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Strategic Debt Service

PIERRE MELLA-BARRAL and WILLIAM PERRAUDIN*

ABSTRACT

When firms experience financial distress, equity holders may act strategically, forcing concessions from debtholders and paying less than the originally-contracted interest payments. This article incorporates strategic debt service in a standard, continuous time asset pricing model, developing simple closed-form expressions for debt and equity values. We find that strategic debt service can account for a substantial proportion of the premium on risky corporate debt. We analyze the efficiency implications of strategic debt service, showing that it can eliminate both direct bankruptcy costs and agency costs of debt.

THE CONTINGENT CLAIMS LITERATURE on corporate debt initiated by Merton (1974) interprets risky debt as a portfolio comprising the safe asset plus a short position in a put option written on the value of the firm's underlying assets. Under appropriate assumptions about asset return distributions, this interpretation implies simple, closed-form expressions for risky bond prices. However, empirical evidence suggests that risk premia on corporate debt significantly exceed those implied by Merton's model, except if one includes unrealistically high bankruptcy costs. Recent research has suggested two approaches whereby the high, observed risk premia may be rationalized.

1. Franks and Torous (1989) document the fact that, in many bankruptcy settlements, junior claimants including equityholders receive a greater share of residual value than is consistent with absolute priority. Longstaff and Schwartz (1995) incorporate such departures from absolute priority in a contingent claims model, taking the allocation of the bankrupt firm's residual value among stakeholders as exogenously given. They show that risk premia prior to bankruptcy may be significantly boosted by expectations of deviations from absolute priority.
2. An alternative approach takes as its starting point the observation that, in many cases, debtholders in distressed firms are persuaded by equityholders to accept concessions prior to formal bankruptcy proceedings. Bergman and Callen (1991) study the extraction of such concessions by equityholders in a static model of capital structure determination. Anderson and Sundaresan (1996) include such concessions in a binomial pricing model and examine the design of debt contracts.

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In this article, we incorporate strategic debt service by equityholders in a standard, contingent claims asset pricing model. Employing continuous time methods, we obtain closed-form analytic expressions for the values of debt and equity when the debt issued is perpetual. Our research yields an explanation of the large default premia observed in actual bond markets. For reasonable parameter values, we find that strategic debt service may account for 30% to 40% of the premium on risky debt. For debt issued by firms whose asset values have low volatility, this percentage can easily become much higher.

The fact that we obtain closed-form solutions for debt and equity values, and the trigger points for state variables, enables us to analyze the efficiency implications of strategic debt service. In our model, debt imposes costs both by introducing direct bankruptcy costs and by creating agency-theoretic "under-investment" problems of the kind discussed by Myers (1977).[1] We show that strategic debt service by equityholders eliminates both sets of costs. This result replicates in a dynamic, continuous time pricing model the earlier insight of Bergman and Callen (1991) that renegotiation of coupon payments may eliminate bankruptcy costs.

In Bergman and Callen, the bargaining framework, based on Rubinstein's (1982) model of alternating offers, is efficient in the sense that value-reducing strategies are threatened by equityholders but never actually implemented. However, the ex post bargaining places a limit on the maximum possible debt service payment that equityholders can be induced to make and hence constrains the amount of debt the firm can issue ex ante. This prevents the firm from fully exploiting the tax advantages of debt and hence reduces the firm's ex ante value. While the firm does have a debt capacity in our model, this does not generate inefficiencies, since we suppose that there is no tax advantage to debt. In this case, when renegotiation is possible, other costs of debt are eliminated and the Modigliani-Miller theorem holds. As we show, strategic debt service has important implications for pricing even in this case, since agents know ex ante that equityholders will use the threat of bankruptcy costs to reduce the coupon payments they make when the firm approaches closure.

It should be stressed that our article fits within a broader literature that has investigated the effects on debt valuation of different assumptions about the strategies open to stakeholders in the firm. Brennan and Schwartz (1984), Mello and Parsons (1992), and Mello, Parsons, and Triantis (1995) study contingent claims models in which real investment and closure decisions interact with debt valuation. Fischer, Heinkel, and Zechner (1989) allow dynamic adjustments in the firm's capital structure. Mauer and Triantis (1994) analyze the interaction of leverage adjustments with dynamic investment and operating decisions. Leland (1994) looks at comparative static properties of infinite maturity coupon bond prices under different assumptions about what

[1] In our case, "under-investment" amounts to the fact that, if debt contracts cannot be renegotiated, the firm is liquidated inefficiently early. Titman (1984) has examined the efficiency of liquidation decisions from a somewhat similar perspective, although in his case the firm's customers suffer agency costs from early liquidation.

triggers bankruptcy. Our research and that of Anderson and Sundaresan (1996) contributes to this literature by examining the effect on valuation of expanding the strategy space open to equityholders by introducing take-it-or-leave-it offers on debt service to the firm's creditors.

The article is structured as follows. Section I presents a continuous time model of a levered firm without renegotiation of the debt contract. We obtain analytic solutions for debt and equity and examine the dependence of total firm value on the level of leverage. In Section II, we allow equityholders to make take-it-or-leave-it offers to debtholders regarding coupon payments. With renegotiation, the firm's service flow varies stochastically in a way that depends on the state variable, the price of the firm's output. We characterize properties of equityholders' optimal service flow function. Finally, we describe the solution of the model when instead debtholders make take-it-or-leave-it offers to reduce the coupon. Section III discusses extensions of the model. Section IV presents numerical simulations that show how changes in firm parameters affect the contribution of renegotiation effects to risk premia. Section V concludes.

I. Debt and Agency Costs

A. Basic Assumptions

Throughout our analysis, we suppose that capital markets are frictionless and free of informational asymmetries and that agents are risk neutral and can borrow and lend freely at a constant, sure interest rate, r. The assumption of risk neutrality represents little loss of generality. If agents are risk averse, the analysis may be developed under risk neutral rather than actual probabilities. (See Harrison and Kreps (1979).)

Consider a firm that produces a unit of output which it sells for a price, p_t. Let p_t be a geometric Brownian motion

$$dp_t = \mu p_t + \sigma p_t dB_t, \tag{1}$$

where μ and σ are constant parameters and B_t is a standard Brownian motion. While in production, the firm incurs costs per period of $w > 0$, so its net earnings flow is

$$p_t - w. \tag{2}$$

Suppose that bankruptcy impairs the firm's efficiency in that, after bankruptcy, new owners of the firm can only generate earnings of

$$\xi_1 p_t - \xi_0 w, \tag{3}$$

where $\xi_1 \leq 1$ and $\xi_0 \geq 1$. At any time, whoever owns the firm can opt for liquidation. The scrapping value of the firm at liquidation is a constant γ.

The expected, discounted value of the decline in earnings that accompanies bankruptcy may be regarded as the *direct cost of bankruptcy*. If the prospect of bankruptcy distorts real investment decisions, one may refer to these as agency costs of debt or *indirect bankruptcy* costs. It will turn out in our model that the firm's financing may in certain circumstances lead to inefficiently early liquidation. Since liquidation may be regarded as a real investment decision, the model contains the possibility of both direct and indirect bankruptcy costs.

B. The Pure Equity Firm

We begin by considering the firm's value under pure equity financing, W_t. Since W_t is the value of an asset with an income flow $p_t - w$, financial market equilibrium under risk neutrality requires that

$$rW_t = p_t - w + \frac{d}{d\Delta} E_t(W_{t+\Delta})\Big|_{\Delta=0}. \tag{4}$$

The stationarity of the payoffs involved implies that W_t depends on time only through its dependence on the state variable, p_t, and that liquidation will occur the first time that p_t hits some constant low level p_c. If $W_t = W(p_t)$ is a twice continuously differentiable function of the state variable, p_t, by Ito's lemma, this function must satisfy the differential equation

$$rW(p) = p - w + \mu p W'(p) + \frac{\sigma^2}{2} p^2 W''(p). \tag{5}$$

The boundary conditions that $W(p)$ must obey are as follows. In the absence of arbitrage, the firm value must equal the scrapping price at closure, $W(p_c) = \gamma$. If asset prices are free of bubbles, as p_t becomes large, $W(p_t)$ must approach the expected discounted integral of future income flows: $\mathrm{E}_t(\int_t^\infty (p_v - w) \times \exp[-r(v - t)]\, dv) = p_t/(r - \mu) - w/r$. Under the assumption of pure equity financing, the trigger point for closure, p_c, will be chosen so as to maximize total firm value, $W(p)$. A necessary condition for this is the smooth-pasting condition, $W'(p_c) = 0$.

An alternative way in which the firm may end up operating with pure equity financing is if it starts with equity and debt, goes bankrupt, and then is run as a pure equity operation by its former debtholders. By our earlier assumptions, after bankruptcy, the former debtholders may either liquidate immediately or maintain it as a going concern in which case the maximum earnings flow they can generate is $\xi_1 p_t - \xi_0 w$. Let $X_t = X(p_t)$ denote the value of the firm in the hands of the new owners. By arguments similar to those given above, $X(p)$, satisfies a differential equation like (5) except that the nonhomogeneous term $p - w$ is replaced with $\xi_1 p - \xi_0 w$. This equation is subject to the boundary conditions $X(p_x) = \gamma$ (no arbitrage), $\lim_{p\to\infty} X(p) = \xi_1 p/(r - \mu) - \xi_0 w/r$ (no bubbles), and $X'(p_x) = 0$ (smooth-pasting), where the last condition implicitly defines the trigger price p_x which maximizes the value of $X(p)$.

As we show in the Appendix, solving the differential equations for $W(p)$ and $X(p)$, one obtains the following result.

PROPOSITION 1: *Let $W(p_t)$ and $X(p_t)$ denote the total value of the pure equity firm in the hands (i) of its initial equityholders and (ii) of other owners after bankruptcy. $W(p)$ and $X(p)$ are equal to:*

$$W(p) = \frac{p}{r-\mu} - \frac{w}{r} + \left[\gamma - \frac{p_c}{r-\mu} + \frac{w}{r}\right]\left(\frac{p}{p_c}\right)^{\lambda} \qquad for \quad p \geq p_c \tag{6}$$

$$X(p) = \frac{\xi_1 p}{r-\mu} - \frac{\xi_0 w}{r} + \left[\gamma - \frac{\xi_1 p_x}{r-\mu} + \frac{\xi_0 w}{r}\right]\left(\frac{p}{p_x}\right)^{\lambda} \qquad for \quad p \geq p_x. \tag{7}$$

Here, λ is the negative root of the quadratic equation $\lambda(\lambda - 1)\sigma^2/2 + \lambda\mu = r$, and p_c and p_x are given by:

$$p_c = -\frac{\lambda}{(1-\lambda)}\frac{w + r\gamma}{r}(r-\mu) \tag{8}$$

$$p_x = -\frac{\lambda}{(1-\lambda)}\frac{\xi_0 w + r\gamma}{\xi_1 r}(r-\mu). \tag{9}$$

For $p < p_c$, $W(p) = \gamma$ and for $p < p_x$, $X(p) = \gamma$.

Using the solutions $W(p)$ and $X(p)$ as building blocks, we now turn to the valuation of the levered firm.

C. Debt and Equity Valuation

Suppose that the firm has issued perpetual debt with principal b/r and a contractual coupon flow b per period of time. Let $\hat{L}_t$ and $\hat{V}_t$ denote the levered firm's debt and equity values. Financial market equilibrium requires that $\hat{L}_t$ and $\hat{V}_t$ satisfy:

$$r\hat{L}(p) = b + \mu p\hat{L}'(p) + \frac{\sigma^2}{2}p^2\hat{L}''(p) \tag{10}$$

$$r\hat{V}(p) = p - w - b + \mu p\hat{V}'(p) + \frac{\sigma^2}{2}p^2\hat{V}''(p). \tag{11}$$

Assume that equityholders are free to cover the firm's operating losses by injecting capital and that, so long as they do this, bankruptcy cannot occur. The timing of bankruptcy is thus determined by the decision of equityholders to cease injecting capital. In adopting this assumption, we abstract from bond covenants that trigger bankruptcy when the firm's net worth deteriorates sufficiently far. Our approach is broadly similar to that of Black and Cox (1976) and Leland (1994) and may be contrasted with that of Merton (1974), which supposes that bankruptcy occurs when net worth becomes negative and implicit debt covenants are triggered.[2]

Again, the stationary nature of the payoffs involved implies that bankruptcy occurs when the output price, p_t, first hits some constant low level, p_b. In the absence of arbitrage and assuming that strict seniority of claims is respected, $\hat{L}$ and $\hat{V}$ must satisfy

$$\hat{L}(p_b) = \min\{X(p_b),\, b/r\} \quad \text{and} \quad \hat{V}(p_b) = \max\{0,\, W(p_b) - b/r\} \tag{12}$$

at the bankruptcy point, p_b. Since equityholders decide the timing of bankruptcy, the bankruptcy trigger, p_b, will be selected so as to maximize the value of equity. It is therefore the root of the smooth-pasting condition, $\hat{V}'(p_b) = 0$. Solving the differential equations, (10) and (11), subject to the boundary conditions just described together with a no-bubbles condition, yields the following result.

PROPOSITION 2: *Assume that the firm has issued perpetual debt with coupon, b, and principal b/r. Suppose that the terms of the debt cannot be renegotiated. If $\hat{V}_t$ and $\hat{L}_t$ denote the total values of the firm's equity and debt under these assumptions, then, if $\gamma < b/r$, the debt is risky and*

$$\hat{V}(p) = \frac{p}{r-\mu} - \frac{w+b}{r} - \left[\frac{p_b}{r-\mu} - \frac{w+b}{r}\right]\left(\frac{p}{p_b}\right)^{\lambda} \tag{13}$$

$$\hat{L}(p) = \frac{b}{r} + \left[X(p_b) - \frac{b}{r}\right]\left(\frac{p}{p_b}\right)^{\lambda}, \tag{14}$$

where p_b is given by

$$p_b = -\frac{\lambda}{(1-\lambda)}\frac{w+b}{r}(r-\mu). \tag{15}$$

Otherwise, if $\gamma \geq b/r$, the debt is riskless and

$$\hat{L}(p) = \frac{b}{r} \quad \text{and} \quad \hat{V}(p) = W(p) - \frac{b}{r}. \tag{16}$$

The solutions, $W(p)$, $\hat{V}(p)$, and $\hat{L}(p)$ from Propositions 1 and 2 are illustrated in Figure 1. The assumption in Proposition 2 that the principal value is b/r is

[2] A third approach, followed by Kim, Ramaswamy, and Sundaresan (1993) and Anderson and Sundaresan (1996), is to suppose that bankruptcy is triggered by illiquidity, i.e., when net earnings after interest fall below zero.

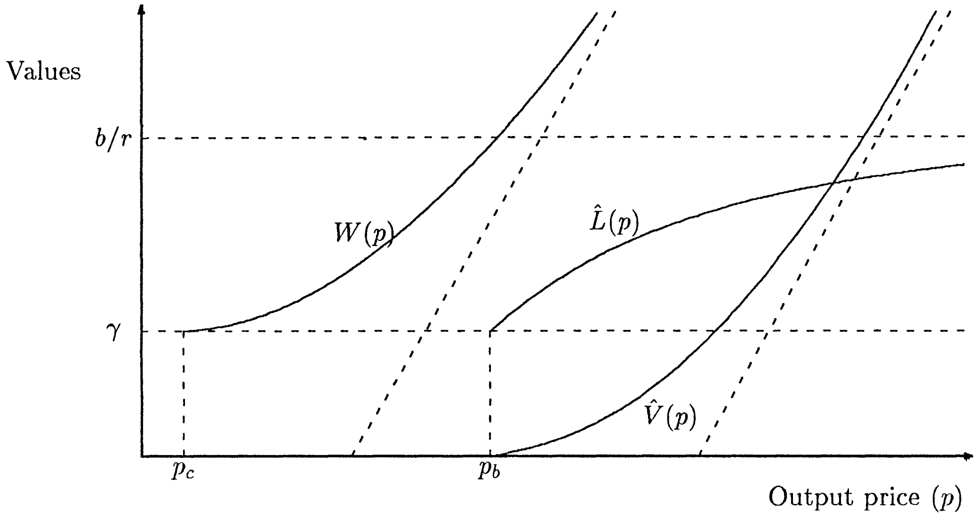

Figure 1. Security valuation with no renegotiation. The values of the firm as a whole under efficient closure at $p_c(W(p))$, and of equity and debt under inefficient closure at p_b ($\hat{V}(p)$ and $\hat{L}(p)$), are shown as functions of the output price, p. Note that $W(p) > \hat{V}(p) + \hat{L}(p)$ reflecting our assumptions of both direct and indirect bankruptcy costs. $W(p)$ smooth pastes to the liquidation value of the firm's assets, γ, at p_c. $\hat{V}(p)$ and $\hat{L}(p)$ value match to zero and γ respectively at p_b. The asymptotic values to which W, $\hat{V}$, and $\hat{L}$ tend as $p \to \infty$ in the absence of bubbles are shown as dashed lines.

adopted for expositional simplicity. Otherwise, the value of debt depends on the bankruptcy trigger, p_b, even when it is riskless.[3]

To interpret the expressions in Proposition 2, note that if $\gamma > b/r$, bankruptcy will never occur since there will always be sufficient assets to meet the firm's liabilities. In this case, when closure occurs, the equityholders are the residual claimants on the firm's assets and hence have an incentive to liquidate the firm at the trigger level, p_c, which maximizes total firm value. If $\gamma < b/r$, the debt is risky and bankruptcy takes place when p_t first hits the level p_b, which maximizes the value of the levered firm's equity.

D. Firm Value and Leverage

To understand the inefficiencies that result from the presence of debt, consider how the total value of the levered firm, $\hat{W}(p) \equiv \hat{V}(p) + \hat{L}(p)$ depends on the contracted coupon flow, b. This dependency is illustrated in Figure 2. When the principal of the debt, b/r, is less than the scrapping value γ, equityholders act as residual claimants even at closure, and the firm is oper-

[3] Furthermore, if the principal exceeds b/r, the value of debt will be decreasing in the state variable, p, in some cases. If the b/r exceeds the principal, for very high p, equityholders may be able to extract concessions from bondholders as we discuss in Section III.C.

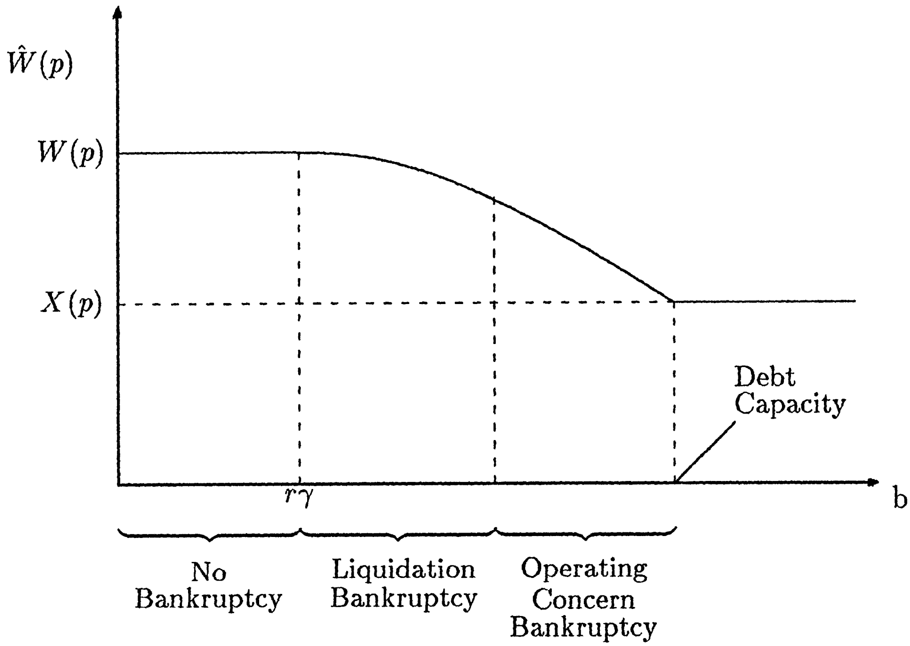

Figure 2. Total firm value and leverage. The total value of the levered firm, $\hat{W}(p)$, is depicted as a function of the contracted coupon flow, b. For low b, existing equityholders are residual claimants and no inefficiency occurs. For b slightly above $r\gamma$ (where r is the interest rate and γ is the liquidation value of the firm's assets) when bankruptcy occurs, liquidation takes place instantaneously and is ex ante inefficient. For large b, when bankruptcy occurs, former bondholders maintain the firm in operation, but earnings are reduced due to direct bankruptcy costs.

ated and liquidated efficiently. Leverage imposes no efficiency losses, and the value of the firm is then the same as under pure equity financing

$$\hat{W}(p) = W(p). \tag{17}$$

When the debt principal, b/r, is greater than the scrapping value γ, bondholders are the residual claimants and the debt is risky. The value of the firm is

$$\hat{W}(p) = \frac{p}{r-\mu} - \frac{w}{r} + \left[X(p_b) - \frac{p_b}{r-\mu} + \frac{w}{r}\right]\left(\frac{p}{p_b}\right)^{\lambda}. \tag{18}$$

Given our assumptions about direct bankruptcy costs (that $\xi_1 < 1$ and $\xi_0 > 1$), to the right of $r\gamma$, there exists an interval for b such that $p_b < p_x$, and bondholders who take over at bankruptcy will prefer to liquidate the firm instantly. In this case, $X(p_b) = \gamma$. For b in this range, leverage is costly because it results in liquidation at p_b rather than at the trigger, p_c. The firm is liable to such a "Liquidation Bankruptcy" in the future if

$$r\gamma < b < \frac{\xi_0 w + r\gamma}{\xi_1} - w. \tag{19}$$

If b exceeds $(\xi_0 w + r\gamma)/\xi_1$ (but $p \geq p_b$ so the levered firm is still operating), then when future bankruptcy occurs, former bondholders will prefer to maintain the firm as a going concern. In the case of such an "Operating Concern Bankruptcy," $X(p_b)$ is given by equation (7). Here, leverage generates efficiency losses from an ex ante point of view (i) because of the direct bankruptcy costs it entails and (ii) because liquidation ultimately occurs at p_x rather than at p_c.

One should note that, when b goes to zero, the firm is purely equity financed and our model becomes very similar to the real options model of irreversible investment studied by Dixit (1989). In fact, when costs of entry are zero and $b = 0$, the two are identical. Another model that, restricted in various ways, closely resembles the model of this section is the one investigated by Mello, Parsons, and Triantis (1995).

II. Pricing with Renegotiation

A. *Service Flows with Equityholder Offers*

Now, consider how the value of the firm's securities is affected if debtholders and equityholders can renegotiate coupon payments. Initially, we suppose that equityholders can make take-it-or-leave-it offers to bondholders. We shall assume that possible strategies for equityholders consist of piecewise right-continuous service flow functions of the state variable, p_t.[4] Let $s(p)$ denote the equityholders' optimal service flow function, and let V_t and L_t denote the value of the firm's equity and debt when equityholders adopt $s(p)$.

Now, what characteristics might one expect the optimal service to have? If the output price falls sufficiently far, one may presume that equityholders will eventually wish to wind up the firm. Let p_c^* denote the trigger price at which this occurs. Consider some price slightly higher than p_c^*. Suppose that equityholders service the debt as originally intended by paying a flow of coupon payments, b, to debtholders. If the value of the firm's debt exceeds $X(p_t)$, then equityholders can extract a surplus by offering debtholders a service flow less than b. Bondholders will not wish to declare bankruptcy, since the value of the firm's assets will be less than the current value of the debt. As p_t increases, equityholders will eventually have an incentive to pay the full, initially contracted service flow, since otherwise the liquidation value will be so large that debtholders will definitely wish to close the firm if given the opportunity. In these circumstances, there exists a price, p_s, which is the lowest price at which equityholders pay the full coupon, b.

The structure of the argument in this and the next two subsections will be as follows. We shall (i) formalize the discussion of the last paragraph as a hypothesis about the optimal service flow function, $s(p)$, (ii) show that this

[4] We assume piecewise continuity in order to avoid technical problems with strategies that involve cutting coupon payments below contracted levels on zero-measure sets. Such cuts in the coupon would allow debtholders to trigger bankruptcy, but would not affect claim values. Right-continuity is assumed in order to ensure uniqueness of the optimal service flow function.

hypothesis uniquely determines $s(p)$, and (iii) verify that the hypothesis is satisfied by an optimal service flow function. As the first stage of this argument, we suppose the following.

HYPOTHESIS 1: *If equityholders can make take-it-or-leave-it offers to bondholders regarding debt service then there exist trigger levels, p_s and p_c^*, such that*

1. *bankruptcy occurs when p_t first hits p_c^*,*
2. *for all $p < p_s$, $s(p) < b$ and $L(p) = X(p)$ (i.e., when debt service is less than the contracted coupon, the value of debt equals that of debtholders' outside option, $X(p)$),*
3. for all p $\geq p_s$, $s(p) = b$ *(i.e., the contracted coupon is paid).*

Now, consider the restrictions placed on $s(p)$ by the above hypothesis. In equilibrium under risk neutrality, $L(p)$ must satisfy an equation like (10) except with b replaced with $s(p)$. If $L(p) = X(p)$ for all $p \in [p_c^*, p_s]$, then we may substitute $X(p)$ and its derivatives for the level and corresponding derivatives of $L(p)$ in this equation. Hence, $s(p)$ must satisfy the equation

$$rX(p) = s(p) + \mu p X'(p) + \frac{\sigma^2}{2} p^2 X''(p). \tag{20}$$

Since $X(p)$ is known exogenously from the analysis of Section I.A. and since $s(p) = b$ for all $p > p_s$, it follows that, for given p_s and p_c^*, equation (20) uniquely determines $s(p)$. Substituting for $X(p)$ and its derivatives in equation (20), we obtain a simple explicit form for $s(p)$:

$$s(p) = \begin{cases} r\gamma & \text{for} \quad p \in [p_c^*, p_{xx}) \\ \xi_1 p - \xi_0 w & \text{for} \quad p \in [p_x, p_s) \\ b & \text{for} \quad p \in [p_s, \infty). \end{cases} \tag{21}$$

An intuitive explanation of the service flows in equation (21) is that equityholders must provide debtholders with an income flow whose capitalized value is sufficient to dissuade them from forcing bankruptcy to obtain their "outside option." Thus, for example, when $p \in [p_c^*, p_x)$, debtholders require a service flow of $r\gamma$ to dissuade them from declaring bankruptcy to obtain the outside option, γ.

B. Debt and Equity Values with Equityholder Offers

To complete the derivation of the service flow function, $s(p)$, we need to solve for the two triggers, p_c^* and p_s. To accomplish this requires that one obtain the equity and debt values, V_t and L_t. Equilibrium with risk-neutral agents implies that $V(p)$ and $L(p)$ satisfy

$$rL(p) = s(p) + \mu p L'(p) + \frac{\sigma^2}{2} p^2 L''(p) \tag{22}$$

$$rV(p) = p - w - s(p) + \mu p V'(p) + \frac{\sigma^2}{2} p^2 V''(p). \tag{23}$$

The absence of arbitrage implies that $V(p_c^*) = 0$ and $L(p_c^*) = X(p_c^*)$. In fact, the construction of the service flow implies that $L(p) = X(p)$ for all $p \in [p_c^*, p_s]$. No bubbles conditions include $\lim_{p\to\infty} L(p) = b/r$ and $\lim_{p\to\infty} V(p) = p/(r - \mu) - (w + b)/r$. Since equityholders are effectively selecting p_c^* by their choice of service flows, p_c^* must be such as to maximize the value of the firm's equity, and hence, $V'(p_c^*) = 0$. Finally, the absence of arbitrage implies that in the range, (p_c^*, ∞), $V(p)$, and $L(p)$ must be continuous both in levels and in first derivatives.

Solving the differential equations in equations (22) and (23) subject to the above boundary conditions, we obtain the following result.

PROPOSITION 3: *Under Hypothesis 1, equityholders adopt the service flow function, $s(p)$, defined in equation (21), and the values of equity, $V(p)$, and debt, $L(p)$, are as follows.*

$$V(p) = W(p) - L(p) \tag{24}$$

where, if $\gamma \geq b/r$, the debt is riskless and $L(p) = b/r$. If $\gamma < b/r$, the debt is risky and

$$L(p) = \begin{cases} b/r + [X(p_s) - b/r](p/p_s)^{\lambda} & \text{if} \quad p > p_s \\ X(p_s) & \text{if} \quad p \leq p_s, \end{cases} \tag{25}$$

where p_s is given by

$$p_s = -\frac{\lambda}{1-\lambda} \frac{b + \xi_0 w}{\xi_1 r} (r - \mu). \tag{26}$$

The firm is liquidated the first time that p_t hits p_c, where p_c is defined as in equation (8).

Equity and debt values with equityholder offers are shown in Figure 3. The last sentence of the proposition makes the point that liquidation occurs at the trigger price, which maximizes the value of the pure equity firm. Since it is also true that prior to liquidation the firm is operated by the original equityholders, it is clear that renegotiation generates the fully efficient outcome. To stress this finding, we state it in the form of a corollary.

COROLLARY 1: *If equityholders adopt the service flow function, $s(p)$, leverage does not reduce the ex ante value of the firm.*

The mathematical explanation for the result in Corollary 1 may be seen directly from equations (24) and (25). With renegotiation, $V(p)$ equals $W(p) - L(p)$. From equation (25), it is clear that $L(p)$ is independent of the liquidation trigger, p_c^*. Hence, the same liquidation trigger maximizes both $V(p)$ and $W(p)$. It follows that $p_c^* = p_c$ where p_c is defined as in equation (8).

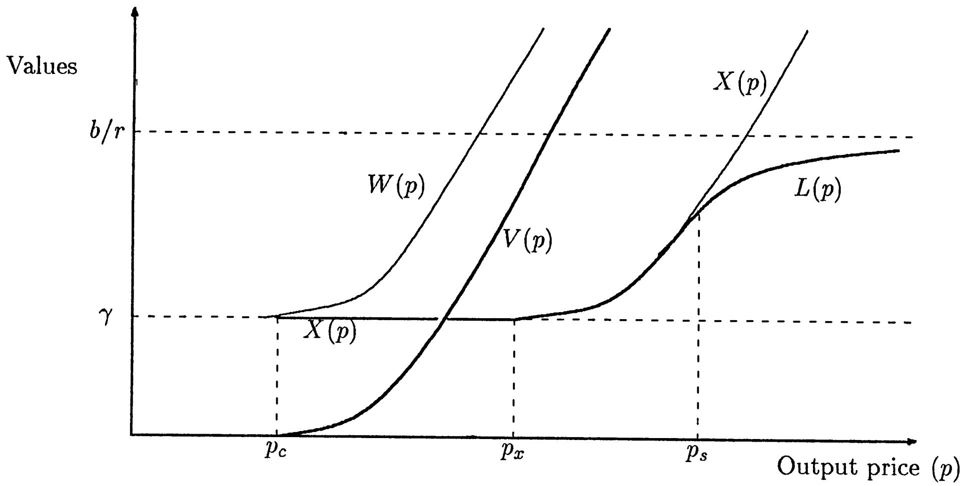

Figure 3. Security valuation with equityholder offers. The values of equity ($V(p)$) and debt ($L(p)$) with equityholder take-it-or-leave-it offers are shown as functions of the output price, p. $L(p)$ and $V(p)$ smooth paste respectively to the liquidation value of the firm's assets, γ, and to zero at the efficient liquidation price, p_c. $L(p)$ value matches to the bondholders' "outside offer," $X(p)$, for all prices less than p_s, the trigger price for renegotiation.

The economic intuition for Corollary 1 is that when equityholders can make take-it-or-leave-it offers to bondholders, they can extract any surplus in the value of bonds over and above the bondholders' exogenously-given outside option, $X(p)$. Equityholders thus become the residual claimants to firm value in all states of the world and have an incentive to maximize the total value of the firm's assets.

A second important corollary of Proposition 3, which follows from its proof, is

COROLLARY 2: *If $\gamma < b/r$, the trigger price for efficient liquidation, p_c, is less than p_s. Hence, if debt is risky and equityholders employ the service flow function, $s(p)$, renegotiation always occurs for some interval of output prices.*

This result shows that *whenever debt is risky*, there is scope for renegotiation. The intuition here is that if debt becomes risky, equityholders are no longer the residual claimants on the firm's income stream in states of the world involving bankruptcy. Hence, the decision to trigger bankruptcy will be made inefficiently. The loss in value associated with this inefficiency creates scope for renegotiation.

C. The Optimality of $s(p)$

In Hypothesis 1, we claim that the optimal service flow function for equityholders who can make take-it-or-leave-it offers consists of the function $s(p)$ such that $L(p) = X(p)$ and $s(p) < b$ in some interval $[p_c^*, p_s]$, and that $s(p) = b$ for all $p \geq p_s$. This subsection provides a formal proof of this claim. While this result is technical it deserves emphasis, which is why we present it in the main body of the text.

Let $\mathscr{A}$ denote the set of pairs consisting of a liquidation price, $p_c^r \in \mathbf{R}^+$ and a piecewise right-continuous, bounded service flow function, $s^r(\cdot)$, defined on $[p_c^r, \infty)$, such that

$$\forall p \geq p_c^r, \quad s^r(p) < b \Rightarrow L^r(p, p_c^r) \geq \min\{X(p), b/r\}, \tag{27}$$

where r indexes members of $\mathscr{A}$. The set $\mathscr{A}$ consists of those bankruptcy price and service flow pairs such that coupon payments fall short of the contracted level only when the value of debt exceeds the postbankruptcy value of the firm.

Let $V(p, p_c)$ and $L(p, p_c)$ denote the equity and debt values derived in II.B.[5] The main result of this subsection is

PROPOSITION 4: *For any $(p_c^r, s^r(\cdot)) \in \mathscr{A}$, $V(p, p_c) \geq V(p, p_c^r)$ for all $p \geq p_c$. Hence, Hypothesis 1 is true.*

This result is quite strong since it shows that, provided equityholders can make take-it-or-leave-it offers to debtholders, for any output price, p_t, the pair $(p_c, s(\cdot))$ derived above will generate a higher equity value than any other feasible $(p_c^r, s^r(\cdot))$ pair.

D. Properties of $s(p)$

Let us study properties of the service flow function, $s(p)$, depicted in Figure 4. Clearly, $s(p)$ is flat and continuous for prices below p_x and above p_s, while it is linearly increasing with slope ξ_1 for $p \in (p_x, p_s)$. At p_s and p_x, $s(p)$ exhibits discontinuities in that

$$\begin{aligned}\lim_{p \downarrow p_x} s(p) - \lim_{p \uparrow p_x} s(p) &= \frac{(r\gamma + \xi_0 w)(r - \mu)}{-r(1 - \lambda)} < 0 \\ \lim_{p \downarrow p_s} s(p) - \lim_{p \uparrow p_s} s(p) &= \frac{(b + \xi_0 w)(r - \mu)}{r(1 - \lambda)} > 0.\end{aligned} \tag{28}$$

To understand why these discontinuities occur, note first that $L(p)$ satisfies the differential equation (22) in the neighborhoods of p_x and p_s. But, in the absence of arbitrage, $L(p)$ and $L'(p)$ must both be continuous at p_x and p_s. From inspection of the differential equation, it follows that $s(p)$ is discontinuous at a given point *if and only if* the second derivative, $L''(p)$, exhibits discontinuities at the same points. What then is the intuition for the discontinuities in the second derivative, $L''(p)$, at p_x and p_s?

Values p_x and p_s are reversible switch points in that the change that occurs when the state variable crosses these levels is reversed if p_t moves back to its original position. It is well known that, in general, the second derivatives of claim values are not continuous at reversible switch points (see Dumas (1991)

[5] This notation serves to stress the dependence on the liquidation trigger price, p_c.

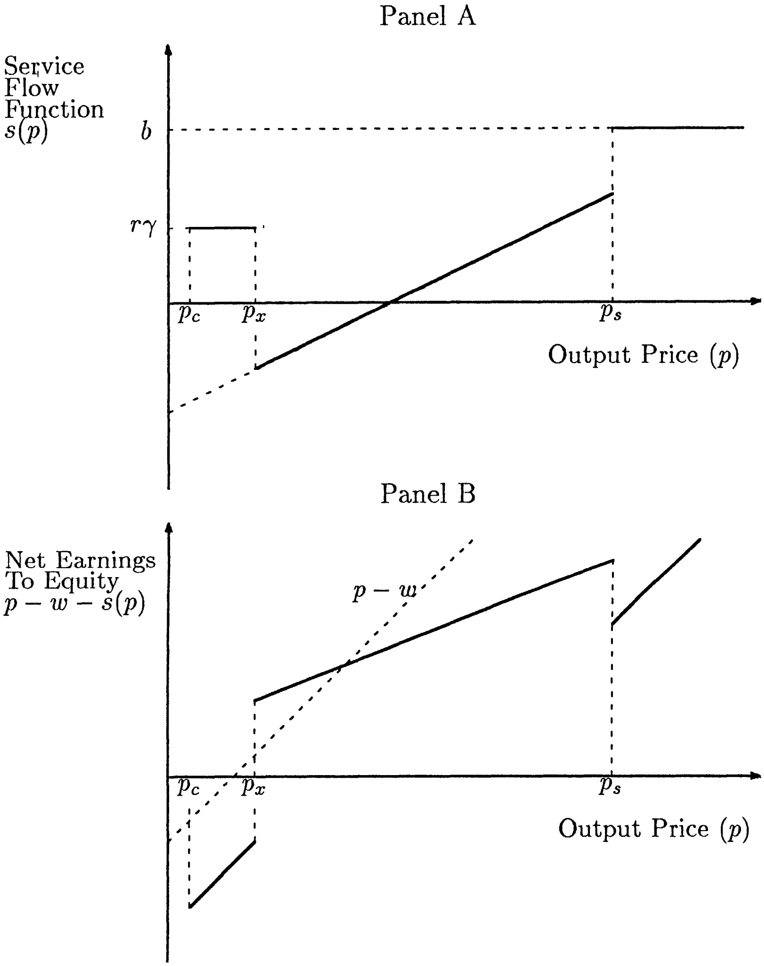

Figure 4. Service flow and equity earnings under equityholder offers. Panel A shows the service flow under equityholder take-it-or-leave-it offers, $s(p)$, as a function of the output price, p. For p greater than p_s (the trigger price for renegotiation) the service flow equals the contracted coupon, b. For $p \in [p_x, p_s]$, (where p_x is the highest price at which liquidation bankruptcies occur) the service flow is linearly increasing. For $p \in [p_c, p_x]$, (where p_c is the trigger price for firm closure), the service flow equals the product, $r\gamma$, of the interest rate, r, and the liquidation value of the firm's assets, γ. Panel B shows equity earnings, $p - w - s(p)$, where w is the firm's flow costs.

for a discussion). Continuity *is* implied if the switch point is chosen optimally so as to maximize the value of the claim.[6] In our application, the switch points are not freely chosen by equityholders to maximize their claim values, but are

[6] Termed by Dumas (1991) a super-contact condition, such continuity of the second derivative is the equivalent for reversible switch-points of the smooth-pasting conditions for permanent switches. The latter are encountered, for example, in the optimal exercise of an American option.

implied by the form of the outside option function, $X(p)$. Hence, there is no reason to expect continuity of the second derivative or of the service flow function, $s(p)$.

Now, consider the sign of the $s(p)$ function. An interesting possibility that arises for certain parameter values is that $s(p)$ actually turns negative for some output prices. In effect, bondholders are then injecting cash into the firm. In fact, one may show that $s(p)$ is negative for some interval of p if and only if $\xi_0 > \rho r\gamma/w$ where the constant ρ is defined as $\rho \equiv 1/[(\lambda - 1)/\lambda(r/(r - \mu))^2 - 1] > 0$. Since a larger ξ_0 corresponds to higher bankruptcy costs, the intuition here is that the greater the cost of bankruptcy to debtholders, the more their reluctance to take over the firm can be exploited by equityholders. Lastly, note that our assumptions that $\xi_0 \geq 1$ and $0 < \xi_1 \leq 1$ imply that for $p \in [p_x, p_s]$, $s(p) = \xi_1 p - \xi_0 w \leq p - w$. Hence, whenever bondholders inject cash, $p - w - s(p) \geq 0$ and equityholders are actually receiving nonnegative dividends.

The above results on the sign of $s(p)$ very much underline the importance of the bankruptcy cost parameters, ξ_0 and ξ_1, in determining how much equityholders can extract from the firm. It is instructive in this regard to consider what happens as $\xi_1 \uparrow 1$ and $\xi_0 \downarrow 1$, i.e., direct bankruptcy costs shrink to zero. Inspection of equations (8), (9), (15), and (26) reveals that $p_s \downarrow p_b$ and $p_x \downarrow p_c$. The limiting solution involves the bondholders receiving the full, contracted coupon flow, b until p_t first hits p_b and then paying $p - w$, the firm's operating losses, until liquidation takes place at p_c. Indeed, evaluation of equations (24) and (25) for the limiting values of ξ_1 and ξ_0 reveals the following result:

PROPOSITION 5: *As $\xi_1 \uparrow 1$ and $\xi_0 \downarrow 1$ (so direct bankruptcy costs are eliminated), the service flow function becomes:*

$$s(p)|_{\xi_1=\xi_0=1} = \begin{cases} p - w & \text{for} \quad p \in [p_c, p_b) \\ b & \text{for} \quad p \geq p_b, \end{cases} \tag{29}$$

$V(p) \rightarrow \hat{V}(p)$ and $L(p) \rightarrow \hat{L}(p)$.

The implication of this proposition is that, when $\xi_1 = \xi_0 = 1$, any surplus to be extracted from bondholders disappears and equityholders can do no better than in the absence of renegotiation, even when they have maximum bargaining power. What drives this result is our assumption that bondholders can exploit the scrapping value of the firm just as well as equityholders. In this case, threatening bondholders with firm closure does not enable equityholders to extract concessions when direct bankruptcy costs are zero since, for all p, equityholders and debtholders are equally able to exploit the firm's productive potential.

E. Bondholder Offers

The above analysis presumes that equityholders have maximum bargaining power and hence can make take-it-or-leave-it offers to bondholders. Consider the alternative polar case in which bondholders choose a service flow function to maximize the value of the firm's debt. Let this optimal service flow function be denoted $q(p)$. One may think of $q(p)$ as embodying an optimal package of concessions by lenders. In making these concessions, debtholders will increase the value of their claims compared to a situation with no renegotiation.

The logic of our analysis of equityholder offers suggests that we look for a $q(p)$ that maximizes the total value of the firm, while minimizing the value of the equity. If bondholders can make take-it-or-leave-it offers in renegotiating coupon payments, we would expect them to behave as residual claimants, maximizing firm value subject to the constraint placed upon them by the "outside option" of equityholders. In this context, the outside option of equityholders is supplied by limited liability, i.e., unless the equity value is nonnegative, equityholders will abandon the firm, precipitating bankruptcy.

Pursuing an analysis parallel to that applied above in the case of equityholder offers, yields the following proposition.

PROPOSITION 6: *When bondholders can make a take-it-or-leave-it offer of concessions to equityholders, their optimal service flow function, $q(p)$ equals*

$$q(p) = \begin{cases} p - w & \textit{for} \quad p \in [p_c, p_b) \\ b & \textit{for} \quad p \geq p_b. \end{cases} \tag{30}$$

The value of equity is the same as in the case of no renegotiation, $\hat{V}(p)$, and the value of the firm's debt is $W(p) - \hat{V}(p)$.

In other words, the best package of concessions that bondholders can make is to insist that equityholders pay the full contracted coupon, b, so long as the output price, p_t, exceeds the trigger for bankruptcy in the case without renegotiation, p_b. For $p < p_b$, bondholders should inject cash to cover the firm's operating losses. (It is easy to show that $p < p_b$ implies $p - w < 0$, so the firm *is* making operating losses at such output prices.) Figure 5 shows $q(p)$ and net earnings to equity order bondholder offers. It is interesting to observe that, in the case of bondholder take-it-or-leave-it offers, the cash flows to debtholders and equityholders and hence the claim values are *exactly the same* as under equityholder offers when $\xi_0 = \xi_1 = 1$.

Another instructive fact is that, for p just greater than p_c, while on the one hand the *value* of debt is *higher* when bond holders have maximum bargaining power, *debt service* is *lower*, i.e., $q(p) < s(p)$. This finding strongly underscores the fact that what matters to agents in this kind of model is the value of their claim and not simply their current income flow. An agent's bargaining power directly affects the claim value he can obtain, but the implications for instantaneous flows can be quite complicated. In the models described above, bondholders with maximum bargaining power inject more for p_t just greater than p_c because this permits them to extract a higher service payment from equityholders when p_t is somewhat higher.

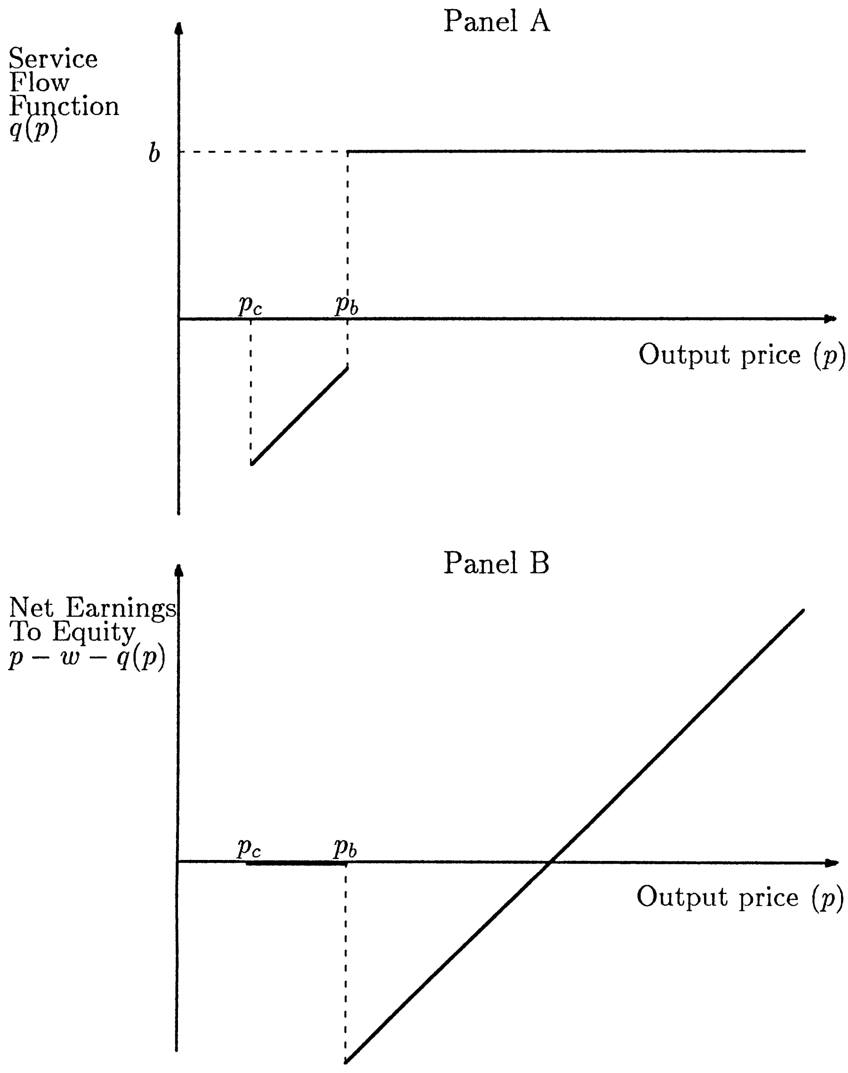

Figure 5. Service flow and equity earnings under bondholder offers. Panel A shows the service flow function under bondholder offers, $q(p)$, as a function of the output price, p. Bondholders demand the contracted service flow, b, so long as p exceeds the bankruptcy trigger price without renegotiation, p_b. Thereafter, bondholders meet the operating losses of the firm until p hits the efficient closure trigger, p_c. The income flow to equity shown in Panel B equals the flow equityholders receive in the absence of renegotiation.

III. Extensions of the Model

Our analysis so far has assumed zero costs of renegotiation and no tax advantages for leverage. The net result, as we have seen, is that firms are indifferent between debt and equity financing. If one wished to develop a model of optimal capital structure, it would clearly be necessary to modify these assumptions. The most obvious approach might be to formulate the model so as to create a tradeoff between renegotiation costs and tax advantages of debt. In this section, we discuss how the model may be generalized to include these two features.

A. Renegotiation Costs

So far, we have presumed that renegotiation is costless. A natural alternative is to suppose that renegotiation of coupon payments imposes a cost δ per unit of time when the coupon is not being fully serviced. To analyze this extension of the model, define $C(p_t|\underline{p}, \bar{p})$ to be the value of a claim that, until p_t first hits a level $\underline{p}$, pays an income flow δ when $p_t \in (\underline{p}, \bar{p})$ and zero when $p_t > \underline{p}$. Here, $\underline{p}$ and $\bar{p}$ are arbitrary constants satisfying $0 < \underline{p} < \bar{p}$. We show in the Appendix that $C(p|\underline{p}, \bar{p})$ is equal to

$$C(p|\underline{p}, \bar{p}) = \begin{cases} \delta/r\varphi(p) & \text{for} \quad p \in [\underline{p}, \bar{p}) \\ \delta/r\varphi(\bar{p})(p/\bar{p})^{\lambda_1} & \text{for} \quad p \in [\bar{p}, \infty), \end{cases} \tag{31}$$

where

$$\varphi(p) \equiv 1 - (p/\underline{p})^{\lambda_1} + \frac{\lambda_1}{\lambda_2 - \lambda_1}[(p/\bar{p})^{\lambda_2} - (\underline{p}/\bar{p})^{\lambda_2}(p/\underline{p})^{\lambda_1}]. \tag{32}$$

Here, λ_1 and λ_2 are the negative and positive roots respectively of the quadratic equation $\lambda(\lambda - 1)\sigma^2/2 + \lambda\mu = r$ already encountered.

Renegotiation costs alter the solutions for debt and equity obtained in earlier sections in a simple fashion. Consider first the case in which equityholders make the offers. Once again, debt values are largely determined by bondholder's outside offers. Let $L(p|\underline{p})$ and $W(p|\underline{p})$ denote the values given in equations (25) and (6) ($L(p)$ and $W(p)$, respectively) except that firm closure occurs at some arbitrary level $\underline{p} > 0$ instead of at p_c. The value of the firm's debt and equity with renegotiation costs and equityholder offers will then equal: $L(p|p_{c1})$ and $V(p|p_{c1}) \equiv W(p|p_{c1}) - C(p|p_{c1}, p_s) - L(p|p_{c1})$ where p_{c1} is the root of the smooth-pasting condition $V'(p_{c1}|p_{c1}) = 0$.

Since debt levels with equityholder offers are again independent of the closure trigger,[7] it follows that the trigger price that maximizes $V(p|p_{c1})$ is the same as that which maximizes $W(p|p_{c1}) - C(p|p_{c1}, p_s)$. In other words, the new closure point is *constrained Pareto optimal*, i.e., efficient subject to the inclusion of renegotiation costs. The fact that debt values are completely unaffected by the inclusion of renegotiation costs under equityholder offers also implies that the entire incidence of the renegotiation cost is on equityholders.

It is not hard to see what the debt and equity values look like with debtholder offers. Again, the incidence will be entirely on the residual claimants, in this case the bondholders. Thus, equity values will equal those obtained

[7] Our notation, $L(p|p_{c1})$ suggests that $L(\cdot\,|\,\cdot)$ does depend on p_{c1}. However, p_{c1} just determines the lower end of the domain on which $L(\cdot\,|\,\cdot)$ is defined and $\partial L(p|p_{c1})/\partial p_{c1} = 0$.

in the model with no renegotiation, and the value of debt will be $W(p|p_{c2}) - C(p|p_{c2}, p_b) - \hat{V}(p)$, where p_{c2} is the root of $W'(p_{c2}|p_{c2}) - C'(p_{c2}|p_{c2}, p_b) = 0$.

B. Corporate Taxes

The above analysis is developed under the assumption that there is no tax advantage to leverage. It is straightforward to incorporate such a tax advantage by supposing, for example, that the income flow to equity is taxed at a rate τ but that interest payments are tax deductible. The firm's debt and equity may be valued in the same way as in previous sections except that the equity value must be multiplied by $1 - \tau$. The deductibility of coupon payments implies that debt values are precisely the same as in the absence of taxes.

The introduction of a tax advantage for debt does have one important implication stressed by Bergman and Callen (1991), however. The fact that agents know coupon payments will be renegotiated imposes an absolute ceiling on the value of debt that the firm can issue. If leverage has a tax advantage, it is optimal to issue as much debt as possible so this ceiling or "debt capacity" will act as a binding constraint. Renegotiation thus limits the degree to which the firm may exploit the tax deductibility of interest payments. In this case, the maximum firm value will be achieved if renegotiation is possible, but bondholders have maximum bargaining power. Increasing bondholder bargaining power raises the debt capacity and hence permits better use of tax deductibility.

C. Debt Principal

In Section I, we adopted the assumption that the debt principal equals b/r. If the firm's debt is risky, its value at flotation will be strictly less than b/r, implying that the debt is sold at a discount. The motivation for this assumption is as follows. So far in our discussion, we have only considered renegotiation when the firm's finances are relatively weak, i.e., in an interval above the closure point. However, if the debt is sold at par, then there will be opportunities for renegotiation when the firm is doing well.[8] In fact, for any $p_t > p_e$ where p_e is the output price prevailing when the firm is set up, $\hat{L}(p_t)$ will exceed the par value if the latter is $\hat{L}(p_e)$ and hence there will be surplus for equityholders to extract.

In reality, call provisions may reduce the scope for renegotiation when the firm is doing well. Technically, it would be possible to incorporate some types of call provisions in our model but this would not add much to the analysis. We, therefore, adopt the simplifying assumption that the debt principal is no less than b/r. Since, for all p_t, $L(p_t) \leq b/r$, this rules out renegotiation at high levels of the state variable.

[8] We thank Anthony Neuberger for pointing this out.

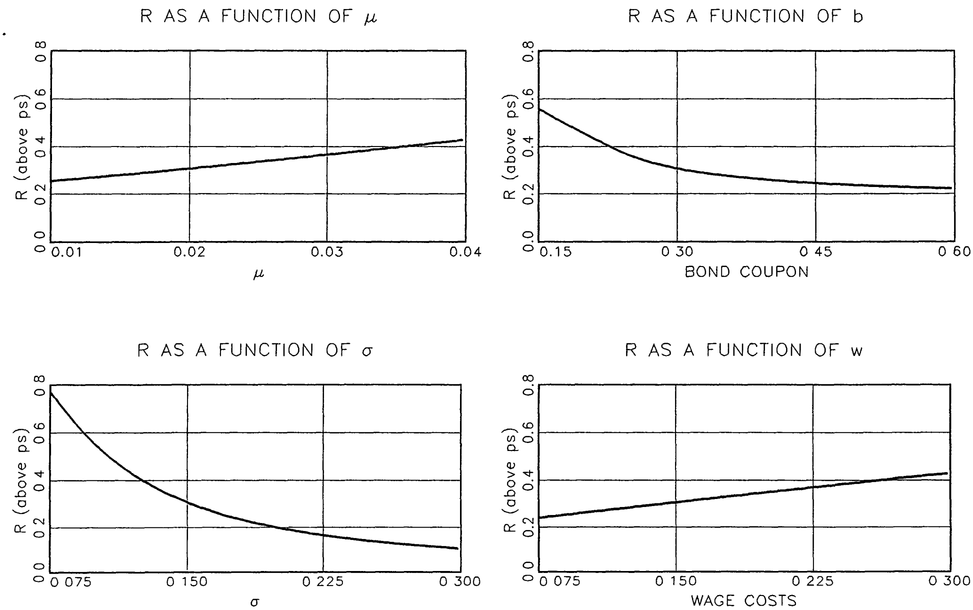

Figure 6. The contribution of strategic debt service to default premia. R is the fraction of the total premium over the safe return attributable to strategic debt service when equityholders can make take-it-or-leave-it offers. R is constant for output prices, p, exceeding the trigger for renegotiation, p_s. The plots show R for $p > p_s$.

IV. Renegotiation and Risk Premia

A. Decomposing the Risk Premium

The fraction of corporate debt risk premia attributable to renegotiation may be measured by the ratio $R \equiv [\hat{L}(p_t) - L(p_t)]/[b/r - L(p_t)]$. Note the interesting fact that, for any $p_t \in [p_s, \infty)$, $R = 1 - (p_s/\hat{p}_b)^{\lambda}[b/r - X(\hat{p}_b)]/[b/r - X(p_s)]$ is independent of p_t. In other words, the proportional contribution of equityholder take-it-or-leave-it offers to corporate risk premia is constant for all prices to the right of the renegotiation region.

Figure 6 shows plots of R against parameter values ranging from half to twice the baseline levels. Our choices of baseline parameters are justified in the Appendix. For the baseline parameter values (see the Appendix for details), R equals just over 0.3. The ratio is highly sensitive to volatility but relatively impervious to changes in other parameters. One may interpret the sensitivity of R to the various parameters as follows.

B. Volatility Effects

For a fixed residual value, $X(p_t)$, higher σ reduces the relative contribution of renegotiation due to three effects. First, one may think of the corporate debt premium as embodying option values associated with coupon cuts and bankruptcy. The latter is further "out of the money" if the firm is at a p_t greater than

p_s. Raising volatility increases the impact of the more "out-of-the-money" component.

Second, as σ increases, the coupon cut that shareholders can extract from debtholders declines since the continuation value of the debt is reduced. Third, increasing σ shifts up the residual, postbankruptcy value of the firm, $X(p)$. This increases the effective bargaining power of debtholders, allowing them better to resist equityholders' demands for coupon reductions, and hence reduces the contribution of strategic debt service to bond risk premia.

Together these three effects mean that the impact of renegotiation shrinks to little more than zero for very high volatility levels. Since all three effects operate in the same direction, it is not surprising that R is highly sensitive to volatility. Even though the baseline R is 0.3, it is very easy to generate much higher values of R by focusing on firms with relatively low asset value volatility.

C. Other Comparative Statics

Another comparative static depicted in Figure 6 is the impact of changes in the coupon flow parameter, b. Increasing b has two effects on the value of the debt without renegotiation. For high p, it raises the debt value for obvious reasons. For low p, a debt Laffer curve effect operates in that coupon increases accelerate bankruptcy and hence reduce debt value. These opposing effects mean that the continuation value of the debt may rise or fall when b increases. For the current parametrization, raising b boosts the continuation value of the debt. Both the size of the renegotiation range and the cut in coupons that equityholders increase and hence the impact of renegotiation on bond risk premia rises.

Lastly, higher wage costs reduce the postbankruptcy value of the firm, which should tend to increase the scope for strategic debt service. Higher wage costs also increase the closure price, p_c, which may partially offset this effect. For the current parametrization, the first effect dominates as one may see in the plot.

V. Conclusion

Our study shows that, if equityholders are able to make debtholders take-it-or-leave-it offers of coupon payments, then when the firm is in financial distress, debt service payments will vary stochastically and debt values will approximate the firm's liquidation value.[9] This situation will persist until the firm is either liquidated or recovers sufficiently to force equityholders to pay the full, initially contracted interest payment.

It is important to stress that our analysis does *not* imply that strategic debt service *only* occurs when the firm's finances are weak. If bankruptcy costs are

[9] Renegotiation can arise even when the firm is quite healthy if the residual value of the firm after bankruptcy is sufficiently low. So long as direct bankruptcy costs are not too high, renegotiation only occurs when the firm is in financial distress.

substantial and scrapping values low, equityholders can extract concessions from creditors even when the firm is making significant profits. In the limit, if bondholders are unable to run the firm profitably without the equityholders' cooperation (say because the direct bankruptcy parameter $\xi_1 \to 0$), then there is no link between the firm's ability to pay and actual debt service.[10] Whatever the level of contracted coupon payments, b, equityholders will pay $r\gamma$, the income flow for which the capitalized value equals the scrapping price of the firm. In this case, the firm's debt capacity is simply the firm's scrapping value, γ. This finding is in conformity with the observation in Myers (1977) that debt capacity is determined by the book value of a firm's assets since the latter is a marker for the liquidation value.

Other interesting findings that emerge from our analysis include the following.

1. Renegotiation of interest payments may eliminate both direct bankruptcy costs and agency costs of debt. This result is demonstrated by Bergman and Callen in a static model of optimal capital structure.
2. In our model, strategic debt service exists for some level of the state variable, p_t, whenever debt is risky and there are nonzero direct costs of bankruptcy.
3. Bankruptcy costs in our model are never actually incurred if renegotiation is possible. They nevertheless can have an important impact on debt pricing, since they determine the size of the concessions equityholders can extract from bondholders.
4. When a firm is not renegotiating its debt service, the proportion of the debt premium attributable to the possibility of strategic debt service is a constant, independent of the state variable. For typical parameter values this proportion is sizeable, being around 30% to 40%.
5. For firms with relatively low asset value volatility, this percentage increases substantially. This squares with the fact that it is the relatively large default premia on high quality corporate bond yields that standard models find particularly hard to replicate.
6. When direct bankruptcy costs go to zero, equityholders can do no better than in the absence of renegotiation even if they are able to make take-it-or-leave-it offers to bondholders and hence have maximum bargaining power.
7. The optimal debt service paid by equityholders who can make take-it-or-leave-it offers, $s(p_t)$, is a piecewise linear function of the output price. $s(p_t)$ exhibits discontinuities at p_s, the price at which renegotiation commences, and at points at which the second derivative of bondholders' outside option changes.

[10] In this, our model resembles typical analyses of sovereign debt. In sovereign defaults, lenders do not even have access to a liquidation value, and borrowers can only be obliged to repay the capitalized value of the costs of default that lenders can inflict upon them.

8. $s(p_t)$ may go negative when the firm is close to liquidation, in which case debtholders are injecting cash into the firm. Whenever this occurs, equityholders receive a nonnegative income flow.
9. For p just greater than the efficient liquidation point, p_c, debt service is higher if bondholders rather than equityholders can make take-it-or-leave-it offers (i.e., $q(p) > s(p)$ for p just larger than p_c). On the other hand, debt values are clearly higher when bondholders have the greater bargaining power. This result underlines the fact that agents care about values (i.e., discounted expected future income flows) and that extracting the highest value may entail accepting a relatively low income flow for some levels of the state variable, p_t.

Appendix

Proof of Proposition 1: One may easily verify that the general solution of equation (5) is

$$W(p) = \frac{p}{r-\mu} - \frac{w}{r} + A_1 p^{\lambda_1} + A_2 p^{\lambda_2}, \tag{A1}$$

where λ_1 and λ_2 are the negative and positive roots, respectively, of the quadratic equation, $\lambda(\lambda - 1)\sigma^2/2 + \lambda\mu = r$ and A_1 and A_2 are constants to be determined from the boundary conditions. The absence of bubbles in asset prices implies that $\lim_{p\to\infty} W(p) = p/(r-\mu) - w/r$. But as $p \to \infty$, p^{λ_2} is explosive, hence A_2 must equal zero. Suppose that liquidation occurs when p_t hits some low level p_c. In the absence of arbitrage (which precludes jumps in asset values), $W(p_c) = \gamma$. Hence, $A_1 = p_c^{-\lambda_1}[p_c/(r-\mu) - (w + r\gamma)/r]$ and for given p_c, $W(p)$ is uniquely determined, and equals the expression in equation (6). If p_c is chosen to maximize $W(p)$, the usual optimality condition implies that the derivative of $W(p)$ at p_c must equal that of the function to which it switches. Since, the scrapping value γ is constant, $W'(p_c) = 0$, which then implies the value for p_c given in equation (8). Applying identical arguments to $X(p)$ and p_x yields the expressions in equations (7) and (9). Q.E.D.

Proof of Proposition 2: If $\gamma > b/r$, bankruptcy can never occur, as the residual value of the firm's assets always exceeds the face value of the debt. In this case, the debt is riskless so $\hat{L}(p) = b/r$. Since equityholders are the residual claimants in all states of the world, the total firm value is $W(p)$. By the additivity of value in frictionless, perfectly competitive asset markets, $\hat{V}(p) = W(p) - \hat{L}(p) = W(p) - b/r$.

Now, consider the valuation of debt and equity when $\gamma < b/r$. Clearly, the firm will go bankrupt if p_t falls sufficiently far, since liquidation will at some point be preferable to continued operation and at this point the residual value of the firm will be insufficient to pay off bondholders. To derive debt and equity values, consider equations (11) and (10). The general solutions to these equations are:

$$\hat{L}(p) = \frac{b}{r} + C_1 p^{\lambda_1} + C_2 p^{\lambda_2} \tag{A2}$$

$$\hat{V}(p) = \frac{p}{r-\mu} - \frac{w+b}{r} + D_1 p^{\lambda_1} + D_2 p^{\lambda_2}. \tag{A3}$$

The asymptotic conditions, $\lim_{p\to\infty} \hat{L}(p) = b/r$, and $\lim_{p\to\infty} \hat{V}(p) = p/(r-\mu) - (w+b)/r$ imply that C_2 and D_2 are zero. Determining C_1 and D_1 from the value-matching conditions at bankruptcy, $\hat{L}(p_b) = X(p_b)$ and $\hat{V}(p_b) = 0$ yields the expressions in equations (14), (13), and (16). Since p_b is chosen to maximize the value of the levered firm's equity, it must satisfy $\hat{V}'(p_b) = 0$. Solving this equation for p_b yields the expression in equation (15). Q.E.D.

Proof of Proposition 3: Again, there are two cases. If $\gamma > b/r$, debt is riskless and the firm is liquidated efficiently by equityholders when p_t first equals p_c. When $\gamma < b/r$, the analysis is more interesting. Suppose that Hypothesis 1 applies and that p_c is less than p_s where p_s is the trigger for renegotiation assumed in the Hypothesis. For $p < p_s$, $L(p) = X(p)$. For $p \geq p_s$, $L(p)$ satisfies equation (22) with $s(p)$ set equal to b. The general solution to equation (22) is that given in equation (A2). Again, a no-bubbles condition implies that $C_2 = 0$. C_1 and p_s are determined by the no arbitrage conditions $L(p_s) = X(p_s)$ and $L'(p_s) = X'(p_s)$. (Note that continuity of $L'(p)$ at p_s has nothing to do with optimality. Claim values must have continuous derivatives at reversible switch points, since kinks permit arbitrage gains.) Simple derivations based on these conditions yields the expression for $L(p)$ and p_s given in equations (25) and (26).

The additivity of claim values implies that $V(p) = W(p|p_c^*) - L(p)$ where $W(p|p_c^*)$ is defined to be the same as $W(p)$ given in equation (6) but with p_c replaced with the liquidation trigger p_c^*. Maximizing $V(p)$ with respect to p_c^* is the same as maximizing $W(p|p_c^*)$ with respect to p_c^* since $L(p)$ is independent of the closure trigger. This implies that $p_c^* = p_c$. Finally, to ensure that the solution makes sense, one must check the ordering of p_c and p_s. When $b/r < \gamma$, inspection of equations (8) and (26) shows that $p_c < p_s$ as required. Q.E.D.

Proof of Proposition 4: To prove Proposition 4, we first need to establish a lemma.

LEMMA 1: *Consider two assets whose prices, $Y_1(p)$ and $Y_2(p)$, are functions of a stochastic process, p_t, with almost surely continuous sample paths. Assetholders receive income flows $f_1(p)$ and $f_2(p)$. Suppose there exist scalars p_1 and p_2 where $p_1 < p_2$ such that $Y_1(p_1) = Y_2(p_1)$ and $Y_1(p_2) = Y_2(p_2)$ and $f_1(p) \geq f_2(p)$ for all $p \in [p_1, p_2]$. If agents are risk neutral, then $Y_1(p) > Y_2(p)$ for all $p \in [p_1, p_2]$.*

Proof of the Lemma: for all $p_t \in [p_1, p_2]$

$$Y_1(p_t) - Y_2(p_t) = E_t\left[\int_t^{\tau} (f_1(p_s) - f_2(p_s))\exp[-r(s-t)]\, ds\right] \tag{A4}$$

where τ is the stopping time at which p_t first hits either p_1 or p_2. The result follows from the nonnegativity of the integrand.[11] Q.E.D.

Proof of the Proposition: Let $L^r(p)$ and $V^r(p)$ denote the values of debt and equity under risk neutrality for some $(p_b^r, s^r(\cdot)) \in \mathcal{A}$. Let $L(p, p_b^r)$ be the value of debt when the service flow is $s(p)$ but liquidation occurs at p_b^r rather than p_b. Note (i) that $L(p_b^r, p_b^r) = L^r(p_b^r, p_b^r) = \min\{X(p_b^r), b/r\}$, and (ii) that as $p \to \infty$, $X(p) \to \infty$, so that $\lim_{p\to\infty} L^r(p, p_b^r) \geq b/r = \lim_{p\to\infty} L(p, p_b^r)$. (i) and (ii), together with continuity, imply that if there exists p such that $L^r(p, p_b^r) < L(p, p_b^r)$, then there exist p_1, p_2 where $p_1 < p_2$ (and where p_2 is possibly ∞), such that for all $p \in [p_1, p_2)$, $L^r(p, p_b^r) < L(p, p_b^r)$ and $L^r(p_1, p_b^r) = L(p_1, p_b^r)$ and $\lim_{p\to p_2} [L^r(p, p_b^r) - L(p, p_b^r)] = 0$. But, if $L^r(p, p_b^r) < L(p, p_b^r)$ for $p \in [p_1, p_2]$, then $s^r(p) = b$ in this range since otherwise debtholders would trigger liquidation. But Lemma 1 implies $L^r(p, p_b^r) \geq L(p, p_b^r)$ for all $p \in [p_1, p_2]$, which is a contradiction. Hence, $L^r(p, p_b^r) \geq L(p, p_b^r)$ for all $p \geq p_b^r$, which implies that $V^r(p, p_b^r) \leq V(p, p_b^r)$ for all $p \geq p_b^r$. But, $V(p, p_b) \geq V(p, p_b^r)$ for all $p \geq p_b$ by the optimality of p_b. Q.E.D.

The Renegotiation Cost Function, $C(p|\underline{p}, \bar{p})$, has the general solution

$$C(p|\underline{p}, \bar{p}) = \begin{cases} \delta/r + C_{11}p^{\lambda_1} + C_{12}p^{\lambda_2} & \text{for} \quad p \in [\underline{p}, \bar{p}) \\ C_{21}p^{\lambda_1} + C_{22}p^{\lambda_2} & \text{for} \quad p \in [\bar{p}, \infty). \end{cases} \tag{A5}$$

Boundary conditions $\lim_{p\uparrow\bar{p}} C(p|\underline{p}, \bar{p}) = C(\bar{p}|\underline{p}, \bar{p})$, $\lim_{p\uparrow\bar{p}} C'(p|\underline{p}, \bar{p}) = C'(\bar{p}|\underline{p}, \bar{p})$, $C(\underline{p}|\underline{p}, \bar{p}) = 0$, and $\lim_{p\to\infty} C(p|\underline{p}, \bar{p}) = 0$, determine the constants C_{11}, C_{12}, C_{21}, and C_{22}. Q.E.D.

Justification of Baseline Parameters for Simulations

The parameter σ is chosen so that the volatility of the equity price when $p_t = p_s$ is approximately 15 percent. This figure is consistent with data on typical annualized equity return standard deviations. Lastly, in selecting w, it is important to recognize that the interpretation of w in the article as irrecoverable flow costs is made simply to aid exposition. In fact, w measures the deviation from log-normality of the distribution of changes in our earnings latent variable (i.e., if $w = 0$, earnings, $p_t - w$, are lognormal). Hence, it is not really appropriate to select w by examining data on costs (as, for example, in Dixit (1989)). We experiment with various values but report a simulation for w equal to 0.15, i.e., for a latent variable reasonably close to log-normality.

[11] Note that the proof of the lemma relies on a dynamic form of first order stochastic dominance.

We employ a scrapping value, γ, of 2 and suppose that the direct costs of bankruptcy are 20 percent (so $\xi_0 = 0.8$ and $\xi_1 = 1.25$). Note that when renegotiation occurs, these costs are never realized but instead simply allow equityholders to extract concessions by threatening bankruptcy. Estimates of realized direct bankruptcy costs are typically much lower for large industrial and commercial companies but this is quite consistent with our model. Lastly, we choose a safe interest rate of 6 percent, approximately equaling the current U.S. prime rate. The share of the bond risk premium associated with strategic debt service, R, is relatively insensitive to the growth rate in the output price, μ, and the coupon flow, b. We therefore simply set μ at a reasonable number of 2 percent and choose a value of b of 0.3.

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