Warrant	4.3	6.4	0.67	5.0	7.3	0.33–1.01
Harmony CA–IB	9.6	40.4	0.63	5.7	24.1	0.45–1.01
Harmony CB	10.2	34.0	0.55	5.7	27.6	0.45–0.65
Anglo American CB	3.6	43.0	0.43	2.1	25.8	0.31–0.58
Anglo American CE–IB	5.5	46.7	0.39	2.8	23.8	0.31–0.58
Anglo American CE–SB	4.3	41.0	0.34	3.9	37.0	0.31–0.45
Ang.Amer. Platinum CB–IB	17.8	51.4	0.35	16.8	48.4	0.28–0.51
Ang.Amer. Platinum CC	17.6	45.5	0.39	24.0	62.0	0.28–0.42
Ang.Amer. Platinum CD	8.1	46.3	0.39	12.2	69.7	0.28–0.42
BHP Billiton CA	1.3	56.9	0.33	1.0	44.4	0.32–0.39
Impala Platinum CA	7.6	9.5	0.46	7.1	9.1	0.35–0.53
Impala Platinum CC	12.5	28.1	0.44	19.7	43.5	0.35–0.46
Kumba Resources C1	1.8	56.4	0.44	1.7	51.7	0.30–0.54
Average		36.2			29.7	

are call options, the optimism of small investors appears to be the most likely explanation for the highly priced options. The issue price of options written by the large banks (that would certainly know about, and probably use, some form of the Black–Scholes (1973) model), is also too high. That is, it is not just a secondary market phenomenon. There are no standard option contracts. Each option has an ad hoc strike price and expiry date. This lack of standardization may have been done intentionally by the issuing banks to avoid a direct comparison of options between different issuers, thus avoiding price competition.

Twelve of these 19 derivatives are never in-the-money compared to 2 of the 15 early twentieth-century warrants. The modern options and one warrant are priced slightly more poorly than the warrants in the early twentieth century, with a weighted average absolute mispricing across all derivatives of 36.2% using the perfect foresight measure of volatility and 29.7% using the historical

measure of volatility.[14] Derivatives that are always out-of-the-money, in particular Goldfields CD, Anglo American Platinum CB-IB, CC, and CD, and Kumba Resources C1 are priced further from Black–Scholes (1973) prices than are the derivatives that are sometimes in-the-money, such as the Harmony Warrant, Goldfields CE - SB, Impala Platinum CA, and Durban Roodepoort CA.

Using the same data set, we then compare the mispricing of 14 call options written on industrial companies in Johannesburg from 2002 to 2003. The market for industrial companies' call options is even less liquid than the market for mining companies' call options, and it is difficult to find options that trade more than a handful of times. Of the 14 call options in our data set, the minimum number of option price observations we have is 9, the maximum is 58, and the average is 26.4. Using these data we find an average absolute percentage mispricing of 41.6% using the perfect foresight measure of volatility and 37.3% using the 90-day backward-looking measure. We find that the prices of industrial companies' call options are further from Black–Scholes (1973) prices than are the prices of mining companies' call options.

We suspect that the reason for the mispricing is partly due to the institutional features of the Johannesburg options market, namely, its small size, illiquidity, unsophisticated retail investors, and lack of direct price competition among option writers. Conversely, the mispricing in contemporary Johannesburg does not appear to be linked to whether or not the options are for industrial or mining companies. Mispricing on the modern JSE is at a comparable level with mispricing in the early twentieth century. This suggests that the development of a formal pricing model has not greatly improved investors' abilities to accurately price derivatives.

To provide some perspective on the mispricing we find on the modern JSE, we contrast these results with a set of 24 call options traded on U.S. markets. We collect 15 call options for resource companies, 3 each for Barrick Gold (ABX), Halliburton (HAL), Harmony Gold (HMY—the same company that is listed on the JSE), Placer Dome (PDG), and Royal Gold (RGLD), and use 9 call options for industrial companies, 3 each for Dell Inc (DELL), Coca-Cola (KO), and Microsoft (MSFT). Our sample covers the period July 2003 to November 2003. We use transaction prices from both the Chicago Board Options Exchange (CBOE) and the International Securities Exchange (ISE) for this data set.

Although the U.S. market is much more liquid and has standardized contracts, the market for individual equity options is still quite small. On the U.S. market there is an average daily volume of fewer than 100 contracts for some of the mining options versus over 1,000 contracts for the industrial companies.

[14] Harmony Warrant is underpriced, on average, by 0.7% using the perfect foresight volatility measure and 1.9% using the 90-day backward-looking volatility measure. Goldfields CE–SB and Impala Platinum CA are underpriced by one measure and overpriced by the other. The weighted average mispricing, whereby underpricings partly offset the majority of overpricings, is slightly lower at 34.4% using the perfect foresight volatility measure and 26.3% using the 90-day backward looking volatility measure.

We collect data on the most liquid call options, which tend to be those that are at-the-money.

Our average absolute percentage mispricing measure for the U.S. market is 13.4% for the mining companies and 11.9% for the industrial companies using a 90-day backward-looking window. This finding indicates that investors in U.S. markets are pricing derivatives more accurately than are investors trading on the JSE. Increased price competition and standardized contracts in the U.S. market partly explain the difference in mispricing.

VI. Conclusion

In this paper we present a new data set of 15 warrants traded in the early 20th century on the JSE and of a broker's quotes of call options written on 112 stocks. Using modern derivative pricing theory, we assess how well investors priced these securities.

We find that, in general, warrant prices were surprisingly accurate in the pre-Black–Scholes era, with an average absolute percentage mispricing of 23.7% when we use a perfect foresight measure of volatility and 27.4% when we use a historical measure of volatility. Comparing these warrants to derivatives trading between 2001 and 2003 on the same exchange and using the same methods to compute volatility, we find that early twentieth-century investors mispriced in a comparable way using a historical measure of volatility and outperformed modern JSE investors using a perfect foresight measure of volatility. Development of the modern theory does not appear to have improved the performance of South African investors.

Between 1908 and 1911, a broker offered to write call options on many mining stocks. We find that these were priced less accurately than were the JSE-traded warrants. We argue that the less accurate performance of call options is due to the absence of market forces in driving prices to fair values. A simple trading strategy suggests that investors would have lost 47% of each pound spent on purchasing call options.

We also assess how well investors priced warrant extensions in the pre-Black–Scholes (1973) era and whether they exercised warrants optimally. We find that extensions were not priced very well, although warrant prices moved in the direction indicated by modern pricing theory. Warrants were generally exercised optimally, although complicated extensions that required partial exercise do not seem to have been dealt with well.

Taken together these results suggest that a competitive market, whether through trading of derivatives on an exchange or competition among rival brokers, has been more important in driving derivative prices to fair values than the development of a formal derivative pricing model by Black and Scholes (1973) and Merton (1973). While development of the formal theory has been invaluable for a complete understanding of the factors that influence the value of derivatives, early twentieth-century investors in Johannesburg appear to have been able to process relevant information and come close to determining the fair value of derivatives.

REFERENCES

Bachelier, Louis, 1900, Théorie de la spéculation, *Annales de l'Ecole Normale Superieure* 17, 21–86. English translation by A. James Boness, 1964, in Paul H. Cootner, ed.: *The Random Character of Stock Market Prices* (MIT Press, Cambridge, MA).

Black, Fischer, and Myron Scholes, 1972, The valuation of option contracts and a test of market efficiency, *Journal of Finance* 27, 399–417.

Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637–654.

Boness, A. James, 1964, Some evidence on the profitability of trading in put and call options, in Paul H. Cootner, ed.: *The Random Character of Stock Market Prices* (MIT Press, Cambridge, MA).

Conway, Thomas Jr., 1913, *Investment and Speculation* (Alexander Hamilton Institute, New York).

Emanuel, David, 1983, Warrant valuation and exercise strategy, *Journal of Financial Economics* 12, 211–235.

Franklin, Charles B., and Marshall R. Colberg, 1958, Puts and calls: A factual survey, *Journal of Finance* 13, 21–34.

Galai, Dan, 1977, Tests of market efficiency of the Chicago Board Options Exchange, *Journal of Business* 50, 167–197.

Galai, Dan, and Meir I. Schneller, 1978, Pricing of warrants and the value of the firm, *Journal of Finance* 33, 1333–1342.

Higgins, Leonard, 1906, *The Put-and-Call* (Aberdeen University Press, Aberdeen).

Kairys, Joseph P., Jr., and Nicholas Valerio III, 1997, The market for equity options in the 1870s, *Journal of Finance* 52, 1707–1723.

Kassouf, Sheen T., 1969, An econometric model for option price with implications for investors' expectations and audacity, *Econometrica* 37, 685–694.

Kruizenga, Richard J., 1964, Introduction to the option contract, in Paul H. Cootner, ed.: *The Random Character of Stock Market Prices* (MIT Press, Cambridge, MA).

Lauterbach, Beni, and Paul Schultz, 1990, Pricing warrants: An empirical study of the Black-Scholes model and its alternatives, *Journal of Finance* 45, 1181–1209.

Macbeth, James D., and Larry J. Melville, 1979, An empirical examination of the Black-Scholes call option pricing model, *Journal of Finance* 34, 1173–1186.

Merton, Robert C., 1973, Theory of rational option pricing, *Bell Journal of Economics and Management Science* 4, 141–183.

Rand Daily Mail, various issues.

South African Mining Journal, various issues.

The Star, various issues.

Sullivan, Edward D., and Timothy M. Weithers, 1991, Louis Bachelier: The father of modern option pricing theory, *Journal of Economic Education* 22, 165–171.