THE JOURNAL OF FINANCE • VOL. LXI, NO. 1 • FEBRUARY 2006

Asset Pricing Implications of Nonconvex Adjustment Costs and Irreversibility of Investment

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ABSTRACT

This paper derives a real options model that accounts for the value premium. If real investment is largely irreversible, the book value of assets of a distressed firm is high relative to its market value because it has idle physical capital. The firm's excess installed capital capacity enables it to fully benefit from positive aggregate shocks without undertaking costly investment. Thus, returns to equity holders of a high book-to-market firm are sensitive to aggregate conditions and its systematic risk is high. Simulations indicate that the model goes a long way toward accounting for the observed value premium.

RECENT EMPIRICAL STUDIES HAVE FOUND BOTH THAT (1) stocks of firms that are characterized by a high book-to-market ratio earn on average higher returns than those of firms characterized by a low book-to-market ratio, and (2) cross-sectional stock returns bear little or no discernible relationship to the capital asset pricing model (CAPM) beta.[1] There is an ongoing debate among finance researchers as to whether the book-to-market ratio proxies for nondiversifiable factor risk or whether security mispricing drives the value premium.

This paper develops a real options model that accounts for the observed value premium. The model ties the firm's book-to-market characteristic to its loadings with respect to risk factors. Intuitively, if capital investment is largely irreversible, the book value of assets of a distressed firm, that is, a firm that has been hit by adverse profitability shocks, remains fairly constant. This firm's

*Ilan Cooper is with the Department of Financial Economics, Norwegian School of Management. This paper is based on a chapter of my doctoral dissertation at the Graduate School of Business of the University of Chicago. I am grateful to my advisors John Heaton, Owen Lamont, Toby Moskowitz, and especially George Constantinides and Steve Davis for their guidance and encouragement. I have benefited immensely from the comments and suggestions of Simon Benninga, Bruno Gerard, Leonid Kogan, Lior Menzly, Richard Priestley and Nahid Rahman. I owe special thanks to Rick Green (the editor) and an anonymous referee, whose insightful comments helped me improve the paper greatly. I am also grateful to John Cochrane, Adlai Fisher, Boyan Jovanovic, Marco Lyrio, Bernt-Arne Odegaard, and seminar participants at the University of Chicago, Federal Reserve Bank of Boston, Free University of Amsterdam, Norwegian School of Management, Copenhagen Business School, University of Haifa, Tel-Aviv University, Hebrew University, Norwegian School of Economics and Business Administration, 2003 EFA annual meeting, University of Leuven, Tilburg University, 2004 AFA annual meeting, and Stockholm School of Economics for helpful comments and suggestions. All remaining errors are my own.

[1] See Fama and French (1992) for summary evidence.

book-to-market ratio is high because its market value falls. Such a firm is sensitive to aggregate shocks. Its excess installed capital capacity allows it to benefit from a positive aggregate shock without undertaking any costly investment. The payoff to its equity holders is high in such a case. In contrast, a low book-to-market firm would have to undertake costly investment in order to fully benefit from the positive shock. Thus, low book-to-market firms are less sensitive to economic conditions and have lower systematic risk. This means they have lower market betas if equilibrium security returns are determined according to the CAPM relationship. Alternatively, if sources of risk beyond movements in the market portfolio are priced, for example factors that covary with recessions, low book-to-market firms have lower loadings with respect to those factors as well. In the model, systematic risk is conditional on the firm's excess capital capacity, which evolves stochastically with shocks to the firm's profitability. Thus, the firm's loadings with respect to risk factors are stochastic. The firm's book-to-market ratio is correlated with its excess capital capacity and therefore serves as a proxy for its systematic risk. Model simulations demonstrate that empirical tests that do not adequately allow for dependence of the market beta on the firm's excess physical capital will tend to reject the CAPM predictions, even if the CAPM, in its conditional form, describes equilibrium security returns. These simulated tests find a statistical relationship between book-to-market and average returns in the cross-section and predictive power for book-to-market in the time series.[2]

Model simulations also facilitate a comparison of the role of two capital investment frictions, namely, investment irreversibility and fixed adjustment costs of investment, in generating a value premium; the intuition for the model provided above is valid when investment is irreversible as well as when in addition to irreversible investment the firm faces fixed adjustment costs of investment. In both cases the firm has an investment threshold, so that only when profitability reaches an upper threshold, investment is triggered.

In the former case, whenever this threshold is crossed the firm undertakes a small investment to keep Tobin's marginal q from exceeding 1. In the latter case investment is lumpy because the firm aims to avoid incurring the fixed costs frequently. This difference in investment behavior implies that the cross-sectional distribution of firms' book-to-market ratios is different in these two cases. Due to the lumpiness of investment in the presence of fixed adjustment costs of investment, a larger fraction of the firms is distant from their investment thresholds, implying that investment is less frequent in this case. In the model, the lower frequency of investment and its larger magnitude in the presence of fixed adjustment costs of investment have opposing effects on the relationship between book-to-market and systematic risk. Intuitively, the lower frequency of investment weakens the book-to-market effect produced by the model because a larger fraction of the firms is distant from their investment threshold. On the other hand, the lumpiness of investment strengthens

[2] Pontiff and Schall (1998) provide evidence that the aggregate book-to-market ratio predicts subsequent market returns.

the book-to-market effect in the model because low book-to-market firms that do invest incur even larger investment costs. Model simulations enable a comparison of the role of these two capital investment frictions in generating a value premium. I find that in the model, irreversibility of investment, not fixed adjustment costs, is the driving force behind the value premium.

The simulations further demonstrate that the relation between book-to-market and systematic risk crucially depends on the degree of investment irreversibility: the larger the degree of investment irreversibility, the stronger the relation between book-to-market and systematic risk. The rationale for this is that if investment is partially reversible, the firm has the real option to disinvest. Negative profitability shocks, while reducing the value of the firm's assets in place and its growth options, increase the value of its option to disinvest. This effect is more pronounced for a firm with excess capital capacity (a high book-to-market firm) for which the option to disinvest is more likely to be exercised. Thus, the sensitivity of firms, and particularly high book-to-market firms, to negative aggregate shocks is lower in case they have the option to disinvest relative to a situation in which no such option exists and investment is completely irreversible. This model implication is directly testable and opens a potentially fruitful area for future empirical research.

This paper is part of a new and growing line of research, pioneered by Berk, Green, and Naik (1999). This literature relates stock return dynamics to firms' real investment decisions. Berk et al. (1999) present a model in which firms that perform well tend to be those that discovered particularly valuable investment opportunities with low systematic risk. In their model good news is associated on average with lower systematic risk, and changes in risk lead to a role for book-to-market in predicting returns. Gomes, Kogan, and Zhang (2003) present a dynamic stochastic one-factor general equilibrium model in which book-to-market serves as a proxy for the systematic risk of the firm's assets in place. In these papers investment is irreversible.

My paper differs from these papers along several dimensions. First, in Berk et al. (1999) and Gomes et al. (2003), when an investment project becomes available the firm undertakes it if the net present value (NPV) of the project is positive. In these models the profitability of the firm's assets in place does not affect the firm's investment decisions. In contrast, in my paper, the firm's investment does depend on the accumulated profitability shocks to its assets in place and is thus path dependent. That is, in my model the value of growth options is affected by shocks to the profitability of the firm's assets in place. As the firm's assets in place become more profitable, the value of its growth options increases and the probability that the firm undertakes investment increases. Second, in Berk et al. (1999) and in Gomes et al. (2003), the firm's systematic risk depends not only on its book-to-market ratio but also on its market value (size). In contrast, in my model there is a single state variable that is related only to book-to-market, implying that my model accounts for the value premium but not for the size effect.

Kogan (2004) develops a general equilibrium model of irreversible investment and shows that investment irreversibility entails time variation in the conditional moments of stock returns. Kogan assumes perfect competition, constant returns to scale in the production technology, and convex adjustment costs of investment. In contrast, in my model firms operate under imperfect competition and decreasing returns to scale, and face fixed adjustment costs of investment.

Zhang (2005) makes an important contribution by presenting an industry equilibrium model of asymmetric convex adjustment costs of investment. In Zhang's model, value firms have more difficulty severing their capital stocks than growth firms, since value firms are typically burdened with more unproductive capital. This is especially important during economic downturns, when value stocks become more risky. Zhang solves his model by applying modern numerical techniques.

My paper differs from Zhang's along several dimensions: First, Zhang models adjustment costs of both investment and disinvestment as convex for the sake of analytical tractability. This implies that optimal investment and disinvestment behavior is smooth over time. However, the empirical literature shows that such behavior is inconsistent with firm-level and plant-level data. Instead, actual investment behavior exhibits periods of inaction interrupted by investment spikes, behavior which is consistent with the existence of nonconvex adjustment costs and irreversibility.[3] My model incorporates exactly these types of costs as well as complete or partial investment irreversibility.

A second difference from Zhang's paper is that Zhang's solution relies purely on numerical methods whereas my model offers a closed-form solution for the relationships between the firm's systematic risk and both its book-to-market ratio and its excess capital capacity. Thus, my model has the advantage of being directly testable: It gives rise to future empirical tests that will involve constructing proxies for the firm's excess capital capacity. Consequently, my model offers unique opportunities to examine the role of economic fundamentals in asset return dynamics. In contrast, Zhang's model yields a relationship only between book-to-market and risk. Therefore, Zhang's explanation for the value premium cannot be distinguished empirically from other explanations (for example, explanations based on behavioral biases).

The mechanisms that generate the value premium in my paper and Zhang's paper give rise to a third difference. Zhang (2005, p. 2) argues that, "As expanding capital is relatively easy, the dividends and returns of growth firms do not covary much with economic booms." In contrast, in my model it is precisely the fact that positive investment is costly and difficult (the firm must pay a constant price for each unit of capital it purchases) that implies that growth firms do not covary much with economic booms.

A final important difference between the two papers is that Zhang's quantitative results hinge on a counter-cyclical price of risk, whereas my model yields

[3] For empirical evidence on nonconvex adjustment costs and irreversibility, see Caballero, Engel, and Haltiwanger (1995), Doms and Dunne (1998), Barnett and Sakellaris (1998), Caballero and Engel (1999), Cooper, Haltiwanger, and Power (1999), Ramey and Shapiro (2001) and Abel and Eberly (2002).

a large dispersion in the quantity of risk of value and growth firms and can therefore account for much of the value premium even when the price of risk is constant. Thus, my model's results do not require any implied restrictions on preferences such as time-varying risk aversion. Empirical evidence in Petkova and Zhang (2005), who find considerable dispersion in the market betas of value and growth stocks, lends support to my model.

Carlson, Fisher, and Giammarino (2004) analyze the effect of operating leverage on expected returns. They demonstrate that a book-to-market effect can be directly related to fixed operating costs. Their paper is distinct from my paper along several dimensions. First, Carlson et al. (2004) use a simulated method of moments to estimate their model whereas I employ the methodology of Fama and French (1992) on simulated data. My simulations gauge the role of several frictions in the capital adjustment technology, most importantly partial reversibility, in generating a value premium, and thereby facilitate future empirical tests that are different from the tests in Carlson et al. Second, in their model there are limits to firms' growth opportunities, which amounts to entailing an independent size effect, whereas my model allows for economic growth but has no size effect.

A related strand of literature focuses on the link between stock returns and aggregate investment returns using a production-based asset pricing model with frictions in the capital adjustment technology; for example, Cochrane (1991) examines the relation between aggregate investment returns and the aggregate stock market. Cochrane (1996) extends the work in Cochrane (1991) by examining the link between cross-sectional as well as time-series variation in expected stock returns and aggregate investment returns. Gomes, Yaron, and Zhang (2002) incorporate costly external finance in an investment-based asset pricing model with adjustment costs and examine whether financing frictions help in pricing the cross-section of expected returns.

Several empirical findings provide support for the explanation of the value premium suggested in this paper: Lewellen (1999) finds that the book-to-market ratio explains time variation in the loadings of firms with respect to the three Fama and French (1993) factors. Liew and Vassalou (2000) use data from 10 countries and find that the value premium is positively related to future GDP growth. Thus, high book-to-market stocks benefit more than low book-to-market stocks from a forthcoming expansion in real activity. The intuition presented above, that is, that firms with a high book-to-market ratio have excess capital and can therefore benefit more than low book-to-market firms from positive shocks, is consistent with this finding.

The rest of the paper is organized as follows. Section I presents an overview of the theoretical and empirical literature on adjustment costs and irreversibility of real investment. The model is presented in Section II. The main implications of the model are presented in Section III. Section IV discusses aggregate economy dynamics, Section V presents simulation results, and the paper concludes with Section VI.

I. Adjustment Costs and Irreversibility

Several forms of adjustment costs are described in the real investment literature. Until recently, the workhorse model of this literature has been a model with convex adjustment costs (often approximated to be quadratic, that is, symmetric). However, recent empirical studies find that nonconvexities and investment irreversibility play an important role in the investment process.[4]

Fixed adjustment costs can take two forms. The first is simply a fixed lump-sum cost whenever the firm increases or decreases its stock of capital. The second takes the form of forgone output during periods of investment or disinvestment. The firm might forgo some of its output due to disruption to production when new capital is installed or uninstalled. This cost is not fixed in the classic sense because it depends on the amount of output the firm would have produced had it not invested. That is, this cost depends on the firm's productivity, capital stock, and other factors of production at the time of investment. Yet, this cost does not vary with the amount of investment and hence is fixed in this sense. I term this cost quasi-fixed. Both fixed and quasi-fixed adjustment costs entail a region of inaction in investment activity—it is optimal for the firm to wait until accumulated shocks to fundamentals are large enough, so that the benefit of adjusting the stock of capital covers the fixed costs of adjusting. When profitability rises to a sufficiently high level relative to the existing capital stock, investment will be triggered. Similarly, disinvestment will be triggered when profitability is sufficiently low.

Irreversibility of investment can be thought of as a form of adjustment costs. It implies that the purchase cost of the new capital is a sunk cost. Thus, irreversibility entails an indirect cost of investment. In contrast, when investment is reversible, part of the purchase cost can be recovered in the future by selling the undepreciated capital. Irreversibility can arise when the resale price of capital is lower than its purchase price (due to capital specificity, "lemon" problems, or market thinness).

Figure 1 presents the different forms of adjustment costs described. This figure depicts the costs of adjustment incurred by the firm as a function of its investment. The convex curve represents asymmetric convex adjustment costs of investment, the form of adjustment costs in Zhang (2003). The flat lines represent fixed and quasi-fixed adjustment costs of investment and disinvestment. The dotted line represents proportional adjustment costs; for positive investment the dotted line represents the purchase cost of capital and for negative investment it represents the revenues from the sales of capital. The different slopes for investment and disinvestment reflect a difference in the purchase and sales price of capital.

II. The Model

In this section, I introduce a continuous time dynamic real options model of the firm. Under tractable conditions and reasonable assumptions I obtain

[4] See Caballero et al. (1995), Eberly (1997), Doms and Dunne (1998), Barnett and Sakellaris (1998), Cooper and Haltiwanger (2000), Ramey and Shapiro (2001), and Abel and Eberly (2002).

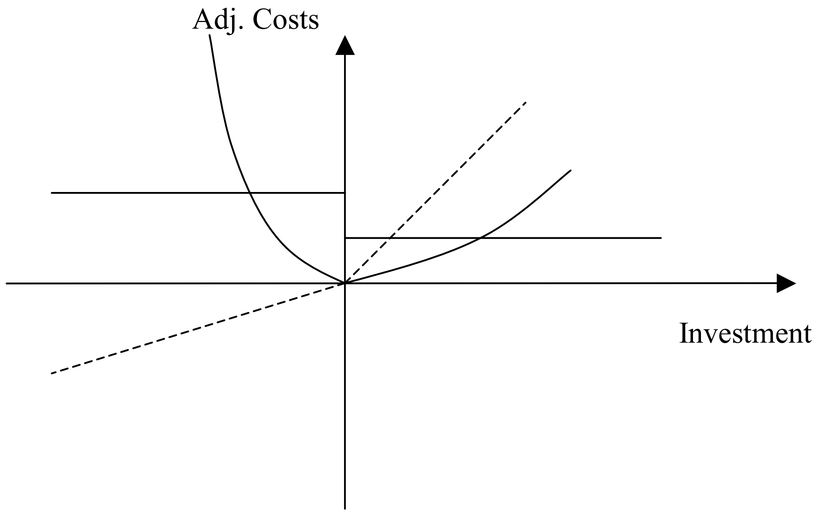

Figure 1. Various forms of adjustment costs as a function of investment. The convex curve represents asymmetric convex adjustment costs of investment, implying that disinvesting is more costly than investing (the form of adjustment costs in Zhang (2003)). The flat lines represent fixed or quasi-fixed adjustment costs of both investment and disinvestment. Notice that quasi-fixed costs of adjustment do not depend on the investment or disinvestment, and therefore are represented by a flat line. In the figure, the fixed and quasi-fixed adjustment costs are larger for disinvestment than for investment. The extreme case in which these costs are infinite for disinvestment leads to irreversible investment. The dashed line represents the purchase cost of the new capital for positive investment, and the sales revenues from disinvestment. Notice that the purchase cost of capital is lower than the sale cost of capital. Abel and Eberly (1994) show that if the sales price of capital is low enough relative to its purchase price, the firm does not disinvest.

a solution for the value of the firm. This solution shows how the empirically observed determinant of average returns, namely, the book-to-market ratio, is related to economically fundamental characteristics of the firm. The solution enables me in the next section to infer the relationship between the firm's book-to-market ratio and its loadings with respect to common risk factors, and it facilitates the simulations in the subsequent section.

The principal assumptions in the model are that (i) the firm's investment is irreversible, and (ii) the firm faces quasi-fixed as well as proportional adjustment costs of investment.[5]

A. Technology

The firm uses capital to produce. Its flow of profits per unit of time is given by

$$\Pi(\theta, K) = \theta^{1-\gamma} K^{\gamma} - mK, \tag{1}$$

[5] Several authors use quasi-fixed adjustment costs of investment rather than purely fixed adjustment costs of investment. Examples are Caballero and Leahy (1996) and Caballero and Engel (1999).

where K is the firm's stock of capital, θ is its productivity parameter, $0 < \gamma < 1$, and m is production costs per unit of capital, which can be thought of as necessary maintenance to keep the capital operational.[6]

When the firm invests, it forgoes a fraction of its profits. The adjustment cost function $C(\theta, K)$ is given by

$$C(\theta, K) = F\theta^{1-\gamma}K^{\gamma}, \tag{2}$$

where $0 < F < 1$. The interpretation of this function is that whenever the firm invests, it forgoes a fraction F of the profits it would have made had it not invested. Modeling the adjustment costs as in (2), that is, as a fraction of the firm's output, ensures that the adjustment costs do not become negligible or overwhelming relative to the size of the firm.

In addition to forgone output, when the firm invests it has to pay for each unit of capital it purchases. The produced good in the economy can be used either for consumption or for investment as a capital good. Therefore, the price of new capital is equal to the price of output. Both prices are assumed to be constant and are normalized to one.

The law of motion for the capital stock is

$$dK = -\delta K\, dt + I, \tag{3}$$

where δ is the capital depreciation rate and I is the firm's investment.

The firm has the option to permanently shut down operations. Shutting down is costless and results in a firm value of zero.

I denote the value of the firm by $J(\theta, K)$. Note that in the model the price of a new unit of capital is constant and is equal to one. Therefore, the firm's capital, K, can be interpreted as its capital measured in historical costs. Put differently, K can be thought of as the book value of the firm's assets. It follows that the ratio $\frac{K}{J(\theta,K)}$ in the model can be interpreted as the firm's book-to-market ratio.

The firm's productivity is given by

$$\theta = \theta_A\theta_i, \tag{4}$$

where θ_A is the aggregate productivity level and θ_i is an idiosyncratic productivity level. Both θ_A and θ_i follow a geometric Brownian motion as follows:

$$\frac{d\theta_A}{\theta_A} = \mu_A\, dt + \sigma_A\, dw_A, \tag{5}$$

$$\frac{d\theta_i}{\theta_i} = \sigma_i\, dw_i, \tag{6}$$

[6] There is no loss of generality in assuming a profit function that is homogeneous of degree one in K and θ: assume that the production function is of the form ψK^{γ}, where ψ is a profitability index. Assume that ψ follows a geometric Brownian motion. Then define a process θ as follows: $\theta \equiv \psi^{\frac{1}{1-\gamma}}$ (implying that $\theta^{1-\gamma} = \psi$). Note that θ follows a geometric Brownian motion as well. Then substitute $\theta^{1-\gamma}$ into the production function in place of ψ.

where μ_A and σ_A are the drift and volatility, respectively, of the growth rate of the aggregate productivity, σ_i is the volatility of the growth rate of the idiosyncratic productivity, and dw_A and dw_i are increments of two independent Wiener processes that represent shocks to the aggregate and idiosyncratic productivity growth rates, respectively. It follows that θ follows a geometric Brownian motion as well:

$$\frac{d\theta}{\theta} = \mu_A \, dt + \sigma \, dw, \tag{7}$$

where $\sigma \equiv \sqrt{\sigma_A^2 + \sigma_i^2}$ and $dw \equiv \frac{\sigma_A dw_A + \sigma_i dw_i}{\sqrt{\sigma_A^2 + \sigma_i^2}}$.

Note that the growth rate of θ follows an aggregate drift and fluctuations around this drift are due to aggregate and idiosyncratic volatilities. Thus, my model incorporates economic growth. This is in contrast to Carlson et al. (2004), in which there are limits to growth.

Let μ be the risk-adjusted drift of the firm's productivity growth process. Using μ in place of μ_A in valuing the firm implies that the firm's future cash flows should be discounted using the risk-free rate. This valuation rule is developed in Constantinides (1978). Appendix A presents the expression for the risk-adjusted drift for the productivity growth rate.

B. Valuation

In this subsection I derive a solution for the value of the firm as the sum of two components, namely, the value of the firm's assets in place and the value of its growth options. I show that an empirically observed determinant of returns, the book-to-market ratio, is related to underlying economic characteristics of the firm.

Note that the profit function $\Pi(\theta, K)$ and the adjustment cost function $C(\theta, K)$ are both homogeneous of degree one in K and θ. It follows that the value of the firm, $J(\theta, K)$ is also homogeneous of degree one in K and θ.

Let $Z \equiv \frac{K}{\theta}$ and redefine (1) and (2) in terms of Z, such that

$$\Pi(\theta, K) = \theta Z^{\gamma} - m\theta Z = \theta\pi(Z), \tag{1$'$}$$

where $\pi(Z) = Z^{\gamma} - mZ$, and

$$C(\theta, K) = \theta F Z^{\gamma} = \theta c(Z), \tag{2$'$}$$

where $c(Z) = FZ^{\gamma}$. The value of the firm can then be written as

$$J(\theta, K) = \theta V(Z). \tag{8}$$

It is now possible to fully characterize the optimization problem of the firm in the space of Z.

Note that $Z = \frac{K}{\theta}$ is positively related to the firm's book-to-market ratio: The numerator of Z, K, can be thought of as the firm's book value of assets, and the denominator of Z, θ, is positively related to the firm's market value of equity.

The Bellman's equation for the firm's problem in the region in which it operates is

$$V^0(Z) = \max_{\tilde{I}\geq 0}\left\{\pi(Z) - \tilde{I} - c(Z)1_{\{I>0\}} + e^{-r\,dt}\int_0^\infty V(Z')\phi(\theta'/\theta)\,d\theta'\right\}, \quad (9)$$

and the value of the firm is

$$V(Z) = \max\{V^0, 0\}, \quad (10)$$

where $\tilde{I} = \frac{I}{\theta}$, r is the risk-free rate, and the indicator function $1_{\{H\}}$ is equal to 1 if the event H occurs and zero otherwise. Note that θ' and Z' represent the future values of θ and Z, respectively, and $\phi(\theta'/\theta)$ is the conditional density function of θ' given θ. Appendix B shows the derivation of (9) and proves the existence of $V(Z)$.

Proposition 1 below shows that given the homogeneity assumptions, there exists a unique $V(Z)$ that satisfies (9). This proposition is proved in Appendix B.

PROPOSITION 1: *There exists a unique solution to the functional equation (9).*

In the region in which no investment takes place, the variable Z follows a geometric Brownian motion:

$$\frac{dZ}{Z} = (\sigma^2 - \mu - \delta)\,dt - \sigma\,dw. \quad (11)$$

The derivation of (11) is in Appendix C.

The nature of the solution to the firm's problem is as follows. First, for a positive Z, since $c(Z)$ is positive even for small adjustments, it is apparent that when Z is close enough to the value for which $V(Z)$ is maximized, the benefit from adjusting Z (which is performed by increasing K) will be smaller than the cost entailed by doing so. Thus, there is a range of inaction in investment. Investment will be triggered only when Z is sufficiently low. That is, investment will be triggered when the productivity θ is high enough relative to the stock of capital K, so that the benefits of adjusting the capital stock cover the cost entailed by doing so. When productivity is low enough such that the firm value is zero, the firm will shut down permanently. Thus, when Z is sufficiently high, the firm will shut down operations.

Let L be the minimum value of Z for which there is no investment and let G be the target level to which Z is reset once it hits L ($L < G$). Let U be the maximum value of Z for which the firm continues operations. The range of inaction in the space of Z is (L,U).

Note that given θ, large values of K imply that the firm is distant from its investment threshold. Therefore the variable $Z(=\frac{K}{\theta})$ contains information about the firm's excess capital capacity. Higher values of Z are associated with higher excess capital capacity. Let $K^G(\theta)$ be the target level of capital stock corresponding to the target level of Z, namely, G. Then $K^G(\theta) = \theta G$. I define the firm's excess capital capacity, EC as the ratio of the firm's actual capital stock to its target capital stock, where EC is given by

$$EC = \frac{K}{K^G(\theta)} = \frac{\left(\dfrac{K}{\theta}\right)}{\left(\dfrac{K^G(\theta)}{\theta}\right)} = \frac{Z}{G}. \tag{12}$$

Note that since G is constant in the model, EC is proportional to Z. Values of EC greater than one correspond to excess capital capacity, whereas when EC is smaller than one, the firm observes a capital shortage.

The solution in the space of Z is characterized by five boundary conditions: a value matching condition, three smooth pasting conditions, and a shutdown condition.[7]

The value matching condition is

$$V(L) = V(G) - FL^{\gamma} - (G - L), \tag{13}$$

and the smooth pasting conditions are

$$V_Z(L) = 1, \tag{14}$$

$$V_Z(G) = 1, \tag{15}$$

$$V_Z(U) = 0. \tag{16}$$

The condition to shut down operations is given by

$$V(U) = 0. \tag{17}$$

Note that $J_K(\theta, K) = \frac{d(\theta V(Z))}{dK} = \theta V_Z Z_K = \theta \frac{1}{\theta} V_Z = V_Z$. Therefore, V_Z is the marginal value of capital.

It is now possible to derive an expression for the value of the firm as a function of the state variable Z. The details of this derivation are in Appendix D. The value of the firm comprises two components. The first component is the value of assets in place. The second is the value of the firm's real options, namely, its growth options and its option to disinvest. The value of the firm in the region in which it operates is given by

$$\begin{aligned} J(\theta, K) &= \theta V(Z) = \theta \left[AZ^{\gamma} - SZ + D_N Z^{\lambda_N} + D_P Z^{\lambda_P} \right] \\ &= \theta \left[V_1(Z) + V_2(Z) \right], \end{aligned} \tag{18}$$

where $A = \frac{1}{r + \gamma\mu + \gamma\delta - \gamma\sigma^2 - \frac{1}{2}\sigma^2\gamma(\gamma-1)}$, $S = \frac{m}{r - \sigma^2 + \mu + \delta}$, $V_1(Z) = AZ^{\gamma} - SZ$, $V_2(Z) = D_1 Z^{\lambda_N} + D_2 Z^{\lambda_P}$, and D_N and D_P are constants related to the value of the firm's option to invest and option to shut down operations, respectively. The values of

[7] See, for example, Dixit (1993) for a very clear explanation of the boundary conditions.

D_N and D_P are found by numerically solving the boundary conditions (equations (D12) through (D16) in Appendix D). Note that λ_N and λ_P are the negative and positive roots of the fundamental quadratic of the geometric Brownian motion (see Appendix D for the solution for λ_N and λ_P).

The function $V_1(Z)$ is the value of the firm's existing assets (assets in place). It is found by solving for the value of the firm as if the firm were unable to change its existing stock of capital (see Appendix D for the derivation of $V_1(Z)$). Thus, we can view $V_2(Z)$ as the value of the firm's options to invest (the value of its growth options) and to shut down. These are real options. The value of these options depends on Z and thus varies within the inaction region. Note that λ_N is negative. Therefore, the value of the firm's growth options declines with Z, the firm's excess capital capacity, and the firm's book-to-market ratio.

III. Book-to-Market, Factor Loadings, and Expected Returns

A. A Firm Facing Irreversibility and Quasi-Fixed Adjustment Costs of Investment

This section presents expressions linking the book-to-market ratio and loadings with respect to the risk factors described in Section II.

The covariance of the return on such a firm with the return on factor s (denoted R_s) in the inaction region is given by

$$\mathrm{Cov}\left(\frac{dJ+\Pi}{J-\Pi}, R_s\right) = \left(\frac{V}{V-\pi}\right)\sigma \mathrm{Cov}(dw, R_s)\left(1-\frac{V_Z Z}{V}\right). \tag{19}$$

(The derivation of (19) is given in Appendix E).[8] The beta (loading) of the firm with respect to the factor return R_s, denoted β_s, is given by

$$\beta_s = \left(\frac{V}{V-\pi}\right)\left[\sigma \mathrm{Cov}(dw, R_s)\left(1-\frac{V_Z Z}{V}\right)\right]\frac{1}{\mathrm{Var}(R_s)}. \tag{20}$$

Note that (20) relates the firm's β with respect to factor s to its book-to-market ratio, because from (8) and the definition of Z, it follows that $\frac{Z}{V} = \frac{(\frac{K}{\theta})}{V} = \frac{K}{\theta V} = \frac{K}{J}$, which is the firm's book-to-market ratio. Therefore, the loadings of the firm with respect to the risk factors are directly related to its book-to-market characteristic:

$$\beta_s = \left(\frac{V}{V-\pi}\right)\left[\sigma \mathrm{Cov}(dw, R_s)\left(1-V_Z\frac{K}{J}\right)\right]\frac{1}{\mathrm{Var}(R_s)}. \tag{21}$$

[8] For simplicity of the notation, in this subsection I omit the arguments of the functions J and V, that is, I write J instead of $J(\theta, K)$ and V instead of $V(\theta, K)$.

The expected return of the firm per unit of time is given by

$$E\left[\frac{dJ+\Pi}{J-\Pi}\right]=E\left[\frac{d(\theta V)+\Pi}{\theta V-\Pi}\right]=\left(\frac{V}{V-\pi}\right)\left[\frac{d\theta}{\theta}+\frac{dV}{V}+\frac{\pi}{V}\right]$$
$$=\mu+\frac{\left(\mu V_Z Z+\frac{1}{2}\sigma^2 V_{ZZ} Z^2\right)}{V}+\frac{\pi}{V}. \tag{22}$$

The firm's expected return varies within the inaction region.

Figures 2 and 3 depict results from a numerical example. Figure 2 describes the relation between market beta and the logarithm of the book-to-market ratio predicted by the model. The range of book-to-market ratios in this figure contains the range of book-to-market ratios of the 10 book-to-market portfolios in Fama and French (1992, Table IV). In this example the choice of parameters is aimed at matching key empirical findings on macroeconomic and microeconomic variables (see Table I for the parameter values used in this example as well as in the model simulations described in the next section). The selection of the parameters is described in detail in the next section. As discussed above, in the model the ratio $\frac{Z}{V}$ can be interpreted as book-to-market. The point Log(BM_L) in Figure 2 represents the logarithm of the book-to-market ratio at the investment threshold L, and the point Log(BM_G) is the logarithm of the book-to-market ratio at the investment target G. In the figure, beta appears as an exponential function of the logarithm of book-to-market shifted upward, implying an approximately linear relation between beta and the book-to-market itself.[9]

Figure 3 describes the relation between market beta and EC, that is, the ratio of the firm's actual capital to its target capital, or the firm's excess capital capacity. Consistent with the intuition provided for the model, beta increases with EC.

This numerical example indicates that the firm's loadings with respect to the risk factors are increasing and convex functions not only of the logarithm of $\frac{Z}{V}$, but also of Z and of its excess capital capacity, EC.

B. Frictionless Investment

I denote by β^* and the term *frictionless beta* the beta of a firm that does not face any adjustment costs or irreversibility of investment. Appendix F shows that for such a firm it would be optimal to keep the ratio of K to θ constant, and therefore Z constant. It follows that

$$\beta^*=\text{constant}\times\frac{\sigma\,\text{Cov}(dw,R_s)+\text{constant}}{\text{Var}(R_s)}. \tag{23}$$

Note that this is a linear transformation of the regression coefficient when regressing the productivity shock's growth rate on factor s returns, scaled by σ.

[9] Robustness checks indicate that the shape of the relation between the firm's beta and its book-to-market ratio is linear for a wide range of parameters.

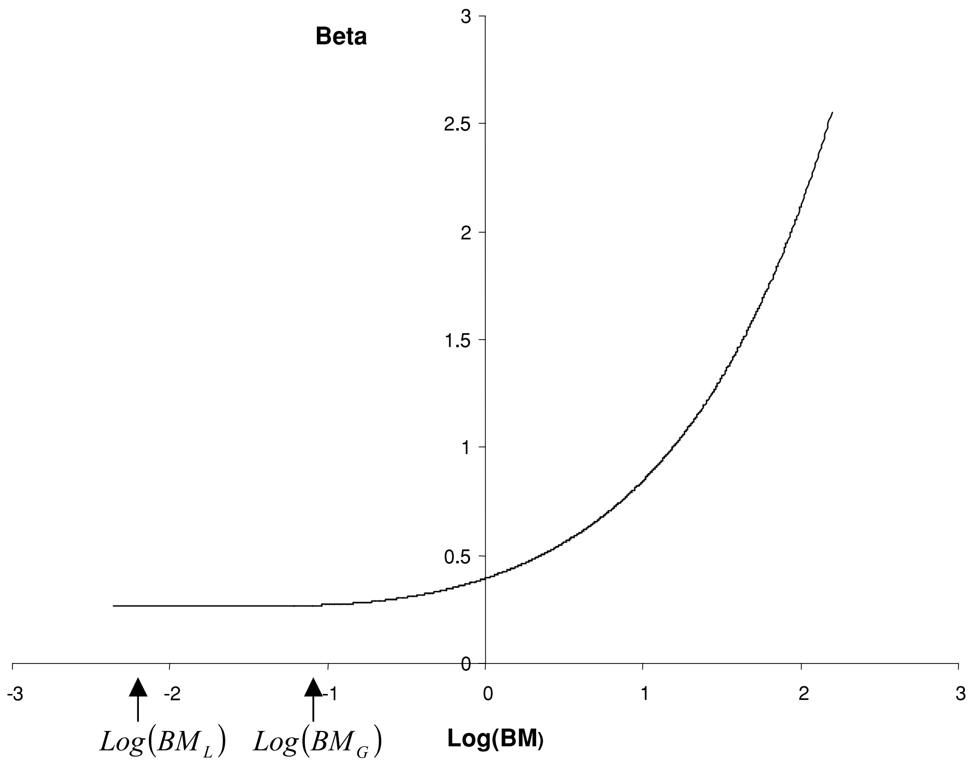

Figure 2. A numerical example of the relationship between the firm's market beta and the logarithm of its book-to-market ratio. The functional form of the relationship between the firm's market beta and its book-to-market ratio is given by

$$\beta_m = \left(\frac{V}{V-\pi}\right)\left[\sigma \operatorname{Cov}(dw, R_m)\left(1 - V_Z \frac{K}{J}\right)\right]\frac{1}{\operatorname{Var}(R_m)}$$

(equation (21) in the text where the factor s is the market portfolio), where $\frac{K}{J}$ is the book-to-market ratio in the model. Note that BM_L is the book-to-market ratio at the investment threshold and BM_G is the firm's book-to-market ratio at the investment target. Define $Z \equiv \frac{K}{\theta}$. The firm will undertake investment when Z is equal to L. Intuitively, when profitability is high enough relative to the existing stock of capital (i.e., when Z is low enough) the firm will invest. The firm will optimally increase its stock of capital setting Z equal to G. Thus, G is the target level for Z. The parameters assumed in this numerical example are presented in Table I. Notice that if the firm does not face any adjustment costs or irreversibility of investment, Z would optimally be kept constant (see Appendix F for a proof).

In the model σ, $\operatorname{Cov}(dw, R_s)$, and $\operatorname{Var}(R_s)$ are assumed to be constant, implying that the frictionless beta is constant as well.

In contrast, when the firm does face adjustment costs and investment irreversibility, beta becomes time varying—the sensitivity of the firm's value to shocks varies within the inaction interval. Note that the firm's β in equations (20) and (21) contains terms that depend on the state variable Z, representing the effect of the firm's location in its inaction interval.

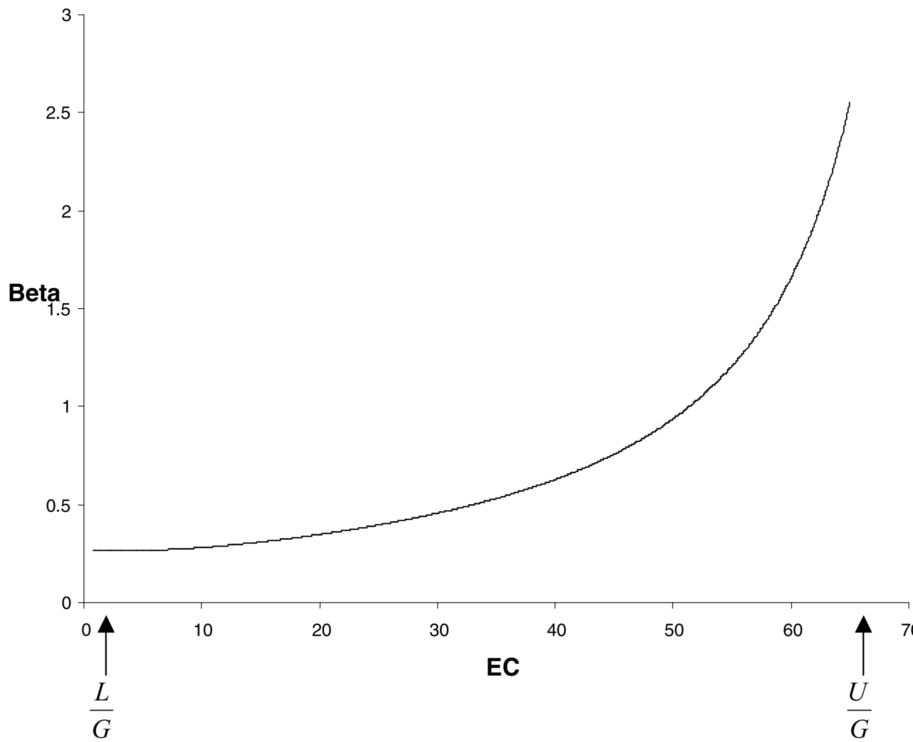

Figure 3. A numerical example of the relationship between the firm's market beta and its excess capital capacity. The functional form of the relationship between the firm's market beta and the state variable Z, where $Z \equiv \frac{K}{\theta}$, is given by

$$\beta_m = \left(\frac{V}{V - \pi}\right)\left[\sigma \operatorname{Cov}(dw, R_m)\left(1 - \frac{V_Z Z}{V}\right)\right]\frac{1}{\operatorname{Var}(R_m)}$$

(equation (20) in the text when the factor s is the market portfolio). The firm's excess capital capacity is proportional to $Z : EC = \frac{Z}{G}$, where G is the investment target. The investment threshold is point L. Intuitively, when profitability is high enough relative to the existing stock of capital (i.e., when Z is low enough), the firm will invest. The firm will optimally increase its stock of capital setting Z equal to G (and $EC = 1$). When $EC > 1$, the firm has excess capital capacity. Values of EC that are in the interval $[\frac{L}{G}, 1)$ imply that the firm's actual capital stock is smaller than the target capital stock, that is, the firm observes capital shortage. The parameters assumed in this numerical example are presented in Table I. Notice that if the firm does not face any adjustment costs or irreversibility of investment, Z would optimally be kept constant (see Appendix F for a proof).

IV. Aggregate Economy Dynamics

Having solved the dynamic optimization problem of a single firm, this section describes the dynamics of the cross-section of firms. The economy consists of a continuum of firms, distributed over the firm-level state space. Each firm is subject to aggregate and idiosyncratic shocks and makes decisions about both its optimal investment and whether to continue or permanently shut down operations.

Table I

Parameter Values for the Simulations (Annualized)

Listed are the values for all parameters required to simulate the model. Group I, presented in Panel A, consists of parameters that are standard in the real business cycle literature: the capital depreciation rate (δ) and the production technology parameter (γ). Panel B presents Group II which consists of cost parameters: the adjustment cost parameter (F) which is the fraction of annual output that is forgone when the firm undertakes investment, the entry cost parameter (C_e), and the fixed production cost parameter (m). Panel C presents the risk-free interest rate (r), the drift of the aggregate productivity growth process (μ_A), the volatility of the aggregate productivity growth process (σ_A), the volatility of the idiosyncratic productivity growth process (σ_i), the Sharpe ratio of the market portfolio (S_{R_m}), and the covariance between the aggregate productivity growth shock and the return on the market portfolio (C_{Am}).

Panel A: Group I					
δ	γ				
0.12	0.85				
Panel B: Group II					
F	C_e	m			
0.012	1.017	0.48			
Panel C: Group III					
r	μ_A	σ_A	σ_i	S_{R_m}	C_{Am}
0.018	0.0262	0.0781	0.2375	0.43	0.32

As the model incorporates permanent shutdown of operations, entry of new firms must be permitted, otherwise there would eventually be no firms in the economy. I follow Carlson et al. (2004) and assume that entry opportunities are created by the exit of existing firms; whenever a firm exits, a potential entrant emerges. Thus, the combined mass of potential entrant and active firms in the economy is held constant. This assumption is appropriate for an economy in which each firm possesses market power and entry is possible only upon the firm's exit.[10]

Entry entails a sunk cost, which is proportional to the aggregate productivity level. Modeling the cost of entry as proportional to the aggregate productivity level ensures that the cost of entry does not become negligibly small or overwhelmingly large. This cost can be thought of as a search cost for scarce resources such as skilled managers, for whom the opportunity cost of working for the entering firm increases with aggregate productivity. In addition, an entering firm needs to purchase new capital to begin operations. Recalling that the price of a new unit of capital is one, the total cost of entry for an entrant indexed by j entering with capital stock K_j is given by $\theta_A C_e + K_j$, where C_e is a constant.

[10] Firms in Carlson et al. (2004) also possess market power: In their model the economy consists of a continuum of monopolistic firms. In the model in this paper, the fact that $0 < \gamma < 1$ can be interpreted as the firm having market power.

Each potential entrant is given a single opportunity to enter. A firm that does not enter is assigned a value of zero and in the next period a new potential entrant takes its place. At any point in time each potential entrant j is given an independent draw for its idiosyncratic productivity level θ_{jt} from the time-t distribution of θ_j (see Appendix G for the expression for the time-t distribution of θ_j). A potential entrant j enters if there is $K > 0$ such that $J(\theta_j, K) \geq \theta_A C_e + K$, in which case the entrant enters with capital stock of $K_j^* = \arg\max_K \{J(\theta_j, K)\}$.

V. Simulation Evidence

In this section I evaluate the model's ability to reproduce key qualitative and quantitative features of the data. The methodology follows Berk et al. (1999) and Gomes et al. (2003).

I simulate the model for two basic cases: first, a case in which investment is completely irreversible and the firm also faces quasi-fixed adjustment costs of investment (case 1), and second, a case in which investment is irreversible but there exist no adjustment costs of investment (case 2). This facilitates a comparison of the role of each type of friction in explaining the value premium. I then conduct sensitivity tests for various magnitudes of adjustment costs and partially reversible investment.

I calibrate parameters at an annual frequency, and then convert them into a monthly frequency given that the empirical tests in Fama and French (1992) and Pontiff and Schall (1998) are conducted using a monthly frequency. Table I summarizes the parameter values in the model simulations. I break down the parameters into three groups. The first group consists of parameters that are commonly used in the business cycle literature and that can be restricted by prior empirical or quantitative studies. The second group includes cost parameters, that is, the cost of production, cost of adjustment, and cost of entry parameters. The third group consists of key asset-pricing parameters.

Panel A of Table I presents parameters that are considered standard in the real business cycle literature. Real business cycle models are generally calibrated with depreciation rates of 8% to 12% per year (see, for example, Kydland and Prescott (1982) and Cooper and Haltiwanger (2000)). I choose a value of 0.12 for the annual depreciation rate parameter, δ, implying a 1% depreciation per month. I select the value of the technology parameter γ to be 0.85. This choice is consistent with estimates of aggregate capital share of 0.3 (see Kydland and Prescott (1982), for example).[11]

Panel B of Table I presents the cost of adjustment, cost of entry, and the production cost parameters. Calibration for this group of parameters has only limited guidance from prior studies and I have certain degrees of freedom in choosing their values.

[11] Note that since the profit function in the model assumes that the choice of labor has already been optimized, γ represents the share of capital divided by one minus the share of labor. The derivation of γ from a production function that includes both capital and labor as arguments is available from the author upon request.

The adjustment cost parameter, F, is the fraction of output that the firm forgoes when it invests. I set this parameter to 1.2% per year. This choice leads on average to investment spikes of approximately 15% each year, in line with the findings of Doms and Dunne (1998, Figure 5) and Abel and Eberly (2002, Table 1) on the median net investment rate of U.S. manufacturing firms. In the simulated cross-sectional tests, I conduct robustness checks for the results by varying the magnitude of the adjustment costs and by examining the case of partial and complete investment irreversibility without adjustment costs.

The entry cost parameter, C_e, is set to 1.04. For this value of C_e the average rate of entry of firms into the economy is equal to the average rate of exit of firms from the economy. This rate in the simulations is approximately 1.8% per year, which is somewhat lower than the average annualized entry and exit rates of 2.5% and 3.62%, respectively, reported by Campbell (1998) for the U.S. manufacturing plants in the period 1972 to 1989.

The value of m, the production costs per unit of capital, affects the market value of the firm. I calibrate m to be 0.48, such that the average book-to-market of firms is 0.72, which is quite similar to the average book-to-market ratio (0.67) of firms in the data as reported by Pontiff and Schall (1998). The interpretation of the value of m is that for every dollar of installed capital, the firm must pay 48 cents in fixed costs annually.

The third group of parameters appears in Panel C of Table I. These parameters include the drift and volatility of the aggregate productivity growth process, the volatility of the idiosyncratic productivity growth process, the Sharpe ratio of the market portfolio, and the correlation between firms' productivity growth shocks and the return on the market portfolio.

The firm's productivity growth drift and volatility are calibrated to match the unconditional moments of the aggregate stock market as follows. Chan, Karceski, and Lakonishok (2003) report that the average annual earnings growth rate for the S&P 500 firms in the period 1951 to 1997 is roughly 3.5%. I first find the drift of the productivity growth process that matches this average earnings growth rate. This drift is 3.7% in annual terms. However, since the value of the firm in the model is obtained using the risk-neutral measure, the appropriate drift for the productivity growth process is the risk-adjusted drift, which is lower than the one matching the data. As seen in equation (A1) in Appendix A, the choice of the risk-adjusted drift involves choices of the market Sharpe ratio and the correlation of the firm's productivity growth process with the market return. I choose the market Sharpe ratio to be 0.43, which is the average annual Sharpe ratio in the postwar sample of Campbell and Cochrane (1999). As seen in equation (A4) in Appendix A, the correlation of the firm's productivity growth process and the return on the market portfolio depends on the volatility of the firm's productivity growth process, σ. I set $\sigma = 0.25$ (or 25% per year). This choice is made to match the standard deviation of the annual earnings growth of U.S. corporate earnings in the period 1929 to 2001 (29%) as reported by Longstaff and Piazzesi (2004). The annual aggregate and idiosyncratic volatilities of the productivity growth process (σ_A and σ_i) are 7.81% and

Table II

Key Moments of the Aggregate Stock Market

This table reports the unconditional mean and standard deviation of the equity premium and the real interest rate. The moments of the equity premium in the data are from Campbell et al. (1997). The real interest rate in the data is the annual return on 6-month commercial paper, rolled over in January and July, as reported in Campbell et al. (1997). In the model, the risk-free rate is assumed to be equal to that in the data. For the market portfolio I simulate 20,000 monthly returns on a value-weighted portfolio of 50 firms.

Equity Premium	Model	Data
Mean	0.052	0.060
Volatility	0.191	0.180
Real interest rate	0.018	0.018

23.75%, respectively. These choices are made in order to match the average R^2 in market model regressions as reported in Fama (1976, chapter 4, Table 4.4). Equation (A4) in Appendix A also shows that given σ_A and σ, the correlation of the firm's productivity growth process and the return on the market portfolio depends only on the covariance of the shock to the aggregate productivity growth and the return on the market portfolio, denoted C_{Am}. I set $C_{Am} = 0.32$, implying that the correlation of the firm's productivity growth process and the return on the market portfolio is 0.1 (see equation (A4)). The choice of C_{Am} is somewhat arbitrary.[12] It implies, according to equation (A1) in Appendix A, that the risk-adjusted drift of any firm's productivity growth rate is 2.62% per year.

A. Time-Series Simulation

I begin by simulating 20,000 months of data for 50 firms. I focus on the steady state cross-sectional distribution of firms by dropping the first 200 observations for each firm. I construct a value-weighted portfolio from the 50 firms and compare the key aggregate moments of this portfolio to those of the data. Table II presents the moments of the key aggregate variables in the data and in the simulated artificial data.[13] The equity premium in the simulation, 5.23%, is close to the 6% reported by Campbell, Lo, and MacKinlay (1997) for the period 1951 to 2000.

Pontiff and Schall (1998) find that the aggregate book-to-market ratio is a significant predictor of the market returns in the time series. I follow their estimation procedure in my simulations. Table III displays results of time-series regressions of the value-weighted portfolio of 50 firms on a 1-month lagged aggregate book-to-market (the aggregate book-to-market is the ratio of the sum of the book values of the firms' capital stocks to the sum of their market values).[14]

[12] Robustness checks for the value of Corr($\frac{d\theta}{\theta}$, R_m) yield quantitatively similar results.

[13] The results in Table I are for case 1. The results for case 2 are very similar qualitatively and quantitatively.

[14] Results are for case 1. The results for case 2 are very similar.

Table III

Book-to-Market as a Predictor of Market Returns

This table presents time-series regression of a simulated value-weighted index return on the simulated aggregate book-to-market.

$$R^{vw}_{t+1} = a + b \times (B/M)_t + \varepsilon_{t+1},$$

where R^{vw}_{t+1} denotes the simulated value-weighted return on a portfolio of 50 firms. B/M is aggregate book-to-market defined as the sum of all firms' capital divided by the sum of the firms' market values. The row "Data" reports the empirical results of Pontiff and Schall (1998). The row "Model" reports the results of 100 simulations of 70 years' worth of monthly data (comparable to the sample size used by Pontiff and Schall). The numbers in square brackets denote the 95% confidence interval around the estimated coefficient. The results in this table are for the case in which the firm faces quasi-fixed adjustment costs and irreversibility of investment. The results are very similar in the case in which the firm faces irreversible investment but no adjustment costs.

	Slope	R^2
Data	0.032	0.010
Model	0.058	0.001
	[0.0092, 0.1084]	

Pontiff and Schall report a coefficient of 3.02% in a univariate regression of the value-weighted index return on book-to-market. This value is well within the 95% confidence interval around the 5.88% simulation-based value. The R^2 in the simulation is smaller than that reported by Pontiff and Schall (both are quite close to zero).

B. Cross-Sectional Simulations

In this subsection I examine whether the cross-sectional predictions of the model can quantitatively match the empirical findings reported by Fama and French (1992).

Each simulated data set consists of 2,000 firms for 600 months. I allow the firms to reach their steady state cross-sectional distribution by dropping the first 200 observations for each firm. This leaves 400 months for the cross-sectional regressions. This panel size is comparable to that in Fama and French (1992), who use an average of 2,267 firms for 318 months. For each simulation I then replicate cross-sectional regression tests reported in Fama and French (1992, Table III). I repeat the simulations 100 times. Further details of the simulation procedure are summarized in Appendix H.

In the simulations I test whether two variables, namely, the CAPM beta (estimated by applying the empirical procedure in Fama and French (1992) and termed hereafter the static CAPM beta) and the book-to-market ratio, can explain the cross-section of average firm returns. Firm size is also included as an explanatory variable.

Table IV presents the empirical results reported by Fama and French (in the second column titled FF) and the results of the simulations (in the third

Table IV

Summary Statistics of Cross-Sectional Regressions over 400 Months and 2,000 Firms, under Quasi-Fixed Adjustment Costs of Investment and Irreversible Investment

Summary statistics are listed for the coefficients and the t-statistics of Fama and French (FF) regressions across 100 independent simulations. The dependent variable is the realized return. Independent variables are market beta (β), the log of the book-to-market ratio (log (B/M)), and the log of the market value (log (MV)). The FF row gives the empirical results obtained by Fama and French (1992, Table III), using the actual returns of 2,267 firms over 318 months. Rows 2 and 3 list the means of the coefficients over the simulations, and their corresponding t-statistics for low and high quasi-fixed adjustment costs, respectively.

Panel A

	β Coefficient	t-Statistic
FF	0.150	0.46
Low quasi-fixed costs (1.2% per year)	−0.004	−1.19
High quasi-fixed costs (12% per year)	−0.003	−1.09

Panel B

	Log (B/M) Coefficient	t-Statistic
FF	0.50	5.71
Low quasi-fixed costs (1.2% per year)	0.09	9.37
High quasi-fixed costs (12% per year)	0.02	3.98

Panel C

	Log (MV) Coefficient	t-Statistic
FF	−0.17	−3.41
Low quasi-fixed costs (1.2% per year)	−0.01	−0.62
High quasi-fixed costs (12% per year)	−0.01	−0.65

Panel D

	β Coefficient	t-Statistic	Log(B/M) Coefficient	t-Statistic
FF	(–)	(–)	(–)	(–)
Low quasi-fixed costs (1.2% per year)	−0.004	−1.12	0.09	9.01
High quasi-fixed costs (12% per year)	−0.004	−1.14	0.02	4.02

and fourth columns) for the case in which the firm's investment is irreversible and there are quasi-fixed adjustment costs of investment. I simulate the model for two magnitudes of adjustment costs. The third column presents results for relatively low adjustment costs (fraction of annual output lost due to investment is 1.2%) and the fourth presents the results for relatively high adjustment costs (fraction of annual output lost due to investment is 12.0%). In Panel A I test the CAPM. Panel A shows that like in the data, the simulated static CAPM beta does not capture the cross-sectional variation of stock returns. This result holds for both low and high adjustment costs. The coefficient on beta is small and statistically indistinguishable from zero. Note that the static CAPM beta has little meaning in the model as it fails to account for the conditional nature of market beta.

Panel B of Table IV presents results from a univariate regression of excess returns on book-to-market. In calculating the book-to-market ratio, the only point of departure between my simulations and the empirical procedure in Fama and French is that they update the book-to-market variable annually, whereas I update it each month, since the full information set is observable monthly in my simulations.[15] When adjustment costs are low, the coefficient on book-to-market is positive and highly statistically significant, albeit quite smaller in magnitude than the one reported by Fama and French (1992). Book-to-market becomes substantially less significant in explaining the cross-section of average stock returns when adjustment costs are high.

Panel C of Table IV reports results from a univariate regression of excess returns on the firm's market value (size). Size is statistically insignificant. The reason is that in the model there is only one state variable, the ratio of the capital stock K to the productivity level $\theta(Z)$. Thus, two firms with the same ratio of capital stock to productivity level will have the same systematic risk and expected returns even if they have different market values.

Panel D presents results for a multivariate regression of returns on the static CAPM beta and book-to-market. The results from the univariate regressions carry over to the multivariate regressions. The coefficient on the static CAPM beta is close to zero, and is statistically insignificant. The book-to-market coefficient is positive and highly significant but small in magnitude.

Table V presents the cross-sectional results for the case in which the firm's investment is irreversible but the firm does not face any adjustment costs of investment. The results in Panel A indicate that like the findings of Fama and French (1992), the cross-section of average firm returns bears no discernible relationship to the static CAPM beta. This holds for both complete irreversibility as well as partial irreversibility. The results in panels B and D of Table V indicate that the model's outcomes appear to conform closer to the empirical results than the results in Table IV. When investment is completely irreversible the coefficients on book-to-market are highly statistically significant and are close in magnitude to the coefficients that Fama and French report. Partial reversibility weakens the relation between book-to-market and the cross-section of average returns. Market value is insignificant in Table V as well. I conclude that real investment irreversibility can account for the observed value premium in stock returns to a large extent.

VI. Concluding Remarks

This paper analyzes the effects of nonconvex adjustment costs, irreversibility of investment, and operating leverage on the behavior of financial asset prices. The model demonstrates that under irreversibility and fixed adjustment costs

[15] Results are quantitatively similar under annual updating.

Table V

Summary Statistics of Cross-Sectional Regressions over 400 Months and 2,000 Firms, under Irreversible Investment and No Adjustment Costs of Investment

Summary statistics are listed for the coefficients and the t-statistics of Fama and French (FF) regressions across 100 independent simulations. The dependent variable is the realized return. Independent variables are market beta (β), the log of the book-to-market ratio (log (B/M)), and the log of the market value (log (MV)). The FF row gives the empirical results obtained by Fama and French (1992, Table III), using the actual returns of 2,267 firms over 318 months. Rows 2 and 3 list the means of the coefficients over the simulations, and their corresponding t-statistics for partial reversibility and complete irreversibility, respectively.

Panel A

	Coefficient	t-Statistic
FF	0.1500	0.46
Partial reversibility	−0.0039	−1.24
Complete irreversibility	−0.0032	−1.21

Panel B

	Log (B/M) Coefficient	t-Statistic
FF	0.50	5.71
Partial reversibility	0.18	8.96
Complete irreversibility	0.32	9.89

Panel C

	Log (MV) Coefficient	t-Statistic
FF	−0.170	−3.41
Partial reversibility	0.005	0.05
Complete irreversibility	0.005	0.09

Panel D

	β Coefficient	t-Statistic	Log(B/M) Coefficient	t-Statistic
FF	(–)	(–)	(–)	(–)
Low quasi-fixed costs (1.2% per year)	−0.0040	−1.39	0.17	8.82
High quasi-fixed costs (12% per year)	−0.0033	−1.28	0.31	9.41

of investment and operating leverage, changes in the firm's excess capital capacity lead to changes in the loadings of the firm with respect to risk factors and therefore in its conditional expected return. In the model book-to-market is correlated with the firm's conditional loadings on risk factors. The resulting relation in the model between book-to-market and expected returns is consistent with empirical findings and with the conditional versions of the CAPM and the ICAPM. Within the model, due to the irreversibility of real investment, a firm that has experienced a sequence of adverse profitability shocks has idle capital and a high book-to-market ratio. Such a firm will benefit from positive aggregate shocks without having to undertake any costly investment. The payoffs to this firm's equity holders are therefore sensitive to aggregate conditions and it has a high systematic risk. Simulation results show that the model can reproduce to a large extent the cross-sectional relation between book-to-market and average stock returns and the predictive ability of aggregate book-to-market in the time series observed in the data. The simulations also indicate that in the model investment irreversibility is the main driving force behind the value premium and that fixed adjustment costs cannot account for it.

Appendix A: Risk-Neutral Valuation

If μ_A is the drift of the productivity growth process, then according to a rule developed by Constantinides (1978), the risk-adjusted drift is given by

$$\mu = \mu_A - \sum_{k=1}^{K} \lambda_k \mathrm{Corr}\left(\frac{d\theta}{\theta}, r_k\right), \tag{A1}$$

where r_k and λ_k are the return and Sharpe ratio of risk factor k, respectively.

The correlation of the firm's productivity growth process and the return on the market portfolio is as follows. The value of the market portfolio, denoted X_m, is assumed to follow a geometric Brownian motion as follows:

$$\frac{dX_m}{X_m} = \mu_m\, dt + \sigma_m\, dw_m, \tag{A2}$$

where μ_m and σ_m are the drift and volatility of the return on the market portfolio and dw_m is the increment of a Wiener process representing shocks to the return on the market portfolio. Then, since $dw \equiv \frac{\sigma_A\, dw_A + \sigma_i\, dw_i}{\sqrt{\sigma_A^2 + \sigma_i^2}}$ and $\sigma \equiv \sqrt{\sigma_A^2 + \sigma_i^2}$, it follows that

$$\begin{aligned} \mathrm{Cov}\left(\frac{d\theta}{\theta}, \frac{dX_m}{X_m}\right) &= \mathrm{Cov}(\sigma\, dw, \sigma_m\, dw_m) = \sigma\sigma_m \mathrm{Cov}(dw, dw_m) \\ &= \sigma\sigma_m \mathrm{Cov}\left(\frac{\sigma_A\, dw_A + \sigma_i\, dw_i}{\sigma}, dw_m\right) \\ &= \sigma_A\sigma_m \mathrm{Cov}(dw_A, dw_m) = \sigma_A\sigma_m C_{Am}\, dt, \end{aligned} \tag{A3}$$

where $C_{Am} \equiv \mathrm{Cov}(dw_A, dw_m)$. The correlation of the firm's productivity growth and the return on the market portfolio is

$$\mathrm{Corr}\left(\frac{d\theta}{\theta}, \frac{dX_m}{X_m}\right) = \frac{\sigma_A\sigma_m C_{Am}\, dt}{\sigma\sqrt{dt}\sigma_m\sqrt{dt}} = \frac{\sigma_A}{\sigma} C_{Am}. \tag{A4}$$

Appendix B: Proof of Proposition 1

The proof follows quite closely the proof of Proposition 1 in Caballero and Leahy (1996). The firm makes its investment decisions in order to maximize the present discounted value of its profits. The Bellman equation for the firm's problem is

$$J(\theta, K) = \max_I \Big\{ \Pi(\theta, K)\,dt - I - C(\theta, K)\,1_{\{I\neq 0\}} + e^{-r\,dt} \int_0^\infty J(\theta + d\theta, K + dK)\,\phi(\theta'/\theta)\,d\theta' \Big\}. \tag{B1}$$

Dividing (B1) by θ yields

$$V(Z) = \max_I \Big\{ \pi(Z)\,dt - \tilde{I} - c(Z)1_{\{I\neq 0\}} + e^{-r\,dt} \int_0^\infty V(Z')\phi(\theta'/\theta)\,d\theta' \Big\}. \tag{B2}$$

This is equation (9) in the text.

It is mathematically convenient to present the problem of the firm in the equivalent form:

$$V(Z) = \max_I \Big\{ \big[\pi(Z) - (r_K + \delta)\,Z\big]\,dt - c(Z)1_{\{I\neq 0\}} + e^{-r\,dt} \int_0^\infty V(Z')\phi(\theta'/\theta)\,d\theta' \Big\}, \tag{B3}$$

where r_K is a rental rate of capital per unit of time. This is correct because it is assumed in the model that there are no borrowing constraints or bankruptcy options. In such a case the problem of the firm is unchanged by replacing the lump-sum payment at the time of purchase of capital with a flow payment. That is, conditional on buying new capital, all that matters to the firm is the present discounted value of payments, not the timing of when these payments take place.

It is inconvenient to work with $V(Z)$ because the current period payoff $\pi(Z) - (r_K + \delta)Z$ is not bounded from below and the cost of adjustment $c(Z)$ is not bounded from above as Z approaches infinity. Instead, I analyze

$$\tilde{V}(Z) \equiv V(Z) - \pi(Z)\,dt + (r_K + \delta)\,Z\,dt + c(Z). \tag{B4}$$

Note that this does not affect the optimization problem since Z is fixed at the time of the investment decision. Note that $\tilde{V}(Z)$ is bounded above by $\frac{1}{r}\max_Z \pi(Z) - (r_K + \delta)Z < \infty$, which is the maximum possible payoff, and is bounded below by $\frac{1}{r}\max_{Z'}\{-c(Z) + E\pi(Z')\} > -\infty$, which is the value of adjusting each period. Note also that

$$\tilde{V}(Z) = \max_{I}\left\{c(Z) - c(Z)1_{\{I\neq 0\}} + e^{-r\,dt}\int_0^\infty V(Z')\phi(\theta'/\theta)\,d\theta'\right\}$$
$$= \max\left\{c(Z) + e^{-r\,dt}\int_0^\infty V(Z')\phi(\theta'/\theta)\,d\theta', \max_{Z'}\int_0^\infty V(Z')\phi(\theta'/\theta)\,d\theta'\right\}. \tag{B5}$$

Let ν be the space of bounded, continuous functions. Given a function $g(Z)$, consider the mapping $g \to Mg$,

$$(Mg)(Z) = \max\left\{c(Z) + e^{-r\,dt}\int_0^\infty [g(Z') + \pi(Z')\,dt - (r_k+\delta)Z'\,dt - c(Z')]\phi(\theta'/\theta)\,d\theta', \max_{Z'} e^{-r\,dt}\int_0^\infty [g(Z') + \pi(Z')\,dt - (r_k+\delta)Z'\,dt - c(Z')]\phi(\theta'/\theta)d\theta'\right\}. \tag{B6}$$

It follows that if $g \in \nu$, $Mg \in \nu$, and $\tilde{V}(Z)$ is a fixed point of M should one exist. Since M satisfies Blackwell's conditions for a contraction mapping, the existence and uniqueness of such a fixed point follow from the contraction mapping theorem. Therefore, $\tilde{V}(Z)$ exists and is unique. It follows that $V(Z)$ exists and is unique.

Appendix C: The Stochastic Process of the State Variable *Z*

From equation (7) in the text and using μ, the risk-adjusted drift for the productivity growth process, it follows that

$$d\log(\theta) = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dw. \tag{C1}$$

Since $Z = \frac{K}{\theta}$, it follows that

$$\log(Z) = \log(K) - \log(\theta). \tag{C2}$$

In the inaction region, $d\log(K) = -\delta\,dt$, so in this region

$$d\log(Z) = \left(\frac{1}{2}\sigma^2 - \mu - \delta\right)dt - \sigma dw. \tag{C3}$$

(C3) then implies that Z follows a geometric Brownian motion as follows:

$$\frac{dZ}{Z} = \mu_Z\,dt - \sigma dw, \tag{C4}$$

where $\mu_Z \equiv (\sigma^2 - \mu - \delta)$.

I next show that the stochastic process Z has a long-run stationary distribution.

Proof: For any starting point $Z_0 \in (L, U)$, with probability one Z will hit either the point L or the point U within finite time (proofs are in Ross (1996)). If $\mu_Z < 0$, Z is expected to hit the lower barrier L before it reaches the upper barrier U. If Z hits L, it is reset to the point G (if the firm faces quasi-fixed costs of adjustment) or reflected at the point L (in the case that investment is irreversible but adjustment costs do not exist). In either case, the process will repeat itself each time L is hit. If $\mu_Z \geq 0$, the upper barrier will be hit with a higher probability relative to the case in which $\mu_Z < 0$. When Z hits the upper barrier U, the process is terminated and the firm shuts down. The process Z will hit the barrier U within a finite time with probability one for any value of μ_Z (proof is in Ross (1996)). Therefore, Z has a long-run stationary distribution (a degenerate one). Q.E.D.

Note that firms in the model are defined as employing a single technology and are incapable of adopting new technologies or entering into new markets. The eventual shutdown of firms in the model is consistent with the fact that in the long run, practically all production processes are replaced by newer technologies.

Appendix D: Derivation of the Value of the Firm

In this appendix I derive an expression for the value of the firm. Specifically, I derive an expression for $V(Z)$.

In the investment inaction region the following Bellman equation holds:

$$V(Z) = \pi(Z)\,dt + e^{-r\,dt} E\left[V\left(Z + dZ\right)\right]. \tag{D1}$$

Using the approximation $e^{-rdt} \approx 1 - rdt$ and applying Ito's lemma, I obtain

$$V(Z) = \pi(Z)\,dt + \left(1 - r\,dt\right) e^{-r\,dt}\left[V(Z) + V_Z \mu_Z Z\,dt + \frac{1}{2}\sigma^2 V_{ZZ} Z^2\,dt\right]. \tag{D2}$$

Dividing both sides by dt and eliminating terms of order higher than dt we obtain

$$0 = \pi(Z) + V_Z \mu_Z Z + \frac{1}{2}\sigma^2 V_{ZZ} Z^2 - rV(Z). \tag{D3}$$

To find a particular solution, I try the following. Begin with $V(Z) = AZ^\gamma - SZ$. Substituting into (D3) yields

$$0 = Z^\gamma + \gamma A Z^\gamma \mu_Z + \frac{1}{2}\sigma^2 \gamma(\gamma - 1) A Z^\gamma - rAZ^\gamma. \tag{D4}$$

Dividing through by Z^{γ} yields

$$0 = 1 + \gamma A\mu_Z + \frac{1}{2}\sigma^2\gamma(\gamma - 1)A - rA. \tag{D5}$$

It follows that

$$A = \frac{1}{r - \gamma\sigma^2 + \gamma\mu + \gamma\delta - \frac{1}{2}\sigma^2\gamma\left(\gamma - 1\right)}. \tag{D6}$$

The complementary solution is the solution to the following equation:

$$0 = V_Z Z\mu_Z + \frac{1}{2}V_{ZZ}Z^2\sigma^2 - rV(Z). \tag{D7}$$

Trying a solution of the form Z^{λ} yields

$$\lambda Z^{\lambda}\mu_Z + \frac{1}{2}\lambda(\lambda - 1)Z^{\lambda}\sigma^2 - rZ^{\lambda} = 0. \tag{D8}$$

Equation (D8) implies

$$\frac{1}{2}\sigma^2\lambda^2 + \left(\mu_Z - \frac{1}{2}\sigma^2\right)\lambda - r = 0. \tag{D9}$$

I denote by λ_P and λ_N the positive and negative roots of this quadratic equation, respectively. Then,

$$\begin{aligned}\lambda_{P,N} &= \frac{\left(\frac{1}{2}\sigma^2 - \mu_Z\right) \pm \sqrt{\left(\mu_Z - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2} \\ &= \frac{\left(\mu + \delta - \frac{1}{2}\sigma^2\right) \pm \sqrt{\left(\frac{1}{2}\sigma^2 - \mu - \delta\right)^2 + 2\sigma^2 r}}{\sigma^2}.\end{aligned} \tag{D10}$$

The particular solution and the complementary solution together imply that

$$V(Z) = AZ^{\gamma} - SZ + D_N Z^{\lambda_N} + D_P Z^{\lambda_P}, \tag{D11}$$

where D_N and D_P are constants to be determined by the boundary conditions. Note that both D_N and D_P are positive as they represent option values.

The constants D_N and D_P are found by solving the boundary conditions (equations (12) through (16) in the text). These conditions translate into the following five nonlinear equations in the five unknowns L, G, U, D_N, and D_P.

$$AL^{\gamma} + D_N L^{\lambda_N} + D_P L^{\lambda_P} = AG^{\gamma} + D_N G^{\lambda_N} + D_P G^{\lambda_P} - FL^{\gamma} - (G - L), \tag{D12}$$

$$\gamma AL^{\gamma-1} + \lambda_N D_N L^{\lambda_n - 1} + \lambda_P D_P L^{\lambda_P - 1} = 1, \tag{D13}$$

$$\gamma A G^{\gamma-1} + \lambda_N D_N G^{\lambda_N - 1} + \lambda_P D_P G^{\lambda_P - 1} = 1, \tag{D14}$$

$$\gamma A U^{\gamma-1} + \lambda_N D_N U^{\lambda_N - 1} + \lambda_P D_P U^{\lambda_P - 1} = 0, \tag{D15}$$

$$A U^{\gamma} + D_N U^{\lambda_N} + D_P U^{\lambda_P} = 0. \tag{D16}$$

Equations (D12) through (D16) must be solved numerically.

If selling the stock of capital is possible, but the resale price of capital is lower than its purchase price, the firm's investment is partially reversible. The purchase price of capital is one, as before. Let P_S denote the resale price of capital. I assume that $0 < P_S < 1$. Let U_S denote the upper threshold, such that when Z reaches U_S the firm sells capital. Note that since there are no fixed costs of disinvestment, U_S is a reflecting barrier. The boundary conditions of the firm include in this case conditions (D12) through (D14) and instead of (D15) and (D16) the following smooth pasting condition:

$$\gamma A U_S^{\gamma-1} + \lambda_N D_N U_S^{\lambda_N - 1} + \lambda_P D_P U_S^{\lambda_P - 1} = P_S. \tag{D17}$$

Appendix E: The Factor Loadings of the Firm

Note that the value of the firm, J, is cum dividend. Therefore, the net rate of return on the firm is given by

$$\begin{aligned} \frac{dJ + \Pi}{J - \Pi} &= \frac{d(\theta V) + \theta\pi}{\theta V - \theta\pi} = \frac{V d\theta + \theta dV + \theta\pi}{\theta(V - \pi)} \\ &= \frac{d\theta}{\theta}\left(\frac{V}{V-\pi}\right) + \frac{dV}{V}\left(\frac{V}{V-\pi}\right) + \frac{\pi}{V-\pi}. \end{aligned} \tag{E1}$$

The covariance of the return on the firm and the return on any common risk factor s is given by

$$\begin{aligned} \mathrm{Cov}\left(\frac{dJ+\Pi}{J-\Pi}, R_s\right) &= \left(\frac{V}{V-\pi}\right)\left[\mathrm{Cov}\left(\frac{d\theta}{\theta}, R_s\right) + \mathrm{Cov}\left(\frac{dV}{V}, R_s\right)\right] \\ &= \left(\frac{V}{V-\pi}\right)\Bigg[\mathrm{Cov}(\mu\, dt + \sigma dw, R_s) \\ &\quad + \mathrm{Cov}\left(\frac{\left(\mu_Z V_Z Z + \frac{1}{2}\sigma^2 V_{ZZ} Z^2\right) dt - V_Z Z \sigma dw}{V}, R_s\right)\Bigg] \\ &= \left(\frac{V}{V-\pi}\right)\left[\mathrm{Cov}(\sigma dw, R_s) - \frac{V_Z Z \sigma}{V}\mathrm{Cov}(dw, R_s)\right] \\ &= \left(\frac{V}{V-\pi}\right)\sigma\left[\mathrm{Cov}(dw, R_s)\left(1 - \frac{V_Z Z}{V}\right)\right], \end{aligned} \tag{E2}$$

and the loading of the firm on factor s is given in equations (19) and (20) in the text.

Appendix F: A Benchmark Case of a Firm That Does Not Face Any Adjustment Costs and Whose Investment Is Fully Reversible

In this appendix I present the solution for the profit maximization problem of a firm that does not face any adjustment costs of investment and whose investment is fully reversible. In such a case, in each period the capital stock is set to the level that maximizes the period's profits. Put differently, the choice of capital does not depend on the firm's expectations for future productivity, as the firm will be able to costlessly change its stock of capital in the future if it decides to do so.

The profit maximization problem of the firm at period t (time subscripts omitted) is:

$$\begin{aligned} \operatorname*{Max}_{dK} \pi(K,\theta) &= (K+dK)^{\gamma}\theta^{1-\gamma} - P_K\,dK \\ &\Rightarrow \gamma(K+dK)^{\gamma-1}\theta^{1-\gamma} = P_K \\ &\Rightarrow K+dK^{*} = \left(\frac{P_K}{\gamma}\right)^{\frac{1}{\gamma-1}}\theta. \end{aligned} \tag{F1}$$

Denote $K^{*} = K + dK$. Then we see that K^{*} is proportional to θ at every period, and therefore $Z = \frac{K}{\theta}$ is constant.

Appendix G: The Conditional Distribution of the Productivity Level

Let θ_{At} and θ_{jt} be the aggregate productivity level and the idiosyncratic productivity level of potential entrant j at time t, respectively. I assume that the economy starts at time zero with $\theta_{A0} = 1$. The logarithm of the idiosyncratic component of a potential entrant at time t is assumed to be distributed as follows:

$$\mathrm{Log}(\theta_{jt}) \sim N\left(-\frac{1}{2}\sigma_j^2 t, \sigma_j^2 t\right). \tag{G1}$$

Thus, the idiosyncratic component of the productivity of a potential entrant is distributed as if its process started at time 0 with a value of $\theta_{j0} = 1$ and it followed a zero drift geometric Brownian motion from time 0 to time t. This assumption implies that the logarithm of the total productivity of a potential entrant is normally distributed as

$$\mathrm{Log}\,(\theta_t) \sim N\left(\left(\mu_A - \frac{1}{2}\sigma^2\right)t, \sigma^2 t\right), \tag{G2}$$

where $\sigma \equiv \sqrt{\sigma_A^2 + \sigma_j^2}$.

Appendix H: Simulation Procedure

Given the selected parameter values and the values of λ_N and λ_P, by numerically solving conditions (D12) through (D16) in Appendix D, I find the values of D_N and D_P in equation (17) in the text. Therefore, I have the solution for $V(Z)$ in (17). Solving conditions (D12) through (D16) also yields the values of the investment threshold L, the investment target G, and the shutdown point U.

I set the initial values of K and θ such that Z (which is defined as $\frac{K}{\theta}$) is in the middle of the interval $[L, G]$. Firms are subject to aggregate and idiosyncratic shocks as described in (5) and (6) in the text and the value of θ evolves according to equation (4) in the text. The values of Z and $V(Z)$ evolve according to these shocks as well. Thus, I am able to find the values of $J(\theta, K)$, $\Pi(\theta, K)$, $C(\theta, K)$, $V(Z)$, $\pi(Z)$, $c(Z)$, and the returns on the firms. Book-to-market is given by $\frac{Z}{V(Z)}$.

I simulate data for 2,000 firms over 600 months. In order to focus on the steady state cross-sectional distribution of firms, I drop the first 200 observations for each of the firms. The cross-sectional regressions begin in month 201. However, because the beta estimates depend on estimates of pre-beta from the previous 60 months, the cross-sectional regressions use data beginning in month 141 (the beta estimation procedure follows Fama and French (1992)). For each of the last 400 months, firms are sorted into one of 10 portfolios by market value. Within each market value portfolio, stocks are sorted into 10 more portfolios by their pre-betas—their betas calculated based on the previous 60 months of data. The beta of each of these 100 portfolios is then calculated using the last 400 months. In each of the last 400 months, I then allocate the portfolio beta to each of the firms within the portfolio. I assign each firm its own book-to-market (B/M) value, following Fama and French (1992). The cross-sectional regressions are then run monthly over the last 400 months using beta estimates as well as log(B/M) of the previous month. The coefficients estimated in each simulation are the average coefficients over the 400 cross-sectional regressions. The t-statistics are these averages divided by the standard error.

REFERENCES

Abel, Andrew B., and Janice C. Eberly, 1994, A unified model of investment under uncertainty, *American Economic Review* 84, 1369–1384.

Abel, Andrew B., and Janice C. Eberly, 2002, Investment and q with fixed costs: An empirical analysis, Working paper, The Wharton School at the University of Pennsylvania.

Barnett, Steven A., and Plutarchos Sakellaris, 1998, Nonlinear response of firm investment to Q: Testing a model of convex and non-convex adjustment costs, *Journal of Monetary Economics* 42, 261–288.

Berk, Jonathan B., Richard C. Green, and Vasant Naik, 1999, Optimal investment, growth options, and security returns, *Journal of Finance* 54, 1553–1607.

Caballero, Ricardo J., and Eduardo M. R. A. Engel, 1999, Explaining investment dynamics in U.S. manufacturing: A generalized (S, s) approach, *Econometrica* 67, 783–826.

Caballero, Ricardo J., Eduardo M. R. A. Engel, and John C. Haltiwanger, 1995, Plant-level adjustment and aggregate investment dynamics, *Brookings Papers on Economic Activity* 2, 1–54.

Caballero, Ricardo J., and John V. Leahy, 1996, Fixed costs: The demise of marginal q, National Bureau of Economic Research Working paper 5508.

Campbell, Jeffrey R., 1998, Entry, exit, embodied technology, and business cycles, *Review of Economic Dynamics* 1, 371–408.

Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.

Campbell, John Y., Andrew W. Lo, and A. Craig MacKinlay, 1997, *The Econometrics of Financial Markets* (Princeton University Press, Princeton, NJ).

Carlson, Murray, Adlai Fisher, and Ron Giammarino, 2004, Corporate investment and asset price dynamics: Implications for the cross-section of returns, *Journal of Finance* 59, 2577–2603.

Chan, Louis K. C., Jason Karceski, and Josef Lakonishok, 2003, The level and persistence of growth rates, *Journal of Finance* 58, 643–684.

Cochrane, John H., 1991, Production-based asset pricing and the link between stock returns and economic fluctuations, *Journal of Finance* 46, 209–237.

Cochrane, John H., 1996, A cross-sectional test of an investment-based asset pricing model, *Journal of Political Economy* 104, 527–621.

Constantinides, George M., 1978, Market risk adjustment in project valuation, *Journal of Finance* 33, 603–616.

Cooper, Russell W., and John C. Haltiwanger, 2000, On the nature of capital adjustment costs, National Bureau of Economic Research Working paper 7925.

Cooper, Russell W., John C. Haltiwanger, and Laura Power, 1999, Machine replacement and the business cycle: Lumps and bumps, *American Economic Review* 89, 921–946.

Dixit, Avinash K., 1993, *The Art of Smooth Pasting* (Harwood Academic Publishers, Chur, Switzerland).

Doms, Mark, and Timothy Dunne, 1998, Capital adjustment patterns in manufacturing plants, *Review of Economic Dynamics* 1, 409–429.

Eberly, Janice C., 1997, International evidence on investment and fundamentals, *European Economic Review* 41, 1055–1078.

Fama, Eugene F., 1976, *Foundations of Finance* (Basic Books, New York).

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Gomes, Joao F., Leonid Kogan, and Lu Zhang, 2003, Equilibrium cross-section of returns, *Journal of Political Economy* 111, 693–732.

Gomes, Joao F., Amir Yaron, and Lu Zhang, 2002, Asset pricing implications of firms' financing constraints, National Bureau of Economic Research Working paper 9365.

Kogan, Leonid, 2004, Asset prices and real investment, *Journal of Financial Economics* 73, 411–431.

Kydland, Finn E., and Edward C. Prescott, 1982, Time to build and aggregate fluctuations, *Econometrica* 50, 1345–1370.

Lewellen, Jonathan, 1999, The time-series relations among expected return risk, and book to market, *Journal of Financial Economics* 54, 3–43.

Liew, Jimmy, and Maria Vassalou, 2000, Can book-to-market, size and momentum be risk factors that predict economic growth?, *Journal of Financial Economics* 57, 221–245.

Longstaff, Francis, and Monika Piazzesi, 2004, Corporate earnings and the equity premium, *Journal of Financial Economics* 74, 401–421.

Petkova, Ralitsa, and Lu Zhang, 2003, Is value riskier than growth? *Journal of Financial Economics* 78, 187–202.

Pontiff, Jeffrey, and Lawrence D. Schall, 1998, Book to market as a predictor of market returns, *Journal of Financial Economics* 49, 141–160.

Ramey, Valerie A., and Mathew D. Shapiro, 2001, Displaced capital: A study of aerospace plant closings, *Journal of Political Economy* 109, 958–992.

Ross, Sheldon M., 1996, *Stochastic Processes* (Wiley Series in Probability and Mathematical Statistics, New York).

Zhang, Lu, 2005, The value premium, *Journal of Finance* 60, 67–103.