THE JOURNAL OF FINANCE • VOL LIII, NO. 2 • APRIL 1998

Resolving the Puzzling Intertemporal Relation between the Market Risk Premium and Conditional Market Variance: A Two-Factor Approach

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ABSTRACT

The existing empirical literature fails to agree on the nature of the intertemporal relation between risk and return. This paper attempts to resolve the issue by estimating a conditional two-factor model motivated by Merton's intertemporal capital asset pricing model. When long-term government bond returns are included as a second factor, the *partial* relation between the market risk premium and conditional market variance is found to be positive and significant. The paper also helps explain the convoluted empirical relation between the market risk premium, conditional market variance, and the nominal risk-free rate previously reported in the literature.

GIVEN THE CENTRAL ROLE played by the market portfolio in financial theory, it is not surprising that dozens of papers have studied the intertemporal relation between the market risk premium and conditional market variance. It is surprising, however, that the empirical nature of this important relationship has not been resolved. Theory generally predicts a positive relation between the market risk premium and conditional market variance if investors are risk averse.[1] Yet, empirical studies to date fail to agree on the sign of this important relation.

Most papers on the intertemporal behavior of the market risk premium employ a conditional single factor model motivated by the static capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965). These conditional single-factor models imply a *simple* linear or proportional relation between the market risk premium and conditional market variance. Estimates of the *simple* risk–return relation range from significant positive (Har-

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[1] Backus and Gregory (1993) argue that the theoretical relation between the market risk premium and conditional market variance is not necessarily a positive, linear function. In general, the function depends on the preferences of the representative agent and the stochastic nature of the economy.

vey (1989), Turner, Startz, and Nelson (1989)) to significant negative (Campbell (1987), Glosten, Jagannathan and Runkle (1993)). Table I summarizes the empirical results of several representative papers from this literature. Given the theoretical and practical importance of the market risk premium, the inconclusive state of the literature is unsatisfactory.

This paper attempts to resolve the issue by using a conditional two-factor model of the market risk premium motivated by Merton's (1973) intertemporal capital asset pricing model (ICAPM). In the conditional two-factor model, the market risk premium is a function of conditional market variance and conditional market covariance with a variable that describes the state of investment opportunities in the economy. The relation between the market risk premium and conditional market variance is a *partial* relation in this model. If the "true" model is a conditional multifactor model, then conditional single-factor models are misspecified and estimates of the *simple* risk–return relation are subject to an omitted variable bias. This paper explores whether a conditional two-factor model explains the intertemporal behavior of the market risk premium better than the conditional single factor models used in the existing literature.

State variables in the ICAPM arise because investors desire to hedge against adverse changes in available investment opportunities. Merton (1973) suggests that "one should interpret the effects of a changing interest rate . . . in the way economists have generally done in the past: namely, as a single (instrumental) variable representation of shifts in the investment opportunity set." Long-term bonds are a natural instrument for hedging interest rate risk because their returns are negatively, albeit not perfectly, correlated with changes in interest rates. I employ excess returns on a long-term government bond as the second variable in a conditional two-factor model. Long-term bond returns have long been used in empirical studies of multifactor asset pricing models. Chen, Roll, and Ross (1986) find that exposure to interest rate risk is significantly priced in the stock market. Shanken (1990) tests a conditional two-factor version of the ICAPM and finds that both stock and bond market exposures are priced. However, few studies have employed multifactor models to examine the intertemporal behavior of the market risk premium.

Recent studies by Ferson and Harvey (1991) and Evans (1994) highlight the importance of understanding the intertemporal behavior of the market risk premium. These papers examine whether predictable variation in portfolio returns is attributable to time-varying betas or to time-varying factor risk premia. Both studies conclude that most of the predictable variation in returns can be attributed to predictable variation in risk premia. The finding that conditional betas are relatively constant over time underscores the importance of furthering our understanding of the intertemporal behavior of the market risk premium.

This paper has two objectives. The first objective is to examine whether a conditional two-factor model explains the intertemporal behavior of the market risk premium better than existing single-factor models. My main empir-

Table I

Summary of the Empirical Literature

This table summarizes the empirical literature on the intertemporal relationship between the monthly market excess return ($r_{M,t}$) and conditional market variance ($\hat{\sigma}^2_{M,t}$). Baillie and DeGennaro (1990) use conditional market standard deviation rather than conditional market variance to estimate the risk–return relationship. Glosten et al.'s (1993) Modified GARCH(1,1)-M model includes the one-month Treasury bill in the conditional variance specification. VWRETD and VWRETX are the CRSP value-weighted equity indices with and without dividends, respectively. Estimation methodologies include maximum likelihood (ML), quasi-maximum likelihood (QML), and generalized method of moments (GMM). Baillie and DeGennaro (1990) estimate each model twice; assuming alternate distributions (normal and t) for residuals.

Paper	Market Proxy	Sample Period (year:month)	Model of Conditional Market Variance	Estimation Methodology	Risk–Return Relation	Empirical Relation between $r_{M,t}$ and $\hat{\sigma}^2_{M,t}$
French, Schwert, and Stambaugh (1987)	CRSP VWRETD	28:2–84:12	GARCH(1,2)-M	ML	Simple linear	Insignificant positive
		28:2–52:12	GARCH(1,2)-M	ML	Simple linear	Insignificant positive
		53:1–84:12	GARCH(1,2)-M	ML	Simple linear	Insignificant positive
Campbell (1987)	CRSP VWRETD	59:2–79:8	Instrumental variables	GMM	Simple linear	Significant negative
Harvey (1989)	CRSP VWRETD	41:9–87:12	Instrumental variables	GMM	Simple proportional	Significant positive
Turner, Startz, and Nelson (1989)	S&P 500	46:1–87:12	Two-state Markov:			
			Certainty	ML	Simple linear	Significant negative
			Uncertainty	ML	Simple linear	Significant positive
			Uncertainty/Learning	ML	Simple linear	Significant positive
Baillie and DeGennaro (1990)	CRSP VWRETX	28:2–84:12	GARCH(1,1)-M	ML (normal)	Simple linear	Significant positive
			GARCH(1,1)-M	ML (t dist)	Simple linear	Insignificant positive
			GARCH(1,2)-M	ML (normal)	Simple linear	Insignificant positive
			GARCH(1,2)-M	ML (t dist)	Simple linear	Insignificant positive
Glosten, Jagannathan, and Runkle (1993)	CRSP VWRETD	51:4–89:12	GARCH(1,1)-M	QML	Simple linear	Insignificant positive
			Mod GARCH(1,1)-M	QML	Simple linear	Significant negative

ical results suggest that it does. I find evidence of a significant positive *partial* relation between market excess returns and conditional market variance. The *partial* relation between market excess returns and conditional covariance between stock and bond returns is negative and significant. I show that estimates of the *simple* risk–return relation obtained from conditional single-factor models may be biased downward due to the omission of an interest rate–related state variable from the conditional market risk premium equation. The magnitude of the bias is sufficient to affect inference regarding the significance of the risk–return relation.

The paper's second objective is to explain the convoluted empirical relationship between the market risk premium, conditional market variance, and the nominal risk-free rate. Campbell (1987) and Glosten et al. (1993) report a strong negative *simple* relation between the market risk premium and conditional market variance when the nominal one-month Treasury bill yield is included in the conditioning information set. The negative risk–return relation is absent when the T-bill yield is removed. Campbell describes this result as "perverse." I show that including the nominal one-month T-bill yield in the conditional market variance equation distorts estimates of the *simple* risk–return relation.

The remainder of the paper is organized as follows. Section I derives testable conditional single- and two-factor models of the market risk premium from Merton's (1973) ICAPM. Section II discusses estimation methodology. I assume that stock and bond returns follow a constant correlation bivariate EGARCH-M process. Section III describes the data and reports the main empirical results. In Section IV, I investigate the convoluted empirical relationship between the market risk premium, conditional market variance, and the nominal risk-free rate reported by Campbell (1987) and Glosten et al. (1993). Section V concludes the paper.

I. Empirical Models of the Market Risk Premium

This section of the paper describes the empirical models I employ to estimate and test the intertemporal relation between the market risk premium and conditional second moments. This paper's primary innovation is the use of a conditional two-factor model motivated by Merton's ICAPM. Conditional single-factor models are shown to be special cases nested within the more general conditional two-factor model. Empirical models of conditional second moments are discussed in Section II.

A. Theoretical Model

The underlying theoretical asset pricing model is a conditional two-factor model motivated by Merton's (1973) ICAPM. The model is set in a continuous-time economy where both asset prices and state variables are assumed to follow diffusion processes. I assume the existence of a risk-averse representative agent with the derived utility of wealth function $J(W(t), F(t), t)$, where

$W(t)$ is wealth and $F(t)$ is a variable describing the state of investment opportunities in the economy. In equilibrium, the expected market risk premium is

$$E_{t-1}[r_{M,t}] = \left[\frac{-J_{WW}W}{J_W}\right]\sigma_{M,t}^2 + \left[\frac{-J_{WF}}{J_W}\right]\sigma_{MF,t}, \tag{1}$$

where $E_{t-1}[\cdot]$ is the expectation operator conditional on information available at time $t - 1$, and $\sigma_{M,t}^2$ and $\sigma_{MF,t}$ are, respectively, market variance and market covariance with the state variable F conditional on information available at time $t - 1$. Subscripts of J denote partial derivatives.

The term $[-J_{WW}W/J_W]$ in equation (1) is the coefficient of relative risk aversion. Because risk aversion implies $J_W > 0$ and $J_{WW} < 0$, the model suggests a positive *partial* relation between the market risk premium and conditional market variance. The sign of the *partial* relation between the market risk premium and conditional market covariance with the state variable is more complicated. Nonsatiation implies that $J_W > 0$, so the sign of $[-J_{WF}/J_W]$ in equation (1) is opposite that of J_{WF}. If marginal utility of wealth is state-independent ($J_{WF} = 0$), the expected market risk premium is solely a function of conditional market variance. However, if $J_{WF} \neq 0$, the expected market risk premium is also a function of conditional market covariance with the state variable F. If $J_{WF} > 0$ and $\sigma_{MF,t} > 0$, or if $J_{WF} < 0$ and $\sigma_{MF,t} < 0$, investors will demand a lower risk premium on the market portfolio. In these cases, the market portfolio tends to pay off in states where marginal utility of wealth is high. Investors will demand a higher risk premium if $J_{WF} > 0$ and $\sigma_{MF,t} < 0$, or if $J_{WF} < 0$ and $\sigma_{MF,t} > 0$. In these cases, the market portfolio tends to pay off in states where marginal utility of wealth is low.

In its most general form, all of the terms in equation (1) could be time-varying. To be empirically tractable, additional assumptions regarding the intertemporal nature of the utility function, market variance, and market covariance with the state variable must be made. I assume that conditional second moments are time-varying and follow EGARCH (exponential generalized autoregressive conditional heteroskedastic) processes. These EGARCH models are described in Section II. This section is concerned with empirical models of the market risk premium that emerge when additional restrictions are placed on equation (1).

B. Conditional Single-Factor Models

The first two empirical models I consider are conditional versions of the familiar Sharpe–Lintner CAPM. Merton (1973) shows that the ICAPM reduces to the Sharpe–Lintner CAPM if the investment opportunity set is static or if investors exhibit logarithmic utility. In terms of equation (1), the conditional CAPM results if it is assumed that $\sigma_{MF,t} = 0$ for all t or that $J_{WF} = 0$.

If it is further assumed that the coefficient of relative risk aversion, $[-J_{WW}W/J_W]$, is an intertemporal constant, then the result is a conditional single-factor model with a *simple proportional* relation between the market risk premium and conditional market variance.

Model 1:

$$r_{M,t} = \lambda_M \sigma_{M,t}^2 + \epsilon_{M,t}. \tag{2}$$

Risk aversion implies that $\lambda_M > 0$. Merton (1980) and Harvey (1989) employ this model.

Model 2 relaxes the restriction that the risk–return relation is proportional by including a constant term in the asset pricing model.

Model 2:

$$r_{M,t} = \lambda_0 + \lambda_M \sigma_{M,t}^2 + \epsilon_{M,t}. \tag{3}$$

The constant, λ_0, is included to account for market imperfections such as taxes, transaction costs, or preferred habitats.[2] In Model 2, there is a *simple linear* relation between the market risk premium and conditional market variance. Risk aversion implies that $\lambda_M > 0$.

Model 2 has been widely employed in studies of the intertemporal relation between the market risk premium and conditional market variance. Examples include Campbell (1987), French et al. (1987), Pagan and Hong (1989), Bodurtha and Mark (1991), and Glosten et al. (1993).

C. Conditional Two-Factor Model

Conditional two-factor models relax the assumption that the investment opportunity set is static or that investors exhibit logarithmic utility. In Model 3, I assume that both $[-J_{WW}W/J_W]$ and $[-J_{WF}/J_W]$ in equation (1) are intertemporal constants. As in Model 2, I include a constant in the model to account for market imperfections.

Model 3:

$$r_{M,t} = \lambda_0 + \lambda_M \sigma_{M,t}^2 + \lambda_F \sigma_{MF,t} + \epsilon_{M,t}. \tag{4}$$

The coefficient λ_M measures the *partial* relation between the market risk premium and conditional market variance. λ_M is the coefficient of relative risk aversion and should be positive. The coefficient λ_F measures the *partial* relation between the market risk premium and conditional market covari-

[2] Consider the case of taxes. Let τ_M and τ_f be the representative investor's marginal tax rates on market portfolio and risk-free asset returns. And let $R_{M,t}$ be the total return on the market portfolio. If Model 1 holds for *after-tax* returns, estimating the risk–return relation with *pretax* returns introduces a constant, $\lambda_0 = (\tau_M R_{M,t} - \tau_f r_{f,t})$, into the asset pricing model. If the tax code gives preferential treatment to stock returns relative to T-bill returns (i.e., $\tau_M < \tau_f$), then the unconditional mean of market excess returns ($\bar{r}_M$) is an estimate of the upper bound of λ_0.

ance with the state variable. The sign of λ_F should be the opposite of the sign of J_{WF}.

If the "true" asset pricing model is a conditional two-factor model such as Model 3, then estimates of the *simple* risk–return relation from a conditional single-factor model such as Model 2 may be biased. For illustration, assume that Model 3 is the "true" model and that the econometrician estimates Model 2. Resulting estimates of the *simple* risk–return relation, $\hat{\lambda}_M$, will have the following bias:

$$\hat{\lambda}_M - \lambda_M = \lambda_F \frac{\operatorname{cov}(\sigma_{M,t}^2, \sigma_{MF,t})}{\operatorname{var}(\sigma_{M,t}^2)}. \tag{5}$$

The direction of the bias is determined by the signs of J_{WF} and $\operatorname{cov}(\sigma_{M,t}^2, \sigma_{MF,t})$.

II. Methodology

A. Empirical Models of Conditional Second Moments

Estimating and testing the models derived in Section I requires specification of an empirical model for conditional second moments. My choice of models is motivated by the empirical literature on market volatility.

I assume that conditional second moments follow EGARCH processes.[3] The GARCH family of models assumes that the market conditions its expectation of market variance on both past conditional market variance and past market return innovations.[4] The EGARCH model, a refinement of the GARCH model, imposes a nonnegativity constraint on $\hat{\sigma}_{M,t}^2$ and allows for conditional variance to respond asymmetrically to return innovations of different signs. An EGARCH-M (EGARCH-in-Mean) system of equations is formed by combining one or more conditional mean equations (based on asset pricing models) with EGARCH equations for conditional second moments.

A.1. Univariate EGARCH-M Models

Estimation of conditional single-factor models requires a univariate model of conditional market variance. Previous empirical studies of conditional market variance generally observe that (1) market volatility is persistent, (2) market volatility responds asymmetrically to lagged return innovations,[5]

[3] Nelson (1991) introduces the EGARCH model.

[4] ARCH and GARCH models are introduced by Engle (1982) and Bollerslev (1986), respectively. ARCH-M (ARCH-in-Mean) models are developed by Engle, Lilien, and Robins (1987). Bollerslev, Chou, and Kroner (1992) survey the use of ARCH models in finance.

[5] Black (1976) observes that market volatility tends to increase in response to negative return innovations ("bad news") and decrease in response to positive return innovations ("good news"). Christie (1982) theorizes that this negative relation between price and volatility is caused by changes in financial leverage ("leverage effect").

and (3) market volatility is related to the level of the nominal risk-free rate.[6]

A modified EGARCH(1,1)-M system is sufficient to model these observed characteristics of market volatility. Models 1 and 2 are nested in the following EGARCH(1,1)-M system:

$$r_{M,t} = \lambda_0 + \lambda_M \hat{\sigma}^2_{M,t} + \hat{\epsilon}_{M,t}$$

$$\ln(\hat{\sigma}^2_{M,t}) = \omega_M + \alpha_M g(\hat{\psi}_{M,t-1}) + \beta_M \ln(\hat{\sigma}^2_{M,t-1}) + \gamma_M r_{f,t}$$

$$g(\hat{\psi}_{M,t-1}) = (|\hat{\psi}_{M,t-1}| - E_{t-1}[|\hat{\psi}_{M,t-1}|]) + \theta_M \hat{\psi}_{M,t-1}. \tag{6}$$

$\hat{\psi}_{M,t-1} = \hat{\epsilon}_{M,t-1}/\hat{\sigma}_{M,t-1}$ is a conditional standardized return innovation; $g(\hat{\psi}_{M,t-1})$ is the "news response function."[7] The news response function permits conditional market variance to respond to both the sign and the magnitude of the lagged return innovation. If $\theta_M = 0$, the news response function is symmetric and $g(\hat{\psi}_{M,t-1})$ is dependent only on the magnitude of the lagged return innovation. The parameter θ_M measures the asymmetric response of conditional variance to the sign of the lagged return innovation. If $0 < \theta_M < 1$ ($-1 < \theta_M < 0$), conditional market variance increases more in response to positive (negative) innovations than to negative (positive) innovations of the same magnitude. If $\theta_M = 1$ ($\theta_M = -1$), the news response function is positive for $\hat{\psi}_{M,t-1} > 0$ ($\hat{\psi}_{M,t-1} < 0$) and zero otherwise. If $\theta_M > 1$ ($\theta_M < -1$), the news response function is upward (downward) sloping in $\hat{\psi}_{M,t-1}$.

The observed relation between market volatility and the level of the nominal risk-free rate is incorporated by including $r_{f,t}$ directly in the conditional market variance equation in equation (6).[8] In order to study the effect of including the short-term interest rate in the conditional variance equation, I estimate Models 1 and 2 twice each. Model 1a imposes the constraints $\lambda_0 = 0$ and $\gamma_M = 0$. Model 1b imposes the constraint $\lambda_0 = 0$, but does not restrict γ_M. Model 2a removes the constraint on λ_0, but does impose the constraint $\gamma_M = 0$. Model 2b imposes no constraints on equation (6). When γ_M is unconstrained, I refer to equation (6) as a modified EGARCH-M model.

By construction, EGARCH models preclude negative estimates of $\hat{\sigma}^2_{M,t}$. This is an attractive feature when modeling the effects of asymmetric news response functions and interest rates on conditional market variance. Glosten et al. (1993) employ a modified GARCH-M model to study these effects. However, negative estimates of $\hat{\sigma}^2_{M,t}$ can result if return innovations are sufficiently large in such a model.[9] The nonnegativity of $\hat{\sigma}^2_{M,t}$ imposed by

[6] Campbell (1987), Breen, Glosten, and Jagannathan (1989), Shanken (1990), and Glosten et al. (1993) report a significant positive relation between market variance and the nominal one-month T-bill yield.

[7] Engle and Ng (1993) introduce the concept of a "news response curve."

[8] Glosten et al. (1993) employ a similar approach.

[9] Glosten et al. (1993) employ several modified EGARCH models of conditional market variance, but these models assume a different form for the news response function.

EGARCH-M models eliminates the need to constrain parameter values during the estimation process.

A.2. Bivariate EGARCH-M Models

Estimation of conditional two-factor models requires a bivariate model of conditional second moments. I employ a constant correlation bivariate EGARCH(1,1)-M model. As in the univariate case, I employ both conventional and modified versions. The modified bivariate EGARCH(1,1)-M system is

$$
\begin{aligned}
r_{M,t} &= \lambda_0 + \lambda_M \hat{\sigma}^2_{M,t} + \lambda_F \hat{\sigma}_{MF,t} + \hat{\epsilon}_{M,t} \\
r_{F,t} &= \mu_F + \hat{\epsilon}_{F,t} \\
\ln(\hat{\sigma}^2_{M,t}) &= \omega_M + \alpha_M g(\hat{\psi}_{M,t-1}) + \beta_M \ln(\hat{\sigma}^2_{M,t-1}) + \gamma_M r_{f,t} \\
\ln(\hat{\sigma}^2_{F,t}) &= \omega_F + \alpha_F g(\hat{\psi}_{F,t-1}) + \beta_F \ln(\hat{\sigma}^2_{F,t-1}) + \gamma_F r_{f,t} \\
\hat{\sigma}_{MF,t} &= \hat{\rho}_{MF} \hat{\sigma}_{M,t} \hat{\sigma}_{F,t} \\
g(\hat{\psi}_{M,t-1}) &= (|\hat{\psi}_{M,t-1}| - E_{t-1}[|\hat{\psi}_{M,t-1}|]) + \theta_M \hat{\psi}_{M,t-1} \\
g(\hat{\psi}_{F,t-1}) &= (|\hat{\psi}_{F,t-1}| - E_{t-1}[|\hat{\psi}_{F,t-1}|]) + \theta_F \hat{\psi}_{F,t-1},
\end{aligned}
\tag{7}
$$

where $g(\hat{\psi}_{M,t-1})$ and $g(\hat{\psi}_{F,t-1})$ are news response functions. Model 3a imposes the constraint $\gamma_M = \gamma_F = 0$. Model 3b(i) imposes the constraint $\gamma_F = 0$. Model 3b(ii) imposes the constraint $\gamma_M = 0$. Model 3b(iii) allows the nominal one-month T-bill yield to enter both conditional variance equations. The definition of conditional covariance in equation (7) makes the simplifying assumption that ρ_{MF}, the conditional correlation between excess market and bond returns, is a constant for the sample period. For simplicity, I assume that the conditional mean excess bond return is a constant (μ_F).

B. Quasi-Maximum Likelihood Methodology

The quasi-maximum likelihood (QML) method is a natural choice for estimating EGARCH-M systems because it provides for simultaneous estimation of the conditional mean and conditional second moment equations. QML estimates are obtained by searching for values of the parameter vector P that maximize the log likelihood function

$$L = \sum_{t=1}^{T} \ell_t(P).$$

For the single-factor models, the conditional log-likelihood function is computed under the assumption that the market return innovations, $\epsilon_{M,t}$, are normally distributed. Ignoring the constant term, the log of the normal density function is

$$\ell_t(P) = -\tfrac{1}{2}\ln(\hat{\sigma}^2_{M,t}) - \tfrac{1}{2}(\hat{\epsilon}^2_{M,t}/\hat{\sigma}^2_{M,t}).$$

For two-factor models, the conditional log-likelihood function is computed under the assumption that the innovation vector $\hat{\epsilon}_t = (\hat{\epsilon}_{M,t} \quad \hat{\epsilon}_{F,t})'$ is bivariate normally distributed.[10] Define the conditional variance–covariance matrix as

$$\hat{H}_t = \begin{bmatrix} \hat{\sigma}^2_{M,t} & \hat{\sigma}_{MF,t} \\ \hat{\sigma}_{MF,t} & \hat{\sigma}^2_{F,t} \end{bmatrix}.$$

Ignoring the constant, the log of the resulting bivariate normal density function is

$$\ell_t(P) = -\tfrac{1}{2}\ln(\det(\hat{H}_t)) - \tfrac{1}{2}\hat{\epsilon}_t'\hat{H}_t^{-1}\hat{\epsilon}_t.$$

QML estimates are so named because they are obtained by maximizing a misspecified conditional log likelihood function. In this case, the log likelihood function assumes normality, even though market returns are known to be skewed and leptokurtic. Bollerslev and Wooldridge (1992) show that QML estimates of P are consistent if

$$E_{t-1}(\hat{\epsilon}_{M,t}/\hat{\sigma}_{M,t}) = 0 \quad \text{and} \quad E_{t-1}(\hat{\epsilon}^2_{M,t}/\hat{\sigma}^2_{M,t}) = 1.$$

Conventional standard errors, however, need to be adjusted for misspecification. Bollerslev and Wooldridge (1992) derive formulas for asymptotic standard errors that are robust to departures from normality. These robust standard errors are used to compute the t ratios reported below.

I employ the Berndt, Hall, Hall, and Hausman (1974) optimization algorithm to estimate the EGARCH-M models described here. Because the conditional variance models are recursive, some assumption regarding initial values is required. I assume that the vector $\epsilon_0 = \mathbf{0}$ and estimate $\hat{\sigma}^2_{M,1}$ and $\hat{\sigma}^2_{F,1}$ as free parameters.

III. Empirical Results

A. Description of the Data

I examine monthly data for the period 1950:3 to 1994:12. This period roughly corresponds to the period 1951:4 to 1989:12 studied by Glosten et al. (1993). Descriptive statistics for the data are reported in Table II. All variables are expressed as continuously compounded monthly returns.

[10] Under the normality assumption, $E_{t-1}[|\psi_{M,t}|] = E_{t-1}[|\psi_{F,t}|] = \sqrt{2/\pi}$.

Table II

Descriptive Statistics

Descriptive statistics for the sample period 1950:3–1994:12 (538 months). Variables include the excess monthly return on the CRSP value-weighted index of NYSE–AMEX stocks ($r_{M,t}$), the excess monthly return on the Ibbotson Associates (1995) long-term government bond total return index ($r_{F,t}$), and the nominal yield on a one-month Treasury bill ($r_{f,t}$).

Panel A: Univariate Statistics

Variable	Mean (×100)	Standard Deviation (×100)	Autocorrelations ρ_1	ρ_2	ρ_3	ρ_{12}
$r_{M,t}$	0.508	4.139	0.066	−0.037	0.010	0.036
$r_{F,t}$	0.001	2.544	0.058	0.000	−0.124[a]	0.000
$r_{f,t}$	0.420	0.250	0.961[a]	0.935[a]	0.918[a]	0.792[a]

Panel B: Correlation Matrix

Variable	$r_{M,t}$	$r_{F,t}$	$r_{f,t}$
$r_{M,t}$	1.000	0.276	−0.146
$r_{F,t}$		1.000	0.012
$r_{f,t}$			1.000

[a] Greater than two standard errors from zero.

$r_{M,t}$ is the excess monthly return on the market portfolio. It measures the total return from the end of month $t - 1$ to the end of month t. The market proxy is the value-weighted portfolio of NYSE–AMEX stocks provided by the Center for Research in Security Prices (CRSP) at the University of Chicago. The excess return is obtained by subtracting the nominal yield on a one-month T-bill from the monthly total return on the index, $r_{M,t} = R_{M,t} - r_{f,t}$.

$r_{F,t}$ is the excess return on the Ibbotson Associates (1995) long-term U.S. Treasury bond total return index. It is measured in the same manner as $r_{M,t}$.

$r_{f,t}$ is the nominal yield on a Treasury bill with one month until maturity. Unlike stock and bond returns, $r_{f,t}$ is known with certainty at the end of month $t - 1$. The series is derived from Ibbotson Associates (1995). Figure 1 plots $r_{f,t}$ for the sample period.

B. Estimates of Conditional Single-Factor Models

QML estimates of conditional single-factor models are reported in Table III. Four variants (Models 1a, 1b, 2a, and 2b) of equation (6) are estimated in order to replicate previous results and facilitate comparison to the conditional two-factor model (equation (7)) estimated in the next subsection. Model 2a is comparable to a model estimated by French et al. (1987).[11] Glosten et al.

[11] A GARCH(1,2)-M model of the market risk premium is estimated in Table 6A of French et al. (1987).

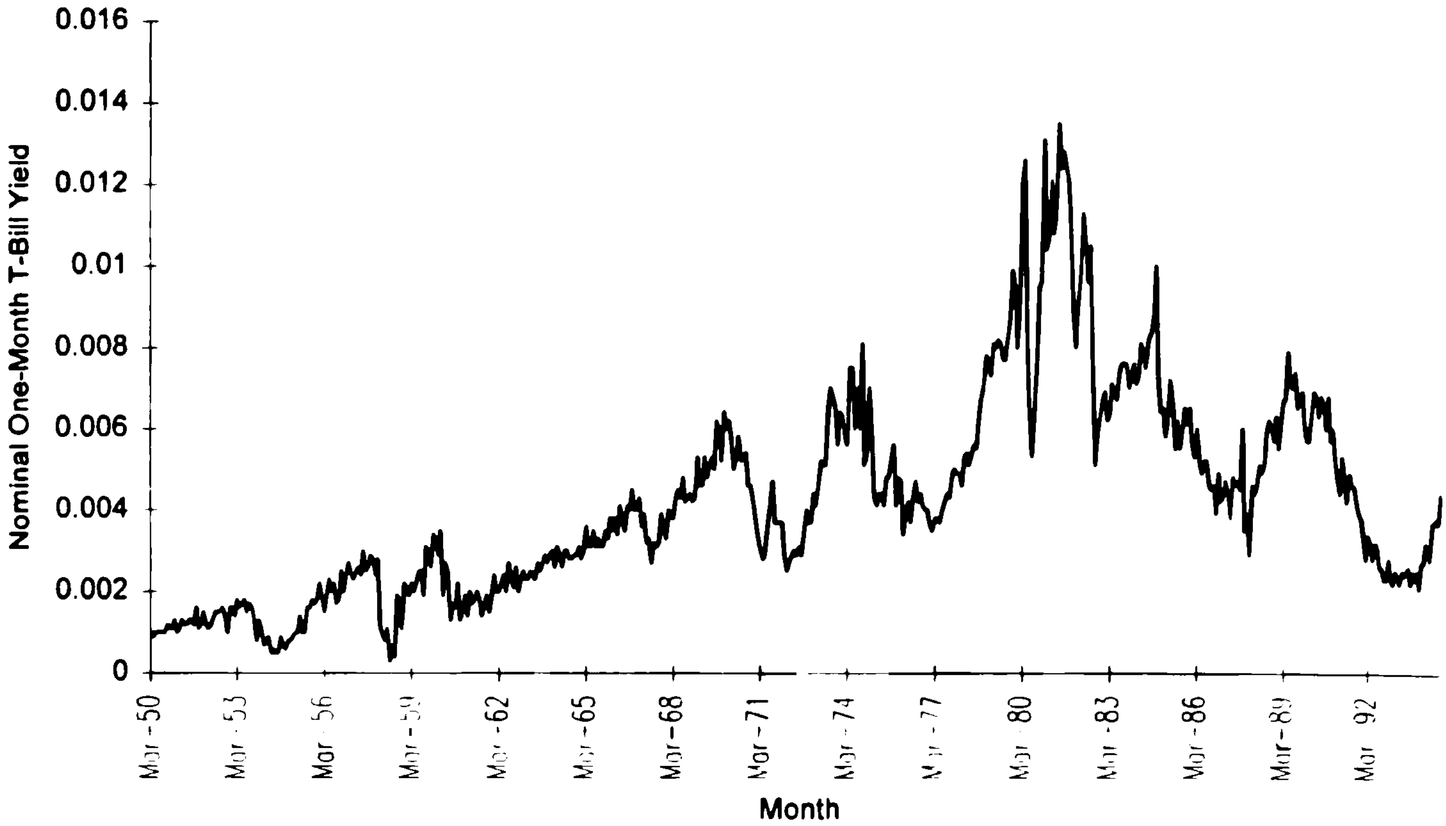

Figure 1. Nominal one-month T-bill yield. The figure plots the nominal one-month T-bill yield ($r_{f,t}$) for the sample period 1950:3 to 1994:12.

(1993) estimate GARCH-M and EGARCH-M models similar to Model 2b. I first discuss estimates of the conditional market variance equation parameters, and then discuss estimates of the conditional mean equation.

Point estimates of the conditional variance parameters are nearly identical for Models 1a and 2a. Estimates of β_M are near 0.9 and highly significant for both models, indicating that conditional market variance is persistent.[12] It is useful to think of volatility persistence in terms of the implied half-life of volatility shocks. A β_M of 0.9 is associated with a half-life of 6.43 months.[13] Estimates of α_M are positive and significant for both Models 1a and 2a. Estimates of the asymmetry parameter, θ_M, are negative, but not significantly different from zero. These results suggest that conditional market variance increases in response to large return innovations, but that the response is symmetric. Figure 2 plots conventional EGARCH(1,1)-M estimates of $\hat{\sigma}^2_{M,t}$ for the period 1950:3 to 1994:12. The values plotted result from QML estimation of Model 2a, but are nearly indistinguishable from those of Model 1a. The conditional heteroskedasticity of market excess returns is evident in Figure 2. Several distinct periods of high market volatility ("vol-

[12] The estimates of β_M are also both significantly less than one, so they satisfy Nelson's (1991) condition for variance stationarity. t ratios for the unit root hypothesis are 3.05 and 2.93 for Models 1a and 2a.

[13] The half-life, h, of return shocks is calculated by solving $\beta_M^h = \frac{1}{2}$. Nelson (1991) estimates a half-life of 1820 trading days (approximately 60 months) using an EGARCH(1,2)-M model of daily market returns for the period 1962:7 to 1987:12.

Table III

Conditional Single-Factor Models of the Market Risk Premium

This table estimates conditional models of the relation between the market risk premium ($r_{M,t}$) and conditional market variance ($\hat{\sigma}^2_{M,t}$) using U.S. monthly data for the period 1950:3 to 1994:12 (538 observations). The models estimated are nested within the following univariate EGARCH(1,1)-M model (equation (8)):

$$r_{M,t} = \lambda_0 + \lambda_M \hat{\sigma}^2_{M,t} + \lambda_f r_{f,t} + \hat{\epsilon}_{M,t}$$

$$\ln(\hat{\sigma}^2_{M,t}) = \omega_M + \alpha_M g(\hat{\psi}_{M,t-1}) + \beta_M \ln(\hat{\sigma}^2_{M,t-1}) + \gamma_M r_{f,t}$$

$$g(\hat{\psi}_{M,t-1}) = \left(|\hat{\psi}_{M,t-1}| - \sqrt{2/\pi}\right) + \theta_M \hat{\psi}_{M,t-1}.$$

Models 1a and 1b impose the constraints $\lambda_0 = 0$ and $\lambda_f = 0$. Models 2a and 2b impose the constraint $\lambda_f = 0$. Two variants of each model are estimated; Models 1a, 2a, and 4a impose the further constraint $\gamma_f = 0$. $r_{M,t}$ is the excess monthly return on the CRSP value-weighted total return index of NYSE–AMEX stocks. $r_{f,t}$ is the yield on a one-month T-bill from Ibbotson Associates (1995). $\hat{\psi}_{M,t} = \hat{\epsilon}_{M,t}/\hat{\sigma}_{M,t}$ is the standardized conditional return innovation. Bollerslev and Wooldridge (1992) robust t ratios are reported in parentheses. L is the log likelihood function.

	Risk–Return Relation					
	Simple Proportional		Simple Linear		Ad Hoc Partial	
Parameter	Model 1a	Model 1b	Model 2a	Model 2b	Model 4a	Model 4b
λ_0 (×100)			0.14	1.57	0.35	0.60
(t ratio)			(0.30)	(5.57)	(0.62)	(1.08)
λ_M	3.25	2.99	2.43	−6.37	7.43	17.07
(t ratio)	(2.81)	(2.53)	(0.84)	(−3.08)	(1.94)	(2.55)
λ_f					−2.44	−6.86
(t ratio)					(−2.93)	(−3.69)
ω_M	−0.66	−6.33	−0.65	−5.60	−0.49	−1.01
(t ratio)	(−2.99)	(−3.82)	(−2.88)	(−3.59)	(−2.10)	(−3.73)
α_M	0.15	0.00	0.15	0.01	0.18	0.10
(t ratio)	(2.34)	(0.01)	(2.32)	(0.16)	(3.81)	(2.52)
β_M	0.90	0.07	0.90	0.20	0.92	0.86
(t ratio)	(26.80)	(0.29)	(25.76)	(0.92)	(26.43)	(22.33)
θ_M	−0.50	−299.97	−0.61	−23.18	−0.25	−0.52
(t ratio)	(−1.20)	(−0.01)	(−1.22)	(−0.15)	(−0.95)	(−1.14)
γ_M		76.03		101.60		24.78
(t ratio)		(1.97)		(2.50)		(3.95)
L	969.64	977.57	969.68	988.07	975.29	988.00

atility clusters") are visible. The smoothness of the plot reflects the persistence of conditional market variance. Large volatility shocks appear to decay within a year. The plot in Figure 2 is very similar to plots of conditional market volatility presented in Schwert and Seguin (1990).[14]

[14] Schwert and Seguin (1990), Figures 3 and 4, plot GARCH predictions of conditional standard deviation for the CRSP equally weighted portfolio.

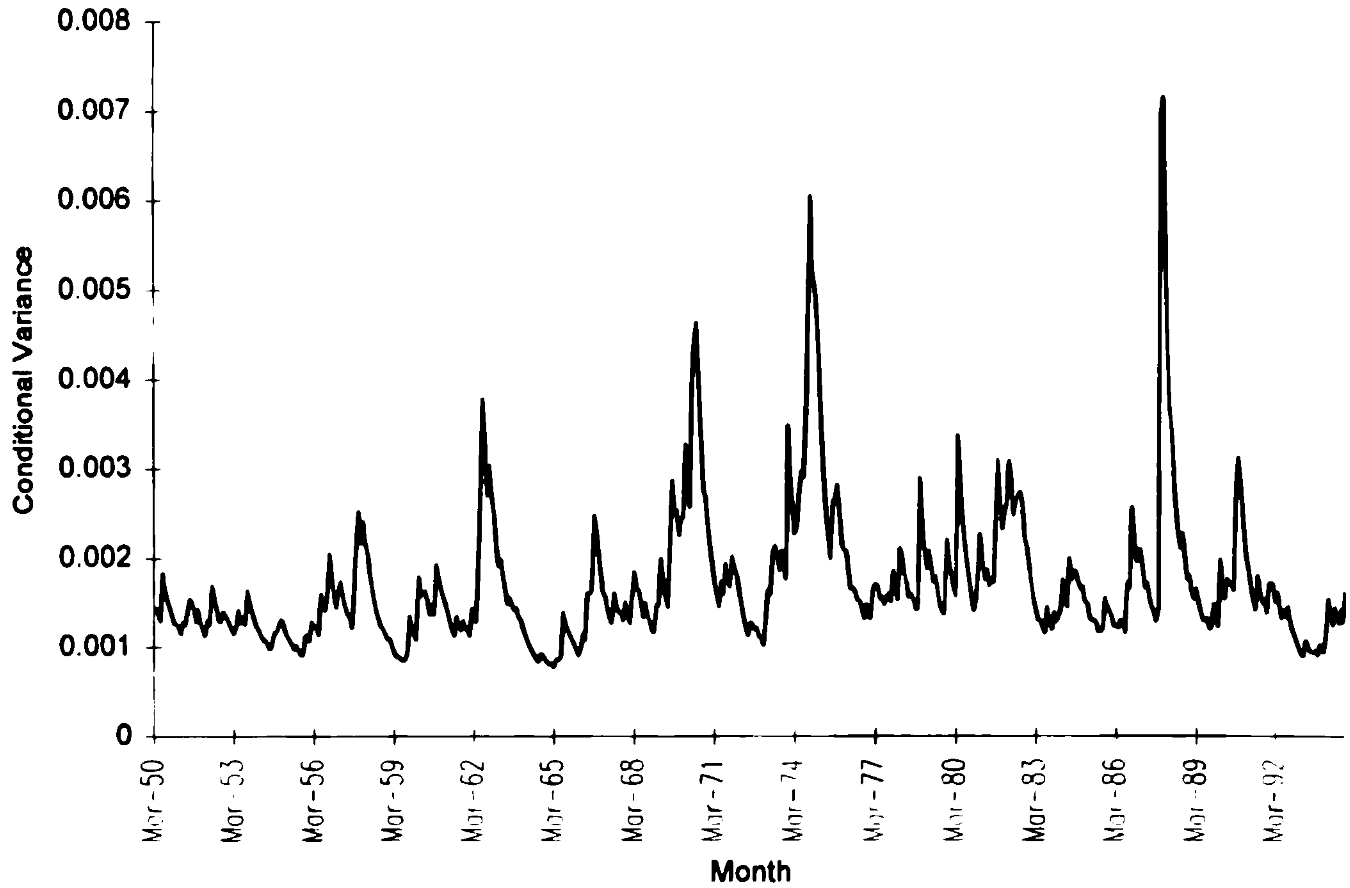

Figure 2. Conditional variance of excess market returns for Model 2a. The figure plots conventional EGARCH(1,1)-M estimates of $\hat{\sigma}^2_{M,t}$ for U.S. monthly data for the sample period 1950:3 to 1994:12. The estimates of $\hat{\sigma}^2_{M,t}$ result from the quasi-maximum likelihood estimation of Model 2a reported in Table III.

Models 1b and 2b employ the modified EGARCH(1,1)-M model that includes $r_{f,t}$ directly in the conditional market variance equation. Estimates of γ_M are positive and significant for both models. This strong positive relation between conditional market variance and the nominal one-month T-bill yield is consistent with results reported by Campbell (1987), Breen et al. (1989), Shanken (1990), and Glosten et al. (1993). However, including $r_{f,t}$ in the conditional market variance equation results in estimates of ω_M, α_M, β_M, and θ_M that are dramatically different from those obtained for Models 1a and 2a. Estimates of α_M, β_M, and θ_M are not significantly different from zero for Models 1b and 2b.

These results suggest that conditional market variance is neither persistent nor responsive to lagged return innovations. Furthermore, these results suggest that much of the variation in conditional market variance is attributable to changes in the level of the nominal one-month T-bill yield. Figure 3 plots the modified EGARCH(1,1)-M estimates of $\hat{\sigma}^2_{M,t}$ for the period 1950:3 to 1994:12. The values of $\hat{\sigma}^2_{M,t}$ plotted result from QML estimation of Model 2b, but are similar for Model 1b. The plot in Figure 3 appears jagged when compared to the plot in Figure 2, reflecting the lower persistence of conditional variance in Model 2b. The values of $\hat{\sigma}^2_{M,t}$ plotted in Figure 3 are correlated with $r_{f,t}$. Although the estimates of α_M, β_M, and θ_M in Model 2b are not significantly different from zero, the effects of large return innovations on conditional market variance are still evident in Figure 3. Because

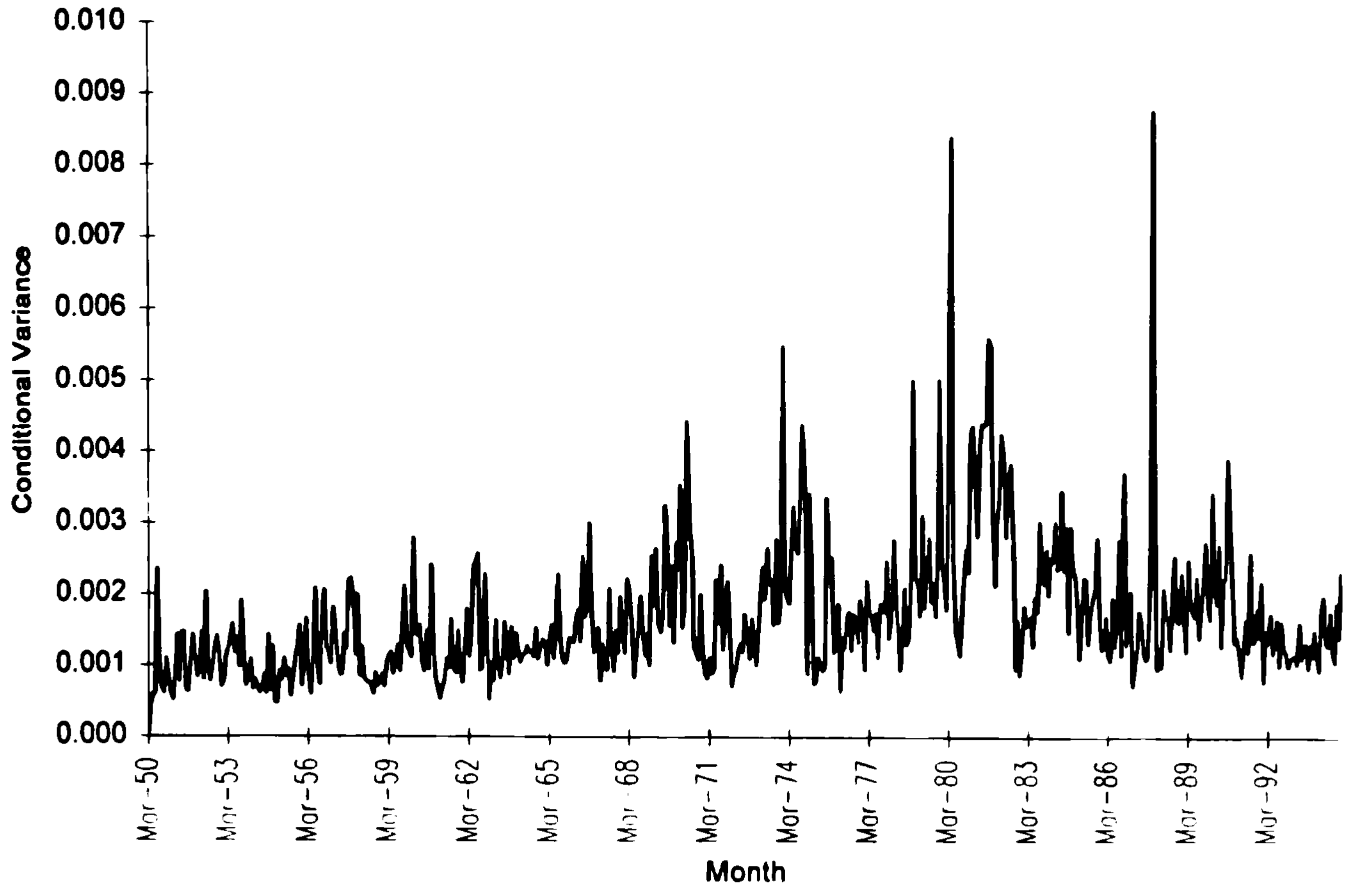

Figure 3. Conditional variance of excess market returns for Model 2b. The figure plots modified EGARCH(1,1)-M estimates of $\hat{\sigma}^2_{M,t}$ for U.S. monthly data for the sample period 1950:3 to 1994:12. The estimates of $\hat{\sigma}^2_{M,t}$ result from the quasi-maximum likelihood estimation of Model 2b reported in Table III.

γ_M is positive and significant, the plot of $\hat{\sigma}^2_{M,t}$ in Figure 3 exhibits some of the features of the plot of $r_{f,t}$ in Figure 1.

The results reported in Table III for Models 2a and 2b are similar to the results reported by Glosten et al. (1993). Based on results from a model similar to Model 2b, they conclude that "conditional volatility of the monthly excess return is not highly persistent," and that "negative residuals are associated with an increase in variance, while positive residuals are associated with a slight decrease in variance."

In conditional single-factor models, λ_M measures the *simple* relation between the market risk premium and conditional market variance. The estimates of λ_M reported in Table III illustrate how model specification can affect inference regarding the relationship between $r_{M,t}$ and $\hat{\sigma}^2_{M,t}$. Two effects stand out: (1) the effect of constraining the intercept term, and (2) the effect of including $r_{f,t}$ in the conditional variance equation. In Models 1a and 1b, where the constraint $\lambda_0 = 0$ is imposed, estimates of λ_M are positive and significant. Since the excess market return is, on average, positive and values of $\hat{\sigma}^2_{M,t}$ are nonnegative by construction, it is not surprising that the slope coefficient is positive when the intercept is forced through the origin. When the constraint $\lambda_0 = 0$ is relaxed in Models 2a and 2b, estimates of λ_M become insignificant and negative, respectively. These results suggest that imposing the constraint $\lambda_0 = 0$ can result in misleading estimates of the *simple* risk–return relation. This may explain the significant positive *simple* risk–return relation reported by Harvey (1989).

Including $r_{f,t}$ in the conditional variance equation also affects estimates of the *simple* relation between $r_{M,t}$ and $\hat{\sigma}^2_{M,t}$. When the conventional EGARCH-M model is employed (Model 2a), the *simple* relation between $r_{M,t}$ and $\hat{\sigma}^2_{M,t}$ is positive, but insignificant. The *simple* relation becomes negative and highly significant when the modified EGARCH-M model is employed (Model 2b). Glosten et al. (1993) obtain results similar to Model 2b when they include $r_{f,t}$ in the conditional variance equation. They conclude that "the relation between conditional mean and conditional variance is negative and statistically significant." I believe it is premature to draw conclusions about the empirical relationships between $r_{M,t}$, $\hat{\sigma}^2_{M,t}$ and $r_{f,t}$ based on the results from these conditional single-factor models. It appears that the significant negative relation between $r_{M,t}$ and $\hat{\sigma}^2_{M,t}$ found in Model 2b might be driven by the significant positive relation between $\hat{\sigma}^2_{M,t}$ and $r_{f,t}$. It is puzzling that the strong persistence in conditional market variance evident in the conventional EGARCH-M models (1a and 2a) vanishes when modified EGARCH-M models (1b and 2b) are estimated. One plausible explanation is that the well-documented negative relation between $r_{M,t}$ and $r_{f,t}$ dominates the estimation of both the conditional variance and conditional mean equations. It appears that the positive relation between $\hat{\sigma}^2_{M,t}$ and $r_{f,t}$ is accentuated and the persistence of conditional market variance is suppressed. The resulting estimates of λ_M in Model 2b are negative and significant. The puzzling relationship between the market risk premium, conditional market variance, and the one-month nominal T-bill yield is further examined in Section IV.

Figures 4 and 5 plot estimates of the expected market risk premium, $E_{t-1}[r_{M,t}]$, which result from QML estimation of Models 2a and 2b, respectively. The difference is dramatic. Since Model 2 implies a simple linear relation between $E_{t-1}[r_{M,t}]$ and $\hat{\sigma}^2_{M,t}$, the plots are closely related to the associated plots of $\hat{\sigma}^2_{M,t}$ in Figures 2 and 3. In Figure 4, the expected market risk premium is positive throughout the sample period. In Figure 5, the negative value of λ_M results in a plot of $E_{t-1}[r_{M,t}]$ that looks like an inverted plot of $\hat{\sigma}^2_{M,t}$. $E_{t-1}[r_{M,t}]$ is negative for 13.94 percent of the sample period. The negative values occur primarily in months following large return surprises (the largest negative values occur in November 1987 and April 1980) and in months when $r_{f,t}$ is high. It is not surprising that a likelihood ratio test rejects Model 2a in favor of Model 2b. The negative values of $E_{t-1}[r_{M,t}]$ result in a better fit of the market excess return series.[15]

C. Estimates of Conditional Two-Factor Models

This section of the paper examines a conditional two-factor model in which the long-term government bond return ($r_{F,t}$) is the second state variable. In Model 3, the market risk premium is a linear function of conditional market

[15] In simple regressions of $r_{M,t}$ on $\hat{\sigma}^2_{M,t}$, the adjusted R^2 was 0.000 for Model 2a and 0.017 for Model 2b.

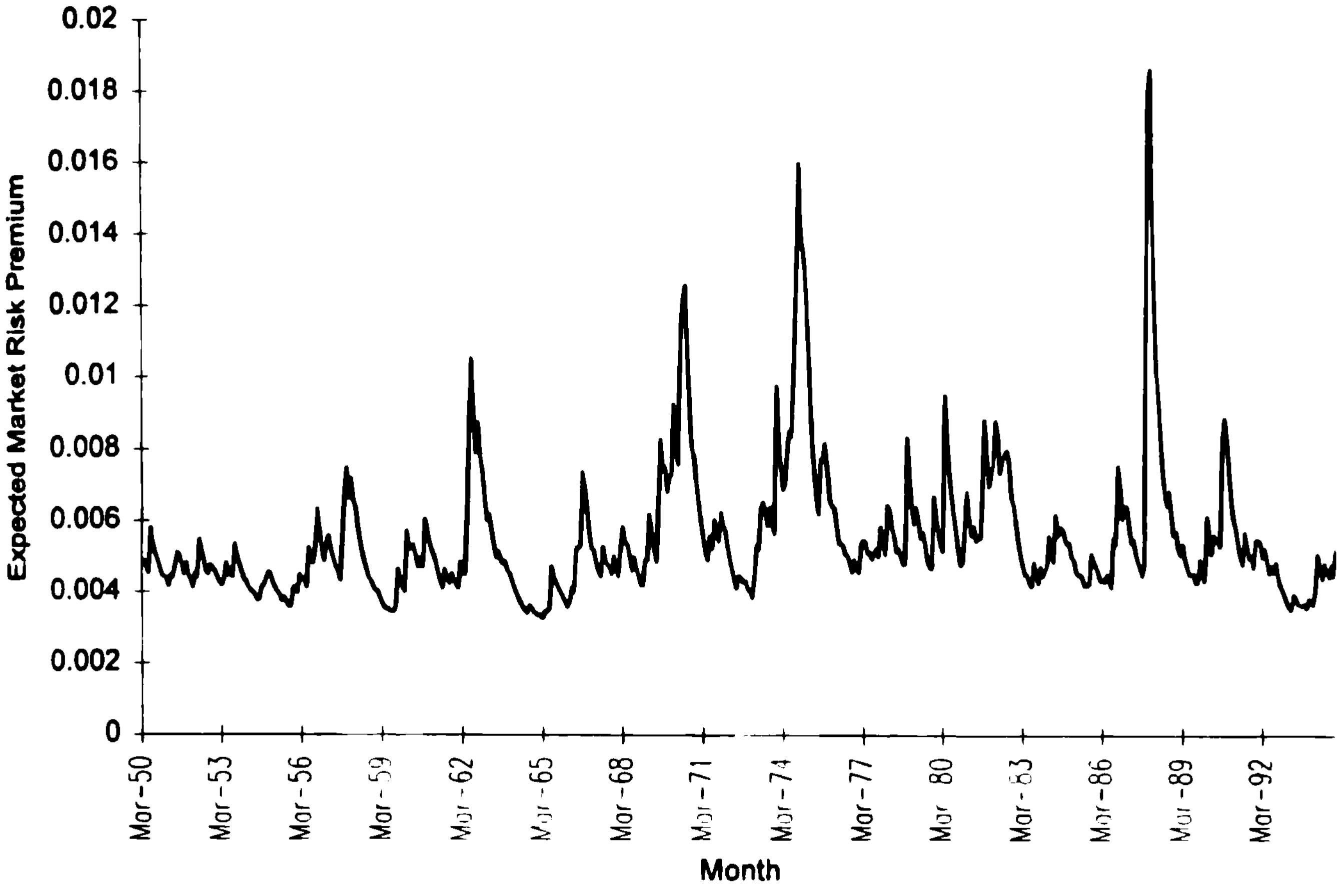

Figure 4. Expected market risk premium for Model 2a. The figure plots conventional EGARCH(1,1)-M estimates of $E_{t-1}[r_{M,t}]$ for U.S. monthly data for the sample period 1950:3 to 1994:12. The estimates of $E_{t-1}[r_{M,t}]$ result from the quasi-maximum likelihood estimation of Model 2a reported in Table III. Model 2a implies a *simple linear* relation between $E_{t-1}[r_{M,t}]$ and the estimates of $\hat{\sigma}^2_{M,t}$ plotted in Figure 2.

variance $(\hat{\sigma}^2_{M,t})$ and conditional market covariance with the return on a long-term government bond $(\hat{\sigma}_{MF,t})$. QML estimates of the bivariate EGARCH(1,1)-M system (equation (7)) are reported in Table IV. The four variants of Model 3 that are estimated differ only in their use of the instrumental variable $r_{f,t}$. Model 3a employs a conventional EGARCH-M model in which the parameters γ_M and γ_F are constrained to be zero. Models 3b(i), 3b(ii), and 3b(iii) employ modified EGARCH-M models of conditional second moments in which the constraints on the parameters γ_M and γ_F are relaxed. In addition to these four two-factor models, I estimate a restricted variant of Model 3a which imposes the constraint $\lambda_F = 0$. I refer to this conditional single-factor model as Model 2a*.

Parameter estimates are very similar for the four variants of Model 3 reported in Table IV. Point estimates of the conditional market variance equation parameters $(\omega_M, \alpha_M, \beta_M,$ and $\theta_M)$ are similar to those obtained for the conventional univariate EGARCH-M models reported in Table III. Estimates of β_M are again near 0.9, and estimates of α_M and θ_M indicate that $\hat{\sigma}^2_{M,t}$ increases symmetrically in response to large market return innovations. Estimates of β_F range from 0.95 to 0.98, indicating that the conditional variance of long-term bond returns is highly persistent. The half-life of a bond volatility shock is approximately 27 months when $\beta_F = 0.97$. Although the estimates of β_F are close to one, the unit root hypothesis is rejected in every case with a t statistic greater than two. Estimates of α_F and θ_F indicate that

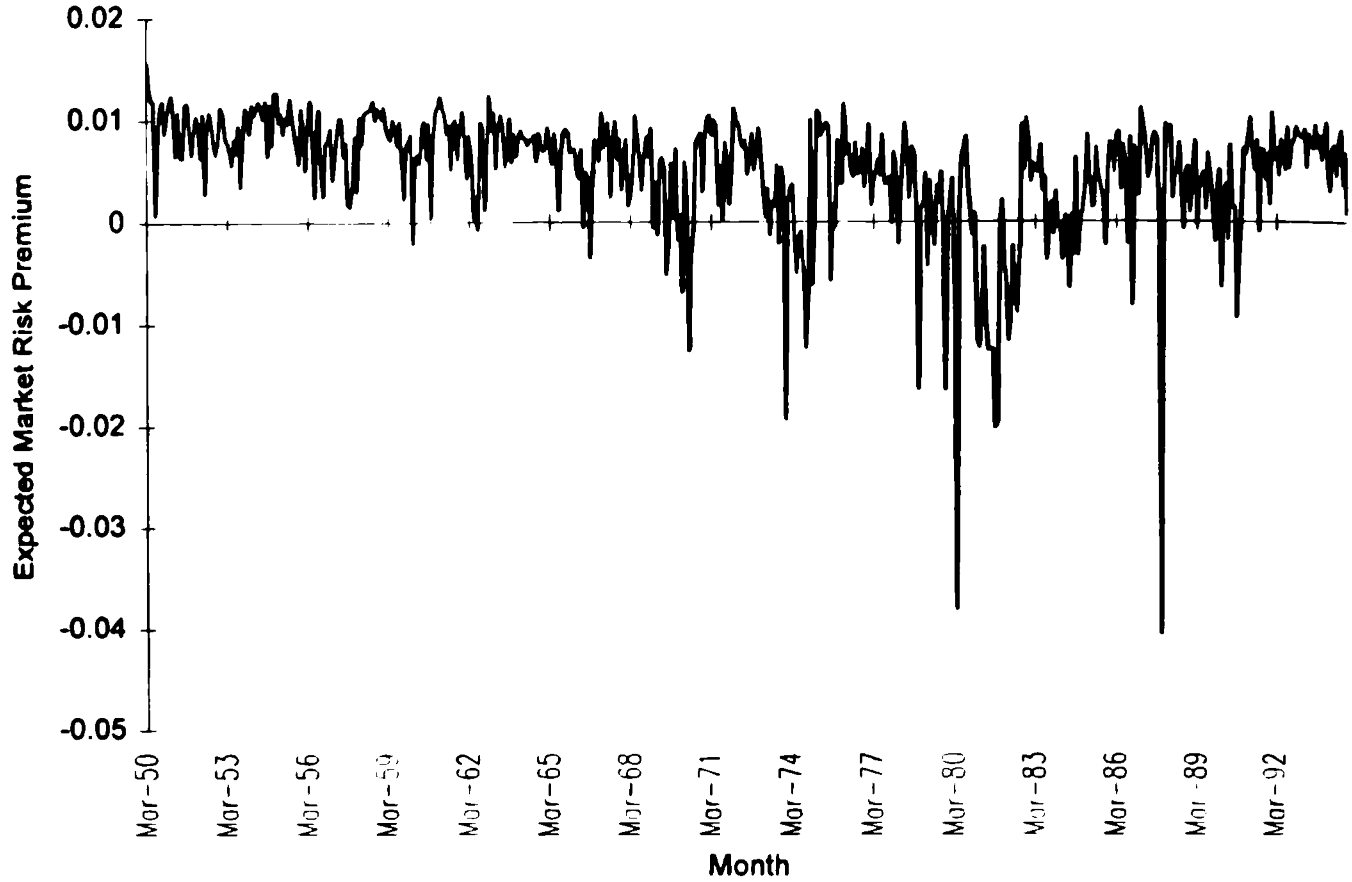

Figure 5. Expected market risk premium for Model 2b. The figure plots modified EGARCH(1,1)-M estimates of $E_{t-1}[r_{M,t}]$ for U.S. monthly data for the sample period 1950:3 to 1994:12. The estimates of $E_{t-1}[r_{M,t}]$ result from the quasi-maximum likelihood estimation of Model 2b reported in Table III. Model 2b implies a *simple linear* relation between $E_{t-1}[r_{M,t}]$ and the estimates of $\hat{\sigma}^2_{M,t}$ plotted in Figure 3.

conditional bond return variance increases in response to large bond return innovations, and that it increases more in response to negative return shocks than to positive return shocks. Estimates of the conditional correlation between market returns and bond returns, ρ_{MF}, are between 0.22 and 0.24. These are close to the sample correlation of 0.276 reported in Table II.

Figures 6, 7, and 8 plot estimates of $\hat{\sigma}^2_{M,t}$, $\hat{\sigma}^2_{F,t}$, and $\hat{\sigma}_{MF,t}$, respectively, that result from QML estimation of Model 3b(iii). The plot of $\hat{\sigma}^2_{M,t}$ in Figure 6 is difficult to distinguish from that in Figure 2. This reflects the similarity of the conditional market variance equation parameter estimates in Models 2a and 3b(iii). The plot in Figure 7 provides graphic evidence of "volatility clustering" in long-term bond returns. The substantial increase in bond market volatility after the Federal Reserve regime change in September 1979 is noteworthy. Due to the constant correlation assumption implicit in equation (7), the plot of $\hat{\sigma}_{MF,t}$ in Figure 8 displays characteristics from both Figures 6 and 7.

Estimates of the conditional mean equation parameters in Table IV provide evidence of a positive *partial* relation between $r_{M,t}$ and $\hat{\sigma}^2_{M,t}$. Estimates of λ_M are positive and significant for every variant of Model 3. Estimates of the *partial* relation between $r_{M,t}$ and $\hat{\sigma}_{MF,t}$ are negative and statistically significant for every model. The intercept, λ_0, is not significantly different from zero. The sign of λ_F suggests that the elasticity of marginal utility of wealth with respect to bond prices is positive. This is intuitively appealing. Because marginal utility of wealth and the required rate of return are inversely related, investors require a *lower* return on the market portfolio at times when $\hat{\sigma}_{MF,t}$ is positive and high. This is consistent with Chen et al. (1986), who find a negative risk premium associated with portfolio betas measuring exposure to interest rate risk.[16]

Estimates of the parameter γ_M in models 3b(i) and 3b(iii) provide further evidence of a significant positive relation between conditional market variance and the nominal one-month T-bill yield. The relation between $\hat{\sigma}^2_{F,t}$ and $r_{f,t}$ is not as strong. Estimates of γ_F are positive, but not significantly different from zero in either Model 3b(ii) or Model 3b(iii). It is noteworthy that inclusion of $r_{f,t}$ in the conditional second-moment equations has no apparent effect on estimates of λ_M and λ_F. A simple likelihood ratio test rejects Model 3a in favor of Model 3b(i) at a 5 percent level of significance.

Model 2a* is estimated in Table IV by imposing the constraint $\lambda_F = \gamma_M = \gamma_F = 0$ on equation (7). Estimates of the conditional second-moment parameters are very similar to those obtained for Model 3a. However, imposing the constraint $\lambda_F = 0$ results in an estimated *simple* risk–return relation that is not statistically significant ($\lambda_M = 0.86$, $t = 0.30$). Model 2a* is nested within Model 3a. A simple likelihood ratio statistic rejects Model 2a* in favor of Model 3a at a 5 percent level of significance.

Most papers on the intertemporal relation between the market risk premium and conditional market variance conclude that the *simple* relation is insignificant or negative. The estimates of Model 3a provide support for the omitted variable explanation of these results. If Model 3a is the "true" model, equation (5) predicts that estimates of the parameter λ_M in Model 2a* would have a bias of -6.67. Such a bias is sufficient to explain the insignificant relation between the market risk premium and conditional market variance found in Model 2a* and in most previous studies of the *simple* risk–return relation.

Figure 9 plots estimates of the expected market risk premium which result from QML estimation of Model 3b(iii). These fitted values are negative for 14.5 percent of the sample period. The negative values occur primarily in the period of high conditional bond market volatility following the Federal Reserve regime change of September 1979. This finding is consistent with Boudoukh, Richardson, and Smith (1993), who report that a negative ex ante risk premium is more likely to occur in states of the world where T-bill rates are high or the term structure is downward-sloping. The existence of a negative market risk premium is puzzling. It is intuitive that risk-averse investors would demand a positive risk premium for bearing nondiversifiable risk. Although theoretically possible, it seems doubtful that hedging demand for the market portfolio could be strong enough to result in a negative risk premium. There are also statistical reasons to treat the negative

[16] The UTS variable employed by Chen et al. (1986) is similar to the excess bond return series used in this paper.

Table IV

Conditional Two-Factor Models of the Market Risk Premium

This table estimates conditional two-factor models of the relation between the market risk premium ($r_{M,t}$), conditional market variance ($\hat{\sigma}^2_{M,t}$), and conditional market covariance with long-term government bond returns ($\hat{\sigma}_{MF,t}$) using U.S. monthly data for the period 1950:3 to 1994:12 (538 observations). The models estimated are nested within the following bivariate EGARCH(1,1)-M system (equation (7)):

$$r_{M,t} = \lambda_0 + \lambda_M \hat{\sigma}^2_{M,t} + \lambda_F \hat{\sigma}_{MF,t} + \hat{\epsilon}_{M,t}$$

$$r_{F,t} = \mu_F + \epsilon_{F,t}$$

$$\ln(\hat{\sigma}^2_{M,t}) = \omega_M + \alpha_M g(\hat{\psi}_{M,t-1}) + \beta_M \ln(\hat{\sigma}^2_{M,t-1}) + \gamma_M r_{f,t}$$

$$\ln(\hat{\sigma}^2_{F,t}) = \omega_F + \alpha_F g(\hat{\psi}_{F,t-1}) + \beta_F \ln(\hat{\sigma}^2_{F,t-1}) + \gamma_F r_{f,t}$$

$$\hat{\sigma}_{MF,t} = \hat{\rho}_{MF} \hat{\sigma}_{M,t} \hat{\sigma}_{F,t}$$

$$g(\hat{\psi}_{M,t-1}) = \left(|\hat{\psi}_{M,t-1}| - \sqrt{2/\pi}\right) + \theta_M \hat{\psi}_{M,t-1}$$

$$g(\hat{\psi}_{F,t-1}) = \left(|\hat{\psi}_{F,t-1}| - \sqrt{2/\pi}\right) + \theta_F \hat{\psi}_{F,t-1}.$$

Models 3a, 3b(i), 3b(ii), and 3b(iii) differ in the constraints they place on γ_M and γ_F. Model 2a* is a conditional single-factor model that imposes the constraints $\lambda_F = 0$ and $\gamma_M = \gamma_F = 0$. $r_{M,t}$ is the excess monthly return on the CRSP value-weighted total return index of NYSE–AMEX stocks, $r_{F,t}$ is the excess monthly return on the Ibbotson Associates (1995) long-term government bond portfolio, and $r_{f,t}$ is the yield on a one-month T-bill from Ibbotson Associates (1995). All returns are continuously compounded. $\hat{\psi}_{M,t} = \hat{\epsilon}_{M,t}/\hat{\sigma}_{M,t}$ and $\hat{\psi}_{F,t} = \hat{\epsilon}_{F,t}/\hat{\sigma}_{F,t}$ are standardized conditional return innovations. Bollerslev and Wooldridge (1992) robust t ratios are reported in parentheses. L is the log likelihood function.

Parameter	Model 2a*	Model 3a	Model 3b(i)	Model 3b(ii)	Model 3b(iii)
		Panel A: Conditional Mean Equation Parameters			
λ_0 (×100)	0.36	−0.11	0.14	−0.12	0.18
(t ratio)	(0.80)	(−0.20)	(0.30)	(−0.22)	(0.38)
λ_M	0.86	10.57	10.15	10.16	9.29
(t ratio)	(0.30)	(2.36)	(2.05)	(2.28)	(1.88)
λ_F		−49.75	−59.83	−47.80	−57.74
(t ratio)		(−2.12)	(−2.03)	(−2.00)	(−1.95)
μ_F (×100)	−0.10	−0.08	−0.09	−0.08	−0.08
(t ratio)	(−2.18)	(−1.86)	(−2.03)	(−1.68)	(−1.66)
		Panel B: Conditional Second Moment Equation Parameters			
ω_M	−0.70	−0.49	−0.74	−0.50	−0.81
(t ratio)	(−2.71)	(−1.98)	(−2.49)	(−1.98)	(−2.64)
α_M	0.16	0.19	0.18	0.19	0.18
(t ratio)	(2.34)	(4.14)	(3.44)	(4.10)	(3.18)
β_M	0.89	0.92	0.89	0.92	0.88
(t ratio)	(22.33)	(24.47)	(20.84)	(24.43)	(19.88)
θ_M	−0.60	−0.17	−0.21	−0.19	−0.26
(t ratio)	(−1.20)	(−0.70)	(−0.73)	(−0.76)	(−0.85)

Table IV—*Continued*

Parameter	Model 2a*	Model 3a	Model 3b(i)	Model 3b(ii)	Model 3b(iii)
		Panel B: *Continued*			
γ_M			11.85		13.85
(t ratio)			(2.09)		(2.20)
ω_F	−0.17	−0.18	−0.17	−0.38	−0.40
(t ratio)	(−2.33)	(−2.45)	(−2.39)	(−2.28)	(−2.40)
α_F	0.38	0.38	0.37	0.37	0.36
(t ratio)	(6.80)	(6.91)	(6.94)	(6.19)	(6.13)
β_F	0.98	0.97	0.98	0.96	0.95
(t ratio)	(105.65)	(102.57)	(108.00)	(56.22)	(55.68)
θ_F	−0.23	−0.27	−0.28	−0.24	−0.24
(t ratio)	(−1.91)	(−2.27)	(−2.36)	(−1.93)	(−1.90)
γ_F				11.75	13.90
(t ratio)				(1.27)	(1.48)
ρ_{MF}	0.23	0.24	0.22	0.23	0.22
(t ratio)	(4.59)	(4.97)	(4.70)	(4.79)	(4.45)
L	3318.53	3321.76	3324.78	3323.29	3326.86

values plotted in Figure 9 with some skepticism. Although estimates of λ_M and λ_F are statistically significant, it is unlikely that they are estimated with sufficient precision to make reliable inferences regarding the existence of a negative risk premium.[17]

IV. Relation to Previous Work

A. *Using the T-Bill Yield as an Instrumental Variable*

This section of the paper investigates the puzzling relationship between $r_{M,t}$, $\hat{\sigma}^2_{M,t}$, and $r_{f,t}$ reported in the empirical literature. Campbell (1987) and Glosten et al. (1993) both report a strong negative relation between the market risk premium and conditional market variance when the nominal one-month T-bill yield is included in the conditioning information set. Similar results are reported in Table III for Model 2b. When $r_{f,t}$ is removed from the conditioning information set (as in Model 2a), the significant negative risk–return relation disappears. Campbell describes similar results as "perverse."

Two distinct interest rate effects have received attention in the empirical literature. The first is the well-established empirical relation between market volatility and the level of short-term interest rates. Campbell (1987), Breen et al. (1989), Shanken (1990), and Glosten et al. (1993) report a significant positive relation between market variance and the nominal one-month T-bill yield. The second effect is the direct relation between the market risk premium and short-term interest rates. Fama and Schwert (1977), Camp-

[17] Fama and Schwert (1977) make a similar argument.

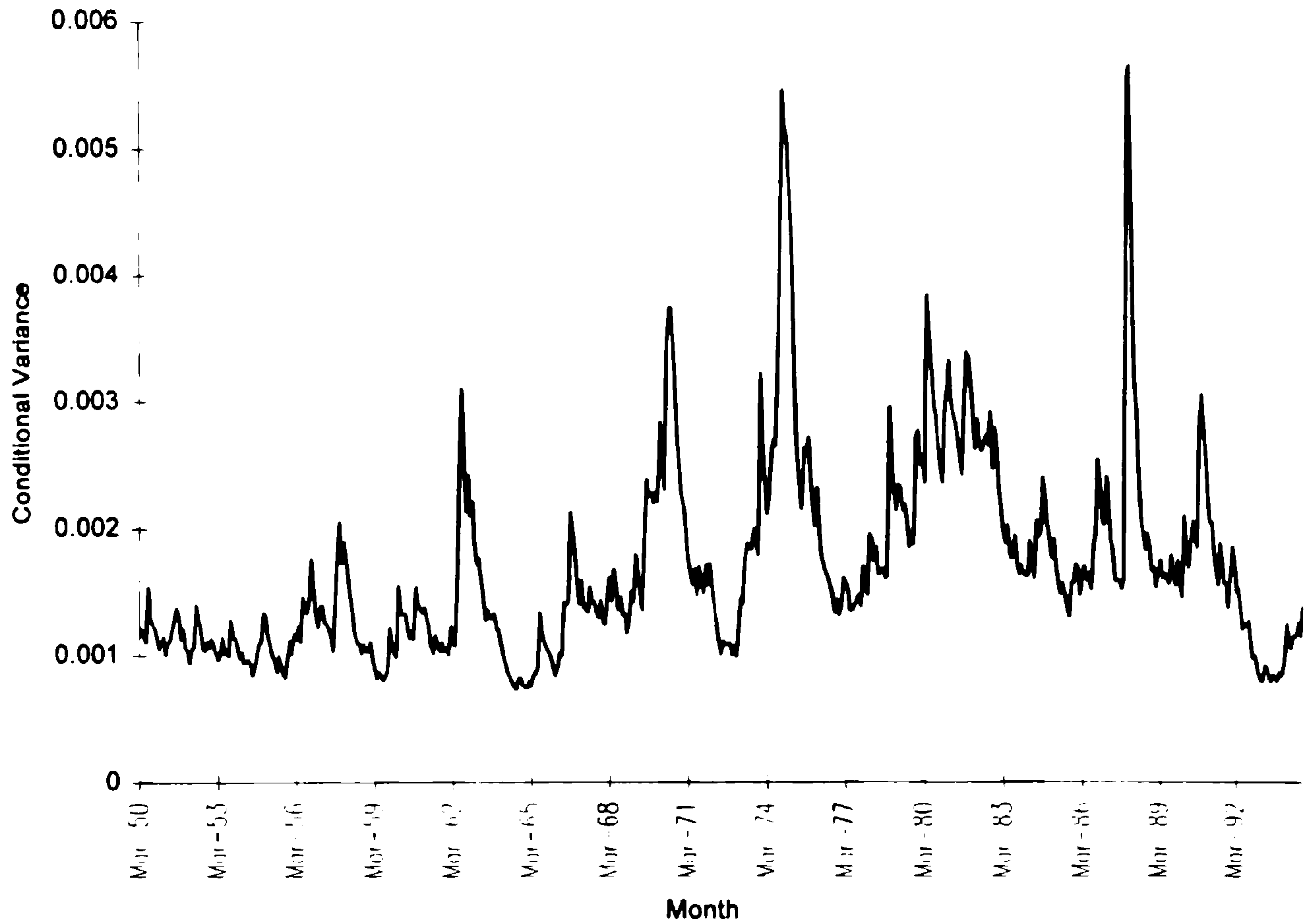

Figure 6. Conditional variance of excess market returns for Model 3b(iii). The figure plots modified bivariate EGARCH(1,1)-M estimates of $\hat{\sigma}^2_{M,t}$ for U.S. monthly data for the sample period 1950:3 to 1994:12. The estimates of $\hat{\sigma}^2_{M,t}$ result from the quasi-maximum likelihood estimation of Model 3b(iii) reported in Table IV.

bell (1987), Ferson (1989), and Shanken (1990) find that nominal T-bill yields are negatively correlated with future stock returns. If there are two distinct interest rate effects, then forcing the interest rate to enter a two-equation system (such as equation (6)) through the conditional variance equation could distort estimates of the conditional mean equation parameters.

I explore this idea by estimating an ad hoc model in which $r_{f,t}$ enters directly into both the conditional mean and conditional variance equations.

$$r_{M,t} = \lambda_0 + \lambda_M \hat{\sigma}^2_{M,t} + \lambda_f r_{f,t} + \hat{\epsilon}_{M,t}$$

$$\ln(\hat{\sigma}^2_{M,t}) = \omega_M + \alpha_M g(\hat{\psi}_{M,t-1}) + \beta_M \ln(\hat{\sigma}^2_{M,t-1}) + \gamma_M r_{f,t}$$

$$g(\hat{\psi}_{M,t-1}) = (|\hat{\psi}_{M,t-1}| - E_{t-1}[|\hat{\psi}_{M,t-1}|]) + \theta_M \hat{\psi}_{M,t-1}. \quad (8)$$

If there are two interest rate effects, estimating the system in equation (8) should uncouple them. I refer to equation (8) as Model 4b. Model 4a imposes the constraint $\gamma_M = 0$. In these models, λ_M measures the *partial* risk–return relation and λ_f measures the *partial* relation between the market risk pre-

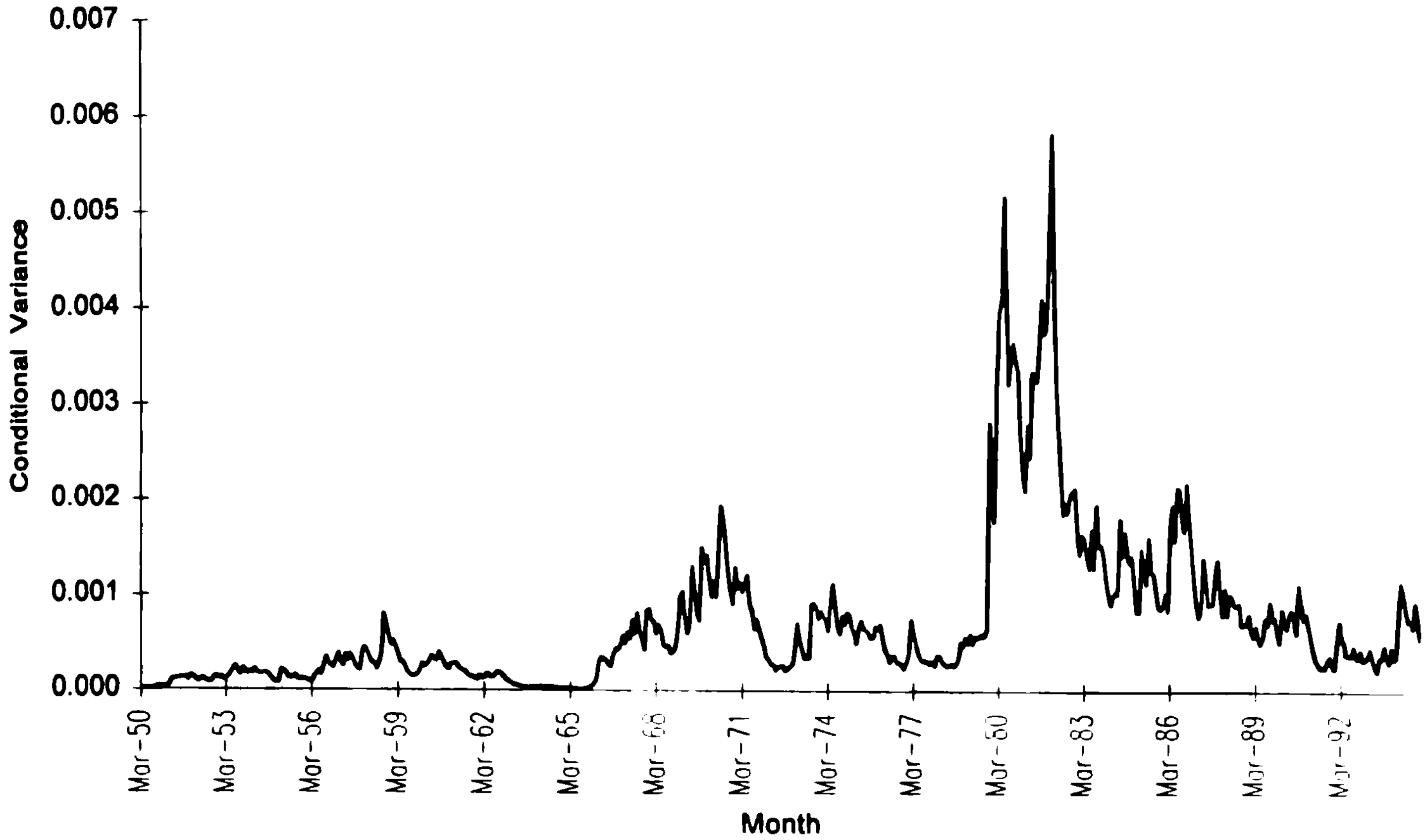

Figure 7. Conditional variance of long-term government bond excess returns. The figure plots modified bivariate EGARCH(1,1)-M estimates of $\hat{\sigma}^2_{F,t}$ for U.S. monthly data for the sample period 1950:3 to 1994:12. The estimates of $\hat{\sigma}^2_{F,t}$ result from the quasi-maximum likelihood estimation of Model 3b(iii) reported in Table IV.

mium and the nominal one-month T-bill yield. Estimates of Models 4a and 4b are reported in Table III to permit comparison with Models 1a, 1b, 2a, and 2b.

Estimates of the conditional variance equation parameters for Model 4a are very similar to those obtained for Models 1a and 2a. However, estimates of the conditional mean equation parameters are very different. The *partial* risk–return relation is positive and marginally significant ($\lambda_M = 7.43$, $t = 1.94$), but the *partial* relation between $r_{M,t}$ and $r_{f,t}$, is negative and significant ($\lambda_f = -2.44$, $t = -2.93$).

In Model 4b, $r_{f,t}$ appears in both the conditional mean and conditional variance equations. The point estimate of β_M is 0.86, which is highly significant ($t = 22.33$) and similar to the point estimates obtained for Models 1a, 2a, and 4a. The news response function parameters, α_M and θ_M, are similar to those for Model 4a. It appears that including $r_{f,t}$ as a second factor in the conditional mean equation restores the persistence of conditional market variance that was evident in Models 1a, 2a, and 4a. Yet, the strong positive relation between $\hat{\sigma}^2_{M,t}$ and $r_{f,t}$ that emerges in Models 1b and 2b is also evident in Model 4b ($\gamma_M = 24.78$, $t = 3.95$). In Model 4b, conditional market variance is *both* highly persistent and related to the level of the nominal risk-free rate. Figure 10 plots the modified EGARCH(1,1)-M estimates of $\hat{\sigma}^2_{M,t}$ which result from QML estimation of Model 4b. The plot in Figure 10 is similar in appearance to the plot in Figure 2. The relative smoothness of both of these plots reflects the persistence of conditional market variance in

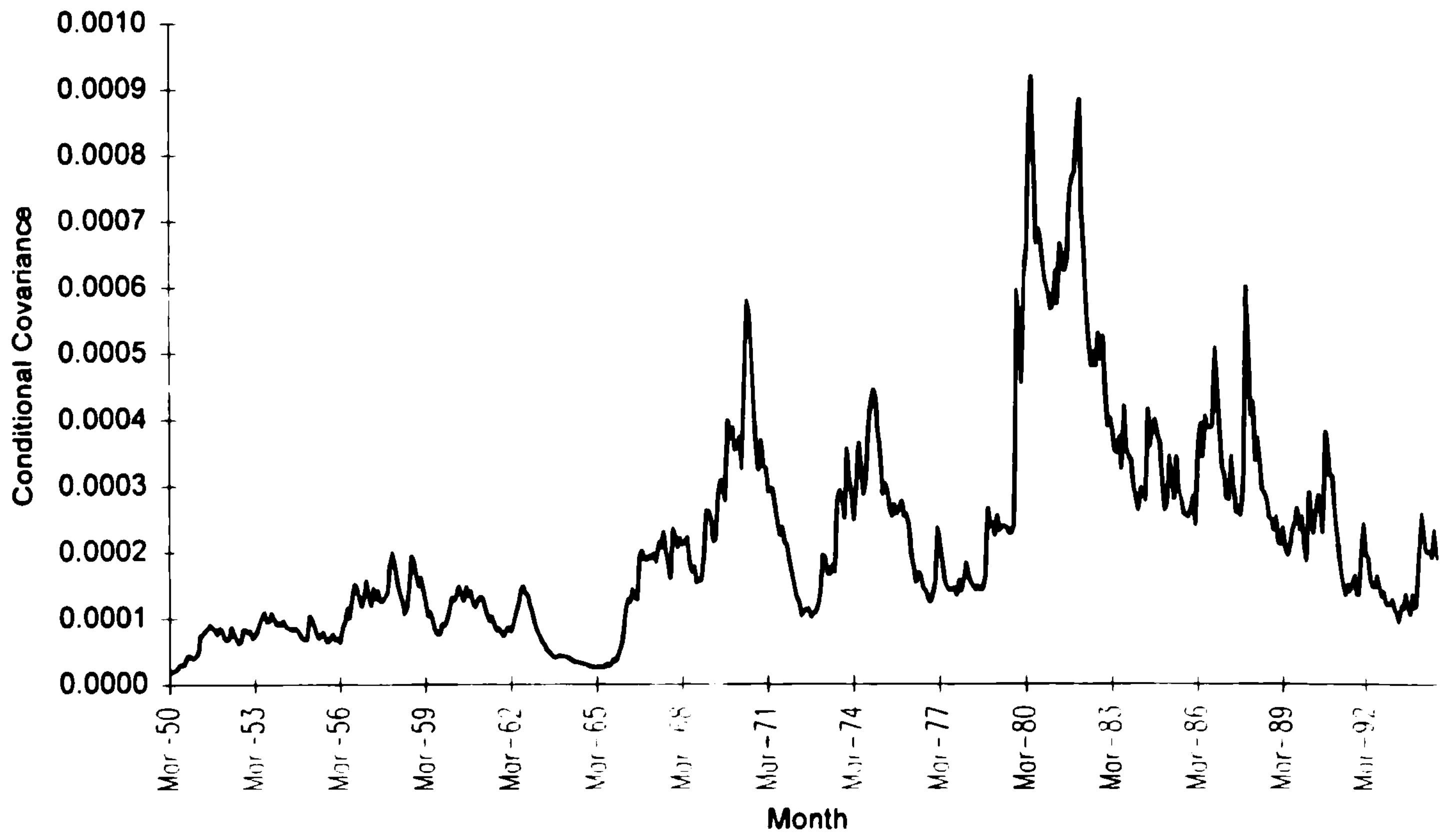

Figure 8. Conditional market covariance with long-term government bond excess returns. The figure plots modified bivariate EGARCH(1,1)-M estimates of $\hat{\sigma}_{MF,t}$ for U.S. monthly data for the sample period 1950:3 to 1994:12. The estimates of $\hat{\sigma}_{MF,t}$ result from the quasi-maximum likelihood estimation of Model 3b(iii) reported in Table IV.

Models 2a and 4b. The primary difference between the plot in Figure 10 and the plot in Figure 2 is elevated market volatility in the 1979–1986 period. Short-term interest rates are at their highest levels during this period.

Estimates of the conditional mean equation parameters in Model 4b are similar to those in Model 4a. The *partial* relation between $r_{M,t}$ and $\hat{\sigma}^2_{M,t}$ is positive and significant ($\lambda_M = 17.07$, $t = 2.55$) and the relation between $r_{M,t}$ and $r_{f,t}$ is negative and significant ($\lambda_f = -6.86$, $t = -3.69$). The intercept term, λ_0, is not significantly different from zero. Although Model 4b is admittedly ad hoc, these results suggest that a conditional two-factor model might resolve the puzzling relationship between $r_{M,t}$, $\hat{\sigma}^2_{M,t}$, and $r_{f,t}$. It appears that two distinct interest rate effects exist in the data: a conditional mean effect and a conditional variance effect. In Model 2b, where these effects are convoluted, volatility persistence appears low and the *simple* relation between $r_{M,t}$ and $\hat{\sigma}^2_{M,t}$ is negative. These are the main results reported by Glosten et al. (1993). In Model 4b, where the two interest rate effects are uncoupled, market volatility is highly persistent and the *partial* relation between $r_{M,t}$ and $\hat{\sigma}^2_{M,t}$ is positive.

B. Identification of the Market Portfolio and Priced Risk Factors

In Merton's ICAPM, as in the traditional CAPM, the market portfolio consists of all assets in the economy. Because the true market portfolio is unobservable, empirical researchers must choose some proxy for market portfolio.

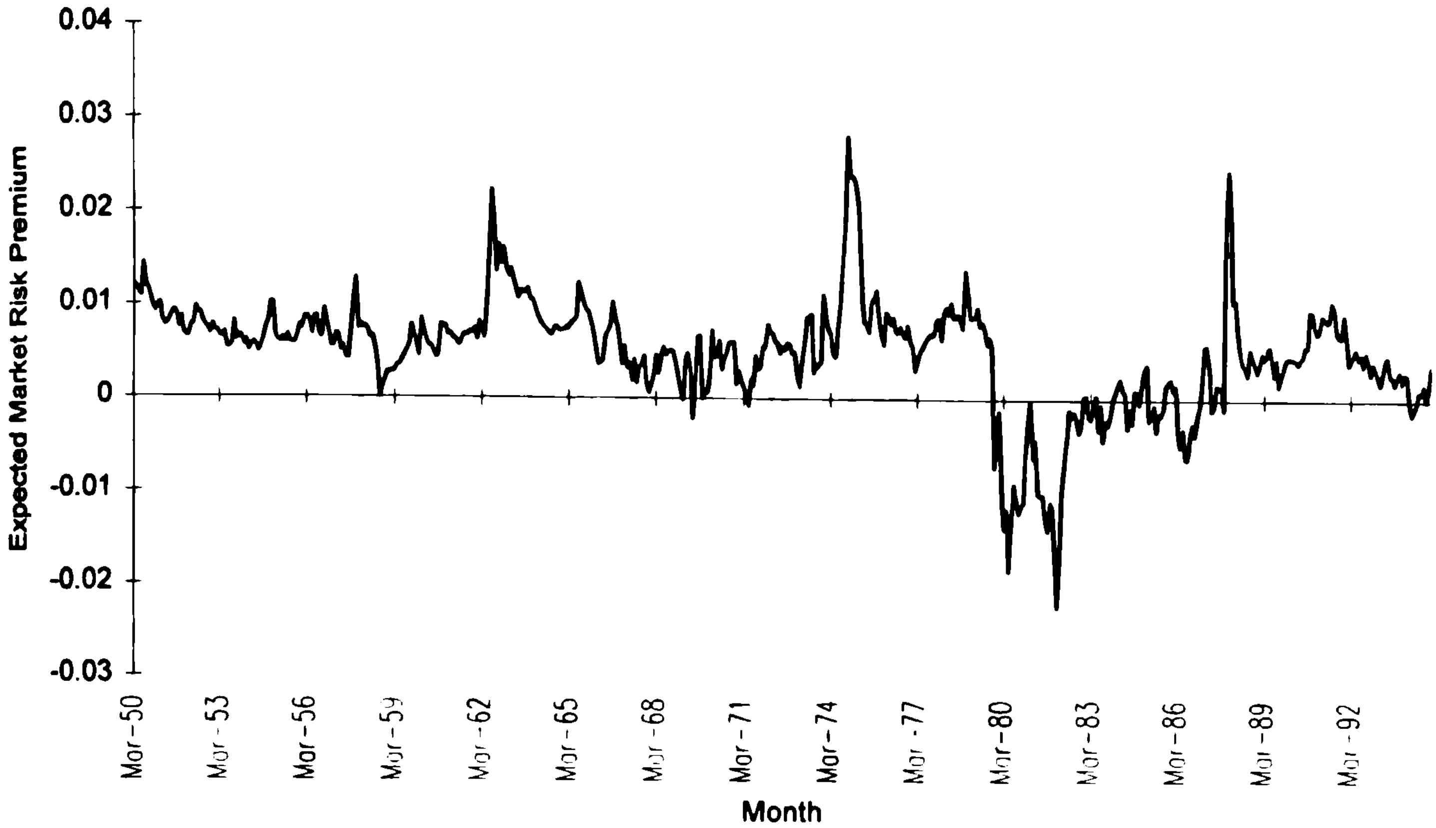

Figure 9. Expected market risk premium for Model 3b(iii). The figure plots modified bivariate EGARCH(1,1)-M estimates of $E_{t-1}[r_{M,t}]$ for U.S. monthly data for the sample period 1950:3 to 1994:12. The estimates of $E_{t-1}[r_{M,t}]$ result from the quasi-maximum likelihood estimation of Model 3b(iii) reported in Table IV. Model 3b(iii) implies that $E_{t-1}[r_{M,t}]$ is a linear function of $\hat{\sigma}^2_{M,t}$ (plotted in Figure 6) and $\sigma_{MF,t}$ (plotted in Figure 8).

This paper's objective is to improve our understanding of the puzzling relationship between the market risk premium and conditional market variance reported in the literature. Since comparability with previous research is desirable, I use the CRSP value-weighted total return index of NYSE–AMEX stocks as a proxy for the market portfolio.

Other researchers have tested the risk–return relation using "broader" proxies for the market portfolio. Chan, Karolyi, and Stulz (1992) examine a model similar to equation (4) in which the two factors are domestic and foreign equity indices.[18] In general, they find that the U.S. market risk premium is positively related to its conditional covariance with a foreign equity index, but unrelated to its own conditional variance. They also estimate several nested models in which the conditional second moments are market-weighted and the "price of risk" coefficients are constrained to be equal across factors. These constrained models can be interpreted as conditional single-factor models in which the market portfolio is an international equity index. They find evidence of a significant positive risk–return relationship and generally fail to reject the conditional single-factor model in favor of the unconstrained conditional two-factor model. These results suggest that future studies of the intertemporal risk–return relation may wish to consider a more broadly defined proxy for the market portfolio.

[18] Chan et al. (1992) examine daily data on the S&P 500, the MSCI EAFE and Japan indices (1980–1989), and the Nikkei 225 index (1979–1989).

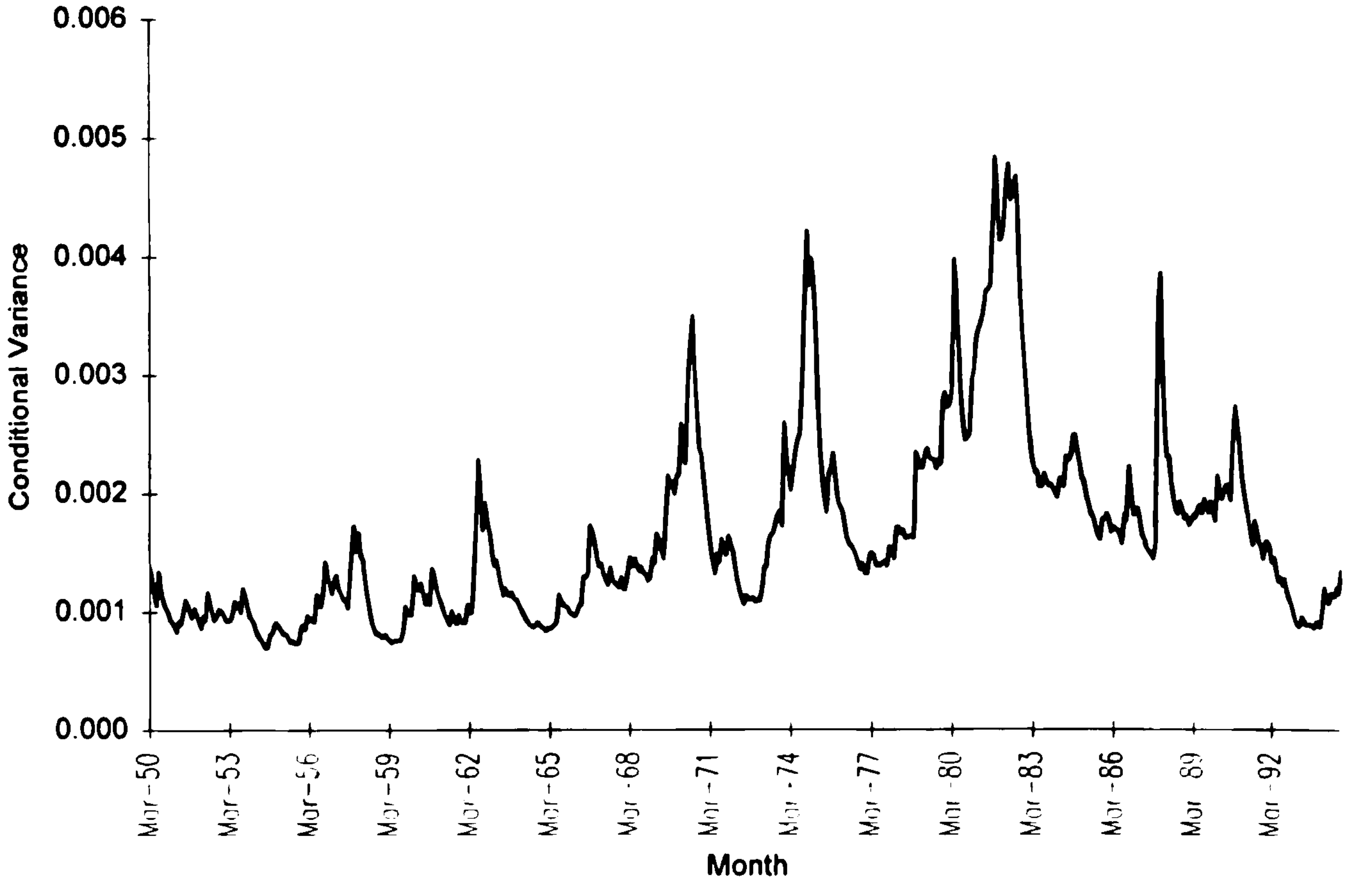

Figure 10. Conditional variance of excess market returns for Model 4b. The figure plots modified EGARCH(1,1)-M estimates of $\sigma^2_{M,t}$ for U.S. monthly data for the sample period 1950:3 to 1994:12. The estimates of $\hat{\sigma}^2_{M,t}$ result from the quasi-maximum likelihood estimation of Model 4b reported in Table III.

Bollerslev, Engle, and Wooldridge (1988) estimate a trivariate version of the conditional CAPM in which the market portfolio consists of three assets: a U.S. equity index, a twenty-year U.S. Treasury bond, and a six-month U.S. Treasury bill. In their conditional single-factor model, each asset's risk is a value-weighted average of its conditional covariances with the three assets in the market portfolio. The authors find that the market price of covariance risk is positive and significant. They do not estimate an unconstrained model in which the price of risk is allowed to vary across asset types. Whether long-term bond returns should be included in the market portfolio or as a second factor in a multifactor model remains an open question.

Asset pricing theory is generally silent regarding the identity of additional priced risk factors. Identification of state variables is largely an empirical matter. The long-term bond return employed as a state variable in this paper has long been used in cross-sectional tests of multifactor asset pricing models. Chen et al. (1986) and Shanken (1990) find that exposure to bond price movements is priced in the stock market. Whether conditional market covariance with other state variables affects the intertemporal behavior of the market risk premium is another open question. In work related to this paper, Scruggs (1996) tests a conditional two-factor model (equation (7)) using alternative state variables motivated by Chen et al. (1986). The state variables tested include the inflation rate, a bond quality premium (the monthly return on a low-grade corporate bond portfolio less the monthly return on a long-term Treasury bond), and the growth rate in industrial production. Conditional covariances with these alternate state variables do not significantly affect the expected market risk premium.

V. Conclusions

This paper investigates the intertemporal relation between the market risk premium and conditional market variance. Previous papers on this topic study the *simple* risk–return relation implied by a conditional single-factor model motivated by the Sharpe–Lintner CAPM. That literature is inconclusive. This paper examines the *partial* risk–return relation implied by a conditional two-factor model derived from Merton's ICAPM.

I find that the *partial* relation between the market risk premium and conditional market variance is positive and significant in a constant correlation bivariate EGARCH-M model that includes government bond returns as a second factor. The *partial* relation between the market risk premium and conditional market covariance with excess long-term government bond returns is negative and significant. I show that omitting the bond factor from the conditional mean equation results in estimates of the *simple* risk–return relation that are biased downward. The magnitude of the bias is sufficient to affect inference regarding the significance of the risk–return relation.

I also explore the negative *simple* relation between the market risk premium and conditional market variance reported by Glosten et al. (1993). I find that including the nominal risk-free rate in the conditional market variance equation distorts estimates of the *simple* risk–return relation suggested by the conditional single-factor model. The strong negative relation between $r_{M,t}$ and $r_{f,t}$ appears to dominate the estimation of both the conditional mean and variance equations. The positive relation between $\hat{\sigma}^2_{M,t}$ and $r_{f,t}$ is accentuated and the persistence of conditional market variance is suppressed. Resulting estimates of the *simple* risk–return relation are negative and significant. When I estimate an ad hoc model in which the two interest rate effects are uncoupled, the sign of the risk–return relation becomes positive and the persistence of conditional market variance is restored.

REFERENCES

Backus, David, and Allan W. Gregory, 1993, Theoretical relations between risk premiums and conditional variances, *Journal of Business and Economic Statistics* 11, 177–185.

Baillie, Richard T., and Ramon P. DeGennaro, 1990, Stock returns and volatility, *Journal of Financial and Quantitative Analysis* 25, 203–214.

Berndt, Ernst R., Bronwyn H. Hall, Robert E. Hall, and Jerry A. Hausman, 1974, Estimation and inference in nonlinear structural models, *Annals of Economic and Social Measurement* 3, 653–665.

Black, Fischer, 1976, Studies of stock price volatility changes, *Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economics Statistics Section*, 177–181.

Bodurtha, James N. Jr., and Nelson C. Mark, 1991, Testing the CAPM with time-varying risks and returns, *Journal of Finance* 46, 1485–1505.

Bollerslev, Tim, 1986, Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics* 31, 307–327.
Bollerslev, Tim, Ray Y. Chou, and Kenneth F. Kroner, 1992, ARCH modeling in finance, *Journal of Econometrics* 52, 5–59.
Bollerslev, Tim, Robert F. Engle, and Jeffrey M. Wooldridge, 1988, A capital asset pricing model with time-varying covariances, *Journal of Political Economy* 96, 116–131.
Bollerslev, Tim, and Jeffrey M. Wooldridge, 1992, Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances, *Econometric Reviews* 11, 143–172.
Boudoukh, Jacob, Matthew Richardson, and Tom Smith, 1993, Is the *ex ante* risk premium always positive?, *Journal of Financial Economics* 34, 387–408.
Breen, William, Lawrence R. Glosten, and Ravi Jagannathan, 1989, Economic significance of predictable variations in stock index returns, *Journal of Finance* 44, 1177–1189.
Campbell, John Y., 1987, Stock returns and the term structure, *Journal of Financial Economics* 18, 373–399.
Chan, K. C., Andrew Karolyi, and René M. Stulz, 1992, Global financial markets and the risk premium, *Journal of Financial Economics* 32, 137–167.
Chen, Nai-Fu, Richard Roll, and Stephen A. Ross, 1986, Economic forces and the stock market, *Journal of Business* 59, 383–403.
Christie, Andrew A., 1982, The stochastic behavior of common stock variances: Value, leverage, and interest rate effects, *Journal of Financial Economics* 10, 407–432.
Engle, Robert F., 1982, Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation, *Econometrica* 50, 987–1007.
Engle, Robert F., and Victor K. Ng, 1993, Measuring and testing the impact of news on variance, *Journal of Finance* 48, 1749–1778.
Engle, Robert F., David M. Lilien, and Russell P. Robins, 1987, Estimating time varying risk premia in the term structure: The ARCH-M model, *Econometrica* 55, 391–407.
Evans, Martin D. D., 1994, Expected returns, time-varying risk, and risk premia, *Journal of Finance* 49, 655–679.
Fama, Eugene F., and G. William Schwert, 1977, Asset returns and inflation, *Journal of Financial Economics* 5, 115–146.
Ferson, Wayne E., 1989, Changes in expected security returns, risk, and the level of interest rates, *Journal of Finance* 44, 1191–1217.
Ferson, Wayne E., and Campbell R. Harvey, 1991, The variation of economic risk premiums, *Journal of Political Economy* 99, 385–415.
French, Kenneth R., G. William Schwert, and Robert F. Stambaugh, 1987, Expected stock returns and variance, *Journal of Financial Economics* 19, 3–29.
Glosten, Lawrence R., Ravi Jagannathan, and David E. Runkle, 1993, On the relation between the expected value and the variance of the nominal excess return on stocks, *Journal of Finance* 48, 1779–1801.
Harvey, Campbell R., 1989, Time-varying conditional covariances in tests of asset pricing models, *Journal of Financial Economics* 24, 289–317.
Ibbotson Associates, 1995, *Stocks, Bonds, Bills and Inflation 1994 Yearbook* (Ibbotson Associates, Chicago, Ill.).
Lintner, J., 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 47, 13–37.
Merton, Robert C., 1973, An intertemporal asset pricing model, *Econometrica* 41, 867–888.
Merton, Robert C., 1980, On estimating the expected return on the market: An exploratory investigation, *Journal of Financial Economics* 8, 323–361.
Nelson, Daniel B., 1991, Conditional heteroskedasticity in asset returns: A new approach, *Econometrica* 59, 347–370.
Pagan, Adrian R., and Y. S. Hong, 1989, Nonparametric estimation and the risk premium, in William A. Barnett, James Powell, and George E. Tauchen, eds.: *Semiparametric and Nonparametric Methods in Econometrics and Statistics* (Cambridge University Press, Cambridge), 51–75.

Schwert, G. William, and Paul J. Seguin, 1990, Heteroskedasticity in stock returns, *Journal of Finance* 45, 1129–1155.

Scruggs, John T., 1996, *Market Returns and Conditional Market Variance in a Dynamic Economy*, Doctoral dissertation, University of Washington.

Shanken, Jay, 1990, Intertemporal asset pricing, *Journal of Econometrics* 45, 99–120.

Sharpe, William F., 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425–442.

Turner, Christopher M., Richard Startz, and Charles R. Nelson, 1989, A Markov model of heteroskedasticity, risk, and learning in the stock market, *Journal of Financial Economics* 25, 3–22.

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