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Author(s): Bruno Biais and Catherine Casamatta
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Optimal Leverage and Aggregate Investment

BRUNO BIAIS and CATHERINE CASAMATTA*

ABSTRACT

We analyze the optimal financing of investment projects when managers must exert unobservable effort and can also switch to less profitable riskier ventures. Optimal financial contracts can be implemented by a combination of debt and equity when the risk-shifting problem is the most severe while stock options are also needed when the effort problem is the most severe. Worsening of the moral hazard problems leads to decreases in investment and output at the macroeconomic level. Moreover, aggregate leverage decreases with the risk-shifting problem and increases with the effort problem.

THE INTERPLAY BETWEEN THE OPERATIONAL CHARACTERISTICS of firms, their financial and investment decisions, and macroeconomic variables such as aggregate investment and the equilibrium cost of capital are analyzed in this paper. Issues at stake include the following: how do the respective roles of different sources of financing (and consequently leverage) vary with technological changes, the business cycle, or phases of economic development? Should decreases in output and investment be attributed to excessive leverage? What is the role played by outside equity or convertible bonds in the financing of growth and innovation? Should the government design public policies relative to these issues?

At the firm and industry level, an important body of empirical research provides us with information about *the determinants of financial leverage*. Barclay, Smith, and Watts (1995) find that leverage is high for regulated firms and firms in low-tech industries, and it is low in high-tech industries. Bradley, Jarrell, and Kim (1984) and Long and Malitz (1985) find that leverage decreases with R&D expenditures. At a more anecdotal level, the contrast between mature industries and high-tech and innovative industries is apparent in this quotation from an executive of Bank of America:

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> In financing an LBO in a standard manufacturing company . . . the amount of the senior debt will be typically 3–4 times EBITDA with sponsor-controlled capital composing 20–25 percent of the capital structure. By contrast in financing recent LBOs of high growth technology driven investments . . . we provided . . . senior debt-to-EBITDA multiples under 2.5, and sponsor-controlled capital represented 35 percent of the capital structure. (Allen (1996), p. 23)

Another line of empirical research focuses on *the determinants of firms' investments*. Fazzari, Hubbard, and Peterson (1988), Hoshi, Kashyap, and Sharfstein (1991), Gertler and Gilchrist (1994), and Lamont (1997) find that investment is positively correlated to internal cash flow, and that this link is stronger for firms experiencing greater barriers to external finance. Brown (1997) finds that this is the case for particularly innovative firms.

At the root of both types of empirical findings are the agency problems between the suppliers of funds and firm managers. Since Jensen and Meckling's (1976) study, these problems have been studied in depth by corporate finance theory. Yet, while Jensen and Meckling solved the agency problem "given that only stocks and bonds can be issued," most of the theoretical analyses endogenizing the financial contracts obtain debt as the optimal source of outside finance.[1] This is at odds with the use, in practice, of outside equity financing. One contribution of the present paper is to propose a theory explaining why and how outside equity, as well as more complex hybrid securities, is used along with outside debt.[2]

Relying on the seminal insights of Jensen and Meckling (1976), we consider entrepreneurs who need outside financing to fund investments in the presence of two dimensions of moral hazard.[3] First, the entrepreneur must exert costly and unobservable effort, leading to an improvement in the distribution of the cash flows generated by the project (in the sense of first-order stochastic dominance). Second, the manager must abstain from switching to risky ventures, leading to a deterioration in the cash flows generated by the project (in the sense of second-order stochastic dominance). We analyze

[1]Analyses of debt as an optimal contract often rely on the costly state verification paradigm initiated by Townsend (1979), Diamond (1984), and Gale and Hellwig (1985). Another interesting approach, emphasizing the inalienability of human capital, is presented by Hart and Moore (1994). An insightful survey of financial contracting is offered by Allen and Winton (1995).

[2] As far as we know, the only other papers where outside equity arises as an optimal contract are Lacker and Weinberg (1989), Chang (1993), Dewatripont and Tirole (1994), Fluck (1998), and Cestone and White (1998)—papers that rely on the costly state verification/incomplete contracts/control rights paradigm. We consider a complete contract model, with two dimensions of moral hazard.

[3] Hellwig (1994) predated us in revisiting the Jensen and Meckling (1976) paradigm in terms of optimal contracts. Our technical assumptions differ from his, however, and consequently our results also differ. Further, some of the issues we discuss, such as the financing of innovation, the role of stock options, or the macroeconomic aspects of the problem, are not considered in Hellwig.

the financial contract, considered as a revenue-sharing rule, which optimally copes with the moral hazard problems. We find that there is a tension between the two moral hazard problems. To induce the manager to exert effort one has to promise her large payoffs when the cash flow generated by the firm is large. Unfortunately, this can make risk-taking too attractive for the manager. When this tension is too strong, it can lead to credit rationing.

Having characterized the optimal financial contract in terms of abstract sharing rules, we study its implementation using instruments observed in the real world, which include debt, equity, and stock options.[4] We find that there are three different regimes.

In the first regime (credit rationing), firms have too little internal cash flow to undertake investment. This is in line with the above-mentioned empirical works on the link between investment and internal cash flows.

In the second regime, where the risk-shifting problem is the major source of moral hazard, the optimal contract can be implemented by a combination of debt and equity. Alternatively, in this case, convertible bonds can be structured to implement the optimal contract.[5] These theoretical results are consistent with the stylized facts on equity financing. Equity and convertible bond financing, provided by venture capitalists, play an important role for young, innovative, and high-tech firms, where the ability to switch to riskier ventures is large. Also, leverage is relatively low for high-tech industries and firms with large R&D expenditures, where the risk-shifting problem is potentially large.

In the third regime, where the effort problem is the major source of moral hazard, more sophisticated option-like claims must be used to enhance the manager's incentives to exert effort. This is consistent with the empirical results of Yermack (1995) on the determinants of the use of stock options. Yermack concludes that, contrary to the implications of previous theoretical research, stock-option incentives are not decreasing with CEOs' shareholdings or with leverage. In this third regime, managers hold shares as well as stock options. Also, the firm is considerably levered, since the incentives provided by stock options complement those induced by leverage when the moral hazard problem related to effort is severe.

At a macroeconomic level, the link between internal cash flows and investment has implications for business cycles.[6] In previous theoretical analyses of this point, however, outside financing is provided under the form of debt. Consequently, one cannot study the interaction between economic conditions and leverage, reflecting the respective weights of equity and debt in

[4] Dybvig and Zender (1991) criticize received financial structure theory by showing that properly designed managerial compensation schemes can often eliminate agency problems. Our analysis is immune from this criticism since our optimal contracting approach allows for general managerial compensation schemes for the entrepreneur.

[5] This is in the spirit of Green (1984).

[6] See, for example, Gertler (1988), Bernanke, Gertler, and Gilchrist (1996), Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Aghion, Banerjee, and Piketty (1997).

outside financing. To study this issue, we set our microeconomic analysis in a simple general equilibrium model, in the same spirit as Holmstrom and Tirole (1997).[7]

We consider a continuum of entrepreneurs. Each one is endowed with one investment project. Entrepreneurs differ in terms of initial wealth, or equivalently, internal cash flows. Financing is supplied by those entrepreneurs who do not invest and save to finance their final period consumption. For simplicity, we assume that if there is no moral hazard, all investment projects have positive net present values. However, in the presence of the two sources of moral hazard discussed above, entrepreneurs with low initial wealth are rationed. In this context, we analyze the joint determination of the aggregate level of investment, the cost of capital, and the leverage of the firms in equilibrium. Within this framework we study the impact of exogenous shocks on the economy.

For example, consider a phase of technological innovation raising the profitability of investments, but at the same time requiring changes in the management and technology of firms and thus enhancing the extent of the risk-shifting problem. In this context, our theory predicts a decrease in the aggregate leverage. It has ambiguous implications, however, on the level of investment. On the one hand, the increase in profitability brought about by the technological innovation should lead to an increase in output and investment. On the other hand, the worsening of the moral hazard problem can potentially offset this first effect.

Alternatively, consider a phase of increased volatility, leading to increased dispersion in the internal cash flows of the firms. This increases the mass of firms falling below the minimum threshold necessary to have access to financing. Consequently, investment is reduced.

One policy implication of our analysis is that government subsidies to small, young, and cash-poor firms in innovative industries could help mitigate credit rationing problems and consequently foster investment and growth. On the other hand, our theoretical analysis warns that public policies should not aim at influencing the financial choices of corporations to the extent that these choices are optimal responses to agency problems. For example, we show that in mature industries where the effort problem is severe, investment is reduced and leverage is high. In this context, however, public policies, wrongly attributing low investment to excessive debt, and seeking to reduce leverage, would be misguided.

The model and assumptions are presented in the next section. Section II analyzes the optimal contract. Section III presents the implementation of this optimal contract with debt, equity, and stock options or convertible bonds. Section IV develops the implications of our analysis for the optimal leverage of corporations and compares them to previous empirical findings. Section V presents the macroeconomic analysis. Concluding comments are in the last section and proofs are in the Appendix.

[7] Our analysis of endogenous financial structure with agency problems in general equilibrium is also related to Bolton and Freixas (1998).

I. Model

Consider the following extension of Holmstrom and Tirole (1997). There is a continuum of risk-neutral entrepreneurs. Each entrepreneur is endowed with one opportunity to invest I, and initial wealth A, which can, for example, be the internal cash flow generated by past activities. Outside financing $I - A$ (assumed to be nonnegative) is needed. If investment is undertaken, it yields revenue R^{θ} contingent on the state of the world θ. State θ occurs with probability p_{θ}. In contrast with the costly state verification paradigm, the outcome is supposed to be perfectly verifiable. In the same spirit as Jensen and Meckling (1976) we consider the case where there are two sources of moral hazard. First, the manager must exert costly unobservable effort to improve the distribution of cash flows (R^{θ}) in the sense of first-order stochastic dominance. Second, the manager must abstain from taking excessive risks, leading to a deterioration of the distribution of cash flows, in the sense of second-order stochastic dominance.

To capture this two-dimensional moral hazard problem in the simplest possible way, assume there are three states of the world, u, m, and d, with corresponding cash flows: $R^u > R^m > R^d$. Assume also that the manager can choose between two levels of risk and two levels of effort. First, consider the case where the manager does not take excessive risk. If she does not exert effort, we assume the three states are equally probable. If she works, however, the distribution of the cash flows is improved in the sense of first-order stochastic dominance: the probability of the good state (u) is increased by ϵ $(< 1/3)$, and the probability of the bad state (d) is decreased by ϵ. The cost of effort for the manager (or, equivalently, her utility from shirking) is B. Denote by $\overline{V}$ the expected outcome under effort, and $\underline{V}$ the expected outcome when shirking. To simplify the analysis, assume the project has positive net present value only if the manager exerts effort. Next consider the possibility for the manager to switch to riskier projects, for which the probability of the medium state (m) is reduced by $\alpha + \beta$, and the probability of the good state (u) is increased by α and that of the bad state (d) by β. We assume that this riskier distribution is dominated in the sense of second-order stochastic dominance, which is equivalent to

$$\alpha(R^u - R^m) \leq \beta(R^m - R^d). \tag{1a}$$

In most of our analysis, we will assume that the inequality is strict; that is,

$$\alpha(R^u - R^m) < \beta(R^m - R^d). \tag{1b}$$

Under this assumption the present value of the project is reduced by risk-taking, which is in line with the textbook presentations of the risk-shifting problems (see, e.g., the examples in Brealey and Myers (1991) or Grinblatt and Titman (1998)). This reflects the view that managers considering excessively risky ventures that lead to an expropriation of the debtholders are tempted to engage in these ventures even if they lead to negative net present value (NPV) investments. This is consistent with the informative case study of risk-shifting in the savings and loans industry by Esty (1997). In his analysis, Esty shows that in addition to raising risk the Twin City's strategy is likely to have also reduced expected cash flows. For example, it included such actions as paying deposit rates well above market rates even on insured deposits, or making commercial loans and mortgages in places as far from its home base as Texas and Florida, where it did not have an experienced commercial lender on staff. Assuming that excessive risk-taking leads to a decrease in the expected cash flow simplifies our theoretical analysis since, under this assumption, the first best solution, corresponding to effort and no risk, is clearly identified. In fact, to simplify matters further, we assume that if the manager switches to excessive risk, the net present value of the project becomes negative. At the end of Section III, we analyze and discuss the case where excessive risk-taking leaves the expected cash flow unchanged. We explain in what sense this essentially amounts to a particular case of our main analysis.

Note that the assumption that effort is socially optimal, that is, $\overline{V} > \underline{V} + B$, is equivalent to $R^d < R^u - (B/\epsilon)$. Further, the assumption that risk-taking leads to second-order dominated distribution and lower expected cash flows is equivalent to $R^d < R^m - (\alpha/\beta)(R^u - R^m)$. Hence, when combined, the two conditions imply

$$R^d < \min\left[R^u - \frac{B}{\epsilon}, \frac{(\alpha+\beta)R^m - \alpha R^u}{\beta}\right]. \tag{2}$$

If $[(\alpha+\beta)R^m - \alpha R^u]/\beta < R^u - (B/\epsilon)$, that is, if $R^m < R^u - [\beta/(\alpha+\beta)](B/\epsilon)$, the condition corresponding to effort is redundant and the risk-shifting problem is said to be dominant. If the reverse inequality holds, the effort problem is said to be dominant. As will be shown in Section III, the distinction between the case where the effort problem is dominant and the case where the risk-shifting problem is dominant will turn out to play an important role in the implementation of the optimal contract.

The contract between the outside investor and the manager is a revenue-sharing rule specifying what fraction of the revenue accrues to each of the parties in each of the states of the world. It is specified by the triplet $\{\delta^\theta\}$, $\theta \in \{u,m,d\}$, where δ^θ is the share of outcome earned by the outside investor in state θ. We assume the manager has limited liability so that $\forall\theta$, $\delta^\theta \leq 1$.

Since the project has positive net present value only if the manager chooses to exert effort and to abstain from taking excessive risk, the project can be undertaken only if the two following incentive compatibility conditions hold. First,

$$\begin{aligned}\left(\tfrac{1}{3}+\epsilon\right)(1-\delta^u)R^u + \tfrac{1}{3}(1-\delta^m)R^m + \left(\tfrac{1}{3}-\epsilon\right)(1-\delta^d)R^d \\ \geq \tfrac{1}{3}(1-\delta^u)R^u + \tfrac{1}{3}(1-\delta^m)R^m + \tfrac{1}{3}(1-\delta^d)R^d + B,\end{aligned} \tag{3}$$

where the left-hand side is the expected revenue of the manager if she works and the right-hand-side is the sum of her expected revenue if she shirks and of the benefits from shirking. Second,

$$\begin{aligned}\left(\tfrac{1}{3}+\epsilon\right)(1-\delta^{u})R^{u}+\tfrac{1}{3}(1-\delta^{m})R^{m}+\left(\tfrac{1}{3}-\epsilon\right)(1-\delta^{d})R^{d}\\ \geq\left(\tfrac{1}{3}+\epsilon+\alpha\right)(1-\delta^{u})R^{u}+\left(\tfrac{1}{3}-(\alpha+\beta)\right)(1-\delta^{m})R^{m}\\ +\left(\tfrac{1}{3}-\epsilon+\beta\right)(1-\delta^{d})R^{d},\end{aligned} \tag{4}$$

where the left-hand side is the expected revenue of the manager if she works and sticks to the relatively safe project, while the right-hand side is her expected revenue if she works but takes excessive risks. In fact, a third constraint should be added to ensure that the manager prefers to choose effort and no risk, rather than no effort and risk, but it is straightforward to show that it is redundant.

Assume for the moment that the cost of capital is r. The supply of funds and equilibrium cost of capital will be endogenized in Section V. The participation constraint of the outside investors is

$$\left(\tfrac{1}{3}+\epsilon\right)\delta^{u}R^{u}+\tfrac{1}{3}\delta^{m}R^{m}+\left(\tfrac{1}{3}-\epsilon\right)\delta^{d}R^{d}\geq(I-A)(1+r). \tag{5}$$

An optimal contract is a triplet $(\delta^u;\delta^m;\delta^d) \in [0,1]^3$ such that the two incentive compatibility conditions of the manager and the participation constraint of the outside investors are satisfied. It is easy to prove that, since the project has positive NPV, the participation constraint of the manager can always be satisfied.

For simplicity we assume there are no bankruptcy costs. Hence, debt financing with face value D (between R^m and R^d) corresponds to $\delta^u = D/R^u$, $\delta^m = D/R^m$, and $\delta^d = 1$, and outside equity financing with dilution δ corresponds to $\delta^u = \delta^m = \delta^d = \delta$.

II. Optimal Financial Contracts

If there is no moral hazard, the manager exerts effort and abstains from excessive risk-taking. Consequently, the project will have positive net present value and be financed. This corresponds to the first best solution. With moral hazard there are two possibilities: either the incentive compatibility conditions of the manager and the participation constraint of the outside investors are mutually consistent, in which case the first best is still attainable, or the incentive compatibility and rationality conditions are not consistent, and the project cannot be financed. In the latter case there is credit rationing.

The incentive compatibility condition under which the manager exerts effort can be rewritten as

$$(1-\delta^{u})R^{u}-(1-\delta^{d})R^{d}\geq\frac{B}{\epsilon}. \tag{6}$$

To induce the manager to work, her increase in reward when working must be at least equal to the cost of effort. This is the case if $(1 - \delta^u)$ is high and $(1 - \delta^d)$ low enough.

The incentive compatibility condition under which the manager abstains from excessive risk-taking can be rewritten as

$$\alpha(1-\delta^u)R^u + \beta(1-\delta^d)R^d \leq (\alpha+\beta)(1-\delta^m)R^m. \tag{7}$$

The left-hand side of equation (7) represents the additional revenue accruing to the manager when she takes risks. The right-hand side is the loss in revenue due to risk-taking. The left-hand side is increasing in both $(1 - \delta^u)$ and $(1 - \delta^d)$, and the right-hand side is increasing in $(1 - \delta^m)$. Hence, to ensure that the manager does not take excessive risk, $(1 - \delta^u)$ and $(1 - \delta^d)$ must be low and $(1 - \delta^m)$ must be high.

Note that the crucial parameter in the risk-shifting problem is α. Taking excessive risks increases the probability of the good state by α. Hence, when α is large, the manager's reward in state u cannot be too large, lest it would encourage risk-taking.

Comparing the two incentive compatibility conditions one observes that although both require a low reward for the manager in the bad state, they conflict relative to the reward in the good state. On the one hand, the condition under which the manager has incentives to exert effort requires that she receives a large reward in the good state. On the other hand, the condition under which she does not engage in excessive risk requires that this reward be low. When the tension is so strong that the two conditions are not consistent with one another, there is credit rationing. We will also show below that this tension between the two incentive compatibility conditions influences the form of the optimal financial contract.

The left-hand side of the participation constraint of the outside investors in equation (5) is increasing in all the δ^θ. Raising δ^d relaxes all constraints. Hence the optimal contract will always allow for $\delta^d = 1$. The participation constraint imposes conditions on δ^u and δ^m that are potentially in conflict with the incentive compatibility conditions of the manager. When the tension is so strong that the different conditions are not consistent with one another, there is credit rationing.

After some manipulations, the three constraints simplify to the condition stated in the following proposition.

PROPOSITION 1: *The first best solution is attained, and the positive NPV project can be financed, if the initial wealth of the entrepreneur (A) is large enough, relative to the cost of effort (B) and the risk-shifting problem (measured by α), in the sense that*

$$A \geq A_0 \equiv I - \frac{1}{1+r}\left[\overline{V} - \frac{B}{\epsilon}\left(\frac{1}{3} + \epsilon + \frac{1}{3}\,\frac{\alpha}{\alpha+\beta}\right)\right]. \tag{8}$$

The proposition states that the initial wealth of the entrepreneur, or internal cash flow of the firm (A), must be larger than a minimum level (A_0) for financing to be granted. When A is large, the outside investors do not need to receive a large fraction of the revenue, because their investment is limited. Consequently, large revenues are left available to compensate the manager in such a way that she will have incentives to work and not to engage in value-destroying risky activities.

As the magnitude of the two moral hazard problems increases (i.e., as B and α increase), more and more positive NPV investment projects fail to be financed and credit rationing becomes more prevalent. A_0 is increasing in the *product* of the cost of effort (B) and a term increasing in the magnitude of the risk–shifting problem (α). This product reflects the interaction between the two sources of moral hazard discussed above.

Our analysis uncovers a trade-off that could arise during phases of technological progress. On the one hand, technological innovation leads to more efficient production (corresponding to an increase in $\overline{V}$). With perfect capital markets this should lead to an increase in investment and output. On the other hand, at times of technological change firms have access to an array of new investment opportunities with different levels of risk. This is likely to correspond to an increase in α. Although the increase in the efficiency of production ($\overline{V}$) tends to lead to a decrease in the minimum level of internal cash necessary to finance investment (A_0), the increase in α tends to increase the hurdle A_0. If the latter effect dominates the former, technological innovation can have a (transiently) negative impact on investment and output.

III. Implementing the Optimal Contract with Outside Debt, Outside Equity, and Stock Options

A. Outside Equity or Outside Debt Only

Restricting one's attention to financing with outside equity or outside debt only amounts to imposing restrictions on the triplet $(\delta^u;\delta^m;\delta^d)$ defining the financial contract. The following lemmas state under which conditions the first best is still attainable in spite of these restrictions.

LEMMA 1: *The first best solution can be attained and the positive NPV project financed with outside equity if the initial wealth of the entrepreneur (A) is large enough relative to the magnitude of the cost of effort (B), in the following sense:*

$$A \geq A_1 \equiv I - \frac{1}{1+r}\left[\overline{V} - \frac{B}{\epsilon}\,\frac{\overline{V}}{R^u - R^d}\right]. \tag{9}$$

If there is no effort problem ($B = 0$), then the condition in the above lemma simplifies to the positive NPV condition. That is, when the only moral hazard problem stems from the ability to engage in value destructive risky ventures, outside equity is the optimal financial contract and always enables one to attain the first best.

The next lemma characterizes the case where only outside debt is available.

LEMMA 2: *The first best solution can be attained and the positive NPV project financed with outside debt if the initial wealth of the entrepreneur (A) is large enough relative to the magnitude of the risk-shifting problem (measured by α) and the cost of effort (B), in the following sense:*

$$A \geq A_2$$
$$\equiv I - \frac{1}{1+r}\left[\overline{V} + \frac{1}{3}(R^u - R^m) - \left(\frac{2}{3} + \epsilon\right)\max\left[\frac{B}{\epsilon}; \frac{\alpha + \beta}{\beta}(R^u - R^m)\right]\right]. \tag{10}$$

When α goes to 0, and the only source of moral hazard is costly effort, then $A_2 < A_1$. That is, debt is more efficient than equity in terms of effort incentives. Indeed, making the manager a residual claimant, as is the case with outside debt, enhances her reward in the good state (which effort makes more likely to occur) and bankruptcy "punishes" her in the bad state (the likelihood of which is reduced by effort).

The following proposition compares the efficiency of pure outside debt and pure outside equity to that of the optimal contract.

PROPOSITION 2: *If $B > 0$ and $\alpha > 0$, the minimal bounds on the internal cash flow under which the project can be undertaken are strictly larger in the case of pure outside debt and pure outside equity than in the case of the optimal contract; that is, $A_1 > A_0$ and $A_2 > A_0$.*

The proposition implies that the incentives provided by pure outside debt or pure outside equity are not always powerful enough. Equity is not efficient enough in terms of incentives to exert effort. On the other hand, debt encourages value-destructive risk-taking, though it is sometimes less efficient than the optimal contract in terms of incentives to exert effort.

B. Implementing the Optimal Contract

Equity is more efficient than debt at aligning incentives regarding the risk-shifting problem. However, debt is more efficient than equity at providing incentives to exert effort. Thus it is natural to investigate the efficiency properties of a combination of outside debt and outside equity. The next step in this line of reasoning is to question whether the optimal combination of debt and equity implements the optimal contract. The following proposition summarizes the answers to these questions.

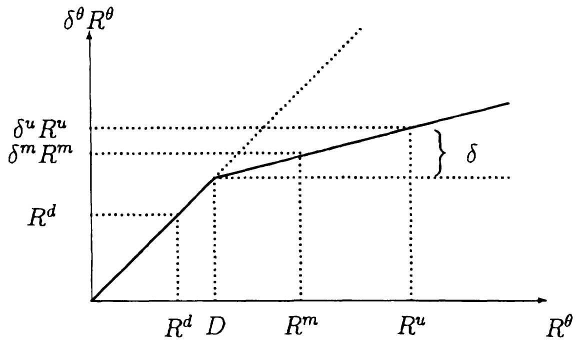

Figure 1. Payoff to the outside financier when the risk-shifting problem is dominant. The figure plots the investor's revenue as a function of the project's outcome. The investor's payoff structure can be implemented by giving him a debt claim with face value D and a share δ of the firm's equity.

PROPOSITION 3: *The optimal financial contract can be implemented with a mix of outside debt and outside equity when the risk-shifting problem is dominant in the sense that*[8]

$$R^m \leq R^u - \frac{\beta}{\alpha + \beta}\frac{B}{\epsilon}. \tag{11}$$

It can be implemented with outside debt, outside equity, and stock-options if the effort problem is dominant.

As discussed above, when the only source of moral hazard is due to the ability of the manager to increase risk, outside equity is the optimal contract. When there is another source of moral hazard, although the main problem remains risk-shifting, the optimal contract can be implemented by adding some outside debt to the outside equity in order to enhance the incentives of the manager to undertake effort. The revenue of the outside investor, as a function of the total revenue generated by the firm, in the case where the risk-shifting problem is dominant, is represented graphically in Figure 1. Note that, in this case, complete markets, spanning the three states of the world, u, m, and d, are not needed to implement the optimal contract because the latter can be obtained simply by combining debt and equity.

On the other hand, when the major problem is due to the cost of effort, the structure of payoffs generated by debt and equity is no longer rich enough and stock options must be allocated to the manager. Stock options increase the responsiveness of the reward of the manager to the cash flows created by the firm. This enhances her incentives to exert effort. The stock options

[8] This is the distinction between the case where the effort problem is dominant and the case we introduced in Section I where the risk-shifting problem is dominant.

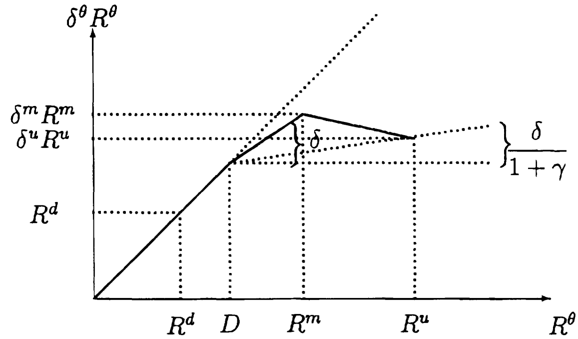

Figure 2. Payoff to the outside financier when the effort problem is dominant. The figure plots the investor's revenue as a function of the project's outcome. When the effort problem is dominant, the investor's payoff structure can be implemented by giving him a debt claim with face value D and a share δ of the firm's equity. In state R^u, γ shares are given to the manager, which dilutes the investor's initial stake.

mentioned in Proposition 3 are structured as follows: If the revenue of the firm is high (and equals R^u), γ additional shares are created and allocated to the manager.[9] This is similar to the "ratchet contracts" commonly used in LBOs. Note that LBOs often occur for mature firms where the effort problem is likely to be dominant. In the case where the effort problem is dominant, the revenue of the outside investor as a function of the total revenue generated by the firm is represented graphically in Figure 2. The decrease in the revenue of the outside investor in the best state of nature is similar to the pattern arising in the "live or die" contract analyzed in Innes (1990).

C. Convertible Bonds

Jensen and Meckling (1976) suggest that convertible debt provides good incentives to induce effort while precluding risk-taking. Green (1984) offers a formal treatment of this point. In the case where the manager has to allocate funds between two investment projects with different levels of risk, Green shows that convertible bonds can provide the incentives to implement the optimal investment policy. This issue can be revisited in the context of our model, where the manager must exert effort and also choose the riskiness of the project.

PROPOSITION 4: *The optimal financial contract can be implemented with convertible bonds when the risk-shifting problem is dominant.*

[9] Note that the value of this component of the compensation of the manager is increasing and convex in the total value of the firm.

In our simple model, optimal convertible bonds can be structured to generate exactly the same revenues as the combination of outside equity and outside debt presented in Proposition 3. This illustrates that, as long as incentive compatibility conditions and participation constraints are satisfied, standard arbitrage arguments apply, so that convertible bonds are equivalent to a replicating portfolio of stocks and bonds.

D. Numerical Example

To illustrate our theoretical analysis we briefly present a numerical example. Suppose that the amount to be invested is: I = \$125, and that the large cash flow is R^u = \$180, the medium one is R^m = \$120, and the lower one is R^d = \$60. Suppose the monetary equivalent of the managerial cost of effort is B = \$12. This is significant, but still reasonable (it is less than 10 percent of the amount invested). Suppose also that the increase in the probability of the good state corresponding to effort is $\epsilon = \frac{1}{6}$, the increase in the probability of the good state due to risk-taking, α, is $\frac{1}{10}$, and the corresponding increase in the probability of the bad state is $\beta = \frac{1}{6}$. Normalizing the interest rate to zero, the NPV (net of the cost of effort) of the project under effort and without risk-shifting is \$3, and without effort or with excessive risk it is negative. Under the optimal contract, the project can be financed if the internal cash flow A is larger than or equal to \$30. Under pure debt financing, this minimum level goes to \$45; under pure equity financing it is equal to \$69. Hence, the initial wealth needed to finance investment is significantly higher in the case of pure debt or pure equity financing than under the optimal contracting scheme. Note also that the parameter values are such that the optimal contract can be implemented by a combination of debt and equity. Supposing that the initial internal cash flow is A = \$35, the optimal contract can be implemented with a combination of outside equity and debt, granting 37.5 percent of the equity to the financier, and with debt service equal to \$60.

E. When Risk-Shifting Does Not Reduce Expected Cash Flow

In the analysis above, we assumed that risk-shifting led to a decrease in the expected cash flow, and was therefore detrimental to social welfare. What if, instead, risk-shifting simply amounted to a mean-preserving spread? Since we consider risk neutral agents, risk-shifting leaves social welfare unaffected. Hence, to analyze financing of the project in this case, we follow the same steps as in the previous case except that we do not need to impose the incentive compatibility condition that the manager refrains from increasing risk. Following this approach it is easy to prove that the project can be funded if and only if the initial wealth of the entrepreneur is large enough, as stated in the next proposition.

PROPOSITION 5: *When risk-shifting does not reduce expected cash flows, the project can be funded if and only if the initial wealth of the entrepreneur is large enough relative to the magnitude of the moral hazard problem, in the following sense:*

$$A \geq A_3 \equiv I - \frac{1}{1+r}\left[\bar{V} - \left(\frac{1}{3} + \epsilon + \alpha\right)\frac{B}{\epsilon}\right]. \tag{12}$$

A_3 is the minimum level of initial wealth for which the participation constraint of the outside investors is consistent with the incentive compatibility conditions of the manager, specifying that she should exert effort and take risk. Note that this minimum level of wealth (A_3) is below the minimum level of wealth such that the participation constraint of the investors is consistent with effort and no risk–taking on the part of the manager (A_0). In that sense, it is easier to induce effort and risk than to induce effort and no risk. This reflects the tension between the incentives to exert effort and the incentives not to engage in risk-shifting.

The nature of the optimal financial contract in the case where risk-shifting does not reduce the expected cash flow, and its implementation, are presented in the next proposition.

PROPOSITION 6: *When risk-shifting does not decrease the expected cash flow of the project, the optimal financial contract entails allocating all of the cash flows to the outside investors in states m and d. If $R^u \geq R^m + (B/\epsilon)$, the optimal contract can be implemented by a simple pure debt contract. If, on the contrary, $R^u < R^m + (B/\epsilon)$, the optimal contract can be implemented by allocating all of the shares to the outside investors while leaving to the manager stock options to be exercised in state u only.*

Allocating all of the cash flows to the outside investors in state m contributes to relaxing both their participation constraint and the incentive compatibility constraint requiring that the manager take risk. Similarly, allocating all the cash flows to the outside investors in state d contributes to relaxing both their participation constraint and the incentive compatibility constraint of the manager relative to effort.

If the cash flow in state u is large (in the sense that $R^u \geq R^m - (B/\epsilon)$), then the optimal contract can be achieved by issuing debt with $D \geq R^m$, and leaving $R^u - D$ to compensate the manager. Relying on debt to finance the project is rather natural because debt financing provides strong incentives to exert effort and because the drawback of debt pointed out by Jensen and Meckling (1976), namely that it induces risk-taking, is not an issue in the present case.

On the other hand, if $R^u < R^m - (B/\epsilon)$, the residual cash flow $R^u - D$ is not sufficient to motivate the manager to exert effort. In that case, larger revenue must be pledged to the manager in state u. This is achieved by allocating stock options to the manager, similarly to the case analyzed in Proposition 3.

Comparing the analysis presented earlier in this subsection to the results presented above, the following remarks can be made. When risk-shifting does not lead to a decrease in the expected cash flows from the project, the moral hazard problem is less severe. Since it is no longer necessary to preclude risk-taking, we face a less constrained optimization problem, and more can be achieved. However, the menu of financial instruments needed to implement the optimal contract is not qualitatively modified.

In the remainder of the paper, we revert to the case where risk-shifting is socially suboptimal because it reduces the expected cash flow.

IV. Optimal Leverage

In this section, we focus on the case where the risk-shifting problem is dominant, in the sense that

$$R^m \leq R^u - \frac{\beta}{\alpha + \beta} \frac{B}{\epsilon}, \tag{13}$$

so that the optimal contract can be implemented by a combination of debt and equity. In this case, the optimal leverage ratio of the firm can be readily computed. We focus on leverage defined as the ratio of outside debt to outside equity, to measure the relative share of debt in the outside financing of the firm.

The optimal financial contract, specified by the triplet $(\delta^u, \delta^m, \delta^d)$, is restricted by three inequalities: the two incentive compatibility conditions and the participation constraint. For $A = A_0$, all constraints are binding, and there is a unique feasible contract corresponding to a unique combination of debt and equity. For $A > A_0$, there is some slack in the constraints. This leaves some flexibility in the choice of the triplet $(\delta^u, \delta^m, \delta^d)$ and of the combination of debt and outside equity with which to finance the firm. Hence, there is a multiplicity of optimal leverage ratios. When A is large enough—that is, larger than A_1 (defined in Lemma 1) and A_2 (defined in Lemma 2)—this multiplicity is such that the optimal leverage can take any value between zero and ∞. (The leverage ratio is defined as the ratio of debt to outside equity, so it is infinite when there is no outside equity.)

For a given level of A, the set of possible leverages implementing the optimal contract is defined by an upper bound l^+ and a lower bound l^-. Within these two bounds leverage is irrelevant, as in the Modigliani–Miller (1958) theorem. The values of the two bounds are given in the following proposition.[10]

[10] The proposition is stated for the case where R_d is normalized to zero. This assumption simplifies the formulas but it does not affect the results qualitatively.

PROPOSITION 7: *When $A \in [A_0;A_2[$, the maximum feasible leverage is*

$$l^+ = \frac{\left(\frac{2}{3}+\epsilon\right)\frac{(\alpha+\beta)R^m - \alpha R^u}{\beta}}{(I-A)(1+r) - \left(\frac{2}{3}+\epsilon\right)\frac{(\alpha+\beta)R^m - \alpha R^u}{\beta}}. \tag{14}$$

When $A > A_2$, the maximum possible leverage is infinite. When $A \in [A_0;A_1[$, the minimum possible leverage is

$$l^- = \frac{\left(\frac{2}{3}+\epsilon\right)\left[R^u[(I-A)(1+r) - \bar{V}] + \bar{V}\frac{B}{\epsilon}\right]}{((I-A)(1+r) - \bar{V})\left[(I-A)(1+r) - \left(\frac{2}{3}+\epsilon\right)\left(R^u - \frac{B}{\epsilon}\right)\right]}. \tag{15}$$

When $A > A_1$, the minimum possible leverage is zero.

In the sense that value-destructive risk-taking can be deterred only if leverage is below this threshold, l^+ is the maximum optimal leverage ratio. Because this maximum leverage ratio reflects the risk-shifting problem, rather than the effort problem, α enters in the expression of l^+ but B does not. On the other hand, l^- is the minimum optimal leverage ratio in the sense that effort can be induced only if leverage is at least as large as this threshold. Correspondingly, B enters the expression of l^- but α does not.

Increasing the intensity of the risk-shifting problem (α) leads to a downward shift in l^+, since, when risk-taking becomes more tempting, leverage must be reduced. This reduces the power of the incentives to exert effort and worsens the credit rationing problem, which is reflected in an increase in A_0.

Since start-ups are characterized by a greater flexibility in defining their activities than are established firms, α is likely to be relatively large in the case of start-ups. It is also likely to be relatively large in high-tech industries during stages of economic development corresponding to the introduction of new technologies or for innovative firms. This gives rise to the following empirical implications. Young, innovative, and high-tech firms should be less levered and rely more on equity financing than other firms. This is consistent with the empirical findings of Bradley et al. (1984), Long and Malitz (1985), and Barclay et al. (1995). Also, these firms are likely to suffer relatively more from credit rationing. This is consistent with the empirical findings of Brown (1997).

Additionally, our theoretical analysis suggests that it could be interesting to explore econometrically the joint determination of investment (as in Brown (1997)) and financial structure (as in Bradley et al. (1984)).

On the other hand, an increase in the cost of effort B leads to an upward shift of the minimum optimal leverage l^-. In that sense, leverage is higher. This is consistent with LBOs taking place in mature sectors where new and better ways of doing business are likely to be hard to find, and where, consequently, B is likely to be large. The upward shift in the minimum possible leverage leads to a greater tension with the maximum possible leverage. This in turn leads to more stringent credit rationing, reflected in an increase in A_0.

V. Aggregate Investment and the Equilibrium Cost of Capital

A. The Economy

So far, we have focused on the problem faced by each individual entrepreneur, taking the cost of capital (r) as given. In this section we endogenize the cost of capital, within the framework of a simple, general equilibrium model.

We consider a unit mass continuum of agents, living and consuming during two periods. For simplicity the agents are risk neutral and have an identical psychological discount rate, denoted ρ.

Each agent is endowed with an investment opportunity. For simplicity, all of the projects are identical and the entrepreneurs differ only in terms of their initial wealth A. Also for simplicity, assume the revenues from the different projects are independent. Hence there is no aggregate uncertainty. The population of entrepreneurs is characterized by the density f of the distribution of initial wealths over $[0,I]$. Its cumulative distribution function is denoted F.

The agents make decisions regarding investment and consumption. Investment opportunities for each agent include lending on the financial market, where the rate of return is r, or investing in his own project, to the extent that he can obtain financing to do so. The rate of return on the financial market stems from the portfolio of all projects that have been funded. Since the returns on the projects are assumed to be independent, and since there is a continuum of investment projects, the rate of return on the financial market r is deterministic.

Consider a given agent, and suppose that he does not invest in his own project. Then his maximization program is

$$\max_{s}(A - s) + \rho\big(s(1 + r)\big), \tag{16}$$

where s is the amount lent on the financial market, $A - s$ is the first period consumption and $s(1 + r)$ is the second period consumption. If $r > (1/\rho) - 1$, the entrepreneur saves A and consumes $A(1 + r)$ at the second period; if $r < (1/\rho) - 1$, the entrepreneur consumes everything at the beginning of the period; and if $r = (1/\rho) - 1(\equiv \underline{r})$ he is indifferent between consuming and saving. Overall, his utility is: $A\max[1,\rho(1 + r)]$.

Now suppose this agent decides to invest in his own project. If he exerts effort and does not take excessive risk, his expected utility is $\overline{V} - (I - A)(1 + r) - B$.

The demand for funds in the financial market stems from all the agents who invest in their own project, and the supply of funds stems from the agents who lend on the financial market. The equilibrium rate of return r on the financial market is set to equate supply and demand.

Assume that at the rate of return equating ρ and $1/(1 + r)$ (i.e., $\underline{r} = (1/\rho) - 1$), the investment projects have strictly positive net present value: $\overline{V} - B > I(1 + \underline{r})$ or, equivalently, $\rho(\overline{V} - B) > I$.

B. Capital Market Equilibrium without Moral Hazard

Momentarily, abstract from the moral hazard problem (i.e., assume that effort and risk choices are observable) so that effort can be obtained and risk avoided without paying agency rents. At the low rate of return $\underline{r}$, all the agents desire to invest in their own project. Indeed,

$$\overline{V} - (I - A)(1 + \underline{r}) - B > A(1 + \underline{r}) \Leftrightarrow \overline{V} - B > I(1 + \underline{r}). \tag{17}$$

So the equilibrium rate of return must be above $\underline{r}$. In fact, equilibrium arises for the value of r such that all investors are indifferent between lending on the market or investing in their own project and earning its internal rate of return $(\overline{V} - B - I)/I$, which we will denote r^*. In this first best equilibrium situation all the wealth available in the economy at the first period is invested in the projects. Hence, aggregate investment is equal to $E(A)$. Since at the first best equilibrium rate of return r^* agents are indifferent between lending on the market and investing in their own project, we can focus, for example, to fix ideas, on the case where all agents with initial wealth below a certain level (A^*) lend, while all the agents with $A > A^*$ invest in their own project. Equilibrium on the financial market requires that

$$\int_{A^*}^{I} (I - A)\, dF(A) = \int_{0}^{A^*} A\, dF(A), \tag{18}$$

where the left-hand side is the demand for funds and the right-hand side is the supply of funds. This market clearing condition is equivalent to

$$A^* = F^{-1}\left(\frac{I - E(A)}{I}\right), \tag{19}$$

where $F^{-1}(.)$ is the inverse of the c.d.f $F(.)$.

C. Capital Market Equilibrium with Moral Hazard

Now we turn to the problem faced by the entrepreneurs under moral hazard. As shown in the previous sections, only entrepreneurs with initial wealth A larger than the minimum level of internal cash A_0 can obtain funding at the equilibrium cost of capital r. Since the minimum level of internal cash necessary to be financed, A_0, depends on the cost of capital, r, we hereafter denote it $A_0(r)$. The incentive compatible demand for funds, stemming from the entrepreneurs who are not rationed, is

$$D^{IC}(r) = \int_{A_0(r)}^{I} (I - A)\, dF(A). \tag{20}$$

Since A_0 is increasing in r, the incentive compatible demand for funds (D^{IC}) is decreasing in the cost of funds (r).

Entrepreneurs with internal cash below A_0 are rationed in the capital market. These agents can either consume in the first period or lend their initial wealth, earn the equilibrium cost of capital r, and consume at the end of the period. For $r < \underline{r}$ they prefer to consume immediately, so that the aggregate supply of funds is $S(r) = 0$. For $r > \underline{r}$, they prefer to lend on the market, so that $S(r) = \int_0^{A_0(r)} A\, dF(A)$. For $r = \underline{r}$, they are indifferent between lending and consuming, and the supply correspondence takes its values in the interval $[0, \int_0^{A_0(\underline{r})} A\, dF(A)]$. In this context, two different regimes can arise on the credit market.

- *The underinvestment regime:* If $A_0(\underline{r}) > A^*$, aggregate investment cannot reach its first best level, for which all available funds are invested. To see this, note that, for all rates $r \geq \underline{r}$, the incentive compatible demand for funds,

  $$\int_{A_0(r)}^{I} (I - A)\, dF(A),$$

  is below its first best level:

  $$\int_{A^*}^{I} (I - A)\, dF(A).$$

  In this case, the equilibrium rate of return is $\underline{r}$, and aggregate investment is

  $$I(1 - F(A_0(\underline{r})),$$

  which is below the first best aggregate investment level,

  $$I(1 - F(A^*)) = E(A),$$

and agents with initial wealth $A < A_0(\underline{r})$ are indifferent between lending on the financial market (to meet the incentive compatible demand for funds) and consuming at the first period.

- *The full investment regime:* On the other hand, if $A_0(\underline{r}) \leq A^*$, the equilibrium rate of return is $\hat{r}$ such that $A_0(\hat{r}) = A^*$. At this rate, the incentive compatible demand for funds,

$$\int_{A_0(\hat{r})}^{I} (I - A)\, dF(A) = \int_{A^*}^{I} (I - A)\, dF(A),$$

and the supply of funds,

$$\int_{0}^{A^*} A\, dF(A) = \int_{0}^{A_0(\hat{r})} A\, dF(A),$$

are the same as in the first best.

Hence the consequences of moral hazard are the following:

1. Some agents are prevented from investing because their initial wealth is too low. If the mass of these credit-constrained agents is large, aggregate investment is lower than in the first best.
2. Because the (incentive compatible) demand curve for funds is lower, the equilibrium cost of capital is also lower.
3. The agents who are able to invest, in spite of the moral hazard problem, because their initial wealth is large enough, earn larger profits than in the first best. On the other hand, the agents who are rationed earn lower profits than in the first best. Hence, moral hazard creates excessive heterogeneity of profits, relative to the first best.

D. Macroeconomic Consequences of Moral Hazard

Having characterized equilibrium on the capital market, we can study how the aggregate investment and the cost of capital vary with changes in the environment.

First consider the situation where risk-taking becomes more appealing for the manager—that is, α increases. In this case, as was shown in the previous section, there is a downward shift in the maximum optimal leverage, and the credit rationing bound A_0 increases. Formally,

$$\frac{\partial A_0}{\partial \alpha} = \frac{B}{\epsilon} \frac{\frac{1}{3}\beta}{(\alpha + \beta)^2} > 0. \tag{21}$$

Similarly, if the cost of effort (B) increases, then the minimum optimal leverage shifts upward and A_0 increases also. Formally,

$$\frac{\partial A_0}{\partial B} = \frac{1}{2}\left(\frac{\frac{1}{3}\alpha}{(\alpha + \beta)} + \frac{1}{3} + \epsilon\right) > 0. \tag{22}$$

Hence, the consequences of worsening the moral hazard problems are transmitted to the economy through A_0. The final impact on the equilibrium cost of capital and aggregate investment is given in the following proposition.

PROPOSITION 8: *An increase in the cost of effort (B) leads to an increase in firms' leverage and a decrease in the equilibrium cost of capital. An increase in the magnitude of the risk-shifting problem (α) leads to a decrease in firms' leverage and in the equilibrium cost of capital. If the worsening of the moral hazard is such that $A_0(\underline{r})$ is raised above A^*, aggregate investment is reduced.*

As the moral hazard problems become more severe, incentive-compatible demand decreases; this lowers the interest rate. Aggregate investment decreases also if the shock on B or α is big enough to switch from the full investment regime to the underinvestment regime.

Our framework can also be used to study the effect of heterogeneity across firms on aggregate economic performance. To address this issue we analyze the consequences of a mean-preserving spread over the distribution of initial wealth across entrepreneurs. A mean-preserving spread can be obtained by shifting mass from the center of the distribution to its extremes. We focus on mean-preserving spreads leading to an increase in the mass of firms in the lower tail of the distribution, below A^*, compensated for by an increase in the mass of firms in the upper tail of the distribution. The initial distribution of wealth across entrepreneurs was denoted $F(.)$; now denote by $G(.)$ the new distribution obtained after this mean-preserving spread. Also, denote by A^{**} the new cut-off level between the agents who, in the first best situation, lend on the financial market and those who invest in their own firm. We then obtain

$$\int_0^{A^{**}} A\,dG(A) = \int_{A^{**}}^{I} (I - A)\,dG(A); \tag{23}$$

that is,

$$G(A^{**}) = \frac{I - E(A)}{I}.$$

Since $G(.)$ is obtained from $F(.)$ by shifting mass to the left of A^*, $A^{**} < A^*$. Now consider a strong increase in the heterogeneity across agents, leading to a significant decrease from A^* to A^{**}, such that $A^* > A_0(\underline{r}) > A^{**}$. This increase in the heterogeneity across agents is accompanied by a switch from the full investment regime to the underinvestment regime.

The intuition is the following: The agents who are credit constrained are those with low initial wealth. Increasing the heterogeneity of the distribution of initial wealth across agents leads to an increase in the mass of people whose initial wealth is so low that they are credit rationed. This can lead to a switch from the full investment regime to the underinvestment regime.

VI. Conclusion

This paper analyzes the optimal financing of investment projects when managers must exert costly unobservable effort and also can engage in value-reducing risky ventures. Thus we revisit the seminal analysis of Jensen and Meckling (1976) within the framework of the theory of optimal contracts. There is a tension between the two sources of moral hazard: coping with the effort problem requires promising large rewards to the manager in good states, but such rewards may be excluded because of the risk-shifting problem. This can lead to rationing in the capital market.[11] If the risk-shifting problem is dominant, the optimal financing scheme is a combination of debt and outside equity. When the effort problem is the major source of moral hazard, stock options awarded to the manager must be added to the array of financial instruments.

The paper also analyzes the macroeconomic implications of this microeconomic analysis, and underlines the consequences of the moral hazard problems for the optimal leverage, the aggregate level of investment, and the equilibrium cost of capital. Worsening moral hazard problems lead to more severe credit rationing, which can reduce aggregate investment. When this shock is due to the risk-shifting problem, it is accompanied by a decrease in leverage; if it is due to an increase in the effort problem it corresponds to an increase in leverage.

Appendix

In the proofs, for the sake of brevity, we will refer to the incentive compatibility condition under which the entrepreneur exerts effort as $(IC)_{effort}$. Similarly, we will refer to the incentive compatibility condition under which the entrepreneur does not engage in value-destructive risky activities as $(IC)_{risk}$, and to the participation constraint of the outside investors as (PC).

Proof of Proposition 1: To establish the proposition we must determine the conditions under which there exists a triplet $(\delta^u, \delta^m, \delta^d)$ in $[0,1]^3$ such that $(IC)_{risk}$, $(IC)_{effort}$, and (PC) hold. First note that there is no loss of generality in considering the case where (PC) holds as an equality, and therefore is equivalent to

$$\delta^u = \frac{(1+r)(I-A) - 1/3\delta^m R^m - (1/3-\epsilon)\delta^d R^d}{(1/3+\epsilon)R^u}. \tag{A1}$$

[11] Thus our paper contributes to meeting the demand for further research on "the way in which different agency problems interact" stated in the conclusion of the survey on corporate financial structure, incentives, and optimal contracting, by Allen and Winton (1995).

Consequently, $\delta^u \in [0,1]$ is equivalent to

$$\delta^m \leq \frac{(1+r)(I-A) - (1/3-\epsilon)\delta^d R^d}{1/3R^m} \tag{A2}$$

and

$$\delta^m \geq \frac{(1+r)(I-A) - (1/3+\epsilon)R^u - (1/3-\epsilon)\delta^d R^d}{1/3R^m}. \tag{A3}$$

Denote by $\kappa(\delta^d)$ the right-hand side of equation (A2). Substituting the value of δ^u obtained from (PC) into $(IC)_{risk}$ and $(IC)_{effort}$ gives

$$\delta^m \geq \frac{(1+r)(I-A) - (1/3+\epsilon)\left(R^u - R^d - \dfrac{B}{\epsilon}\right) - 2/3\delta^d R^d}{1/3R^m}, \tag{A4}$$

$$\delta^m \leq \frac{\alpha(1+r)(I-A) + (1/3+\epsilon)(\beta(R^m - R^d) - \alpha(R^u - R^m)) - [\alpha(1/3-\epsilon) - \beta(1/3+\epsilon)]\delta^d R^d}{[\alpha(2/3+\epsilon) + \beta(1/3+\epsilon)]R^m}. \tag{A5}$$

Denote by $\varphi(\delta^d)$ the right-hand side of equation (A4) and $\phi(\delta^d)$ the right-hand side of equation (A5). Note that equation (A3) is redundant given equation (A4). Hence, there exists an admissible contract if and only if $\exists(\delta^m,\delta^d) \in [0,1]^2$ such that (PC), equations (A2), (A4), and (A5) hold. This is equivalent to $\exists\delta^d \in [0,1]$ and $\delta^m \in \left[\max[\varphi(\delta^d),0];\min[\phi(\delta^d),\kappa(\delta^d),1]\right]$ such that (PC) holds. Now, $\max[\varphi(\delta^d),0] < \min[\phi(\delta^d),\kappa(\delta^d),1]$ iff

$$\delta^d \geq \frac{(1+r)(I-A) - \overline{V} + R^d + \dfrac{B}{\epsilon}\left(1/3 + \epsilon + 1/3\dfrac{\alpha}{\alpha+\beta}\right)}{R^d}, \tag{A6}$$

$$\delta^d \geq \frac{(1+r)(I-A) - \overline{V} + 2/3R^d + \dfrac{B}{\epsilon}(1/3+\epsilon)}{2/3R^d}, \tag{A7}$$

$$\delta^d \leq \frac{(1+r)(I-A)}{(1/3-\epsilon)R^d}. \tag{A8}$$

Denote by ϱ the right-hand side of equation (A6), ξ the right-hand side of equation (A7), and ς the right-hand side of equation (A8). There exists an optimal contract defined by:

$$\begin{cases} \delta^u = \dfrac{(1+r)(I-A) - 1/3\delta^m R^m - (1/3-\epsilon)\delta^d R^d}{(1/3+\epsilon)R^u}, \\ \delta^m \in \left[\max[\varphi(\delta^d),0];\min[\phi(\delta^d),\kappa(\delta^d),1]\right], \\ \delta^d \in \left[\max[\varrho,\xi,0];\min[\varsigma,1]\right], \end{cases} \tag{A9}$$

iff $\max[\varrho,\xi,0] < \min[\varsigma,1]$. This is the case iff

$$A \geq I - \frac{1}{1+r}\left[\overline{V} - \frac{B}{\epsilon}\left(\frac{1}{3} + \epsilon + \frac{1}{3}\frac{\alpha}{\alpha+\beta}\right)\right].$$

Q.E.D.

Proof of Lemma 1: A pure equity contract is defined by $\delta^u = \delta^m = \delta^d = \delta$. Replace δ^u, δ^m, and δ^d by δ in all the constraints. Note that $(IC)_{risk}$ always holds. Hence an optimal contract with outside equity financing is possible iff $\exists\delta \in [0,1]$ such that $(IC)_{effort}$ and (PC) hold. The former can be rewritten as $\delta \leq 1 - \{B/[\epsilon(R^u - R^d)]\}$, and the latter can be rewritten as $\delta = [(1+r)(I-A)]/\overline{V}$. Note that $[(1+r)(I-A)]/\overline{V}$ always lies between 0 and 1. We can thus conclude that the project can be financed with outside equity iff

$$A \geq I - \frac{1}{1+r}\left[\overline{V} - \frac{B}{\epsilon}\frac{\overline{V}}{R^u - R^d}\right]. \tag{A10}$$

Q.E.D.

Proof of Lemma 2: In order to relax (PC) as much as possible, the amount of debt to be repaid must be set as large as possible. Note, however, that very risky debt (i.e., with face value D greater than R^m) is impossible because of the risk-shifting problem: If the manager is only compensated in state u, she will always choose to take risks. So we are looking for a debt contract with $R^d \leq D < R^m$. Replacing δ^u, δ^m, and δ^d by the expressions defined at the end of Section I in all constraints, it follows that an optimal contract with outside debt financing exists iff $\exists D \in [R^d,R^m[$ such that

$$D = \frac{(1+r)(I-A) - (1/3-\epsilon)R^d}{(2/3+\epsilon)}, \tag{A11}$$

$$D \leq R^u - \frac{B}{\epsilon}, \tag{A12}$$

$$D \leq R^m - \frac{\alpha}{\beta}(R^u - R^m). \tag{A13}$$

Note that $D < R^m$ is always satisfied when equation (A13) is. Substituting equation (A11) into equations (A12) and (A13), it is possible to finance the project with outside risky debt iff[12]

$$A \geq A_2 \equiv I - \frac{1}{1+r}\left[\overline{V} + \frac{1}{3}(R^u - R^m) - \left(\frac{2}{3} + \epsilon\right)\max\left[\frac{B}{\epsilon}; \frac{\alpha+\beta}{\beta}(R^u - R^m)\right]\right]. \tag{A14}$$

Q.E.D.

Proof of Proposition 3: We consider two cases: an optimal mix of debt and equity, then such a mix complemented by stock options.

Case 1. Implementing the optimal contract by an optimal mix of debt and equity:

(i) The first step in the proof is to specify the conditions under which the abstract contract δ^u, δ^m, and δ^d translates into a mix of debt with face value D and outside equity with dilution δ. These conditions are

$$\delta^u R^u = D + \delta(R^u - D), \tag{A15}$$

$$\delta^m R^m = D + \delta(R^m - D), \tag{A16}$$

$$\delta^d = 1,$$

$$\delta \in [0,1], \tag{A17}$$

$$D \in [R^d, \delta^m R^m]. \tag{A18}$$

Combining equations (A15) and (A16) gives

$$\delta = \frac{\delta^u R^u - \delta^m R^m}{R^u - R^m}, \tag{A19}$$

$$D = \frac{(\delta^m - \delta^u) R^u R^m}{(1 - \delta^u) R^u - (1 - \delta^m) R^m}. \tag{A20}$$

Note that equation (A16) and $\delta^m \leq 1$ imply $D \leq \delta^m R^m$. Substituting these values of δ and D in conditions (A17) and (A18), it follows that a contract can be interpreted as a mix of debt and equity iff

[12] If $A > I - (R^d/(1+r))$, that is if A is very large and very little outside financing is needed, safe debt, with $D < R^d$, is issued.

$$\delta^u \leq \frac{\delta^m (R^u - R^d) + R^d - R^u \dfrac{R^d}{R^m}}{\left(1 - \dfrac{R^d}{R^m}\right) R^u}, \tag{A21}$$

$$\delta^u \geq \frac{\delta^m R^m}{R^u}, \tag{A22}$$

$$\delta^u \leq \frac{\delta^m R^m + (R^u - R^m)}{R^u}. \tag{A23}$$

Note that equation (A21) implies equation (A23).

(ii) The second step in the proof is to examine when these conditions on δ^u, δ^m, and δ^d are consistent with the incentive compatibility and participation constraints. To do this we rewrite the incentive and participation constraints with $\delta^d = 1$ and substitute the value of δ^u obtained by saturating (PC) into equations (A21) and (A22). It follows that an optimal contract can be implemented by a mix of debt and equity iff $\exists \delta^m \in \big[\max[\varphi(1),0];\min[\phi(1),\kappa(1),1]\big]$ such that

$$\delta^m \leq \frac{(1+r)(I-A) - (1/3-\epsilon)R^d}{(2/3+\epsilon)R^m}, \tag{A24}$$

$$\delta^m \geq \frac{(R^m - R^d)[(1+r)(I-A) - (1/3-\epsilon)R^d] + (1/3+\epsilon)R^d(R^u - R^m)}{[(1/3+\epsilon)(R^u - R^d) + 1/3(R^m - R^d)]R^m}. \tag{A25}$$

Denote by J the right-hand side of equation (A24) and K the right-hand side of equation (A25). Note that $\kappa(1) > J$. The optimal contract can be implemented by a mix of debt and equity iff the interval $\big[\max[\varphi(1);K;0];\min[\phi(1);J;1]\big]$ exists. After some manipulations, one can find that it is the case iff

$$\begin{cases} A \geq I - \dfrac{1}{1+r}\left[\overline{V} - \dfrac{B}{\epsilon}\left(\dfrac{1}{3} + \epsilon + \dfrac{1}{3}\dfrac{\alpha}{\alpha+\beta}\right)\right] \\ \quad \text{if } R^m \leq R^u - \dfrac{\beta}{\alpha+\beta}\dfrac{B}{\epsilon} \\ A \geq I - \dfrac{1}{1+r}\left[\overline{V} + 1/3(R^u - R^m) - \dfrac{B}{\epsilon}(2/3+\epsilon)\right] \\ \quad \text{if } R^m > R^u - \dfrac{\beta}{\alpha+\beta}\dfrac{B}{\epsilon}. \end{cases} \tag{A26}$$

Note that the minimum cash needed is equal to A_0 when $R^m \leq R^u - [\beta/(\alpha + \beta)](B/\epsilon)$; in this case, the optimal contract can be implemented by a mix of debt and equity, whatever the level of internal cash. When $R^m > R^u - [\beta/(\alpha + \beta)](B/\epsilon)$, the minimum cash needed to be financed with a mix of debt and equity is strictly greater than A_0. In that case, when $A \in \left[A_0, I - [1/(1 + r)][\bar{V} + 1/3(R^u - R^m) - (B/\epsilon)(2/3 + \epsilon)]\right]$ the optimal contract cannot be implemented by a mix of debt and equity.

Case 2. Implementing the optimal contract by an optimal mix of debt and equity complemented by stock options: In this case, in addition to the outside debt (D) and dilution δ, the contract now specifies that γ additional shares will be created and will be handed out free to the manager if the high cash flow R^u is obtained. Hence the previous contract is modified as follows:

$$\delta^u R^u = D + \frac{\delta}{1+\gamma}(R^u - D), \tag{A27}$$

$$\gamma \geq 0\,. \tag{A28}$$

Note that conditions (A16), (A17), and (A18) are still required. Combining conditions (A16) and (A17) gives

$$\delta = \frac{\delta^m R^m - D}{R^m - D}, \tag{A29}$$

$$\gamma = \frac{(\delta^m R^m - D)(R^u - D) - (\delta^u R^u - D)(R^m - D)}{(\delta^u R^u - D)(R^m - D)}. \tag{A30}$$

With δ defined below, it is easy to see that equation (A17) is always satisfied by equation (A18) and $\delta^m \leq 1$. Replacing γ by its value in condition (A28) gives the following:

(i) if $\delta^u R^u < D$, then

$$(\delta^m R^m - D)(R^u - D) + (D - \delta^u R^u)(R^m - D) < 0, \tag{A31}$$

which is not compatible with $D \leq \delta^m R^m$.

(ii) if $\delta^u R^u > D$, then

- if $(1 - \delta^u)R^u - (1 - \delta^m)R^m > 0$,

$$D < \frac{(\delta^m - \delta^u)R^u R^m}{(1 - \delta^u)R^u - (1 - \delta^m)R^m}; \tag{A32}$$

- if $(1 - \delta^u)R^u - (1 - \delta^m)R^m < 0$,

$$D > \frac{(\delta^m - \delta^u)R^u R^m}{(1 - \delta^u)R^u - (1 - \delta^m)R^m}. \tag{A33}$$

It can be shown easily that equation (A33) in case $(1 - \delta^u)R^u - (1 - \delta^m) R^m < 0$ is not consistent with $D \leq \delta^m R^m$. Hence there is only one case to consider. There exists a contract with outside debt, dilution, and stock options iff

$$\exists D \in \left[R^d ; \min\left[\delta^m R^m ; \frac{(\delta^m - \delta^u)R^u R^m}{(1 - \delta^u)R^u - (1 - \delta^m)R^m} ; \delta^u R^u \right]\right],$$

with $(1 - \delta^u)R^u - (1 - \delta^m)R^m > 0$. Then we proceed as before, searching for conditions on δ^m and δ^u so that such an interval exists. It follows that to implement this contract, δ^m and δ^u must verify the following:

$$\delta^m \geq \frac{R^d}{R^m}, \tag{A34}$$

$$\delta^u \geq \frac{R^d}{R^u}, \tag{A35}$$

$$\delta^m \geq \frac{\delta^u R^u (R^m - R^d) + R^d (R^u - R^m)}{(R^u - R^d)R^m}, \tag{A36}$$

$$\delta^m \geq \frac{\delta^u R^u - (R^u - R^m)}{R^m}. \tag{A37}$$

Substituting the value of δ^u obtained from saturating (PC), we obtain

$$(A35) \Leftrightarrow \delta^m \leq \frac{(1 + r)(I - A) - 2/3 R^d}{1/3 R^m}$$

denoted $\delta^m \leq L$.

$$(A36) \Leftrightarrow \delta^m \geq \frac{(R^m - R^d)[(1 + r)(I - A) - (1/3 - \epsilon)R^d] + (1/3 + \epsilon)(R^u - R^m)R^d}{(\overline{V} - R^d)R^m}$$

already denoted $\delta^m \leq K$.

$$(A37) \Leftrightarrow \delta^m \geq \frac{(1 + r)(I - A) - \overline{V} + (2/3 + \epsilon)R^m}{(2/3 + \epsilon)R^m}$$

denoted $\delta^m \leq M$.

Hence the optimal contract can be implemented by outside debt, outside equity, and stock options iff

$$\exists \delta^m \in \left[\max\left[\frac{R^d}{R^m}; K; M; \varphi(1)\right]; \min[1; L; \phi(1)]\right].$$

It is easy to check that such an interval exists iff $A \geq A_0$. Q.E.D.

Proof of Proposition 4: The contract specifies the amount of debt repayment D and the fraction of shares δ the investor can get if he chooses to convert his bonds. Hence, for each state θ, the investor gets $\max\left[\min[R^\theta;D];\delta R^\theta\right]$. Consider the case where debt is risky—that is, $D \in [R^d;R^m[$ (as before, riskless debt will not implement the optimal contract for low values of A). D and δ can be set such that either the investor converts in both states m and u or he only converts in state u. It can be shown easily that the first type of convertible bond cannot implement the optimal contract for low values of A, hence we will focus on the second case. The contract can be interpreted as convertible bonds iff

$$\delta^d = 1,$$

$$\delta^m R^m = D,$$

$$\delta^u R^u = \delta R^u,$$

$$\delta \in [0,1], \tag{A38}$$

$$D \in [R^d;R^m], \tag{A39}$$

$$D \geq \delta R^m, \tag{A40}$$

$$D \leq \delta R^u. \tag{A41}$$

Hence we have $D = \delta^m R^m$ and $\delta = \delta^u$. Replacing D and δ by their value in the conditions above and substituting (PC) in all constraints, we find that the optimal contract can be implemented by convertible bonds iff

$$\begin{cases} \delta^m \geq \dfrac{(1+r)(I-A) - (1/3-\epsilon)R^d}{\overline{V} - (1/3-\epsilon)R^d} \\[2ex] \delta^m \leq \dfrac{(1+r)(I-A) - (1/3-\epsilon)R^d}{(2/3+\epsilon)R^m} \\[2ex] \delta^m \geq \dfrac{R^d}{R^m} \\[2ex] \delta^m \geq \varphi(1) \\ \delta^m \leq \phi(1). \end{cases} \tag{A42}$$

When solving for the existence of this interval, one finds that convertible bonds can implement the optimal contract iff

$$\begin{cases} A \geq I - \dfrac{1}{1+r}\left[\overline{V} - \dfrac{B}{\epsilon}\left(\dfrac{1}{3} + \epsilon + \dfrac{1}{3}\dfrac{\alpha}{\alpha+\beta}\right)\right] \\ \quad \text{if } R^m \leq R^u - \dfrac{\beta}{\alpha+\beta}\dfrac{B}{\epsilon} \\ A \geq I - \dfrac{1}{1+r}\left[\overline{V} + 1/3(R^u - R^m) - \dfrac{B}{\epsilon}(2/3+\epsilon)\right] \\ \quad \text{if } R^m > R^u - \dfrac{\beta}{\alpha+\beta}\dfrac{B}{\epsilon}. \end{cases} \tag{A43}$$

This is exactly the same condition as needed to implement the contract with a mix of outside debt and outside equity. Q.E.D.

The proofs of Propositions 5 and 6 can easily be established by following the steps of the proofs of Propositions 1 and 3. They are omitted here for the sake of brevity.

Proof of Proposition 7: Let $\kappa(D)$ be the expected value of debt repayment. Leverage is defined by $l = \kappa(D)/[(1 + r)(I - A) - \kappa(D)]$. See that leverage is maximal when $\kappa(D)$ is maximal. Assuming for simplicity that $R^d = 0$, $\kappa(D)$ can be written as $(2/3 + \epsilon)D$ (remember that we always have $D < R^m$ because of the risk-shifting problem).

(i) Determining l^+: Recall that

$$D = \frac{(\delta^m - \delta^u)R^u R^m}{(1-\delta^u)R^u - (1-\delta^m)R^m}.$$

Replacing D by its value into $\kappa(D)$ and δ^u by (PC) gives

$$\kappa(D) = (2/3+\epsilon)R^m\left[\frac{(1+r)(I-A) - \overline{V}\delta^m}{(1+r)(I-A) - (1/3+\epsilon)(R^u - R^m) - (2/3+\epsilon)\delta^m R^m}\right]. \tag{A44}$$

For a given A, as δ^m lies in an interval $[\delta^{m-};\delta^{m+}]$, there exists a continuum of possible values for $\kappa(D)$. We look for the value of δ^m such that $\kappa(D)$ is maximized. See that $\partial\kappa(D)/\partial\delta^m > 0$. Hence, to determine l^+ we must compute $\kappa(D)$ at the point δ^{m+} in the interval of choice of δ^m. From the proof of Proposition 3, we know that $\delta^m \in \big[\max[\varphi(1);K;0];\min[\phi(1);J;1]\big]$. It is easy to see that

$$\delta^{m+} = \phi(1) \quad \text{for } A \leq A_2, \tag{A45}$$

$$\delta^{m+} = J \qquad \text{for } A > A_2. \tag{A46}$$

Replacing δ^{m+} into $\kappa(D)$ and $\kappa(D)$ into l^+, we obtain the following:

- for $A \in [A_0;A_2]$,

$$l^+ = \frac{(2/3+\epsilon)\,\dfrac{(\alpha+\beta)R^m - \alpha R^u}{\beta}}{(1+r)(I-A) - \left[(2/3+\epsilon)\,\dfrac{(\alpha+\beta)R^m - \alpha R^u}{\beta}\right]}; \tag{A47}$$

- for $A > A_2$, $\kappa(D) = (1+r)(I-A)$ and l^+ is infinite; that is, the firm is 100 percent levered.

(ii) Determining l^-: l^- is determined by δ^{m-}. Assume again that $R^d = 0$. Then,

$$\delta^{m-} = \varphi(1) \quad \text{for } A \leq A_1, \tag{A48}$$

$$\delta^{m-} = K \qquad \text{for } A > A_1. \tag{A49}$$

Note that for $A > A_1$ a pure equity financing is possible; it is therefore the contract that minimizes leverage. In this case, see that $\delta^{m-} = K = [(1+r)(I-A)]/\overline{V}$. Replacing δ^{m-} into $\kappa(D)$ and l^- gives

- for $A \in [A_0;A_1]$,

$$l^- = \frac{\left(\dfrac{2}{3}+\epsilon\right)\left[R^u[(I-A)(1+r) - \overline{V}] + \overline{V}\dfrac{B}{\epsilon}\right]}{((I-A)(1+r) - \overline{V})\left[(I-A)(1+r) - \left(\dfrac{2}{3}+\epsilon\right)\left(R^u - \dfrac{B}{\epsilon}\right)\right]}. \tag{A50}$$

- for $A > A_1$, $l^- = 0$ and the firm is 100 percent equity-financed. Q.E.D.

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