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The Price of Options Illiquidity

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ABSTRACT

The purpose of this paper is to examine the effect of illiquidity on the value of currency options. We use a unique dataset that allows us to explore this issue in special circumstances where options are issued by a central bank and are not traded prior to maturity. The value of these options is compared to similar options traded on the exchange. We find that the nontradable options are priced about 21 percent less than the exchange-traded options. This gap cannot be arbitrated away due to transactions costs and the risk that the exchange rate will change during the bidding process.

ONE IMPORTANT ASPECT OF RESEARCH on the microstructure of financial markets is the effect of liquidity on financial assets. The early microstructure literature views liquidity as a determinant of transactions costs, affecting the bid-ask spreads but not equilibrium prices.¹ More recent studies that deal with the effect of liquidity on the prices of financial assets present evidence that illiquidity has an adverse effect on asset values (e.g., Amihud and Mendelson (1991) on bond prices, Silber (1991) on restricted stocks, and Longstaff (1995) on liquidity, volatility, and price).²

These papers deal with the effect of liquidity on stocks and bonds. How about options? Does illiquidity of an option affect its price? Thus far, to our knowledge, this issue has not been discussed in the microstructure literature.³

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¹ See Demsetz (1968) and more recent studies by Christie and Huang (1994) and Huang and Stoll (1996) on the relationship between liquidity and bid-ask spreads.

² See also Kadlec and McConnell (1994) and Amihud, Mendelson, and Lauterbach (1997).

³ Leland (1985), Boyle and Vorst (1992), and Toft (1994) introduced transactions costs into the B-S model. Their analysis, however, is concerned with the costs of trading the underlying asset and its effect on the option. We are concentrating on the liquidity of the option itself.

Though illiquidity in the options market has been mentioned with regard to the bid-ask spread, it has not been discussed in the context of the option price itself, partly due to the fact that the concept of liquidity is not trivial when it comes to options.⁴ For example, to price an option in the Black-Scholes (B-S) world, we need a market where the option payoffs can be replicated. Thus, if the market for the underlying asset trades continuously (i.e., is liquid) we can price any option, whether it trades continuously or not. If, however, the underlying asset is not very liquid and/or the price process is not a diffusion process (e.g., it is a jump process or one with stochastic volatility) we cannot price the option by replication. Under these circumstances how would an illiquid option fare compared to a liquid option? Should the illiquid option, when issued, command a premium or sell at a discount, compared to the liquid option? In a market where options are created by market participants, illiquid options should sell, if at all, for about the same price as liquid options because buyers will never pay more for illiquid ones and sellers will never sell for less. However, should illiquid options sell for less in case the writer of these (nontradable) options is a central bank, which sells the options in an auction to the highest bidder? Clearly, if these options could be replicated costlessly with liquid options, then they should sell for the same price. Because the liquid options in our case do not have the same characteristics (they differ by strike price and expiration dates), replicating the illiquid options with the liquid ones would generate transactions costs. In such a case, these options should sell at a discount determined mainly by the transactions costs.

The objective of this study is to examine empirically the effect of illiquidity on option values. Are illiquid options selling at a discount? Is the discount fully accounted for by the transactions costs? The Israeli currency market provides a unique opportunity for testing the effect of illiquidity on options prices. There are two types of options on the U.S. dollar (paid in Israeli currency—NIS): options issued and traded on the Tel-Aviv Stock Exchange (TASE) and nonnegotiable options sold weekly by the Central Bank of Israel (BI) in sealed-bid auctions. The main differences between these two types of options are: (1) although the exchange-traded (ET) options are created by market participants and everyone is a potential buyer or writer, the sole writer of the BI options is the central bank; and (2) the ET options are continuously traded whereas the central bank options are issued in an auction and cannot be traded prior to expiration.⁵

The remainder of the paper is organized as follows. Section I describes the data and the methodology. In Section II we present and discuss the results of the statistical tests. The main findings are summarized in Section III.

⁴ Examples of illiquid options include options granted to executives, tailor-made OTC options, and so forth.

⁵ Commercial banks offer nonstandardized over-the-counter options issued sporadically depending on the demand for FX options.

I. Data and Methodology

A. Data

The sample period starts with the inception of trading in currency options on the TASE in April 1994, ending in June 1997.⁶ The BI options data include the three-month at-the-money (ATM) call options for the corresponding period, the three-month at-the-money-forward (ATMF) calls, and the six-month ATMF calls. The BI option prices that we use are the average price in each auction. The ET options have fixed exercise prices at 0.05 spreads (e.g., $K = 3.45$, $K = 3.50$, $K = 3.55$) and a given calendar expiration date, whereas the BI options are always issued ATMF, or ATM, with 90 or 180 days to expiration. Only on rare occasions will the expiration date and/or the exercise price of the two options match.

The total number of observations of BI options is 566. Of those, 272 are ATMF options with 90 days to expiration, 127 are ATMF options with 180 days to expiration, and 167 of them are ATM options with 90 days to expiration.⁷ For the ET options, we used the transaction prices of trades done on the same days that auctions were held (Tuesdays and Thursdays).⁸ In addition to the option premium (price), we also collected the daily NIS/\$ exchange rate, the domestic short-term interest rate, and the three-month Euro-dollar rate. Other inputs are given by the specification of the option contracts. The BI options data were obtained from the Bank of Israel and the ET options data were obtained from the TASE.

An important aspect of the data is the auction process. The BI options are issued in a sealed-bid discriminating auction. The process starts at 9:00 a.m. with an electronic announcement of the central bank to all banks on the details of the auction, notional amount, strike price, days to expiration, and minimum bidding price. This announcement goes out immediately from the banks to all their clients. By 11:00 a.m., all interested clients submit their bids through their banks (branch offices) directly to the central bank. At the same time, the banks submit their bids for their own account, independent of the branch offices, to the central bank.⁹ The results are known within half an hour (it used to be one hour). Because only the banks can participate in the auction, the clients pay a commission, charged by the banks, of 0.25–0.50 percent (lower for preferred clients) of the options premium. Every bank is submitting the bids of many clients such that in every auction there are potentially many participants (the Bank of Israel is not making public the number of bids submitted by all the banks). The options cannot be traded

⁶ See the Appendix for a detailed description of the currency market in Israel.

⁷ During the sample period, there were no bidders in nine auctions of the ATMF options and in 105 auctions of the ATM three-month options.

⁸ We use the average quotes, for the ET options, because most foreign exchange transactions are done at the “representative” rate, based on mid-day bank quotes.

⁹ The bids submitted by the clients through branch offices of their banks are not known to the main office, which deals with the bank’s own account (Chinese walls).

prior to maturity and the sole writer is the Bank of Israel. During the sample period, the daily average notional amount was 5.27 million dollars for the BI options and 9.98 million dollars for the ET options.

B. Methodology

Our objective is to study the effect of illiquidity on option prices. Is an illiquid option valued less than a corresponding liquid option? The answer to this question is not clear because both the writer and the buyer face the same problem, lack of liquidity. If, however, we can identify clearly the options' writers and claim that they are indifferent to liquidity but the buyers are not, then it can be argued that the price should be *lower* the less liquid the option is. This discount, however, should be a function of the extent to which one can replicate the payoffs of the illiquid option. The nontradable BI options provide a unique opportunity to test the effect of liquidity on options' prices compared to the ET liquid options.

The options offered by the Bank of Israel are issued in an auction process where the bank is a passive writer and their premiums should reflect the buyers' concern with them being nontradable. It can be argued that the options buyers could write similar ET options to lock in arbitrage profits. However, a perfect arbitrage, which is riskless to expiration, is possible only if the ET options have exactly the same characteristics as the BI options. Because they differ by strike prices and time to maturity, the only possible arbitrage is to replicate the BI options with the ET options and rebalance the position as the market changes. The replication is costly and therefore should be reflected in the discount.

The difference in strike prices and time to maturity requires special care when the BI options are compared to the ET options. To deal with this problem, we start with a simple test, which we consider an indication test, that compares the Implied Standard Deviations (ISD) computed from the ET options to those computed from the BI options. We use the same parameters for both options on days that the BI options are issued except for strike prices and days to expiration. The ISD is weighted by *vega* in the following way:

$$WISD = \sum_{i=1} W_i ISD_i \quad (1)$$

where n is the number of ET options at date t , $W_i = W_i^* / \sum_{i=1} W_i^*$, $W_i^* = \partial C_i / \partial \sigma_i$ and ISD_i is the ISD from option i .

The justification for vega weighting is as follows. Because ATM options are more liquid than away from the money, we used a weighting scheme that gives larger weight to ATM options, which are more sensitive to volatility. Also, the larger weight given to ATM options reduces the effect, if any, of a volatility "smile."

Table I

Sample Statistics of Weighted Implied Standard Deviations (WISD)

The weighted standard deviation is computed as follows: $WISD = \sum_{i=1}^n w_i ISD_i$ where ISD is the implied standard deviation from previous day market call prices $w_i = w_i^* / \sum_{i=1}^n w_i^*$, and $w_i^* = \partial C_i / \partial ISD_i$. The exchange-traded (ET) WISD is computed using all options traded on auction days on the Tel-Aviv Stock Exchange (TASE). The Bank of Israel (BI) WISD is computed using all options auctioned in the sample period, December 4, 1994, to June 30, 1997. Diff. represents the average differences between ISDs for each day BI option was auctioned. A p value for the differences in means is based on a t test. A p value for the differences in medians is based on a Kruskal-Wallis (KW) nonparametric test.

	Mean	Median	S.D.	Maximum	Minimum
All sample ($n = 566$)					
TASE	0.0693	0.0629	0.0147	0.1335	0.0342
Bank of Israel	0.0492	0.0474	0.0122	0.1173	0.0230
Diff.	0.0148	0.0147	0.0116	0.0605	-0.0199
p value	(0.000)	(0.000)			
Three-month ATM ($n = 167$)					
TASE	0.0653	0.0638	0.0154	0.1335	0.0342
Bank of Israel	0.0535	0.0525	0.0138	0.1773	0.0264
Diff.	0.0118	0.0113	0.0132	0.0605	-0.0199
p value	(0.000)	(0.000)			
Three-month ATMF ($n = 272$)					
TASE	0.0625	0.0611	0.0146	0.1335	0.0342
Bank of Israel	0.0470	0.0445	0.0117	0.0921	0.0231
Diff.	0.0155	0.0149	0.0103	0.0563	-0.0065
p value	(0.000)	(0.000)			
Six-month ATMF ($n = 127$)					
TASE	0.0653	0.0637	0.0139	0.1062	0.0349
Bank of Israel	0.0482	0.0455	0.0102	0.0755	0.0249
Diff.	0.0171	0.0165	0.0111	0.0516	-0.0056
p value	(0.000)	(0.000)			

The results of our initial test, are presented in Table I, which compares the ISD computed from the ET options to those computed from the BI options.¹⁰

The mean ISD (and the median) of the BI options is significantly lower than the ET options, 0.0492 versus 0.0639, using the entire sample. Note that although the ATMF options show a difference of 1.5 percent to 1.7 percent on average, the ATM options show a smaller difference, about 1.2 percent. One possible explanation for the fact that the ATM options show less of a deviation than the ATMF options is the presence of a selection bias. Although in all auctions (except nine) for three-month ATMF options and

¹⁰ We exclude ET options that violate the basic arbitrage (Merton's) conditions, because these violations are usually for out-of-the-money options that trade infrequently and are not synchronous with the exchange rate. Because the BI options are issued ATM or ATMF and are auctioned with a minimum price, the arbitrage conditions are never violated for these options.

for six-month ATMF options there were bidders (in most cases the demand was much larger than the supply), in about 40 percent of the ATM options, there were no bidders. The main reason that there were not many bidders for these options was the fact that, given the high interest rate differential between the domestic and the foreign rate, the ATM options were, in fact, deep-in-the-money options and most clients were not interested in options that were practically forward contracts.¹¹ A second reason was the difference in the minimum price that was set for the ATM options and ATMF options. When we examined the minimum price series for both options, we found that the minimum price of the ATMF options implied, on the average, a volatility of four percent whereas the minimum price set for the ATM options in many cases implied a volatility of almost six percent. For these reasons, there was less interest in the ATM options. Thus, the ATM options are a biased sample because they exclude all potential participants who did not bid because of the above mentioned reasons. The ATMF options, on the other hand, had, most of the time, demand much larger than the amount offered by the Bank of Israel. We show later that the demand for the ATMF options was about four times larger, on the average, than the amount offered. We, therefore, focus on the ATMF options in the tests that follow.

The problem with the use of implied standard deviations is that the assumptions of the B-S model do not strictly hold; in particular, the assumption of a log-normal distribution of the underlying asset and the requirement that all securities trade continuously. We argue that the dynamics of the exchange rate (NIS/US\$) in our sample has a negligible effect on the empirical results for the following reasons. First, the exchange rate bands, imposed by the central bank, are on a basket of currencies including the dollar. Although the dollar's weight in the basket is about 55 percent, the band is not as restrictive vis-à-vis the dollar. Second, empirical evidence indicates that for bands larger than 10 percent, the value of ATM options is only slightly different from the B-S value which assumes no FX bands (see Ingersoll (1997)). Third, in a study on the Israeli currency market HajYehia (1997) claims that the B-S model is robust to the NIS/\$ exchange rate dynamics during the sample period that we used.

In an attempt to deal with the dynamics of the exchange rate, especially the issue of FX bands, which is not consistent with a B-S environment, we use two alternative methods. In both methods, we compare the premium of the BI options to a synthetic premium obtained from the ET options. Whereas the first method uses the ISD of ET options to compute the synthetic option price, which is similar to comparing the ISDs, the second method uses the B-S model to compute only the *weight*, in creating the synthetic price, such that the effect of the B-S model on our tests is minimal.

¹¹ Several banks that regularly participate in these auctions and that were asked to explain their lack of interest in the ATM options raised this point. This is also the reason why the Bank of Israel recently stopped issuing the ATM options.

Table II

**The Difference Between the Price of an Exchange-traded Option
and the Price of a Bank of Israel Option: An Example**

The example is based on two options traded on the Tel-Aviv Stock Exchange (TASE) on June 18, 1996, and their terms (K, T) are the closest to the Bank of Israel (BI) options. The inputs were: $S = 3.261$ (NIS/\$), $r_{IS} = 16.9$ percent, $r_{US} = 5.4$ percent where S is the exchange rate, r_{IS} is the yield on an Israeli three-month T-bill, and r_{US} is the yield on a U.S. three-month T-bill. We computed a weighted implied standard deviation (WISD) using two at-the-money options traded on the exchange. This WISD was used in computing the price of the synthetic exchange-traded (ET) option. The percentage difference between the Central Bank of Israel option price and the synthetic ("market") ET price is what we call "price of illiquidity" or Illiquidity Discount (ILD).

	Option Price	Time to Expiration	Exercise Price	Implied standard deviation
TASE				
Option 1	0.117	144 days	3.30	6.03%
Option 2	0.061	71 days	3.30	7.47%
Bank of Israel Option	0.0438	91 days	3.34	
Exchange market price	0.0501	91 days	3.34	6.66%
Price of illiquidity	ILD = $ (0.0438/0.0501) - 1 = 0.126$			

B.1. Method 1

The first method is illustrated with option prices recorded on June 18, 1996. Table II provides an example using two ET options that differ by maturity only. These options straddle the maturity of the BI options that are issued with three months to maturity. We use their ISD to compute a weighted ISD and use this figure as an input to compute the "theoretical" price of the BI options. The auction price of the BI option (0.0438 NIS per 1 \$US), on June 18, 1996, is compared to the synthetic option (0.0501 NIS per 1 \$US). In this example, the percentage difference is interpreted as the price of illiquidity or "Illiquidity Discount," which is about 12.6 percent (i.e., the price of the illiquid option, the BI option, is 12.6 percent lower than the price of the ET liquid option).

B.2. Method 2

The second method is designed to deal with the concern that the results of our test may depend upon the model used. We follow a method offered by Brenner and Galai (1993) that allows us to estimate the illiquidity discount in a manner that is minimally dependent upon the B-S model. The test uses more information than the previous method to create a 90-day ATMF synthetic call interpolated from prices of calls from four series with *weights* based on the B-S model. We use two shorter-term call options with strike prices that bracket the strike price of the BI option and two longer-term call

options with strike prices that similarly bracket the strike prices of the BI options. We then create two synthetic calls, which are close to the money, and finally combine the two to create a 90-day ATMF synthetic call as follows.

Denote the currency forward exchange rate by $Se^{(r-r^*)T}$, the strike prices by K_1 and K_2 , where $K_1 < Se^{(r-r^*)T} < K_2$. C^A and C^M are the actual (A) and model (M) prices. First, we compute

$$C(T = 90, K_1) = C_1^A(K_1)w + C_2^A(K_1)(1 - w), \quad (2)$$

where $C_1^A(K_1)$ is the actual price of the short maturity option, $C_2^A(K_1)$ is the actual price of the long maturity option, and

$$w = \frac{C^M(T = 90, K_1) - C_2^M(K_1)}{C_1^M(K_1) - C_2^M(K_1)}, \quad (3)$$

where the weight w is based on model prices for the corresponding short, long, and 90-day maturities and $C(T = 90, K_1)$ is the value of a 90-day call with the strike K_1 . The model prices use the ATM implied volatility.

Second, we repeat this procedure to compute $C(T = 90, K_2)$. We then combine these two synthetic calls to compute our synthetic ATMF 90-days call option, denoted C^* .

$$C^* = C(T = 90, K_1)v + C(T = 90, K_2)(1 - v) \quad (4)$$

where

$$v = \frac{C^M(T = 90, K = Se^{(r-r^*)T}) - C^M(T = 90, K_2)}{C^M(T = 90, K_1) - C^M(T = 90, K_2)}, \quad (5)$$

and C^* is a synthetic option created from ET options that is compared to the BI option. Thus, according to this method, we take a weighted average of four ET options to create a synthetic price that is compared to the price of the BI option.

II. Results

A. The Illiquidity Discount: Method 1

The main hypothesis is that nontradable options should be priced lower than similar liquid options. In Table I we use the B-S model to compute the implied standard deviations and compare them. Another way to measure and present the results of this test is to compare the untransformed BI options premiums to those of synthetic options based on the prices of the ET options.

Table III
The Effect of Liquidity on the Pricing of Currency Options—
Method 1

The average illiquidity discount (ILD) is measured by the percentage difference between the price of an option traded on the Tel-Aviv Stock Exchange (TASE), C_{ET} , and that issued by the Bank of Israel, C_{BI} , using Method 1 (Table II).

$$ILD = |(C_{BI}/C_{ET}) - 1|$$

The numbers presented in column I are based on weighted implied standard deviation (WISD) using the closing prices of all options traded on the exchange in the previous day. The numbers presented in column II are based on WISD using the three closest at-the-money options traded on the TASE in the same day. The numbers presented in column III are based on ISD using the closest at-the-money option traded on the exchange. In the last two columns, the options used are also those with the closest time to expiration of the option issued by the Bank of Israel on that day. All ILDs are significantly different from zero at the one percent level.

Period			Illiquidity Discount					
			I		II		III	
From	To	N	Mean	S.D.	Mean	S.D.	Mean	S.D.
All sample		399	0.210	0.104	0.204	0.125	0.181	0.127
4/12/94	5/15/95	132	0.274	0.102	0.297	0.116	0.231	0.151
5/16/95	2/15/96	111	0.189	0.109	0.195	0.117	0.151	0.132
2/16/96	6/30/97	156	0.171	0.073	0.148	0.092	0.160	0.079

We follow the procedure outlined in Table II, where we compute the value of an option that has the same exercise price and maturity as that of the BI option, but uses the volatility implied by TASE options. This model value is compared to the auction price of the BI option. We then compute the illiquidity discounts (ILD), measured by the percentage difference in option premiums between ET options and the nontradable BI options. Table III displays the results of the illiquidity discount. Because WISD, using vega weighting, may not fully compensate for the possibility of the known volatility smile, we have used three estimates of volatility. Only the first alternative (column I) uses vega weighting across strike prices and maturities based on all ET options traded on day t . The second alternative (column II) uses vega weighting across strike prices only, based on three ET options closest to ATMF and closest to maturity of the BI options, which is effectively similar to equal weighting. The third alternative (column III) uses only one option (no weighting at all) that its strike price is the closest to ATMF and is the closest in maturity.¹²

¹² We tested if there is a volatility smile for the options that we used. The test compares ISD of options which are out-of-the-money with options that are in-the-money. The following results indicate that there is no significant difference between ISDs of the options that were used for interpolation.

	Out-of-the-money	In-the-money	t -value	p -value
ISD	0.0648	0.0617	1.27	0.20

We find that the BI option price is on the average about 21 percent lower than the ET option price (column I). The smallest difference, obtained in the most recent period, is about 17 percent. Similar results were obtained when we used alternative estimates of volatility displayed in columns II and III. It seems that the illiquidity discount is insensitive to the method of estimating the volatility.¹³ All the results are statistically significant at the one percent level.

When we examined the change from the earlier period (1994 to 1995) to the latter period (1996 to 1997), we observe that on average, there was a decline in the illiquidity discount that could be related to the increased liquidity in the FX spot and FX options markets. The increased liquidity in these markets is a result of a change in the central bank intervention policy (to nonintervention inside the FX band) combined with lower transactions costs for trading options.¹⁴

In Table IV we present the results by option maturity as auctioned by the Bank of Israel. We use the same method as in Table III (Method 1). Here we find that both the three- and six-month ATMF options are discounted by about 21 percent compared to the ET options. The null hypothesis of zero difference, no liquidity discount, is rejected at the one percent level for both types of options. Here, too, we observe a decline in the discount from 26.5 percent in the early period (1994 to 1995) to 16.7 percent in the later period (1996 to 1997).¹⁵

Another indication that the discount that we find is due to the illiquidity of the BI options is the relationship between volatility and the illiquidity discount (see, e.g., Longstaff (1995)). We have tested this relationship by regressing the illiquidity discount (ILD) against the implied volatility (ISD). The results are:

$$ILD_i = -0.04325 + 4.0024ISD_i, \quad R^2 = 21.1\% \quad (6)$$

(0.086) (0.000)

¹³ Because we use transaction prices that blend bid and ask prices, it may be argued that the bid-ask spread can explain most of the illiquidity discount. However, as it turns out, the bid-ask spread in this market is rather narrow and its effect on our results is negligible. As described in the Appendix, in over 80 percent of the trades, the effective bid-ask spread is one tick (half of it is about one percent of the premium). Even if we use two ticks, half of the spread for an average premium of 500 NIS would amount to two percent of the option premium, which will reduce the average discount from 21 percent to 19 percent.

¹⁴ We also use two additional alternative inputs of the underlying exchange rate; the previous day FX rate and the next day FX rate. The need for these alternative inputs stems from a potential bias in our estimates due to a synchronization issue of the exchange rate with the option prices, when we use representative exchange rates set by the central bank reflecting mid-day transactions. The results are essentially the same as in Table III.

¹⁵ It could be argued that there is a tax effect. Under the tax code, individuals are exempt from taxes on TASE securities but not on the BI options. Other market participants (corporations, money managers) are not exempt. However, because only a fraction of trading in the FX market is done by individuals, we believe that the tax effect is negligible and cannot explain the difference that we observe.

Table IV
The Effect of Illiquidity on the Pricing of Currency Options—
Method 1 by Option Type

In this table, we present the average illiquidity discount (ILD) that is measured by the percentage difference between the price of an option traded on the Tel-Aviv Stock Exchange (TASE), C_{ET} , and that issued by the Bank of Israel, C_{BI} , using Method 1 (Table II).

$$ILD = |(C_{BI}/C_{ET}) - 1|$$

The numbers presented are based on WISD using the three most at-the-money options traded on the TASE in the previous day. The numbers presented in column I are the illiquidity discounts of at-the-money-forward options issued by the Bank of Israel with three months to maturity. The numbers presented in column II are the illiquidity discount of at-the-money-forward options issued by the Bank of Israel with six months to maturity. All ILDs are significantly different from zero at the one percent level.

Period		Illiquidity Discount					
		I—three-month ATMF			II—six-month ATMF		
		<i>N</i>	Mean	S.D.	<i>N</i>	Mean	S.D.
All sample		272	0.210	0.105	127	0.211	0.174
4/12/94	5/15/95	98	0.265	0.106	34	0.302	0.086
5/16/95	2/15/96	73	0.197	0.111	38	0.174	0.106
2/16/96	6/30/97	101	0.167	0.071	55	0.180	0.076

The numbers in parenthesis are *p* values. We use the *ISD* of an exchange option that is the closest in maturity and strike price to the BI option. Though the variation in the implied volatility is not large, we found a positive and significant relationship.¹⁶

B. Illiquidity Discount: Method 2

In the next test, we examine the robustness of the results to the method used. We estimate the illiquidity discount in an alternative way, described in the previous section (Method 2). This method uses two short-term call options with strike prices that bracket the forward price and similar two next-term call options. We then create two synthetic calls, which are close to the money, and combine the two to create a 90-days ATMF synthetic call. The value of this call is compared to the premium paid on the option issued by the central bank.

Table V presents the mean percentage difference between the synthetic options, based on ET options data, and the BI options. The results are consistent with the earlier results. For the entire sample period, the mean difference is about 19.4 percent and is significant at the 1 percent level. In the

¹⁶ This particular test was suggested to us by the referee.

Table V
The Effect of Liquidity on the Pricing of Currency Options—
Method 2

The average illiquidity discount (ILD) measured by the percentage difference between the price of an option traded on the Tel-Aviv Stock Exchange (TASE), C_{ET} , and the one issued by the Bank of Israel (BI), C_{BI} , using Method 2 described in Section II. The BI at-the-money-forward (ATMF) option is compared to a synthetic call option that is obtained by interpolating four series of exchange-traded (ET) options with weights based on the Black-Scholes model. The number of observations shrinks to 165 BI options (auctions) because we could not always obtain all four series of ET options, especially for the six-month ATMF. All mean ILDs are significantly different from zero at the one percent level.

Period			Illiquidity Discount	
From	To	<i>N</i>	Mean	S.D.
All sample		165	0.194	0.098
4/12/94	5/15/95	56	0.240	0.111
5/16/95	2/15/96	37	0.212	0.098
2/16/96	6/30/97	72	0.150	0.064

most recent period the difference is still highly significant, but lower, about 15 percent. The results of Method 2 are indistinguishable from the results of Method 1.

C. Illiquidity Discount and Transaction Costs

In this section, we examine whether the cost of replication could fully account for the 21 percent discount. A well-designed arbitrage would require replicating the BI option with the lowest possible costs. Because any replication of the BI options is not perfect, dynamic replication will incur some costs. Because in this market, selling and buying foreign exchange funds is more costly than selling and buying options, we replicate the BI options with ET options. The replication method is as follows: when a BI option is offered, we calculate the option's delta using WISD of the three closest ATMF exchange-traded options with the closest expiration date, computed on the same day. Against the BI option that we buy in the auction, we sell on the first day x_t of ET options that result in a delta-neutral position. From then on, we recalculate the BI option's delta, each day, in the same manner and sell or buy Δx_t of ET options to regain a delta-neutral position. The transactions costs incurred using this replicating procedure amount to $1.5 \sum_{t=1}^T |\Delta x_t|$ when there are T days to expiration. The 1.5 NIS is the cost of buying or selling ET options by members of the exchange, paid to the clearing corporation. The percentage transactions costs is estimated by $1.5 \sum_{t=1}^T |\Delta x_t| / C_{BI}$, where C_{BI} is the BI call price. The idea is to maintain a combined position that is essentially riskless. In doing so we incur transactions costs, which account for 12 percent out of the 21 percent. These results are presented in

Table VI
The Illiquidity Discount and Transactions Costs

Transactions costs are calculated by replicating the Bank of Israel (BI) options with exchange-traded (ET) options in the following way. When a BI option is offered, we calculate the option's delta using weighted implied standard deviation (WISD) of the three closest at-the-money-forward (ATMF) ET options at day $t - 1$. Against the BI option, we sell on the first day x_t ET options, which results in a delta-neutral position. Each day, we recalculate the BI option's delta in the same manner and sell or buy Δx_t to regain a delta neutral position. The transaction costs incurred using this replicating procedure amounts to $1.5 \sum_{t=1}^T |\Delta x_t|$ when there are T days to expiration, assuming that buying or selling an ET option costs 1.5 NIS. The percentage transaction cost is estimated by $1.5 \sum_{t=1}^T |\Delta x_t| / C_{BI}$.

Period		N	Illiquidity Discount	
From	To		Mean	S.D.
All sample		319	0.1246	0.0846
4/12/94	5/15/95	114	0.1327	0.0644
5/16/95	2/15/96	99	0.1631	0.1148
2/16/96	6/30/97	106	0.0804	0.0378

Table VI. The remaining gap, about nine percent, is still significantly large and is about the same in the most recent period. This gap can be explained by the uncertainty associated with the time span between the submission of the bid and the announcement of the winners (between half an hour and an hour). An indication that this is the case is given by the significant positive relationship between the illiquidity discount and implied volatility reported earlier. This relationship suggests that when the volatility is larger, arbitrageurs are more concerned about the move of the exchange rate during the time gap and therefore they demand a larger discount. Also, there is no evidence of excessive arbitrage activity on auction expiration days compared with other days.¹⁷ This is another indication that the nine percent remaining gap is due to the uncertainty mentioned above.

D. The Illiquidity Discount and the Auction Process

How competitive is the bidding process and to what extent does the auction process drive these sizable discounts? To address this question we first examined the auction data regarding the rate of over- and undersubscription at the Bank of Israel auctions. The rate of subscription is measured by the amount of options demanded divided by the amount of options offered. In Table VII, we find that across all 399 auctions, the mean rate of subscription is 3.92 (the median is 4.2). The maximum is over 10 and the minimum is 2 percent. Eighty-three percent of the auctions are oversubscribed.

¹⁷ The average daily volume on auction expiration days was 993 contracts, compared with 897 contracts on other days. The difference is statistically insignificant (p value of 0.287).

Table VII
The Rate of Subscription at Bank of Israel Auctions

Rate of subscription is calculated as a ratio of the amount of options demanded at the auctions divided by the amount offered. Number of undersubscribed represents the number of times the auctions were undersubscribed.

Period		N	Mean	S.D.	Median	Min	Max	No. of Undersubscribed
From	To							
All sample		399	3.92	2.32	4.20	0.02	10.32	66
4/12/94	5/15/95	132	3.47	2.54	3.52	0.02	10.32	34
5/16/95	2/15/96	111	3.30	2.37	3.78	0.04	7.83	30
2/16/96	6/30/97	156	4.74	1.79	4.78	0.15	9.00	2

Another indication of the competitiveness of the auction process is the deviation of the minimum price (C_{BI}^{MIN}) set for each auction from the average auction price (C_{BI}). In Table VIII, we find that for the overall sample, C_{BI}^{MIN} is 17 percent lower than C_{BI} . We also find that it is 35 percent lower than C_{ET} . The results in both tables cannot explain the large discounts and do not support the possibility that the discounts are driven by the auction process.

Nevertheless, the fact that 17 percent of the auctions were undersubscribed and that the deviation, C_{BI}^{MIN} from C_{BI} , ranges from 6 percent in the early period to 31 percent in the third period may still indicate that the discounts may be an outcome of the auction process. Because of this concern, we examine the relationship between the discount (ILD) and the rate of subscription (RS) using a regression test. If the discounts are a result of the auction process we should find a negative relationship between RS and ILD . The following results show that there is no significant relationship between ILD and RS .

$$ILD_i = 0.1972 + 0.0034RS_i \quad R^2 = 0.3\% \quad (7)$$

(0.000) (0.135)

Moreover, we computed the average discount for all auctions that were oversubscribed (333) and for those that were undersubscribed (66). The average discount is *larger* (21.6 percent) for the oversubscribed than the average discount for the undersubscribed (18.1 percent). These results provide further evidence that the observed discounts are not driven by the auction process.

III. Summary and Conclusions

Given the growing interest among academics and practitioners in the effect of liquidity on the values of financial assets, we examine here the effect of illiquidity on the value of currency options. In the standard case, illiquid-

Table VIII

Descriptive Statistics of C_{BI}^{MIN} relative to C_{BI} and C_{ET}

C_{BI}^{MIN} is the minimum price set by the Bank of Israel (BI) for the auction. C_{BI} is the average auction price. C_{ET} is the price of the exchange-traded (ET) option. C_{ET} is calculated using Method 1 (Table II). $|C_{BI}^{MIN}/C_{BI} - 1|$ measures the percent C_{BI}^{MIN} is lower than C_{BI} . $|C_{BI}^{MIN}/C_{ET} - 1|$ measures the percent C_{BI}^{MIN} is lower than C_{ET} .

Period			$ C_{BI}^{MIN}/C_{BI} - 1 $		$ C_{BI}^{MIN}/C_{ET} - 1 $	
From	To	<i>N</i>	Mean	S.D.	Mean	S.D.
All sample		399	0.172	0.160	0.351	0.139
4/12/94	5/15/95	132	0.062	0.084	0.318	0.125
5/16/95	2/15/96	111	0.103	0.118	0.275	0.130
2/16/96	6/30/97	156	0.314	0.125	0.432	0.113

ity should not affect a derivative asset because options are a zero-sum game. In this paper, however, we use a unique dataset that allows us to explore this issue in circumstances where the question is relevant. We look at currency options issued by a central bank that are not traded until maturity. We argue that prices of such options may be affected by illiquidity. We test our hypothesis by comparing these options to similar exchange-traded options. The results are significant in all cases. We reject the hypothesis that liquidity has no effect on the price of the options. We find that the nontradable options are discounted by about 21 percent on the average. There was, however, a gradual decline in the discount from 27 percent in the first period to about 17 percent in the last period. In general, illiquid options should not be selling at a discount even when the underlying asset is not liquid. The exception is in cases when the options are sold at an auction, as done by some central banks. The discount should be a function of the cost of replicating the illiquid option. In our case the discount can be explained by transactions costs associated with replication and by the time gap during the bidding process.

Appendix: The Currency Market in Israel

A. The Spot and Forward Currency Market

The Israeli Foreign Exchange market has undergone a major transformation in recent years, from a market largely controlled by the central bank to an active interbank market that trades continuously but not in a large volume. However, due to several restrictions imposed by the central bank,¹⁸ market participants are mainly corporations who use foreign exchange in their transactions. The banks act as “market makers” because all transactions must go through them. At the end of each trading day, the central bank

¹⁸ Recently the central bank has dropped many of these restrictions.

publishes a settlement (representative) exchange rate for each currency that is based on the quotes obtained from the major banks. This is used in settling options at expiration.

Since the early 1990s, the exchange rate regime was a "crawling" peg system. In this regime, a band was imposed around an upward sloping exchange rate, on a basket of five currencies, reflecting the difference in inflation between Israel and the "basket" economies. The central bank is committed to intervene only when the band is breached. In May 1995, the width of the band was increased from 10 percent to 14 percent and in June 1997, it was increased again to 30 percent. Also, until February 1996, the central bank intervened occasionally in the FX market to keep the exchange rate around the center of the band. It stopped its intervention on February 16, 1996. In our tests, we have divided the whole sample period into three subperiods in line with the changes in the policy of the central bank. The second subperiod starts when the FX band was increased from 10 percent to 14 percent. The third subperiod starts when the central bank stopped its intervention inside the exchange rate band.

B. Currency Options on the TASE

Currency options on the TASE started trading on April 1, 1994. In the first 20 months, the volume of trading was not high and amounted to 400 contracts a day, on the average. During 1996, the daily volume increased dramatically from 331 contracts in January to 5,674 in December of that year. Open interest increased over the same interval, from 10,677 contracts in the beginning of the year to 168,212 contracts in December.

The underlying asset is the dollar denominated in shekel (the domestic currency). The options are European puts and calls. They are cash settled, where the settlement price is the so-called representative exchange rate published daily by the Bank of Israel. The maturity cycle is two, four, and six months.

The options are traded by members of the TASE in an open outcry system, with about 20 to 30 traders in the pit, governed by rules and regulations set by the Exchange. There is no designated market maker that is committed to quote prices. The tick size for options with a premium of 200 to 2,000 NIS is 10 NIS, and is 20 NIS for larger than 2,000 NIS. The premium for a wide range of options around the money, an average of 90-day maturity, and a volatility of 5 percent to 10 percent are in the range of 200 to 1,000. Over 80 percent of the trades in these options are done within one tick, implying that the effective bid-ask spread is one tick.

C. Options Issued by the Bank of Israel

The Bank of Israel started issuing call options (European type) on the dollar in November 1989. Initially, the main reason that the central bank engaged in this activity was to enhance the development of the markets for derivatives in Israel. At that time, no other derivative was traded on the Stock Exchange.

There are two classes of three-month (13 weeks) options auctioned twice a week (Tuesday and Thursday). They are issued with two strike prices; exactly at-the-money-forward and at-the-money. They also expire on Tuesdays and Thursdays. A six-month (26 weeks) at-the-money-forward option is auctioned once a week (Thursday). In all auctions, there is a floor (minimum) price that is typically set below the comparable price on exchange-traded options.

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