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Source: *The Journal of Finance*, Vol. 52, No. 5 (Dec., 1997), pp. 2145-2170
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2329479
Accessed: 25/11/2009 10:13

THE JOURNAL OF FINANCE • VOL. LII, NO. 5 • DECEMBER 1997

# An Examination of Uncovered Interest Rate Parity in Segmented International Commodity Markets

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**ABSTRACT**

We examine the effect of segmented commodity markets on the relation between forward and future spot exchange rates in a dynamic economy. We calculate the slope coefficient in our theoretical economy from regressing exchange rate changes on forward premia. With reasonable parameter values, the slope coefficient is less than unity. However, even for extreme parameters the slope is not less than zero, as found in the data. A negative slope coefficient in a nominal version of the model requires the covariance between monetary shocks and relative output shocks to be significantly negative, in contrast to the covariance in the data.

ONE OF THE MOST STUDIED relations in international finance is that between forward exchange rates and future spot rates. Empirical tests typically find that when one regresses changes in spot rates on forward premia the slope coefficient is less than one, and often negative.[1] This result is called the forward bias anomaly, and implies that the returns from investing in forward exchange contracts are predictable. Our objective is to examine the effect of segmentation of international commodity markets on the relation between the forward premium and the change in the spot rate. Segmentation of commodity markets is modeled by introducing a cost for transferring goods across countries, giving rise to deviations from purchasing power parity (PPP). In this article, we first derive the theoretical implications of the effect of segmentation on the forward bias in a dynamic general equilibrium model and then analyze a calibrated version of the model. Our analysis serves as a useful benchmark to study how much of the predictable deviations from uncovered interest parity (UIP) can be explained by endogenous deviations from PPP.

* We thank the Social Sciences and Humanities Research Council of Canada and the Center of International Business Studies at the University of British Columbia for financial support, and Lorenzo Garlappi, Khang Min Lee, Steve Milos, David Peterson, and Jim Storey for research assistance. We thank Michael Adler, Jonathan Berk, Gordon Bodnar, Michael Brennan, Mick Devereux, Kathy Dewenter, Glen Donaldson, Bernard Dumas, Rick Green, Vasant Naik, Piet Sercu, Stan Zin, and participants at various seminars and conferences. We thank two referees and the editor, René Stulz, for comments. The usual disclaimer applies.

[1] For example, Froot and Thaler (1990) report that the average coefficient on the forward premium, across 75 studies, is $-0.88$. For excellent summaries of the literature on the relation between spot and forward exchange rates, see Baillie and McMahon (1989), Engel (1996), Froot and Thaler (1990), Hodrick (1987), and Lewis (1995).

Deviations from PPP are an important feature of exchange rate data. Empirical studies such as that of Bekaert (1994), Canova (1991), Gokey (1994), Levine (1989, 1991), and Mishkin (1984) find that predictable deviations from PPP are highly correlated with predictable violations of UIP; Engel (1996) provides a discussion of these tests. Mishkin (1984), in his study of international differences in real interest rates, reports that (p. 1345) "The evidence suggests that it is worth studying open economy macroeconomic models which allow: 1) domestic real rates to differ from world rates, 2) time-varying risk premiums in the forward market, or 3) deviations from ex-ante relative PPP." We analyze a model where all three of these features arise endogenously.

Our contribution is to derive the exchange risk premium and the expected change in the exchange rate as functions of foreign and domestic interest rates in a general equilibrium economy with endogenous deviations from PPP. This characterization of the exchange rate and the risk premium in terms of interest rates permits tests of the reduced form of the model using financial data.[2] We numerically analyze the model and find that, while the real model generates a slope coefficient that is less than unity, it is unlikely to generate a slope coefficient that is negative, even with extreme risk aversion and shipping costs. When we extend the real model to nominal quantities, we find that to obtain a negative UIP slope coefficient, the covariance between monetary shocks and the relative output shocks must be significantly negative.

There is a large literature that studies the relation between spot and forward exchange rates. Using the assumption of rational expectations, various authors have tested the UIP relation by regressing changes in the log of the exchange rate on the forward premium and various predetermined variables. The UIP hypothesis implies that the coefficient on the forward premium should be unity, the intercept should be zero, and any predetermined variables used to predict the change in the spot rate should have a coefficient of zero. Studies typically find, however, that the coefficient on the forward premium is smaller than one, and often negative. Two main hypotheses have been advanced to explain these results:[3] either market participants make systematic expectational errors (Lewis (1989)), or there are time-varying risk premia in the forward exchange market.[4] Fama (1984) makes the important empirical observation that the risk premium for holding forward contracts is more variable than the expected change in the spot rate, and the correlation between the two is negative. Models used to study the risk premium are typically based

[2] Empirical tests of the model can be obtained from the authors.

[3] Other explanations that have been advanced include the Jensen's inequality effect (found to be unimportant by McCulloch (1975)), and the effect of central bank intervention in exchange markets (McCallum (1994)). Also, Bekaert and Hodrick (1993) find that measurement errors due to bid-ask spreads cannot explain deviations from UIP.

[4] There are several articles studying time-varying risk premia. Examples include Backus, Foresi, and Telmer (1996), Backus, Gregory, and Telmer (1993), Baillie and Bollerslev (1989), Bansal, Gallant, Hussey, and Tauchen (1995), Bekaert, Hodrick, and Marshall (1994), Bekaert (1994, 1996), Canova and Marrinan (1993), Cumby (1988), Domowitz and Hakkio (1985), Engel (1992), Fama (1984), Hansen and Hodrick (1983), Hodrick (1987), Hodrick and Srivastava (1984), Korajczyk and Viallet (1992), Macklem (1991), Mark (1985) and McCurdy and Morgan (1991).

either on international versions of the capital asset pricing model (CAPM) with time-varying conditional moments, or on versions of the consumption-based asset pricing models of Lucas (1982) and Svensson (1985).[5] Generally, these studies find it difficult to explain all major features of the data.

Our article differs from the work described above in the following manner. Rather than to abandon the rational expectations view, we examine UIP in a general equilibrium economy with optimizing agents, but where the optimization is subject to the segmentation of the commodity markets. This emphasis on the imperfection of the real sectors also distinguishes our work from that of Bekaert (1994, 1996) and Bansal, Gallant, Hussey, and Tauchen (1995), where the focus is on monetary disturbances. In contrast to these models, where PPP is assumed to hold, the segmentation of the commodity markets in the model we study leads investors from the home and foreign country to behave differently; thus, the equilibrium is one without perfect-pooling, which leads to endogenous deviations from PPP and predictable deviations from UIP.[6]

Stulz (1981, 1987) also theoretically studies the effect of deviations from PPP on UIP. In his models, deviations from PPP arise as a consequence of nontraded goods. In our model, the deviations from PPP arise as a consequence of the shipping cost, which leads to deviations from the law of one price (LOP). Empirically, Engel (1993) and Rogers and Jenkins (1995) find that most of the variation in the real exchange rate arises from deviations from LOP. Further, Engel and Rogers (1995) show that within-country deviations from LOP are much smaller than cross-country deviations. This evidence is consistent with the shipping cost for trading goods across countries in our model.

The rest of this article is organized as follows. In Section I, we describe the model. In Section II, we derive the theoretical implications of the real model for spot and forward exchange rates. In Section III, we use numerical methods to study the population moments of the real economy calibrated to U.S. data. In Section IV, we use the results from our analysis of the real economy to derive implications for the nominal economy. Our conclusions are presented in Section V. Proofs for all results are contained in the Appendix.

## I. The Economy

The model we work with is a version of the economy in Dumas (1992). In this economy, there are two countries, each having a stochastic constant-returns-to-scale production technology with a country-specific output shock. The output in each country may be used either for consumption or as an input in the production process in either the foreign or the domestic country. There is, however, a proportional cost for transferring goods between the two countries that segments the real sectors of the two countries. Our objective is to analyze the effect of this segmentation on the UIP relation. We now describe the preferences and the production technology in the two countries. We use the

[5] For a recent overview of international asset pricing models, see Stulz (1995).

[6] For a review of the empirical evidence on deviations from PPP, see Froot and Rogoff (1995).

superscript "*" to refer to foreign variables, while variables without the superscript refer to their domestic counterparts.

The consumption rate of the representative domestic investor is given by $c(t)$ and that of the foreign investor by $c^*(t)$. Each investor is constrained to consume only from the stock of the good located in her own country. Preferences of the representative agents in the domestic and foreign country are given by constant relative risk averse utility functions, with relative risk aversion given by RRA $\equiv 1 - \gamma$, where $\gamma < 1$. We also assume that the representative consumers in the two countries have the same subjective discount rate, $\rho$, and start with equal endowments.

We denote the stock of goods in the home country at time $t$ by $K(t)$ and in the foreign country by $K^*(t)$. The constant returns to scale production technologies in both countries are assumed to have the same instantaneous expected rate of return, $\alpha$, and instantaneous volatility, $\sigma$. For analytical convenience, we assume that the innovations in the domestic and foreign production processes are uncorrelated.[7]

The only factor that distinguishes one country from another is that production in each country is subject to a country-specific shock and it is costly to transfer goods across countries. It is assumed that a proportional cost of $\theta$ is incurred in exporting goods from one country to another; that is, for each unit of good that is exported from one country only $1 - \theta$ arrives in the other. This cost may be interpreted as either a physical cost for shipping goods or as a dissipative tariff that has to be paid for importing goods from abroad.[8] To account for the export of the good from one country to another, we introduce two processes, $X(t)$ and $X^*(t)$, with $X(0) = X^*(0) = 0$. The processes $X(t)$ and $X^*(t)$ represent the cumulative amounts of the goods exported, since time 0, from the home to the foreign location, and from the foreign to the home location, respectively.

We assume that financial markets are complete and frictionless so that a Pareto optimal allocation is obtained.[9] This implies that the equilibrium consumption, investment, and shipping rules are given by the solution to the following central planner's problem.

$$\hat{V}[K(t), K^*(t), t] \equiv \max_{\{c,c^*,\Omega\}} E_t\left\{ \int_t^\infty e^{-\rho(u-t)}\left[\frac{1}{\gamma}c(u)^\gamma + \frac{1}{\gamma}c^*(u)^\gamma\right] du \right\}, \tag{1}$$

subject to the controlled output processes:

$$dK(t) = [\alpha K(t) - c(t)]dt + \sigma K(t)dz_K(t) - dX(t) + (1-\theta)dX^*(t),$$

$$dK^*(t) = [\alpha K^*(t) - c^*(t)]dt + \sigma K^*(t)dz_{K^*}(t) - dX^*(t) + (1-\theta)dX(t),$$

[7] All our results are robust to allowing the innovations to be correlated across the two countries.

[8] See Deardorff and Stern (1990) for the nature and magnitude of the various barriers to trade.

[9] Uppal (1993) provides the contingent claim holdings that are consistent with the Pareto optimal allocation.

where $dz_K(t)$, $dz_{K^*}(t)$ are two independent Wiener processes, the control $\Omega$ is an open region of $(K, K^*)$ space and $E_t$ is used to denote expectations conditional on all information up to and including time $t$. The boundary conditions that the value function must satisfy are given in equations (A1)–(A4) in the Appendix.

Since the objective function in equation (1) is homothetic, and the constraints are linear, one can show that the value function is homogeneous of degree $\gamma$. The homogeneity and symmetry of the problem imply that the set $\Omega$, and the optimal shipping policy, are defined by a single constant, $\lambda$, such that it is optimal to export goods from the home to the foreign country when $K(t)/K^*(t) = \lambda$ and import goods when $K(t)/K^*(t) = 1/\lambda$. Also, since the above problem is stationary,[10] we work with the undiscounted value function, $V[K(t), K^*(t)]$, where $\hat{V}[K(t), K^*(t), t] = e^{-\rho t}V[K(t), K^*(t)]$. For expositional ease, we will often suppress the arguments $K(t)$ and $K^*(t)$ and use $V(t)$ to denote the value function $V[K(t), K^*(t)]$.

To solve the planner's problem, we use the first-order condition for domestic consumption which is obtained from the Hamilton-Jacobi-Bellman equation in the Appendix, (A6). This condition equates the marginal utility of consumption with the marginal indirect utility of home capital,

$$c(t)^{\gamma-1} = V_1[K(t), K^*(t)], \tag{2}$$

where subscripts denote partial derivatives. A similar condition holds for foreign consumption. Further, the homogeneity of the value function enables us to reduce the dimension of the problem from two state variables, $K(t)$ and $K^*(t)$, to one state variable, $\omega(t) \equiv \ln[K(t)/K^*(t)]$.[11] Substituting the first-order condition for consumption into equation (A6), yields an ordinary differential equation in $\omega$, that, subject to the boundary conditions in equations (A1)–(A4), can be solved numerically.

Using the solution to the planner's problem, and Itô's Lemma, one can then obtain the process for $\omega(t)$,

$$d\omega(t) = \left[ -\frac{c(t)}{K(t)} + \frac{c^*(t)}{K^*(t)} \right] dt + \sqrt{2}\sigma dz_\omega(t), \tag{3}$$

where $dz_\omega(t) = [dz_K(t) - dz_{K^*}(t)]/\sqrt{2}$ is a standardized Weiner process and $\omega(t)$ is controlled so that it remains between $-\ln\lambda$ and $\ln\lambda$.

A characteristic of this process that will be important for our analysis is that its drift is positive. That is, capital imbalances tend to increase rather than decrease.[12] To understand the reason for this, consider the situation where the

[10] The derivation of the condition for existence, and the proof of the assertion that the value function is stationary and homogeneous of degree $\gamma$, are given in Theorem 1 of Uppal (1993).

[11] As in Dumas (1992), we choose this definition of the state variable because its process has constant volatility; thus, the behavior of the process is fully characterized by its drift and its boundary conditions.

[12] Dumas (1992) offers a similar explanation to the one provided below, and Uppal (1993) gives the portfolio strategies that are consistent with our explanation.

home capital stock increases relative to that abroad. In the decentralized equilibrium, the foreign investor owns a fraction of the domestic capital stock to diversify output risk; thus, the increase in the domestic capital stock should lead to a higher dividend being paid out to the foreign investor. However, because of the shipping cost, it is not optimal to ship the foreign investor's share of the dividend. Instead, the foreign investor invests this dividend in the domestic production process, leading to an increase in the already larger domestic capital stock. This additional investment has the effect of increasing the foreign investor's wealth; consequently, she decreases investment locally, thereby decreasing the already smaller foreign capital stock. The capital imbalance has the reverse effect on the domestic investor, which serves to accentuate the imbalance between the capital stocks. However, the imbalance is bounded because eventually it is optimal to transfer goods.

## II. Implications of the Real Model for Uncovered Interest Parity

We now derive the theoretical implications of this model for the relation between the real exchange rate and the domestic and foreign real interest rates. In doing this, we derive the stochastic process that the domestic and foreign pricing kernels follow, and use these to interpret the effect of segmentation on the premium for real exchange rate risk.

We start by deriving the real interest rates. In our economy, $q(t) \equiv V_1(t)$ can be used to price assets in terms of the domestic consumption good and $q^*(t) \equiv V_2(t)$ gives prices in terms of the foreign consumption good. The domestic price at time $t$, $P_\delta(t)$, of an asset with stochastic payoff stream $\delta(t)$ in domestic consumption units is: $P_\delta(t) = E_t[\int_t^\infty e^{-\rho(u-t)} [q(u)/q(t)]\, \delta(u)\, du]$. Applying this to price an instantaneously riskless bond that pays one unit of the domestic consumption good yields, as in Cox, Ingersoll, and Ross (1985):

LEMMA 1. *The domestic real interest rate, $r(t)$, is*

$$r(t) = \rho - \frac{E_t[dV_1(t)]}{V_1(t)dt} = \alpha + \frac{V_{11}(t)}{V_1(t)} K(t)\sigma^2, \tag{4}$$

*and the foreign interest rate is given similarly.*

The real exchange rate is defined to be the shadow price of goods abroad relative to the price of domestic goods: $V_2[K(t), K^*(t)]/V_1[K(t), K^*(t)] = q^*(t)/q(t)$. In the absence of a cost for transporting goods from one country to another, the marginal valuation of output would be equated across countries at all points in time, implying that the real exchange rate would be constant at one. However, in the presence of the proportional shipping cost, goods are transferred between countries only when $V_2(t)/V_1(t) = 1 - \theta$ or $V_2(t)/V_1(t) = 1/(1 - \theta)$. Thus, with positive shipping costs, the real exchange rate will vary stochastically between $1 - \theta$ and $1/(1 - \theta)$.

To derive the process for the real exchange rate, we need the processes for $q(t)$ and $q^*(t)$. To understand the effect of segmentation on the pricing kernels, we have decomposed the stochastic shocks in the pricing kernels into two *uncorrelated* sources of uncertainty.[13] The first source of uncertainty, $dz_{K(t)} + dz_{K^*}(t)$, is the innovation to the return on the aggregate capital stock in an economy where the capital stocks are always perfectly balanced; that is, an economy with no shipping cost. The second source is the shocks due to innovations in the relative capital stocks, $dz_{\omega}(t)$.

LEMMA 2. *The stochastic processes for the domestic and foreign pricing kernels in this economy can be expressed as:*

$$\frac{dq(t)}{q(t)} = [\rho - r(t)]dt - \sigma \frac{1-\gamma}{2}[dz_K(t) + dz_{K^*}(t)] + \sqrt{2}\sigma\left(\frac{1-\gamma}{2} - \frac{\alpha - r(t)}{\sigma^2}\right)dz_{\omega}(t), \quad (5)$$

$$\frac{dq^*(t)}{q^*(t)} = [\rho - r^*(t)]dt - \sigma \frac{1-\gamma}{2}[dz_K(t) + dz_{K^*}(t)] - \sqrt{2}\sigma\left(\frac{1-\gamma}{2} - \frac{\alpha - r^*(t)}{\sigma^2}\right)dz_{\omega}(t), \quad (6)$$

*where* $cov([dz_K(t) + dz_{K^*}(t)], dz_{\omega}(t)) = 0$

To study the effect of shipping costs on the pricing kernel, we examine two polar cases. In an economy with no shipping costs, it is optimal to rebalance the stock of goods in the two countries at each instant because of symmetry between the home and foreign economies. This implies that $(1 - \gamma)/2 = [\alpha - r(t)]/\sigma^2$.[14] In this case, the coefficient on $dz_{\omega}(t)$ in equation (5) is zero, indicating that the only risk priced in this economy is that of the world capital stock. On the other hand, with 100 percent shipping costs, $[\alpha - r(t)]/\sigma^2 = 1 - \gamma$.[15] Hence, the domestic pricing kernel reduces to $dq(t)/q(t) = [\rho - r(t)]dt - \sigma[\alpha - r(t)]/\sigma^2 dz_K(t)$, so that the only risk priced is that of the home capital stock. In the general case, where shipping costs lie between zero and 100 percent, both the aggregate capital stock risk and the risk of capital stock imbalances are priced.

We are now ready to state the relation between the log of the real exchange rate, denoted $e(t) \equiv \ln[q^*(t)/q(t)]$, and the domestic and foreign real interest rates.[16]

[13] A similar decomposition is possible in the case where the shocks to the production processes are correlated or have difference volatilities.

[14] With zero shipping costs, it is optimal to have $K(t) = K^*(t)$ for all $t$, so that $V_{11}(t) = V_{12}(t)$. Using the homogeneity of the value function, equation (A10), $(\gamma - 1)/2 = [V_{11}(t)/V_1(t)]\,K(t) = [V_{22}(t)/V_2(t)]\,K^*(t)$. Then, from (4), $[\alpha - r(t)]/\sigma^2 = [\alpha - r^*(t)]/\sigma^2 = (1 - \gamma)/2$.

[15] This can be shown either using the homogeneity of the value function, and the fact that across perfectly segmented countries $V_{12}(t) = 0$, or from the closed-form expression for the value function with 100 percent shipping costs.

[16] Dumas (1992) also derives an equation for the expected change in the log of the real exchange rate. His result is in terms of the derivative of the log of the real exchange rate with respect to the

PROPOSITION 1. *The relation between the log of the real exchange rate and the domestic and foreign real interest rates in the no-shipping region is:*

$$\begin{aligned} de(t) &= [r(t) - r^*(t)] \times [1 + \phi(t)]dt - \sqrt{2}\sigma\phi(t)dz_\omega(t), \\ &= [r(t) - r^*(t)]dt - rp^{\text{real}}(t)dt - \sqrt{2}\sigma\phi(t)dz_\omega(t), \end{aligned} \tag{7}$$

*where*

$$rp^{\text{real}}(t) \equiv -\phi(t)[r(t) - r^*(t)], \tag{8}$$

$$\phi(t) \equiv \left[(1-\gamma) - \left(\frac{\alpha - r(t)}{\sigma^2} + \frac{\alpha - r^*(t)}{\sigma^2}\right)\right]. \tag{9}$$

From equation (7),

$$E_t[de(t)] = [r(t) - r^*(t)] \times [1 + \phi(t)]dt. \tag{10}$$

The standard test for UIP (expressed in real terms) is

$$E_t[de(t)] = a^{\text{real}} + b^{\text{real}}[r(t) - r^*(t)]dt, \tag{11}$$

where the null hypothesis is that $a^{\text{real}} = 0$ and $b^{\text{real}} = 1$. Comparing equations (10) and (11), we see that the effect of segmentation of commodity markets on the relation between real exchange rate changes and the real interest rate differential is captured by the term $\phi(t)[r(t) - r^*(t)]$.

The quantity $\phi(t)$ can be interpreted in two ways: one in terms of the local stock market risk premia, and the other in terms of the risk premium for capital imbalances across countries. The first interpretation follows from observing that $K(t)$ and $K^*(t)$ are equal to the value of the local stock market portfolio in each country,[17] so that the expected rate of return on capital, $\alpha$, equals the expected rate of return on the local stock market portfolio; hence, the term $[\alpha - r(t)]/\sigma^2$ is the local stock market risk premium. Thus, $\phi(t)$ is the difference between the relative risk aversion of the representative agents, $1 - \gamma$, and the sum of the local market risk premia in the two countries. In an economy with no shipping costs, the local market risk premium is equal to $(1 - \gamma)/2$, as shown previously, and thus $\phi(t) = 0$ in this case. Hence, the deviation of $\phi(t)$ from zero is a measure of the risk arising from market segmentation.

The second interpretation of $\phi(t)$ is as the sum of the home and foreign premia for the risk that capital stocks in the two countries are not perfectly balanced due to the shipping cost. The risk premium for any risky asset is given by the negative of the covariance of the asset's return and the pricing kernel. Using equation (5) for the pricing kernel and calculating the risk premium for an asset whose return is perfectly correlated with the imbalance

imbalance of the quantity of goods at home and abroad. In contrast, the equation we derive is in terms of interest rates.

[17] This interpretation follows from the constant-returns-to-scale process used to model the return on the capital stock.

in the capital stock, $\omega(t)$, and performing similar calculations for the foreign investor, whose measure of the capital imbalance is $-\omega(t)$, leads to the following result.

LEMMA 3. *The quantity $\phi(t)$ is proportional to the sum of the home and foreign risk premia for capital imbalances, or equivalently, proportional to the covariance of relative consumption growth rates and the capital imbalance.*

$$\phi(t)dt \propto -\left[\operatorname{cov}\left(\frac{dq(t)}{q(t)}, d\omega(t)\right) + \operatorname{cov}\left(\frac{dq^*(t)}{q^*(t)}, -d\omega(t)\right)\right]$$

$$= (1-\gamma)\operatorname{cov}\left(d \ln\left(\frac{c(t)}{c^*(t)}\right), d\omega(t)\right). \tag{12}$$

We now use our two interpretations of $\phi(t)$ to explain that with shipping costs $\phi(t)$ is nonpositive. Thus, from equation (10) this model has the potential of generating a UIP slope coefficient that is less than unity. First, we compare $\phi(t)$ in a perfectly integrated economy to an economy with shipping costs. In a perfectly pooled economy, the production risk premium in each of the countries is $(1-\gamma)/2$, as explained above. Thus, $\phi(t)$ is equal to zero. However, in an economy with segmented markets, it is not possible to diversify production optimally across the two countries. Thus, given that this economy has more risk than an economy with zero shipping costs, there is a relatively higher demand for the riskless assets; hence, the interest rates are lower in this economy, compared to one with zero shipping costs. From equation (9), thus implies that $\phi(t)$ will be lower in an economy with shipping costs; thus $\phi(t) \leq 0$.[18]

The second explanation for $\phi(t) \leq 0$ is in terms of Lemma 3, and is similar to the explanation for the drift of $\omega(t)$ being positive. For this lemma, the sign of $\phi(t)$ is the same as the covariance between changes in relative consumption rates and changes in the capital imbalance, $d\omega(t)$. To explain why this covariance is negative, consider the experiment where the world capital stock is fixed, and $\omega(t)$ increases, so that the home capital stock increases relative to that abroad. In equilibrium, the foreign investor owns a fraction of the domestic capital stock to diversify output risk. Because of the shipping cost, the foreign investor's share of the dividend from the increase in the domestic capital stock is not immediately shipped to the foreign country; instead, this dividend is reinvested in the domestic production process. This leads to an increase in the foreign investor's wealth; consequently she decreases investment in her local (the foreign) production function, and increases consumption

[18] With 100 percent shipping costs, the production risk premia in each country are constant and equal to $1-\gamma$, as explained previously. Thus, in this case, $\phi(t) = -(1-\gamma) < 0$. From equation (10), it may appear that with 100 percent shipping cost the model can generate a negative coefficient in the UIP regression. However, this is not true; with 100 percent shipping cost, the real interest rates in the two countries are constant and equal to one another because in both countries the production technologies and preferences are identical and the output shocks are independently and identically distributed. Thus, in this case the risk premium, $-\phi(t)[r(t) - r^*(t)] = 0$.

from the stock of capital in her own country. By the same logic, the capital imbalance has the reverse effect on the domestic investor, leading to a decrease in home consumption. Hence, the covariance between shocks to the capital imbalance and the relative consumption rates is negative, implying that $\phi(t) \leq 0$. Our numerical analysis of the model in Section III confirms this intuition.

In the next section of the article, we study the ability of our model to explain deviations from UIP found in the data.

## III. Population Moments in the Real Economy

In this section, using a calibrated version of the real model, we investigate whether the real model can generate predictable deviations from UIP of the magnitude observed in the data. In the next section, we study the UIP relation in nominal terms. Analyzing the UIP relation first in the real economy and then in nominal terms facilitates the interpretation of the results, since it is convenient to express the nominal regression coefficient in terms of the UIP regression based on real terms. It also allows us to identify the source of the deviations in UIP in our model, and explains which features of the model would have to change in order to explain the data.

Our numerical analysis of the model in this section is divided into two parts. First, in Section III.A, we explain the approach we adopt for studying the calibrated version of the model. Then, in Section III.B, we compute the regression coefficients for the UIP relation in real terms.

### A. Methodology and Parameter Values

The standard test for UIP (expressed in real terms) is based on the regression equation in (11). Comparing (11) to (10), we see that the population regression coefficient in equation (11) is given by

$$b^{\text{real}} = \frac{\operatorname{cov}(E_t[de(t)] + rp^{\text{real}}(t),\ E_t[de(t)])}{\operatorname{var}(E_t[de(t)] + rp^{\text{real}}(t))}, \tag{13}$$

where var and cov are the unconditional variance and covariance operators. The unconditional moments in equation (13) are calculated in two steps. In the first step, we solve the problem defined in equation (1), subject to the boundary conditions given in equations (A1)–(A4). The solution to this problem yields the optimal consumption, investment and shipping policies. From these policies, we compute the process for the state variable, $\omega(t)$, in equation (3).

The second step is to use the functions for the drift and diffusion of $\omega(t)$ to compute its stationary (unconditional) density. Using the results in Karlin and Taylor (1981, equation (5.34)), and letting $\mu_\omega(\omega)$ and $\sigma_\omega(\omega)$ denote the drift and diffusion of $\omega(t)$, its stationary density, $\psi(\omega)$, is

$$\psi(\omega) = g(\omega)[C_1 H(\omega) + C_2],$$

where,

$$H(x) = \int_{-\ln\lambda}^{x} h(y)\ dy, \qquad \text{for} \quad -\ln \lambda < x < \ln \lambda,$$

$$h(y) = \exp\left\{ -\int_{-\ln\lambda}^{y} \left[ \frac{2\mu_{\omega}(u)}{\sigma_{\omega}^{2}(u)} \right] du \right\}, \qquad \text{for} \quad -\ln \lambda < y < \ln \lambda,$$

$$g(x) = \frac{1}{\sigma_{\omega}^{2}(x) h(x)}, \qquad \text{for} \quad -\ln \lambda < x < \ln \lambda,$$

and $C_1$ and $C_2$ are constants of integration that are chosen so that $\psi(\omega) \geq 0$ for $-\ln\lambda < \omega < \ln\lambda$, $\int_{-\ln\lambda}^{\ln\lambda} \psi(\omega)\ d\omega = 1$, and the stationary density is symmetric around $\omega = 0$. The unconditional variance and covariance terms are then obtained by numerically integrating the appropriate functions against the stationary density for $\omega(t)$.

Another way to compute the unconditional moments would be to use Monte Carlo simulations of the model solution. In the context of our model, it is better to compute the stationary density directly instead of using Monte Carlo simulations for two reasons. First, Monte Carlo simulations require that to obtain a series of observations that are stationary, larger and larger draws need to be generated as the shipping cost increases. This is because, as the region of no-shipping increases the controls are applied less frequently, and thus, the distribution of the state variable comes closer to being a random walk. Second, using the stationary density avoids the problems that can arise as a consequence of discretizing the time-step in the Monte Carlo simulations, especially at the boundaries of the no-shipping region where the controls are applied.[19]

To calibrate the model, one needs to choose a set of values for the parameters of the output process governing the average (real) rate of return on investment, $\alpha$, and the volatility of this return, $\sigma$. In addition, one needs to choose parameters for investor's preferences: the rate of impatience, $\rho$, and the degree of relative risk aversion (RRA). The parameters we choose must satisfy the existence condition,

$$\rho > \gamma[\alpha - 0.25(1 - \gamma)\sigma^2], \tag{14}$$

which ensures that the lifetime utility defined in equation (1) is bounded.[20] For a given $\rho$, $\alpha$, and $\sigma$, the above condition puts an upper bound on the degree of relative risk aversion, $1 - \gamma$.

The choice of parameters for the risk and expected return of the output process depends on whether one chooses to interpret $K(t)$ and $K^*(t)$ as the value

[19] The issues of convergence that arise when simulating controlled diffusion processes with discrete Markov chains are discussed in Fleming and Soner (1993).

[20] This condition is similar to that derived in Merton (1990, equation (4.40)).

of the local market portfolios[21] or as the real output in each country. Given that the results for the two cases are similar, we report results only for the case where the model is calibrated to financial data. We base our estimates on the return for the U.S. economy reported in Ibbotson and Sinquefield (1988). We assume that the average real rate of return on the market is 8 percent per annum, and the volatility is 10 percent per annum.[22] We assume that $\rho$ is equal to 0.02, as is typically done in macroeconomic studies. For this set of parameter values, the largest RRA for which equation (14) is satisfied is 32.356. We allow relative risk aversion to range from 2 to 30. For the case where the parameters are calibrated to the real economy, the largest feasible value for RRA is 129. The results we report below are similar to those for the case where we allow RRA to be as large as 128.

Deardorff and Stern (1990, Table A7) report that the average tariff in industrialized countries is about 6.6 percent. Deardorff (1989, Table 4 and B5) reports the low and high estimates of the ad valorem equivalent of nontariff barriers for individual industrial sectors in the U.S.: for example, in the case of machinery, the low and high estimates are both 0 percent, while for food products they range from 14.5 to 20.5 percent. To analyze the behavior of the model for different shipping costs, we consider costs from a low of 5 percent to a high of 70 percent. We allow for such large costs so that we can determine if the model can deliver negative UIP slopes under extreme parameter values.

### B. The UIP Regression Coefficient in the Real Economy

The unconditional regression coefficients for the case where the model is calibrated to financial data are presented in Figure 1. This figure presents the population regression coefficient defined in equation (13), for different values of RRA and shipping costs ($\theta$). The figure shows that, consistent with the findings of empirical studies, the model always generates a slope coefficient that is less than unity. However, for the set of parameter values we examine, the slope coefficient is never negative—not even for extreme levels of risk aversion or transactions costs. For example, even when the shipping cost is 70 percent and RRA is 15, the slope coefficient is 0.48.

In Figure 1, we also see that the UIP slopes do not decrease monotonically with an increase in transactions costs. This is because an increase in transactions costs has two countervailing effects: first, it leads to a decrease in $\phi(\omega)$; second, the economy spends more time close to the shipping region. The first effect tends to decrease the UIP slope, since the coefficient of the forward premium is given by $[1 + \phi(\omega)]$. The second effect tends to increase the slope, since near the shipping region, the risk premium is small. The two effects can be seen below in our analysis of the model for different levels of transactions costs.

[21] As mentioned before, this is consistent with the constant-returns-to-scale process used to model the return on investment.

[22] Obstfeld (1994) uses similar values in a calibration exercise in a different context. These values are reported in his Table 8.

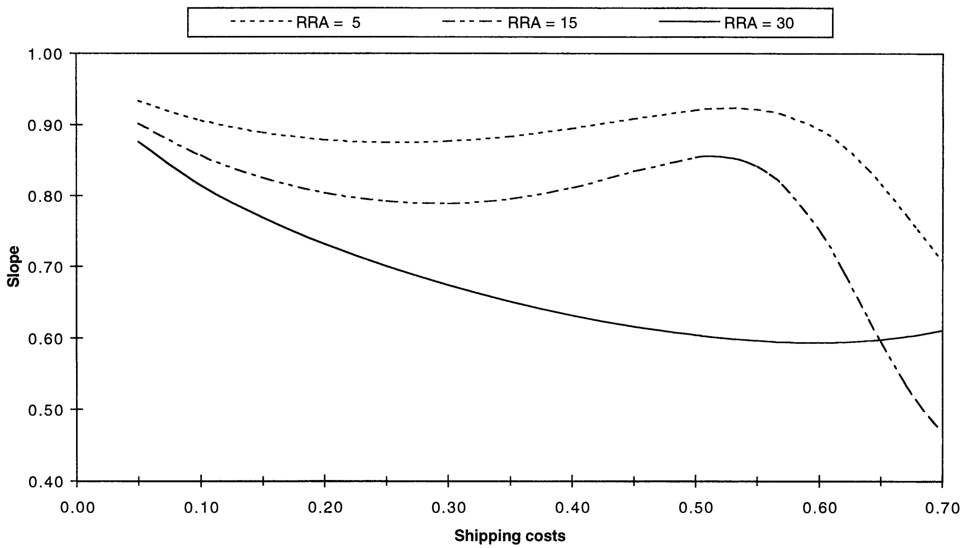


**Figure 1. UIP Regression Slope in the Real Model.** In this figure, we graph the real uncovered interest parity (UIP) slope coefficient against shipping costs. The model is calibrated to stock market data. The volatility of real returns in the stock market, $\sigma$, is assumed to be 10 percent per annum and the expected real return on the market, $\alpha$, is assumed to be 8 percent per annum. The subjective discount rate, $\rho$, is 0.02. RRA is the relative risk aversion parameter. The graph shows that the slope coefficient is less than unity for all the parameter values for which it is calculated, but never negative.

To understand the results in Figure 1 in terms of our model, we use the analysis in Fama (1984). In this article, Fama derives the necessary conditions to match the negative UIP regression. These conditions, in real terms, are:

$$\mathrm{var}(rp^{\mathrm{real}}(t)) > \mathrm{var}(E_t[de(t)]), \tag{15}$$

$$\mathrm{corr}(E_t[de(t)],\ rp^{\mathrm{real}}(t)) < 0. \tag{16}$$

Recall that $rp^{\mathrm{real}}(t) = -\phi(t)[r(t) - r^*(t)]$, and the only state variable in our model is the international capital imbalance, $\omega(t)$. To study the ability of our model to satisfy the above conditions, we examine three quantities as functions of $\omega$. These are (a) the measure of capital imbalance and of real exchange rate risk, $\phi(\omega)$; (b) the expected change in the real exchange rate, $E_t[de(\omega)]$; and (c) the premium for real exchange rate risk, $rp^{\mathrm{real}}(\omega)$. The Fama conditions are in terms of unconditional moments. To provide information about the unconditional moments, we also plot the stationary density of $\omega$. These graphs are presented for two sets of parameter values—in Figure 2 with 5 percent shipping costs, and in Figure 3 where shipping costs are 25 percent.

In Figure 2, the parameter values used are $\rho = 0.02$, $\alpha = 0.08$, $\sigma = 0.10$, RRA $= 5$, and $\theta = 5$ percent. The population UIP slope coefficient is 0.93. Plot

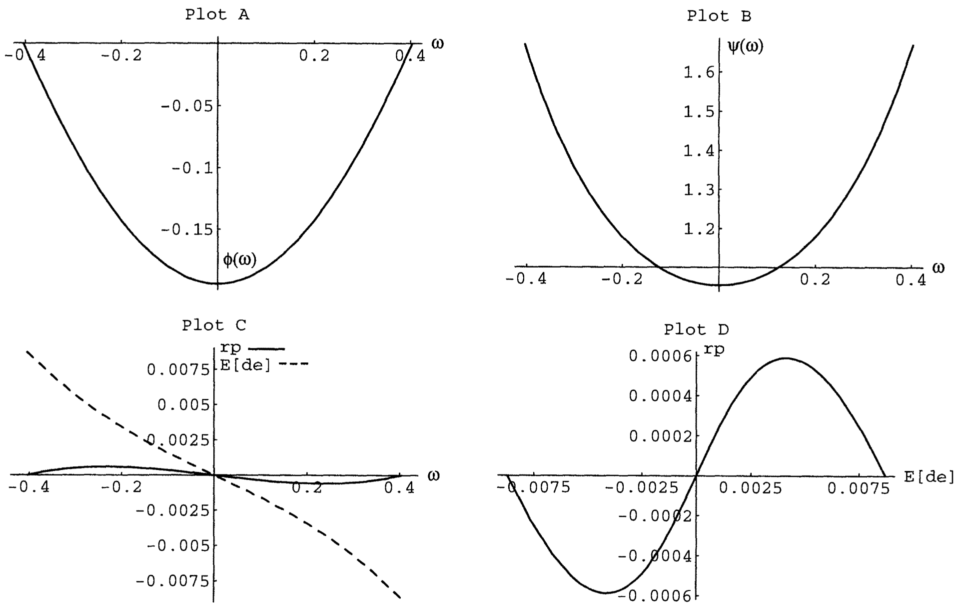


**Figure 2. Analysis of the Real Model: The Base Case.** Plot *A* shows the function $\phi(\omega)$, which is a measure of the degree of segmentation of the home and foreign commodity markets. Plot *B* shows the stationary density of the state variable, $\omega = \ln[K/K^*]$. In Plot *C*, the real risk premia, $-\phi(\omega)[r(\omega) - r^*(\omega)]$ (solid line), and the expected change in the real exchange rate (dashed line) are plotted against $\omega$, and in Plot *D* the risk premia is plotted against the expected change in the exchange rate. The parameters for the base case are the following: relative risk aversion, *RRA*, is 5, the impatience parameter, $\rho$, is 0.02, the mean return on investment, $\alpha$, is 0.08, the volatility of the return processes, $\sigma$, is 0.10, and the shipping cost, $\theta$, is 0.05.

A of Figure 2 shows $\phi(\omega)$, which is nonpositive for the reasons given in Section II. The plot shows that $\phi(\omega)$, the measure of segmentation of the two commodity markets, is zero at the boundaries of $\omega$ and attains its minimum when the capital stocks are equal across countries, $\omega = 0$. At the extreme values of $\omega$, where shipping takes place, the commodity markets abroad and at home are integrated, thus $\phi = 0$. At these points, the consumption growth rates across the two countries are perfectly correlated, and deviations from relative PPP are small. On the other hand, when the capital stocks are perfectly balanced, so that $\omega = 0$, the likelihood of shipping is the smallest, and at this point, $\phi$ takes its smallest value. At this point, the correlation in consumption growth rates across the countries is the lowest, so that the economy is highly segmented; that is, shocks to output are least likely to be diversified across countries.

In Plot B, we present the stationary density of the state variable $\omega$. As explained before, the drift of $\omega$ is positive. Consequently, the stationary density is U-shaped, indicating that the mass is concentrated away from the center and the economy spends a large proportion of its time at the boundaries, where shipping takes place.

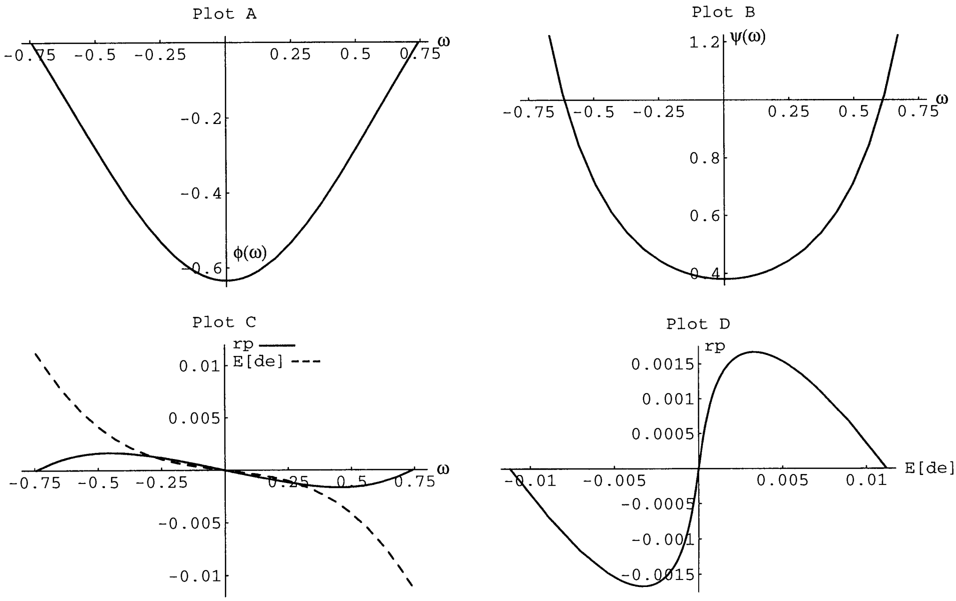


**Figure 3. Analysis of the Real Model: Increased Shipping Costs.** Plot *A* shows the function $\phi(\omega)$, which is a measure of the degree of segmentation of the home and foreign commodity markets. Plot *B* shows the stationary density of the state variable, $\omega = \ln[K/K^*]$. In Plot *C*, the real risk premia, $-\phi(\omega)[r(\omega) - r^*(\omega)]$ (solid line), and the expected change in the real exchange rate (dashed line) are plotted against $\omega$, and in Plot *D* the risk premia are plotted against the expected change in the exchange rate. The parameters are the following: relative risk aversion, RRA, is 5, the impatience parameter, $\rho$, is 0.02, the mean return on investment, $\alpha$, is 0.08, the volatility of the return processes, $\sigma$, is 0.10, and the shipping cost, $\theta$, is 0.25 compared to 0.05 in the previous figure.

In Plot C of Figure 2, we graph the exchange risk premium (solid line) and the expected change in the real exchange rate (dashed line) as functions of the state variable, $\omega$. When the capital stocks across the two countries are equal, $\omega = 0$, international real interest rates are identical, and so from equations (7) and (8), the real risk premium and expected changes in the real exchange rate are zero. At the boundaries where shipping takes place, $\phi = 0$, and so the real risk premium is also zero. The first Fama condition requires that the risk premium be more variable than the expected change in the exchange rate. However, from Plot C, we see that the risk premium is less variable than the expected change in the exchange rate. Further, the stationary density given in Plot B indicates that the model economy spends a large proportion of time at the boundaries, where the risk premium is close to zero, so that the deviations from UIP are small on average.

In Plot D, we show the risk premium as a function of the expected change in the real exchange rate. The second Fama condition requires that the expected change in the exchange rate be negatively correlated to the risk premium. However, from Plot D, we see that on average the risk premium is positively related to the expected change in the real exchange rate.

To understand how the predictions of the model are affected by a change in shipping costs, we examine the above quantities when the shipping cost is increased to 25 percent from 5 percent, while all other parameter values are the same. With these parameters, the population UIP slope coefficient drops to 0.88. To understand this decline, we observe from Plot A of Figure 3 that the region of no-shipping is wider. This is because with a larger shipping cost it is optimal to ship only for larger imbalances in the relative capital stocks. Thus, production in this economy is riskier, implying that $\phi$ is smaller for all values of $\omega$, relative to the previous case. For example, when capital stocks are balanced, $\phi(0)$ drops from approximately $-0.19$ in the previous case to $-0.64$.

With higher transaction costs, Plot B shows that the economy spends an even larger amount of time at the boundaries, where shipping takes place. Moreover, Plot C shows that the variability of the risk premia relative to that of the expected change in the exchange rate has increased, while Plot D shows that the correlation between these two quantities has decreased. These changes explain the drop in the UIP regression coefficient. However, from Plots C and D, we see that even with the higher transaction costs, neither Fama condition is satisfied, and thus the slope coefficient is still positive.

To summarize, our analysis of the UIP regression in the real economy shows that the model generates a slope coefficient that is less than unity but far from negative. The intuition underlying this result is the following. The segmentation of commodity markets leads to large imbalances in the stock of goods at home and abroad, implying that there are large deviations from *absolute* PPP. However, the imbalance in the stock of goods is persistent, and therefore, the deviations from *relative* PPP are not substantial. There are two reasons for the persistence in the imbalance in the stock of goods at home and abroad—the first one explains why the imbalance tends to increase rather than decrease, and the second one explains why the imbalance does not get corrected. Imbalances in capital tend to increase rather than decrease because the drift of $\omega(t)$ is positive.[23] The correction to even large imbalances, associated with large deviations from absolute PPP, is small because with proportional costs it is optimal to ship only infinitesimal amounts. Given that there are only small deviations from *relative* PPP, the premium for real exchange rate risk that the model generates is too small to obtain the negative UIP slope coefficient found in the data.

## IV. The UIP Relation in Nominal Terms

The model analyzed above is of a real economy. Typical empirical tests of UIP are based on nominal data. Hence, in this section we extend the relation between the exchange rate and interest rates from real to nominal quantities. We proceed by first deriving the nominal UIP slope coefficient in terms of the moments from the real economy and a term that captures the interaction between the monetary and real sectors. We then use our numerical analysis of

[23] The explanation for why the drift is positive is given in the text below equation (3).

the real model to determine the magnitude of this interaction term required for the model to match the UIP regression in the data.

Given that our focus is on the effect of imperfections in the real sector, rather than that of the monetary sector, we introduce money so that it has no effect on the real allocations but it still can generate inflation risk premia in nominal interest rates. We assume, as in Lucas (1982), that investors face cash-in-advance constraints:

$$c(t)p(t) \leq M(t), \qquad c^*(t)p^*(t) \leq M^*(t), \tag{17}$$

where $c(t)$ is consumption at home, $p(t)$ is the domestic price level, and $M(t)$ is the domestic money holdings. The timing conventions in Lucas (1982) are such that uncertainty about real variables is resolved before investors decide on the money balances to hold, implying a unitary velocity of money. Introducing money in this manner allows us to obtain the nominal quantities using the solutions to the real model in the previous section, while maintaining our focus on the effect of real exchange rate risk on the UIP relation.

Alternative approaches to introducing monetary factors in the model would be to impose a cash-in-advance constraint with the timing convention in Svensson (1985), as done in Bekaert (1994); or, to introduce a stochastic demand for money with a transaction technology, as in Bansal *et al.* (1995), Bekaert (1996), and Bekaert, Hodrick, and Marshall (1994). We decided not to adopt these approaches to introducing monetary factors for two reasons. First, our objective is to see how much of the predictable deviations from UIP can be explained by the real exchange rate risk we model rather than by monetary sources. Second, these approaches introduce additional state variables so that it is no longer possible to characterize the equilibrium in our economy without making other approximations.

We assume the process for money in the domestic economy is,

$$dM(t) = \mu_M M(t)dt + \sigma_M M(t)dz_M(t), \tag{18}$$

where $dz_M(t)$ is a Wiener process. The process for the foreign money supply is given analogously. We denote the covariances between the shock to the money supply and the shocks to domestic and foreign output as $\sigma_{KM}$ and $\sigma_{K^*M}$. The conditional covariance of domestic money and the capital imbalance is denoted by $\sigma_{\omega M}$; the covariance of foreign money and the capital balance from the foreign perspective is $\sigma_{-\omega M^*}$. Given our focus on the risk arising from the segmentation of commodity markets, we have restricted the conditional moments of the money supplies in both countries to be constants. In order to simplify the analysis, we also assume that the effect of the imbalance on domestic and foreign money supplies is symmetric, that is, $\sigma_{\omega M} = \sigma_{-\omega M^*}$. Economically, this implies that the response of each country's monetary policy to real exchange rate shocks is the same. From a modeling standpoint, this assumption imposes discipline on our choice of parameters by restricting the degrees of freedom in the model.

Assuming that the nominal interest rates at home and abroad are positive, the cash-in-advance constraints bind in equilibrium. Then, from equation (17), the process for domestic prices is

$$d\ \ln p(t) = d\ \ln M(t) - d\ \ln c(t), \tag{19}$$

and similarly for foreign prices. We denote the nominal spot exchange rate by $S(t)$, and define

$$s(t) \equiv \ln S(t) = e(t) + \ln p(t) - \ln p^*(t).$$

We show in the Appendix that the stochastic process for the log of the nominal exchange rate is as follows.

LEMMA 4. *The process for the log of the spot rate is:*

$$ds(t) = ([\mu_M - \mu_{M^*}] - \tfrac{1}{2}[\sigma_M^2 - \sigma_{M^*}^2])dt + \sigma_M dz_M(t) - \sigma_{M^*} dz_{M^*}(t) - \frac{\gamma}{1-\gamma} de(t). \tag{20}$$

In this economy, we can use $V_1(t)/p(t)$ to price domestic currency payoffs in terms of domestic currency. As in Lemma 1, this implies that the instantaneously riskless nominal interest rate is $i(t) = \rho - E_t[dV_1(t)/p(t)]/[V_1(t)/p(t)]dt$. We have already obtained the processes for $p(t)$ and $V_1(t) \equiv q(t)$. Using these processes, we can derive the nominal interest rates in the home and foreign countries. This allows us to derive the difference between the domestic and foreign nominal interest rates.

PROPOSITION 2. *The nominal forward premium is*

$$\begin{aligned} fp^{\text{nom}}(t)dt &= i(t) - i^*(t)dt \\ &\equiv E_t[ds(t)] + rp^{\text{nom}}(t)dt, \end{aligned} \tag{21}$$

*where*

$$\begin{aligned} rp^{\text{nom}}(t) \equiv {} & \left(\frac{\gamma}{1-\gamma}\right)^2 rp^{\text{real}}(t) - \tfrac{1}{2}(\sigma_M^2 - \sigma_{M^*}^2) + \gamma[\sigma_{K^*M} - \sigma_{KM^*}] \\ & - \left(\frac{\gamma}{1-\gamma}\right)\left(\frac{\sigma_{\omega M}}{\sigma^2}\right)[r(t) - r^*(t)], \end{aligned} \tag{22}$$

$$E_t[ds(t)] = ([\mu_M - \mu_{M^*}] - \tfrac{1}{2}[\sigma_M^2 - \sigma_{M^*}^2])dt - \frac{\gamma}{1-\gamma} E_t[de(t)]. \tag{23}$$

From equation (22), we see that the risk premium for nominal exchange rate risk consists of three parts. The first term captures the effect of real factors on deviations from UIP. The second term is the conditional volatility of money supplies. The last two terms measure the interaction between the real and monetary sectors and they reflect the conditional covariances between the real pricing kernel and the money supply processes. If real interest rates were constant, then any predictable deviations from UIP must come from predictability in the conditional variances and covariances involving money supplies.

To see the role of the shipping costs, consider the two polar cases. With either no shipping costs (perfect pooling) or 100 percent cost, the real interest rates in the two countries would be constant and equal to each other. Hence, the predictable deviations from UIP in equation (21) would be driven solely by the volatility of monetary shocks and the conditional covariances between money and output shocks. Thus, it is only when $0 < \theta < 1$ that the real exchange rate has an impact on the nominal risk premium.

We wish to study the slope coefficient in the nominal UIP regression,

$$b^{\text{nom}} = \frac{\text{cov}(E_t[ds(t)] + rp^{\text{nom}}(t),\, E_t[ds(t)])}{\text{var}(E_t[ds(t)] + rp^{\text{nom}}(t))}. \tag{24}$$

From equations (22) and (23), we see that for a given set of parameters for the real model, $b^{\text{nom}}$ depends on only two sets of terms. The first set of terms are the moments of the real model, and the second set consists of the covariance between the capital imbalances and money supplies, $\sigma_{\omega M}$. Our calibration exercise in the previous section provides us with $E_t[de(t)]$ and $rp^{\text{real}}(t)$. We use these to infer the value of $\sigma_{\omega M}$ required to obtain a UIP regression coefficient of $-2$, as is approximately found in the data. Before doing this, we provide some intuition for the magnitude and sign of the parameter $\sigma_{\omega M}$ required to obtain a UIP slope coefficient of $-2$. To do this, we consider an analytical approximation to the required $\sigma_{\omega M}$ for the special case where risk aversion is high.

With high relative risk aversion, $\gamma/(1-\gamma) \approx -1$. Then, equations (22) and (23) simplify to

$$\begin{aligned} rp^{\text{nom}}(t) \approx {} & rp^{\text{real}}(t) - \tfrac{1}{2}(\sigma_M^2 - \sigma_{M^*}^2) + \gamma[\sigma_{K^*M} - \sigma_{KM^*}] \\ & + \left(\frac{\sigma_{\omega M}}{\sigma^2}\right)[r(t) - r^*(t)], \end{aligned} \tag{25}$$

$$E_t[ds(t)] \approx ([\mu_M - \mu_{M^*}] - \tfrac{1}{2}[\sigma_M^2 - \sigma_{M^*}^2])dt + E_t[de(t)]. \tag{26}$$

Substituting these terms into the definition of $b^{\text{nom}}$ in equation (24),

$$\begin{aligned} b^{\text{nom}} & \approx \left(\frac{1}{1 + (\sigma_{\omega M}/\sigma^2)}\right) \frac{\text{cov}(r(t) - r^*(t),\, [r(t) - r^*(t)][1 + \phi(t)])}{\text{var}(r(t) - r^*(t))} \\ & = \left(\frac{1}{1 + (\sigma_{\omega M}/\sigma^2)}\right) b^{\text{real}}. \end{aligned} \tag{27}$$

We can use this equation to solve for the required $\sigma_{\omega M}$ as:

$$\frac{\sigma_{\omega M}}{\sigma^2} = \sqrt{2}\left(\frac{\sigma_M}{\sigma}\right)\mathrm{corr}(\omega,\, M) = \left[\frac{b^{\mathrm{real}}}{b^{\mathrm{nom}}} - 1\right], \tag{28}$$

where $b^{\mathrm{real}}$ can be obtained from our numerical analysis of the real model, while $b^{\mathrm{nom}}$ is estimated from the data. For example, if $b^{\mathrm{real}} = b^{\mathrm{nom}}$, then equation (28) implies that the required $\sigma_{\omega M}$ is zero, so that no interaction is required between the real and monetary sectors to generate the negative slope coefficient. From our numerical analysis of the real model, the smallest $b^{\mathrm{real}}$ we have found is approximately 0.5. To achieve a negative nominal slope coefficient of $-2$ requires that $\sigma_{\omega M}/\sigma^2 = \sqrt{2}(\sigma_M/\sigma)\mathrm{corr}(\omega, M) = -(5/4)$. Given that the correlation coefficient is bounded below by $-1$, this requires that the volatility of the money supply process be larger than roughly $5/4\sqrt{2} = 88.4$ percent of the volatility of output shocks.

To obtain the exact value of $\sigma_{\omega M}$ for which the nominal model delivers a UIP slope coefficient of $-2$ involves solving a quadratic equation. We find that for the real slopes reported in Figure 1, where $\sigma^2 = 0.01$, the value of $\sigma_{\omega M}$ required to get a UIP regression coefficient of $-2$ is approximately equal to $-0.01$. This value is not very sensitive to either the level of shipping costs or RRA. This value for $\sigma_{\omega M}$ implies that, from the processes of $\omega$ and money supplies in equations (3) and (18),

$$\frac{\sigma_{\omega M}}{\sigma^2} = \frac{-0.01}{0.01} = -1 = \sqrt{2}\left(\frac{\sigma_M}{\sigma}\right)\mathrm{corr}(\omega,\, M).$$

This indicates that the correlation between money supplies and the capital imbalance, $\mathrm{corr}(\omega, M)$, must be negative in each country. We estimate this correlation to be small in the data.[24] Our estimates of this correlation also imply that to obtain a negative UIP slope coefficient, the volatility of monetary shocks supply must be much larger than the volatility of output shocks. We find this is not true in the data.

## V. Conclusions

We have derived the relation between exchange rate changes and interest rates in a general equilibrium model where deviations from PPP arise endogenously from the segmentation of commodity markets. The model can generate predictable deviations from UIP, and we derive the functional form for these in terms of domestic and foreign interest rates. We use numerical methods to calculate the UIP regression coefficient in the theoretical model, both in terms

[24] These estimates are obtained from estimating VARs for relative output growth and money supply growth using quarterly data for the US, Canada, Japan, and Germany. The estimates of the correlation range from $-0.15$ (for U.S. money supply and relative output growth in Canada and the United States) to 0.09 (for Japanese money supply and relative output growth in Japan and the United States).

of real interest rates and exchange rates, and in terms of nominal variables. We find that the real model, calibrated to U.S. data, can generate a regression coefficient that is less than unity; however, even for extreme values of shipping costs and risk aversion, the real model fails to generate a negative UIP slope coefficient. On the other hand, introducing monetary factors through a cash-in-advance constraint can generate a negative regression coefficient only if money supply shocks and relative returns on capital in the two countries are significantly negatively correlated. We find that this correlation is small in the data.

The real model fails to generate a slope coefficient that is substantially less than unity even though the segmentation of commodity markets leads to large imbalances in the stock of goods at home and abroad, giving rise to large deviations from *absolute* PPP. This is because the imbalance in the stock of goods and at home is persistent, and hence, the deviations from *relative* PPP are not substantial. The reason for the persistence in the imbalance in the stock of goods at home and abroad is that the shipping cost considered in our model is proportional. Thus, when it is optimal to transfer goods from one country to another, the optimal amount to ship is infinitesimal. The small deviations from *relative* PPP imply that the premium for real exchange rate risk that the model generates is too small to obtain a UIP slope coefficient as negative as that found in the data.

An important economic characteristic of the model we study is that, holding the world capital stock fixed, a relative increase in the capital stock of the home country leads to a decrease in home consumption and an increase in foreign consumption. Thus, the capital imbalance and relative consumption rates are negatively correlated. This is a direct consequence of the shipping cost that segments the two countries. To get a negative UIP slope coefficient, this correlation needs to be significantly negative. Our numerical analysis shows that our model fails to generate a sufficiently negative correlation.

To generate a sufficiently negative correlation the model considered in this article could be extended to allow for shipping costs that are fixed rather than proportional. One could also introduce time variation in the return to capital. These modifications may allow for greater deviations from relative PPP by limiting the time spent near the shipping region. A third way to extend the model is to introduce money in such a way that it affects the allocation of real resources. One could then study whether the interaction between the monetary sector and the segmented real sectors can generate the risk premia required to explain the UIP regression.

## Appendix

*Boundary Conditions That the Value Function Must Satisfy.*

The value-matching boundary conditions that the solution to the problem in equation (1) must satisfy are:

$$V_1[K(t), K^*(t)] - (1-\theta)V_2[K(t), K^*(t)] = 0, \qquad \text{for} \quad K(t)/K^*(t) = \lambda, \tag{A1}$$

$$(1-\theta)V_1[K(t), K^*(t)] - V_2[K(t), K^*(t)] = 0, \qquad \text{for} \quad K(t)/K^*(t) = 1/\lambda, \tag{A2}$$

where $\lambda$ is that ratio of $K(t)/K^*(t)$ for which it is optimal to ship goods from the domestic country to the foreign country, and $1/\lambda$ is the ratio for which it is optimal to import goods. The smooth-pasting boundary conditions are:

$$V_{11}[K(t), K^*(t)] - (1-\theta)V_{12}[K(t), K^*(t)] = 0, \qquad \text{for} \quad K(t)/K^*(t) = \lambda, \tag{A3}$$

$$V_{22}[K(t), K^*(t)] - (1-\theta)V_{12}[K(t), K^*(t)] = 0, \qquad \text{for} \quad K(t)/K^*(t) = 1/\lambda. \tag{A4}$$

*Proof of Lemma 1.* Defining $q(t) \equiv V_1[K(t), K^*(t)]$, and using Itô's Lemma, in the no-shipping region:

$$\begin{aligned} dq(t) = {} & (q_1(t)[\alpha K(t) - c(t)] + \tfrac{1}{2} q_{11}(t)\sigma^2 K(t)^2)dt + q_1(t)\sigma K(t) dz_K(t) \\ & + (q_2(t)[\alpha K^*(t) - c^*(t)] + \tfrac{1}{2} q_{22}(t)\sigma^2 K^*(t)^2)dt \\ & + q_2(t)\sigma K^*(t) dz_{K^*}(t), \end{aligned} \tag{A5}$$

where $q_1(t)$ is the derivative of $q(t)$ with respect to $K(t)$ and $q_2(t)$ is the derivative of $q(t)$ with respect to $K^*(t)$. The Hamilton-Jacobi-Bellman equation for the problem defined in equation (1) in the no-shipping region is given by:

$$\begin{aligned} 0 = \max_{c,c^*} \bigg( & \frac{c^{\gamma}(t)}{\gamma} + \frac{c^{*\gamma}(t)}{\gamma} - \rho V(t) + V_1(t)[\alpha K(t) - c(t)] + V_2(t)[\alpha K^*(t) - c^*(t)] \\ & + 0.5\sigma^2 V_{11}(t)K(t)^2 + 0.5\sigma^2 V_{22}(t)K^*(t)^2 \bigg). \end{aligned} \tag{A6}$$

Differentiating the above equation with respect to $K$, and using the envelope condition gives:

$$\begin{aligned} 0 = {} & -\rho q(t) + q_1(t)[\alpha K(t) - c(t)] + q(t)\alpha + q_2(t)[\alpha K^*(t) - c^*(t)] \\ & + \tfrac{1}{2}\sigma^2 q_{11}(t)K(t)^2 + \sigma^2 q_1(t)K(t) + \tfrac{1}{2}\sigma^2 q_{22}(t)K^*(t)^2. \end{aligned} \tag{A7}$$

Substituting (A7) in (A5),

$$\begin{aligned} \frac{dq(t)}{q(t)} = {} & \left[\rho - \alpha - \frac{q_1(t)}{q(t)}\sigma^2 K(t)\right]dt + \frac{q_1(t)}{q(t)}\sigma K(t) dz_K(t) \\ & + \frac{q_2(t)}{q(t)}\sigma K^*(t) dz_{K^*}(t). \end{aligned} \tag{A8}$$

From Cox, Ingersoll, and Ross (1985),

$$\frac{E_t[dq(t)]}{q(t)} = [\rho - r(t)]dt. \tag{A9}$$

Equating the $dt$ terms in equations (A8) and (A9), we get the result in equation (4).

*Proof of Lemma 2.* The homogeneity of degree $\gamma$ of $V[K(t), K^*(t)]$ implies that,

$$\gamma - 1 = \frac{V_{11}(t)}{V_1(t)} K(t) + \frac{V_{12}(t)}{V_1(t)} K^*(t) = \frac{q_1(t)}{q(t)} K(t) + \frac{q_2(t)}{q(t)} K^*(t). \qquad \text{(A10)}$$

Using equations (4) and (A10) to re-write (A8), gives equation (5).

*Proof of Proposition 1.* From the definition of $e(t)$, in the no-shipping region:

$$de(t) = d \ln V_2(t) - d \ln V_1(t) = d \ln q^*(t) - d \ln q(t), \qquad \text{(A11)}$$

where $q(t) \equiv V_1[K(t), K^*(t)]$ and $q^*(t) \equiv V_2[K(t), K^*(t)]$. Using Itô's Lemma, and equation (5),

$$d \ln q(t) = \left[\rho - r(t) - \sigma^2\left(\left(\frac{r(t) - \alpha}{\sigma^2}\right)^2 + (1 - \gamma)\left(\frac{r(t) - \alpha}{\sigma^2}\right) + 0.5(1 - \gamma)^2\right)\right]dt$$
$$+ \sigma\left(\frac{r(t) - \alpha}{\sigma^2}\right)dz_K(t) - \sigma\left((1 - \gamma) + \frac{r(t) - \alpha}{\sigma^2}\right)dz_{K^*}(t). \qquad \text{(A12)}$$

Subtracting equation (A12) from its foreign counterpart, and simplifying, we get the result.

*Proof of Lemma 3.* This follows from the processes for the capital imbalance, equation (3), and the pricing kernels, equations (5) and (6).

*Proof of Lemma 4.* Substituting equation (2) in (19) gives,

$$d \ln p(t) = d \ln M(t) - \frac{1}{\gamma - 1} d \ln V_1(t). \qquad \text{(A13)}$$

Using equation (A13), and the definition $s(t) \equiv \ln S(t) = e(t) + \ln p(t) - \ln p^*(t)$,

$$\begin{aligned} ds(t) &= de(t) + d \ln p(t) - d \ln p^*(t) \\ &= de(t) + d \ln M(t) - d \ln M^*(t) - \frac{1}{\gamma - 1}[d \ln V_1(t) - d \ln V_2(t)] \\ &= d \ln M(t) - d \ln M^*(t) + \frac{\gamma}{\gamma - 1} de(t), \end{aligned}$$

where we use $e(t) \equiv \ln V_2(t) - \ln V_1(t)$, in the last step. Using the process for the money supplies, equation (18), gives the result.

*Proof of Proposition 2.* From equations (19) and (2),

$$\frac{V_1(t)}{p(t)} = V_1(t) \times \frac{c(t)}{M(t)} = V_1(t) \times \frac{V_1(t)^{1/(\gamma-1)}}{M(t)} = \frac{V_1(t)^{\gamma/(\gamma-1)}}{M(t)},$$

which implies that

$$\frac{d[V_1(t)/p(t)]}{V_1(t)/p(t)} = \frac{\gamma}{\gamma - 1} d \ln V_1(t) - d \ln M(t) + \frac{1}{2}\left(\frac{\gamma}{\gamma - 1} \mathrm{d} \ln V_1(t) - d \ln M(t)\right)^2. \tag{A14}$$

We also know, from Cox, Ingersoll, and Ross (1985), that

$$\rho - i(t) = \frac{E_t[dV_1(t)/p(t)]}{[V_1(t)/p(t)]dt}. \tag{A15}$$

From equations (A14) and (A15),

$$i(t) = \rho - \frac{1}{dt} E_t\left[\frac{\gamma}{\gamma - 1} d \ln V_1(t) - d \ln M(t) + \frac{1}{2}\left(\frac{\gamma}{\gamma - 1} d \ln V_1(t) - d \ln M(t)\right)^2\right], \tag{A16}$$

and similarly for the foreign nominal interest rate. Equation (21) is obtained by subtracting the expression for the foreign nominal interest rate from equation (A16).

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