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Options on Leveraged Equity: Theory and Empirical Tests

KLAUS BJERRE TOFT and BRIAN PRUCYK*

ABSTRACT

We develop an option pricing model for calls and puts written on leveraged equity in an economy with corporate taxes and bankruptcy costs. The model explains implied Black–Scholes volatility biases by relating them to the firm's structural characteristics such as leverage and debt covenants. We test the model by comparing predicted pricing biases with biases observed in a large cross-section of firms with liquid exchange traded option contracts. Our empirical study detects leverage related pricing biases. The magnitudes of these biases correspond to those predicted by our model. We also find significant pricing biases for firms financed primarily by short-term debt. This supports our model because short-term debt introduces net-worth hurdles similar to net-worth covenants.

THE DIRECT LINK BETWEEN financing decisions at the firm level and the pricing of derivative securities has, with few exceptions, been ignored in the theoretical and empirical option pricing literature. Most finance literature treats the pricing of securities in the firm's capital structure and the pricing of options on these securities as separate research areas. Securities in the capital structure are priced using an assumption about the value process of the firm's assets; hedging arguments are invoked and the firm's debt and equity are priced as contingent claims.[1] Most equity option pricing literature, on the other hand, uses an exogenously specified equity price process as the basis for an option pricing formula.[2]

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[1] The most notable studies of this type include articles by Black and Scholes (1973), Merton (1974), Black and Cox (1976), Geske (1977), Ingersoll (1977a, 1977b), Brennan and Schwartz (1978, 1980, 1984), Nielsen, Saá-Requejo, and Santa-Clara (1993), Leland (1994), Longstaff and Schwartz (1995), and Leland and Toft (1996).

[2] The classic studies by Black and Scholes (1973), Merton (1973), Cox and Ross (1976), and Rubinstein (1983) fall into this category. Examples of more recent models in this class are the

Different primitive assumptions are thus made for different purposes, and the resulting security price processes may permit arbitrage opportunities.[3] All security values related to the same underlying real assets should be derived from primitive assumptions that create a set of consistent price processes that preclude arbitrage opportunities. Following this philosophy, Geske (1979) models option prices as compound options, where equity itself is treated as an option on the firm's assets. Geske uses the Black–Scholes (1973) and Merton (1974) capital structure model with zero coupon debt of finite maturity. Geske's model leads to an option pricing formula that is consistent with both equity and debt price processes. However, this model assumes abrupt changes in the capital structure when the debt matures.[4]

In this article, we propose a set of closed form equity option pricing formulas based on the capital structure model developed by Leland (1994). Unlike Geske (1979), Leland's model allows for taxes and bankruptcy in an economy where equity holders may maximize firm value by choosing both leverage ratio and bankruptcy point endogenously. The firm's capital structure consists of equity and perpetual debt with constant continuous coupon payments. The equity price process derived by Leland (1994) is characterized by an endogenous stochastic volatility function that depends on a set of structural variables. Equity option prices must therefore also be determined by these structural variables. The use of this rich and endogenous equity price process distinguishes our article's theoretical contributions from existing literature.

We show that the asset value at which bankruptcy is declared affects both option values and option sensitivities. This has implications for the hedging of far out-of-the money call options. For example, if bankruptcy is triggered by a strict net-worth covenant, a call option's delta may not converge to zero as equity becomes worthless. Furthermore, the mechanism behind bankruptcy is important in determining the option's sensitivities. If bankruptcy is determined endogenously by equity holders, an increase in the asset volatility increases option values directly due to convexity, but also indirectly through an increase in equity values caused by a lower endogenous bankruptcy point.[5] This indirect effect does not exist when bankruptcy is determined exogenously.

The structural option pricing model presented in this article *explains* observed Black–Scholes implied volatility biases not only by leverage, but also by other capital structure characteristics such as debt covenants.[6] This distin-

stochastic volatility models developed by Johnson and Shanno (1987), Hull and White (1987), Scott (1987), Wiggins (1987), Stein and Stein (1991), and Heston (1993).

[3] For example, if the value of the firm's unleveraged assets follows a geometric Brownian motion, then the possibility of bankruptcy prevents a leveraged firm's equity value process from following a geometric Brownian motion. Consequently, arbitrage opportunities will exist if the Black–Scholes model is used to price equity options.

[4] A more recent article by Bensoussan, Crouhy, and Galai (1994) approximates stochastic equity volatility functions in an economy similar to that of Geske (1979).

[5] See Leland and Toft (1996) for a detailed discussion of the comparative statics of optimal bankruptcy triggers.

[6] Rubinstein's (1983) displaced diffusion model can also be calibrated to exhibit leverage effects.

guishes our model from those developed by Dupire (1994), Derman and Kani (1994), Rubinstein (1994), and Derman, Kani, and Chriss (1996), who construct price processes that *fit* all observed options prices.[7]

We test our model by comparing predicted cross-sectional pricing biases with observed biases in a large sample of firms. Other authors have investigated implied volatilities for different degrees of moneyness and time to maturity. See for example Macbeth and Merville (1979), Emanuel and Macbeth (1982), and Rubinstein (1985).[8] These studies focus on pricing differences for a *cross-section of options*. We investigate and explain structural differences in option prices using a large *cross-section of firms*. We use intraday bid and ask quotes for all liquid equity option contracts traded on the Chicago Board Options Exchange during 1993 and 1994 to show that implied volatility skews are indeed related to firm leverage.[9] Furthermore, the magnitudes of observed implied volatility skews correspond to those predicted by our model. These findings are related to Christie (1982), who analyzes *time-series* of leverage and stock price data to show that historical volatilities are positively related to financial leverage.[10] We, on the other hand, employ *cross-sectional* regressions of *implied volatility skews* on firm characteristics. This allows us to investigate other relationships predicted by our model. In particular, we show that options on equity in firms with large ratios of short-term debt to total debt exhibit significantly more pronounced volatility skews than those of firms with mainly long-term debt financing. This finding supports our model because short-term debt introduces a net-worth hurdle similar to that of a net-worth covenant.

The remainder of the article is organized as follows: Section I presents our assumptions and reviews the underlying capital structure theory. Section II develops and interprets the pricing formulas for options on leveraged equity. Section III presents the empirical evidence, and Section IV concludes the article.

I. Assumptions and Capital Structure Theory

Leland's (1994) model provides the foundation for the equity option pricing formulas. The value of the firm's assets is assumed to follow an Ito-process

$$dV = (\eta(V, t) - \delta)V\,dt + \sigma V\,dw(t), \tag{1}$$

[7] Recent evidence by Dumas, Fleming, and Whaley (1996) indicates that these models lack predictive power.

[8] Recently, Longstaff (1995) detected biases related to transactions costs and liquidity in the market for S&P 100 index options.

[9] Throughout the article we use the term "skew" to refer to the situation where observed implied Black–Scholes volatilities are larger for options with low striking prices than for options with high striking prices.

[10] Black (1976) and Schmalensee and Trippi (1978) find relationships between volatility changes and underlying asset price changes that are consistent with leverage effects. Franks and Schwartz (1991) find similar effects in the FTSE index using volatilities implied from index option prices. It is, however, difficult to determine causality from these studies. Perhaps it is a higher volatility that results in a lower equilibrium price, and not a lower equilibrium price that implies a higher volatility.

where $dw(t)$ is the increment of a standard Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, $\eta(V, t)$ is the instantaneous drift rate, δ is the proportional payout rate, and σ is the asset volatility. The value of the firm's assets is assumed to be independent of the capital structure. This is in tradition with the classic capital structure literature pioneered by Modigliani and Miller (1958), Merton (1974), Black and Cox (1976), and Brennan and Schwartz (1978).

The firm has issued debt that pays bond holders a continuous stream of coupon payments C per year. Bond covenants allow for liquidation of assets at a rate δ of the total value of the firm's assets. The payout $\delta V dt$ can be interpreted as the after-tax cash-flow available to both bond and equity holders. When the after-tax value of the coupon payments $(1 - \tau)C dt$ is deducted from this cash-flow, the remaining cash-flow is the net-payout, or dividends, received by equity holders. Using this interpretation, the payout rate δ can be determined so that the after-tax net payout to equity holders, $(\delta V - (1 - \tau)C)dt$, corresponds to an exogenously given dividend payment over the time interval dt.

Assuming no arbitrage opportunities, Leland (1994) derives the value of equity as

$$E(V) = V - A + B\left(\frac{V}{V_b}\right)^{-x} = V - A + Bp_b, \tag{2}$$

where

$$A = \frac{(1 - \tau)C}{r}, \qquad B = (A - V_b), \qquad x = \frac{m + \mu}{\sigma^2},$$

$$m = \sqrt{\mu^2 + 2r\sigma^2}, \qquad \mu = r - \delta - \frac{1}{2}\sigma^2, \qquad p_b = \left(\frac{V}{V_b}\right)^{-x}. \tag{3}$$

V_b is the value of the underlying assets at which equity holders are either forced or decide to declare bankruptcy, r is the riskfree rate of interest, and τ is the corporate tax rate.

The bankruptcy trigger V_b is either exogenously given by a net-worth covenant or it may be determined endogenously if it is within the equity holder's discretion to declare bankruptcy. The asset value at which bankruptcy is optimal for the equity holders is derived by Leland (1994) as

$$V_b = \frac{(1 - \tau)Cx}{r(x + 1)}. \tag{4}$$

This model of equity values implies that the volatility of equity is time independent, yet highly dependent on leverage: as the value of the firm's assets falls to a relatively low level, equity returns become highly volatile. For high asset values, the equity volatility does not change significantly with changes in

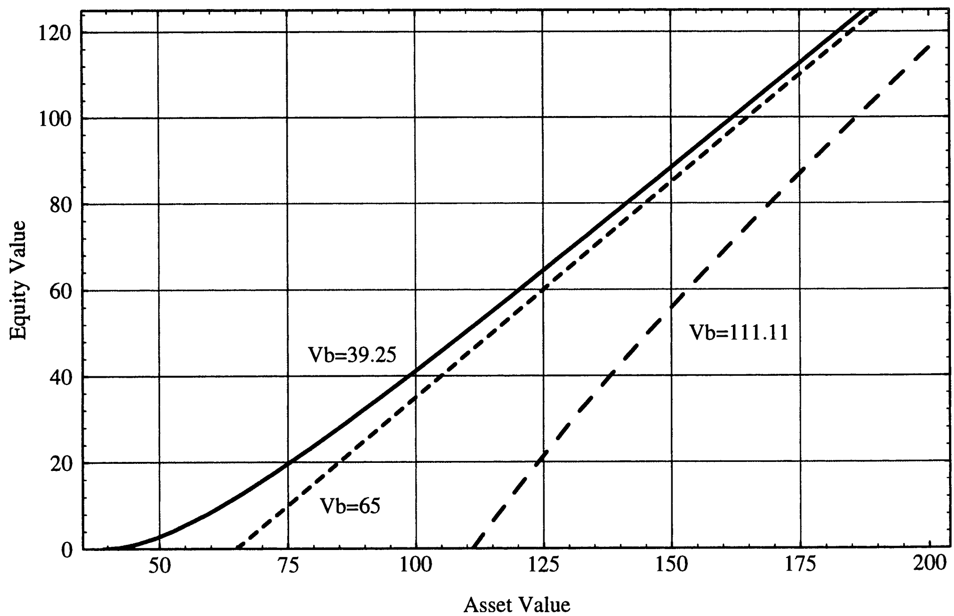

Figure 1. Equity values as functions of underlying asset value. This figure shows equity values as functions of underlying asset value for three different bankruptcy triggers: $V_b = C/(r(1 - \alpha)) = 111.11$ (strict net-worth covenant). $V_b = (1 - \tau)C/r = 65$, and $V_b = (1 - \tau)Cx/(r(x + 1)) = 39.25$ (endogenous bankruptcy). The following parameters are used: $C = 8$, $r = 0.08$, $\sigma = 0.2$, $\tau = 0.35$, $\alpha = 0.1$, and $\delta = (1 - \tau)C/100 + 0.03 = 0.082$.

equity values. This "leverage effect" has important implications for option prices. Furthermore, as we will see later, the exact structure of the equity value function, as determined by the level of the bankruptcy trigger, affects the pricing of stock options.

It is therefore useful to examine equity values as functions of the value of the firm's assets for various bankruptcy triggers. Figure 1 plots equity values as functions of the value of the firm's assets for an example with $C = 8$, $r = 0.08$, $\tau = 0.35$, $\delta = (1 - \tau)C/100 + 0.03 = 0.082$, and $\sigma = 0.2$ for three different bankruptcy trigger values. The payout rate $\delta = 0.082$ implies a 3 percent current dividend yield on equity when the value of the underlying assets is 100.

Figure 1 shows equity value functions that are concave, linear, and convex in the value of the firm's assets. The function labeled $V_b = 111.11$ plots the equity value of a firm where debt is protected by a strict net-worth covenant that effectively makes the debt riskfree. This covenant triggers bankruptcy when the value of the firm's assets hits $V_b = C/(r(1 - \alpha))$, where $\alpha = 0.1$ is a proportional bankruptcy cost levied on the firm's assets when V reaches V_b. This net-worth covenant results in a concave equity value function. An option interpretation can explain the concavity of this equity value function. The term $V - A$ in the equity value function (2) represents the value of the firm's unleveraged assets, less the after-tax value of the promised coupon payments.

This expression is linear in V. The term Bp_b is the value of a perpetual cash-at-hit option where the equity holders surrender the firm in exchange for being released from the obligation to honor all future coupon payments when the firm value $V = V_b$.[11] B is negative when $A < V_b$ and equity holders are essentially short a perpetual cash-at-hit option with a cash payment of $V_b - A$. It follows that equity is a concave function of the asset value.

The function labeled $V_b = 65$ shows the equity value as a linear function of the asset value. This structure is obtained when bankruptcy is triggered immediately when the asset value reaches $(1 - \tau)C/r$. We will subsequently show that the option pricing formulas in this special case simplify substantially. Finally, the function labeled $V_b = 39.25$ represents the equity value as a function of asset value when bankruptcy is declared optimally by equity holders. When V_b is determined endogenously, equity is a convex function that satisfies a smooth pasting condition at $V = V_b$.

We have shown that Leland's model is flexible enough to produce endogenous leveraged equity values that are either a convex or a concave function of asset value. This stands in sharp contrast to Black and Scholes' (1973) and Merton's (1974) classic interpretation of equity as a long call option on the firm's assets. This classic model restricts equity to be a convex function of the firm's assets. This, in turn, limits the flexibility one has to fit equity option prices to observed market data.

II. Options on Leveraged Equity

We now turn to the valuation of options written on leveraged equity. We assume that the equity value process is given by equation (2). Leland (1994) derives this process under the assumption that it is possible to trade a security with a price process that is perfectly correlated with the value of the firm's underlying assets. Because this economy is dynamically complete, we can also determine option values by calculating the discounted expected payoff under the equivalent martingale measure.[12]

A. Valuation

The value of a call option written on equity with default risk can be expressed as[13]

$$\text{CALL} = e^{-rT}\mathrm{E}^Q[\max[E(V(T)) - K,\ 0]1_{(s \geq T)}], \tag{5}$$

where s is the stopping time defined as the first time the value of the firm's assets reaches the bankruptcy trigger: $s = \inf\{t \geq 0 : V(t) \leq V_b\}$. E^Q is the

[11] The term p_b is the value of a perpetual cash-at-hit option with a payoff of \$1. This term can also be calculated as $p_b = \int_0^\infty f(t)e^{-rt}\,dt$, where $f(t)$ represents the first passage time density for V hitting V_b, where V follows a geometric Brownian motion with drift rate $r - \delta$ and volatility σ.

[12] See Harrison and Kreps (1979) and Harrison and Pliska (1981).

[13] Technically, this is an option on the firm. We can use homogeneity to obtain the value of an equity option on one share of stock if the option is not payout protected.

expectation under the equivalent martingale measure, $E(V(T))$ is the equity value expressed as a function of the asset value at time T, K is the striking price, and $1_{(\cdot)}$ is the indicator function. If $s < T$, equity becomes worthless prior to the option's expiration date. This implies an option value of zero. The expectation in equation (5) can be computed using techniques similar to those used to price barrier options.[14]

$$\begin{aligned}\text{CALL} = {} & Ve^{-\delta T}\left[N(y^* + \sigma\sqrt{T}) - \left(\frac{V_b}{V}\right)^{2\mu/\sigma^2+2} N\left(y^* + \sigma\sqrt{T} + \left(\frac{2b}{\sigma\sqrt{T}}\right)\right)\right] \\ & + B\left(\frac{V_b}{V}\right)^{x}\left[N(y^* - x\sigma\sqrt{T}) - \left(\frac{V_b}{V}\right)^{2\mu/\sigma^2-2x} N\left(y^* - x\sigma\sqrt{T} + \left(\frac{2b}{\sigma\sqrt{T}}\right)\right)\right] \\ & - (A + K)e^{-rT}\left[N(y^*) - \left(\frac{V_b}{V}\right)^{2\mu/\sigma^2} N\left(y^* + \left(\frac{2b}{\sigma\sqrt{T}}\right)\right)\right],\end{aligned} \tag{6}$$

where $N(\cdot)$ denotes the standard normal distribution function, and y^* is the smallest solution to the following equation in y:[15]

$$\begin{aligned}K = {} & V\exp[(r - \delta - \tfrac{1}{2}\sigma^2)T + \sigma\sqrt{T}(-y)] - A \\ & + B\left(\frac{V_b}{V}\right)^{x}\exp[-x(r - \delta - \tfrac{1}{2}\sigma^2)T - x\sigma\sqrt{T}(-y)],\end{aligned} \tag{7}$$

and

$$b = \ln\left(\frac{V_b}{V}\right).$$

The implicit equation (7) equates the striking price and equity value at the option's expiration date, or equivalently it is used to find the critical value of y where the call option expires exactly at-the-money.

If $A > V_b \geq (1 - \tau)Cx/(r(x + 1))$, there are two solutions to equation (7). However, the larger solution does not have any economic meaning. It can only be reached by crossing the bankruptcy boundary. To see this, note that when the bankruptcy trigger V_b is between the optimal bankruptcy trigger $(1 - \tau)Cx/(r(x + 1))$ and A, then B is positive. Outside the bankruptcy region, $E(V(T))$ is increasing in the value of the firm's assets, and therefore decreasing

[14] See Rubinstein and Reiner (1991) and Rich (1994). For standard barrier options, we must distinguish between cases where the striking price is above or below the barrier. For the compound down-and-out barrier option considered in this article, hitting the barrier implies an equity value of zero. Consequently, for options with $K \geq 0$, the barrier will always be at or below the asset value that equates the equity value and the striking price.

[15] When $V(T) \equiv V(0)\exp[(r - \delta - \tfrac{1}{2}\sigma^2)T + \sigma\sqrt{T}(-y)]$, $-y$ represents a standard normal random variable, measurable with respect to $\mathcal{F}_T$, which completely describes the value of the firm's asset at time T. We use the minus sign in this definition to avoid an excessive number of minus signs in the formula for the call option price.

in y. Hence, one searches for the smallest value of y that satisfies the implicit equation (7). If $V_b > A$, then B is negative. In this case the implicit equation has only one solution for y.

Although slightly more complicated, we see that the option pricing formula (6) is related to the Black-Scholes formula. The equity option's value is written as a function of the value of the firm's assets. The equity value (2) contains two stochastic terms that contribute to the option's payoff if it expires in-the-money. These two stochastic terms are equivalent to the first and third terms on the right-hand side of equation (7). The structure of the first term on the first line of formula (6) is similar to that of the first term in the Black–Scholes formula; it represents the value of the payoff from the first term on the right-hand side of the implicit equation (7), the value of the underlying assets, if the option expires in-the-money. The second term on the first line adjusts for the possibility of hitting the bankruptcy trigger. The second line of equation (6) represents the value of the payoff in the risk-neutral economy from the third term on the right-hand side of the implicit equation (7) in the case of exercise. This is the value of a security that pays the owner a perpetual cash-at-hit option with a payoff $(A - V_b)$ if the value of equity is higher than the option's striking price at the expiration date. Again, the second term on the second line of equation (6) adjusts for the possibility of bankruptcy. Finally, the last line of equation (6) is the value of the striking price and the after-tax riskfree promised debt payments, which are the option owner's liability if the option finishes in-the-money.

It is important to note the difference between the option pricing formula with and without endogenous bankruptcy. First, endogenous bankruptcy is declared when $V_b = ((1 - \tau)Cx)/(r(x + 1))$, which is less than the present value of the promised after-tax payments on the debt. Second, even if a net-worth covenant requires bankruptcy to be declared at $V_b = (1 - \tau)Cx/(r(x + 1))$, the option pricing formula with exogenously determined bankruptcy exhibits distinct differences from the pricing formula with endogenous bankruptcy. While option prices are the same regardless of the mechanism that triggers bankruptcy, their sensitivities to changes in the coupon payments, interest rate, proportional payout, volatility, and corporate tax rate are different. For example, an increase in the corporate tax rate has several effects when bankruptcy is determined endogenously by equity holders. First, the equity value changes simply because the value of the tax shield changes. Second, bankruptcy will be declared at a lower asset value. This lower bankruptcy point increases equity values and thus influences option values. An increase in the volatility σ of the firm's assets affects the option value in similar ways. First, a change in the option value follows directly from the change in equity value given by equation (2). In addition, bankruptcy is declared at a lower asset value, which produces a secondary change in the equity value. Similar interpretations hold for changes in the riskfree rate of interest r and the proportional payout rate δ.

The call option pricing formula simplifies to well known formulas in two special cases. First, if the bankruptcy point is determined by a net-worth covenant that requires the firm's assets to be worth at least $V_b = (1 - \tau)C/r$, then the call option pricing formula (6) simplifies to the Black–Scholes type formula for a down-and-out call option, where the option is written on the underlying asset value, has a striking price of $A + K$, and a knock-out boundary at $V_b = A$,[16]

$$\mathrm{CALL} = Ve^{-\delta T}\left[N(z_1) - \left(\frac{A}{V}\right)^{2\mu/\sigma^2+2} N\left(z_1 + \left(\frac{2b}{\sigma\sqrt{T}}\right)\right)\right] - (A+K)e^{-rT}\left[N(z_2) - \left(\frac{A}{V}\right)^{2\mu/\sigma^2} N\left(z_2 + \left(\frac{2b}{\sigma\sqrt{T}}\right)\right)\right], \quad (8)$$

where

$$z_1 = \frac{\ln(V/(K+A)) + (r - \delta + {}^1\!/\!_2\sigma^2)T}{\sigma\sqrt{T}},$$

and

$$z_2 = z_1 - \sigma\sqrt{T}.$$

This special case simplifies because the equity value is a linear function of the asset value above the knock-out barrier.

Second, as $C \to 0$ the call value (6) converges to Merton's (1973) version of the Black–Scholes formula,

$$\mathrm{CALL} = Ve^{-\delta T}N(z_1) - Ke^{-rT}N(z_2), \quad (9)$$

where $A = 0$ in the expressions for z_1 and z_2. The formulas (6), (8), and (9) yield different degrees of flexibility to explain observed market prices. The Black-Scholes formula (9) is the least flexible model. Formulas (8) and (6) add layers of flexibility that are related to the firm's structural characteristics.

Pricing and hedging characteristics of models (6) and (8) are illustrated in Figures 2 and 3. These figures show call option values and deltas as functions of equity value for three different bankruptcy triggers: $V_b = (1 - \tau)Cx/(r(x + 1)) = 39.25$ (endogenous bankruptcy), $V_b = (1 - \tau)C/r = 65$, and $V_b = C/(r(1 - \alpha)) = 111.11$ (strict net-worth covenant). The striking price is 40 and the options expire in 6 months.

Option values are calculated as functions of equity values by inverting equation (2) which yields V as a function of E for $E > 0$. The resulting asset values, along with the remaining parameters, are then used in the call option pricing formulas (6) and (8). The functions plotted in Figures 2 and 3 correspond to the equity value functions in Figure 1. Note that the value of a call option on equity in a firm with exogenous bankruptcy at $V_b = 65$ or 111.11 is higher than the value of a call option on equity in a firm with an endogenous bankruptcy trigger. This phenomenon can be explained by the higher equity

[16] See for example Cox and Rubinstein (1985).

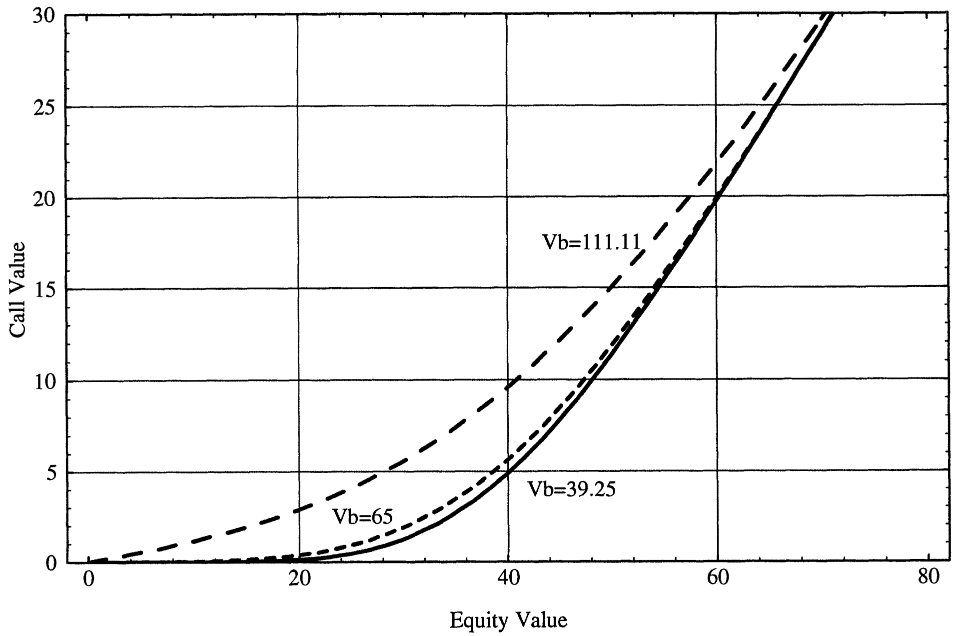

Figure 2. Call option values as functions of equity value. This figure shows call option values as functions of equity value for three different bankruptcy triggers: $V_b = C/(r(1 - \alpha)) = 111.11$ (strict net-worth covenant), $V_b = (1 - \tau)C/r = 65$, and $V_b = (1 - \tau)Cx/(r(x + 1)) = 39.25$ (endogenous bankruptcy). The following input parameters are used: $C = 8$, $r = 0.08$, $\sigma = 0.2$, $\tau = 0.35$, $\alpha = 0.1$, $\delta = (1 - \tau)C/100 + 0.03 = 0.082$, $K = 40$, and $T = 0.5$.

volatility of firms with larger bankruptcy triggers. The higher volatility is caused by two effects. First, given higher asset values, a constant σ implies larger absolute changes in asset values. Second, the larger partial derivative of the equity value function with respect to asset value translates the higher asset volatility into an even higher equity volatility.

Figure 3 illustrates the impact a correctly modeled capital structure has on an equity option's hedge parameters. For example, if the firm's debt is subject to a strict net-worth covenant ($V_b = 111.11$), the call option delta does not converge to zero as the value of equity goes to zero. This nonstandard behavior follows from the characterization of equity as a perpetual down-and-out option.

Above, we concentrated on the valuation and analysis of call options. Put option values can be derived from put-call parity. However, due to the possibility of bankruptcy great care must be taken when accounting for dividend payments between the present and the option's expiration date. The value of a call option and a zero-coupon bond with a face value equal to the striking price must equal the value of a put option plus a contract that delivers equity at the option's expiration date. The value of this contract is identical to the value of a call option with a striking price of zero. Consequently, we can determine the value of a put option from the value of the corresponding call option plus the

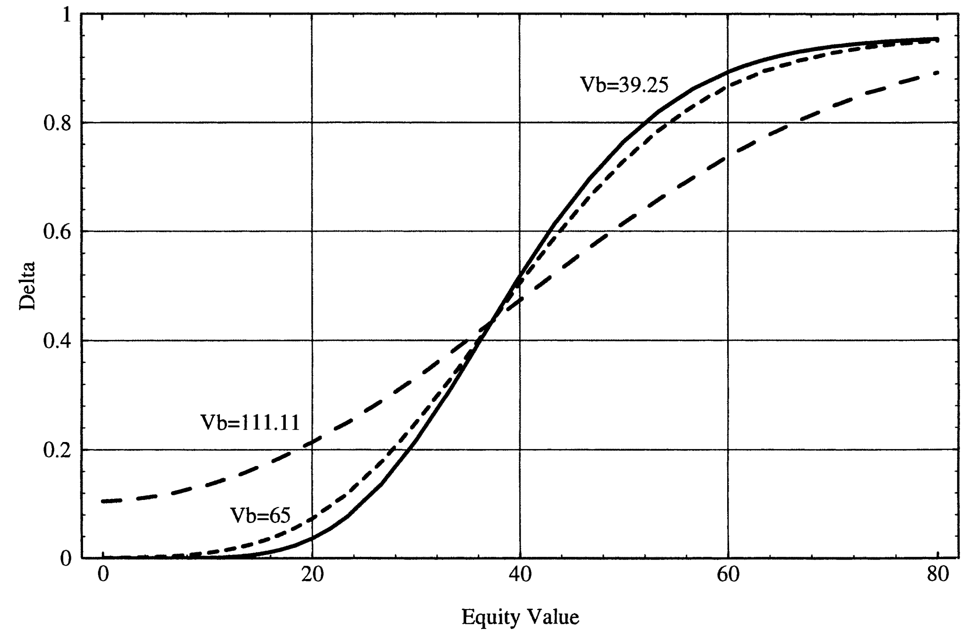

Figure 3. Call option deltas as functions of equity value. This figure shows call option deltas as functions of equity value for three different bankruptcy triggers: $V_b = C/(r(1 - \alpha)) = 111.11$ (strict net-worth covenant), $V_b = (1 - \tau)C/r = 65$, and $V_b = (1 - \tau)Cx/(r(x + 1)) = 39.25$ (endogenous bankruptcy). The following input parameters are used: $C = 8$, $r = 0.08$, $\sigma = 0.2$, $\tau = 0.35$, $\alpha = 0.1$, $\delta = (1 - \tau)C/100 + 0.03 = 0.082$, $K = 40$, and $T = 0.5$.

present value of the striking price minus the value of a call option with a striking price of zero.

B. Comparison to Black-Scholes Values

It is instructive to compare option values derived from our model to the corresponding Black–Scholes values. Given that the proposed model for options on leveraged equity and the Black–Scholes model are functions of different but closely related parameters, we must construct these to be mutually consistent.

First, we choose six different sets of input parameters that represent a variety of capital structures: 20 percent, 40 percent, and 60 percent leveraged firms with either unprotected (endogenous bankruptcy declared by equity holders) or protected debt (bankruptcy triggered by a strict net-worth covenant). Firms with unprotected debt declare bankruptcy at $V_b = (1 - \tau)Cx/(r(x + 1))$, whereas firms with protected debt are forced into bankruptcy when the firm value reaches $V_b = C/(r(1 - \alpha))$. The payout rate δ is chosen so that the expected dividend to equity holders during the next month has a value equiv-

Table I

Parameter Values for Leveraged Capital Structures

This table shows parameter values for 20 percent, 40 percent, and 60 percent leveraged capital structures. Leverage is defined as $D/(D + E)$, where E is the market value of equity, and D is the market value of all outstanding debt. D is determined using the pricing result in Leland (1994). The parameters $V = 100$, $r = 0.08$, $\sigma = 0.2$, $\alpha = 0.1$, and $\tau = 0.35$ are identical for all leverage ratios. The asset liquidation rate, δ, is determined so that the dividend payable to equity holders during the next month is equivalent to a 3 percent dividend yield on equity. Bankruptcy is determined endogenously at $V = V_b = (1 - \tau)Cx/(r(x + 1))$ when debt is unprotected. Bankruptcy is triggered at $V = V_b = C/(r(1 - \alpha))$ when debt is protected by a strict net-worth covenant.

	Unprotected Debt			Protected Debt		
Leverage	20%	40%	60%	20%	40%	60%
C	1.72	3.77	6.42	1.72	3.58	5.22
δ	3.70%	4.54%	5.66%	3.69%	4.34%	4.69%
V_b	10.17	21.61	35.24	23.84	49.76	72.48
E	86.01	69.61	49.79	85.83	67.17	43.49
D	21.50	46.41	74.69	21.46	44.78	65.24

alent to a 3 percent dividend yield.[17] Parameter values for the six capital structures are shown in Table I.

Second, we use the parameters in Table I to compute prices of options on leveraged equity with striking prices ranging from 80 percent to 120 percent of the equity price, and with maturities of two weeks, one month, three months, and six months. Assume for the following analysis that these prices are the theoretically correct option prices.

Third, we convert the parameters in Table I into input variables used in the Black–Scholes formula. Equation (2) determines the equity value. The dividend yield, d, used in the Black–Scholes formula is identified by solving the equation $F = Ee^{(r-d)T}$ for d, where F is the forward price of equity with delivery at time T.[18] This choice of d guarantees that values of far-in-the-money options computed using the Black–Scholes model equal those computed by our model. For the one-month option, d equals 3 percent. For other maturities d differs slightly from 3 percent. The remaining parameters (time to maturity, riskfree interest rate, and striking price) require no conversion.

We now find the implied volatilities that equate the Black–Scholes option prices with the theoretically correct option prices. To facilitate comparisons across striking prices and times to maturity, we divide the matrix of implied volatilities by the implied volatility of the one-month at-the-money option.

[17] Note that this payout rate is close to, but not necessarily identical to $(1 - \tau)C/V + 0.03$. When bankruptcy is imminent, the computed payout rate may be quite different from $(1 - \tau)C/V + 0.03$. This is caused by the high likelihood of bankruptcy.

[18] This forward price is computed as $\text{CALL}e^{rT}$, where CALL is the value of a call option with a striking price of zero.

Table II

Relative Implied Black–Scholes Volatilities as Functions of Moneyness and Time to Expiration

This table shows relative implied Black–Scholes volatilities as functions of moneyness (defined in this table as striking price divided by equity price) and time to expiration for 20 percent, 40 percent, and 60 percent leveraged firms with protected and unprotected debt. Leverage is defined as $D/(D + E)$, where E is the market value of equity and D is the market value of all outstanding debt. D is determined using the pricing result in Leland (1994). Bankruptcy is determined endogenously when the firm's debt is unprotected. Bankruptcy is triggered at $V = V_b = C/(r(1 - \alpha))$ when debt is protected by a strict net-worth covenant. The relative implied Black–Scholes volatility is defined as the volatility implied from the Black-Scholes formula when the option price is determined by the proposed model for options on leverage equity, divided by the corresponding implied volatility for the one-month at-the-money option. Other input parameters are shown in Table I.

	Unprotected Debt				Protected Debt			
Moneyness	2 Weeks	1 Month	3 Months	6 Months	2 Weeks	1 Month	3 Months	6 Months
				20 Percent Leverage				
80%	1.017	1.017	1.016	1.015	1.021	1.020	1.020	1.019
90%	1.008	1.008	1.007	1.006	1.009	1.009	1.009	1.008
100%	1.000	1.000	1.000	0.999	1.000	1.000	1.000	0.999
110%	0.994	0.994	0.993	0.992	0.993	0.993	0.992	0.992
120%	0.988	0.988	0.988	0.987	0.986	0.986	0.986	0.985
				40 Percent Leverage				
80%	1.034	1.034	1.033	1.031	1.069	1.069	1.070	1.071
90%	1.016	1.016	1.014	1.013	1.031	1.031	1.031	1.032
100%	1.000	1.000	0.999	0.997	1.000	1.000	1.000	1.001
110%	0.987	0.987	0.986	0.984	0.974	0.974	0.975	0.975
120%	0.975	0.975	0.974	0.973	0.953	0.953	0.953	0.953
				60 Percent Leverage				
80%	1.049	1.048	1.046	1.042	1.111	1.112	1.116	1.123
90%	1.023	1.022	1.020	1.017	1.050	1.051	1.055	1.061
100%	1.000	1.000	0.998	0.995	0.999	1.000	1.003	1.009
110%	0.981	0.981	0.979	0.976	0.955	0.956	0.959	0.964
120%	0.964	0.964	0.962	0.959	0.917	0.918	0.920	0.925

Table II shows the resulting six matrices of relative implied Black–Scholes volatilities.[19] It illustrates several interesting results: First, the implied volatility structure is more negatively skewed for highly leveraged firms than for moderately leveraged firms. This can be explained by leverage effects—as the value of the firm's assets decreases, equity volatility increases. This makes in-the-money call options and out-of-the-money put options relatively more valuable. On the other hand, as the asset value increases, the equity volatility decreases. This makes out-of-the-money call and in-the-money put options

[19] We conducted similar calculations for other dividend yield, interest rate, and asset volatility assumptions. Alternative parameter values result in similar conclusions.

relatively less valuable. We summarize these observations by our first testable hypothesis:

LEVERAGE EFFECT HYPOTHESIS: *The volatility skew is negatively related to leverage, ceteris paribus.*

Second, the implied volatility structure of equity options on a firm with protected debt is more negatively skewed than that of options on equity in a firm with unprotected debt. This can be explained by returning to Figure 1. The value of equity in a firm with unprotected debt satisfies a smooth pasting condition as $E \rightarrow 0$. This implies a decreasing absolute equity volatility as $E \rightarrow 0$. This decrease in the absolute volatility partially offsets the increase in the relative volatility caused by leverage effects. Consequently, the volatility skew becomes less pronounced. However, with protected debt in the capital structure, a change in the value of the underlying assets translates into an even larger change in the equity value when V is close to V_b. The relative volatility is therefore amplified both by leverage and by an increased absolute volatility. We hypothesize that there are significant differences in the volatility skews of options on equity in firms restricted by net-worth covenants. This is formalized in our second testable hypothesis:

COVENANT EFFECT HYPOTHESIS: *The volatility skew is more pronounced (more negative) for options on equity in a firm where debt is protected by net-worth covenants than for options on equity in a firm with debt without these covenants, ceteris paribus.*

Third, Table II allows us to compare the effect of net-worth covenants for firms with different leverage ratios. It illustrates that protective covenants have little impact on the anatomy of the volatility skew when the firm's leverage is low. Options on equity in firms with high leverage ratios, on the other hand, exhibit significantly more pronounced implied volatility skews when debt is protected by a net-worth covenant. We formalize these findings in our third testable hypothesis:

STRATIFIED COVENANT EFFECT HYPOTHESIS: *Differences in volatility skews caused by net-worth covenants are more pronounced for options on equity in highly leveraged firms than for options on equity in firms with low leverage, ceteris paribus.*

Finally, note that the magnitudes of the theoretical volatility skews are relatively small. For example, a 10 percent in-the-money call option on a 40 percent leveraged firm with protected debt has an implied volatility that is approximately 6 percent larger than that of a 10 percent out-of-the money call option. For a firm with a 40 percent at-the-money volatility, the implied volatility of the in-the-money option would be approximately 41.2 percent while the out-of-the-money option's implied volatility would be approximately 38.8 percent. We will subsequently see that implied volatility skews of this magnitude were indeed observed during 1993 and 1994.

III. Empirical Evidence

We test our hypotheses by regressing observed implied volatility skews in a cross-section of firms on structural characteristics such as leverage and a proxy for net-worth covenants. We use all firms with actively traded and quoted option contracts listed on the Chicago Board Options Exchange in 1993 and 1994. We test the leverage effect hypothesis by regressing a measure of the implied volatility skew on a measure of financial leverage:

$$\text{Volatility Skew}_{it} = a + b_l \text{Leverage}_{it} + \varepsilon_{it}, \qquad \forall i,\ t,$$

where i and t index the firm and fiscal year end, respectively. Our definition of the volatility skew measures the relative difference between the implied volatility of an option with a high striking price and the implied volatility of an option with a low striking price.[20] The null hypothesis (Black–Scholes) predicts that no relationship exists between leverage and implied volatility skews, that is, $b_l = 0$. The leverage effect hypothesis, our alternative, predicts a negative relationship, that is, $b_l < 0$. We expect a negative sign because higher leverage makes the relative difference between the implied volatility of a high striking price option and that of a low striking pricing option more negative.

We use a similar regression to test the covenant effect hypothesis. We regress observed implied volatility skews on a proxy for net-worth covenants:

$$\text{Volatility Skew}_{it} = a + b_c \text{Covenant Proxy}_{it} + \varepsilon_{it}, \qquad \forall i,\ t.$$

The null hypothesis (Black–Scholes) predicts no relationship between the covenant proxy and the implied volatility skew, $b_c = 0$. Our alternative, the covenant effect hypothesis, predicts a negative relationship, $b_c < 0$. The covenant effect hypothesis predicts a negative relationship because a larger proxy value for net-worth covenants results in a more negatively skewed implied volatility structure. Finally, we investigate the stratified covenant effect hypothesis by allowing the slope coefficient of the covenant proxy to differ for firms with low and high leverage ratios. The stratified covenant effect hypothesis predicts that the slope coefficient is more significant (and more negative) for highly leveraged firms than for firms with low leverage.

A. Construction of Proxies for the Independent Variables

While the proposed regressions are straightforward, care must be taken when constructing both the dependent and independent variables. The first explanatory variable is leverage. We define the leverage variable, LEV, as total debt plus preferred stock, divided by the sum of total debt, preferred stock, and the market value of outstanding common equity. Debt and preferred stock are

[20] Exact definitions and the construction of both the implied volatility skew and the leverage measures are described in detail in the following sections.

valued at book. We thus assume that book values of both debt and preferred stock are adequate proxies for market values.[21]

It is more complicated to construct a proxy for protective net-worth covenants using balance sheet data. Fortunately, the maturity structure of the firm's debt can be used as a proxy for the existence of net-worth hurdles. Leland (1994) argues that short-term debt can be associated with an exogenous bankruptcy trigger V_b that equals the market value of debt on the issue date. Leland and Toft (1996) show that it may even be optimal for equity holders to default on short-term debt at an asset value which is larger than the debt's face value. Long-term debt, on the other hand, results in an endogenous bankruptcy point which is significantly below this value. Intuitively, this indicates that firms with a large proportion of debt due in the immediate future must pass a net-worth hurdle. Otherwise they are unable to roll over their debt (or renew their line of credit). Firms primarily financed by long-term debt need not overcome such a strict net-worth hurdle. We therefore use the ratio of long-term debt due in one year plus notes payable to total debt as our covenant proxy, CVNT.

B. Data and Construction of the Dependent Variable

The data set for this study is obtained by merging information from the Berkeley Options Data Base for the years 1993 and 1994 with data from Standard and Poor's COMPUSTAT database, the Center for Research in Security Prices (CRSP), Standard and Poor's Dividend Record, and The Federal Reserve Bank of New York.

We identify a total of 319 firms with options listed on the Chicago Board Options Exchange that trade at least once a month during both 1993 and 1994. For each of these firms we identify all 10-minute time intervals with a set of simultaneous option quotes. A set of simultaneous quotes is recorded when bid and ask quotes are posted for the two nearby call and the two nearby put options with striking prices bracketing the most recently updated stock price.[22] We then select all firms with at least one set of simultaneous option quotes during each calendar month. This liquidity criterion reduces the sample to 186 firms.

From this sample of 186 firms we eliminate all firms with no or incomplete data in the CRSP and COMPUSTAT databases (6 firms). We exclude all foreign firms (13 firms) and firms with primary business activities in finance, insurance, and real estate (21 firms). We also exclude firms with fiscal years ending in October and November (2 firms). We exclude these two firms because we do not have option data for a period of 13 weeks immediately following these firms' fiscal year ends in 1994. Finally, we exclude firms with stock splits or large dividend payments that make it impossible to determine the appropriate striking price adjustments (6 firms). This exclusion is necessary because

[21] This is a standard assumption in the empirical corporate finance literature (see for example Titman and Wessels (1988) and Opler and Titman (1995)).

[22] We use the second nearby contract if the nearby contract has one week or less to expiration.

the adjusted striking price (due to extraordinarily large dividends or splits) cannot always be recovered from the Berkeley Options Data Base.

Our procedure thus identifies 138 firms with both relatively liquid listed option contracts and balance sheet data available from COMPUSTAT. We compute the leverage variable, LEV, and the covenant proxy, CVNT, as defined above, for each of these 138 firms using the two fiscal years ending in the period from December 1992 through October 1994. We then exclude observations with leverage ratios of less than 10 percent. We exclude these observations because our theory cannot identify a relationship between the covenant proxy and the implied volatility skews for firms with low leverage ratios.[23]

Next, we collect sets of simultaneous bid and ask option quotes, as defined above, for all firms in our sample for each 10-minute interval between 9:30 A.M. and 3:00 P.M. Central Time for a period of 13 weeks following the firms' fiscal year ends. We exclude quotes time-stamped earlier than 9:30 A.M. because we suspect that the opening rotation has a significant effect on the quoted prices, and we exclude bid-ask quotes after 3:00 P.M. because the major U.S. equity markets close at 3:00 P.M. Central Time.[24] We also excluded all observations containing zero quotes and quotes where the average of the bid and the ask violates static arbitrage bounds.[25]

Our selection criteria yield a set of nearly simultaneous arbitrage-free quotes, if any, for four option contracts for each 10-minute interval. We calculate one implied volatility for each of these four contracts using the average of the bid and the ask quotes.[26] Because all options in our sample are of the American type, we must use a valuation procedure that allows for optimal early exercise. We use a 100-step binomial model with a Black–Scholes modification and a two-step Richardson extrapolation as proposed by Broadie and Detemple (1996).[27] Broadie and Detemple's method is modified to accom-

[23] The choice of a 10 percent leverage ratio reflects a tradeoff between the desire to have as many data points as possible, and our theory that predicts cross-sectional differences for leveraged firms. If we restrict ourselves to firms with leverage ratios of more than 20 percent, then the estimated relationships are stronger than those we report subsequently.

[24] Chan, Chung, and Johnson (1995) show that bid-ask spreads for S&P 100 index options are relatively wide during the first hour of trading, and relatively narrow during the final 15 minutes of trading. This leads us to suspect that a similar relationship exists for individual equity options.

[25] Bids of zero occur infrequently and only for out-of-the-money options with low volatility and short time to maturity. Implied volatilities calculated from the average of two quotes of which one is zero tend to be significantly lower than implied volatilities calculated from quotes not including a zero. We do not believe a zero quote reflects a price. Furthermore, the deletion of these quotes should not bias our estimate of the volatility skew because they occur for both out-of-the-money call and put options.

[26] We do not use transaction prices because the bid-ask bounce (Roll (1984)) introduces both bias and noise in the implied volatility estimate. Furthermore, due to price impact, transaction prices may be affected by the trade size.

[27] Broadie and Detemple (1996) show that this method is about two orders of magnitudes faster than a standard binomial tree with similar precision when dividends are paid at a constant rate. Values computed on a 100 step binomial tree with these modifications are as precise as those computed on a 1000 step standard binomial tree. Our numerical experiments show similar improvements for a binomial model with lump sum dividends.

modate lump-sum dividends.[28] Dividend information is obtained from the CRSP 1994 stock files and Standard and Poor's Dividend Record. We assume that option market participants have perfect foresight about future dividends. This assumption is likely to be correct because the sample only consists of short-term options. Interest rate data are obtained from the Federal Reserve Bank of New York. We use a daily series of 3-month T-bill rates in our option pricing model.[29]

The resulting implied volatilities are initially computed on a calendar day basis and subsequently converted into a trading day basis.[30] We then compute one implied volatility for each of the two striking prices by averaging the implied volatility of the call and the put option. As discussed in Whaley (1993) and Fleming, Ostdiek, and Whaley (1995), this procedure reduces biases resulting from both stale stock price information and a misspecified riskfree interest rate.

The implied volatilities for the two different striking prices are weighted to obtain implied volatilities for three standardized options: a 10 percent in-the-money, an at-the-money, and a 10 percent out-of-the-money option. We use a weighting scheme similar to the one proposed by Cox and Rubinstein (1985), and subsequently used by Whaley (1993):

$$\sigma_{is,x\%} = \sigma_{is,K_b}\left(\frac{K_a - S_{is}(1 + x\%)}{K_a - K_b}\right) + \sigma_{is,K_a}\left(\frac{S_{is}(1 + x\%) - K_b}{K_a - K_b}\right),$$

where $\sigma_{is,x\%}$ is an index constructed to measure the implied volatility of an option on firm i in the 10-minute interval indexed by s and a striking price which is x percent larger than the most recently reported stock price S_{is}. The terms σ_{is,K_b} and σ_{is,K_a} are the implied volatilities of options with a striking price below, K_b, and a striking price above, K_a, the underlying asset price, respectively.[31]

[28] We use the method outlined by Hull (1997).

[29] We do not know if the 3-month T-bill rate is higher or lower than the rate used by market participants to price options. If the 3-month rate is higher (lower) than market participants' financing rate then the implied call option volatilities will be biased upwards (downwards). Implied volatilities for put options are biased in the opposite direction. Averaging call and put implied volatilities reduces any potential implied volatility bias.

[30] This conversion is necessary because equity volatilities over nontrading days such as weekends are significantly lower than during trading days (see French and Roll (1986)). Omitting this conversion would introduce a significant day-of-the-week effect into estimated implied volatilities.

[31] If the striking price x percent above the current value of the underlying asset is outside the range $[K_b, K_a]$, this procedure extrapolates implied volatilities from the implied volatilities of options with striking prices K_b and K_a. We thereby avoid using far out-of-the-money and far in-the-money options, which tend to be illiquid and have implied volatilities that are highly sensitive to pricing errors caused by imperfections.

Table III

Regression Tests of the Leverage and the Covenant Effect Hypotheses

This table reports the results of regressing the median implied volatility skew, MSKEW (observed during the 13-week period following the firm's fiscal year end), on the leverage variable (LEV) and the covenant proxy (CVNT). LEV equals total debt plus preferred stock divided by the sum of total debt, preferred stock, and the market value of outstanding common equity. CVNT is the ratio of long-term debt due in one year plus notes payable to total debt. In Panel A (B) we include observations with median skews computed from at least 10 (100) individual volatility skews (Sobs $\geq$ 10 (100)). Nobs is the number of firm years with available data. The numbers in parentheses below the estimated coefficients are t-statistics. The numbers in parentheses below the F-statistics are p-values. One-sided tests are conducted for the leverage variable and the covenant proxy.

Dependent Variable	Intercept	LEV	CVNT	Nobs	F-Stat	Adj. R^2
			Panel A: Sobs $\geq$ 10			
MSKEW	0.0265	−0.1065**		138	5.561*	3.2%
	(1.697)	(−2.358)			(0.020)	
	0.0120		−0.1013**	136	6.913**	4.2%
	(1.129)		(−2.629)		(0.010)	
	0.0348*	−0.0824*	−0.0899*	136	5.156**	5.8%
	(2.120)	(−1.811)	(−2.322)		(0.007)	
			Panel B: Sobs $\geq$ 100			
MSKEW	0.0373	−0.2188**		53	13.525**	19.4%
	(1.846)	(−3.678)			(0.001)	
	0.0099		−0.1589**	53	11.844**	17.3%
	(0.661)		(−3.442)		(0.001)	
	0.0486*	−0.1681**	−0.1157**	53	10.624**	27.0%
	(2.465)	(−2.796)	(−2.513)		(0.000)	

** and * denote significance at the 1 percent and 5 percent level, respectively.

We use the implied volatilities of these three standardized options to construct our measure of the volatility skew. One observation of the implied volatility skew at time s for firm i is defined by

$$\mathrm{SKEW}_{is} = \frac{\sigma_{is,10\%} - \sigma_{is,-10\%}}{\sigma_{is,0\%}}.$$

Finally, for each firm i in our sample, we compute the median volatility skew, MSKEW_{it}, for the 13-week period immediately following the firm's fiscal year end t. This yields a set of median volatility skews that we pair with the explanatory variables obtained from COMPUSTAT.[32]

[32] We use median volatility skews because average skews are likely to be biased by outliers.

C. Regression Results

Table III reports the results of regressing the median implied volatility skew (MSKEW) on the leverage variable (LEV) and the covenant proxy (CVNT). These regressions test the leverage and the covenant effect hypotheses.[33]

In these and subsequent regressions we exclude median volatility skews computed from less than 10 (or 100) observed volatility skews, that is, $\text{Sobs}_{it} < 10$ (or $\text{Sobs}_{it} < 100$), where Sobs_{it} is the number of observed implied volatility skews for firm i in the 13-week period following the firm's fiscal year t. We exclude these observations because we expect the medians computed from few observations to be relative noisy estimates of true implied volatility skews. This would, in turn, reduce the power of our statistical tests.

The slope coefficient on the leverage variable LEV is significantly less than zero at the 1 percent level in both univariate regressions on the leverage variable LEV. We notice that the t-statistic is more significant when volatility skews are estimated from at least 100 individual implied volatility skews (Panel B, Sobs $\geq$ 100). The increased significance is accompanied by an increase in the regression's explanatory power.

We can also interpret the magnitude of the regression coefficients: consider the regression involving implied volatility skews computed from at least 10 individual skew observations (Panel A, Sobs $\geq$ 10). The regression coefficient on the leverage variable equals approximately -10 percent. For a 40 percent leveraged firm this indicates that a 10 percent out-of-the-money call option has an implied volatility that is approximately 4 percent less than that of a 10 percent in-the-money call option.[34] The magnitude of this estimate corresponds closely to that predicted by the numerical experiments in Table II. Table II showed that the corresponding theoretical implied volatility skew is approximately -3 percent for equity options on a 40 percent leveraged firm with unprotected debt while the theoretical skew is -6 percent if the firm's debt is protected by net-worth covenants. These numbers bracket those observed in our sample with Sobs $\geq$ 10. When Sobs $\geq$ 100, the regression coefficient doubles. While the magnitude of this coefficient is larger than that predicted using our theory, we nevertheless believe that the results presented in Table III support the leverage effect hypothesis.

We now turn to testing of the covenant effect hypothesis. The regression coefficient of the covenant proxy is significantly less than zero at the 1 percent level in both univariate regressions involving the covenant proxy, CVNT. The estimated coefficient on the covenant proxy equals -10 percent when the implied volatility skews are calculated from at least 10 individual implied volatility skews (Panel A, Sobs $\geq$ 10). This indicates that a 10 percent out-of-the-money call option on equity in a firm with a covenant proxy equal to 1 (all debt is short-term debt) has an implied volatility that is approximately 10

[33] We also performed a full set of regressions using the average of the implied volatility skews for the 13-week period immediately following the firm's fiscal year. The results are similar to those presented in Tables III through V.

[34] Note that this number is a percentage of the volatility.

percent less than that of a 10 percent in-the-money call option, ceteris paribus. Our theory predicts that this difference should be approximately −3 percent for a 40 percent leveraged firm and −5.5 percent for a 60 percent leveraged firm. The average leverage ratio of the firms in our sample with Sobs $\geq$ 10 equals 30 percent. This indicates that the magnitudes of the estimated coefficients are larger than those predicted by our theory. This difference suggests that our covenant proxy also captures other important explanatory variables that are not explicitly incorporated into our model.

Table III also reports the results of regressing the implied volatility skews on both the leverage variable and the covenant proxy. The covenant proxy remains significant at the 1 percent level in one of the regressions. In the other regression it is now significant at the 5 percent level. The leverage variable is also significant at the 1 percent level in one of the two regressions. In the remaining multivariate regression, the leverage variable is significant at the 5 percent level. The adjusted R^2s are larger in these multivariate regressions than in the corresponding univariate regression. This indicates that both the leverage and the covenant proxy provide separate explanations for the cross-sectional differences in implied volatility skews. Overall we interpret the evidence presented in Table III to support both the leverage and the covenant effect hypotheses.

We now turn to testing the stratified covenant effect hypothesis. We test this hypothesis by estimating regressions in which we allow the intercept and the slope coefficient on the covenant proxy to differ for firms with high and low leverage. These tests are reported in Table IV.

The independent variable D1 in Table IV is a dummy variable containing 1 if the leverage ratio is greater than the median leverage ratio, and 0 otherwise. The variable D1CVNT (D2CVNT) contains the covenant proxy if the leverage ratio is greater than (less than or equal to) the median leverage ratio, and zero otherwise. Both Panels A and B in Table IV report four regressions. Regression (1) in Panels A and B are identical to the two univariate regressions on CVNT reported in Table III. They are included to facilitate comparisons. The second regression allows for different intercepts for firms with high and low leverage. The third regression allows for different slope coefficients on the covenant proxy while maintaining the same intercept. Finally, the fourth regression allows for both intercepts and slopes to differ for firms with high and low leverage.

As predicted by the stratified covenant effect hypothesis, Table IV shows large differences in the slope coefficients on the covenant proxy for firms with high and low leverage ratios. Indeed, regressions (3) and (4) in both Panels A and B show that the coefficient on the covenant proxy for firms with low leverage, D2CVNT, is never significant. This result is consistent with the stratified covenant effect hypothesis because the firms in the low leverage group all have leverage ratios of 25 percent or less. For leverage ratios this low, our model predicts that net-worth covenants have only a small effect on the anatomy of the volatility skew. In contrast, the coefficient on the covenant

Table IV

Regression Tests of the Stratified Covenant Effect Hypothesis

This table reports the results of regressing the median implied volatility skew (MSKEW) on the covenant proxy when the slope and intercept coefficient are allowed to differ for firms with high and low leverage ratios. D1 is a dummy variable containing 1 if the firm's leverage ratio is above the median value of the leverage ratios and zero otherwise. D1CVNT (D2CVNT) contains the covenant proxy CVNT if the firm's leverage ratio is above (at or below) the median value of the leverage ratios and zero otherwise. CVNT is the ratio of long-term debt due in one year plus notes payable divided by total debt. Nobs is the number of firm years with available data. RSS is the residual sum of squares. The numbers in parentheses below the estimated coefficients are t-statistics. The numbers in parentheses below the F-statistics are p-values. One-sided tests are conducted for the covenant proxies CVNT, D1CVNT and D2CVNT.

Dependent Variable		Intercept	D1	CVNT	D1CVNT	D2CVNT	Nobs	F-Stat	Adj. R^2	RSS
				Panel A: Sobs ≥ 10						
MSKEW	(1)	0.0120		−0.1013**			136	6.913**	4.2%	1.113
		(1.129)		(−2.629)				(0.010)		
	(2)	0.0177	−0.0114	−0.1007**			136	3.711*	3.9%	1.109
		(1.341)	(−0.730)	(−2.609)				(0.027)		
	(3)	0.0127			−0.1604**	−0.0455	136	5.632**	6.4%	1.079
		(1.210)			(−3.356)	(−0.973)		(0.004)		
	(4)	0.0038	0.0184		−0.1846**	−0.0246	136	4.004**	6.3%	1.073
		(0.262)	(0.875)		(−3.341)	(−0.467)		(0.009)		
				Panel B: Sobs ≥ 100						
MSKEW	(1)	0.0099		−0.1589**			53	11.844**	17.3%	0.313
		(0.661)		(−3.442)				(0.001)		
	(2)	0.0205	−0.0297	−0.1436**			53	6.929**	18.6%	0.302
		(1.220)	(−1.350)	(−3.043)				(0.002)		
	(3)	0.0082			−0.2211**	−0.0492	53	10.152**	26.0%	0.274
		(0.583)			(−4.463)	(−0.819)		(0.000)		
	(4)	0.0017	0.0175		−0.2437**	−0.0343	53	6.799**	25.1%	0.272
		(0.0925)	(0.595)		(−3.889)	(−0.524)		(0.001)		

** and * denote significance at the 1 and 5 percent levels, respectively.

proxy for highly leveraged firms, D1CVNT, is always significantly less than zero at the 1 percent level. This is consistent with the stratified covenant effect hypothesis because this hypothesis predicts that net-worth covenants will have a strong effect on the anatomy of the implied volatility skew for highly leveraged firms.

We also conducted simple F-tests to determine whether regressions (3) and (4) reported in Table IV, Panels A and B, are significantly better than regressions (1) and (2), respectively. These tests determine whether incremental explanatory power is obtained by allowing the coefficient on the covenant proxy to differ for firms with different leverage ratios. The F-statistics testing for increased explanatory power of regression (3) versus (1) are significant at the 5 percent level in both Panels A and B. Regression (4) is also always significantly better than regression (2) at the 5 percent level. Overall, these tests indicate that covenant proxies of highly leveraged firms explain more of the cross-sectional variation of implied volatility skews than those of firms with low leverage. This supports the stratified covenant effect hypothesis.

D. Robustness of the Regression Results

The previous section provided empirical support for our three testable hypotheses by showing that the dependent and independent variables are cross-sectionally correlated. In this section, we investigate whether the signs, magnitudes, and significance levels of the estimated regression coefficients remain unchanged when controlling for size, book-to-market value of equity, industry, and volatility of equity. We control for these firm characteristics by including dummy variables in the regressions.

We control for firm size and the book-to-market value of equity because these variables have been associated with cross-sectional variations in realized rates of return (see for example Banz (1981), Rosenberg, Reid, and Lanstein (1985), and Fama and French (1992)). Our model does not predict any cross-sectional relationship between these firm characteristics and implied volatility skews. However, one may expect firm size and the book-to-market ratio to proxy for characteristics related to the risk-neutral asset value distribution. One can argue that larger, more diversified firms generally have more symmetric asset value distributions due to the central limit theorem. This contrasts with smaller firms that may have more positively skewed asset value distributions. Using this argument one would expect the implied volatility skew to be negatively related to size. Furthermore, a low book-to-market ratio may indicate that large upward price movements are unlikely. Therefore, the asset return distributions for high book-to-market ratio firms may be more positively skewed than those of firms with low book-to-market ratios. Our third control variable is the firm's primary standard industrial classification (SIC) code as reported by COMPUSTAT. We classify firms with SIC codes 2830–2839, 3500–3699, 4800–4899, and 7370–7379 as "hightech" firms, and assign 1 to the HIGHDUM dummy variable if the firm operates primarily in these industries, and 0 otherwise. These firms' risk-neutral asset value distributions may differ because of frequent technological innovations. Our last control variable is the median implied at-the-money volatility for the 13-week period following the firms' fiscal year end. This variable is included because the implied volatility is the main determinant of option values. It is therefore possible that cross-sectional pricing biases may be related to the value of this variable.

Table V reports the results of regressing the median volatility skew (MSKEW) computed from at least 100 observations (Sobs $\geq$ 100) on the explanatory variables while including the four dummy variables.[35] SIZEDUM equals 1 if the market value of equity is greater than the median value in our

[35] Regressions involving median volatilities computed from at least 10 observation yield similar results.

Table V
Regression Results—Including Control Dummies

This table reports the results of regressing the median implied volatility skew, MSKEW, on the independent variables when four different control dummy variables are included as independent variables. The median volatility skews are computed from at least 100 observations (Sobs $\geq$ 100). SIZEDUM equals 1 if the market value of equity is greater than the median market value in our sample, and 0 otherwise. BEMEDUM equals 1 if the book value of equity divided by the market value of all outstanding common shares of stock at the fiscal year end is greater that the median value in our sample, and 0 otherwise. HIGHDUM equals 1 if the firm's primary Standard Industrial Classification (SIC) code assigns the firm to an industry that we classify as "hightech." VOLDUM equals 1 if the median implied at-the-money volatility for the 13-week period following the fiscal year end is greater than the cross-sectional median implied volatility in our sample, and 0 otherwise. The leverage ratio, LEV, is defined as total debt plus preferred stock, divided by the sum of total debt, preferred stock, and the market value of outstanding common equity. The covenant proxy, CVNT, equals the ratio of long-term debt due in one year plus notes payable divided by total debt. D1CVNT (D2CVNT) contains the covenant proxy CVNT if the firm's leverage ratio is above (at or below) the median value of the leverage variable, and 0 otherwise. The regressions are estimated using the 53 observations for which data is available. The numbers in parentheses below the estimated coefficients are t-statistics. The numbers in parentheses below the F-statistics are p-values. One-sided tests are conducted for the variables LEV, CVNT, D1CVNT, and D2CVNT. Two-sided tests are conducted for all control dummies.

Intercept	SIZEDUM	BEMEDUM	HIGHDUM	VOLDUM	LEV	CVNT	D1CVNT	D2CVNT	F-Stat	Adj. R^2
					Panel A					
0.0373					−0.2188**				13.525**	19.4%
(1.846)					(−3.678)				(0.001)	
0.0402	−0.0215				−0.1924**				7.151**	19.1%
(1.963)	(−0.908)				(−2.902)				(0.002)	
0.0241		0.0483*			−0.2553**				9.915**	25.5%
(1.189)		(2.279)			(−4.298)				(0.000)	
0.0506*			−0.0244		−0.2301**				7.419**	19.8%
(2.161)			(−1.117)		(−3.821)				(0.002)	
0.0378				−0.0024	−0.2168**				6.637**	17.8%
(1.794)				(−0.105)	(−3.430)				(0.003)	
0.0441	−0.0416	0.0527*	−0.0334	0.0114	−0.2328**				4.865**	27.1%
(1.786)	(−1.391)	(2.497)	(−1.507)	(0.394)	(−3.599)				(0.001)	

Table V—Continued

Panel B										
0.0099						−0.1589**			11.844**	17.3%
(0.661)						(−3.442)			(0.001)	
0.0187	−0.0282					−0.1367**			6.734**	18.1%
(1.133)	(−1.227)					(−2.768)			(0.003)	
0.0029		0.0124				−0.1548**			6.002**	16.1%
(0.147)		(0.564)				(−3.287)			(0.005)	
0.0093			0.0018			−0.1596**			5.810**	15.6%
(0.548)			(0.082)			(−3.376)			(0.005)	
0.0105				−0.0018		−0.1577**			5.810**	15.6%
(0.619)				(−0.076)		(−3.186)			(0.005)	
0.0086	−0.0544	0.0214	−0.0073	0.0266		−0.1255*			2.988*	16.0%
(0.368)	(−1.708)	(0.931)	(−0.285)	(0.833)		(−2.250)			(0.020)	
Panel C										
0.0486*					−0.1681**	−0.1157**			10.624**	27.0%
(2.465)					(−2.796)	(−2.513)			(0.000)	
0.0493*	−0.0073				−0.1608**	−0.1118*			6.988**	25.7%
(2.462)	(−0.312)				(−2.473)	(−2.324)			(0.001)	
0.0365		0.0366			−0.2054**	−0.0937*			8.331**	29.7%
(1.769)		(1.710)			(−3.266)	(−1.995)			(0.000)	
0.0550*			−0.0128		−0.1769**	−0.1091*			7.108**	26.1%
(2.437)			(−0.592)		(−2.839)	(−2.287)			(0.000)	
0.0464*				0.0122	−0.1757**	−0.1225**			7.085**	26.0%
(2.286)				(0.551)	(−2.830)	(−2.553)			(0.000)	
0.0440	−0.0344	0.0423	−0.0205	0.0201	−0.2036**	−0.0726			4.417**	28.3%
(1.795)	(−1.141)	(1.894)	(−0.854)	(0.682)	(−3.003)	(−1.333)			(0.001)	
Panel D										
0.0082							−0.2211**	−0.0492	10.152**	26.0%
(0.583)							(−4.463)	(−0.819)	(0.000)	
0.0095	−0.0037						−0.2165**	−0.0494	6.643**	24.6%
(0.577)	(−0.151)						(−3.688)	(−0.813)	(0.001)	
−0.0081		0.0284					−0.2198**	−0.0249	7.479**	27.2%
(−0.435)		(1.344)					(−4.472)	(−0.399)	(0.000)	
0.0071			0.0033				−0.2223**	−0.0502	6.644**	24.6%
(0.447)			(0.154)				(−4.390)	(−0.823)	(0.001)	
0.0027				0.0165			−0.2382**	−0.0518	6.880**	25.3%
(0.167)				(0.726)			(−4.326)	(−0.857)	(0.001)	
−0.0129	−0.0270	0.0291	0.0066	0.0326			−0.2222**	−0.0323	3.824**	24.6%
(−0.547)	(−0.843)	(1.326)	(0.266)	(1.075)			(−3.398)	(−0.500)	(0.004)	

** and * denote significance at the 1 and 5 percent level, respectively.

sample, and 0 otherwise. BEMEDUM equals 1 if the book value of equity divided by the market value of all outstanding common shares of stock at the fiscal year end is greater that the median value in our sample, and 0 otherwise. The definition of our third dummy variable, HIGHDUM, is described above. Finally, VOLDUM equals 1 if the median implied at-the-money volatility for the 13 week period following the fiscal year end is greater than the cross-sectional median implied volatility, and 0 otherwise.

Panel A shows the results of including the control dummy variables in the univariate regression on the leverage ratio. We observe that the sign, significance, and magnitude of the regression coefficient on the leverage variable are unaltered by the inclusion of the dummy variables. The slope coefficient on the leverage variable is always significant at the 1 percent level. This conclusion does not change when all four dummy variables are included in the regression. Of the control variables, only the book to market dummy is significant at the 5 percent level, but the magnitude of the regression coefficient is small. The sign of the coefficient indicates that firms with high book-to-market ratios have slightly more positively skewed implied volatility structures than firms with low book-to-market ratios. This indicates that the option market assigns relatively large risk-neutral probability weights to the upside tails of the firms' risk-neutral asset price distributions.

Panel B indicates that the covenant variable becomes marginally less significant when the size dummy is included in the regression. However, the covenant proxy remains significant at the 1 percent level in four of the five regressions on the dummy variables. The significance level drops to 5 percent only when the implied volatility skew is regressed on all the dummy variables. However, none of the dummy variables is significant in this case.

Panel C shows that both the coefficient on the leverage variable and the covenant proxy remain significant when the control dummies are included in the multivariate regression: the leverage variable is always significant at the 1 percent level while the covenant proxy is significant at the 1 percent level in one regression and significant at the 5 percent level in three regressions involving a single dummy variable. Only in the regression including all dummy variables is the covenant proxy insignificant. This is of no great concern because none of the dummy variables is significant.

Finally, panel D allows for different slope coefficients on the covenant proxy for high and low leverage firms. None of the control variables is significant in these regressions. As predicted by the stratified covenant effect hypothesis, the covenant proxy for the highly leveraged firms is always negative and significant at the 1 percent level. The covenant proxy for firms with low leverage ratios remains negative and insignificant. Overall, Table V shows that the leverage effect, the covenant effect, and the stratified covenant effect are robust to the inclusion of regression variables that are designed to control for cross-sectional firm characteristics that may be related to the risk-neutral distribution of asset values.

IV. Conclusion

We have presented a set of equity option pricing formulas that are consistent with the firm's capital structure in an economy with corporate taxes and bankruptcy costs. Our equity option pricing formula simplifies to a Black–Scholes type formula for down-and-out barrier options if bankruptcy is declared when the value of the firm's assets equals the after-tax value of all future promised debt payments. Furthermore, our model encompasses the standard Black–Scholes pricing formula. Combined with Leland's (1994) capital structure model, our model of options on leveraged equity represents a comprehensive set of consistent, closed form pricing formulas for debt, equity, and equity option values. These formulas link equity option price processes and sensitivities to leverage ratios and debt covenants.

Our model provides *structural* explanations for observed implied volatility skews. It predicts that options on highly leveraged equity will exhibit more pronounced implied volatility skews than options on equity in firms with low leverage. We investigate this hypothesis empirically and find that implied volatility skews for the largest possible cross-section of firms with equity options traded on the Chicago Board Options Exchange during the years 1993 and 1994 were indeed related to leverage. In addition, the magnitudes of the observed implied volatility skews correspond to those predicted by our model. The observed implied volatility skews are small relative to those observed in equity index option markets by Rubinstein (1994), Jackwerth and Rubinstein (1996), and Dumas, Fleming, and Whaley (1996) during the period subsequent to the market crash of 1987. This may indicate that the market component of the implied risk-neutral equity return distribution is significantly skewed to the left, while the idiosyncratic component appears to be more symmetric or even positively skewed.[36]

Our model also predicts that implied volatility skews are more pronounced (more negative) for options on equity in firms financed by debt with protective net-worth covenants than for options on equity in firms with debt without these covenants. We test this prediction by constructing a covenant proxy as the ratio of short-term debt to total debt and regressing implied volatility skews on this covenant proxy. These regressions reveal a strong negative cross-sectional relationship between the covenant proxy and the implied volatility skew. Furthermore, as predicted by our theory, the net-worth covenant proxy is highly significant for highly leverage firms, while it is insignificant for firms with low leverage. Finally, our empirical investigation shows that all estimated relationships are unchanged when we control for firm size, the book-to-market value of equity, industry, and the level of the implied volatility.

We have thus provided both theoretical and empirical support for our hypothesis that cross-sectional differences in firms' capital structures explain cross-sectional differences in implied volatility skews. This also implies that there are cross-sectional differences in the implied risk-neutral distributions of

[36] The authors thank Mark Rubinstein for pointing out this explanation.

future equity returns. It would be interesting to investigate if these differences can explain cross-sectional differences in expected equity returns.

Finally, if we are willing to interpret our empirical results with a broad perspective, our study supports the contingent claims approach to modeling a firm's capital structure. This indicates that more detailed models of the firm, such as those developed by Geske (1977), Cox and Rubinstein (1985), Ericsson and Reneby (1995), and Leland and Toft (1996) may explain more of the cross-sectional variation in implied volatility structures.

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