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Monitoring and Structure of Debt Contracts

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ABSTRACT

This paper presents a theory of optimal debt structure when the moral hazard problem is severe. The main idea is that the optimal debt contract delegates monitoring to a single senior lender and that seniority allows the monitoring senior lender to appropriate the full return from his monitoring activities. The theory explains (i) why debt contracts are prioritized, (ii) why short-term debt is senior to long-term debt, and (iii) why financial intermediaries usually hold short-term senior debt whereas long-term junior debt is widely held. Another implication of the theory is that covenant and maturity structures will be set to conform to the seniority structure.

When a firm becomes highly leveraged, its debt contracts are often prioritized into several classes, so that, in the event of bankruptcy liquidation, some lenders will have claims on the firm's assets before others do. The most common financing pattern consists of senior bank debt with public debt subordinate to it. Another common characteristic is that senior bank debt usually has a shorter maturity and a more restrictive set of covenants than junior public debt.[1]

When we consider banks as delegated monitors who specialize in information gathering, it seems puzzling that banks are usually senior. If, as has been argued (Fama (1990)), junior lenders have stronger incentives than senior lenders to monitor default risks, then we would expect monitoring specialists such as banks to be junior, not senior. Furthermore, if covenants

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[1] Welch (1997) quotes Carey (1995), who finds that 99 percent of bank loans made between 1986 and 1993 and recorded in the Loan Pricing Corporation *Dealscan* database contain a senior priority clause. See also Smith and Warner (1979), Berlin and Mester (1992), and Carey, Prowse and Rea (1993). According to Tufano (1993), who studies mergers and acquisitions of the late 1980s, the median maturity of bank loans (excluding bridge loans) is about five years, whereas the median maturity of public debt is 10 years.

are used to reduce contracting costs (Smith and Warner (1979)), we would expect junior debt contracts to have more restrictive covenants because junior debt is inherently riskier than senior debt because of its lower priority and longer maturity.[2] We might also ask why short-term debt is further protected through seniority. These issues beg the question of why firms prioritize their debt in the first place.

This paper seeks to answer these questions by developing a theory of the structure of debt contracts when the borrower's moral hazard problem is severe. As such, our theory can be used to understand the structure of debt contracts employed in the highly leveraged transactions of the 1980s. By explicitly modeling a costly monitoring technology in a simple model of asset substitution (Jensen and Meckling (1976)), we show how prioritized debt contracts with observed characteristics emerge as a way to minimize contracting costs. In our model, all agents are risk neutral, and renegotiation is limited only by incentives, so that the theory depends neither on risk-sharing motives nor on the exogenously imposed difficulty of renegotiation.

We study a model where the borrower's moral hazard problem is so severe that no borrowing and lending occur unless lenders expend resources to monitor the borrower. Monitoring deters moral hazard because it enables lenders to detect a borrower's opportunistic behavior and to punish it either by liquidation or through renegotiation. If lenders have strong incentives to monitor, a borrower is therefore less likely to behave opportunistically. Put another way, a well-structured contract can serve as a commitment mechanism for the borrower by providing lenders with strong incentives to monitor. Because contracting costs will be lower if lenders have stronger incentives to monitor, the borrower will structure debt contracts to maximize the lenders' incentives to monitor.[3]

Other things being equal, a lender's incentive to monitor is maximized when he appropriates the full return from monitoring. However, if there are lenders senior to him, the return will go to these senior lenders first. The lender will then have to share what is left of the return with other lenders of the same seniority. Therefore, for a lender's incentive to monitor to be maximized, there must be no lender senior to him, and he must be the only

[2] Smith and Warner (1979) and Berlin and Mester (1992) explain this in terms of difficulty of renegotiation with public lenders. The Federal Trust Indenture Act, which requires unanimous consent for votes on principal or interest concessions for publicly traded debt, is certainly a factor here. However, Carey et al. (1993) note that the covenants of the new 144A securities, or *tradable private debt contracts,* are very similar to those of public debt. These are securities held by and traded among private lenders such as insurance companies, and as such they are not subject to the Federal Trust Indenture Act. Therefore, the difficulty of renegotiation alone does not fully explain the relative restrictiveness of covenants.

[3] Maximizing *incentives* to monitor is not the same as maximizing the *actual* amount of monitoring carried out by lenders. Because monitoring is costly, the borrower wants to minimize the actual amount of monitoring. Maximizing the incentive to monitor is equivalent to maximizing the *threat* of monitoring *when the borrower is suspected of misbehavior*. Another way to understand this is that it costs less to induce a lender to monitor if he has a stronger incentive to monitor.

lender in his seniority class. In other words, the lender must either be the sole lender financing the whole project or the only lender in the most senior class. In the senior lender case, however, the senior claim must be big enough to be impaired by liquidation. Otherwise, because his claim is risk free, the senior lender would have no incentive to monitor. It follows that maximization of incentives to monitor leads the borrower to choose between single-lender contracts and prioritized contracts with an impaired senior lender.

To figure out which contract is better, note that if the project continues, an impaired senior lender will get less than a sole lender simply because his claim is smaller. On the other hand, if the project is liquidated, an impaired senior lender will get the same amount as a sole lender, the full liquidation value. Therefore, a small piece of bad information may prompt the impaired senior lender to choose liquidation over continuation, whereas it takes far worse information to induce the sole lender to seek liquidation. In other words, the impaired senior lender is more sensitive to his information and thus has a stronger incentive to monitor. Indeed, as the old saying describes; "Borrow a little, and you will get a nagging creditor; borrow a lot, and you will get a generous partner."[4] A contract with a nagging impaired senior lender is a better deterrent to moral hazard.

The above discussion shows that an impaired senior lender's incentive to monitor is stronger if his stake is smaller. This is not as counterintuitive as it sounds. If monitoring is meant to raise the going concern value of a project, an investor's incentive to monitor is stronger when his stake is bigger. In contrast, monitoring in our model serves as a deterrent to moral hazard, and as such, it is carried out to identify and liquidate a bad project, not to raise the going concern value of a project. An increase in the impaired senior lender's claim would make him more reluctant to liquidate the project and punish the borrower because it would make his claim *more impaired* by liquidation. It follows that the optimal debt structure will be the one where there is only one senior lender who contributes the smallest possible amount that would keep him impaired by liquidation.

Note that under the optimal contract, junior lenders get nothing if the project is liquidated. Therefore, no matter how badly the borrower behaves, the junior lenders have no incentive to liquidate the project and thus have no incentive to monitor. In other words, *even though their claim is very risky, it does not follow that junior lenders have a stronger incentive to monitor than a senior lender*. Under the optimal contract, only the senior lender will monitor the borrower, and seniority allows the monitoring lender to appropriate the full return from his monitoring.[5]

[4] I am indebted to Milt Harris for this quote.

[5] The following quote gives us an example where other lenders depend on the senior bank to discipline the borrower. When Security Pacific Bank, after renegotiation with Campeau, waived covenants prohibiting new borrowing on Allied stocks, a source from Prudential was quoted to have said, "We have been relying on the intelligence of the banks. The banks were stupider than we thought" (Rothchild 1992, p. 142).

Once we note that only the senior lender monitors, then other features of contracts are easy to explain. First, senior debt will be best held by lenders with the lowest monitoring costs. We interpret these lenders to be financial intermediaries. Second, because junior lenders do not monitor, it does not matter how many classes of junior debt are issued. Therefore, issuing two classes is enough. Third, to maximize the monitoring incentives of the senior lender, the senior claim will have the most restrictive covenants and the shortest possible maturity. As hypothesized by Fama (1990), *the seniority, covenant, and maturity structures of debt contracts are not separate.* They are all set to maximize the monitoring incentives of the senior lender. Although our theory is directly applicable to cases where asset substitution may occur immediately after financing, it also has implications about cases where it is not certain when and whether asset substitution will occur. These and other implications of the model will be discussed later in the paper.

There are several recent papers dealing with the debt structure problem. Diamond (1993a, 1993b) studies a model with adverse selection and derives an optimal debt structure based on a trade-off between informational sensitivity and preservation of control rent. His theory, though, does not explain why firms need multiple creditors holding different securities. Using the model of Hart and Moore (1998), Berglöf and von Thadden (1994) explain this by showing that the optimal contract drives a wedge between short-term lenders and long-term lenders. However, these two papers do not model monitoring and thus cannot answer questions such as why banks should hold senior debt.

Rajan (1992) and Repullo and Suarez (1998) employ similar models and explicitly deal with monitoring. Whereas Rajan (1992) concludes that monitoring lenders must be junior to limit the informational monopoly rent, Repullo and Suarez (1998) abstract from the informational monopoly problem and arrive at an opposite seniority rule where informed lenders are senior. Both papers share the assumption that a lender has the same incentive to monitor whether he is senior or junior. On the other hand, our paper endogenizes monitoring incentives of lenders and shows that putting a monitoring lender in a junior position would make the informational monopoly problem worse. Rajan and Winton (1995) and Chemmanur and Fulghieri (1994) also model monitoring, but neither derives the optimal debt structure.

Harris and Raviv (1990) study the informational role of debt payments and derive implications for capital structure. Whereas their main concern is information generated by the state of default, information in our model is generated by lenders before default occurs. Moreover, they neither distinguish different kinds of lenders nor consider different seniority classes. Other related papers are Winton (1995), Dewatripont and Tirole (1994), Hart and Moore (1995, 1998), Bolton and Scharfstein (1996), Welch (1997), and O'Connor (1996). None of these papers deal with monitoring, and each has a focus different from ours.

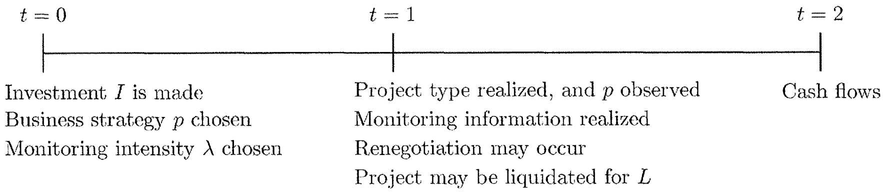

Figure 1. Time line of events. At date zero, financing is done, and strategies are chosen. At date one, all the information is revealed, and the project may be liquidated with or without renegotiation. At date two, cash flows occur.

The rest of the paper proceeds as follows. Section I lays out our model and its assumptions. Section II considers the case when lenders do not monitor and shows that borrowing and lending do not occur in equilibrium. Section III shows how monitoring helps a project to get financed and derives the optimal single-class debt contract. Although this is not the optimal contract, the results of this section will be used repeatedly in later sections. We then derive the optimal two-class debt contract in Section IV. Section V combines results from Sections III and IV to derive the optimal debt contract and then discusses the implications. The paper concludes in Section VI. Some proofs are omitted either because they are very simple or because they take up too much space. All the missing proofs are contained in Park (1995) and are also available from the author upon request.

I. Model and Assumptions

We consider a model in which all agents are risk neutral and the risk-free interest rate is zero. The model has three dates. At date zero, an entrepreneur contacts investors to finance his project, which requires an investment of I dollars. In the case of leveraged buyouts, the project is an acquisition, and the variable I is the acquisition price. After financing is done, and before date one, the entrepreneur chooses his *business strategy*, and each lender chooses his *monitoring intensity* (more on this below). At date one, the project type is realized, and the project may be liquidated for a value of L. If not liquidated at date one, the project returns cash flows at date two. There is no cash flow at date one, and there is no liquidation value at date two. The time line is depicted in Figure 1.

A. Project Type and Monitoring Technology

The entrepreneur's project is either *safe* or *risky*. A safe project is always a success and returns G for sure. A risky project is a success with probability $\pi \in (0,1)$ and returns B when it is a success and nothing otherwise. We assume that only success or failure of the project is verifiable but not the actual cash flows. Therefore, G represents the verifiable component of a successful project's return, and G is the most investors can get from the entrepreneur.[6] We assume

$$\pi B < L < I < G < B. \tag{P1}$$

Thus, only a safe project has a positive NPV, and liquidation is better than continuation if the project is risky.

As of date zero, the project type is not yet determined. The project type is realized and becomes known to the borrower at date one. The probability $p \in [0,1]$ of the project being safe is determined by how the project is managed or by the borrower's business strategy. In a leveraged buyout, the business strategy would include decisions regarding whether, when, and at what prices to sell some assets. With little loss of generality, we can view the probability p itself as the borrower's business strategy and assume the borrower chooses p at date zero. Therefore, if the project is not to be liquidated at date one, then by choosing $p \in [0,1]$, the borrower expects to get

$$p(G - D) + (1 - p)\pi(B - D) = pG + (1 - p)\pi B - q(p)D, \tag{1}$$

where D is his repayment obligation and $q(p) \equiv [p + (1 - p)\pi]$ is the probability of repayment at p. The value p is not observable when it is chosen, but it becomes *publicly observable* at date one, so that covenants can be written against it.[7]

The contract design problem arises because the borrower may pursue a very risky strategy and shift the risk to the lenders. We make the following assumption of asset substitution:

$$(G - \alpha I) < \pi(B - \alpha I), \tag{P2}$$

where $\alpha \equiv G(I - L)/I(G - L) < 1$. Assumption ($P2$) states that even if the borrower has to finance only $\alpha I < I$, he prefers a risky project to a safe one. The motivation behind this assumption is further explained in Proposition 1 in the next section. In short, if assumption ($P2$) is not satisfied, there are contracts that can implement the efficient solution of $p = 1$ *without the*

[6] This assumption allows us to concentrate on debt contracts. If the actual cash flows are verifiable, issuing equity shares would completely eliminate the moral hazard problem. We will show how prioritized debt contracts can reduce effective debt burden by creating equity-like debt, namely, junior debt.

[7] What is assumed here is that the covenant variable p is under perfect control of the borrower. A more general setup would allow the variable p to be only partially controlled by the borrower. However, that would not affect any of our results qualitatively.

lenders having to monitor the borrower, and the converse is also true. Therefore, assumption ($P2$) is the necessary and sufficient condition for monitoring to play any role in deterring moral hazard.[8]

Because $0 < \alpha < 1$, our assumption ($P2$) is stronger than the usual assumption:

$$(G - I) < \pi(B - I), \qquad (P2')$$

which says that the borrower prefers a risky project even if he can borrow I at the risk-free rate. Because assumption ($P2$) is stronger, our model can be considered as dealing with cases where the moral hazard problem is severe. One way to see this is to look at assumption ($P2$) as a combination of ($P2'$) and the following assumption ($P2''$):

$$I \text{ is very close to } G. \qquad (P2'')$$

If I is very close to G, then $\alpha = G(I - L)/I(G - L)$ will be close to one, so that $\alpha I \simeq I$. By continuity, assumptions ($P2'$) and ($P2''$) together imply assumption ($P2$).

In the context of leveraged buyouts, assumption ($P2''$) describes a market situation where acquisitions are priced very tightly. If managed in a very safe way, the value of the acquired firm, G, is very close to the acquisition price I, so that most of the return would go to the lenders. Therefore, the acquirer has additional incentives to pursue a risky strategy, say, holding on to assets for too long or expanding recklessly. In other words, the asset substitution problem (Jensen and Meckling (1976)) described by assumption ($P2'$) is exacerbated by the underinvestment problem (Myers (1977)) of the safe project described by assumption ($P2''$). In this sense, our model under assumption ($P2$) deals with a situation where the borrower's moral hazard problem is severe.[9]

There are many investors. They will accept any contract that has nonnegative expected return. Once an investor becomes a lender, he gains access to a monitoring technology. However, *outsiders do not have such access*, which implies that incumbent lenders may be better informed than outsiders. The borrower will thus be faced with the ex post informational monopoly problem studied by Sharpe (1990) and Rajan (1992). At date zero, a lender chooses the *intensity* of his monitoring activities, denoted by $\lambda \in [0,1]$.

[8] If assumption ($P2$) is not satisfied, then two-class debt contracts can achieve the efficient solution $p = 1$ without monitoring. In this case, the senior claim must be smaller than the liquidation value L and will be refinanced at date one. The moral hazard problem is completely resolved without monitoring because the borrower is faced with high refinancing costs if he chooses a very low p. However, if assumption ($P2$) holds, the high refinancing cost is not enough to induce the borrower to choose a high p. In this sense, we are here dealing with a severe moral hazard problem. For details, see the companion paper Park (1996).

[9] Note that in our model, asset substitution occurs right after the financing is done. However, the model also has implications about situations where it is not certain when and whether asset substitution will occur. See Section V for details.

Then, at date one, he will learn the true project type with probability λ; with probability $(1 - \lambda)$, he will get no further information beyond what is publicly available, p. Monitoring of intensity λ costs λc dollars, where $c > 0$ is the unit monitoring cost.[10] Lenders choose their monitoring intensity without knowing the borrower's choice of p, and monitoring intensity λ is not verifiable. We also assume that at date one, it is common knowledge that the borrower knows whether or not each lender is informed of the true project type. This assumption is made for convenience and does not affect our analysis.

For lenders to carry out monitoring, monitoring should provide returns high enough to cover costs. We assume

$$(1 - \hat{p})(L - \pi B) > c, \tag{P3}$$

where $\hat{p}$ is defined by $q(\hat{p})G = I$ or $\hat{p} = (I - \pi G)/(1 - \pi)G$. If the borrower is expected to choose $p = \hat{p}$, then identifying and liquidating a risky project will yield a gain of $(1 - \hat{p})(L - \pi B)$, so that assumption ($P3$) states that the unit monitoring cost c is small relative to the return from monitoring at $p = \hat{p}$. If assumption ($P3$) is violated, lenders will not monitor for any $p \geq \hat{p}$. However, because the value $\hat{p}$ is the lowest p at which investors could possibly recover their initial investment I, no borrowing and lending occur unless the lenders expect the borrower to choose $p \geq \hat{p}$. Therefore, assumption ($P3$) is the necessary condition for economical monitoring in the relevant range of p.

B. Contracts and Renegotiation

In our framework, each contract induces a game to be played by the borrower and lenders, and the equilibrium of the game depends on the form of the contract. For example, suppose that the borrower issues long-term debt that matures at date two. If the contract specifies only the face value D, then no investor would accept the contract because the borrower will then choose $p = 0$, leaving only $\pi D (< I)$ for the lenders. The problem with such a contract is that lenders have no control over the borrower.

There are two ways to give some control rights to the lenders. One is to issue long-term debt with covenants attached. In our model, the value of p can be used to set up a covenant of the form $p \geq p^c$ in which lenders have the right to liquidate the project if and only if the observed p is below p^c. Another is to issue short-term debt, so that the loan has to be refinanced at date one. Because short-term debt is equivalent to long-term debt with the most restrictive covenant $p^c = 1$, we can safely consider only long-term debt with a covenant p^c and denote a contract by a pair (p^c, D).[11]

[10] For now, we assume all investors have the same monitoring costs. Implications of differences in the unit monitoring cost among investors will be discussed later.

[11] The point is that when we consider the *effective maturity* of debt, maturity cannot be discussed separately from covenants. If long-term debt has covenants more restrictive than short-term debt, it can be considered to have a shorter effective maturity.

Once a contract (p^c,D) is issued, the equilibrium of the game depends on how likely it is that renegotiation will occur at date one. If the covenant is to be enforced without renegotiation, the borrower will choose $p = p^c$ to avoid liquidation, and the efficient solution $p = 1$ can be implemented by setting $p^c = 1$. On the other hand, if the borrower believes renegotiation is very likely, he may not choose $p = p^c$, expecting the covenant to be waived. This paper assumes that *renegotiation and refinancing possibilities are limited only by the incentives of the parties concerned* and no exogenous restriction is imposed on them. Therefore, a covenant violation does not necessarily lead to liquidation of the project. By making this assumption, this paper attempts to show how to structure debt contracts when lenders cannot credibly commit to the no-renegotiation policy and covenants alone are not enough to prevent moral hazard.[12]

II. Equilibrium without Monitoring

Before proceeding to the model with monitoring, let us first see what happens if lenders do not monitor. Suppose that a contract (p^c,D) is issued. If the most restrictive covenant $p^c = 1$ is used, lenders have the right to force liquidation for all $p < 1$. However, they do not have the incentive to liquidate as long as $q(p)D \geq L$, where $q(p) \equiv [p + (1 - p)\pi]$ is the probability of repayment at p. As a result, the borrower can choose $\bar{p}$ such that $q(\bar{p})D = L$, have the covenant waived, and avoid liquidation. Because investors then get only L, they would not accept the contract (p^c,D) in the first place. More generally, we have the following proposition.

Proposition 1: *Suppose lenders do not monitor. If assumption* ($P2$) *holds, then there is no contract that allows borrowing and lending to occur in equilibrium. In particular, issuing multiple classes of debt would not help. If assumption* ($P2$) *does not hold, then there are contracts that can achieve* $p = 1$.

Proof: See the Appendix.

This proposition is the motivation behind assumption ($P2$). To see what is involved, rewrite assumption ($P2$) into

$$(G - I) < (1 - \underline{p})\pi(B - G), \tag{2}$$

[12] There are two reasons for our assumption. First, in reality, covenants are renegotiated and waived very often. It is not rare that bondholders renegotiate with borrowers (See Chen and Wei (1992) and Rosenberg (1992)). Second, theoretically, it is difficult to justify exogenous restrictions. It is one thing to show that renegotiation breaks down because of, say, a free-rider problem but another to assume that renegotiation does not occur. Another thing to note is that even when renegotiation is unlikely, the borrower may choose to violate the covenant if he believes that cheap refinancing will be available. That is why we have to consider both renegotiation *and* refinancing.

where $\underline{p}$ satisfies $q(\underline{p})G = L$ or $\underline{p} = (L - \pi G)/(1 - \pi)G$. The left-hand side is the most the borrower could get by playing the strategy $p = 1$. According to assumption ($P2$), this payoff $(G - I)$ is so small that the borrower would rather choose $p = \underline{p}$ and pay G. Because G is the highest possible face value, no contract would induce the borrower to choose $p = 1$ over $p = \underline{p}$. On the other hand, if assumption ($P2$) does not hold, $p = 1$ can be implemented by the contract under which the lender has the liquidation right for all p and the borrower pays I when $p = 1$ and G otherwise. In other words, assumption ($P2$) is necessary and sufficient for monitoring to play any role. Note here that the proof considers all feasible contracts, including contracts with face values contingent on p. This is necessary because renegotiation turns even a straight debt contract into a contingent contract.

III. Single-Class Debt with Monitoring

When lenders do not monitor, the borrower can avoid liquidation as long as $q(p)D \geq L$. When lenders monitor, a risky project could be identified and liquidated. Therefore, to reduce the probability of liquidation, the borrower chooses a higher p. This section shows that borrowing and lending occur when lenders monitor and then characterizes the optimal single-class debt contract. The results of this section will be used repeatedly to derive the optimal debt contract in later sections of this paper.

A. Single-Lender Case

We start with the simplest case where a single lender finances the whole project. Because seniority then would not matter, we assume that only one contract (p^c, D) is issued. We also assume that p^c satisfies $q(p^c)D > L$; contracts with $q(p^c)D \leq L$ are inferior (see the Appendix for a proof).

To analyze the game induced by the contract (p^c, D), we use *the perfect Bayesian equilibrium*, which requires, among other things, that beliefs be consistent with the equilibrium strategies and that each subgame be played in a sequentially rational manner. A given contract (p^c, D) generates a continuum of subgames, one for each combination of p and monitoring information. Outcomes of these subgames will determine the date zero equilibrium strategy pair (p, λ) and equilibrium payoffs. The borrower will then choose the contract that maximizes his equilibrium payoff. Therefore, to derive the optimal contract, we first have to know what happens in each subgame.

Consider first cases when the lender at date one learns that the project is safe. Recall that it is common knowledge that the borrower knows the lender's informational status. Because the lender has no incentive to liquidate the project, neither liquidation nor renegotiation will occur; if the lender demands renegotiation, the borrower will simply not respond. Therefore, for all values of p, the project will continue under the original contract when the lender knows the project is safe.

On the other hand, if the lender knows the project is risky, the project will be liquidated for all values of p. If the covenant is violated, or if $p < p^c$, this is obvious: the lender forces liquidation and gets L.[13] If the covenant is not violated, then the lender has to renegotiate with the borrower to have the project liquidated. Because both parties know the project is risky, and $(L - \pi B) > 0$, they will agree to liquidate the project and split the surplus $(L - \pi B)$. Let $Z(D)$ denote the renegotiated borrower's share of the liquidation value. Then, no matter what kind of a bargaining game they play, its outcome $Z(D)$ must satisfy

$$\pi(B - D) \leq Z(D) \leq (L - \pi D). \tag{3}$$

The inequalities come from individual rationality conditions.[14]

When the lender is uninformed, things are a bit more complicated because the borrower may try to reveal his project type in an attempt to affect the lender's decisions. However, that will not happen in equilibrium.

LEMMA 1: *Suppose the lender is not informed of the project type. Then, the borrower's information will not be revealed in equilibrium, and the lender's decisions will be based solely on the observed value of p. Specifically, if $q(p)D \geq L$, the project continues without renegotiation, and if $q(p)G < L$, the project is liquidated without renegotiation. If $q(p)D < L$ and $q(p)G \geq L$, renegotiation occurs, and the project continues under a new contract.*

We now explain this in detail. Essentially, what happens is that there is no equilibrium separating the two types of projects because one type of borrower will always mimic the other type. The lender therefore should not infer anything new from the borrower's behavior and should base his decisions solely on his own information, p.

If the covenant is not violated, or $p \geq p^c$, then it is a safe borrower who mimics a risky borrower. Because $(L - \pi B) > 0$, a risky borrower would seek to reveal the project type and liquidate the project under a new contract. Such a renegotiated liquidation will occur if and only if the lender is persuaded that the project is risky with a high enough probability. Suppose that the lender is somehow persuaded and agrees to give, say, Z' out of the liquidation value L. Then, $Z' \geq \pi(B - D)$ must hold. However, once the lender agrees to liquidate the project, a safe borrower will pretend to be risky and try to get the same deal Z' because $Z' > \pi(B - D) > (G - D)$. Therefore, the

[13] Due to the adverse selection problem, the borrower will not be able to refinance D from outsiders and avoid liquidation. Because the incumbent lender is better informed, a safe borrower finds it better to negotiate with him. Knowing this, outsiders, whenever they see the borrower in the public market at $p < p^c$, will have such a belief that it is impossible for the borrower to refinance D. For example, if $p < p^c$ and $q(p)D \geq L$, then outsiders believe that the project is risky with probability one.

[14] The amount $Z(D)$ does not depend on p because both parties are informed of the project type, making information contained in p irrelevant. Note that our analysis is robust to how we model the renegotiation process, and the exact functional form of $Z(\cdot)$ does not matter at all.

fact that the borrower proposes liquidation reveals no new information, and the lender's belief should stay the same at p. Because $q(p)D > L$ holds at $p \geq p^c$, the project will continue under the original contract.

If the covenant is violated, or $p < p^c$, then a risky borrower will always mimic a safe borrower to avoid liquidation. If $q(p)D \geq L$, an uninformed lender has no incentive to liquidate the project, and thus the project will continue under the original contract. If $q(p)D < L$, an uninformed lender has an incentive to liquidate the project, but there is room for renegotiation if $q(p)G \geq L$. Suppose the bargaining power of each party is such that the lender accepts $S(p,D)$ as the new face value and allows the project to continue. Then,

$$q(p)S(p,D) \geq L \quad \text{and} \quad S(p,D) \leq G \tag{4}$$

must hold for all p in this range, and the project continues under the new contract $S(p,D)$. If $q(p)G < L$, there is no room for renegotiation, and an uninformed lender will liquidate the project without renegotiation.[15]

The following summarizes our discussion so far and gives equilibrium payoffs of the borrower $U(\cdot)$ and of the lender $V(\cdot)$ under a single-lender debt contract (p^c, D). The critical values $\bar{p}$ and $\underline{p}$ are defined by $q(\bar{p})D = L$ and $q(\underline{p})G = L$, respectively.

$$U(p,\lambda) = \begin{cases} p(G-D) + (1-\lambda)(1-p)\pi(B-D) + \lambda(1-p)Z(D) & \text{if } p \geq p^c \\ p(G-D) + (1-\lambda)(1-p)\pi(B-D) & \text{if } \bar{p} \leq p < p^c \\ \lambda p(G-D) + (1-\lambda)\{p(G-S) + (1-p)\pi(B-S)\} & \text{if } \underline{p} \leq p < \bar{p} \\ \lambda p(G-D) & \text{if } p < \underline{p}, \end{cases} \tag{5}$$

$$V(p,\lambda) = \begin{cases} q(p)D + \lambda\{(1-p)(L - Z(D) - \pi D) - c\} & \text{if } p \geq p^c \\ q(p)D + \lambda\{(1-p)(L - \pi D) - c\} & \text{if } \bar{p} \leq p < p^c \\ q(p)S + \lambda\{(1-p)(L - \pi S) - p(S-D) - c\} & \text{if } \underline{p} \leq p < \bar{p} \\ L + \lambda\{p(D-L) - c\} & \text{if } p < \underline{p}. \end{cases} \tag{6}$$

[15] Suppose that the borrower does not know whether the lender is informed or not. Then the outcome of the game when $q(p)D < L \leq q(p)G$ will change because a lender who knows the project is safe may get $S(p,D)$ by pretending to be uninformed. However, the outcome will not change for other values of p. For example, if $q(p)D \geq L$ and $p < p^c$, the lender has an incentive to liquidate the project only when he knows the project is risky. Thus, the borrower will simply stay put and let the lender decide what to do. This will result in the same outcome as in the text. Because $q(p)D < L$ will not be observed in equilibrium, it does not matter whether the borrower knows the lender's informational status or not.

Using equations (5) and (6), we can now characterize the best contract when a single lender finances the whole project, which we call the optimal single-lender contract.

PROPOSITION 2 (The Optimal Single-Lender Contract): *Monitoring allows borrowing and lending to occur. There exists a unique optimal single-lender debt contract* (p^{c*}, D^*), *and the associated equilibrium strategy pair* (p^*, λ^*) *is also unique. They are characterized by*

$$p^{c*} = 1 \tag{7}$$

$$(1 - p^*) = c/(L - \pi D^*) \tag{8}$$

$$(1 - \lambda^*) = (G - D^*)/\pi(B - D^*) \tag{9}$$

$$p^* D^* + (1 - p^*)L - c = I. \tag{10}$$

The equilibrium payoff to the borrower is $(G - D^*)$ *under the optimal single-lender contract.*

Proof: See the Appendix.

Monitoring performs different functions at different values of p. When $q(p)D \geq L$, monitoring identifies and liquidates a risky project. When $q(p)D < L$, monitoring identifies and saves a safe project. The borrower wants to choose a low p value because it dilutes the value of the lender's claim. However, if p is so low that $q(p)D < L$ holds, then such a dilution will not occur either because of liquidation or because of renegotiation, which forces the borrower to raise the face value and pay at least L. Therefore, in equilibrium, the borrower chooses a p with $q(p)D \geq L$, and monitoring will serve as a deterrent of moral hazard. When lenders monitor, borrowing and lending occur because the borrower chooses a high enough p to avoid liquidation and renegotiation.

The two equations (8) and (9) are the familiar marginal conditions for an optimum. The borrower's equilibrium business strategy p^* is determined by the lender's incentive to monitor, which is represented by the marginal return from monitoring, $(1 - p)(L - \pi D) - c$. On the other hand, the equilibrium monitoring intensity λ^* is determined by the borrower's incentive to choose a risky strategy or by the marginal return from a risky strategy, $(G - D) - (1 - \lambda)\pi(B - D)$. Optimality requires these two expressions to be zero.

The optimal single-lender contract has the most restrictive covenant $p^{c*} = 1$, and the borrower violates it by choosing $p^* < 1$. Therefore, the lender then has the right to demand an immediate repayment at date one. One interpretation is that the optimal single-lender contract is equivalent to short-term debt that has to be refinanced at date one and that if a single lender finances the whole project, it is best to issue short-term debt.[16]

Because the covenant is going to be violated, it may seem odd that the covenant is set as high as $p^{c*} = 1$. One can show that if the covenant is low enough, the borrower will never violate it. However, because renegotiation is costless, a violation of the covenant is not a bad thing in itself; liquidation decisions are always made efficiently. The question is rather whether the covenant provides the lender with enough incentives to monitor. A contract with $p^c < 1$ is not optimal because it weakens the lender's incentive to monitor by giving too little to the lender when the project is liquidated. Furthermore, because liquidation gives the borrower $Z(D) > \pi(B - D) > (G - D)$, liquidation is not a punishment but a reward for the borrower; he wants to choose a risky strategy and get the project liquidated. In short, a contract with a less restrictive covenant $p^c < 1$ is not optimal because it distorts incentives.[17]

B. Many-Lender Case

Suppose now that there are N lenders of the same class, with lender i holding a contract (p_i^c, D_i), $i = 1, \ldots, N$. Let θ_i be the fraction of debt held by lender i, so that $D_i = \theta_i D$, where $D = \sum D_i$. In case of liquidation, lender i gets $\theta_i L$. For a given number of lenders N and for a given set of contracts $\{(p_i^c, D_i)\}_{i=1}^N$, we can construct a *single* lender holding a contract (p^c, D), where $p^c = \max_i p_i^c$ and $D = \sum_i D_i$. Call this single lender *the equivalent single lender*. In other words, the equivalent single lender is a single lender who holds all the contracts $\{(p_i^c, D_i)\}_{i=1}^N$.

Because renegotiation is costless and lenders are not going to have any contradictory information, the number of lenders would not matter as far as date one decisions are concerned. We therefore state the following without a formal proof.

[16] What is important is not whether $p^c = 1$ or not but whether or not the lender gets the control rights at date one. Therefore, contracts with $p^c < 1$ can also be considered as short-term debt if the borrower always violates it. In a more general setting where the variable p is not perfectly controlled by the borrower and there is interim cash flow at date one, the covenant could incorporate the probability of coupon payments being made at date one, with contracts with $p^c = 0$ being pure discount bonds with no other covenants attached. This interpretation of covenants as debt maturity is also pursued by Rajan and Winton (1995).

[17] The Appendix shows that if the covenant p^c is very close to but smaller than one, contracts will violate the lender participation constraint and thus are not acceptable. Contracts with $p^c = p^*$ are one such example. The reason for this is that when p^c is very close to one, the lender's incentive to monitor is so weak near $p = p^c$. Technically speaking, there is a discontinuity near $p^c = 1$ because of the function $Z(D)$, which disappears if $p^c = 1$.

LEMMA 2: *The following is the unique equilibrium outcome. If there is at least one lender who is informed of the project type, then lenders collectively make the same date one decisions as those of an informed equivalent single lender. If no lender is informed of the project type, then lenders make the same date one decisions as those of an uninformed equivalent single lender.*

In other words, *a single informed lender can induce others to agree to the optimal action for the informed lender.* This is the case because lenders of the same class have the same induced preferences. Consider, for example, the case where a p with $q(p)D > L$ is observed and lender i argues for liquidation. Because $q(p)D_i = q(p)\theta_i D \geq \theta_i L$, lender i would not argue for liquidation unless he knows the project is risky. Other lenders will infer this and agree to liquidate the firm. This is the basis of the uninformed lenders' beliefs that support the above outcome as an equilibrium. Also note that only the most restrictive covenant $p^c = \max_i p_i^c$ matters, and it does not matter who has the most restrictive covenant.[18]

To describe the equilibrium payoffs, we assume that *lenders' monitoring activities are independent of each other*, so that the event that lender i is informed is independent of the event that lender j is informed ($i \neq j$). Each lender chooses his monitoring intensity λ_i at the cost of $\lambda_i c$, where $i = 1,2,\ldots,N$. We use the same notation, λ, to denote *the aggregate monitoring intensity* or the probability that at least one lender is informed of the project type. Then, by definition,

$$(1 - \lambda) = (1 - \lambda_1)(1 - \lambda_2)\ldots(1 - \lambda_N). \tag{11}$$

Using Lemma 2 and results from the single-lender case, we have the following equilibrium payoff to lender i:

$$V_i(p, \boldsymbol{\lambda}) = \begin{cases} q(p)\theta_i D + \lambda\theta_i(1-p)(L - Z - \pi D) - \lambda_i c & \text{if } p^c \leq p \leq 1 \\ q(p)\theta_i D + \lambda\theta_i(1-p)(L - \pi D) - \lambda_i c & \text{if } \bar{p} \leq p < p^c \\ q(p)\theta_i S + \lambda\theta_i[(1-p)(L - \pi S) - p(S - D)] - \lambda_i c & \text{if } \underline{p} \leq p < \bar{p} \\ \theta_i L + \lambda\theta_i p(D - L) - \lambda_i c & \text{if } 0 \leq p < \underline{p}, \end{cases} \tag{12}$$

[18] Here we are ruling out the possibility of the borrower bribing the lender who has the most restrictive covenant p^c. The only way to bribe the lender and avoid liquidation is to raise the face value of that lender's claim. However, once the face value is raised, the lender still has incentive to liquidate because his share of the liquidation value also goes up. Therefore, a bribe will not work unless the lender signs a binding contract with the borrower. If such a contract is legal and enforceable, other lenders at the initial stage of financing will demand either a covenant restricting *all* new debt issue, or the same covenant p^c to block such a deal. Because bribery is not optimal ex ante (wrong liquidation decisions will be made), optimal contracts will have such provisions. Either way, we can safely rule out the possibility of the side payment.

where $\boldsymbol{\lambda} = (\lambda_1,\ldots,\lambda_N)$, $Z \equiv Z(\boldsymbol{D})$ is the borrower's share of the liquidation value in a renegotiated liquidation, and $S \equiv S(p,\boldsymbol{D})$ is the renegotiated total face value of debt, where $\boldsymbol{D} = (D_1,\ldots,D_N)$.[19] The borrower's equilibrium payoff is the same as in the single-lender case. We therefore make the following proposition.

PROPOSITION 3: *When a firm issues only one class of debt, it is best to have only one lender. Therefore, the optimal single-lender contract is the best unprioritized contract.*

Proof: See the Appendix.

To understand this proposition, rewrite the equation for $p \in [\bar{p}, p^c)$ to get

$$V_i(p,\boldsymbol{\lambda}) = q(p)\theta_i D + \lambda_{-i}\theta_i(1-p)(L-\pi D) + \lambda_i\{\theta_i(1-\lambda_{-i})(1-p)(L-\pi D) - c\}, \tag{13}$$

where $(1 - \lambda_{-i})$ is the probability that no lender other than lender i is informed of the project type. The terms in the bracket show that each lender's return from monitoring is lower than that of the equivalent single lender. First, the return is generated only when no other lenders are informed. Second, lender i will get only θ_i fraction of the return. The best contract with many lenders will delegate monitoring to a single lender, say, lender i. Then, $\lambda_{-i} = 0$ holds. However, even the delegated monitor has to share the return with other lenders and gets only $\{\theta_i(1-p)(L-\pi D) - c\}$ as his share of the return. The delegated monitor's incentive to monitor is maximized when his share of the return is maximized. Therefore, it is best when $\theta_i = 1$ or when there is only one lender.[20]

IV. Two-Class Debt Contracts

This section studies contracts with two seniority classes and characterizes the optimal two-class debt contract. The main result of this section is that two-class debt contracts dominate the optimal single-lender contract if there is a single senior lender who will be impaired in liquidation. Because he has a smaller stake in the project, an impaired senior lender has stronger incentive to liquidate the project and, therefore, a stronger incentive to

[19] The formula above assumes that lenders are treated proportionally when there is renegotiation. Thus, lender i gets $\theta_i(L - Z(\boldsymbol{D}))$ at $p \in [p^c,1]$ and $\theta_i S(p,\boldsymbol{D})$ at $p \in [\underline{p},\bar{p})$. This assumption is made for convenience and does not affect our results in any way.

[20] If the monitoring intensity λ is verifiable, lenders can devise a cost sharing scheme to allow the delegated monitor to get $\theta_i[(1-p)(L-\pi D) - c]$. In that case, the delegated monitor will have an incentive to monitor as strong as a single lender. Because λ is not verifiable, such a scheme is not feasible.

monitor. It is also shown here that putting a monitoring lender in the junior position would make things worse because of the informational monopoly problem. A short discussion of properties of the optimal two-class debt contract is also provided.

Suppose the borrower raises I by issuing one senior debt (p_S^c, D_S) and one junior debt (p_J^c, D_J). Let I_S and I_J, respectively, denote the amount raised from the senior lender and the junior lender, so that $I = I_S + I_J$. Denote $D = D_S + D_J$, the total face value, and $\boldsymbol{D} = (D_S, D_J)$, the face value pair. In this section, we assume that there is only one lender in each class and the two lenders are different. The next section will discuss the optimality of multiple lenders and cross holdings of debt.

As before, we have to calculate the equilibrium of the second period game determine the optimal two-class debt contract. What happens in that game depends both on the face values D_S and D_J and on the relative restrictiveness of the covenants p_S^c and p_J^c. There are two distinctive cases, depending on whether or not senior debt is impaired in liquidation: For each case, there are two subcases, depending on which of the covenants is more restrictive. Because cases with $p_J^c > p_S^c$ can be discussed independent of the size of the senior lender's claim, we have the following three cases: (i) unimpaired and restrictive senior debt ($D_S \leq L$ and $p_S^c \geq p_J^c$), (ii) restrictive junior debt ($p_J^c > p_S^c$), and (iii) impaired and restrictive senior debt ($D_S > L$ and $p_S^c \geq p_J^c$). We discuss them in that order.

A. *Unimpaired and Restrictive Senior Debt: $D_S \leq L$ and $p_S^c \geq p_J^c$*

In this case, senior debt is so small that the senior lender will get paid in full if the firm is liquidated under the original contract. Therefore, his optimal strategy is to demand liquidation or to demand full repayment of D_S in every state when $p < p_S^c$. As a result, senior debt has to be refinanced when $p < p_S^c$. Moreover, because his optimal strategy does not depend on his information, the senior lender has no incentive to monitor. Monitoring has to be carried out by the junior lender. *In short, when $D_S \leq L$* and $p_S^c \geq p_J^c$, senior debt is redundant, and the contract looks very much like a single-class debt contract with the junior lender acting as the only lender. We can prove the following proposition.

PROPOSITION 4: *Two-class debt contracts with $D_S \leq L$ and $p_S^c \geq p_J^c$ are not better than the optimal single-lender contract.*

Proof: Available from the author upon request.

Although the intuition behind this proposition is quite simple, the exact proof is rather complicated because we have to consider several options available to the borrower and the junior lender. Faced with the senior lender's repayment demand, the borrower can do one of the following: (i) make an exchange offer with the junior lender, (ii) directly renegotiate with the senior lender, or (iii) refinance D_S in the public market. Note that the last two options are basically the same thing because either way, the borrower is replacing old senior debt with new senior debt issued to lenders with no monitoring information.[21] On the other hand, the junior lender may (a) agree to do an exchange offer, (b) allow the borrower to raise D_S in the public market, or (c) buy all of senior debt himself.

Because no outside investor is informed of the project type, a safe borrower would find it better to negotiate with the junior lender. In contrast, a risky borrower would rather refinance senior debt from uninformed sources than face an informed junior lender. Therefore, a borrower is more likely to be in the public market when the junior lender knows the project is risky. This creates an adverse selection problem and makes it difficult for even a safe borrower to refinance senior debt from outside sources. As a result, the borrower is forced to deal with the junior lender and is faced with the informational monopoly problem discussed by Rajan (1992).

However, our result is quite the opposite of Rajan's. In his model, putting a monitoring lender in the junior position works better because it would put a limit on the monopoly rent the informed lender can extract from the borrower. The reason for the difference is that, unlike Rajan (1992), we endogenize a lender's incentive to monitor. Thus, a lender's incentive to monitor depends on the seniority of his claim.

The key to understanding our result is that when the informational monopoly problem is present, the borrower is punished both ways. If the project is risky, he will be punished. If the project is safe, he will again be punished with the monopoly rent he has to pay to the junior lender. Contracts with $D_S \leq L$ and $p_S^c \geq p_J^c$ are worse than a single-lender contract because a risky borrower is less likely to be punished and a safe borrower is more likely to be punished. Compared to a single lender, a junior lender is less likely to punish a risky borrower because the return from doing so will go to the senior lender first. A junior lender is more likely to punish a safe borrower because he appropriates the full monopoly rent whereas a single lender cannot get any monopoly rent; a single lender's threat to liquidate is not credible when he knows the project is safe.[22] To sum up, contracts with $D_S \leq L$

[21] If junior debt has a covenant restricting new senior debt issue, the last two options are available to the borrower only when the junior lender waives the covenant. If the junior contract does not have such a covenant, then the borrower may be able to refinance D_S and avoid liquidation even when the project is risky. That is a real possibility when the senior claim is very small. Because, ex ante, the borrower does not want wrong liquidation decisions to be made, the best contract among those with $D_S \leq L$ and $p_S^c \geq p_J^c$ will put such a covenant in the junior debt contract. See Park (1995) for a proof.

[22] A single lender has a credible threat to liquidate only when $q(p)D < L$ and the borrower does not know whether the lender is informed or not. However, because p with $q(p)D < L$ will not be observed in equilibrium, a single lender cannot extract any monopoly rent. Rajan (1992) does not discuss whether the single lender's threat to liquidate is credible or not. Instead, he assumes that to refinance D, the borrower holds an auction, in which outsiders may bid even when the incumbent lender chooses not to bid. In our model, outsiders will never bid when the informed lender does not.

and $p_S^c \geq p_J^c$ induce the borrower to choose a riskier strategy than single-lender contracts because such contracts provide the wrong incentives both for the borrower and the junior lender. This is the intuition behind the proposition.

B. More Restrictive Junior Debt: $p_J^c > p_S^c$

The problem with contracts with unimpaired restrictive debt seems to be that the senior lender has an excessive incentive to liquidate the project, forcing the borrower to refinance senior debt for all $p < p_S^c$. Therefore, one might conjecture that giving the senior lender a less restrictive covenant than the junior lender would work better. However, we can show that is not the case.

PROPOSITION 5: *Irrespective of the face values D_S and D_J, two-class debt contracts with $p_J^c > p_S^c$ are strictly worse than the optimal single-lender debt contract.*

Proof: Available from the author upon request.

To understand the logic behind this proposition, consider contracts with $p_S^c = 0$ and $D_S > L$; other cases can be dealt with similarly. If the project is liquidated under the original contract, the junior lender ends up with nothing. Therefore, he has no incentive to liquidate the project while he is the only lender who has the right to liquidate the project. Therefore, the only way for a senior lender to liquidate the project is to promise the junior lender more than πD_J out of the liquidation value L. In effect, renegotiation of this sort disrupts the seniority structure. The distinction between the senior lender and the junior lender is blurred, and the monitoring return will be shared by the two lenders. Therefore, contracts will look more like a single-class debt contract with two lenders, which is worse than the optimal single-lender contract.

It might seem that the senior lender is given too much protection; he is senior and at the same time has more restrictive covenants. However, as the proposition above shows, the seniority rule alone would not make a senior lender truly senior unless the covenant structure gives him the control rights. If the covenant structure is to disrupt the seniority structure, it would be better not to prioritize debt in the first place. In other words, *the covenant structure and, thus, the maturity structure must be set in conformity with the seniority structure.*

C. Impaired and Restrictive Senior Debt: $D_S > L$ and $p_S^c \geq p_J^c$

This is where we will find the optimal two-class contract. The way each subgame is played makes these contracts very similar to a single-lender contract. Therefore, we use the same method as in the single-lender contracts.

We first establish that *we can safely assume $p_J^c = 0$* and that the junior lender does not monitor at all under these contracts. To see why, note that because the junior lender gets nothing if the firm is liquidated under the original contract, he will seek to avoid liquidation as much as possible. Therefore, even if he is the only lender who knows the project is risky, he will not reveal that information; if convinced, the senior lender can and will unilaterally liquidate the project, leaving nothing for the junior lender. Similarly, when he knows the project is safe, the junior lender has no way to convince the senior lender of that because the junior lender wants even a risky project to continue. Because the junior lender has no use for the monitoring information, we can safely assume that the junior lender does not monitor. Furthermore, because the junior lender will not exercise his control rights, we can also assume that the junior debt contract has the least restrictive covenant $p_J^c = 0$.[23]

As in the single-lender contracts, we will first consider the case where the senior lender is informed of the true project type. When the project is safe, it will continue under the original contracts for all p. If the project is risky, then the project will be liquidated for all p. Liquidation occurs under the original contracts if $p < p_S^c$. If $p \geq p_S^c$, then the project will be liquidated after renegotiation because the senior lender has to seek the borrower's approval. Let $Z_i(\boldsymbol{D})$ be the renegotiated share of the liquidation value for party $i = S,J,B$. Then, no matter how we model the renegotiation game, the resulting outcome (Z_S,Z_J,Z_B) must satisfy

$$\pi D_k \leq Z_k \leq D_k, \qquad \pi(B - D) \leq Z_B, \qquad \text{and} \quad Z_S + Z_J + Z_B = L, \tag{14}$$

where $k = S,J$, and $D = D_S + D_J$. The inequalities $Z_k \leq D_k$ for $k = J,S$ a reconsistent with the impairment rule of the bankruptcy code.

Now, suppose that the senior lender is not informed about the project type. Lemma 1 of Section III applies here again, and the senior lender will make decisions based solely on his own information, namely, on p. If $q(p)D_S \geq L$, then an uninformed senior lender has no incentives to liquidate, and the project continues under the original contracts. If $q(p)G < L$, the senior lender has incentive to liquidate, and there is no room for renegotiation, so the project will be liquidated under the original contracts.

The case with $q(p)D_S < L$ and $q(p)G \geq L$ needs some explanation. Because $q(p)D_S < L$, an uninformed senior lender has an incentive to liquidate the project, but the borrower can avoid liquidation by raising the face value of the senior claim $(q(p)G \geq L)$. Suppose that the bargaining power of each player is such that the senior lender agrees to let the project continue in exchange for a new face value $S(p,\boldsymbol{D})$, where $\boldsymbol{D} = (D_S,D_J)$. Then, the new face value must satisfy

$$q(p)S(p,\boldsymbol{D}) \geq L \quad \text{and} \quad S(p,\boldsymbol{D}) \leq G. \tag{15}$$

[23] The only time the junior lender might have incentives to monitor is when he expects $p \in [p_S^c,1]$ to be played in equilibrium. However, because the optimal contract has $p_S^c = 1$, this is irrelevant.

Note here that because $S(p,\boldsymbol{D}) > D_S$, promising $S(p,\boldsymbol{D})$ amounts to issuing more senior debt. However, it does not matter whether the junior debt contract has a covenant prohibiting new senior debt. If it does, the junior lender has no other choice but to waive the covenant and allow the renegotiation to proceed.

Under the new contract, the borrower's total indebtedness is $T(p,\boldsymbol{D}) \equiv \min\{S(p,\boldsymbol{D}) + D_J, G\}$, and the junior lender gets $J(p,\boldsymbol{D}) \equiv \min\{D_J, G - S(p,\boldsymbol{D})\}$. Therefore, by waiving the covenant and allowing renegotiation to proceed, the junior lender is effectively making concessions because his claim is diluted when $D_J > G - S(p,\boldsymbol{D})$.

Besides renegotiation, there are two other ways to avoid liquidation: refinancing senior debt from outside investors or making an exchange offer to the junior lender. However, neither option will work because whenever the borrower tries either of the two options, both outsiders and the junior lender will believe the project is risky for sure. If they believe the project is safe with a positive probability $r > 0$, then the borrower will come to them only when the senior lender's belief is below r. Conditional on the senior lender's belief being lower than r, the correct belief of both outsiders and the junior lender must be lower than r. In other words, because the senior lender is the only lender who could be informed, all beliefs $r > 0$ are self-defeating. Once outsiders and the junior lender have the belief $r = 0$, it is impossible to raise D_S from them because $D_S > L > \pi G$. Even the junior lender, who has a vested interest in the project, will not accept any exchange offer because the most he can get from providing $D_S > L$ is $\pi G < L$.[24]

Summarizing our discussion, we now can describe equilibrium payoffs to the borrower $U(p,\lambda)$, and to the lenders–$V_S(p,\lambda)$ to the senior lender and $V_J(p,\lambda)$ to the junior lender. Because only the senior lender monitors, λ is the senior lender's monitoring intensity. The formulas below also use abbreviations such as $T \equiv T(p,\boldsymbol{D})$, $S \equiv S(p,\boldsymbol{D})$, and $J \equiv J(p,\boldsymbol{D})$. The critical values $\bar{p}_S$ and $\underline{p}$ are defined by $q(\bar{p}_S)D_S = L$ and $q(\underline{p})G = L$, respectively.

$$U(p,\lambda) = \begin{cases} p(G-D) + (1-\lambda)(1-p)\pi(B-D) + \lambda(1-p)Z_B & \text{if } p \geq p_S^c \\ p(G-D) + (1-\lambda)(1-p)\pi(B-D) & \text{if } \bar{p}_S \leq p < p_S^c \\ \lambda p(G-D) + (1-\lambda)\{p(G-T) + (1-p)\pi(B-T)\} & \text{if } \underline{p} \leq p < \bar{p}_S \\ \lambda p(G-D) & \text{if } p < \underline{p} \end{cases} \tag{16}$$

[24] Because the senior lender gets $q(p)S(p,\boldsymbol{D})$ from the renegotiation, the borrower may have to raise only $q(p)S(p,\boldsymbol{D})$, not D_S. Because $q(p)S(p,\boldsymbol{D}) \geq L$, raising $q(p)S(p,\boldsymbol{D})$ is not possible either.

$$V_S(p,\lambda) = \begin{cases} q(p)D_S + \lambda\{(1-p)(Z_S - \pi D_S) - c\} & \text{if } p \geq p_S^c \\ q(p)D_S + \lambda\{(1-p)(L - \pi D_S) - c\} & \text{if } \bar{p}_S \leq p < p_S^c \\ q(p)S + \lambda\{(1-p)(L - \pi S) - p(S - D_S) - c\} & \text{if } \underline{p} \leq p < \bar{p}_S \\ L + \lambda\{p(D_S - L) - c\} & \text{if } p < \underline{p} \end{cases} \tag{17}$$

$$V_J(p,\lambda) = \begin{cases} q(p)D_J + \lambda(1-p)(Z_J - \pi D_J) & \text{if } p \geq p_S^c \\ q(p)D_J - \lambda(1-p)\pi D_J & \text{if } \bar{p}_S \leq p < p_S^c \\ q(p)J - \lambda(1-p)\pi J & \text{if } \underline{p} \leq p < \bar{p}_S \\ \lambda p D_J & \text{if } p < \underline{p} \end{cases} \tag{18}$$

Contracts with $p_S^c < 1$ again are dominated by contracts with $p_S^c = 1$ because they weaken the senior lender's incentives to monitor and do not punish the borrower enough. Therefore, we assume $p_S^c = 1$ in what follows.

Note here that the borrower's payoff is the same as that of the single-lender contract, except for the facts that $\bar{p}_S$ replaces $\bar{p}$ and that we use different notations such as $Z_B(\boldsymbol{D})$ and $T(p,\boldsymbol{D})$. The senior lender's payoff is also similar to that of the single lender. Moreover, because the junior lender does not monitor, his payoff is irrelevant. Therefore, the equilibrium strategy pair (p^*, λ^*) will be quite similar to that of the single-lender contracts. For example, the borrower never chooses p with $q(p)D < L$ under the single-lender contract. Here, the borrower again chooses $p \geq \bar{p}_S$ even though he could dilute the junior claim by choosing $p < \bar{p}_S$. The reason is that dilution of the junior claim occurs only when the borrower makes the maximum payment or when $T(p,\boldsymbol{D}) = G$, and it thus would hurt the borrower also.

As explained in the case of the single-lender contract, each player's equilibrium strategy is determined by the other player's incentives. First, the borrower's equilibrium business strategy p^* will be determined by the senior lender's incentives to monitor. Given that the borrower chooses p in $[\bar{p}_S, 1)$, the senior lender's incentive to monitor is $(1-p)(L - \pi D_S) - c$, the marginal return from monitoring at $p \in [\bar{p}_S, 1)$. At the equilibrium value p^*, the marginal return must be zero. Similarly, the senior lender's optimal monitoring intensity λ^* is determined by equating the marginal return of a risky strategy at $p \in [\bar{p}_S, 1)$ to zero or by the relation $(G - D) = (1 - \lambda^*)\pi(B - D)$. Thus, the equilibrium strategy pair (p^*, λ^*) satisfies

$$(1-p^*)(L-\pi D_S)=c. \tag{19}$$

$$(1-\lambda^*)=(G-D)/\pi(B-D), \tag{20}$$

and the borrower's equilibrium payoff is $(G-D)$.[25]

Because the borrower enjoys $(G-D)$ in equilibrium, he will choose a contract that minimizes the total face value $D=D_S+D_J$. Using this and the equation $(1-p^*)(L-\pi D_S)=c$, we can rewrite the lender participation constraint as the following equalities:

$$p^*D_S+(1-p^*)\pi D_S=I_S, \tag{21}$$

$$p^*D_J+(1-\lambda^*)(1-p^*)\pi D_J=I-I_S. \tag{22}$$

Equations (19)–(22) allow us to express $D=D_S+D_J$ as a function of the amount raised by senior debt, I_S. By minimizing D by the choice of I_S, we get the following characterization of the optimal two-class debt contract.

PROPOSITION 6 (Optimal Two-Class Debt Contract): *There exists a unique optimal two-class contract* $\{(p_S^{c*},D_S^*),\ (p_J^{c*},D_J^*)\}$. *It is characterized by*

$$p_S^{c*}=1, \tag{23}$$

$$I_S^*=L, \tag{24}$$

$$(D_S^*-L)(L-\pi D_S^*)=(1-\pi)cD_S^*, \tag{25}$$

$$(B-D_S^*-D_J^*)(D_J^*-I+L)(L-\pi D_S^*)=(B-G)cD_J^*, \tag{26}$$

where D_S^* *and* D_J^* *are the smallest values satisfying equations* (25) *and* (26). *Under the optimal contract, the borrower's payoff is* $(G-D^*)$. *Furthermore, (i) it does not matter whether or not the junior debt contract has a covenant restricting new senior debt issue, nor does the value* p_J^c *matter*; *and* (*ii*) *the optimal two-class debt contract strictly dominates the optimal single-lender contract.*

Proof: See the Appendix.

[25] Although there are other equilibria of the game, they are easily eliminated as unreasonable by the forward induction criterion of Kholberg (1989) and the noise-proofness criterion of Carlsson and Dasgupta (1997). However, when $I_S=L$, the equilibrium λ is not unique, and any $\lambda\in[\bar{\lambda},\lambda^*]$ is an equilibrium for some $\bar{\lambda}>0$; see the Appendix for details. We choose to discuss only λ^* for two reasons. First, contracts whose prices are based on $\lambda'<\lambda^*$ will not be accepted by the junior lender if he believes the senior lender will choose $\lambda\in(\lambda',\lambda^*]$. Second, because the borrower's payoff function is decreasing in λ, the borrower's payoff $(G-D)$ that is generated by λ^* is the worst he would get. Because our purpose is to show that the above contracts are better than others, the choice of λ^* as an equilibrium only strengthens our result.

Under the optimal two-class debt contract, the senior lender contributes L, the full liquidation value, which is the smallest amount that will keep him impaired by liquidation. Equations (25) and (26) determine the optimal face value of each debt and are simply equations (21) and (22) with $I_S = L$ and equations (19) and (20) plugged in.

As can be seen from equations (21) and (22), *senior debt is cheaper than junior debt*. The interest rate charged on junior debt is $1/[q(p^*) - \lambda^*(1-p^*)\pi]$, whereas the rate on senior debt is only $1/q(p^*)$. Junior debt is more expensive because monitoring could make junior debt worthless by letting liquidation occur.

Even though senior debt is cheaper, using *less* senior debt is better. As more senior debt is used, the senior lender's incentive to monitor gets weaker, and costs of *both* classes of debt will rise. This is the case because a bigger contribution from the senior lender would increase the size of his claim and, therefore, his share of the going concern value of the project. Because what he gets from liquidation does not change, the senior lender will thus be more reluctant to liquidate the project and less likely to monitor. In other words,

$$\frac{\partial D}{\partial I_S} > 0 \tag{27}$$

holds. The characterization of the optimal two-class contract is an immediate consequence of this.

Because the optimal single-class debt contract corresponds to the case where $I_S = I$, inequality (27) also proves that not just the optimal two-class contract but *any* two-class contract with $D_S > L$ and $p_S^c = 1$ dominates the optimal single-lender contract. Two effects are at work here. First, an impaired senior lender has stronger incentives to monitor than a single lender because he has stronger incentives to liquidate the project. Second, as long as $I_S < I$, $q(p)D_S < L$ holds whenever $q(p)D \le L$, and it can happen that $q(p)D_S < L$ holds but not $q(p)D < L$. In other words, even when they are not informed of the true project type, an impaired senior lender has incentives to liquidate the project more often and earlier than a single lender. Both effects force the borrower to choose a higher p than under the optimal single-lender contract.

Under the optimal two-class contract, *only the senior lender monitors, and the senior debt contract has the most restrictive covenant,* $p_S^{c*} = 1$. Another characteristic of the optimal two-class contract is the following: *Only the senior lender will be involved in renegotiation with the borrower, whereas the junior lender is in no position to bargain with the borrower*. In other words, at some observations of p, there will be a senior lender–borrower coalition, which will sometimes do harm to the junior lender through dilution of the junior claim. This is the setup used by Bulow and Shoven (1978), and our theory explains why this is what we should expect.

Moreover, *under the optimal two-class contract, it does not matter whether renegotiation with junior lenders is very difficult or not.* If renegotiation with junior lenders is costly, the borrower will simply give the junior lender the least restrictive covenant, $p_J^c = 0$, and no covenant prohibiting new senior debt issue. One testable implication of our theory is thus that whether or not the junior claim has these covenants should not affect the pricing of junior debt (net of renegotiation costs). Brealey and Myers (1996) note that during the 1980s most bond issues did not have covenants restricting new debt issue. Diamond (1993a) makes a similar observation.

As for the face value D^* of the optimal contract, note that at optimum the borrower gets $(G - D^*)$. Because $(G - I)$ is what he is going to get if there is a commitment mechanism ensuring the choice of $p = 1$, the difference $(D^* - I)$ represents *agency costs* of debt. Rewrite equations (21) and (22) using equation (19) and get an expression

$$(D^* - I) = \lambda^* c + (1 - p^*)\{D^* - [\lambda^* L + (1 - \lambda^*)\pi D^*]\}. \tag{28}$$

Therefore, agency costs have two components: the monitoring costs $\lambda^* c$ and *a default premium*. The second term is the expected loss to the lender due to a low choice of p; if $p^* = 1$, then it disappears. The optimal face value D^* is the sum total of the minimum required return, the monitoring costs, and the default premium for the lender.

V. Optimal Debt Structure and Implications

A. Optimal Debt Structure

We now discuss cases where there are more than two classes of debt. Suppose that there are n lenders with lender i holding the ith most senior debt of face value D_i, $i = 1,2,\dots,n$. When $D_1 \le L$, let k be the largest integer with $D_1 + \cdots + D_{k-1} \le L$. Then, the first $(k - 1)$ lenders have no incentive to monitor, and we can consider them as belonging to the same (most senior) class with a total face value of $D_S \equiv D_1 + \cdots + D_{k-1}$. Because lenders $i > k$ also have no incentive to monitor (they get nothing in liquidation), it is lender k who has to monitor and refinance D_S if necessary. In other words, this type of contract is equivalent to contracts with $D_S \le L$ and thus is dominated by the optimal two-class contract. If $D_1 > L$, then lender one monitors, and it does not matter how many classes of junior debt are issued. Because this holds true for all n, we have the following proposition.

PROPOSITION 7 (Optimal Number of Seniority Classes): *Issuing the optimal two-class debt contract is enough. Issuing more than two classes cannot do any better than the optimal two-class debt contract.*

So far, we have assumed that there is only one lender in each class. It must be obvious now that the number of lenders in a given class matters only when monitoring is carried out by a lender of that class. Therefore, the number of junior lenders does not matter because they do not monitor under the optimal contract. Moreover, as Proposition 3 of Section III indicates, it is best to make the monitoring lender the only lender in his class. Because only the senior lender monitors under the optimal debt structure, it is best to have only one senior lender.

PROPOSITION 8 (Optimal Number of Lenders): *It is best to have only one senior lender whereas the number of junior lenders does not matter. Therefore, junior debt can be widely held.*

The monitoring costs in our discussion are those that the contract itself has to cover. If investors have other ways to be compensated for some of their monitoring costs, then those costs are not included in our costs. For example, as emphasized by Black (1975), if a bank gets some information about a borrower by managing his deposit accounts, then costs of getting that information will be charged on the deposit accounts, and the debt contract does not have to cover those costs. Therefore, even if investors have the same monitoring technology, financial intermediaries such as banks would have some advantage over others because they are already in the business of monitoring potential borrowers and part of the monitoring costs are incurred outside the contract. Therefore, we have the following proposition.

PROPOSITION 9: *Financial intermediaries will hold senior debt, whereas junior debt can be held widely.*

This combination of senior bank loan and junior public debt is the most commonly observed pattern of debt finance. In most leveraged buyouts (LBOs), banks or other financial intermediaries provide senior credit, and the rest of the financing is provided by others. Our theory predicts that the senior lender will be the one who has an ongoing relationship with the borrower.

Note also that our theory is in line with Diamond's theory of financial intermediaries as delegated monitors (Diamond (1984)). Here, banks are delegated monitors for the junior lenders in the sense that it is the senior lender's monitoring that allows borrowing and lending to occur in the first place. However, once the contracts are in place, junior lenders do not want the senior lender to monitor because monitoring raises the chances of the firm being liquidated. Therefore, *although the senior lender acts like a delegated monitor for the junior lender, there is no need for the junior lender to monitor the delegated monitor.*

The discussion above makes it clear that the optimal debt contract drives a wedge between the senior and junior lender. Therefore, it is necessary that the two lenders are different. If the senior lender holds some junior debt as well, then his monitoring incentives will be weaker, and the borrower has to pay higher financing costs.

PROPOSITION 10: *It is never optimal for a senior lender to hold part of the junior debt. If a bank is the senior lender to a firm, then the bank will not hold any junior debt contracts.*

This suggests that even when they are allowed, banks will not be holding junior securities such as subordinated debt or equities. Even though we do not know of any empirical studies on this, casual observations indicate that it is at best rare that a senior lender also buys subordinated debt at the initial financing stage. One might consider the prevalent use of strip financing in the early 1980s as a counterexample of this prediction. However, as reported in Kaplan and Stein (1993), most strip financing takes the form of subordinated debtholders buying equity, and it is rare that the senior lender provides equity capital. One testable implication of our theory is that junior lenders buying equity will not affect the terms of debt contracts because the senior lender's monitoring incentives will not change.

B. Debt Maturity Structure

One way to interpret the covenant p^c is in terms of debt maturities. The most restrictive covenant $p_S^{c*} = 1$ gives the senior lender the right to force liquidation, except for the case when $p = 1$ is observed. Because the borrower always violates this covenant in equilibrium, senior debt in our model is effectively short-term debt maturing at date one. On the other hand, junior debt in our model can be considered as long-term debt in the sense that its loan covenants do not matter and junior lenders never get to exercise their control rights even if they have such rights.[26]

PROPOSITION 11: *Senior debt will be short term and junior debt will be long term.*

Therefore, financial intermediaries will hold short-term senior debt whereas the public will hold long-term junior debt. Diamond (1993a, 1993b) makes the same prediction. Consistent with our theory, Tufano (1993) finds that in the acquisition financing done in the late 1980s, bank loans had a median maturity of 48 months, whereas public debt had a median maturity of 120 months.[27]

In our model, asset substitution occurs right after the debt agreement is signed, and short-term debt matures immediately after that. In reality, lenders may not be certain when and whether asset substitution will occur, and short-term debt may mature *before* asset substitution becomes a real possibility. For these cases, the theory predicts that the initial short-term debt will be so large that whenever it is repaid in full, the possibility of asset substitution is significantly reduced. In other words, short-term debt is not

[26] Because our model is a simple two-period model, it cannot distinguish between covenants and maturities. For example, our model does not allow for long-term debt with more restrictive covenants than short-term debt. However, from our analysis, one can easily deduce that such a long-term debt contract is never optimal. Because covenants and maturities are set to maximize the senior lender's incentives to monitor, senior debt will have the most restrictive covenants *and* the shortest possible maturity. Otherwise, junior long-term debt will have a shorter *effective maturity*, and the seniority structure will be destroyed.

[27] We do not argue that the control aspect completely determines debt maturity choice. Diamond (1991, 1993a) shows that informational sensitivity is another determinant of debt maturity. For an excellent survey on both theoretical and empirical fronts, see Ravid (1996).

meant to be repaid but to be rolled over. That way, junior lenders can still count on senior short-term lenders monitoring the borrower when asset substitution becomes a real possibility.[28]

C. Comparative Statics

Suppose that borrowers differ in their investment requirement I, whereas other parameters are the same. A borrower with a higher I is riskier because the closer I gets to G, the more likely the borrower is to choose a risky strategy. In the context of an LBO, the variable I, with G and B being fixed represents the price/cash flow ratio Kaplan and Stein (1993) use. Therefore, a borrower who pays a higher price I per one dollar of cash flows has greater incentives to pursue risky strategies. However, this increase in risk would affect neither the amount I_S raised by senior debt nor the rate charged on senior loans because both I_S and the rate are determined solely by the size of the liquidation value L.

PROPOSITION 12: *The riskiness of the borrower defined by the initial investment I affects neither the amount raised by senior debt nor the terms of the senior debt contract. However, as I increases, more junior debt is used, and junior debt becomes more expensive.*

As the investment requirement I increases, the senior lender's incentive to monitor does not change; his contribution and face value are determined solely by the size of the liquidation value. Because the borrower's choice of p depends on the monitoring incentives of the senior lender, the equilibrium p value will not be affected by an increase in I. On the other hand, the senior lender's *actual* monitoring intensity depends on the borrower's incentive to choose a risky strategy, which is stronger due to an increase in I. Therefore, the senior lender has to carry out more monitoring to maintain the same level of p. Note here the distinction we make between monitoring incentives and the actual monitoring (see footnote 3). This makes junior debt more expensive because liquidation is now more likely; higher λ^* is chosen at the same p^*. Moreover, because the amount I_S raised by senior debt stays the same, junior lenders have to contribute more.

In the context of LBO transactions, an increase in I occurs when the acquisition price goes up relative to earnings potential. Kaplan and Stein (1993) report that in LBOs done in the late 1980s, senior debt pricing did not show much sensitivity to earnings potential. Although they interpret this evidence as suggesting an overheated market, our analysis suggests a different inter-

[28] Repayment of large short-term debt reduces the chances of asset substitution because the company now has a smaller amount of debt. Moreover, only well-performing borrowers will be able to do that, making asset substitution less likely. Sinking fund provisions, for example, can be used to force the company to use cash flows to reduce long-term debt and not to repay short-term debt in full unless the company has huge cash inflows. If retiring short-term debt is not feasible, the borrower's performance is not stellar, and short-term lenders will have to stay and monitor the borrower.

pretation. Consistent with our theory, Kaplan and Stein (1993) also find that as the buyout prices increased, more junk bonds were used than before. The same prediction is obtained by Diamond (1993a) via a different route.

PROPOSITION 13: *As the liquidation value increases, more senior debt will be used. Both senior debt and junior debt become cheaper, and the total face value decreases.*

Proof: See the Appendix.

When the liquidation value L increases by \$1, the senior lender contributes \$1 more and gets \$1 more when he liquidates the project. However, if the senior lender allows the project to continue, his payoff will increase by less than \$1 because of the risk. Therefore, as L increases, the senior lender has a stronger incentive to liquidate and thus to monitor, and the borrower will choose a higher p^*. That, in turn, lowers the interest costs of both debt and the borrower's total indebtedness. Our prediction is consistent with the findings of Alderson and Betker (1995), who find that firms with a smaller liquidation value relative to the going concern value (higher liquidation costs) use more public, unsecured debt.

VI. Conclusion

The main idea of this paper is that it is best to delegate monitoring to a senior lender. The observed structure of debt contracts can be explained as a way to maximize the senior lender's incentives to monitor. Seniority, covenants, and maturities are all chosen to accomplish that objective. We have also shown that having junior lenders monitor the borrower is not as effective as a senior lender carrying out monitoring because of distorted incentives.

According to our theory, what is required of a senior lender is expertise in gathering information about the borrower and making the right liquidation decisions. The role of banks as a senior lender is to be tough on the borrower ex post when things go wrong, so that the moral hazard problem can be prevented ex ante. Supposedly, borrowing from banks has the advantage of easy renegotiation. However, we seldom see banks forgive debt out of renegotiation. Although regulations may explain part of the bank's behavior, our theory suggests their actions are indeed the way they are meant to be.

On the other hand, junior lenders are in a very weak bargaining position. They stand to lose a lot if the project is liquidated. Therefore, if concessions ever occur, they are likely to come from junior lenders. The combination of a tough senior lender and soft junior lenders makes it more convenient and easy for the borrower to form a coalition with the senior lender. This is the presumption of Bulow and Shoven (1978) and is consistent with findings of Asquith, Gertner, and Scharfstein (1994).

If the borrower controls only the probability distribution of the covenant variable p, all the $p \in [0,1]$ are observable even though p near p^* will be observed more often. Under this broad interpretation, our theory also sheds light on the renegotiation outcome of financially distressed firms. For example, our theory suggests that the senior lender will be better informed than junior lenders, so that total restructuring of debt involving both classes of lenders will be difficult because of the informational asymmetry. This point is consistent with the findings of Asquith et al. (1994). The informational asymmetry would also make it very difficult for a financially distressed firm to refinance even part of its debt in the public market.

We assume that the liquidation value is known in advance. If the liquidation value is stochastic, the same analysis goes through. However, if the liquidation value fluctuates over time, then the size of senior short-term debt has to adjust accordingly. How that is achieved will be an interesting extension of the model. Another possible extension of the model is to allow for investment decisions at date one. If the borrower can take some actions at date one, we might see some debt forgiveness by lenders.

Our theory is in line with the recent research program on debt contracts, namely, *the control aspect of debt*. When renegotiation is possible, exercise of the control rights becomes more subtle because covenants may be waived and may not be enforced because of lack of incentives on the lender's part. However, our analysis shows that properly structured debt contracts can effectively control the borrower's moral hazard problem. Although our theory also touches upon problems such as debt maturity structure and informational monopoly, further research is needed.

Appendix

Proof of Proposition 1: Fix (p^0, D^0) where $I \leq D^0 < G$ and consider the following contract. For all p, the lender has the right to liquidate the project and gets L if he liquidates the project. The contract has a face value contingent on p that is given by

$$D(p) = \begin{cases} D^0 & \text{when } p = p^0 \\ G & \text{when } p \neq p^0. \end{cases} \tag{A1}$$

This contract is designed to give the borrower the strongest possible incentives to choose $p = p^0$ for some prescribed p^0. If this contract does not induce the borrower to choose $p = p^0$, then no other contract will do so. It is easy to show that as long as $q(p^0)D^0 \geq L$, this contract is renegotiation proof and the borrower's payoff will be

$$U(p) = \begin{cases} p^0(G - D^0) + (1 - p^0)\pi(B - D^0) & \text{when } p = p^0 \\ (1 - p)\pi(B - G) & \text{when } p \geq \underline{p} \text{ and } p \neq p^0 \\ 0 & \text{when } p < \underline{p}, \end{cases} \tag{A2}$$

where $\underline{p}$ satisfies $q(\underline{p})G = L$. If p^0 is chosen as prescribed, the lender gets $q(p^0)\bar{D}^0$. To give the lender the required return, $q(p^0)D^0 \geq I$ must hold. If assumption ($P2$) is satisfied, then

$$\begin{aligned} U(p^0) &= p^0G + (1-p^0)\pi B - q(p^0)D^0 \\ &\leq (G-I) < (1-\underline{p})\pi(B-G) = U(\underline{p}), \end{aligned} \tag{A3}$$

and the borrower chooses $\underline{p}$ over p^0, leaving only L for the lender. Therefore, no investor would accept the contract. Because this holds true for any (p^0,D^0) with $q(p^0)D^0 \geq I$, no contract, including multiple-class debt contracts, will be acceptable. On the other hand, if assumption ($P2$) does not hold, then the above contract with $(p^0,D^0) = (1,I)$ can implement $p = 1$. Q.E.D.

Proof of Proposition 2: Because contracts with $p^c = 1$ are very different from contracts with $p^c < 1$, we first find the equilibrium outcome of each contract. First recall that $\bar{p}$ and $\underline{p}$ are defined by $q(\bar{p})D = L$ and $q(\underline{p})G = L$, respectively.

CLAIM 1: *If $p^c > \bar{p}$, then for all values of λ, the borrower's optimal choice of p lies in $[\bar{p},p^c]$. If $p^c \leq \bar{p}$, then the borrower chooses $p = p^c$.*

Proof of Claim 1: Obviously p^c dominates all $p \in [0,\underline{p})$. Because $Z(D) \geq \pi(B-D)$, $U(p,\lambda)$ is strictly decreasing in p when $p \in [p^c,1]$, so that p^c also dominates $p > p^c$. Suppose $p^c \geq \bar{p}$. Then, $q(p)S(p,D) \geq L = q(\bar{p})D$ and $G > \pi B$ imply the following: for all $p \in [\underline{p},\bar{p})$ and $\lambda \in [0,1]$,

$$\begin{aligned} U(p,\lambda) &= \lambda p(G-D) + (1-\lambda)\{[pG + (1-p)\pi B] - q(p)S(p,D)\} \\ &< \lambda\bar{p}(G-D) + (1-\lambda)\{[\bar{p}G + (1-\bar{p})\pi B] - q(\bar{p})D\} = U(\bar{p},\lambda), \end{aligned} \tag{A4}$$

so that $\bar{p}$ dominates all $p \in [\underline{p},\bar{p})$. In sum, if $p^c \geq \bar{p}$, only $p \in [\bar{p},p^c]$ can be optimal. Similarly, if $p^c \leq \bar{p}$, then p^c dominates all $p \in [\underline{p},p^c)$. Therefore, if $p^c \leq \bar{p}$, only p^c is optimal. Q.E.D. of Claim

Definition: Within each range of p, the function $V(p,\lambda)$ is linear in λ. The coefficient of λ is a function of p and is denoted by $m(p)$. For example, $m(p) = (1-p)(L-\pi D) - c$ when $p \in [\bar{p},p^c)$. We call this function $m(\cdot)$ *the incentive-to-monitor function.*

CLAIM 2: *A contract with $p^c = 1$ is acceptable only if $m(\bar{p}) > 0$. Each acceptable contract $(1,D)$ has a unique equilibrium (p^*,λ^*) such that $(1-p^*) = c/(L-\pi D)$ and $(1-\lambda^*) = (G-D)/\pi(B-D)$. The best contract with $p^c = 1$ has a face value D^* as the smaller of the two solutions to the equation*

$$f(D) = (D-I)(L-\pi D) - (1-\pi)cD = 0. \tag{A5}$$

The borrower's equilibrium payoff from the contract $(1,D^*)$ is $(G-D^*)$.

Proof of Claim 2: For any face value D, each player's optimal response function is as follows:

$$p(\lambda) = \begin{cases} 1 & \text{if } \lambda > \lambda^* \\ \text{any } p \in [\bar{p},1] & \text{if } \lambda = \lambda^* \\ \bar{p} & \text{if } \lambda < \lambda^*, \end{cases} \qquad \lambda(p) = \begin{cases} 1 & \text{if } m(p) > 0 \\ \text{any } \lambda \in [0,1] & \text{if } m(p) = 0 \\ 0 & \text{if } m(p) < 0 \end{cases}, \tag{A6}$$

where $(1 - \lambda^*) = G - D/\pi(B - D)$. Suppose $m(\bar{p}) \leq 0$. Then, $m(p) < 0$ and $\lambda(p) = 0$ for all $p > \bar{p}$, which implies $p > \bar{p}$ is not an equilibrium. Therefore, the only equilibrium occurs at $\bar{p}$ and satisfies $\lambda m(\bar{p}) = 0$. Because the lender then gets only $V(\bar{p}) = q(\bar{p})D + \lambda m(\bar{p}) = L$, contracts with $m(\bar{p}) \leq 0$ are not acceptable.

Now, suppose $m(\bar{p}) > 0$. Then, $\lambda(\bar{p}) = 1$ holds, and thus $\bar{p}$ is not an equilibrium. Neither is $p = 1$ because $\lambda(1) = 0$. Therefore, the only equilibrium is (p^*, λ^*), where $p^* \in [\bar{p},1]$ and satisfies $m(p^*) = 0$ so that $\lambda(p^*) = \lambda^*$. Because the borrower gets $(G - D)$ in equilibrium, he will choose the smallest D with $q(p^*)D \geq I$. Rewriting the equation $q(p^*)D = I$ using $m(p^*) = 0$ gives us equation (A5). Because $f(I) < 0$, and $f(G) > 0$ by the assumption ($P3$), the smaller solution to the equation $f(D) = 0$ lies in (I,G).

Q.E.D. of Claim

CLAIM 3: *A contract with $p^c < 1$ is acceptable only if $m(p^c) \geq 0$ holds and the borrower chooses $p = p^c$. The maximum he can get from a contract with $p^c < 1$ is*

$$U^C \equiv p^c G + (1 - p^c)L - (I + c). \tag{A7}$$

Proof of Claim 3: Suppose $m(p^c) < 0$. If $p^c \leq \bar{p}$, the borrower chooses p^c for all λ. Because the lender then gets only $V(p^c, \lambda) = q(p^c)D + \lambda m(p^c) \leq q(p^c)D \leq q(\bar{p})D = L$, he will not accept the contract. If $\bar{p} < p^c < 1$, the borrower chooses p^c only if $\lambda \leq \bar{\lambda}$, where $\bar{\lambda}$ satisfies $U(p^c, \bar{\lambda}) = U(\bar{p}, \bar{\lambda})$; otherwise, he chooses $\bar{p}$. Because $m(p^c) < 0$ implies $\lambda(p^c) = 0$, the only equilibrium is $\bar{p}$ with $m(\bar{p}) \leq 0$. Then again, the lender gets less than L, and the contract is not acceptable. Thus, an acceptable contract must have $m(p^c) \geq 0$.

Therefore, the borrower chooses $p = p^c$ in equilibrium and gets

$$\begin{aligned} U(p^c, \lambda) &= \{p^c G + (1 - p^c)(1 - \lambda)\pi B + (1 - p^c)\lambda L\} - (I + \lambda c) \\ &\leq \{p^c + (1 - p^c)\pi B\} + \lambda\{(1 - p^c)(L - \pi B) - c\} - I \\ &\leq p^c G + (1 - p^c)L - (I + c) \equiv U^C, \end{aligned} \tag{A8}$$

where the last inequality comes from

$$(1-p^c)(L-\pi B)-c \geq (1-p^c)(L-\pi D-Z(D))-c \equiv m(p^c) \geq 0 \qquad \text{(A9)}$$

because $Z(D) \geq \pi(B-D)$. This proves the claim. Q.E.D. of Claim

CLAIM 4: *The contract* $(1, D^*)$ *dominates all the acceptable contracts with* $p^c < 1$.

Proof of Claim 4: The contract $(1, D^*)$ gives the borrower

$$U^* \equiv p^*G + (1-p^*)(1-\lambda^*)\pi B + (1-p^*)\lambda^* L - (I + \lambda^* c), \qquad \text{(A10)}$$

where p^*, λ^*, and D^* are defined in Claim 2. Take the difference, and get

$$(U^* - U^c) \geq (p^* - p^c)(G-L) - (1-\lambda^*)[(1-p^*)(L-\pi B)-c]. \qquad \text{(A11)}$$

Because $(1-p^*)(L-\pi D^*) = c$, we have $(1-p^*)(L-\pi B)-c < 0$. On the other hand, from equation (A9), $(1-p^c)(L-\pi B)-c \geq 0$. Thus, $p^* > p^c$ holds, and so does $U^* > U^C$. Q.E.D. of Claim

Proof of Proposition 3: We can safely ignore contracts with $p^c = \max_i p_i^c < 1$ because they are dominated by contracts with $p^c = 1$. Suppose there are n lenders $i_1, i_2, \ldots, i_n$ monitoring the borrower in equilibrium and let $M = \{i_1, i_2, \ldots, i_n\}$ be the set of monitoring lenders. As in the case of the single-lender contract, any acceptable many-lender contract with $p^c = 1$ has a unique equilibrium characterized by

$$(1-\lambda^M) = (G-D)/\pi(B-D), \qquad \text{(A12)}$$

$$\theta_i(1-p^M)(1-\lambda_{-i}^M)(L-\pi D)-c = 0, \quad \text{for } i \in M, \qquad \text{(A13)}$$

$$\theta_i(1-p^M)(1-\lambda_{-i}^M)(L-\pi D)-c < 0, \quad \text{for } i \notin M, \qquad \text{(A14)}$$

where $(1-\lambda_{-i}^M)$ is the probability that none of the lenders other than i is informed of the project type. Because $(1-\lambda^M) = (G-D)/\pi(B-D)$, the borrower gets $(G-D)$ in equilibrium. From the last two equations, and from the lender participation constraint, we can derive

$$p^M D + (1-p^M)L - (1-p^M)(1-\lambda^M)(L-\pi D)\sum_{i \notin M} \theta_i \geq I + nc. \qquad \text{(A15)}$$

On the other hand, the equilibrium (p^S, λ^S) for the equivalent single lender satisfies

$$(1-\lambda^S) = (G-D)/\pi(B-D), \qquad \text{(A16)}$$

$$(1-p^S)(L-\pi D)-c = 0, \qquad \text{(A17)}$$

and the borrower gets the same payoff of $(G - D)$. Because $\theta_i(1 - \lambda^M_{-i}) < 1$, $p^S > p^M$, and $\lambda^S = \lambda^M$, the equivalent single lender gets in equilibrium

$$p^S D + (1 - p^S)L - c > p^M D + (1 - p^M)L - c$$
$$\geq I + (n-1)c + (1 - p^M)(1 - \lambda^M)(L - \pi D) \sum_{i \notin M} \theta_i > I, \tag{A18}$$

where we used equation (A15). Therefore, the equivalent single lender's participation constraint is also satisfied, and the contract (p^c, D) is an acceptable single-lender contract. Because the optimal single-lender contract is unique and satisfies $p^* D^* + (1 - p^*)L - c = I$, it strictly dominates the equivalent single-lender contract and thus the above many-lender contract; recall the borrower's equilibrium payoff is the same. This holds true for any N and M and for any initial holding $\{\theta_1, \ldots, \theta_N\}$. Therefore, the claim is proved. Q.E.D. of Claim

Proof of Proposition 6: To prove Proposition 6, we first prove the following lemma.

LEMMA 1: *For an acceptable contract, there is a unique equilibrium p^* that satisfies the forward induction criterion of Kholberg (1989) and the noise-proofness of Carlsson and Dasgupta (1997). It is characterized by equation (19). If $I_S > L$, λ^* characterized by equation (20) is the unique equilibrium. If $I_S = L$, the equilibrium λ lies in an interval $[\bar{\lambda}, \lambda^*]$ for some $\bar{\lambda} > 0$.*

Proof of Lemma 1: The proof of the lemma is divided into five steps. Recall that $\bar{p}_S$ is defined by $q(\bar{p}_S)D_S = L$.

Step 1: Characterization of the optimal response of the borrower and the senior lender.

Derivation: Because no $p < \underline{p}$ will be optimal, consider $p \in [\underline{p}, \bar{p}_S)$ first. If $S(p, \boldsymbol{D}) + D_J \leq G$, then the junior claim is not diluted. Calculations similar to the one used in Claim 1 of Proposition 2 show that a p with $S(p, \boldsymbol{D}) + D_J \leq G$ is dominated by $\bar{p}_S$. Among ps that dilute junior debt, the best choice is $\underline{p}$. Therefore, the borrower's choice will be in $\{\underline{p}\} \cup [\bar{p}_S, 1]$.

Recall $(1 - \lambda^*) = G - D/\pi(B - D)$. Define $\Delta(\lambda) \equiv U(\bar{p}_S, \lambda) - U(\underline{p}, \lambda)$ and denote by $\bar{\lambda}$ the solution to $\Delta(\lambda) = 0$. The function $\Delta(\lambda)$ is linear and satisfies $\Delta(\lambda^*) > 0$ and $\Delta(0) < 0$ by assumption $(P2)$. Thus, the existence and uniqueness of $\bar{\lambda} > 0$ are guaranteed. Using these notations, we can characterize the optimal response function $p(\lambda)$:

$$p(\lambda) = \begin{cases} 1 & \text{when } \lambda > \lambda^* \\ \text{any } p \in [\bar{p}_S, 1] & \text{when } \lambda = \lambda^* \\ \bar{p}_S & \text{when } \bar{\lambda} \leq \lambda < \lambda^* \\ \underline{p} & \text{when } \lambda \leq \bar{\lambda}. \end{cases} \tag{A19}$$

For the senior lender, the optimal response function $\lambda(p)$ is the same as before.

Step 2: For the contract to be acceptable, $m(\bar{p}_S) \geq 0$ must hold. When $m(\bar{p}_S) \geq 0$, the only candidates for equilibrium are $p^* \in [\bar{p}_S,1)$ with $m(p^*) = 0$, and $\underline{p}$ with $m(\underline{p}) \leq 0$.

Proof of Step 2: Because $m(1) < 0$ implies $\lambda(1) = 0$, $p = 1$ cannot be an equilibrium. Thus, the equilibrium p lies in $\{\underline{p}\} \cup [\bar{p}_S,1)$. If $m(\bar{p}_S) < 0$, then $m(p) < 0$ for all $p \in [\bar{p}_S,1]$. Therefore, the only equilibrium is $\underline{p}$ with $\lambda \in [0,\bar{\lambda}]$. Because lenders collectively get only L at $\underline{p}$, the contract is not acceptable. Suppose $m(\bar{p}_S) \geq 0$ holds. Then, $p \in [\bar{p}_S,1)$ is an equilibrium if and only if $m(p) = 0$; otherwise the senior lender would not choose $\lambda \in [\bar{\lambda}, \lambda^*]$. Similarly, for $\underline{p}$, $m(\underline{p}) \leq 0$ must hold. Q.E.D. of Step

Step 3: The senior lender's contribution I_S must satisfy $I_S \in [L,I]$.

Proof of Step 3: If $I_S < L$ and $D_S > L$ hold, the senior lender gets $L > I_S$ by not monitoring and demanding liquidation all the time. Knowing this, the junior lender would not accept the contract because without the senior lender monitoring, he will not get the minimum required return. Therefore, for the contracts with $D_S > L$ to be acceptable, $L \leq I_S \leq I$ must hold.

Q.E.D. of Step

Step 4: Suppose that $m(\bar{p}_S) \geq 0$ and $I_S > L$ hold. Once the contracts are accepted, the only equilibrium that survives the forward induction criteria is $p^* \in [\bar{p}_S,1)$ with $m(p^*) = 0$.

Proof of Step 4: When $m(\bar{p}_S) \geq 0$, there can be at most two equilibria, $p^* \in [\bar{p}_S,1)$ with $m(p^*) = 0$, or $p = \underline{p}$. If the senior lender chooses $\lambda \in [0,\bar{\lambda}]$, then $\underline{p}$ is an equilibrium, but he gets only L. According to the forward induction criterion, a senior lender contributing more than L would not play an equilibrium that gives him only L. Q.E.D. of Step

Step 5: Suppose that $m(\bar{p}_S) \geq 0$ and $I_S = L$. Then, only $p^* \in [\bar{p}_S,1)$ with $m(p^*) = 0$ survives the noise-proofness criterion of Carlsson and Dasgupta (1997).

Proof of Step 5: The pair $(p,\lambda) = (\underline{p},0)$ is an equilibrium only because an uninformed senior lender gets $S(\underline{p},\boldsymbol{D}) = G$. Now, suppose that upon receiving a renegotiation demand, the borrower thinks that there is a small probability $\epsilon > 0$ that his inference about the senior lender's information is wrong. Then, a safe borrower would not renegotiate with the senior lender at $p < \underline{p}$; if he does not renegotiate, he will get $\epsilon(G - D) > 0$, whereas he gets nothing if he renegotiates. Because a risky borrower would not follow a different strategy, neither type of borrower will renegotiate.

This perturbation will change the game in the following ways: (i) there will be no change in equilibrium payoffs except for $p \in [\underline{p},\bar{p}_S)$; and (ii) there will be no dilution of the junior claim when $p \in [\underline{p},\bar{p}_S)$ because dilution occurs only when the borrower pays out G. Therefore, the only equilibrium in the perturbed game is $p^* \in [\bar{p}_S, 1)$ with $m(p^*) = 0$. By letting $\epsilon \to 0$, we can eliminate the equilibrium $\underline{p}$. This is the noise-proofness argument of Carlsson and Dasgupta (1997). Q.E.D. of Step

Steps 6 and 7 will complete the proof of Proposition 6.

Step 6: The optimal amount raised by the senior debt is the liquidation value L. Therefore, it is better to have a two-class debt contract.

Proof of Step 6: Let $I_S = k \in [L, I]$ be the amount raised by senior debt. Denote by $S(k)$ and $J(k)$, respectively, the face value of senior and junior debt as a function of k. Then, we have

$$F_S(k) \equiv (S(k) - k)(L - \pi S(k)) - (1 - \pi)cS(k) = 0, \tag{A20}$$

$$F_J(k) \equiv (B - S(k) - J(k))(J(k) + k - I)(L - \pi S(k)) - (B - G)cJ(k) = 0. \tag{A21}$$

Because the borrower gets $(G - D)$, he will choose $k \in [L, I]$ to minimize $D(k) = S(k) + J(k)$. After tedious computations, we get

$$\begin{aligned} D'(k) &= S'(k) + J'(k) \\ &= \frac{c}{F'_S(k)F'_J(k)}\{(1 - \pi)L(B - D(k)) - (B - G)(L - \pi D(k))\}. \end{aligned} \tag{A22}$$

Multiply both sides by $F'_S(k)F'_J(k)$ and get the inequality

$$\begin{aligned} F'_S(h)F'_J(k)D'(k) &> c[(1 - \pi)L(B - D(k)) - (1 - \pi)L(B - G)] \\ &= c(1 - \pi)L(G - D(k)) \geq 0. \end{aligned} \tag{A23}$$

Because the borrower will choose smaller solutions to equations (A20) and (A21), $F'_S(k) > 0$ and $F'_J(k) > 0$ hold. Therefore, inequality (A23) implies $D'(k) > 0$. Because the optimal single-lender contract is equivalent to $k = I$, $D'(k) > 0$ implies the following: (i) it is strictly dominated by any two-class contract with $D_S > L$ and $p_S^c = 1$; and (ii) the minimization of $D(k)$ occurs at $k^* = L$. Q.E.D. of Step

Step 7: The optimal two-class debt contract exists and is unique.

Proof of Step 7: Plug $k = L$ into equation (A20) and get the following equation:

$$H_S(x) = (x - L)(L - \pi x) - (1 - \pi)cx = 0. \tag{A24}$$

Note that $H_S(0) = -L^2 < 0$ and $H_S(d) = (I - L)(L - \pi d) > 0$. Here, d is the face value of the optimal single-lender contract, so that $(d - I)(L - \pi d) = (1 - \pi)cd$. Therefore, there exists a unique $x^* \in (0,d)$ with $H_S(x^*) = 0$. Plug $k = L$ and $S(k) = x^*$ into equation (A21) and get

$$H_J(z) = (B - x^* - z)(z - I + L)(L - \pi x^*) - (B - G)cz = 0. \tag{A25}$$

Note $H_J(0) = (B - x^*)(L - I)(L - \pi x^*) < 0$. Also we have $H_J(G - x^*) > 0$ because

$$\begin{aligned} H_J(G - x^*) &= (B - G)(G - x^* - I + L)(L - \pi x^*) - (B - G)c(G - x^*) \\ &= (B - G)\{(G - I)(L - \pi x^*) - c(G - \pi x^*)\} \\ &> (B - G)\{(1 - \pi)cG + \pi(G - x^*)(G - I) - c(G - \pi x^*)\} \\ &= (B - G)\pi(G - x^*)(G - I - c) > 0, \end{aligned} \tag{A26}$$

by assumption ($P3$). Therefore, there exists a unique $z^* \in (0, G - x^*]$ with $H_J(z^*) = 0$. This proves the existence and the uniqueness. Q.E.D. of Step

Proof of Proposition 13: From the equations $q(p^*)D_S = L$ and $(D_S - L)(L - \pi D_S) = (1 - \pi)cD_S$, we have

$$\frac{\partial q(p^*)}{\partial L} = \frac{1}{D_S^2 F_S'(L)}(D_S - L)(L - \pi D_S) > 0, \tag{A27}$$

where $F_S(\cdot)$ is defined in equation (A20). Because $1/q(p^*)$ is the rate charged on senior debt, senior debt gets cheaper as L goes up. To show junior debt gets cheaper as well, we will show

$$\frac{\partial}{\partial L}\{q(p^*) - \lambda^*(1 - p^*)\pi\} = [(1 - \pi) + \lambda^*\pi]\frac{\partial p^*}{\partial L} - (1 - p^*)\pi\frac{\partial \lambda^*}{\partial L} > 0. \tag{A28}$$

Because $\partial p^*/\partial L > 0$, it suffices to show $\partial\lambda^*/\partial L < 0$. By differentiating equations (A20) and (A21) with respect to L, we get

$$\begin{aligned} \frac{\partial D}{\partial L} &= \frac{c}{\Delta}\{(1 - \pi)(B - D)(2L - D_S + D_J - I) - (B - G)(2L - D_S + D_J - \pi D)\} \\ &< -\frac{c}{\Delta}(B - G)(I - \pi D) < 0, \end{aligned} \tag{A29}$$

where $\Delta = F'_S(L)F'_J(L) > 0$. The inequality comes from $(1 - \pi)(B - D) < (B - G)$. Because $(1 - \lambda^*) = G - D/\pi(B - D)$, we have $\partial\lambda^*/\partial D > 0$, and thus,

$$\frac{\partial\lambda^*}{\partial L} = \frac{\partial\lambda^*}{\partial D}\frac{\partial D}{\partial L} < 0. \tag{A30}$$

The claim is proved. Q.E.D.

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