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ANTONIO ROMA and WALTER TOROUS*

ABSTRACT

This article investigates the behavior of the term structure of interest rates over the business cycle. In contrast to prior studies that measure the business cycle by the simple growth in aggregate economic activity, we consider the deviation of aggregate economic activity from its potentially stochastic trend. We show that incorporating both an independent trend and cyclical component in consumption improves the efficiency in estimating consumption-based asset pricing models. We also find that the term spread is more informative about future changes in stochastically detrended real gross domestic product (GDP) than future growth rates in real GDP.

THE NOTION OF SYSTEMATIC variation in interest rates over the business cycle is a familiar one. Beginning with the work of Kessel (1965), which describes the behavior of long- and short-term interest rates with respect to aggregate economic activity, numerous authors have investigated the relation of the term structure of interest rates to changes in macroeconomic variables such as gross domestic product (GDP), consumption, and inflation. In particular, Fama (1986), C. Harvey (1988, 1989), and Estrella and Hardouvelis (1991), among others, demonstrate that the current term structure provides valuable ex ante information about expected economic growth.

Following Lucas (1978) and Brock (1982), recent general equilibrium term structure models, for example Cox, Ingersoll, and Ross (1985), Breeden (1986), and Sun (1992), have explored the bond pricing implications of stochastic growth models.[2] In these term structure models real interest rates depend, through a representative investor's marginal utility, on consumption and its dynamics. Faced with changing consumption opportunities, the representative investor uses available financial assets to smooth lifetime consumption. Given specific assumptions about the utility function and the dynamics of the underlying state variables, explicit bond pricing formulae obtain that link the behavior of long and short term interest rates. C. Harvey (1988) uses this framework to investigate the relation between real interest rates and consumption growth; the latter is taken as a proxy for the business cycle.

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[1] Evidence for countries other than the U.S. is provided by C. Harvey (1991) and Plosser and Rouwenhorst (1994).

[2] Bakshi and Chen (1996) and Foresi (1990), among others, have modeled the term structure of interest rates in monetary economies.

With constant returns to scale production technology and logarithmic preferences, it is well known that these general equilibrium term structure models imply that both consumption and output levels are integrated of order one ($I(1)$) and consumption is proportional to output. Real business cycle models, in contrast, use this same general equilibrium framework and demonstrate under constant intertemporal elasticity of consumption (King, Plosser, and Rebelo (1988)) that although consumption is no longer proportional to output, permanent innovations in technology will give rise to cointegrated output and consumption levels (King, Plosser, Stock, and Watson (1991)). In particular, consumption dynamics will be characterized by a stochastic trend arising from these technological shocks to output. There is a large body of empirical evidence consistent with macroeconomic variables, like consumption, containing stochastic trends (see Stock and Watson (1988)). The business cycle is then defined as the deviations of a macroeconomic variable from its stochastic trend (Lucas (1976)).

The purpose of this article is to empirically investigate the relation between deviations of consumption from its stochastic trend and the behavior of real interest rates. We follow Hansen and Singleton (1983), C. Harvey (1988), and other empirical studies of consumption-based asset pricing models by exogenously specifying these consumption dynamics in a discrete time exchange economy with a representative investor who is characterized by constant relative risk aversion. By positing that the consumption process contains a stochastic trend, we are assuming that this exchange economy can be supported by a general equilibrium production economy with permanent innovations in technology.

Recognizing that the business cycle is itself unobservable, we use Watson's (1986) UC-ARIMA (unobserved components autoregressive integrated moving average) methodology to stochastically detrend U.S. macroeconomic data and estimate a cyclical component. UC-ARIMA models are nested cases of ARIMA models. But, as noted by Watson, the estimated versions of these alternative business cycle specifications display different properties at different frequencies and, as a result, may exhibit a fundamentally different empirical relation with interest rates.

Incorporating our business cycle measure into C. Harvey's (1988) framework allows us to focus on the precise relation between the real term structure and consumption's cyclical component. Empirical evidence suggests that the term structure is steeper at business cycle troughs and flatter at business cycle peaks. We investigate the plausibility of various real term structure models and demonstrate that in our framework a non-Markovian model is required to capture this countercyclical behavior.

We also provide a maximum likelihood method for estimating the consumption asset pricing model which explicitly recognizes the presence of an independent trend and cyclical component in consumption dynamics. Hansen and Singleton's (1983) maximum likelihood method is nested in our framework as a special case. The fit of the consumption-based model to real bond holding period returns is improved, especially for short term bills. We also find that the variability of consumption's cyclical component plays an important role in jointly explaining real holding period returns to short term bills and long term bonds.

The plan of this article is as follows. Section I introduces our more precise measure of the business cycle. A multivariate Kalman filtering framework is used in Section II to simultaneously estimate consumption's cyclical component as well as its relation to real interest rates. This estimation is compared to extant maximum likelihood techniques. We also use the framework of Hansen and Jagannathan (1991) to further assess our specification of consumption dynamics for consumption-based asset pricing models. To demonstrate additional implications of our approach, we show in Section III that the term spread is more significantly related to real GDP's UC-ARIMA estimated cyclical component than growth rates in real GDP. Section IV concludes the article.

I. The Term Structure of Interest Rates and the Business Cycle

Expected real interest rates should vary with the business cycle. Intuitively, if an investor expects a recession (expansion) next year, he or she will be more (less) willing to demand a one-year bond today, thereby decreasing (increasing) current one-year interest rates. As a consequence, current interest rates should provide information about the business cycle.

However, growth rates in consumption or other macroeconomic variables are imprecise measures of the business cycle. Most macroeconomic time series exhibit a tendency to trend over time, and the business cycle is more properly viewed as stationary deviations about this potentially stochastic trend. Simply first differencing a macroeconomic time series that contains both growth (nonstationary) and cyclical (stationary) components confounds the contribution of each (Stock and Watson (1988)). As a result, by imprecisely measuring the business cycle, previous empirical research may not have thoroughly characterized the relation between interest rates and the business cycle.

A. Modeling the Business Cycle

Empirical evidence suggests that macroeconomic time series are integrated of order one ($I(1)$) (Nelson and Plosser (1982)); that is, their first differences are stationary. As noted by Beveridge and Nelson (1981), integrated series contain a stochastic trend. In particular, any ARIMA(p, 1, q) model may be represented as a stochastic trend plus a stationary component.

We recognize that while the logarithm of a macroeconomic variable, y_t, is itself observable, its additive components, the trend, τ_t, and the cycle, c_t, are individually unobservable:

$$y_t = c_t + \tau_t. \tag{1}$$

The cycle c_t is assumed characterized by a stationary autoregressive process with homoscedastic errors,

$$\Phi(L)c_t = e_t^c, \qquad \mathrm{var}(e_t^c) = \sigma_c^2, \tag{2}$$

where $\Phi(L)$ denotes a polynomial in the lag operator L. The trend τ_t is assumed to follow a random walk with drift,

$$\tau_t = \mu + \tau_{t-1} + e_t^\tau, \qquad \mathrm{var}(e_t^\tau) = \sigma_\tau^2. \tag{3}$$

This specification implies that the macroeconomic variable's logarithmic long term average growth rate is constant at μ. We assume that e_t^τ and e_t^c are normally distributed. More significantly, we follow Watson (1986) and decompose the observed series, y_t, using a UC-ARIMA model in which the trend and stationary innovations are uncorrelated,

$$\mathrm{cov}(e_t^\tau, e_{t-k}^c) = 0 \qquad \forall k.$$

That is, the economic factors giving rise to trend innovations are assumed to be unrelated to the economic sources of business cycle movements. In contrast, if we assume that e_t^τ and e_t^c are perfectly correlated then, from Beveridge and Nelson (1981), an ARIMA model obtains.

B. Modeling the Real Term Structure

We now explore the relation between the term structure of real interest rates and the business cycle in the context of a consumption-based framework that explicitly recognizes the presence of an independent cyclical component in consumption dynamics.

We consider a discrete-time exchange model (Lucas (1978)) in which the representative investor maximizes the expected utility of lifetime consumption. To provide a simple yet testable model, we follow Hansen and Singleton (1983) and others and make the following two assumptions which are retained throughout: (i) conditional on information available at time t, I_t, the logarithm of consumption, $y_{t+j} \equiv \ln C_{t+j}$, is normally distributed, with the variance of $y_{t+j} - y_t$ denoted by $\sigma_y^2(j)$; (ii) the representative investor has a time-separable utility function $\mathrm{U}(C_{t+j}) = e^{-\delta j}(C_{t+j}^{(1-\alpha)} - 1)/(1-\alpha)$ where α is the coefficient of relative risk aversion and δ is the rate of patience.

Given assumptions (i) and (ii), the yield at time t on a j-period zero coupon bond, $r(t,j)$, is

$$r(t,j) = -\frac{\ln B_t(t+j)}{j} = \delta + \left(\frac{\alpha}{j}\right)\mathrm{E}[y_{t+j} - y_t|I_t] - \frac{1}{2}\left(\frac{\alpha^2}{j}\right)\sigma_y^2(j), \tag{4}$$

where $B_t(t+j)$ is the price at time t of a real zero coupon bond paying one unit at time $t+j$. Expression (4) represents a simple linear version of the consumption asset pricing model. Breeden (1986) demonstrates that if consumption is not lognormally distributed then equation (4) holds as a second order approximation.

To explicitly link real interest rates to the business cycle, we adopt the UC-ARIMA model for consumption dynamics. Note that since the logarithm of consumption is assumed normally distributed, the stochastic model given by expressions (1), (2), and (3) is entirely consistent with expression (4).

From equations (1) and (3) it follows that

$$
\begin{aligned}
r(t,j) &= -\frac{\ln B_t(t+j)}{j} \\
&= \delta + \alpha\mu - \frac{\alpha^2}{2}\sigma_\tau^2 + \left(\frac{\alpha}{j}\right)\mathrm{E}[c_{t+j} - c_t|I_t] - \frac{1}{2}\left(\frac{\alpha^2}{j}\right)\sigma_c^2(j), \qquad (5)
\end{aligned}
$$

where $\sigma_c^2(j) = \mathrm{var}(c_{t+j} - c_t)$. Expression (5) implies that real interest rates (as well as returns) are related to the expected change in consumption's cyclical component, *not* to (log) consumption itself. Intuitively, under constant relative risk aversion and homoscedastic optimal consumption, the representative investor smooths only the fluctuations of consumption around its trend and not the random walk component itself.

In this framework, the consol rate will be constant at $\ell = \delta + \alpha\mu - (\alpha^2/2)\sigma_\tau^2$. We may avoid a constant consol rate by assuming that the consumption drift, μ, follows a stochastic process (say, a random walk), or by simply assuming that μ is not known by the representative investor who then must estimate it every period. In either case, the spread $r(t,i) - r(t,j)$, $i > j$, does not depend on the consol rate ℓ.

The expected 1-period holding period return on a j-maturity bond can be expressed as

$$
\begin{aligned}
\mathrm{E}_t[R_{j,t+1}] &= \mathrm{E}_t\left[\ln\left(\frac{B_{t+1}(t+j)}{B_t(t+j)}\right)\right] \\
&= \ell + \alpha\mathrm{E}[c_{t+1} - c_t|I_t] + \frac{\alpha^2}{2}[\sigma_c^2(j-1) - \sigma_c^2(j)]. \qquad (6)
\end{aligned}
$$

Expected holding period returns on bonds of differing maturities differ only in the term $[\sigma_c^2(j-1) - \sigma_c^2(j)]$ involving the variance of consumption's cyclical component.

B.1. The Countercyclical Behavior of the Term Spread

We require a non-Markovian representation of consumption's cyclical component (i.e., an autoregressive process of order $p > 1$) to generate the empirically observed countercyclical behavior of the term spread.

If the cyclical component follows an AR(1) process with autoregressive parameter ρ, then the following simple term structure formula obtains:

$$r(t,j) = -\frac{\ln B_t(t+j)}{j} = \ell - \frac{1}{2}\left(\frac{\alpha^2}{j}\right)\sigma_c^2(1-\rho^{2j})(1-\rho^2)^{-1} - \left(\frac{\alpha}{j}\right)(1-\rho^j)c_t. \tag{7}$$

Expression (7) is similar to Vasicek's (1977) term structure model in which bond prices are lognormally distributed and interest rates follow an AR(1) process.

The AR(1) model, however, implies that the real term structure is steeper at business cycle peaks and flatter at business cycle troughs and is inconsistent with the empirical evidence of Kessel (1965) and others. To see this procyclical behavior, note that the difference between the real rate for maturity $i > 1$ and the one period real rate is given by:

$$r(t,\, i) - r(t,\, 1) = \left(\frac{\alpha^2\sigma_c^2}{2}\right)\left(1 - \frac{1-\rho^{2i}}{i(1-\rho^2)}\right) + \alpha\left(\frac{(i-1)-(i\rho-\rho^i)}{i}\right)c_t,$$

where the coefficient on c_t is strictly positive. Given constant σ_c^2 and σ_τ^2, it follows that the AR(1) model's real term structure is upward sloping at a business cycle expansion ($c_t > 0$) and vice versa. Similarly, Breeden's (1986) term structure model also implies procyclical behavior of the term spread.

The key to why higher order term structure models imply countercyclical term spread behavior, at least for finite maturities, lies in the behavior of the first order term $E[c_{t+j} - c_t|I_t]/j$ to which the j-period real rate is proportional to. For example, if we are in a recession ($c_t < 0$) then business conditions (c_{t+j}) must be expected to improve *fast enough*, i.e., faster than j, to generate an upward sloping term structure. However, the absolute value of this first order term is always decreasing in j if c_t follows an AR(1) process. In other words, we expect business conditions to improve, but not fast enough to generate an upward sloping term structure. If c_t is gaussian and homoscedastic, a non-Markovian process is needed to generate expectations of a sufficiently strong business recovery; an AR(2) specification, for example, is more appropriate.

The behavior of $E[c_{t+j}|c_t, c_{t-1}]$ as a function of j when $c_t, c_{t-1} < 0$ is displayed in Figure 1. It is clear that under the AR(1) specification $E[c_{t+j} - c_t|I_t]$ increases at a decreasing rate. Under the AR(2) specification, in contrast, this expectation increases at an increasing rate for a number of periods and, as a result, the term structure will be increasing at a business cycle trough.

B.2. Misspecification of Previous Tests

The presence of a stochastic trend in consumption dynamics has implications for testing the relation between real returns and consumption growth rates. This empirical analysis has traditionally been carried out (for example,

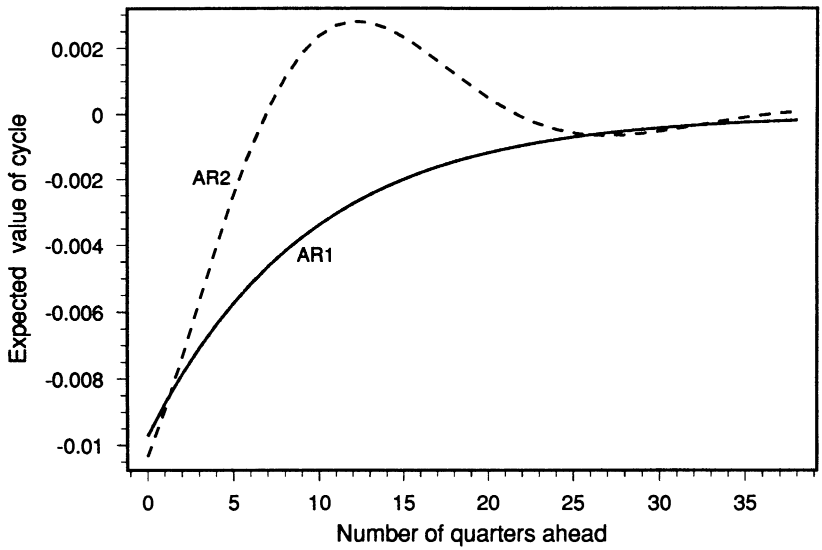

Figure 1. The behavior of $\mathbf{E}[c_{t+j}|c_t, c_{t-1}]$. This figure illustrates the behavior of the expected value of consumption's cyclical component at future quarters $j > 0$ for alternative autoregressive specifications. The AR(1) specification is indicated by the solid line and is characterized by $\rho = 0.90$. The AR(2) specification is indicated by the dashed line and is characterized by $\rho_1 = 1.77$ and $\rho_2 = -0.82$. The AR(1) specification is initialized at $c_t = -0.012$ while the AR(2) specification is initialized at $c_{t-1} = c_t = -0.012$.

C. Harvey (1988, 1989), and Estrella and Hardouvelis (1991)) by regressing ex post realized growth rates in aggregate economic activity against either real interest rates or the term spread. In the presence of a stochastic trend, however, these tests may be adversely affected by consequent measurement errors.

Without loss of generality, we illustrate this by considering one period changes in log real consumption. From equations (1), (2), and (3), we have:

$$y_{t+1} - y_t = \mu + \mathrm{E}_t[c_{t+1} - c_t] + e^c_{t+1} + e^\tau_{t+1}. \tag{8}$$

Notice that in the presence of a stochastic trend, the ex post change in log real consumption will be a noisy proxy for the predictable component $\mathrm{E}_t[c_{t+1} - c_t]$ since it includes both disturbance terms e^c_{t+1} and e^τ_{t+1}. The latter is related to the stochastic change in trend and may be quite relevant. For example, the standard deviation of e^τ is approximately three times as large as the standard deviation of e^c for post-World War II U.S. real log consumption data (see footnote 10). Over longer horizons the measurement error, $\Sigma_j\, e^\tau_{t+j}$, will become increasingly important as its variance grows linearly with the forecast horizon j. In the UC-ARIMA specification, e^τ_t and e^c_t are independent implying that the unanticipated change, e^τ_t, may be filtered out.

II. Consumption Dynamics and the Maximum Likelihood Estimation of the Consumption Asset Pricing Model

The consumption asset pricing model has previously been estimated by the generalized method of moments (GMM) (Hansen and Singleton (1982)), maximum likelihood estimation of an appropriately restricted vector autoregression (VAR) (Hansen and Singleton (1983)), linear regression (C. Harvey (1988)), or by simple calibration (Mehra and Prescott (1985)).

Under the maintained assumptions of normality and constant relative risk aversion, Hansen and Singleton (1983) (H&S) posit a consumption asset pricing model in which returns are proportional, through the coefficient of relative risk aversion, to the growth in real consumption. In their framework, the representative investor's information set, consisting of past consumption growth rates as well as past returns, is assumed to follow a VAR with gaussian disturbances. In particular, changes in the logarithm of real consumption are described by the VAR, while one period realized returns are proportional to the growth in real consumption plus an error term.

Defining Δy_{t+1} as the change in the logarithm of real consumption from quarter t to quarter $t + 1$ and $R_{j,t+1}$ as the corresponding one-quarter real holding period return on a zero coupon bond with maturity j, H&S consider the following estimation framework:

$$R_{j,t+1} = \mu_r + \alpha \Delta y_{t+1} + v^r_{t+1}, \tag{9}$$

$$\Delta y_{t+1} = \mu + \sum_{i=1}^{m} \zeta_i \Delta y_{t+1-i} + \sum_{i=1}^{m} \gamma_i R_{j,t+1-i} + v^y_{t+1}. \tag{10}$$

The disturbances v^r and v^y are assumed normally distributed with constant covariance and H&S estimate this system by maximum likelihood.

However, as previously argued, the decomposition of real consumption into an independent temporary and permanent component implies that real consumption growth rates provide a noisy proxy for consumption's cyclical component. Since real returns and real consumption growth rates are both linearly related to the relevant state variable, this suggests a statistically efficient strategy for estimating the consumption asset pricing model.

Consumption dynamics, defined by expressions (1), (2), and (3), may be written as:[3]

$$\Delta y_{t+1} = \mu + \Delta c_{t+1} + e^{\tau}_{t+1}, \tag{11}$$

[3] Given equation (2), $\Phi(L)c_t = e^c_t$, where $\Phi(L)$ is a lag polynomial, we assume $\Phi(L)$ can be factored so that $\Phi(L) = \hat{\Phi}(L)(1 - L)$, and this gives equation (12), $\hat{\Phi}(L)(1 - L)c_t = e^c_t$. See Stock and Watson (1991). Although we could have carried out the analysis in terms of the level of the cycle, this transformation makes our model comparable to that of H&S.

$$\Delta c_{t+1} = \mu_c + \sum_{\iota=1}^{m} \rho_i \Delta c_{t+1-\iota} + e^c_{t+1}. \tag{12}$$

The asset pricing model then links interest rates to consumption's cyclical component. In particular, expression (6) implies a linear relation between expected real returns and expected changes in consumption's cyclical component. For realized returns we have,

$$R_{j,t+1} = \mu_r + \alpha \Delta c_{t+1} + z_{t+1}, \tag{13}$$

where z is a disturbance term, assumed gaussian. This model may be estimated by maximum likelihood using a multivariate Kalman filter: expressions (11) and (13) provide two measurement equations, while expression (12) gives the transition equation.

According to this specification, real returns and real consumption growth rates have a common predictable factor, Δc. However, returns and (log) consumption are not cointegrated: while y contains a stochastic trend and is stationary only in first differences, real returns do not contain a stochastic trend and are stationary in levels.

Our estimation strategy explicitly recognizes measurement error arising from an independent random walk component in real consumption and that the relevant explanatory variable, Δc, is not observed. We simultaneously estimate this unobservable component as well as the parameters of the consumption asset pricing model which are consistent with our specification of consumption dynamics.[4] Furthermore, in the case of a single asset, for $\sigma_\tau = 0$

[4] In addition, we must allow for potential correlation between the error made in forecasting the cyclical component, e^c, and the pricing error, z. We still maintain that the trend and cycle disturbances are independent. That is, $\text{cov}(e^\tau_t, e^c_{t-k}) = 0 \; \forall k$ and $\text{cov}(e^\tau_t, z_{t-k}) = 0 \; \forall k$, $\text{cov}(z_t, e^c_t) \neq 0$. Our model as it stands, however, does not satisfy the assumption of independence between the transition and measurement equations required by the Kalman filter and necessitates the following modification. Define the state vector α, the transition matrix T, and the measurement matrix Z. Write the model in compact form as

$$\mathbf{Y}_t = \mathbf{d} + \mathbf{Z}\boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\alpha}_{t+1} = \mathbf{T}\boldsymbol{\alpha}_t + \boldsymbol{\eta}_t,$$

where $Y_t = [R_{1,t+1}\; R_{2,t+1} \dots \Delta y_{t+1}]'$, $\mathbf{d}$ is a constant vector, $\text{Var}(\boldsymbol{\varepsilon}) = H$ and $\text{Var}(\boldsymbol{\eta}) = Q$ are both diagonal matrices. In general, however,

$$G = \text{E}[\boldsymbol{\eta}\boldsymbol{\varepsilon}'] = \begin{bmatrix} \sigma_{zc} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

To accommodate the correlation between $\boldsymbol{\varepsilon}$ and $\boldsymbol{\eta}$, we follow A. Harvey (1989), page 113, and use the equivalent transition equation:

$$\boldsymbol{\alpha}_{t+1} = \mathbf{T}^*\boldsymbol{\alpha}_t + GH^{-1}Y_t + \boldsymbol{\eta}^*_t,$$

where $\mathbf{T}^* = \mathbf{T} - GH^{-1}\mathbf{Z}$ and $\boldsymbol{\eta}^*_t = \boldsymbol{\eta}_t - GH^{-1}\boldsymbol{\varepsilon}_t$. By construction, the error terms in the measurement and transformed transition equations are now independent, permitting the application of the Kalman filter in calculating the corresponding likelihood function.

our estimation framework, expressions (11), (12), and (13), collapses to that of H&S, expressions (9) and (10).[5]

A. Data

Interest rate data are quarterly holding period returns on 3-, 6-, and 12-month[6] Treasury bills and 3- and 5-year zero coupon Treasury bonds. The sample period is 1960:3 to 1992:3.

Quarterly holding period returns on the Treasury bills are computed from the Center for Research in Security Prices (CRSP) Fama file. In particular, monthly holding period returns on 3- and 6-month bills are taken from the Fama 6-month file, while monthly holding period returns on 12-month bills are taken from the Fama 12-month file. Quarterly holding period returns on these bills are calculated by appropriately summing corresponding monthly holding period returns.

Quarterly holding period returns on 3- and 5-year zero coupon Treasury bonds are computed from CRSP's Fama-Bliss file. We calculate these quarterly holding period returns by using the beginning-of-period bond price in the Fama-Bliss file and approximating its end-of-period price by discounting the bond's face value with a rate obtained by interpolating between the nearest maturity yields in the Fama-Bliss yield file.

To measure real consumption over our sample period, we use quarterly observations on the seasonally adjusted real consumption of non-durables and services contained in the National Income and Product Accounts, expressed in constant (1987) dollars.

We align this data so that the return for a particular quarter is realized between the last dates of the previous and current quarters and corresponds to the consumption flow over that quarter given by the logarithmic difference between the previous and current quarters' consumption. To deflate returns in a particular quarter, we use the logarithmic change in the Consumer Price Index (CPI) from the final month in the previous quarter to the month in which the return is realized.

As noted by Heaton (1993) and others, the quarterly consumption data that we use is time averaged, giving rise to a temporal aggregation problem which may result in misleading inferences regarding the consumption-based model's performance. For example, this averaging will make consumption appear less volatile than it actually is. Because of this, C. Harvey (1988) uses an average of *daily* interest rates over the quarter as opposed to the rate prevailing at the end of the quarter.[7]

[5] For comparison purposes, we do not use past returns in H&S's estimation procedure. However, these past returns could also be included in our framework, expression (12).

[6] We actually use quarterly holding period returns to 11-month bills since there are numerous missing values in the time series of monthly holding period returns to the 12-month bill reported in the Fama 12-month file.

[7] However, we examine the average of *monthly* interest rates over the quarter and find that they do not differ significantly from the end of quarter rates themselves. Grossman, Melino, and

B. Single Asset Estimation Results

We now turn our attention to the maximum likelihood estimation of the relation between real holding period returns to a single bond and consumption dynamics.[8] The observable series are demeaned and both μ and μ_r are concentrated out of the likelihood function. The resultant parameters $\psi = \{\sigma_\tau, \sigma_c, \sigma_z, \rho_1, \rho_2, \rho_3, \alpha, \sigma_{cz}\}$ are estimated by maximizing the likelihood function initialized at a vague prior.[9]

Panel A of Table I reports the results of this maximum likelihood estimation for 3-, 6-, and 12-month Treasury bill returns and for 3- and 5-year Treasury bond returns. The estimated coefficient of relative risk aversion, $\hat{\alpha}$, is positive and significantly different from zero for the 3- and 6-month returns. These estimates of 8.107 and 6.713, respectively, are larger than C. Harvey's (1988) estimates based on short term bills, which range from -0.57 to 0.96, since consumption's cyclical component is less variable than consumption itself. However, $\hat{\alpha}$ is insignificantly different from zero for the 12-month and 3- and 5-year returns.

The model's ability to explain these holding period returns, as measured by STD(z), worsens with increasing maturity; for example, STD(z) = 2.112 for 3-month Treasury bills but STD(z) = 14.939 for 5-year Treasury bonds. Conversely, the model's ability to explain real consumption growth rates, as measured by STD(e^τ), tends to improve with increasing maturity as the influence of highly variable bond returns in this estimation diminishes.

In comparison, Panel B of Table I summarizes the results of the H&S estimation procedure in which we impose $\sigma_\tau = 0$. Notice that when restricting our attention to the real holding period returns to a single bond, $\hat{\alpha}$ is now consistently negative and statistically insignificant. Compared with the H&S

Shiller (1987) conclude that even after explicitly accounting for the time-averaged nature of consumption data, the consumption asset pricing model assuming time additive preferences still could not be reconciled with their data.

[8] We carry out a specification analysis to determine the order of the AR process in (12). We estimate our model assuming different AR(p) specifications, $p = 1, 2, 3, 5$, and 10. The Akaike Information Criterion (AIC) is maximized for $p = 3$. Taking into account the covariance between e^c and z, σ_{cz}, the resultant model in state-space form is:

$$\begin{bmatrix} R_{j,t+1} \\ \Delta y_{t+1} \end{bmatrix} = \begin{bmatrix} \mu_r \\ \mu \end{bmatrix} + \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta c_{t+1} \\ \Delta c_t \\ \Delta c_{t-1} \\ \mu_c \end{bmatrix} + \begin{bmatrix} z_{t+1} \\ e^\tau_{t+1} \end{bmatrix},$$

$$\begin{bmatrix} \Delta c_{t+1} \\ \Delta c_t \\ \Delta c_{t-1} \\ \mu_c \end{bmatrix} = \begin{bmatrix} \rho_1 & \rho_2 & \rho_3 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta c_t \\ \Delta c_{t-1} \\ \Delta c_{t-2} \\ \mu_c \end{bmatrix} + \begin{bmatrix} \sigma_{cz}/\sigma_z^2 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} R_{j,t} \\ \Delta y_t \end{bmatrix} + \begin{bmatrix} e^{*c}_{t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

[9] This model is identified since the necessary and sufficient conditions required by Otter's (1988) Theorem 1 can be shown to be satisfied.

Table I

Maximum Likelihood Estimation of Models of Real Holding Period Returns and Real Consumption Dynamics

We estimate both single and multiple asset versions of our real holding period returns model. In Panel A we assume the presence of an independent stochastic trend in real consumption dynamics, while in Panel B we assume no stochastic trend, $\sigma_\tau = 0$.

In the single asset case the following specification is estimated:

$$R_{j,t+1} = \mu_r + \alpha\Delta c_{t+1} + z_{t+1}, \qquad \Delta y_{t+1} = \mu + \Delta c_{t+1} + e^{\tau}_{t+1},$$

$$\Delta c_{t+1} = \mu_c + \rho_1\Delta c_t + \rho_2\Delta c_{t-1} + \rho_3\Delta c_{t-2} + e^{c}_{t+1},$$

where R_j is the real one-quarter holding period return to a j-maturity bond, Δy is the change in the logarithm of the real consumption of nondurables and services, and Δc is the change in real consumption's unobservable cyclical component. The coefficient of relative risk aversion is denoted by α, while ρ_1, ρ_2, and ρ_3 characterize the cyclical component's autoregressive specification. We cast this model in state space form and evaluate the likelihood function using a Kalman filter initialized at a vague prior. Asymptotic t-statistics, based on the inverse of the Hessian matrix, are in parentheses. The sample period is 1960:3 to 1992:3 with the first six observations used for the filter's warm-up. Holding period returns to the 12-month bill are proxied by the 11-month bill's holding period returns and are only available beginning in 1963:1 with the first sixteen observations used for the filter's warm-up. A modified transition equation is used to take account of correlation between z and e^c. Both μ and μ_r are concentrated out of the likelihood function.

In the multiple asset case the following specification is estimated:

$$R_{3m,t+1} = \ell - \sigma_c^2\alpha^2/2 + \alpha\Delta c_{t+1} + z_{1,t+1}, \qquad R_{5Y,t+1} = \ell + \alpha\Delta c_{t+1} + z_{2,t+1},$$

$$\Delta y_{t+1} = \mu + \Delta c_{t+1} + e^{\tau}_{t+1}, \qquad \Delta c_{t+1} = \mu_c + \rho_1\Delta c_t + \rho_2\Delta c_{t-1} + \rho_3\Delta c_{t-2} + e^{c}_{t+1},$$

where R_{3m} is the real one-quarter holding period return to a 3-month Treasury bill, and R_{5Y} is the real one-quarter holding period return to a 5-year Treasury bond. A modified transition equation is used to take account of correlation between z_1, z_2, and e^c. The consol rate ℓ as well as μ are concentrated out of the likelihood function.

The value of the likelihood statistic for testing the null hypothesis $\sigma_\tau = 0$ against the alternative hypothesis $\sigma_\tau \neq 0$ is given by LR.

	Single Asset Estimation					Multiple Asset Estimation
	3-Month Bill	6-Month Bill	12-Month Bill	3-Year Bond	5-Year Bond	3-Month Bill and 5-Year Bond
Panel A: Assuming a Stochastic Trend in Real Consumption Dynamics						
α	8.107	6.713	−0.309	−0.722	−0.838	0.206
	(1.90*)	(2.05**)	(−1.20)	(−1.07)	(−0.67)	(7.32***)
ρ_1	0.795	0.841	0.317	0.348	0.318	0.729
	(4.61)	(2.90)	(3.59)	(1.86)	(0.78)	(3.63)
ρ_2	0.518	0.426	0.066	0.020	0.048	0.137
	(2.15)	(1.00)	(0.72)	(0.10)	(0.11)	(0.59)
ρ_3	−0.405	−0.379	0.247	0.267	0.238	−0.132
	(−2.41)	(−1.87)	(2.86)	(1.54)	(0.79)	(−1.20)
STD(z)	2.112	3.096	5.539	10.807	14.939	3.570[a] 15.145[b]
STD(e^τ)	1.943	1.931	4.45×10^{-5}	0.874	0.716	1.410
STD(e^c)	0.098	0.123	1.713	1.493	1.587	0.854
cov(z, e^c)	0.206	0.381	2.418	4.418	5.480	−0.060[a] 5.143[b]

procedure, the UC-ARIMA model is also better in explaining these holding period returns as STD(z) is smaller under the UC-ARIMA specification for every maturity considered. However, for shorter term bills the H&S procedure provides a better fit to real consumption growth rates.

Table I—continued

	Single Asset Estimation					Multiple Asset Estimation
	3-Month Bill	6-Month Bill	12-Month Bill	3-Year Bond	5-Year Bond	3-Month Bill and 5-Year Bond
	Panel B: Assuming No Stochastic Trend in Real Consumption Dynamics					
α	−0.041	−0.119	−0.303	−0.362	−0.639	0.111
	(−0.28)	(−0.73)	(−1.14)	(−0.69)	(−0.61)	(4.52***)
ρ_1	0.259	0.260	0.317	0.291	0.294	0.262
	(3.23)	(3.05)	(5.27)	(3.47)	(2.23)	(5.69)
ρ_2	0.105	0.107	0.066	0.107	0.257	0.102
	(1.23)	(1.24)	(1.07)	(1.14)	(0.98)	(2.02)
ρ_3	0.207	0.201	0.245	0.257	0.063	0.183
	(2.58)	(2.48)	(4.45)	(3.22)	(0.36)	(4.10)
STD(z)	3.040	3.713	5.541	10.839	14.939	3.582[a] 15.172[b]
STD(e^c)	1.756	1.755	1.674	1.726	1.919	1.763
cov(z, e^c)	0.813	1.248	2.762	4.404	5.607	0.585[a] 3.890[b]
LR	33.24***	10.12***	0.31	1.71	4.07**	2.86*

* Denotes significance at the 10% level, ** denotes significance at the 5% level, and *** denotes significance at the 1% level.

[a] Result for the 3-month bill in the multiple asset estimation.

[b] Result for the 5-year bond in the multiple asset estimation.

To see the influence of returns on the properties of the estimated cyclical component, Figure 2 plots estimates of Δc based on holding period returns to alternative bonds against changes in the cyclical estimate obtained when only using consumption data.[10] Estimates of Δc using short term bills exhibit little correlation with changes in the univariate estimated cyclical component. Recall that in this case the linear relation between Δc and returns is statistically significant and hence these returns heavily influence this estimation. On the other hand, returns on longer term bonds are not significantly related to Δc, resulting in estimates much closer to those obtained when only using consumption data.

Table I also provides likelihood ratio statistics (LR) for asymptotically testing the null hypothesis that $\sigma_\tau = 0$ based on the real holding period returns to a single bond. We see evidence of an independent stochastic trend in consump-

[10] We use quarterly real consumption data on non-durables and services over the sample period 1947:1 to 1992:3. Different AR(p) specifications ($p = 1, 2, 3, 5$, and 10) are assumed and the Akaike Information Criterion (AIC) is maximized at $p = 2$. The maximum likelihood estimates of the resultant model (asymptotic t-statistics in parentheses) are as follows:

$$
\begin{aligned}
y_t &= c_t + \tau_t, \\
c_t &= \underset{(18.60)}{1.7836}c_{t-1} - \underset{(-8.97)}{0.8348}c_{t-2}, &\quad \sigma_c = 0.0015, \\
\tau_t &= \underset{(23.42)}{0.0086} + \tau_{t-1}, &\quad \sigma_\tau = 0.0043,
\end{aligned}
$$

The likelihood function is evaluated using a Kalman filter initialized at a vague prior with a warm-up period of 24 quarters.

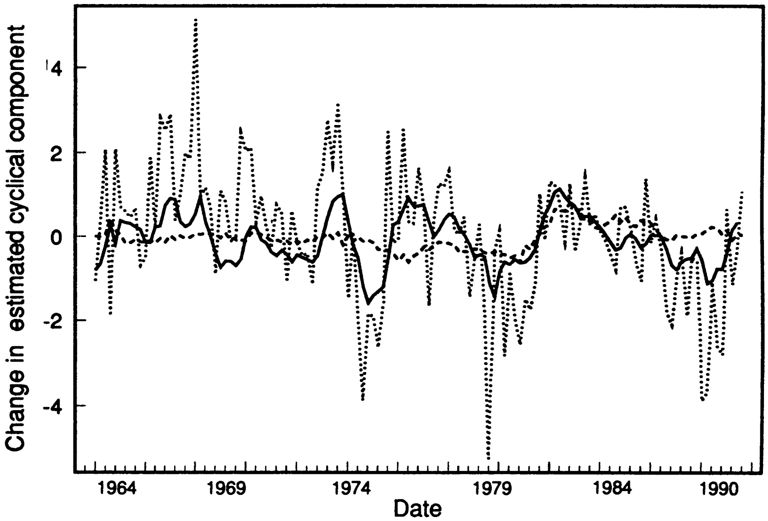

Figure 2. Estimated changes in real consumption's cyclical component using a single asset. This figure compares the time series behavior of the changes in real consumption's cyclical component estimated using quarterly real holding period returns to 3-month Treasury bills (indicated by the dashed line) and estimated using quarterly real holding period returns to 5-year Treasury bonds (indicated by the dotted line) with changes in the univariate estimate of real consumption's cyclical component based on real nondurables and services data only (indicated by the solid line).

tion dynamics. The greatest improvement occurs when using 3- and 6-month returns (LR = 33.24 and LR = 10.12, respectively) and is consistent with these returns being linearly related to the estimated cyclical component. Notice that there is also improvement when using 5-year returns (LR = 4.07). Since these returns are not significantly related to the estimated cyclical component, this improvement follows from consumption dynamics being better explained by their decomposition into an independent trend and cyclical component.

C. Multiple Asset Estimation

When more than one bond is included in the estimation, a cross-sectional restriction on expected returns is introduced. Unlike the single bond case in which its expected return could be concentrated out, this now is no longer possible since expected returns are a function of model parameters.

From equation (6), the *unconditional* expected return on a 1-period bond is given by $\ell - (\alpha^2/2)\sigma_c^2$. Because of the cyclical component's stationarity, the unconditional expected return increases to ℓ as the maturity of the bond increases. If we jointly consider the short and long bonds, the empirical model may be written as:

$$R_{s,t+1} = \ell - \frac{\alpha^2}{2}\sigma_c^2 + \alpha\Delta c_{t+1} + z_{s,t+1}, \tag{14}$$

$$R_{L,t+1} \simeq \ell + \alpha \Delta c_{t+1} + z_{L,t+1}, \tag{15}$$

$$\Delta y_{t+1} = \mu + \Delta c_{t+1} + e^{\tau}_{t+1}, \tag{16}$$

$$\Delta c_{t+1} = \mu_c + \sum_{i=1}^{m} \rho_i \Delta c_{t+1-i} + e^{c}_{t+1}, \tag{17}$$

where, assuming quarterly observations, R_s is the holding period return to a 3-month bill, while R_L is the holding period return to the longest term bond.

In the presence of a stochastic trend in consumption dynamics, the cross-sectional difference in expected returns on bonds of differing maturities depends explicitly on the cyclical component's variance. In particular, if we consider only the shortest and longest bonds,

$$\mathrm{E}[R_L] - \mathrm{E}[R_s] \simeq \alpha^2 \frac{\sigma_c^2}{2}.$$

Neglecting the stochastic trend, that is, setting $\sigma_\tau = 0$, will bias the estimation of α. This follows since in the absence of a stochastic trend the total variance of (log) consumption, σ_y^2, rather than σ_c^2 appears in the preceding expression and, given that $\sigma_c^2 \leq \sigma_y^2$, a smaller value of α is needed to explain the difference in these average returns.

C.1. Multiple Asset Estimation Results

We estimate the model, expressions (14)–(17), using quarterly real holding period returns to the 3-month bill and to the 5-year bond.[11] Table I also reports these multiple asset results.

[11] The resulting state-space model is

$$\begin{bmatrix} R_{3m,t+1} \\ R_{5Y,t+1} \\ \Delta y_{t+1} \end{bmatrix} = \begin{bmatrix} \ell - \dfrac{\alpha^2}{2}\sigma_c^2 \\ \ell \\ \mu \end{bmatrix} + \begin{bmatrix} \alpha & 0 & 0 & 0 \\ \alpha & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta c_{t+1} \\ \Delta c_t \\ \Delta c_{t-1} \\ \mu_c \end{bmatrix} + \begin{bmatrix} z_{1,t+1} \\ z_{2,t+1} \\ e^{\tau}_{t+1} \end{bmatrix},$$

$$\begin{bmatrix} \Delta c_{t+1} \\ \Delta c_t \\ \Delta c_{t-1} \\ \mu_c \end{bmatrix} = \begin{bmatrix} \rho_1 - \alpha \bar{K} & \rho_2 & \rho_3 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta c_t \\ \Delta c_{t-1} \\ \Delta c_{t-2} \\ \mu_c \end{bmatrix} + \begin{bmatrix} \dfrac{\sigma_{cz_1}}{\sigma_{z_1}^2} & \dfrac{\sigma_{cz_2}}{\sigma_{z_2}^2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} R_{3m,t} \\ R_{5Y,t} \\ \Delta y_t \end{bmatrix} + \begin{bmatrix} e^{*c}_{t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

where $\sigma_{z_1}^2$ and $\sigma_{z_2}^2$ denote the variance of the measurement disturbances, σ_{cz_1} and σ_{cz_2} denote the covariance between e^c and z_1, and e^c and z_2, respectively,

$$\bar{K} = \left(\frac{\sigma_{cz_1}}{\sigma_{z_1}^2} + \frac{\sigma_{cz_2}}{\sigma_{z_2}^2} \right) \text{ and } \mathrm{VAR}(e^{*c}) = \sigma_c^2 - \left(\frac{\sigma_{cz_1}^2}{\sigma_{z_1}^2} + \frac{\sigma_{cz_2}^2}{\sigma_{z_2}^2} \right).$$

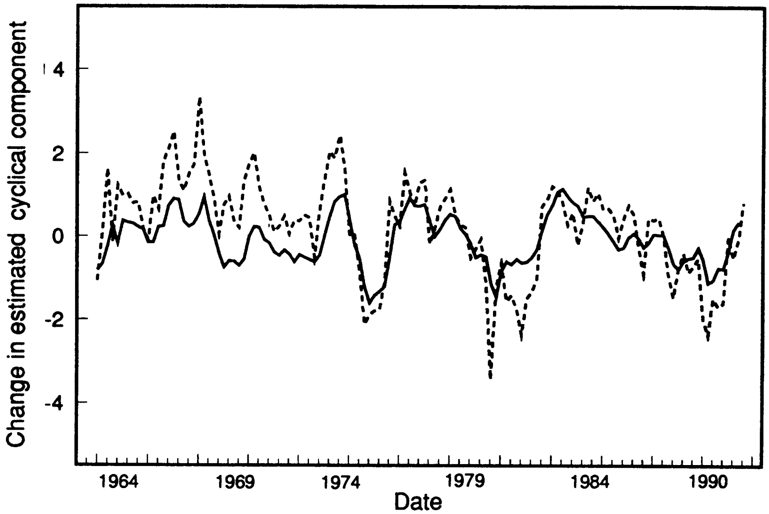

Figure 3. Estimated changes in real consumption's cyclical component using multiple assets. This figure compares the time series behavior of the changes in real consumption's cyclical component estimated using quarterly real holding period returns to both 3-month Treasury bills and 5-year Treasury bonds (indicated by the dotted line) with changes in the univariate estimate of real consumption's cyclical component based on real nondurables and services data only (indicated by the solid line).

From Panel A we see a positive and significant relation between the estimated cyclical component and real bond holding period returns when we assume a stochastic trend in consumption dynamics. The coefficient of relative risk aversion estimated simultaneously from the long and short bonds $\hat{\alpha} = 0.206$, ($t = 7.32$), is smaller (in absolute value) than the positive coefficient estimated from the short bond itself or the negative coefficient estimated from the long bond itself. While the multiple asset model's ability to explain these real holding period returns is now slightly worse, the resultant estimated Δc is less influenced by returns and consequently is more comparable to the changes in the cyclical component estimated using consumption data only (see Figure 3).

By setting $\sigma_\tau = 0$ in Panel B we obtain, as expected, a smaller estimated coefficient of relative risk aversion, $\hat{\alpha} = 0.111$, ($t = 4.52$) when using real holding period returns to the 3-month bill and 5-year bond. A likelihood ratio test, however, allows us to reject at the 10% significance level the null hypothesis that $\sigma_\tau = 0$ (LR = 2.86).[12]

[12] In the case of a single bond we can nest the H&S procedure in our framework because the bond's expected return could be concentrated out. Strictly speaking, this nesting is no longer possible with more than one bond because, in addition to specifying consumption dynamics, we must also cross-sectionally restrict expected returns across the bonds to be consistent with these assumed dynamics.

D. Nonlinear Consumption Asset Pricing Models

We now consider arbitrarily distributed consumption growth rates while maintaining the assumption of constant relative risk aversion. The Kalman filter still provides the best linear estimator of consumption's cyclical component even if consumption growth rates are not normally distributed (A. Harvey (1989)). Hence, our estimation framework can also be used to investigate the non-linear specification of Hansen and Singleton (1982).

In this case the intertemporal marginal rate of substitution is given by

$$\mathrm{MRS}_t = \beta\left\{\frac{C_t}{C_{t-1}}\right\}^{-\alpha}, \tag{18}$$

where $\beta = e^{-\delta}$. Relying on expressions (1) and (3) we have

$$y_t = c_t + \tau_t, \qquad \tau_t = \mu + \tau_{t-1} + \eta_t,$$

but now $\eta_t \sim$ i.i.d. $(0, \sigma_\eta^2)$ and it follows that

$$\begin{aligned}\mathrm{MRS}_t = \beta\left\{\frac{C_t}{C_{t-1}}\right\}^{-\alpha} &= \beta\left\{\frac{\exp(c_t + \mu + \tau_{t-1} + \eta_t)}{\exp(c_{t-1} + \tau_{t-1})}\right\}^{-\alpha} \\ &= \beta\{\exp[-\alpha(c_t - c_{t-1})]\}\xi^{-\alpha},\end{aligned}$$

where $\xi^{-\alpha}$ is a positive and approximately i.i.d. random variable and is unrelated to asset returns. Consequently, returns related variation in the intertemporal marginal rate of substitution arises only from variation in consumption's cyclical component. We approximate the intertemporal marginal rate of substitution by

$$\widehat{\mathrm{MRS}}_t = \beta\exp[-\alpha(c_t - c_{t-1})], \tag{19}$$

with consumption's cyclical component estimated, for example, using a univariate Kalman filter or the Hodrick–Prescott filter (Hodrick and Prescott (1981)). GMM estimation can be based on orthogonality conditions involving $\widehat{\mathrm{MRS}}_t$.

D.1. Hansen–Jagannathan Bounds

As a further diagnostic in assessing the fit of our model we investigate the Hansen–Jagannathan (1991) bound for the volatility of the representative investor's intertemporal marginal rate of substitution. We estimate this lower bound using one-quarter real holding period returns to the 3-month Treasury bill and to the 5-year Treasury bond. The Hansen–Jagannathan bound is then compared to the mean and standard deviation of the approximate intertemporal marginal rate of substitution given by expression (19), $\widehat{\mathrm{MRS}}_t$, for

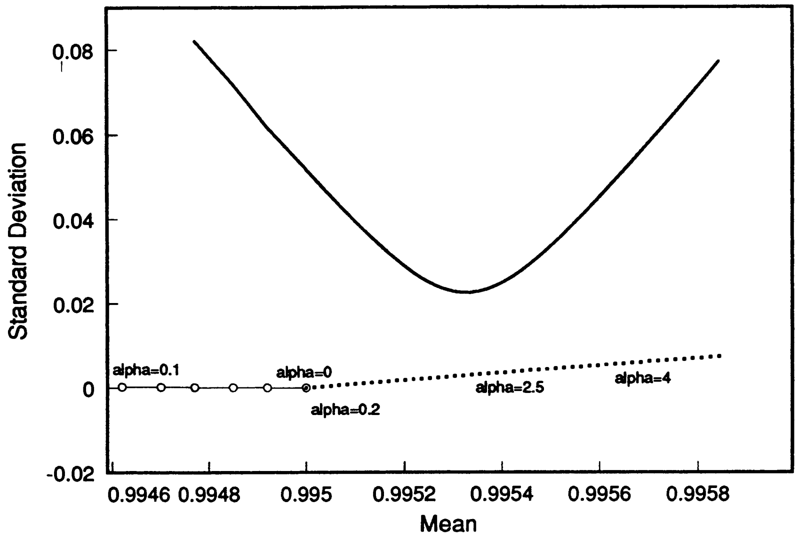

Figure 4. Hansen–Jagannathan bound and the moments of MRS and $\widehat{\mathbf{MRS}}$. The Hansen–Jagannathan bound based on quarterly real holding period returns to 3-month Treasury bills and 5-year Treasury bonds (indicated by the solid line) is compared to the intertemporal marginal rate of substitution MRS_t (indicated by circles) and to the approximate intertemporal marginal rate of substitution $\widehat{MRS}_t$ (indicated by dots) for $\beta = 0.995$ and a variety of α values. MRS_t is based on real consumption growth rates while $\widehat{MRS}_t$ is based on real consumption's UC-ARIMA estimated cyclical component.

β 0.995 and a range of α values.[13] All else equal, we also make this comparison assuming the intertemporal marginal rate of substitution is given by expression (18), MRS_t.

Notice in Figure 4 that as α increases the moments of $\widehat{MRS}_t$ initially move towards but do not touch the Hansen–Jagannathan bound. In contrast, the moments of MRS_t initially move away from the bound and, as pointed out by Hansen and Jagannathan (1991), only subsequently move closer to the bound for extremely large α values.

However, a visual comparison of these moments with the Hansen–Jagannathan bound may be misleading because sampling error is ignored. To quantify the effects of sampling error, Table II presents the results of testing whether MRS_t (Panel A) and $\widehat{MRS}_t$ (Panel B) violate the Hansen–Jagannathan bound for $\beta = 0.995$ and α values ranging from 0.1 to 5 by using the vertical

[13] We use $\beta = 0.995$ since the average real quarterly holding period return on the long term (5-year) Treasury bond over our sample period is approximately 0.005. Also, for illustrative purposes, we use the univariate estimate of consumption's cyclical component detailed in footnote 10. In testing whether the Hansen–Jagannathan bound is violated we ignore the uncertainty in consumption's estimated cyclical component and hence underestimate the uncertainty in the moments of the approximate intertemporal rate of substitution $\widehat{MRS}_t$. As a result, we are biased toward rejecting the null hypothesis that the Hansen–Jagannathan bound is not violated.

Table II

Tests of Hansen–Jagannathan Bound Violations

We test whether the Hansen–Jagannathan bound is violated by the representative investor's intertemporal marginal rate of substitution given by

$$\mathrm{MRS}_t = \beta\left\{\frac{C_t}{C_{t-1}}\right\}^{-\alpha} \quad \text{(Panel A)} \qquad \text{and} \qquad \widehat{\mathrm{MRS}}_t = \beta\exp[-\alpha(c_t - c_{t-1})] \quad \text{(Panel B)}$$

where β denotes the intertemporal coefficient, α is the coefficient of relative risk aversion, C_t is consumption at time t while c_t is log consumption's cyclical component. The bound is constructed using quarterly real holding period returns to 3-month Treasury bills and to 5-year Treasury bonds over the sample period 1960:3 to 1992:3. Consumption is the seasonally adjusted real consumption of nondurables and services and consumption's cyclical component is estimated using a univariate UC-ARIMA procedure. We fix β at 0.995 and examine a range of α values from 0.1 to 5. We estimate the mean ($\hat{\mu}_{\mathrm{MRS}}$) and standard deviation ($\hat{\sigma}_{\mathrm{MRS}}$) of the corresponding intertemporal marginal rate of substitution, as well as the Hansen–Jagannathan volatility bound ($\hat{\sigma}_{\mathrm{HJ}}$). The statistic t − *ratio* tests the null hypothesis that $\sigma_{\mathrm{MRS}} = \sigma_{\mathrm{HJ}}$ and is Burnside's (1994) vertical distance test.

α	$\hat{\mu}_{\mathrm{MRS}}$	$\hat{\sigma}_{\mathrm{MRS}}$	$\hat{\sigma}_{\mathrm{HJ}}$	$t-$ ratio
		Panel A: Using Real Consumption Growth Rates		
0.1	0.994	0.001	0.156	−0.774
0.2	0.994	0.001	0.262	−1.229
0.3	0.993	0.002	0.369	−1.640
0.4	0.992	0.002	0.476	−2.004**
0.5	0.991	0.003	0.583	−2.324***
1.0	0.987	0.005	1.115	−3.421***
2.0	0.980	0.010	2.171	−4.347***
3.0	0.973	0.015	3.217	−4.711***
4.0	0.966	0.019	4.251	−4.897***
5.0	0.959	0.024	5.273	−5.001***
		Panel B: Using Real Consumption's Cyclical Component		
0.1	0.995	0.002×10^{-1}	0.050	−0.292
0.2	0.995	0.003×10^{-1}	0.048	−0.283
0.3	0.995	0.005×10^{-1}	0.046	−0.273
0.4	0.995	0.006×10^{-1}	0.044	−0.264
0.5	0.995	0.008×10^{-1}	0.042	−0.255
1.0	0.995	0.002	0.032	−0.218
2.0	0.995	0.003	0.023	−0.206
3.0	0.996	0.005	0.035	−0.168
4.0	0.996	0.006	0.056	−0.231
5.0	0.996	0.008	0.080	−0.296

** Denotes significance at the 5% level.
*** Denotes significance at the 1% level.

distance test of Burnside (1994). The results should be interpreted bearing in mind that the power of the Burnside test may be low.

From Panel A we see evidence at the 5 percent significance level that MRS_t violates the Hansen–Jagannathan bound for α values greater than 0.3. The fact that MRS_t is statistically indistinguishable from the bound for α values less than 0.3 is consistent with the estimate $\hat{\alpha} = 0.111$ previously obtained when using these real holding period returns and setting $\sigma_\tau = 0$.

In contrast, we see in Panel B that $\widehat{\mathrm{MRS}}_t$ is statistically indistinguishable from the Hansen–Jagannathan bound for all of the α values considered. As in Figure 4, the vertical distance between the moments of $\widehat{\mathrm{MRS}}_t$ and the bound is smallest, but not zero, at the bottom of the bound. Even though the standard error associated with the vertical distance is also smallest at the bottom of the bound, these standard errors are relatively large so that we statistically conclude that the moments of $\widehat{\mathrm{MRS}}_t$ do not lie significantly below the Hansen–Jagannathan bound.

III. The Term Spread's Informativeness

As empirically documented by C. Harvey (1988, 1989), Estrella and Hardouvelis (1991) and others, the current term structure provides valuable ex ante information about future aggregate economic activity. In this section we demonstrate that the term spread is better at forecasting the deviations of aggregate economic activity from its stochastic trend than corresponding growth rates.

A. Data

To investigate the relation between interest rates and the cyclical component of aggregate economic activity requires a comprehensive measure of economic activity. We use quarterly seasonally adjusted data on Gross Domestic Product (GDP) over the sample period 1964:1 to 1992:3, expressed in constant (1987) dollars.

The term spread is measured by the difference between the annualized yields on 5- and 1-year Treasury bonds, taken from the Fama-Bliss file.

B. UC-ARIMA Model of Real GDP's Cyclical Component

Recognizing real GDP's potentially stochastic trend, we fit a UC-ARIMA[14] model to log real GDP, y^{gdp}. The resultant cyclical component of log real GDP, c^{gdp}, is plotted in Figure 5. Notice that the estimated peaks and troughs

[14] We estimate the UC-ARIMA model for real GDP using a Kalman filter initialized at a vague prior with a warm-up period of 16 quarters. Different AR(p) specifications ($p = 1, 2, 3, 5$, and 10) are assumed and the Akaike Information Criterion (AIC) is maximized at $p = 2$. Results of the maximum likelihood estimation are as follows (asymptotic t-statistics in parentheses):

$$y_t^{gdp} = c_t^{gdp} + \tau_t^{gdp},$$

$$\underset{(9.43)}{c_t^{gdp} = 1.6790}\, c_{t-1}^{gdp} - \underset{(4.01)}{0.7212}\, c_{t-2}^{gdp}, \qquad \sigma_c = 0.0041,$$

$$\tau_t^{gdp} = \underset{(11.78)}{0.0082} + \tau_{t-1}^{gdp} \qquad \sigma_\tau = 0.0070.$$

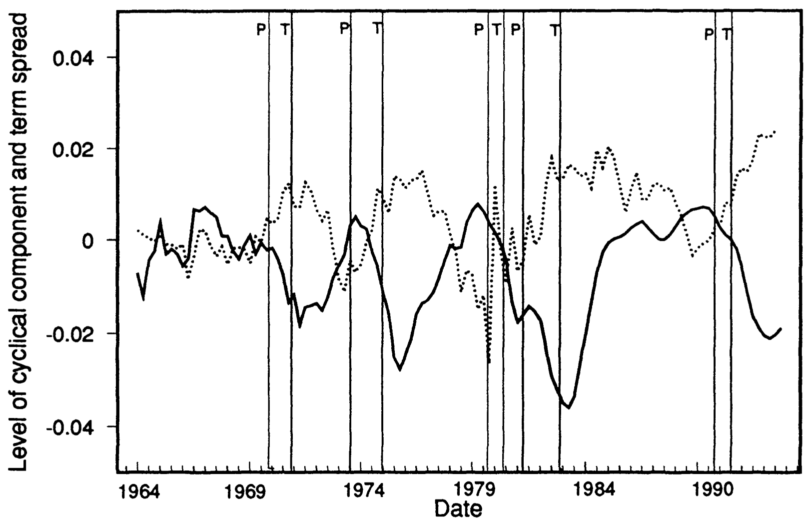

Figure 5. Real GDP's estimated cyclical component and the spread between 5-year and 1-year Treasury yields. This figure illustrates the UC-ARIMA estimated cyclical component of real GDP (indicated by the solid line) and the term spread as measured by the difference between the yields on 5- and 1-year Treasury bonds (indicated by the dotted line). For comparison purposes National Bureau of Economic Research business cycle peaks, denoted by **P**, and troughs, denoted by **T**, are also presented.

correspond well to the National Bureau of Economic Research documented business cycle peaks and troughs. The countercyclical behavior of the term spread is also evident in Figure 5.

C. Regression Evidence

Table III presents univariate regression evidence investigating the term spread's informativeness regarding future economic activity using quarterly data on log real GDP. In Panel A we run regressions of the form:

$$\Delta y^{\mathrm{gdp}}_{t,t+i} = a_i + b_i ts_t + u_t, \tag{20}$$

where ts_t is the term spread prevailing at quarter t and $\Delta y^{\mathrm{gdp}}_{t,t+i}$ is the i-quarter growth rate in real GDP starting at quarter t. In Panel B we use corresponding i-quarter changes in the UC-ARIMA cyclical estimate:

$$\Delta c^{\mathrm{gdp}}_{t,t+i} = a_i + b_i ts_t + u_t. \tag{21}$$

Given these maximum likelihood parameter estimates, the Kalman filter also provides an estimate of real GDP's cyclical component at each time point in our sample. The estimated cyclical component at time t is based only on information available up to time t.

Table III

The Term Spread and Future Changes in Aggregate Economic Activity

Aggregate economic activity is proxied by the logarithm of real GDP, denoted by y^{gdp}, and log real GDP's cyclical component is denoted by c^{gdp}. We estimate the regressions

$$\Delta y^{\text{gdp}}_{t,t+i} = a_i + b_i ts_t + u_t \quad \text{(Panel A)} \qquad \text{and} \qquad \Delta c^{\text{gdp}}_{t,t+i} = a_i + b_i ts_t + u_t \quad \text{(Panel B)}$$

where ts_t is the term spread prevailing at quarter t, $\Delta y^{\text{gdp}}_{t,t+i}$ is the i-quarter growth rate in real GDP from quarter t given by the i-quarter change in log real GDP from quarter t, and $\Delta c^{\text{gdp}}_{t,t+i}$ is the i-quarter change beginning at quarter t in the UC-ARIMA estimated cyclical component of log real GDP. The term spread is measured as the difference between the annualized yields on five year and one year zero coupon Treasury bonds taken from CRSP's Fama–Bliss file. The sample period is 1964:1 to 1992:3. We also document each regression's standard error, denoted by SE, and adjusted R^2. In parentheses are t-statistics computed from corresponding Newey–West (1987) standard errors assuming a lag length of 20 quarters.

Quarters, i	a_i	b_i	SE	R^2
Panel A: Using Real GDP Growth Rates				
1	0.005 (4.05***)	0.345 (2.99***)	0.009	0.10
2	0.010 (3.85***)	0.775 (3.288***)	0.014	0.20
3	0.015 (4.06***)	1.139 (3.59***)	0.018	0.23
4	0.021 (4.34***)	1.504 (3.81***)	0.021	0.27
5	0.026 (4.60***)	1.853 (4.04***)	0.024	0.30
6	0.031 (4.96***)	2.132 (4.11***)	0.026	0.32
7	0.037 (5.37***)	2.387 (4.35***)	0.028	0.33
8	0.043 (5.91***)	2.580 (4.69***)	0.029	0.35
9	0.049 (6.48***)	2.643 (4.96***)	0.031	0.34
10	0.055 (6.95***)	2.714 (5.11***)	0.035	0.30
Panel B: Using Real GDP's Cyclical Component				
1	0.000 (−0.90)	0.050 (1.12)	0.004	0.01
2	−0.001 (−1.35)	0.131 (1.60)	0.006	0.03
3	−0.002 (−1.91)	0.297 (2.55***)	0.008	0.09
4	−0.003 (−2.41***)	0.521 (3.64***)	0.009	0.18
5	−0.004 (−2.58***)	0.728 (4.56***)	0.010	0.26
6	−0.005 (−2.71***)	0.928 (5.48***)	0.011	0.33
7	−0.005 (−2.77***)	1.115 (6.39***)	0.011	0.40
8	−0.006 (−2.76***)	1.273 (6.91***)	0.012	0.46
9	−0.006 (−2.75***)	1.368 (7.18***)	0.012	0.48
10	−0.006 (−2.81***)	1.436 (7.69***)	0.012	0.50

*** Denotes significance at the 1% level

As can be seen in Table III, the informativeness of the term spread varies with the forecasting horizon. In general, we expect a positive relation between the term spread and subsequent changes in economic activity as the steepening (flattening) of the term structure implies that an expansion (recession) is imminent.

From Panel A, consistent with previous research, we see a statistically significant relation between the term spread and subsequent growth in real GDP. The regressions' adjusted R^2s initially increase with the forecasting horizon, peak in the vicinity of 8-quarters, then subsequently decline.

Panel B of Table III also shows a statistically significant positive relation between the term spread and subsequent changes in the UC-ARIMA estimate of real GDP's cyclical component. The adjusted R^2s increase with the forecasting horizon throughout so that at 10 quarters the term spread explains approximately 50 percent of the subsequent change in this estimate. The goodness of fit of the regressions in Panel B, as measured by their adjusted R^2s, exceed those of Panel A at all forecasting horizons beyond 6 quarters, indicating that the term spread provides reliable information about subsequent long-run changes in economic activity proxied by real GDP's UC-ARIMA estimated cyclical component.

IV. Summary and Conclusions

Macroeconomic variables, like real consumption and real GDP, grow or trend over time. However, their rate of growth is not constant but rather varies. Consequently, the dynamics of these macroeconomic variables are better described by stationary cyclical deviations about a stochastic trend. This paper explores the relation between this more appropriate measure of the business cycle and the behavior of real interest rates.

The presence of an independent trend and cyclical component in real consumption dynamics improves the fit of a consumption-based model to real bond holding period returns, especially for short term bills. Real holding period returns to longer term bonds are simply too variable to be explained by this cyclical component. However, we obtain a statistically efficient estimate of the coefficient of relative risk aversion when real holding period returns to both short term bills and longer term bonds are used in our maximum likelihood estimation framework. Finally, using this more precise measure of the business cycle improves the term spread's informativeness regarding future economic activity.

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