THE JOURNAL OF FINANCE • VOL. LIX, NO. 5 • OCTOBER 2004

Investment–Cash Flow Sensitivities: Constrained versus Unconstrained Firms

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ABSTRACT

From the existing literature, it is not clear what effect financing constraints have on the sensitivities of firms' investment to their cash flow. I propose an explanation that reconciles the conflicting empirical evidence. I present two models: the unconstrained model, in which firms can raise external funds, and the constrained model, in which firms cannot do so. Using low dividends to identify financing constraints in my generated panel of data produces results consistent with those of Fazzari, Hubbard, and Petersen; using the constrained model produces results consistent with those of Kaplan and Zingales.

THE LITERATURE DOCUMENTING the sensitivity of firms' investment to fluctuations in their internal funds, initiated by Fazzari, Hubbard, and Petersen (1988), is large and growing. The sensitivity is measured by the coefficient obtained from regressing investment on cash flow, controlling for investment opportunities using Tobin's Q. Fazzari et al. view firms as constrained when external financing is too expensive. In that case, firms must use internal funds to finance their investments rather than to pay out dividends. Fazzari et al. identify firms with low dividends as "Most constrained" and firms with high dividends as "Least constrained." As reported in Table I, "Most constrained" firms have investments that are *more* sensitive to cash flows than "Least constrained" firms. Kaplan and Zingales (1997) disagree with the interpretation of the result. Their identification of financially constrained firms is based on the qualitative and quantitative information contained in the firms' various reports. Kaplan and Zingales identify firms without access to more funds than needed to finance their investment as "Likely constrained" and firms with access to more funds than needed to finance their investment as "Never constrained." In contrast to Fazzari et al., Kaplan and Zingales do not consider firms that choose to pay low dividends even though they could pay out more as constrained. As Table I indicates, the investments of "Likely constrained" firms are *less* sensitive to cash flows than the investments of "Never constrained" firms.

The regression results discussed above depend crucially on the criterion used to identify whether a firm experiences financing constraints. To explain the

*Moyen is from the Leeds School of Business at the University of Colorado. I would like to thank Martin Boileau, Murillo Campello, Gilles Chemla, João Gomes, Rob Heinkel Burton Hollifield, Chris Leach, Jaime Zender, an anonymous referee, and participants at the American Finance Association meetings and at the Western Finance Association meetings for helpful comments.

Table I
Regression Results from the Literature

The measure K denotes the capital stock, CF the cash flow, and Q Tobin's average q. Standard errors are in parentheses.

	CF/K	Q
Fazzari et al. (1988):		
Most constrained	0.461	0.001
	(0.027)	(0.000)
Least constrained	0.230	0.002
	(0.010)	(0.000)
Kaplan and Zingales (1997):		
Likely constrained	0.340	0.070
	(0.042)	(0.018)
Never constrained	0.702	0.009
	(0.041)	(0.006)

empirical evidence, I construct two models: an unconstrained model, in which firms have perfect access to external financial markets, and a constrained model, in which firms have no access. Section I presents the two models, and Section II describes the calibration necessary to solve the models. Series are simulated from the two models and pooled to represent the theoretical sample. Using this laboratory, I investigate whether the empirical results can be replicated. I find that they can. Section III discusses two main findings. First, using low dividends to identify firms with financing constraints leads to Fazzari et al.'s result that low-dividend firms' investment is more sensitive to cash flow than high-dividend firms' investment. Second, using the constrained model to identify firms with financing constraints leads to Kaplan and Zingales's result that constrained firms' investment is less sensitive to cash flow than unconstrained firms' investment.

The fact that the cash flow sensitivity of firms described by the constrained model is lower than the cash flow sensitivity of firms described by the unconstrained model can be easily explained. In both models, cash flow is highly correlated with investment opportunities. With more favorable opportunities, both constrained and unconstrained firms invest more. With more favorable opportunities, unconstrained firms also issue debt to fund additional investment. Because the effect of debt financing on investment is not taken into account by the regression specification, it magnifies the cash flow sensitivity of unconstrained firms. Also, given more cash flow, unconstrained firms use debt to increase both their investment and their dividend payment. Constrained firms choose whether to allocate their cash flows to more investment or more dividends. The link between investment and cash flow is therefore weaker for constrained firms. In accord with Kaplan and Zingales's result, the cash flow sensitivity of constrained firms is lower than that of unconstrained firms.

The fact that the cash flow sensitivity of low-dividend firms is higher than the cash flow sensitivity of high-dividend firms can also be easily explained. Firms from the unconstrained model invest more than firms from the constrained model. Because unconstrained firms can adjust their debt levels through time, they also take on more debt. Debt claimants of unconstrained firms account for a greater portion of the firms than debt claimants of constrained firms. Equity claimants of unconstrained firms receive smaller dividends than equity claimants of constrained firms. Using low dividends to identify firms with financing constraints leads to Fazzari et al.'s result. Low-dividend firms are mostly firms from the unconstrained model, which exhibit a higher cash flow sensitivity than do firms from the constrained model.

While the empirical debate between Fazzari et al. and Kaplan and Zingales remains unresolved, a number of theoretical papers have investigated the sensitivity of investment to cash flow fluctuations.[1] Using a neoclassical framework, Gomes (2001) and Alti (2003) show that cash flow sensitivities can be generated from an environment without any financing friction. Both Gomes and Alti conclude that cash flow sensitivities do not necessarily indicate the presence of financing constraints. In contrast, I investigate whether financing constraints are sufficient to replicate the empirical evidence underlying the Fazzari et al. and Zingales debate.

Other papers use a finance framework to examine the cash flow sensitivity controversy. Almeida and Campello (2001) develop a one-period model in which firms may face credit constraints. Unconstrained firms show no cash flow sensitivity, while credit-constrained firms exhibit a positive cash flow sensitivity. The sensitivity of credit-constrained firms increases with their available collateral. Instead of imposing credit constraints on investment, Povel and Raith (2001) develop a one-period model in which investment is not observable by the market. They find that the relation between investment and cash flow is U-shaped, and that more information asymmetry generally increases the cash flow sensitivity. Like Povel and Raith, Dasgupta and Sengupta (2002) assume that investment is not observable by the market. Using a two-period model, they also find that the relation between investment and cash flow is not monotonic. In contrast, I focus on the dynamic behavior of firms' investment policies in an environment with complete information.

I. The Models

I compare the investment behavior of two types of firms. The first type of firm faces no financing constraint and trades off the costs and benefits of external financing, while the second type of firm is completely shut out of external financial markets. Using such extreme types of firms maximizes the effect of

[1] Some empirical papers provide support for Fazzari et al., while others obtain results consistent with those of Kaplan and Zingales. The papers providing support for Fazzari et al. (1988) include Allayannis and Mozumdar (2001), Fazzari, Hubbard, and Petersen (2000), Gilchrist and Himmelberg (1995), Hoshi, Kashyap, and Scharfstein (1991), Oliner and Rudebusch (1992), and Schaller (1993). See Hubbard (1998) for an extensive literature review. The papers providing support for Kaplan and Zingales (1997) include Cleary (1999), Kadapakkam, Kumar, and Riddick (1998), and Kaplan and Zingales (2000).

financing constraints on the cash flow sensitivity. In other words, the cash flow sensitivity of constrained firms is as different as possible from that of unconstrained firms.

A. The Unconstrained Firm Model

The unconstrained firm model characterizes the investment and financing decisions of firms that face no financing constraint. Firms trade off a tax benefit of debt against a default cost of debt. Because estimating the cash flow sensitivity requires panel data, the unconstrained firm model presented below is dynamic and can represent different firms. The model describes investment and financing decisions within an infinite-horizon discrete-time dynamic stochastic framework. Different firms are characterized by different realizations of the stochastic process.

The firm maximizes the equity value subject to fairly pricing any debt issue by choosing its dividend, investment, and debt policies. All claimants, equity and debt, are risk-neutral. The unconstrained equity value V^u is

$$V_t^u = \max\left\{0, D_t^u + \frac{1}{1+r} E_t\left[V_{t+1}^u\right]\right\}, \tag{1}$$

where the superscript u denotes unconstrained firms, r is the discount rate, and E_t is the conditional expectation at period t. For simplicity, dividends and capital gains are assumed to be untaxed.

Equation (1) shows that the equity value is the sum of the expected discounted stream of dividends, D^u. Equation (1) also shows that equity claimants are protected by limited liability. Equity claimants default whenever $D_t^u + \frac{1}{1+r} E_t[V_{t+1}^u] \leq 0$. The firm may ask its equity claimants to contribute additional funds ($D_t^u < 0$), but equity claimants may choose to relinquish their equity claim rather than contribute more. In the case where the equity issue is not justified by the expected discounted future equity value ($D_t^u + \frac{1}{1+r} E_t[V_{t+1}^u] \leq 0$), equity claimants exercise their option of not contributing additional funds to the firm and trigger default instead.

The firm's sources-and-uses-of-funds equation defines the dividend

$$D_t^u = (1-\tau_f) f(K_t;\theta_t) + \tau_f \delta K_t - I_t + \Delta B_{t+1} - (1-\tau_f)\iota_t B_t, \tag{2}$$

where τ_f is the firm's tax rate, K_t is the capital stock, θ_t describes the firm's underlying income shock, $(1-\tau_f)f(K_t;\theta_t)$ is the after-tax operating income before depreciation, δ is the depreciation rate, $\tau_f \delta K_t$ is the depreciation tax shield, I_t is the investment, ΔB_{t+1} is the new debt issue, ι_t is the interest rate, B_t is the debt level, and $(1-\tau_f)\iota_t B_t$ is the after-tax interest payment. The depreciation rate used for tax purposes is assumed to be equal to the true economic depreciation rate of the capital stock.

The firm's operating income before depreciation is the difference between its revenues and expenses:

$$f(K_t;\theta_t) = \theta_t K_t^{\alpha} - F. \tag{3}$$

Revenues exhibit decreasing returns to scale when $0 < \alpha < 1$. In order to avoid modeling the labor decision, I represent labor (and other) expenses by a fixed cost, F.[2]

This period's depreciated capital stock and investment constitute next period's capital stock. The capital accumulation is thus represented as

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{4}$$

The new debt issue is the difference between the new debt level chosen this period B_{t+1} and the beginning-of-the-period debt level B_t:

$$\Delta B_{t+1} = B_{t+1} - B_t. \tag{5}$$

The debt is specified with a maturity of one period, but can be viewed as longer term debt with a floating rate. Each period the firm can roll over its existing debt $\Delta B_{t+1} = 0$, retire some debt $\Delta B_{t+1} < 0$, or issue more debt $\Delta B_{t+1} > 0$ at the current interest rate, ι_{t+1}. Because the unconstrained model does not financially constrain firms in any way, it does not include a recapitalization cost for debt (and equity) issues.

Fairly pricing the debt requires that

$$\begin{aligned}\frac{1}{1+r}E_t\big[&(1+(1-\tau_\iota)\iota_{t+1})B_{t+1}1_{(V^u_{t+1}>0)}\\ &+(R(K_{t+1};\theta_{t+1})-XB_{t+1})\big(1-1_{(V^u_{t+1}>0)}\big)\big] = B_{t+1}.\end{aligned} \tag{6}$$

Equation (6) shows that debt claimants demand an interest rate such that the debt lent to the firm this period equals next period's expected discounted payoff. The payoff on the debt claim consists of the face value B_{t+1} and the after-tax interest payment $(1-\tau_\iota)\iota_{t+1}B_{t+1}$ if equity claimants do not default, or the net residual value $R(K_{t+1};\theta_{t+1}) - XB_{t+1}$ if they default, where τ_ι is the debt claimants' interest income tax rate, X is the deadweight default cost as a proportion of the debt face value, and the function $1_{(V>0)}$ indicates no-default:

$$1_{(V>0)} = \begin{cases} 1 & \text{if } V > 0, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

The residual accruing to debt claimants upon default $R(K;\theta)$ is the reorganized value of the firm. Debt claimants may then recapitalize the firm in an optimal

[2] Rather than representing labor expenses as a fixed cost F, one could explicitly model the firm's labor demand decision. The firm would gain another instrument to maximize its value. The firm's operating income would become more responsive to the income shock θ_t. The firm would hire more labor in periods of high income shock and less labor in periods of low income shock. Not explicitly modeling the labor decision makes the firm's income less responsive to its shock.

manner. In fact, $R(K;\theta)$ takes into account the optimal recapitalization from that unlevered state

$$R(K_t;\theta_t) = (1-\tau_f)f(K_t;\theta_t) + \tau_f \delta K_t - I_t + B_{t+1} + \frac{1}{1+r}E_t\big[V^u_{t+1}\big]. \qquad (8)$$

By definition, the net residual $R(K_t;\theta_t) - XB_t$ accruing to debt claimants upon default (when $D^u_t + \frac{1}{1+r}E_t[V^u_{t+1}] \le 0$) is always less than the no-default principal and after-tax interest payment $(1+(1-\tau_\iota)\iota_t)B_t$. Using the definition of the residual $R(K_t;\theta_t)$ in Equation (8), we can express the equity value in terms of the residual:

$$D^u_t + \frac{1}{1+r}E_t\big[V^u_{t+1}\big] = R(K_t;\theta_t) - (1+(1-\tau_f)\iota_t)B_t \qquad (9)$$

Recognizing the tax benefit of debt financing, that is, the fact that the interest payment is deductible by the firm at a higher rate than the interest income is taxable to debt claimants ($\tau_f > \tau_\iota$), implies that the residual is smaller than the principal and after-tax interest accruing to debt claimants when no default occurs:

$$R(K_t;\theta_t) \le (1+(1-\tau_f)\iota_t)B_t < (1+(1-\tau_\iota)\iota_t)B_t. \qquad (10)$$

The firm's income shock is represented by a first-order autoregressive process with persistence ρ and volatility σ:

$$\ln\theta_{t+1} = \rho\ln\theta_t + \sigma\epsilon_{t+1}, \qquad (11)$$

where $\epsilon_t \sim$ i.i.d. $N(0, 1)$. Because the persistence parameter ρ is not zero, the income shock is somewhat predictable. The firm anticipates the income shock it will face next period and chooses its investment and debt policies accordingly.

The firm cannot perfectly anticipate the income shock it will face next period. Although the firm positions itself to limit the possibility of default next period, default occurs when next period's income shock θ_{t+1} turns out to be so much lower than expected that the equity claim becomes worthless. The income shock at which equity claimants trigger default $\bar{\theta}^u(K_t, B_t, \iota_t)$ is defined by $D^u_t + \frac{1}{1+r}E_t[V^u_{t+1}] = 0$. If we substitute equations (2)–(5) into this expression, the default point is implicitly defined by

$$(1-\tau_f)\big(\bar{\theta}^u(K_t, B_t, \iota_t)K_t^\alpha - F\big) + (1-(1-\tau_f)\delta)K_t - K_{t+1} + B_{t+1}$$
$$-(1+(1-\tau_f)\iota_t)B_t + \frac{1}{1+r}E_t\big[V^u_{t+1}\,\big|\,\theta_t = \bar{\theta}^u(K_t, B_t, \iota_t)\big] = 0. \qquad (12)$$

Because ϵ_t is normally distributed, the income shock θ_t follows a log-normal distribution. Hence the probability of default is represented by $\Phi[\bar{\theta}^u(K_t, B_t, \iota_t)]$, where $\Phi[\cdot]$ is the log-normal cumulative density function.

Equations (2)–(6) are the only constraints facing the firm. Dividends D^u_t are not restricted to be nonnegative. Negative dividends are interpreted as equity issues. The firm decides on the amount of dividend or equity issue that is optimal. If equity claimants do not find it worthwhile to provide the equity financing, they trigger default. Also, investments I_t and debt issues ΔB_{t+1} are not restricted to be nonnegative. The firm is allowed to sell some assets and to retire debt. There is no need for a stock of cash in the model. The firm can effectively manage its probability of default by buying and selling its capital stock and by changing its financial structure.

The firm chooses how much dividend D_t^u to pay, how much to invest I_t, and how much debt to issue ΔB_{t+1} at the interest rate ι_{t+1} that satisfies the bond-pricing equation (6), in order to maximize the equity value in equation (1) subject to equations (2)–(5). The firm makes these decisions after observing the beginning-of-the-period value for the income shock θ_t and last period's choices of capital stock K_t, debt B_t, and interest rate ι_t. The Bellman equation describing the firm's intertemporal problem is

$$V^u(K_t, B_t, \iota_t; \theta_t) = \max_{\{D_t^u, I_t, \Delta B_{t+1}, \iota_{t+1}\}} \max\left\{0, D_t^u + \frac{1}{1+r} E_t\left[V^u\left(K_{t+1}, B_{t+1}, \iota_{t+1}; \theta_{t+1}\right)\right]\right\}, \quad (13)$$

subject to equations (2)–(6).

The firm's investment decision is characterized by

$$\begin{aligned} &\frac{1}{1+r} E_t\left[(1-\tau_f)\theta_{t+1}\alpha K_{t+1}^{\alpha-1} + (1-(1-\tau_f)\delta)\right] \\ &+ \frac{1}{1+r} E_t\left[(\tau_f - \tau_\iota) B_{t+1} 1_{(V_{t+1}^u > 0)}\right] \frac{\partial \iota_{t+1}}{\partial K_{t+1}} - \frac{1}{1+r}((\tau_f - \tau_\iota)\iota_{t+1} + X) B_{t+1} \\ &\times \left(\frac{\partial \Phi\left[\bar{\theta}^u(K_{t+1}, B_{t+1}, \iota_{t+1})\right]}{\partial K_{t+1}} + \frac{\partial \Phi\left[\bar{\theta}^u(K_{t+1}, B_{t+1}, \iota_{t+1})\right]}{\partial \iota_{t+1}} \frac{\partial \iota_{t+1}}{\partial K_{t+1}}\right) = 1, \end{aligned} \quad (14)$$

where $((\tau_f - \tau_\iota)\iota_{t+1} + X)B_{t+1}$ is the forgone tax benefit and the default cost if equity claimants default on their debt obligation. The effects of investment on the interest rate $\partial \iota_{t+1}/\partial K_{t+1}$ and on the probability of defaulting

$$\left(\frac{\partial \Phi\left[\bar{\theta}^u(K_{t+1}, B_{t+1}, \iota_{t+1})\right]}{\partial K_{t+1}}, \frac{\partial \Phi\left[\bar{\theta}^u(K_{t+1}, B_{t+1}, \iota_{t+1})\right]}{\partial \iota_{t+1}}\right)$$

are defined in the Appendix.

Equation (14) states that the firm invests up to the point where the cost of one unit of capital equals next period's expected discounted marginal contribution to dividends. Equation (14) also shows that the firm accounts for the effects of its investment decision on the interest rate requested by debt claimants and on the default probability. A higher interest rate ι_{t+1} promised to debt claimants translates into a larger tax benefit to the firm, but this higher rate also increases the probability that equity claimants will default on their debt obligation.

The firm's debt policy is characterized by

$$\begin{aligned}&\frac{1}{1+r}E_t\big[(\tau_f-\tau_\iota)\iota_{t+1}1_{(V^u_{t+1}>0)}\big]-\frac{1}{1+r}E_t\big[X\big(1-1_{(V^u_{t+1}>0)}\big)\big]\\&\quad+\frac{1}{1+r}E_t\big[(\tau_f-\tau_\iota)B_{t+1}1_{(V^u_{t+1}>0)}\big]\frac{\partial\iota_{t+1}}{\partial B_{t+1}}-\frac{1}{1+r}((\tau_f-\tau_\iota)\iota_{t+1}+X)B_{t+1}\\&\quad\times\left(\frac{\partial\Phi\big[\bar{\theta}^u(K_{t+1},B_{t+1},\iota_{t+1})\big]}{\partial B_{t+1}}+\frac{\partial\Phi\big[\bar{\theta}^u(K_{t+1},B_{t+1},\iota_{t+1})\big]}{\partial\iota_{t+1}}\frac{\partial\iota_{t+1}}{\partial B_{t+1}}\right)=0.\end{aligned}\tag{15}$$

Equation (15) states that the expected tax benefit of one unit of debt if equity claimants do not default on their debt obligation equals the expected default cost of the unit of debt if default occurs, accounting for the effects of the debt on the interest rate required by debt claimants $\partial\iota_{t+1}/\partial B_{t+1}$ and on the probability of defaulting

$$\left(\frac{\partial\Phi\big[\bar{\theta}^u(K_{t+1},B_{t+1},\iota_{t+1})\big]}{\partial B_{t+1}},\frac{\partial\Phi\big[\bar{\theta}^u(K_{t+1},B_{t+1},\iota_{t+1})\big]}{\partial\iota_{t+1}}\right),$$

defined in the Appendix.

The model cannot be solved analytically. The solution is approximated using numerical methods. Once decision rules are obtained, a panel of firms is simulated and studied.

B. The Constrained Firm Model

Without access to external markets, the model is simpler. The firm's problem is to choose its dividend D_t and investment I_t policies to maximize the equity value. The firm finances itself neither with debt ($B_{t+1}=B_t=0$) nor with equity ($D_t\geq 0$). The model recognizes, however, that constrained firms are levered, as has been documented by Fazzari et al. and by Kaplan and Zingales. It is possible that a constrained firm has had access to external capital markets at some point in the past, no longer has access, but is still servicing an existing debt structure. The debt in place is assumed to be a perpetuity. The interest payment c of the constrained firm does not vary through time, because the firm no longer has access to external capital markets. The firm can rely only on the debt pricing conditions offered when it had access to external markets. This contrasts with the interest payment $\iota_t B_t$ of the unconstrained firm, which varies through time to reflect its debt financing choices.

The constrained firm chooses how much dividend D_t^c to pay and how much to invest I_t, after observing the beginning-of-the-period value for the income shock θ_t and last period's choice of capital stock K_t, where the superscript c denotes constrained firms. The Bellman equation describing the firm's intertemporal problem is

$$V^c(K_t;\theta_t)=\max_{\{D_t^c,I_t\}}\max\left\{0,D_t^c+\frac{1}{1+r}E_t\big[V^c(K_{t+1};\theta_{t+1})\big]\right\}\tag{16}$$

subject to the dividend equation

$$D_t^c = (1-\tau_f)f(K_t;\theta_t) + \tau_f\delta K_t - I_t - (1-\tau_f)c \geq 0, \tag{17}$$

the income equation (3), and the capital accumulation equation (4).

The investment decision is characterized by

$$\frac{1}{1+r}E_t\big[\big\{(1-\tau_f)\theta_{t+1}\alpha K_{t+1}^{\alpha-1} + (1-(1-\tau_f)\delta)\big\}(1+\lambda_{t+1})1_{(V_{t+1}^c>0)}\big] = 1+\lambda_t, \tag{18}$$

where λ_t is the multiplier disallowing equity issues. Equation (18) states that the firm invests up to the point where the shadow cost of one unit of capital equals next period's expected discounted marginal contribution to dividends if equity claimants do not default.

Default may occur when the income shock θ_{t+1} turns out to be much lower than expected and the future expected discounted value is overwhelmed by the interest payment c, so that $D_t^c + \frac{1}{1+r}E_t[V^c(K_{t+1};\theta_{t+1})] = 0$. Using equations (3), (4), and (17), we implicitly define the income shock at which equity claimants of the constrained firm trigger default $\bar{\theta}^c(K_t)$ by

$$\begin{aligned}(1-\tau_f)\big(\bar{\theta}^c(K_t)K_t^\alpha - F\big) + (1-(1-\tau_f)\delta)K_t - K_{t+1} - (1-\tau_f)c \\ + \frac{1}{1+r}E_t\big[V_{t+1}^c \,|\, \theta_t = \bar{\theta}^c(K_t)\big] = 0.\end{aligned} \tag{19}$$

As with the unconstrained model, the solution is approximated numerically.

II. The Calibration

The numerical method is detailed in the Appendix. The method requires parameter values for $r, \delta, \tau_f, \tau_\iota, X, \alpha, \rho, \sigma, c$, and F.

Following most dynamic investment studies since Kydland and Prescott (1982), I set the discount rate r so that $\frac{1}{1+r} = 0.95$, and the depreciation rate δ to 0.1. The tax rates are calibrated to reflect the U.S. corporate and personal tax rates of 40% and 20%: $\tau_f = 0.4$ and $\tau_\iota = 0.2$. The default cost is set to $X = 0.1$, as a compromise between Fischer, Heinkel, and Zechner (1989) and Kane, Marcus, and McDonald (1984), who calibrate this cost to 5% and 15% of the debt face value.

Moyen (1999) estimates the sensitivity parameter α, the persistence ρ, and the volatility σ from annual COMPUSTAT data using the firm's income equation (3) and the autoregressive income shock process of equation (11). Accordingly, the sensitivity of the firms' income-to-capital stock variations is set to $\alpha = 0.45$, the persistence to $\rho = 0.6$, and the volatility to $\sigma = 0.2$.

The long-term coupon is set to $c = 0.15$ to maximize the ex ante constrained firm value $V_0^c + L_0$, where the perpetual debt value L_t is defined in the next section. The coupon is expressed in relation to the numéraire, which is the value of a unit of income $f(K_t, \theta_t)$. For example, if income were scaled to $\$f(K_t, \theta_t)$ millions, the coupon per period to be paid by the firm would be \$150,000.

The fixed cost representing labor and other expenses F is calibrated to obtain a reasonable average debt-to-capital stock ratio of 0.6. In the constrained firm model, this debt-to-capital stock ratio is obtained with a fixed cost of 0.45. In the unconstrained firm model, this debt-to-capital stock ratio is obtained with a higher fixed cost of 1.35.[3] A higher fixed cost simply reduces the debt level that can be serviced by the firm. As is discussed below, constrained firms turn out to be smaller on average than unconstrained firms. When I calibrate the fixed labor cost to obtain a reasonable debt-to-capital stock ratio, smaller constrained firms appropriately face a smaller cost than larger unconstrained firms.

Given these parameter values, the two models are solved numerically as described in the Appendix. The resulting series I_t, ΔB_{t+1}, ι_{t+1}, and V_t are simulated from random outcomes of the income shock ϵ_t. A sample of 1,000 unconstrained firms and 1,000 constrained firms is generated, where each series for which no default $V_t > 0$ occurs for at least 10 consecutive periods defines a firm. I simulate an equal number of constrained and unconstrained firms because it is difficult to know how many firms in the data currently have easy access to external markets. The robustness of the results to different proportions of these two types of firms is discussed in the next section.

Unconstrained firms sometimes default. For example, 0.69% of the unconstrained firms default in periods 11–20. Equity claimants sometimes choose a debt level that is too burdensome to service when next period's income shock turns out to be much lower than expected. Constrained firms do not default. The perpetual coupon chosen ex ante to maximize the firm value is not high enough to generate zero dividends forever.

III. Results

A. Investment–Cash Flow Sensitivities

Table II shows that the cash flow sensitivity of firms identified as experiencing financing constraints may be higher or lower than that of firms identified as experiencing no constraint, depending on the identification criterion used. The simulated sample of 2,000 firms over 10 periods is divided into two groups, firms with financing constraints and firms without constraint, on the basis of five criteria. Firms are alternatively identified as experiencing financing constraints if they have low dividends, if they have low cash flows, if they are indeed constrained as described by the constrained firm model, if they are described by the constrained firm model and exhaust their internal funds when investing, or if they have low Cleary's (1999) index values.

[3] The results are robust to different labor cost F values. A sensitivity analysis is available upon request.

Table II

Regression Results from Simulated Series

The measure K denotes the capital stock, CF cash flow, Q Tobin's average q, D^+ dividend, λ the multiplier disallowing equity issues in the constrained firm model, and Z_{FC} the financial constraints index developed by Cleary (1999). Standard errors appear in parentheses. The portions of firm-year observations identified as experiencing financing constraint, appear in brackets. The difference in cash flow sensitivities between firms with financing constraints and firms with no constraint is statistically significant at the 5% level for all regressions.

	CF/K	Q
Firms identified by dividends:		
Financing constraints—firms with low $\frac{D^+}{K}$	0.399	−0.029
	(0.026)	(0.001)
		[0.544]
No constraint—firms with high $\frac{D^+}{K}$	0.293	−0.153
	(0.012)	(0.003)
Firms identified by cash flows:		
Financing constraints—firms with low $\frac{CF}{K}$	0.730	−0.053
	(0.040)	(0.001)
		[0.533]
No constraint—firms with high $\frac{CF}{K}$	0.324	−0.124
	(0.013)	(0.003)
Firms identified by models:		
Financing constraints—firms from the constrained model	0.248	0.131
	(0.006)	(0.005)
		[0.500]
No constraint—firms from the unconstrained model	1.443	−0.163
	(0.030)	(0.006)
Firms identified by models and multiplier:		
Financing constraints—firms from the constrained model	0.980	0.079
with a binding multiplier $\lambda > 0$	(0.027)	(0.005)
		[0.004]
No constraint—firms from the unconstrained model and	0.655	−0.072
firms from the constrained model with a non-binding multiplier $\lambda = 0$	(0.017)	(0.001)
Firms identified by Cleary's index:		
Financing constraints—firms with low Z_{FC}	0.292	−0.055
	(0.009)	(0.003)
		[0.413]
No constraint—firms with high Z_{FC}	1.007	−0.089
	(0.032)	(0.002)

Following the literature, I estimate the regression specification

$$\frac{INV_{it}^{+}}{K_{it}} = \beta_0 + \beta_1 Q_{it-1} + \beta_2 \frac{CF_{it}}{K_{it}} + \epsilon_{it} \tag{20}$$

for firms experiencing financing constraints and for firms without constraint, where $INV_{it} = K_{it+1} - K_{it}$, and the superscript $+$ refers to the max operator $INV_{it}^{+} = \max\{0, INV_{it}\}$ (a superscript $-$ refers to the min operator, e.g.,

$INV_{it}^{-} = \min\{0, INV_{it}\}$). Cash flow CF_{it} for constrained and unconstrained firms represents the beginning-of-the-period funds:

$$CF_{it}^{c} = (1-\tau_f)(f(K_{it};\theta_{it}) - c) + \tau_f \delta K_{it} \tag{21}$$

and

$$CF_{it}^{u} = (1-\tau_f)(f(K_{it};\theta_{it}) - \iota_{it} B_{it}) + \tau_f \delta K_{it}. \tag{22}$$

A firm's beginning-of-the-period capital stock K_{it} standardizes both its investment and its cash flow. Investment opportunities are measured by the beginning-of-the-period Tobin's Q_{it-1}, defined by the market value of the assets, equity and debt, over the book value:

$$Q_{it}^{c} = \frac{V_{it}^{c} + L_{it}}{K_{it+1}} \tag{23}$$

and

$$Q_{it}^{u} = \frac{V_{it}^{u} + (1 + (1-\tau_{\iota})\iota_{it})B_{it}1_{(V_{it}^{u}>0)} + (R(K_{it};\theta_{it}) - XB_{it})\left(1 - 1_{(V_{it}^{u}>0)}\right)}{K_{it+1}}. \tag{24}$$

The debt value of the constrained firm is constructed as the sum of all expected future discounted after-tax coupon payments if the firm does not default or the permanently unlevered firm value if it does default:

$$\begin{aligned} L_t = {} & (1-\tau_{\iota})c1_{(V_t^c>0)} + \frac{1}{1+r}E_t\left[(1-\tau_{\iota})c1_{(V_t^c>0)}1_{(V_{t+1}^c>0)}\right] + \cdots \\ & + U_t\left(1 - 1_{(V_t^c>0)}\right) + \frac{1}{1+r}E_t\left[U_t 1_{(V_t^c>0)}\left(1 - 1_{(V_{t+1}^c>0)}\right)\right] + \cdots, \end{aligned} \tag{25}$$

where the value of the firm that becomes unlevered permanently is

$$U_t = (1-\tau_f)f(K_t;\theta_t) + \tau_f \delta K_t - I_t + \frac{1}{1+r}E_t[U_{t+1}]. \tag{26}$$

Cash flow CF_{it} and Tobin's Q_{it-1} are predetermined at the time of the investment decision INV_{it}^{+} because they contain only variables determined at time $t-1$.

First, in the spirit of Fazzari et al., the simulated sample of 20,000 observations (2,000 firms over 10 periods) is divided into two groups based on dividends paid out to equity claimants D_{it}^{+}/K_{it}. I compute the average dividend payment for these 20,000 firm-year observations. Lower than average dividend payments identify firm-year observations with financing constraints, while higher than average payments identify firm-year observations without constraint. Depending on its dividend payout, a firm can be identified as experiencing financing constraints in a year but without constraint the following year. The results of this identification criterion appear under "Firms Identified by Dividends" in Table II. In accord with the regression results of Fazzari et al., the cash flow sensitivity of low-dividend firms (0.399) is higher than that of high-dividend firms (0.293).

Second, cash flow is also used as a proxy for financing constraints, as in Allayannis and Mozumdar (2001). Because dividends D_{it}^{+} represent the residual of the firm's cash flow CF_{it} after the firm's investment INV_{it} and financial $(\Delta B_{it+1}, D_{it}^{-})$ decisions, dividends reflect both the state in which the firm finds itself at the beginning of the period and its decisions taken during that period. Using cash flow focuses on the firm's beginning-of-the-period funds and abstracts from the current period's decisions. I compute the average cash flow CF_{it}/K_{it} for the 20,000 firm-year observations. Lower than average cash flows identify firm-year observations with financing constraints, while higher than average cash flows identify firm-year observations without constraint. The results of this identification criterion appear under "Firms Identified by Cash Flows" in Table II. Low–cash flow firms (0.730) have a higher cash flow sensitivity than high cash flow firms (0.324), again in accord with the findings of Fazzari et al.

As will be discussed below, dividends and cash flows are highly correlated for all firms, those described by the constrained model and those described by the unconstrained model. Thus, one gets very similar results whether one uses low dividends or low cash flows to identify firms as experiencing financing constraints.

Third, the constrained model identifies firms with financing constraints and the unconstrained model identifies firms without constraint. I apply this criterion to my simulated sample. The results of this identification criterion appear under "Firms Identified by Models" in Table II. In accord with the findings of Kaplan and Zingales, constrained firms (0.248) have investment policies that are less sensitive to cash flow fluctuations than those of unconstrained firms (1.443).

Fourth, the models can be used to identify firms with financing constraints more specifically as those that invest less because they do not have enough internal funds in the current period and have no external funds. In fact, the constrained firm is constrained whether or not its investment is currently limited by its available funds. When the constraint is binding, the firm does not have access to enough internal funds to finance its desired level of investment. When the constraint is not binding, the firm takes special care to save up internal funds to ease the constraint in the future. The firm always behaves under constraint, whether or not the constraint is currently binding.

Nonetheless, Fazzari et al. and Kaplan and Zingales distinguish between firms that have access to more funds than needed to finance their investment and firms that do not. Fazzari et al. explain why they identify low-dividend firms as experiencing financing constraints as follows: "One reason why firms might pay low dividends is that they require investment finance that exceeds their internal cash flow and retain all of the low-cost internal funds they can generate" (p. 158). Kaplan and Zingales disagree with Fazzari et al.'s use of low dividends to identify firms with financing constraints: "in only 15 percent of firm-years [from Fazzari, Hubbard, and Petersen's most constrained group] is there some question as to a firm's ability to access internal or external funds to increase investment" (p. 171).

Firms from the constrained model with a binding multiplier disallowing equity issues $\lambda_{it} > 0$ identify firm-year observations with financing constraints, while firms from the unconstrained model and firms from the constrained model with a nonbinding multiplier $\lambda_{it} = 0$ identify firm-year observations without constraint. The results of this identification criterion appear under "Firms Identified by Models and Multiplier" in Table II. Fazzari et al.'s result obtains. Binding-multiplier firms (0.980) have a higher cash flow sensitivity than other firms (0.655).

For constrained firms with $\lambda_{it} > 0$, investment is by definition equal to cash flow and asset sales. In other words, only the asset sales variable differentiates investment from cash flow. For constrained firms with $\lambda_{it} = 0$, the relationship between investment and cash flow is not as strong, as it depends on more variables than asset sales. The sensitivity of investment to cash flow of constrained firms with $\lambda_{it} > 0$ is thus higher.

Fifth, I use Cleary's index Z_{FC} to identify firms with financing constraints. Cleary views firms that increase dividends as firms without constraint. Cleary explains firms' decision to increase dividends $1_{(D_{t+1}>D_t)}$ by the fixed charge coverage ratio $FCCov_t$, the net income margin $NI\%_t$, the sales growth rate $SalesGrowth_t$, and the debt ratio $Debt_t$, where

$$1_{(D_{t+1}>D_t)} = \begin{cases} 1 & \text{if } D_{t+1} > D_t \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

and

$$Z_{FC} = \gamma_1 FCCov + \gamma_2 NI\% + \gamma_3 SalesGrowth + \gamma_4 Debt. \tag{28}$$

The explanatory variables for constrained and unconstrained firms are defined as

$$FCCov_t^c = \frac{f(K_t;\theta_t) - \delta K_t}{c}, \qquad FCCov_t^u = \frac{f(K_t;\theta_t) - \delta K_t}{\iota_t B_t}, \tag{29}$$

$$NI\%_t^c = \frac{(1-\tau_f)(f(K_t;\theta_t) - \delta K_t - c)}{\theta_t K_t^\alpha},$$

$$NI\%_t^u = \frac{(1-\tau_f)(f(K_t;\theta_t) - \delta K_t - \iota_t B_t)}{\theta_t K_t^\alpha}, \tag{30}$$

$$SalesGrowth_t^c = \frac{\theta_t K_t^\alpha}{\theta_{t-1} K_{t-1}^\alpha} - 1, \qquad SalesGrowth_t^u = \frac{\theta_t K_t^\alpha}{\theta_{t-1} K_{t-1}^\alpha} - 1, \tag{31}$$

and

$$Debt_t^c = \frac{L_t}{K_t},$$

$$Debt_t^u = \frac{(1+(1-\tau_\iota)\iota_t)B_t 1_{(V_t^u>0)} + (R(K_t;\theta_t) - XB_t)(1 - 1_{(V_t^u>0)})}{K_t}. \tag{32}$$

Cleary also uses the current assets-to-current liabilities ratio and the slack-to-asset ratio, but finds that these two ratios are not significant in explaining firms' decision to increase dividends. These two ratios have no representation in the constrained and unconstrained firm models presented in Section I. I use the slack-to-asset ratio only in the financial slack subsection below.

I estimate the probit model of dividend increases using my simulated sample of firms. I then compute the average index value Z_{FC} for the 20,000 firm-year observations. Lower than average index values identify firm-year observations with financing constraints, while higher than average values identify firm-year observations without constraint. The results of this identification criterion appear under "Firms Identified by Cleary's Index" in Table II. Low index value firms (0.292) have a lower cash flow sensitivity than high index value firms (1.007). These results are consistent with the empirical findings of Cleary as well as with those of Kaplan and Zingales.

Cleary uses the fact that dividends are sticky and identifies firms with financing constraints as those that are predicted to reduce dividends. Dividends are sticky in the data, but the stickiness is not modeled here. Dividend reductions in the model simply reflect lower income shocks. Nevertheless, lower income shocks are able to capture most of the negative aspect of reducing dividends in the data.

If the stickiness were modeled, simulated firms with low income shocks would do all they could to avoid reducing dividends, and firms with high income shocks would delay raising dividends in order to avoid a subsequent dividend reduction. Simulated firms that reduced dividends would behave very differently from simulated firms that raised dividends. With sticky dividends, Cleary's index generated by the models would presumably lead to a larger difference in cash flow sensitivities between firms with financing constraints and firms without constraint.

For all regressions in Table II, the sensitivity of investment to Tobin's Q is always small. Tobin's Q should embed all the information publicly known about the firm, its investment opportunities and its access to external markets alike. Accordingly, the cash flow sensitivity should be zero while Tobin's Q sensitivity should be one. However, it takes a very specific economic environment for Tobin's Q to fully explain investment within Fazzari et al.'s regression specification. Hayashi (1982) shows that Tobin's average Q is equal to Tobin's marginal q only with constant returns to scale in production and a capital adjustment cost function that is linearly homogeneous in investment and capital. Neither of these two features is present in the constrained and unconstrained models. Instead the calibration indicates that firms exhibit decreasing returns to scale in capital, and the models recognize the tax benefit of debt financing. The constrained and unconstrained firms' results are consistent with Poterba's (1988) discussion that cash flow may explain investment because of measurement errors in Tobin's Q.[4]

[4] This possibility has been empirically investigated by Erickson and Whited (2000) and Gilchrist and Himmelberg (1995).

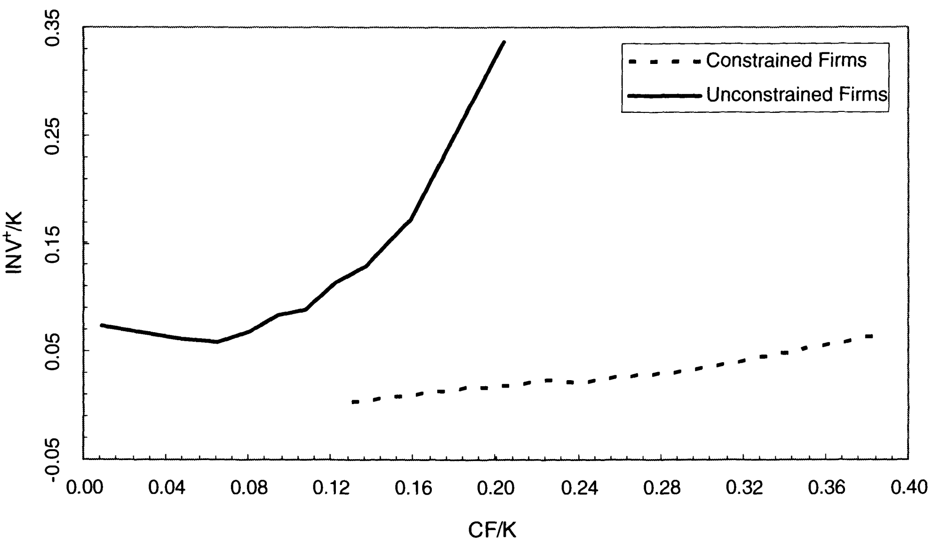

Figure 1. Investment and cash flow.

Figure 1 explains the difference in results shown in Table II. The 1,000 simulated cash flow series over 10 periods for firms described by the unconstrained model are divided into 10 groups, from the lowest 1,000 outcomes to the highest 1,000 outcomes, and averaged to form 10 cash flow grid points. At each of these 10 cash flow points, the average of the corresponding 1,000 investment outcomes is computed and reported in Figure 1. The same is done for firms described by the constrained model.

Two patterns stand out. First, the range of investment realizations is much wider for firms described by the unconstrained model than for firms described by the constrained model. The plot for unconstrained firms dips slightly, then increases steadily. The plot for constrained firms is much flatter. The higher average slope for unconstrained firms than for constrained firms is reflected in the cash flow sensitivities (1.443 for unconstrained firms versus 0.248 for constrained firms). Second, the range of cash flow realizations is shifted to the left for unconstrained firms. If the criterion identifying firms as experiencing financing constraints is low–cash flows rather than the constrained model, then this sample of low-cash flow firms consists mostly of unconstrained firms. Likewise, the sample of high–cash flow firms consists mostly of constrained firms. Hence the results differ when the criterion identifying firms as experiencing financing constraints is low cash flows rather than the constrained model.

It is not surprising that the investment range is wider for firms described by the unconstrained model. With access to external markets, firms can finance more investment. It is also not surprising that the range of cash flows is shifted to the left for unconstrained firms. Because unconstrained firms can respond to different income shocks by varying their debt level, unconstrained firms take on more debt than constrained firms. Unconstrained firms generate lower

Table III

Portions of Simulated Firms Represented under Other Financing Constraint Criteria

The measure K denotes the capital stock, CF cash flow, D^+ dividend, λ the multiplier disallowing equity issues in the constrained firm model, and Z_{FC} the financial constraint index developed by Cleary (1999).

	Constr. Model	Unconstr. Model	Low Z_{FC} Cleary	High Z_{FC} Cleary
Firms identified by dividends:				
Financing constraints—firms with low $\frac{D^+}{K}$	0.221	0.868	0.233	0.764
No constraint—firms with high $\frac{D^+}{K}$	0.779	0.132	0.767	0.236
Firms identified by cash flows:				
Financing constraints—firms with low $\frac{CF}{K}$	0.169	0.896	0.214	0.756
No constraint—firms with high $\frac{CF}{K}$	0.831	0.104	0.786	0.244
Firms identified by models:				
Financing constraints—firms from constr. model	1	0	0.934	0.194
No constraint—Firms from unconstr. model	0	1	0.066	0.806
Firms identified by models and multiplier:				
Financing constraints—firms from constr. model with $\lambda > 0$	0.007	0	0.001	0.006
No constraint—firms from unconstr. model and firms from constr. model with $\lambda = 0$	0.993	1	0.999	0.994
Firms identified by Cleary's index:				
Financing constraints—firms with low Z_{FC}	0.772	0.054	1	0
No constraint—firms with high Z_{FC}	0.228	0.946	0	1

cash flows (and lower dividends) because they service a higher interest obligation than constrained firms.

In accord with Figure 1, Table III reports that the cash flow criterion classifies 83.1% of firms from the constrained model as experiencing no financing constraint and 89.6% of firms from the unconstrained model as experiencing constraints. Likewise, Table III reports that the dividend criterion classifies 77.9% of firms from the constrained model as experiencing no financing constraint and 86.8% of firms from the unconstrained model as experiencing constraints. Table III also reports that Cleary's index identifies firms in line with the constrained and unconstrained firm models. Cleary's index classifies 77.2% of firms from the constrained model as experiencing financing constraints and 94.6% of firms from the unconstrained model as experiencing no such constraint. This suggests that Cleary's index is a useful empirical proxy for identifying firms with financing constraints.

B. Descriptive Statistics

The investment behavior of constrained and unconstrained firms is further analyzed using a more general context than the cash flow sensitivity

Table IV

Statistics of Simulated Firms Identified by Models

The size of the firm is represented by its capital stock K_{it}. Investment opportunities are measured by the underlying income shock θ_{it} and Tobin's Q_{it-1}. Sources of funds include cash flow CF_{it}, asset sales $-INV_{it}^{-}$, equity issues $-D_{it}^{-}$, and debt issues ΔB_{it+1}^{+}. Uses of funds consist of investment INV_{it}^{+}, dividends D_{it}^{+}, and debt retirements $-\Delta B_{it+1}^{-}$.

Panel A: Financing Constraints—Firms from the Constrained Model

	K_{it}	θ_{it}	Q_{it-1}	$\frac{CF_{it}}{K_{it}}$	$\frac{-INV_{it}^{-}}{K_{it}}$	$\frac{INV_{it}^{+}}{K_{it}}$	$\frac{D_{it}^{+}}{K_{it}}$
Means	3.735	1.027	3.521	0.240	0.023	0.026	0.138
Correlations							
K_{it}	1.000						
θ_{it}	0.588	1.000					
Q_{it-1}	−0.366	−0.167	1.000				
$\frac{CF_{it}}{K_{it}}$	0.546	0.998	−0.164	1.000			
$\frac{-INV_{it}^{-}}{K_{it}}$	0.389	−0.339	−0.085	−0.376	1.000		
$\frac{INV_{it}^{+}}{K_{it}}$	−0.385	0.391	0.172	0.439	−0.463	1.000	
$\frac{D_{it}^{+}}{K_{it}}$	0.974	0.672	−0.303	0.630	0.344	−0.308	1.000

Panel B: No Constraint—Firms from the Unconstrained Model

	K_{it}	θ_{it}	Q_{it-1}	$\frac{CF_{it}}{K_{it}}$	$\frac{-INV_{it}^{-}}{K_{it}}$	$\frac{-D_{it}^{-}}{K_{it}}$	$\frac{\Delta B_{it}^{+}}{K_{it}}$	$\frac{INV_{it}^{+}}{K_{it}}$	$\frac{D_{it}^{+}}{K_{it}}$	$\frac{-\Delta B_{it}^{-}}{K_{it}}$
Means	6.504	1.029	0.912	0.103	0.083	0.025	0.131	0.119	0.031	0.091
Correlations										
K_{it}	1.000									
θ_{it}	0.602	1.000								
Q_{it-1}	0.294	0.185	1.000							
$\frac{CF_{it}}{K_{it}}$	0.553	0.976	0.206	1.000						
$\frac{-INV_{it}^{-}}{K_{it}}$	0.420	−0.334	0.120	−0.338	1.000					
$\frac{-D_{it}^{-}}{K_{it}}$	−0.316	−0.740	−0.136	−0.796	0.549	1.000				
$\frac{\Delta B_{it}^{+}}{K_{it}}$	−0.365	0.359	−0.143	0.387	−0.464	−0.347	1.000			
$\frac{INV_{it}^{+}}{K_{it}}$	−0.366	0.357	−0.144	0.385	−0.463	−0.346	0.999	1.000		
$\frac{D_{it}^{+}}{K_{it}}$	0.172	0.822	0.058	0.822	−0.415	−0.463	0.699	0.698	1.000	
$\frac{-\Delta B_{it}^{-}}{K_{it}}$	0.421	−0.333	0.121	−0.336	0.999	0.547	−0.464	−0.463	−0.415	1.000

regression analysis. Table IV reports means and correlation coefficients for four types of variables. The first type of variable is the size of the firm, measured by its capital stock K_{it}. The second type includes variables describing investment opportunities: the underlying income shock θ_{it} and Tobin's Q_{it-1}. The third type consists of possible sources of funds: cash flows CF_{it}, asset sales $-INV_{it}^{-}$, equity issues $-D_{it}^{-}$, and debt issues ΔB_{it+1}^{+}. The fourth type consists of possible uses of funds: investment INV_{it}^{+}, dividends D_{it}^{+}, and debt retirements

$-\Delta B^{-}_{it+1}$. All tables report asset sales $-INV^{-}_{it}$, equity issues $-D^{-}_{it}$, and debt retirements $-\Delta B^{-}_{it+1}$ as positive values, which explains the negative sign in the notation.

Table IV shows that the capital stock K_{it} increases with the income shock θ_{it}. The correlation coefficients are positive for both constrained (0.588) and unconstrained (0.602) firms. Both constrained and unconstrained firms expand with improving investment opportunities, but the constrained firm remains smaller than the unconstrained firm. Equation (18) shows that the constrained firm invests up to the point where the cost of one unit of capital equals next period's expected discounted marginal product of capital. The unconstrained firm has additional incentives to invest. Equation (14) shows that the unconstrained firm also takes into account the effects of its investment decision on the default probability and on the interest rate requested by debt claimants. Investing more reduces both the probability that the firm defaults next period and the interest rate charged by debt claimants on the new debt issued. Investing more therefore decreases the probability that the firm will lose its tax shield.

Investing more to decrease the probability of default is especially important because the unconstrained firm levers up to benefit from the tax shield. The high debt level triggers a high interest payment next period. The high debt level must be balanced by a high investment level generating high revenues next period to keep the probability of default at a reasonable level. The unconstrained firm is therefore much larger than the constrained firm, with means of 6.504 and 3.735.

Constrained firms have a higher Tobin's Q than unconstrained firms, with means of 3.521 and 0.912. Because of decreasing returns to scale $0 < \alpha < 1$, the relation between a firm's value and its capital stock K_{it} is not one-to-one. Although constrained firms are less valuable than unconstrained firms, constrained firms are much smaller, resulting in a higher value-to-capital stock ratio, that is, a higher Tobin's Q.

The strong correlation between the capital stock K_{it} and the income shock θ_{it} explains why the sensitivity of investment to Tobin's Q is small and negative in most regressions in Table II. The numerator of Tobin's Q, the firm value, varies with the income shock, and its effect on investment is thus already accounted for by the cash flow variable. The denominator, the capital stock, however, also strongly varies with the income shock. Because the capital stock appears in the denominator, it generates the small and negative sensitivity of investment to Tobin's Q. Fazzari et al. and Kaplan and Zingales also obtain a small sensitivity of investment to Tobin's Q, but their sensitivity coefficients are positive. The effect of the capital stock may not be as large in the data.

The strong correlation between the capital stock K_{it} and the income shock θ_{it} also explains why beginning-of-the-period Tobin's Q_{it-1} is not a good proxy for investment opportunities as represented in the models by the income shock θ_{it}. Table IV shows that the correlation between firms' Tobin's Q and income shock θ_{it} is weak, with coefficients of -0.167 for constrained firms and 0.185 for unconstrained firms. Again, the numerator of Tobin's Q_{it-1}, the firm value, varies with the income shock, but its denominator, the capital stock, also varies with the income shock. The result is a weak correlation between Tobin's Q_{it-1} and the income shock θ_{it}.

While Tobin's Q is a poor proxy for investment opportunities, cash flow is an excellent proxy. Cash flow is an increasing function of the income shock. Table IV shows that the correlation between cash flow CF_{it}/K_{it} and the income shock θ_{it} is very high, with coefficients of 0.998 for constrained firms and 0.976 for unconstrained firms. These last two correlations are consistent with measurement errors in Tobin's Q, so that cash flow becomes a significant variable in explaining firms' investment policies because it contains information about investment opportunities.

The high correlation between cash flow and the income shock further illuminates Figure 1. An increase in cash flow reflects an increase in the underlying income shock. Any firm, constrained or unconstrained, facing more favorable conditions invests more. This explains the positive slope of the two curves. Unconstrained firms actually invest more than constrained firms because they can raise additional funds on external financial markets. This explains their steeper slope, which in turn magnifies their cash flow sensitivity.

Cash flow CF_{it}/K_{it} is also highly correlated with dividends D^{+}_{it}/K_{it}. Table IV reports coefficients of 0.630 for constrained firms and 0.822 for unconstrained firms. This explains why one gets similar results whether one uses low dividends or low cash flows to identify firms as experiencing financing constraints.

Panel B of Table IV reports a striking correlation between uses and sources of funds of unconstrained firms. Investments INV^{+}_{it}/K_{it} are highly correlated with debt issues $\Delta B^{+}_{it+1}/K_{it}$, and asset sales $-INV^{-}_{it}/K_{it}$ are highly correlated with debt retirements $-\Delta B^{-}_{it+1}/K_{it}$. In both instances, the correlation coefficient is 0.999. Without a doubt, unconstrained firms rely on debt markets to fund their investment.

Because of the tax benefit, debt is the cheapest source of financing. The unconstrained firm chooses a low debt level in periods of low income shock when default is most likely and increases its leverage with the income shock. Investment follows the same pattern. The firm invests less in periods of low income shock when the marginal product of capital next period is low. The firm increases its investment with the income shock. Investing and raising debt thus respond similarly to the underlying income shock. In fact, investment represents about 90% of debt issues, irrespective of the income shock. Whenever the unconstrained firm raises more (less) debt it also invests more (less), so that the proportion of investment to debt issues does not vary much.

Debt issues magnify unconstrained firms' cash flow sensitivity. Because firms choose their investment and financing simultaneously, adding an external funds variable to the cash flow sensitivity regression specification would lead to a simultaneity bias and an inconsistent estimation. Instead, I regress investment net of debt issues on Tobin's Q and cash flow. The large cash flow sensitivity vanishes. Unconstrained firms invest more and raise more debt when the underlying income shock increases. The effect of debt issues in magnifying unconstrained firms' cash flow sensitivities is undeniably large.

Table IV shows that the sources and uses of funds of constrained and unconstrained firms behave similarly, except for the correlation between investment and dividend. While constrained firms must choose between investing more and paying larger dividends, unconstrained firms invest more when they also pay larger dividends. Because the investment–dividend correlation coefficients differ so much between constrained and unconstrained firms, the models suggest that this correlation coefficient may provide a useful way to identify constrained firms in the data. In the models, equity repurchases are treated as dividends. In the data, the investment–dividend correlation must account both for dividends and equity repurchases. The investment–equity payout correlation that accounts for repurchases may well provide a useful way to identify constrained firms in the data.

In sum, unconstrained firms issue debt and finance more investment in periods of high cash flow. This generates a large sensitivity between unconstrained firms' investment and their cash flow. Unlike unconstrained firms, constrained firms at times use their cash flows to pay out more dividends rather than investing. This decreases the sensitivity between constrained firms' investment and their cash flow. Whether the simulated firms are divided into groups according to dividends, cash flows, or Cleary's index values, the constrained and unconstrained firm models are able to explain the various cash flow sensitivity results documented in the literature.

Overall, simulated series are reasonably similar to the data. Table V presents means and correlation coefficients for four groups of firms: Panel A describes low-dividend simulated firms; Panel B, high-dividend simulated firms; Panel C, low-dividend COMPUSTAT firms; and Panel D, high-dividend COMPUSTAT firms. For both simulated and COMPUSTAT firms, low-dividend firms also have lower cash flows than high-dividend firms. The correlation between cash flow and dividend is high for all groups (except that it is not defined for COMPUSTAT firms without dividend). The correlation between investment and debt issues is positive for all groups.

Because low-dividend firms consist mostly of unconstrained firms and unconstrained firms exhibit a higher cash flow sensitivity because of their access to external markets, low-dividend firms should be more active on external markets than high-dividend firms. Panels A and B of Table V show that low-dividend firms are indeed more active on external markets than high-dividend firms. Low-dividend firms issue more debt ($0.068 > 0.063$), issue more equity ($0.023 > 0$), and retire more debt ($0.083 > 0.001$) on average than high-dividend firms. Panels C and D of Table V show that the same is true in the data. Low-dividend COMPUSTAT firms issue more debt ($0.515 > 0.343$), issue more equity ($0.429 > 0.100$), and retire more debt ($0.479 > 0.285$) on average than high-dividend COMPUSTAT firms. See the Appendix for a description of the COMPUSTAT data.

Although the constrained and unconstrained firm models explain many observed facts, the models do not succeed on all fronts when simulated series are grouped according to their dividends. First, while in most areas simulated and COMPUSTAT firms display similar patterns, the magnitudes are different.

Table V

Statistics of Firms Identified by Dividends

The size of the firm is represented by its capital stock K_{it}. Investment opportunities are measured by the underlying income shock θ_{it} and Tobin's Q_{it-1}. Sources of funds include cash flow CF_{it}, asset sales $-INV^-_{it}$, equity issues $-D^-_{it}$, and debt issues ΔB^+_{it+1}. Uses of funds consist of investment INV^+_{it}, dividends D^+_{it}, and debt retirements $-\Delta B^-_{it+1}$.

	K_{it}	θ_{it}	Q_{it-1}	$\frac{CF_{it}}{K_{it}}$	$\frac{-INV^-_{it}}{K_{it}}$	$\frac{-D^-_{it}}{K_{it}}$	$\frac{\Delta B^+_{it}}{K_{it}}$	$\frac{INV^+_{it}}{K_{it}}$	$\frac{D^+_{it}}{K_{it}}$	$\frac{-\Delta B^-_{it}}{K_{it}}$
Panel A: Financing Constraints—Simulated Firms with Low D^+_{it}/K_{it}										
Means	5.802	0.939	1.450	0.109	0.077	0.023	0.068	0.072	0.022	0.083
Correlations										
K_{it}	1.000									
θ_{it}	0.643	1.000								
Q_{it-1}	−0.529	−0.216	1.000							
$\frac{CF_{it}}{K_{it}}$	−0.058	0.528	0.678	1.000						
$\frac{-INV^-_{it}}{K_{it}}$	0.571	−0.129	−0.283	−0.368	1.000					
$\frac{-D^-_{it}}{K_{it}}$	−0.041	−0.603	−0.357	−0.755	0.564	1.000				
$\frac{\Delta B^+_{it}}{K_{it}}$	−0.229	0.111	−0.293	−0.141	−0.347	−0.178	1.000			
$\frac{INV^+_{it}}{K_{it}}$	−0.336	0.119	−0.130	0.022	−0.411	−0.237	0.972	1.000		
$\frac{D^+_{it}}{K_{it}}$	−0.128	0.425	0.434	0.680	−0.451	−0.544	0.209	0.262	1.000	
$\frac{-\Delta B^-_{it}}{K_{it}}$	0.581	−0.109	−0.302	−0.366	0.998	0.566	−0.340	−0.403	−0.465	1.000
Panel B: No Constraint—Simulated Firms with High D^+_{it}/K_{it}										
Means	4.303	1.134	3.133	0.246	0.024	0	0.063	0.073	0.159	0.001
Correlations										
K_{it}	1.000									
θ_{it}	0.555	1.000								
Q_{it-1}	−0.820	−0.440	1.000							
$\frac{CF_{it}}{K_{it}}$	−0.151	0.694	0.316	1.000						
$\frac{-INV^-_{it}}{K_{it}}$	−0.158	−0.517	0.271	−0.391	1.000					
$\frac{-D^-_{it}}{K_{it}}$	NaN	NaN	NaN	NaN	NaN	NaN				
$\frac{\Delta B^+_{it}}{K_{it}}$	0.486	0.344	−0.843	−0.258	−0.239	NaN	1.000			
$\frac{INV^+_{it}}{K_{it}}$	0.455	0.423	−0.817	−0.142	−0.310	NaN	0.983	NaN		
$\frac{D^+_{it}}{K_{it}}$	−0.060	0.468	0.224	0.641	0.252	NaN	−0.085	−0.086	1.000	
$\frac{-\Delta B^-_{it}}{K_{it}}$	0.314	0.111	−0.110	−0.027	0.078	NaN	−0.016	−0.021	−0.055	1.000

(continued)

Second, while Panels A and B report that low-dividend simulated firms are larger than high-dividend simulated firms, Panels C and D show that low-dividend COMPUSTAT firms are smaller than high-dividend COMPUSTAT firms. Note, however, that in Table IV firms from the constrained model are smaller than firms from the unconstrained model. Third, while Panels A and

Table V—*Continued*

	K_{it}	Q_{it-1}	$\frac{CF_{it}}{K_{it}}$	$\frac{-INV_{it}^-}{K_{it}}$	$\frac{-D_{it}^-}{K_{it}}$	$\frac{\Delta B_{it}^+}{K_{it}}$	$\frac{INV_{it}^+}{K_{it}}$	$\frac{D_{it}^+}{K_{it}}$	$\frac{-\Delta B_{it}^-}{K_{it}}$
Panel C: Financing Constraints—COMPUSTAT Firms with Low D_{it}^+/K_{it}									
Means	67.590	5.098	0.314	0.028	0.429	0.515	0.368	0	0.479
Correlations									
K_{it}	1.000								
Q_{it-1}	−0.113	1.000							
$\frac{CF_{it}}{K_{it}}$	−0.003	0.139	1.000						
$\frac{-INV_{it}^-}{K_{it}}$	−0.004	−0.029	−0.031	1.000					
$\frac{-D_{it}^-}{K_{it}}$	−0.047	0.225	−0.167	0.012	1.000				
$\frac{\Delta B_{it}^+}{K_{it}}$	−0.015	0.099	0.002	−0.001	0.035	1.000			
$\frac{INV_{it}^+}{K_{it}}$	−0.051	0.372	0.171	0.043	0.342	0.101	NaN		
$\frac{D_{it}^+}{K_{it}}$	NaN	NaN	NaN	NaN	NaN	NaN	NaN	1.000	
$\frac{-\Delta B_{it}^-}{K_{it}}$	−0.018	0.119	0.044	0.024	0.080	0.753	0.027	NaN	1.000
Panel D: No Constraint—COMPUSTAT Firms with High D_{it}^+/K_{it}									
Means	800.890	4.008	0.471	0.020	0.100	0.343	0.260	0.139	0.285
Correlations									
K_{it}	1.000								
Q_{it-1}	−0.185	1.000							
$\frac{CF_{it}}{K_{it}}$	−0.080	0.359	1.000						
$\frac{-INV_{it}^-}{K_{it}}$	−0.003	0.008	0.029	1.000					
$\frac{-D_{it}^-}{K_{it}}$	−0.061	0.228	−0.042	0.001	1.000				
$\frac{\Delta B_{it}^+}{K_{it}}$	−0.051	0.199	0.014	0.038	0.092	1.000			
$\frac{INV_{it}^+}{K_{it}}$	−0.086	0.319	0.273	0.058	0.339	0.138	1.000		
$\frac{D_{it}^+}{K_{it}}$	−0.069	0.343	0.459	0.016	0.207	−0.020	0.258	1.000	
$\frac{-\Delta B_{it}^-}{K_{it}}$	−0.051	0.215	0.085	0.063	0.099	0.812	0.072	0.008	1.000

B of Table V show that low-dividend simulated firms have lower Tobin's Q than high-dividend simulated firms, Panels C and D show that low-dividend COMPUSTAT firms have higher Tobin's Q than high-dividend COMPUSTAT firms. Also note that Tobin's Q is very large in all the tables because the book value of assets is measured by the capital stock K_{it} for simulated firms and by the amount of property, plant, and equipment for COMPUSTAT firms.

C. Financial Slack

An important source of funds not considered in the constrained and unconstrained firm models is slack. There is no need for slack in the unconstrained firm model. Unconstrained firms can effectively manage their probability of default by buying and selling their capital stock and by changing their financial structure. There is, however, a role for slack in the constrained firm model. Constrained firms are sometimes restricted from investing as much as they might desire and instead limit their investments to their available funds. With slack, constrained firms are able to relax the constraint they face.

The Bellman equation describing the constrained firm's intertemporal problem becomes

$$V^s(K_t, M_t; \theta_t) = \max_{\{D_t^s, I_t, M_{t+1}\}} \max \left\{0, D_t^s + \frac{1}{1+r} E_t\left[V^s(K_{t+1}, M_{t+1}; \theta_{t+1})\right]\right\} \quad (33)$$

subject to the dividend equation

$$\begin{aligned} D_t^s = {} & (1-\tau_f) f(K_t; \theta_t) + \tau_f \delta K_t - I_t - M_{t+1} \\ & + (1 + (1-\tau_f) r) M_t - \chi M_{t+1}^2 - (1-\tau_f) c \geq 0, \end{aligned} \quad (34)$$

the stock of cash being nonnegative $M_{t+1} \geq 0$, the income equation (3), and the capital accumulation equation (4), where the superscript s denotes constrained firms with slack, and the cost of slack χ is set to 0.05. The cost can be viewed as arising from an agency problem: Managers may divert resources away from value-maximizing activities because the cash is so easily accessible. The cost can alternatively represent the double taxation of revenues generated from cash: Unlike debt financing, slack dissipates funds because the interest earned on cash investments is first taxed upon receipt by the firm and taxed again upon distribution to its equity claimants. A convex cost ensures a well-defined firm problem, even when the probability of default is small. At the equilibrium, the marginal cost of a cash investment equals the marginal benefit of relaxing the financing constraint. I solve the model and simulate the series. The expression ΔM_{it+1}^+ measures cash invested for future use, while $-\Delta M_{it+1}^-$ measures the cash made available for funding the firm's activities this period, where $\Delta M_{it+1} = M_{it+1} - M_{it}$.

Although cash funds are much smaller on average than investment ($0.006 < 0.026$ in Table VI), slack has an important effect on constrained firms in relaxing their constraint. With slack, constrained firms do not need to invest as much in periods of low income shock just to generate enough revenues next period so as to avoid the binding constraint. Constrained firms can invest less in periods of low income shock, accumulate cash, and instead invest more in periods of high income shock. As before, cash flows CF_{it}/K_{it} and income shocks θ_{it} are highly correlated. The correlation coefficient is 0.979. An increase in cash flow therefore reflects an increase in the underlying income shock. Constrained firms with slack thus have investment policies that are more sensitive to cash flow fluctuations than constrained firms without slack. Slack magnifies the cash flow sensitivity of constrained firms to 0.775 in Table VII, from 0.248 in Table II where constrained firms cannot accumulate slack.

The ability to accumulate a stock of cash does not change the cash flow sensitivity results, except for the cash flow criterion. Table VII shows that firms identified as experiencing financing constraints if they have low cash flows (0.636) now have a lower sensitivity than high–cash flow firms (0.980). Figure 2 explains why the cash flow sensitivity result obtained with the cash flow criterion

Table VI

Statistics of Simulated Firms from the Constrained Model with Slack

The size of the firm is represented by its capital stock K_{it}. Investment opportunities are measured by the underlying income shock θ_{it} and Tobin's Q_{it-1}. Sources of funds include cash flow CF_{it}, asset sales $-INV_{it}^{-}$, and cash funds $-\Delta M_{it+1}^{-}$. Uses of funds consist of investment INV_{it}^{+}, dividends D_{it}^{+}, and cash investment ΔM_{it+1}^{+}.

	K_{it}	θ_{it}	Q_{it-1}	$\frac{CF_{it}}{K_{it}}$	$\frac{-INV_{it}^{-}}{K_{it}}$	$\frac{-\Delta M_{it+1}^{-}}{K_{it}}$	$\frac{INV_{it}^{+}}{K_{it}}$	$\frac{D_{it}^{+}}{K_{it}}$	$\frac{\Delta M_{it+1}^{+}}{K_{it}}$
Means	5.524	1.027	2.771	0.214	0.029	0.006	0.026	0.119	0.004
Correlations									
K_{it}	1.000								
θ_{it}	0.492	1.000							
Q_{it-1}	−0.893	−0.410	1.000						
$\frac{CF_{it}}{K_{it}}$	0.323	0.979	−0.256	1.000					
$\frac{-INV_{it}^{-}}{K_{it}}$	0.512	−0.280	−0.428	−0.396	1.000				
$\frac{-\Delta M_{it+1}^{-}}{K_{it}}$	−0.346	−0.062	0.233	0.003	−0.276	1.000			
$\frac{INV_{it}^{+}}{K_{it}}$	0.028	0.734	−0.001	0.787	−0.303	0.023	1.000		
$\frac{D_{it}^{+}}{K_{it}}$	0.719	0.141	−0.621	0.023	0.663	−0.031	−0.364	1.000	
$\frac{\Delta M_{it+1}^{+}}{K_{it}}$	0.083	−0.452	−0.060	−0.496	0.474	−0.330	−0.268	−0.061	1.000

is not robust to slack. Compared to Figure 1, Figure 2 plots a different relationship between cash flow and investment for constrained firms. In all but the highest cash flow realizations, constrained firms do not increase their investment by much in response to higher cash flows. Instead, constrained firms accumulate a stock of cash. They use their cash to invest only when investment is most productive. If the cash flow criterion is used to identify financing constraints, then the sample of high–cash flow firms consists mostly of constrained firms that invest more using their accumulated stock of cash. Slack significantly increases the cash flow sensitivity of high–cash flow firms.

It is not surprising that the financing constraint criteria based on dividends and based on cash flows no longer yield similar cash flow sensitivity results. The correlation coefficient between dividends and cash flows has decreased from 0.630 for constrained firms without slack (Panel A of Table IV) to 0.023 for constrained firms with slack (Table VI).

In sum, most of the qualitative results are robust to slack. Even the correlation coefficient between investment and dividend is still negative (−0.364), indicating that the coefficient (accounting for equity repurchases) may well prove useful in identifying constrained firms in the data.

Because most of the qualitative results are robust to slack, the results are also robust to different cost of slack χ parameters. An increase in the cost of slack brings the results closer to those generated from the constrained firm model without slack. An increase in the cost of slack decreases the firm's ability to accumulate cash and use it to invest when investment is most productive. The resulting plot between investment and cash flow plot becomes flatter, migrating

Table VII

Regression Results from Simulated Series with Slack

The measure K denotes the capital stock, CF cash flow, Q Tobin's average q, D^+ dividend, λ the multiplier disallowing equity issues in the constrained model with slack, and Z_{FC} the financial constraint index developed by Cleary (1999). Standard errors appear in parentheses. The portions of firm-year observations identified as experiencing financing constraints appear in brackets. The difference in cash flow sensitivities between firms with financing constraints and firms with no constraint is statistically significant at the 5% level for all regressions.

	CF/K	Q
Firms identified by dividends:		
Financing constraints—firms with low $\frac{D^+}{K}$	0.589	−0.045
	(0.021)	(0.002)
		[0.469]
No constraint—firms with high $\frac{D^+}{K}$	0.299	−0.184
	(0.024)	(0.003)
Firms identified by cash flows:		
Financing constraints—firms with low $\frac{CF}{K}$	0.636	−0.070
	(0.042)	(0.002)
		[0.504]
No constraint—firms with high $\frac{CF}{K}$	0.980	−0.150
	(0.022)	(0.003)
Firms identified by models:		
Financing constraints—firms from the constrained model with slack	0.775	0.064
	(0.012)	(0.002)
		[0.500]
No constraint—firms from the unconstrained model	1.443	−0.163
	(0.030)	(0.006)
Firms identified by models and multiplier:		
Financing constraints—firms from the constrained model with slack and a binding multiplier $\lambda > 0$	1.972	0.225
	(0.093)	(0.042)
		[0.002]
No constraint—firms from the unconstrained model and firms from the constrained model with slack and a non-binding multiplier $\lambda = 0$	0.960	−0.106
	(0.020)	(0.002)
Firms identified by Cleary's index:		
Financing constraints—firms with low Z_{FC}	0.912	−0.052
	(0.019)	(0.002)
		[0.511]
No constraint—firms with high Z_{FC}	1.634	−0.170
	(0.039)	(0.003)

from Figure 2 towards Figure 1. An increase in the cost of slack therefore generates a lower cash flow sensitivity for constrained firms.

D. Sensitivity Analysis

The above results are fairly robust to different assumptions.[5] For example, consider simulating a smaller number of constrained firms than unconstrained

[5] The results are robust to different parameter values. A sensitivity analysis with respect to the sensitivity of revenues to the capital stock α, the persistence of the income shock ρ, its volatility σ, the corporate tax rate τ_f, and the default cost X is available upon request.

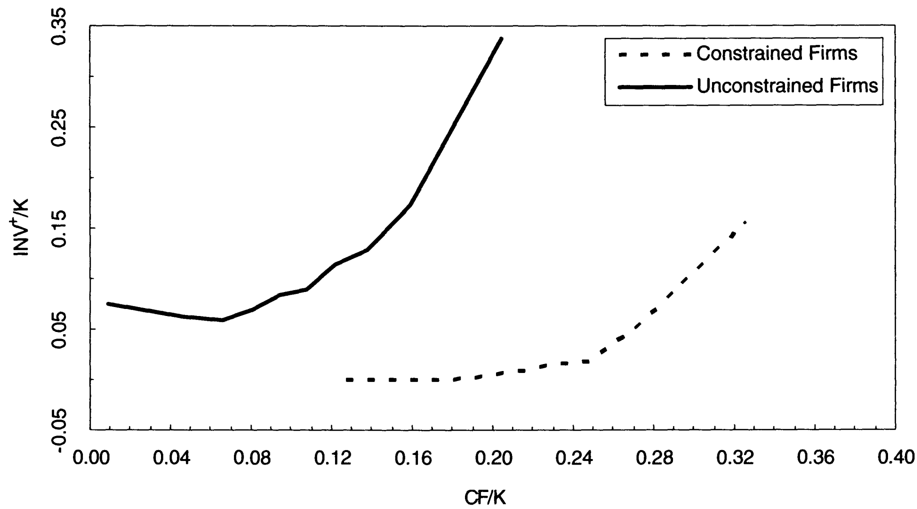

Figure 2. Investment and cash flow, with slack.

firms. Figure 1 indicates that the flatter constrained firm plot would have less weight than the steep unconstrained firm plot. As a result, high–cash flow (high-dividend) firms would exhibit a larger cash flow sensitivity than in Table II, eventually reversing Fazzari et al.'s result.

Now consider classifying a smaller number of firms as low–cash flow (low-dividend) firms. Figure 1 indicates that the negative slope portion of the unconstrained firm plot would become more important for these firms. As a result, the low–cash flow (low-dividend) firms would exhibit a smaller cash flow sensitivity than in Table II, again eventually reversing Fazzari et al.'s result.

However, Fazzari et al. discard firms that do not have enough income to invest or pay out. Similarly, excluding the lowest cash flow firms restores Fazzari et al.'s result. For the highest cash flow firms, the unconstrained firm plot is steeper everywhere than the constrained firm plot. Figure 1 shows that the cash flow sensitivity of high–cash flow firms is a weighted average of the steepest part of the unconstrained firm plot and the flatter constrained firm plot with more weight on the flatter plot, while the cash flow sensitivity of the low–cash flow firms puts more weight on the steeper unconstrained firm plot. High–cash flow (high-dividend) firms would exhibit a lower cash flow sensitivity than low–cash flow (low-dividend) firms, restoring Fazzari et al.'s result.

IV. Conclusion

It is hard to identify firms with financing constraints. I examine various criteria: low dividends, low cash flows, the constrained firm model, the constrained model in which a firm's investment exhausts its internal funds, and Cleary's index. If we use the constrained model criterion or Cleary's index, Kaplan and

Zingales's result obtains: Firms identified as experiencing financing constraints exhibit a lower cash flow sensitivity than firms identified as experiencing no constraint. If we use the other three criteria, Fazzari et al.'s result obtains: Firms identified as experiencing financing constraints exhibit a higher cash flow sensitivity.

Kaplan and Zingales's result obtains if firms with financing constraints are indeed described by the constrained model. Because cash flow is an excellent proxy for firms' underlying income shocks, higher cash flows lead to more investment. Unconstrained firms also borrow more when they experience more favorable income shocks. Because the regression specification does not account for the effect of these external funds on investment, the cash flow sensitivity of unconstrained firms is magnified. Also, unlike unconstrained firms, constrained firms must choose either to pay dividends or to invest with their cash flows. This weakens the link between constrained firms' cash flows and their investments. Constrained firms therefore have investment policies that are less sensitive to cash flow fluctuations than those of unconstrained firms. In addition, because Cleary's index identifies financing constraints in line with the models, Kaplan and Zingales's result also obtains if firms experiencing financing constraints are identified by low Cleary index values.

Fazzari et al.'s result obtains if firms with financing constraints are identified as those with low dividends (or low–cash flows). Firms from the unconstrained model maintain a higher debt burden than firms from the constrained model. Unconstrained firms are therefore more likely to be associated with lower dividends (or lower cash flows) than constrained firms. Low-dividend (or low–cash flow) firms, which are mostly unconstrained firms, have investment policies that are more sensitive to cash flow fluctuations than those of high-dividend (or high–cash flow) firms, which are mostly constrained firms.

Fazzari et al.'s result also obtains if firms with financing constraints are described not only by the constrained model but also by an investment policy that currently exhausts their internal funds. When constrained firms do not have sufficient funds to invest as much as they desire, their investment equals cash flow and asset sales. In that case, the link between investment and cash flow is very strong. Constrained firms without funds to invest more have investment policies that are more sensitive to cash flow fluctuations than those of other firms.

The constrained and unconstrained firm models may prove useful in identifying firms with financing constraints in the data. The investment–dividend correlations generated from the two models differ dramatically: Constrained firms exhibit a negative investment–dividend correlation, while unconstrained firms exhibit a positive correlation. Empirical analysis could help to evaluate different explanations of the observed cash flow sensitivities. For example, Alti shows in his model that learning can generate these sensitivities: Firms uncertain about their quality use their cash flow realizations to resolve their uncertainty; hence younger firms' investments are more sensitive to cash flow fluctuations than older firms' investments. It remains to be seen whether the sensitivities in the data are generated more by learning or by financing constraints. On one hand, young firms should exhibit a higher sensitivity than old firms. On the other hand, firms with negative correlations between investment and equity payout (accounting for repurchases) should exhibit a lower sensitivity than firms with positive correlations.

Appendix

A.1. Effects of the Firm's Decisions on Its Default Probability

The firm's probability of defaulting $\Phi[\bar{\theta}]$ is influenced by the firm's investment choice I and its debt policy ΔB and ι:

$$\frac{\partial \Phi(\bar{\theta}_{t+1})}{\partial K_{t+1}} = \phi[\bar{\theta}_{t+1}]\frac{\partial \bar{\theta}_{t+1}}{\partial K_{t+1}} < 0, \tag{A1}$$

$$\frac{\partial \Phi(\bar{\theta}_{t+1})}{\partial B_{t+1}} = \phi[\bar{\theta}_{t+1}]\frac{\partial \bar{\theta}_{t+1}}{\partial B_{t+1}} > 0, \tag{A2}$$

$$\frac{\partial \Phi(\bar{\theta}_{t+1})}{\partial \iota_{t+1}} = \phi[\bar{\theta}_{t+1}]\frac{\partial \bar{\theta}_{t+1}}{\partial \iota_{t+1}} > 0, \tag{A3}$$

where ϕ is the log-normal probability density function such that

$$\frac{\partial \bar{\theta}_{t+1}}{\partial K_{t+1}} = -\left.\frac{V_{K_{t+1}}}{V_{\theta_{t+1}}}\right|_{\bar{\theta}_{t+1}} < 0, \tag{A4}$$

$$\frac{\partial \bar{\theta}_{t+1}}{\partial B_{t+1}} = -\left.\frac{V_{B_{t+1}}}{V_{\theta_{t+1}}}\right|_{\bar{\theta}_{t+1}} > 0, \tag{A5}$$

$$\frac{\partial \bar{\theta}_{t+1}}{\partial \iota_{t+1}} = -\left.\frac{V_{\iota_{t+1}}}{V_{\theta_{t+1}}}\right|_{\bar{\theta}_{t+1}} > 0, \tag{A6}$$

and

$$V_{K_{t+1}} = (1-\tau_f)\theta_{t+1}\alpha K_{t+1}^{\alpha-1} + (1-(1-\tau_f)\delta) > 0, \tag{A7}$$

$$V_{B_{t+1}} = -(1+(1-\tau_f)\iota_{t+1}) < 0, \tag{A8}$$

$$V_{\iota_{t+1}} = -(1-\tau_f)B_{t+1} < 0, \tag{A9}$$

$$\begin{aligned} V_{\theta_t}^u = {} & (1-\tau_f)K_t^\alpha + \frac{1}{1+r}E_t\left[(\tau_f - \tau_\iota)B_t 1_{(V_{t+1}>0)}\frac{\partial \iota_{t+1}}{\partial \theta_t}\right] \\ & + \frac{1}{1+r}E_t\left[V_{\theta_{t+1}}^u \frac{\partial \theta_{t+1}}{\partial \theta_t}\right] - \frac{1}{1+r}((\tau_f - \tau_\iota)\iota_{t+1} + X)B_{t+1} \\ & \times \left(\frac{\partial \Phi[\bar{\theta}_{t+1}]}{\partial \theta_t} + \frac{\partial \Phi[\bar{\theta}_{t+1}]}{\partial \iota_{t+1}}\frac{\partial \iota_{t+1}}{\partial \theta_t}\right), \end{aligned} \tag{A10}$$

$$V^c_{\theta_t} = (1-\tau_f)K^\alpha_t + \frac{1}{1+r}E_t\left[V^c_{\theta_{t+1}}\frac{\partial\theta_{t+1}}{\partial\theta_t}\right]. \tag{A11}$$

A.2. Effects of the Firm's Decisions and the Income Shock on the Interest Rate

These effects are characterized by totally differentiating the fair bond-pricing equation (6):

$$\frac{\partial\iota_{t+1}}{\partial K_{t+1}} = \frac{-\frac{1}{1+r}E_t\left[\left\{(1-\tau_f)\theta_{t+1}\alpha K^{(\alpha-1)}_{t+1} + (1-(1-\tau_f)\delta)\right\}\left(1-1_{(V_{t+1}>0)}\right)\right] - \frac{1}{1+r}((\tau_f-\tau_\iota)\iota_{t+1}+X)B_{t+1}\frac{\partial\Phi[\bar\theta_{t+1}]}{\partial K_{t+1}}}{\frac{1}{1+r}E_t\left[(1-\tau_\iota)B_{t+1}1_{(V_{t+1}>0)}\right] - \frac{1}{1+r}((\tau_f-\tau_\iota)\iota_{t+1}+X)B_{t+1}\frac{\partial\Phi[\bar\theta_{t+1}]}{\partial\iota_{t+1}}}, \tag{A12}$$

$$\frac{\partial\iota_{t+1}}{\partial B_{t+1}} = \frac{-\frac{1}{1+r}E_t\left[(1+(1-\tau_\iota)\iota_{t+1})1_{(V_{t+1}>0)}\right] - \frac{1}{1+r}E_t\left[X\left(1-1_{(V_{t+1}>0)}\right)\right] - 1 - \frac{1}{1+r}((\tau_f-\tau_\iota)\iota_{t+1}+X)B_{t+1}\frac{\partial\Phi[\bar\theta_{t+1}]}{\partial B_{t+1}}}{\frac{1}{1+r}E_t\left[(1-\tau_\iota)B_{t+1}1_{(V_{t+1}>0)}\right] - \frac{1}{1+r}((\tau_f-\tau_\iota)\iota_{t+1}+X)B_{t+1}\frac{\partial\Phi[\bar\theta_{t+1}]}{\partial\iota_{t+1}}}, \tag{A13}$$

and

$$\frac{\partial\iota_{t+1}}{\partial\theta_t} = -\frac{\frac{1}{1+r}E_t\left[V_{\theta_{t+1}}\left(1-1_{(V_{t+1}>0)}\right)\frac{\partial\theta_{t+1}}{\partial\theta_t}\right] - \frac{1}{1+r}((\tau_f-\tau_\iota)\iota_{t+1}+X)B_{t+1}\frac{\partial\Phi[\bar\theta_{t+1}]}{\partial\theta_t}}{\frac{1}{1+r}E_t\left[(1-\tau_\iota)B_{t+1}1_{(V_{t+1}>0)}\right] - \frac{1}{1+r}((\tau_f-\tau_\iota)\iota_{t+1}+X)B_{t+1}\frac{\partial\Phi[\bar\theta_{t+1}]}{\partial\iota_{t+1}}}. \tag{A14}$$

A.3. Numerical Method

The model is solved numerically using finite-element methods as described in Coleman's (1990) algorithm. Accordingly, the policy functions I_t, ΔB_{t+1}, ι_{t+1}, and V_t are approximated by piecewise linear interpolants of the state variables K_t, B_t, ι_t, and θ_t. Since the beginning-of-the-period debt states, B_t and ι_t, do not appear in the Euler equations, the state space reduces to two dimensions, K_t and θ_t.

The numerical integration involved in computing expectations is approximated with a Gauss–Hermite quadrature rule. Two quadrature nodes are used, reducing the stochastic process to a binary process in which an up move of σ occurs with probability 1/2 and a down move of $-\sigma$ occurs with probability 1/2.

This state space grid consists of five uniformly spaced points for each of the two state variables. The unconditionally lowest outcome of the income shock is specified as $\exp(\frac{-\sigma}{1-\rho})$ and its unconditionally highest outcome as $\exp(\frac{+\sigma}{1-\rho})$.

The approximation coefficients of the piecewise linear interpolants are chosen by collocation, that is, to satisfy the relevant system of equations at all grid points. The approximated policy interpolants are substituted in the equations, and the coefficients are chosen so that the residuals are set to zero at all grid points. The time-stepping algorithm is used to find these root coefficients. Given initial coefficient values for all grid points, the time-stepping algorithm finds the optimal coefficients that minimize the residuals at one grid point, taking coefficients at other grid points as given. In turn, optimal coefficients for all grid points are determined. The iteration over coefficients stops when the maximum deviation of optimal coefficients from their previous values is lower than a specified tolerance level (e.g., 0.0001).

A.4. Data

I use annual COMPUSTAT data from the 1987 to 2001 period. Firms must survive for at least half of the sample period to be included in the sample.

I measure the variables presented in panels C and D of Table V as follows. Assets K are represented by property, plant, and equipment data item 8. Tobin's Q is the long-term debt data item 9 and stockholders' equity data item 216 divided by property, plant, and equipment. Cash flow CF is the sum of income before extraordinary items data item 123, depreciation and amortization data item 125, and deferred taxes data item 126. Asset sales $-INV^-$ are measured by the sale of property, plant, and equipment data item 107, equity issues $-D^-$ by the sale of common and preferred stock data item 108, debt issues ΔB^+ by the long-term debt issuance data item 111. Investments INV^+ are represented by the capital expenditures data item 128, dividends D^+ by cash dividends data item 127, and debt retirements $-\Delta B^-$ by long-term debt reduction data item 114.

I apply the same data filters as Cleary. The sample includes firm-year observations with positive values for sales data item 12, total assets data item 6, assets K_{it}, and Tobin's Q_{it}. The sample includes agricultural, mining, forestry, fishing, and construction firms (SIC codes from 1 to 1999), manufacturing firms (SIC codes from 2000 to 3999), retail and wholesale firms (SIC codes from 5000 to 5999), and service firms (SIC codes 7000 to 8999). In addition, I winsorize the data like Cleary: Tobin's Q between 0 and 10; cash flow CF/K between -5 and 5; investment INV^+/K between 0 and 2; and asset sales $-INV^-/K$, equity issues $-D^-/K$, debt issues ΔB^+, dividends D^+, and debt retirements $-\Delta B^-$ between 0 and 5.

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