



American Finance Association

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Source: *The Journal of Finance*, Vol. 51, No. 4 (Sep., 1996), pp. 1499-1522

Published by: Blackwell Publishing for the American Finance Association

Stable URL: <http://www.jstor.org/stable/2329402>

Accessed: 25/11/2009 10:19

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Relative Pricing of Eurodollar Futures and Forward Contracts

MARK GRINBLATT and NARASIMHAN JEGADEESH*

ABSTRACT

Past research explains observed spreads between futures and forward Eurodollar yields as being due to the futures contract's mark-to-market feature. We derive closed form solutions for this yield spread and show that, theoretically, it should be small. Also, differences in liquidity, taxation, and default risk cannot account for the large spreads observed. We also present evidence that the spreads, which are nonnegligible primarily in the first half of the sample period, are likely to be attributable to the mispricing of futures contracts relative to the forward rates and that the mispricing was gradually eliminated over time.

FINANCE ACADEMICS AND PRACTITIONERS have long been interested in understanding the relative pricing of "twin" futures and forward contracts.¹ In addition to an abundant theoretical literature, exemplified by Cox, Ingersoll, and Ross (1981), there are numerous empirical studies of relative pricing for several commodities and financial instruments. Significant differences between futures and (generally implicit) forward prices, however, have been documented only in the case of interest rate futures, which we focus on here.²

The extant empirical literature on futures and forward interest rate pricing differences does not dispute that a significant price difference exists, but it disagrees about the explanation offered for the phenomenon. Elton, Gruber,

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¹ The primary contractual difference between futures and forward contracts is that futures contracts are marked-to-market on a daily basis while the forward contracts are settled only at the maturity date.

² Examples of studies that examine the relative prices of futures and forward contracts besides interest rate contracts are French (1983) (copper and silver contracts), Cornell and Reinganum (1981) (foreign exchange), and Cornell and French (1983) (stock index). Cornell and French report differences between the stock index forward and futures prices, but studies that cover more recent sample periods (e.g., MacKinlay and Ramaswamy (1988)) do not find significant differences between forward and futures stock index prices.

and Rentzler (1984), Park and Chen (1985), and Kolb and Gay (1985) attribute significant Treasury bill futures-forward price differences to market inefficiency. More recent studies, however, argue that differences between futures and forward interest rates arise either from the mark-to-market feature of the futures contracts (e.g., Meulbroek (1992) and Sundaresan (1991) for Eurodollar contracts) or from differences in liquidity (e.g., Kamara (1988) for Treasury Bill contracts). The paper by Muelbroek, which is closest to ours, notes that the futures-forward yield spread is broadly consistent with the Cox, Ingersoll, and Ross (1981) futures-forward model. In particular, she finds that the percentage yield difference is correlated with the covariance between changes in T-bill prices (a proxy for the risk-free rate) and forward prices from the prior month.

This paper compares Eurodollar futures prices with the forward prices implied by the term structure of spot London Interbank Offered Rates (LIBOR) over the 1982–1992 period. It finds that the yields computed from all but the nearest maturing futures contracts exceeded the implicit forward yields computed from the term structure of spot LIBOR by over 100 basis points in 1982. This yield spread trended downward, albeit somewhat erratically, yet remained surprisingly large until 1985–1987 (depending on the maturity of the futures and forward contract). Since then, LIBOR futures and forward rates have been virtually the same.

The article also provides a detailed analysis of different hypotheses that could potentially explain both the futures-forward yield difference and its pattern over time. By employing term structure models, we analytically show that the theoretical differences between the futures and forward Eurodollar rates due to the mark-to-market feature are small. Differential default and liquidity risk are shown to imply a futures-forward yield differential with a sign opposite to that observed, and thus cannot explain these differences either (although differential default risk accounts for a very small, but statistically significant proportion of the weekly variation in the yield spread). Finally, we show that differential taxation has little effect on the futures-forward yield spread.

These findings indicate that relative mispricing is the likely explanation for the large futures-forward yield differences that were observed in the early part of the sample period. To further assess this mispricing hypothesis, we examine the timeliness of information flow across the futures and forward markets using lead-lag regressions. We find that in the first part of our sample period, lagged changes in futures yields predict changes in forward yields. These findings are consistent with market inefficiency in the first part of our sample period. No such lead-lag relation is found in the later part of our sample period, which suggests more timely information flow across the two markets in recent times.

The organization of the article is as follows: Section I presents data on the futures-forward yield spread and how it has evolved over time. Section II derives theoretical futures-forward yield spreads under two popular term structure models and shows that they cannot explain the magnitude of the spreads observed in Section I. Section III analyzes default risk, liquidity, and

taxation differences as possible explanations for the yield spread. Section IV presents evidence, supportive of an inefficient markets explanation, based on a delay in the flow of information between the futures and forward markets. Section V discusses market frictions that might limit the extent of the arbitrage. Section VI briefly concludes the article.

I. Futures-Forward Yield Differences

A. Data and Notation

We use Eurodollar futures prices and 1-, 3-, 6-, 9-, and 12-month LIBOR quotations in our analysis. Daily Eurodollar futures prices are obtained from the Chicago Mercantile Exchange, and daily spot LIBOR quotations are obtained from Data Resources, Inc. (DRI).³ Our sample period is from April 1982 through December 1992.⁴

The following notation (with time measured in years) is used in the article:

$L_s(y)$ = y -year LIBOR rate prevailing at time s ($s = 0$ if the subscript is omitted).⁵

$d(s, \tau)$ = LIBOR conversion factor; defined as $360/(\text{number of days between } s \text{ and } \tau)$, $\tau > s$.

$P(s, \tau)$ = $1/[1 + L_s(\tau - s)/d(s, \tau)]$. The time s price of \$1 paid at τ in the Eurodollar market, $\tau > s$.

$f(s, \tau)$ = Annualized Eurodollar forward rate at time 0 for the interval s to τ .

$F(s, \tau; t)$ = Annualized Eurodollar futures rate at t for the interval s to τ . (t is omitted when $t = 0$.)

$r(t)$ = Annualized instantaneous interest rate at time t (i.e., the short rate).

B. Forward and Futures Rates

Three-month implied forward rates are computed from LIBOR spot quotations using the formula⁶

$$f(s, s + 0.25) = d(s, s + 0.25)[P(0, s)/P(0, s + 0.25) - 1]. \quad (1)$$

For comparisons with the futures rates, we compute the 3-month forward rates three months hence, $f(0.25, 0.5)$, six months hence, $f(0.5, 0.75)$, and nine

³ The DRI data are averages of LIBOR quotations from several banks, collected through Reuters.

⁴ The closing prices of the fourth nearest contract, which we need for our analysis, are consistently available beginning in April 1982.

⁵ LIBOR is conventionally quoted on a 360-day simple interest basis.

⁶ For notational convenience, we are loosely using 0.25, 0.5, and 0.75 years in the formula to respectively represent three months, six months, and nine months, whereas our actual calculations are based on proper day counts.

months hence, $f(0.75, 1)$, using spot quotations for 3-, 6-, 9-, and 12-month LIBOR.⁷

We use the daily closing price of the futures contract that matures on date s to compute the futures rate from the expression

$$F(s, s + 0.25; t) = 1 - \text{Futures Price}_t/100. \quad (2)$$

The futures contracts mature in a quarterly cycle on fixed dates, while the daily spot rate quotations, used to compute the forward rates, are available only for fixed terms. The futures rate intervals, therefore, do not in general coincide with the forward rate intervals.

Two interpolation methods are used to align the intervals. With the "futures interpolation method," we fit a cubic spline to the futures rates of the four nearest maturing contracts to construct an interpolated "term structure of futures rates."⁸ From this curve, we pick interpolated futures rates for intervals that coincide with the forward rate intervals. The interpolated futures rates, $F(0.25, 0.5)$, $F(0.5, 0.75)$, and $F(0.75, 1)$, are then compared with the implied forward rates, $f(0.25, 0.5)$, $f(0.5, 0.75)$, and $f(0.75, 1)$. The second interpolation method, "the spot LIBOR interpolation method," is consistent with the procedure used in Meulbroeck. This method applies a cubic spline to the spot LIBOR curve to obtain the entire "term-structure of spot rates." Each interpolated LIBOR rate corresponds to a zero coupon bond price. Each of the three nearest maturing futures contracts, with futures rate $F(s, s + 0.25)$, is then compared with an implied forward rate, $f(s, s + 0.25)$, computed from these zero coupon bond prices using equation (1).

Table I presents results on the Eurodollar futures-forward yield differences. We denote DIFF0.25_0.5, DIFF0.5_0.75, and DIFF0.75_1 to be the differences between the corresponding 3-month futures and forward rates when the futures interpolation method is used. For example,

$$\text{DIFF0.25_0.5} = F(0.25, 0.5) - f(0.25, 0.5).$$

Similarly, we denote DIFF1, DIFF2, and DIFF3 to be the difference for the nearest, second nearest, and third nearest maturing futures contract when the forwards are computed from interpolated spot rates. The mean values of DIFF0.25_0.5 and DIFF1 over the entire sample period are 3.64 and -2.57 basis points (bp), respectively. The average spread becomes large and positive as we increase maturity. For DIFF0.5_0.75 and DIFF2, the mean values are

⁷ LIBOR payments due are typically settled on the same day they are due via wire transaction. Some contracts are settled via the Fed wire. During the early and most interesting part of our sample period, most of the spot LIBOR obligations were settled with the NY Clearing House Interbank Payments System, (CHIPS).

⁸ Cubic splines are commonly used for interpolation in the term structure literature (e.g., McCulloch (1971)). The cubic spline used in this article is fitted with the IMSL subroutine. To check the accuracy of the cubic spline interpolation, we generate the futures term structures under the Cox, Ingersoll, Ross, and Vasicek models for a variety of parametrizations. In all cases, the differences between the interpolated values and the theoretical futures rates are less than one basis point.

Table I
Futures-Forward Yield Differences

This table presents the differences between the futures and forward Eurodollar yields expressed in basis points. It employs weekly (Thursday) data from April 1982 through December 1992.

In Panel A, implied forward yields are computed from quoted LIBOR rates and futures yields are obtained by interpolating between the futures transaction prices. DIFF0.25_0.5 is the time t difference (with time expressed in years) between the annualized futures and forward yields for the interval $t + 0.25$ to $t + 0.5$ and DIFF0.5_0.75 and DIFF0.75_1 are the corresponding yield differences for the intervals $t + 0.5$ to $t + 0.75$ and $t + 0.75$ to $t + 1$, respectively. N is the number of observations. The t -statistics are presented in parentheses.

In Panel B, futures rates are computed directly from the first three Eurodollar futures contracts, and the implied forward yields over the corresponding period are computed from the interpolated LIBOR yield curve. DIFF1 is the difference between the annualized 3-month futures and forward yields on the date of maturity of the nearest maturity futures contract; DIFF2 is the difference between annualized 3-month futures and forward yields on the date of maturity of the next-to-nearest maturity futures contract, etc. The t -statistics are presented in parentheses.

Panel A.							
Year	DIFF0.25_0.5		DIFF0.5_0.75		DIFF0.75_1		N
	Mean	Median	Mean	Median	Mean	Median	
8204-9212	3.64 (5.25)	2.62	17.81 (12.06)	8.32	35.03 (21.26)	23.12	561
8204-8706	7.84 (6.57)	6.43	36.16 (14.93)	26.64	60.26 (24.46)	56.06	273
8707-9212	-0.35 (-0.53)	-0.81	0.40 (0.44)	0.30	11.11 (12.71)	11.57	288
Panel B.							
Year	DIFF1		DIFF2		DIFF3		
	Mean	Median	Mean	Median	Mean	Median	
8204-9212	-2.57 (-4.18)	-2.71	11.69 (11.01)	6.21	26.60 (16.60)	15.26	
8204-8706	-2.08 (-1.87)	-2.93	23.84 (13.77)	17.00	48.82 (19.00)	42.78	
8707-9212	-3.05 (-5.28)	-2.63	0.18 (0.22)	-0.28	5.53 (6.87)	5.54	

17.81 and 11.69 bp respectively and for DIFF0.75_1 and DIFF3, the respective means are 35.03 and 26.60 bp. Thus, the futures rates are typically significantly larger than the corresponding forward rates. The only exception is DIFF1, which is negative, but small, in almost every year.

Table I also presents the results for two subperiods: April 1982 to June 1987 and July 1987 to December 1992. The first subperiod roughly corresponds to the sample period covered by Meulbroek (1992). While the average difference between futures and forward rates appears to be fairly large in the earlier sample period, it is significantly smaller in the latter period.

Figure 1 plots the 12-week moving average of DIFF0.25_0.5, DIFF0.5_0.75, and DIFF0.75_1 along with their 95 percent confidence intervals. The

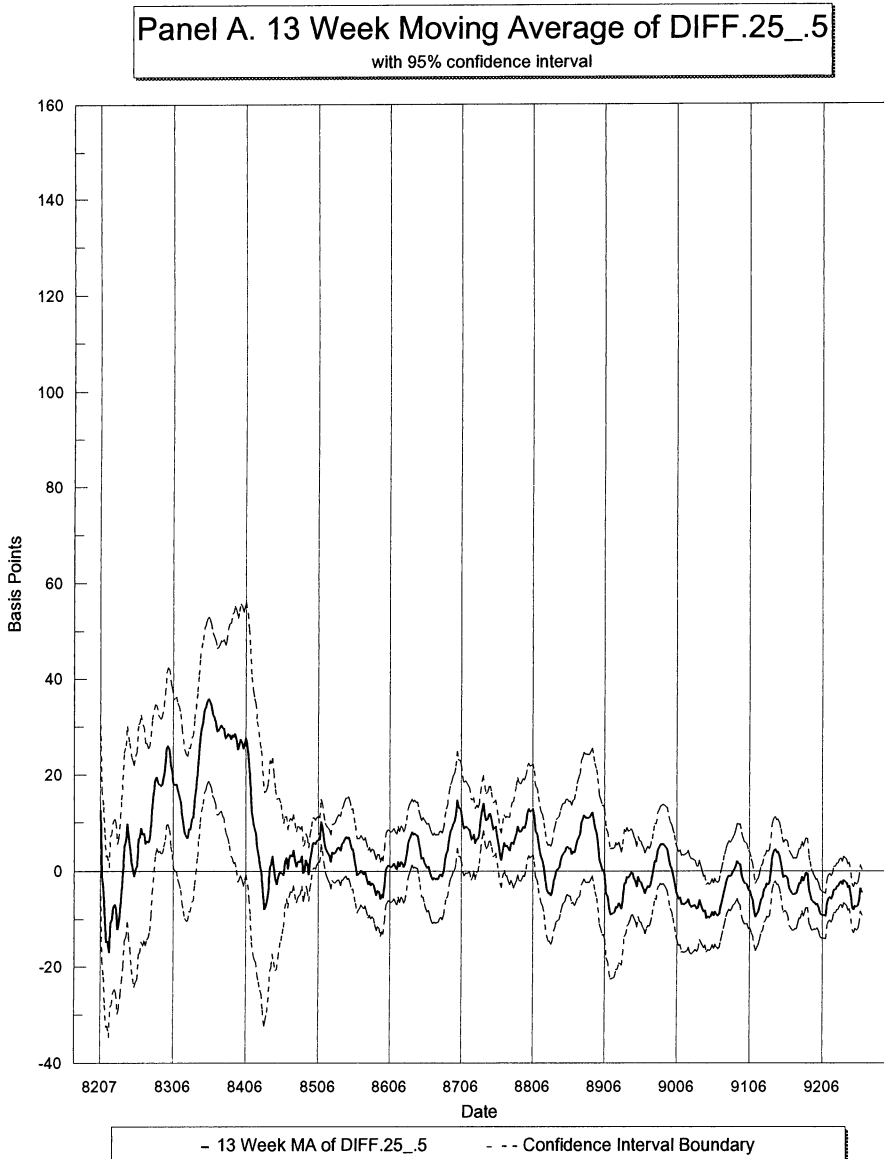
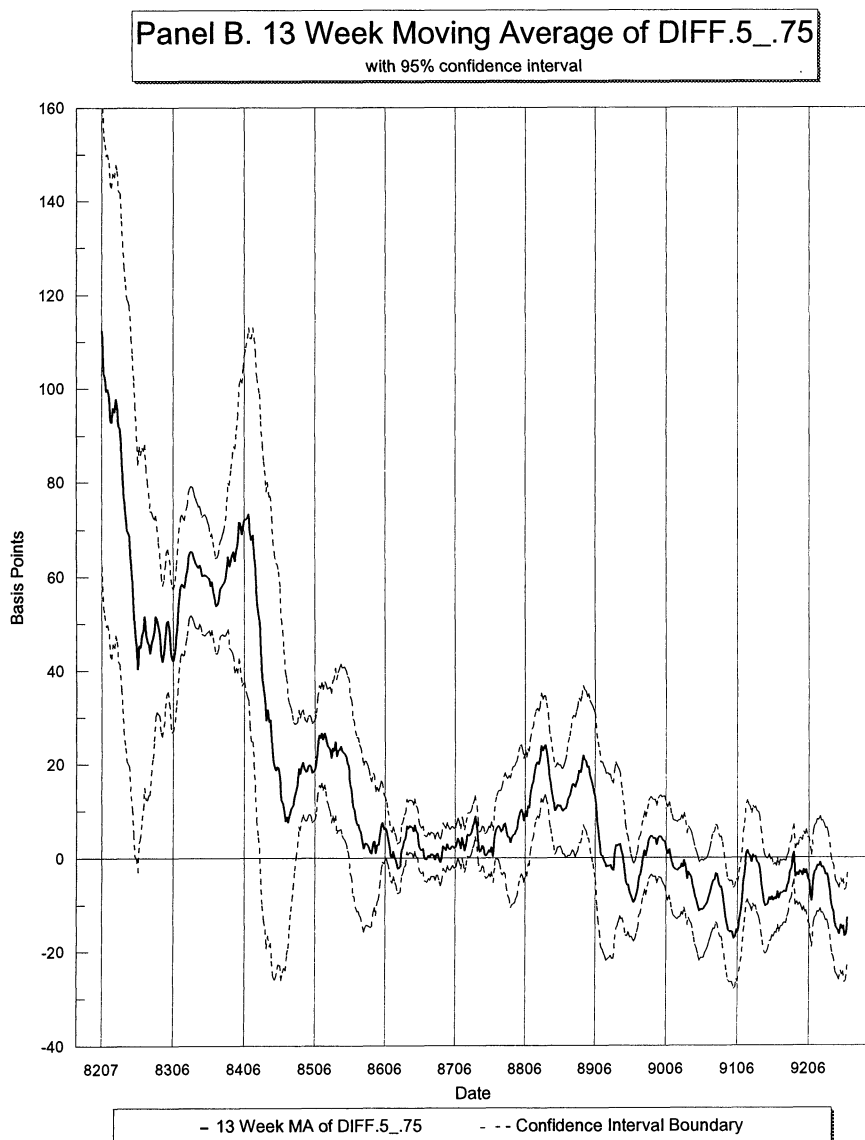
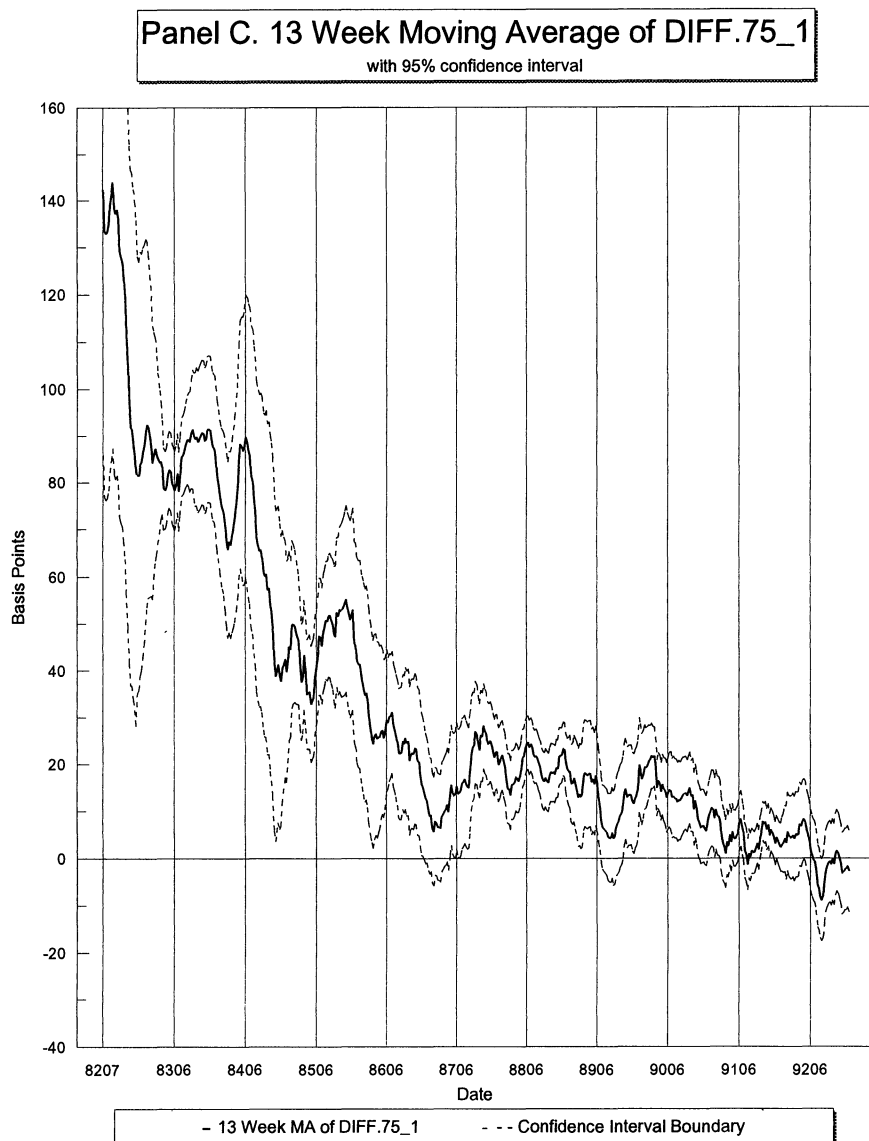


Figure 1. This figure plots a thirteen-week moving average of DIFF0.25_0.5 (Panel A), DIFF0.5_0.75 (Panel B), and DIFF0.75_1 (Panel C) using Thursday weekly closing data. DIFF0.25_0.5 is the time t difference (with time expressed in years) between the annualized futures and forward yields for the interval $t + 0.25$ to $t + 0.5$ and DIFF0.5_0.75 and DIFF0.75_1 are the corresponding yield differences for the intervals $t + 0.5$ to $t + 0.75$ and $t + 0.75$ to $t + 1$, respectively. Also plotted are 95 percent confidence intervals. The confidence intervals, computed from the moving average data, are distanced two standard deviations on either side of each moving average data point, and are computed using the 52 weeks that are nearest to the data point. The data for computing the moving average begin on April 7, 1982 and end on December 30, 1992.

**Figure 1—continued**

confidence intervals, computed from moving average data, are distanced two sample standard deviations on either side of each moving average, and are computed using the 52 weeks that are nearest to the data point (which means they are centered around the data point, except for the first 26 and last 26 observations). As this figure illustrates, the differences between the futures and forward rates for the longer maturities are remarkably large in the early years. For example, the moving averages of DIFF0.5_0.75 and DIFF0.75_1 in

**Figure 1—continued**

Panels *B* and *C* are over 100 basis points at the beginning of the sample period, and they gradually decline to close to zero over time.

The most interesting aspect of Panels *B* and *C* of Figure 1 is the general trend. After mid-1982, both series trend downward at a rapid rate until early 1985 for DIFF0.5_0.75 and early 1987 for DIFF0.75_1. At these points in time, the yield differences are fairly close to zero, and there is no noticeable trend later on. The evidence in the first subperiod led to the conclusions in earlier work that the

mark-to-market feature of the futures contract has significant pricing implications for interest rate contracts.

The level of the futures-forward rate difference and its variation over time can potentially be explained by a number of fundamentally different hypotheses. We explore these in the next two sections of the article.

II. The Mark-to-Market Feature and the Relative Pricing of Futures and Forward Contracts

This section examines the magnitude of the futures-forward rate differences that can potentially be attributed to the mark-to-market feature. We first compute closed-form solutions for the Eurodollar futures-forward rate difference using the Cox, Ingersoll, and Ross (1985) and Vasicek (1977) term structure models. We then compare plausible values of the theoretical futures-forward differences, based on empirical estimates of the models' parameters, with corresponding yield differences in the data.

The Cox, Ingersoll, and Ross (1985) model assumes that $r(t)$ follows the square root process,

$$dr = \kappa(\mu - r)dt + \sigma \sqrt{r} dz, \text{ where}$$

dr = the change in the instantaneous interest rate r ,
 μ = the unconditional mean of this "short rate,"
 κ = the parameter that governs the speed of mean-reversion, and
 σ = the instantaneous volatility parameter.

Cox, Ingersoll, and Ross (1985) show that under this process, the time t_1 prices of zero coupon bonds paying one dollar at time t_2 are given by:

$$P(t_1, t_2) = A(t_2 - t_1) \exp(-B(t_2 - t_1)r(t_1)), \quad (3)$$

where

$$A(x) = \left[\frac{2\gamma \exp[(\kappa^* + \gamma)x/2]}{(\kappa^* + \gamma)(\exp(\gamma x)) - 1} + 2\gamma \right]^{2\kappa^*\mu^*/\sigma^2},$$

$$B(x) = \left[\frac{2(\exp(\gamma x) - 1)}{(\kappa^* + \gamma)(\exp(\gamma x) - 1) + 2\gamma} \right],$$

$$\gamma = [(\kappa^*)^2 + 2\sigma^2]^{1/2},$$

$$\kappa^* = \kappa + \lambda,$$

$$\mu^* = \kappa\mu/(\kappa + \lambda), \text{ and}$$

$$\lambda = \text{the market price of interest rate risk.}^9$$

⁹ The λ parameter in the CIR model is a constant given the CIR log utility assumption.

Equation (3) implies that the forward rates in the CIR model are given by:

$$\begin{aligned} f(s, \tau) &= d(s, \tau)[P(0, s)/P(0, \tau) - 1] \\ &= d(s, \tau)\{[A(s)/A(\tau)]\exp[(B(\tau) - B(s))r(0)] - 1\} \end{aligned}$$

To avoid ambiguity in exposition, we will compare the forward rate with what we term the “futures rate,” which is the price of a futures contract on the LIBOR rate, i.e., a contract, (as defined earlier in Equation (2)), that is settled at $L_s(0.25)$ at maturity date s .¹⁰ Since there is no cash outlay at the time a futures contract is initiated, the expected change in the futures rate equals the premium for bearing interest rate risk. Therefore,

$$E(dF) = \lambda F_r r dt,$$

where subscripts denote partial derivatives. By Ito’s Lemma,

$$E(dF) = (F_r \kappa (\mu - r) + F_t + 1/2 F_{rr} \sigma^2 r) dt.$$

Hence, under the CIR interest rate process, the equilibrium futures rate should (using the notation above) satisfy the following partial differential equation:

$$F_r \kappa^* (\mu^* - r) + F_t + 1/2 F_{rr} \sigma^2 r = 0. \quad (4)$$

Proposition 1 solves this partial differential equation subject to the boundary condition

$$F(s, \tau; s) = L_s(\tau - s). \quad (5)$$

PROPOSITION 1: *Under the assumptions of the CIR model, the no-arbitrage Eurodollar futures rate at date 0 is*

$$F(s, \tau) = d(s, \tau)\{[1/A(\tau - s)]E[\exp(B(\tau - s)r(s))]\} - 1, \quad (6)$$

where

$$E[\exp(B(\tau - s)r(s))] = \frac{\exp[B(\tau - s)\exp(-\kappa^*s)r(0)/(1 - B(\tau - s)\sigma^2 b_C(s)/2)]}{[1 - B(\tau - s)\sigma^2 b_C(s)/2]^{2\kappa^*\mu^*/\sigma^2}}$$

and where

$$b_C(s) = (1/\kappa^*)[1 - \exp(-\kappa^*s)],$$

¹⁰ For analytic tractability, we assume that the futures contracts are marked-to-market every instant and are traded continuously in frictionless markets.

Proof: The solution to the partial differential equation (4) is given by the risk-neutral expectation of the terminal futures price.¹¹ In this case, with the boundary condition given by (5), the solution is:

$$F(s, \tau) = E[L_s(\tau - s)]$$

where E denotes expectation taken with respect to the distribution of $r(s)$ conditional on $r(0)$, with the dynamics of r given by the following "risk-neutral" process:

$$dr = \kappa^*(\mu^* - r)dt + \sigma\sqrt{r} dz.$$

CIR show that the distribution of $r(s)$ conditional on $r(0)$ given this stochastic process is a noncentral chi-square. The moment generating function for the non-central chi-square distribution has been computed by (among others) Richard (1978). The τ -s month LIBOR interest rate at futures maturity is given by

$$\begin{aligned} L_s(\tau - s) &= d(s, \tau)[1/P(s, \tau) - 1] \\ &= d(s, \tau)\{[1/A(\tau - s)][\exp(B(\tau - s)r(s))] - 1\}, \end{aligned}$$

Thus, $E[L_s(\tau - s)]$, as defined above, is a linear function of the moment generating function of $r(s)$ conditional on $r(0)$. Applying the results of Richard (1978) yields Equation (6). Q.E.D.

Intuitively, the mark-to-market feature resets the value of the futures contract to zero at each instant. By backward induction, it follows that the initial futures rate must equal the risk neutral expectation.

The Vasicek model assumes that $r(t)$ follows the Ornstein-Uhlenbeck process,

$$dr = \kappa(\mu - r)dt + \sigma dz.$$

Vasicek (1977) shows that under this process the prices of discount bonds are given by:

$$P(t_1, t_2) = a(t_1 - t_2)\exp(-b(t_1 - t_2)r(t_1)), \quad (7)$$

where

$$b(x) = (1/\kappa)[1 - \exp(-\kappa x)],$$

$$a(x) = \exp\{[b(x) - x][\mu^* - \sigma^2/(2\kappa^2)] - (\sigma^2/4\kappa)b(x)^2\},$$

$$\mu^* = \mu - \lambda\sigma/\kappa, \text{ and}$$

λ = the market price of interest rate risk per unit of σ , assumed constant.

¹¹ See Friedman (1975), Cox and Ross (1976), or Harrison and Kreps (1979).

Equation (7) implies that the Vasicek forward rate is

$$\begin{aligned} f(s, \tau) &= d(s, \tau)[P(0, s)/P(0, \tau) - 1] \\ &= d(s, \tau)\{[a(s)/a(\tau)]\exp[(b(\tau) - b(s))r(0)] \} \end{aligned}$$

The futures rate is provided below:

PROPOSITION 2: *Under the assumptions of the Vasicek model, the “no arbitrage” Eurodollar futures rate at date 0 is*

$$F(s, \tau) = d(s, \tau)\{[1/a(\tau - s)]E[\exp(b(\tau - s)r(s))] - 1\},$$

where

$$E[\exp(b(\tau - s)r(s))] = \exp\{b(\tau - s)E[r(s)] + 0.5b(\tau - s)^2\text{var}[r(s)]\},$$

$$E[r(s)] = \exp(-\kappa s)r(0) + (1 - \exp(-\kappa s))\mu^* \text{ and}$$

$$\text{var}[r(s)] = (\sigma^2/2\kappa)(1 - \exp(-2\kappa s)).$$

Proof: Analogous to Proposition 1.

We now calibrate the differences between the theoretical futures and forward rates using empirical estimates of the CIR and Vasicek models. Our goal is not to determine the best parameter estimates for these models, but rather to obtain a reasonably wide interval within which we can expect the parameters of the interest rate process to fall. The starting point for these parameters are estimates for the Vasicek model obtained with 1-, 3-, 6-, 9-, and 12-month LIBOR rates over the 1982–1992 sample period.

We cast the Vasicek term-structure model in a state-space framework and obtain the maximum likelihood parameter estimates using Kalman filtering. The discrete time state equation under the Vasicek model is

$$r_t = a + br_{t-\delta} + e_t$$

where δ is the discrete time interval, r_t is the unobservable instantaneous interest rate at time t . The relation between the Vasicek model parameters and the discrete-time state equation parameters are given by $a = 1 - e^{-\kappa\delta}$, $b = e^{-\kappa\delta}$, and $\sigma_e^2 = (\sigma^2/2\kappa)(1 - e^{-2\kappa\delta})$. The natural logarithm of annualized LIBOR determines the system of measurement equations. We use weekly observations (i.e., $\delta = 7/365$) of Thursday LIBOR for the five maturities (1, 3, 6, 9, and 12 months) to estimate the model parameters and impose the local expectations hypothesis (i.e., $\lambda = 0$) and assumptions of independent measurement errors for the five maturities in our estimation.¹² The Vasicek model estimates are $\kappa = 0.167$, $\mu = 0.074$, and $\sigma = 0.017$. The parameters κ and μ are comparable

¹² The algorithm used does not converge when λ is a free parameter. Therefore, we constrain this parameter under the local expectations hypothesis during estimation. In our later sensitivity analysis, however, we allow λ to vary over a wide range.

in the Vasicek and CIR models. To keep the other parameters consistent across the two models (so that we can compare the sensitivity of our results across models), we use the same κ and μ values for both models. A somewhat comparable volatility parameter for the CIR model is obtained from the formula

$$\sigma_C = \sigma_V / \sqrt{\mu},$$

where the subscripts C and V denote the parameters for the CIR and Vasicek processes, respectively. Using this relation, our estimate of $\sigma_C = 0.063$. As in the case of the Vasicek model, we assume the local expectations hypothesis and set $\lambda_C = 0$.

Table II presents the theoretical futures-forward yield spreads for different parametrizations of the CIR and Vasicek processes. We compute the differences, not only for the parameter values estimated above, but also for parameter values of κ , μ , and σ that are both zero and twice those estimated with the procedures above.^{13,14} Panels A and B present the futures-forward rate differences under the CIR and Vasicek models, respectively. The values presented assume $r(0) = 0.08$.

Using the aforementioned estimates of the parameters of the interest rate process, the CIR model theoretical values of DIFF0.25_0.5, DIFF0.5_0.75, and DIFF0.75_1 are respectively 0.290, 0.776, and 1.460 basis points. The futures-forward rate differences change by less than half a basis point when the parameters κ , μ , and λ are varied as discussed above. Changes in σ , however, have a significant effect on the rate differences. When σ is doubled, these differences increase by a factor of four.

The values of DIFF0.25_0.5, DIFF0.5_0.75, and DIFF0.75_1 under the Vasicek model are 0.256, 0.658, and 1.186 basis points respectively for the base case. These differences are of the same magnitude as that for the CIR model, and they vary in the same manner with changes in the parameters of the model. Here again, the futures-forward yield differences quadruple when we double the volatility parameter. In fact, when the futures and forward rates are expressed in continuously compounded form, one can show that there is an exact linear relation between the yield difference and the instantaneous variance of the short rate.

A comparison of Panels A and B of Table II indicates that the theoretical futures-forward yield differences are fairly similar under both the CIR and Vasicek models. Additional models explored, but not reported here, indicate that this result is not sensitive to the choice of a specific interest rate model.

The large yield differences observed here can be justified only under much larger interest rate volatility than the range we consider in Table II. The variance of interest rate changes would have to be 23 to 30 times larger

¹³ When $\kappa = 0$ for the Vasicek model, or when $\kappa = \lambda = 0$ for the CIR model, the expression for discount bond prices and the futures rates under the model are modified to their limiting values, which are analytically computed.

¹⁴ Since our base case λ_V and λ_C parameters equal zero, the variations we consider are λ_V equal to -0.01 and -0.02 and λ_C equal to -0.5 and -1 , which contains most of the estimates of these parameters found in the literature.

Table II

Theoretical Differences Between Futures and Forward Yields Under the Cox, Ingersoll, Ross, and Vasicek Models

This table presents the theoretical differences (in basis points) between the futures and forward yields under the assumptions that the interest rate dynamics are generated by the Cox, Ingersoll, and Ross model (Panel A) and the Vasicek model (Panel B). The parameters for the base case interest rate processes are estimated with Kalman filter maximum likelihood techniques for the Vasicek model under the local expectations hypothesis. The sample period is from April 1982 through December 1992. The Vasicek parameter estimates are then translated to comparable parameters for the CIR model. We also consider variations from the base case for the estimated parameters by setting each of the parameters to twice the empirical estimates in the base case and to zero. DIFF0.25_0.5, DIFF0.5_0.75, and DIFF0.75_1 are the time t differences between the annualized futures and forward yields over the intervals $t + 0.25$ to $t + 0.5$, $t + 0.75$, and $t + 1$, respectively.

Panel A: Cox, Ingersoll, and Ross Model							
The interest rate dynamics are given by:							
$dr = \kappa(\mu - r)dt + \sigma\sqrt{r}dz.$							
where r is the instantaneous interest rate.							
	κ	μ	λ	σ	DIFF0.25_0.5	DIFF0.5_0.75	DIFF0.75_1
Base Case:	0.167	0.074	0	0.063	0.29	0.78	1.46
κ Variation	0.334	0.074	0	0.063	0.28	0.73	1.35
	0	0.074	0	0.063	0.31	0.83	1.59
μ Variation	0.167	0.148	0	0.063	0.28	0.74	1.36
	0.167	0	0	0.063	0.30	0.81	1.57
λ Variation	0.167	0.074	-0.5	0.063	0.33	0.87	1.64
	0.167	0.074	-1	0.063	0.36	0.96	1.79
σ Variation	0.167	0.074	0	0.126	1.16	3.09	5.80
	0.167	0.074	0	0	0	0	0

Panel B: Vasicek Model							
The interest rate dynamics are given by:							
$dr = \kappa(\mu - r)dt + \sigma dz,$							
where r is the instantaneous interest rate.							
	κ	μ	λ	σ	DIFF0.25_0.5	DIFF0.5_0.75	DIFF0.75_1
Base Case:	0.167	0.074	0.000	0.017	0.26	0.66	1.19
κ Variation	0.334	0.074	0.000	0.017	0.24	0.59	1.02
	0	0.074	0	0.017	0.28	0.74	1.38
μ Variation	0.167	0.148	0.000	0.017	0.26	0.66	1.19
	0.167	0	0	0.017	0.26	0.66	1.18
λ Variation	0.167	0.074	-0.100	0.017	0.26	0.66	1.19
	0.167	0.074	-0.200	0.017	0.26	0.66	1.19
σ Variation	0.167	0.074	0	0.034	1.02	2.63	4.75
	0.167	0.074	0	0	0	0	0

(depending on model and forward horizon) than that observed to generate the observed average difference for DIFF0.5_0.75 and DIFF0.75_1 solely from the mark-to-market feature. Even time varying volatility for $r(t)$ cannot account for the size of the observed futures-forward yield difference. Moreover, the yield differences do not appear to be monotonic functions of interest rate volatility. For example, the interest rate volatility in 1983 is substantially smaller than that in 1984 and 1987. However, the average futures-forward yield differences in 1983 are significantly larger than the corresponding differences in later years.

III. Default Risk, Liquidity, and Taxes as Explanations for the Difference

A. The Default Risk Explanation

One possible explanation for pricing differences between contracts on organized exchanges, like futures markets, and their “over-the-counter” market equivalents is that the over-the-counter market participants need to factor in the default risk of their counterparties. Since futures contracts are guaranteed by the clearinghouse and backed by margin, default risk is negligible. A short term LIBOR loan, by contrast, has the possibility that the borrower may default on the loan. A synthetic three-month forward contract, created with a position in two LIBOR contracts that differ in loan maturity by three months, has default risk in both loans, but the longer term contract has relatively more default risk, since it contains all of the default risk of the three-month loan, plus the default risk from months three to six. It follows that the forward rate has a default risk premium built into it for the possibility of default between months three and six.

We can illustrate this more formally with a model. Let $\pi(t)$ be the default premium (unannualized) attached to LIBOR borrowing with maturity t , a function which is increasing in t . Let $l(t)$ represent the annualized LIBOR borrowing rate that would exist in the absence of default risk. Assuming that the mark-to-market feature is negligible, it follows that the futures rate would be

$$F(s, \tau) = d(s, \tau)\{[1 + l(\tau)/d(0, \tau)]/[1 + l(s)/d(0, s)] - 1\}. \quad (8)$$

Since, by the definition of the default premium, the unannualized LIBOR rate from 0 to t is

$$l(t)/d(0, t) + \pi(t),$$

the annualized forward rate implied by the term-structure is

$$\begin{aligned} f(s, \tau) &= d(s, \tau)[P(0, s)/P(0, \tau) - 1] \\ &= d(s, \tau)\{[1 + l(\tau)/d(0, \tau) + \pi(\tau)]/[1 + l(s)/d(0, s) + \pi(s)] - 1\}. \end{aligned} \quad (9)$$

One can most easily see that $f(s, \tau) > F(s, \tau)$ with Taylor series expansions of Equations (8) and (9).

$$F(s, \tau) \approx d(s, \tau)\{l(\tau)/d(0, \tau) - l(s)/d(0, s)\}, \text{ while}$$

$$\begin{aligned} f(s, \tau) &\approx d(s, \tau)\{[l(\tau)/d(0, \tau) + \pi(\tau)] - [l(s)/d(0, s) + \pi(s)]\} \\ &\approx F(s, \tau) + d(s, \tau)[\pi(\tau) - \pi(s)] > F(s, \tau). \end{aligned}$$

Thus, the default risk model argues that the forward rate should be larger than the futures rate.¹⁵

However, the regressions exhibited in Table III,

$$\Delta DIFF_t = a + b\Delta TED \text{ Spread}_t + e_t, \text{ where}$$

$\Delta DIFF_t$ = one week change in the futures-forward yield difference at week t

and $\Delta TED \text{ Spread}_t$ = difference between the 3-month Eurodollar and T-bill yields, a measure of default risk

exhibit significantly negative slope coefficients in the first subperiod. While the negative coefficients are consistent with the default risk hypothesis, the regressions explain only a small part of the variation in the week to week change in the futures-forward yield difference. Moreover, as suggested above, this hypothesis cannot account for the sign and magnitude of this yield difference in the first half of our sample period.

B. The Liquidity Premium Explanation

Kamara (1988) presents a model in which the futures-forward differences observed in the T-bill market are related to a liquidity premium. His model assumes that the transaction costs associated with a combined short term spot rate investment from 0 to s and a futures investment from s to τ are lower than those of a long term spot rate investment from 0 to τ . In his model, investors with a preference for long horizon investments will invest in short term bonds and simultaneously buy a futures contract unless the forward rates are larger than the futures rates, because of the higher transaction costs for the long term bonds. This implies that the equilibrium forward rates will be larger than the futures rates, consistent with his findings in the T-Bill market. In the case of

¹⁵ This argument also implies that the sign of the yield spread observed cannot be explained by sovereign risk, (the risk of failure in repatriating the spot LIBOR funds), nor by the risk of settlement failure, due to differences in the way spot LIBOR is settled (via wiring through CHIPS), and futures are settled, (daily margin payments). Moreover, we note that CHIPS has never failed. A bank officer quoted in Stigum (1983) observes, "If CHIPS fails to settle, I jump out my [14th story] window. CHIPS cannot not settle because, if it were to do so, it would destroy confidence in the money market internationally—create a worldwide financial panic, perhaps worldwide depression."

Table III

**The Relation Between Changes in the Futures-Forward Yield
Difference and Changes in the TED Spread**

This table presents estimates, based on weekly data, of the slope coefficients in the following regression:

$$\Delta \text{DIFF}_t = a + b \Delta \text{TED Spread}_t + e_t,$$

where ΔDIFF is the one-week change in the futures forward yield difference. DIFF0.25_0.5 , DIFF0.5_0.75 , and DIFF0.75_1 are the time t differences between the annualized futures and forward yields over the intervals $t + 0.25$ to $t + 0.5$, $t + 0.5$ to $t + 0.75$, and $t +$ and $t + 0.75$ to $t + 1$, respectively. DIFF1 is the difference between the annualized 3-month futures and forward yields on the date of maturity of the nearest maturity futures contract; DIFF2 is the difference between annualized 3-month futures and forward yields on the date of maturity of the next-to-nearest maturity futures contract, etc. TED Spread is the difference between 3-month LIBOR and the 3-month T-bill rate and $\Delta \text{TED Spread}$ is the one-week change in the TED Spread.

Dependent Variable	Sample Period		
	8204–9212	8204–8706	8707–9212
$\Delta \text{DIFF0.25_0.5}$	–0.2152 (–8.38)	–0.2956 (–6.59)	–0.1403 (–5.45)
Adj- R^2 (in percent)	10.99	13.41	9.06
$\Delta \text{DIFF0.5_0.75}$	–0.1359 (–3.20)	–0.1666 (–2.15)	–0.1073 (–2.81)
Adj- R^2	1.62	1.30	2.33
$\Delta \text{DIFF0.75_1}$	–0.1100 (–3.08)	–0.2337 (–3.97)	0.0053 (0.13)
Adj- R^2	1.49	5.10	–0.34
ΔDIFF1	–0.1487 (–5.78)	–0.2622 (–5.70)	–0.0429 (–1.85)
Adj- R^2	5.47	10.31	0.84
ΔDIFF2	–0.1392 (–4.23)	–0.1228 (–2.07)	–0.1545 (–4.98)
Adj- R^2	2.92	1.18	7.63
ΔDIFF3	–0.1143 (–2.78)	–0.1887 (–2.44)	–0.0449 (–1.39)
Adj- R^2	1.19	1.78	0.32

the Eurodollar market, however, the futures rate is generally larger than the forward rate.

C. Taxation Differences

The major difference in the taxation of futures and forwards is due to what is known as the 60/40 rule, which has been in effect since 1982. Under this rule, all exchange-regulated futures contracts have 60 percent of their gains and losses taxed at the long term capital gains rate. The synthetic forward, by contrast, has interest receipts and deductions that are taxed at the income tax rate. Since 1982–1986, a period of large futures-forward yield differences, coincided with a period where the income tax and long term capital gains rates

differed, and 1986–1990, a period where the yield differences were much smaller, coincided with the period where the two tax rates were identical, there appears to be at least some possibility that taxes may play a role here. While this hypothesis sounds plausible, it is not a tenable explanation for the futures-forward yield differences, as we prove in the following proposition:

PROPOSITION 3: *If the no-arbitrage futures-forward rate difference is negligible in a no tax world, then it is negligible when tax rates on the profits earned from these investments differ.*

Proof: Let T_F denote the tax rate on profit from futures, T denote the tax rate on interest received, $L_s(0.25)$ denotes 3-month LIBOR at the maturity date s of the futures and forward contracts, which may have a distribution that depends on tax rates, and $E[\cdot | T, T_F]$ denotes the no arbitrage pricing operator, which also may depend on tax rates. If marking to market is irrelevant for pricing (as we have shown) in a no-tax world, then, since the futures and forward contracts have present values of zero, the futures and forward prices F and f must satisfy

$$E[L_s(0.25) - F | 0, 0] = E[L_s(0.25) - f | 0, 0] = 0,$$

implying

$$F = f = E(L_s(0.25) | 0, 0)$$

and thus $F - f = 0$. In a world with taxes, the pricing condition is

$$E[(L_s(0.25) - F)(1 - T_F) | T, T_F] = E[(L_s(0.25) - f)(1 - T) | T, T_F] = 0,$$

which implies

$$F = f = E[L_s(0.25) | T, T_F]$$

and thus $F - f$ equals zero. Q.E.D.

Both futures and forward contracts have zero present values at the time they are initiated. Since the tax effects are symmetric with respect to gains and losses, they only affect the scale of the payoff at maturity, and not the expected value of the payoff. Therefore, the futures and forward rates are not affected by their tax exposures.

IV. Timeliness of Information Flow across Futures and Forward Markets

None of the hypotheses considered so far is consistent with our empirical evidence. Thus, the large futures-forward yield differences in the early years of the futures contract seem to be attributable to market inefficiency. To further explore whether the market was efficient, we investigate the timing of the flow of information across the futures and forward markets. If changes in one rate

predict future changes in the other rate, then the information relevant for future interest rates is not simultaneously incorporated into both futures and forward rates.

We first examine whether changes in the weekly futures rate (computed using the futures interpolation method) lead the weekly changes in the (actual) forward rate using the regression

$$\Delta_{t+1,t}f(s, \tau) = a_{\delta} + b_{\delta}\Delta_{t,t-\delta}F(s, \tau) + e_{t,\delta},$$

where $f(s, \tau)$ and $F(s, \tau)$ are the Eurodollar forward and futures yields from s to τ , and

$$\Delta_{t2,t1}X(s, \tau) = \text{the change in } X(s, \tau) \text{ from week } t1 \text{ to week } t2.$$

Since we do not have strong priors about how long it takes for information to be transmitted across the futures and forward markets, we consider a set of univariate regressions with one to four week lagged changes in the futures rates as regressors.

We measure the weekly changes in the futures rates based on Wednesday's closing prices.¹⁶ The forward rates are computed using the Thursday DRI quotes. By skipping a day between the futures and forward quote dates, we avoid potential biases that may arise from same-day nonsynchronous quotes in the forecasting regression.

Table IV presents the estimates of the regression slope coefficients. Over the full sample, in the 3-month and 6-month forward rate ($\Delta f(0.25, 0.5)$ and $\Delta f(0.5, 0.75)$) regressions, the slope coefficient is significant when the regressor is the two- or three-week lagged change in corresponding futures rates. In the case of $\Delta f(0.75, 1)$, however, the slope coefficients in all regressions are statistically significant. It appears that the flow of information from the futures prices to forward rates is completed in two to three weeks.¹⁷ A similar pattern is observed when the first subperiod is considered separately but virtually none of the regression coefficients in the second subperiod is reliably different from zero.

The next set of tests are based on the following regression:

$$\Delta_{\tau+1,\tau}F_i = a_{\delta} + b_{\delta}\Delta_{\tau,\tau-\delta}f_i + e_{\tau,\delta}, \text{ where}$$

$\Delta_{\tau+1,\tau}F_i$ is the change in the futures rate from week τ to week $\tau + 1$ implied by the i^{th} nearest futures contract, and f_i is the corresponding forward rate (computed with the spot interpolation method). These regressions examine whether the information in the forward rates is incorporated into the futures prices instantaneously. Here again, to avoid potential biases due to nonsyn-

¹⁶ There is no day of the week effect in the results. We have tested this regression using other days of the week and obtain essentially the same results.

¹⁷ Note that the variance of the independent variable increases with δ . Therefore, the point estimate of the slope coefficient will decline with δ if the information spillover declines as the lag length is increased.

Table IV
The Relation Between Changes in Eurodollar Forward Yields and Lagged Futures Yields

This table presents estimates, based on weekly data, of the slope coefficients in the following regression:

$$\Delta_{\tau+1,\tau}f(s, t) = a_{\delta} + b_{\delta}\Delta_{\tau,\tau-\delta}F(s, t) + e_{\tau,\delta} \quad \delta = 1, 2, 3, \text{ and } 4,$$

where $f(s, t)$ and $F(s, t)$ are the Eurodollar forward and futures yields over the interval s to t . $\Delta_{\tau+1,\tau}f(s, t)$ is the change in $f(s, t)$ from week τ to week $\tau + 1$ and $\Delta_{\tau,\tau-\delta}F(s, t)$ is the change in $F(s, t)$ from week $\tau - \delta$ to week τ .

The forward yields are computed from the LIBOR term structure, and the futures yields are estimated by interpolating between the yields implied by the futures prices. The futures yield changes are measured from closing transaction prices on Wednesdays and the forward yields are computed from the closing LIBOR term structure quotes on Thursdays. The t -statistics are reported in parentheses. The sample period is from April 1982 through December 1992.

Dependent Variable	δ			
	1	2	3	4
8204-9212				
$\Delta f(0.25, 0.5)$	0.0327 (0.76)	0.0759 (2.37)	0.0700 (2.56)	0.0241 (1.05)
$\Delta f(0.5, 0.75)$	0.0769 (1.56)	0.0883 (2.40)	0.0753 (2.43)	0.0334 (1.28)
$\Delta f(0.75, 1)$	0.1266 (2.76)	0.1108 (3.34)	0.0949 (3.44)	0.0551 (2.35)
8204-8706				
$\Delta f(0.25, 0.5)$	0.0251 (0.41)	0.0850 (1.86)	0.0917 (2.35)	0.0285 (0.89)
$\Delta f(0.5, 0.75)$	0.1081 (1.52)	0.1042 (1.95)	0.0969 (2.16)	0.0425 (1.12)
$\Delta f(0.75, 1)$	0.1306 (2.09)	0.1175 (2.60)	0.1220 (3.26)	0.0560 (1.79)
8707-9212				
$\Delta f(0.25, 0.5)$	0.0505 (0.79)	0.0531 (1.16)	0.0190 (0.49)	0.0153 (0.44)
$\Delta f(0.5, 0.75)$	0.0091 (0.13)	0.0561 (1.12)	0.0320 (0.75)	0.0140 (0.38)
$\Delta f(0.75, 1)$	0.1160 (1.65)	0.0975 (1.91)	0.0426 (1.01)	0.0539 (1.46)

chronous same-day futures and forward quotes, we skip a day and measure the futures rate changes from Friday's closing prices.

In this set of regressions, presented in Table V, virtually all of the slope coefficients are insignificantly different from zero in the full sample period and in both subperiods. These results indicate that there are no delays in the flow

Table V

The Relation Between Changes in Eurodollar Futures Yields and Lagged Forward Yields

This table presents estimates, based on weekly data, of the slope coefficients in the following regression:

$$\Delta_{\tau+1,\tau}F_i = a_\delta + b_\delta \Delta_{\tau,\tau-\delta}f_i + e_{\tau,\delta} \quad \delta = 1, 2, 3, \text{ and } 4,$$

where $\Delta_{\tau+1,\tau}F_i$ is the change in futures yield implied by the i^{th} futures contract from week τ to week $\tau + 1$. $i = 1$ is the nearest maturity futures contract, $i = 2$ is the next to nearest maturity futures contract and so on. $\Delta_{\tau,\tau-\delta}f_i$ is similarly defined for forward yields where f_i is the three-month forward yield from the date of maturity of the i^{th} futures contract. The forward yields are computed from the LIBOR term structure. The futures yield changes are measured from closing transaction prices on Fridays, and the forward yields are computed from the closing LIBOR term structure quotes on Thursdays. The t -statistics are reported in parentheses. The sample period is from April 1982 through December 1992.

Dependent Variable	δ			
	1	2	3	4
8204-9212				
ΔF_1	-0.0533 (-1.42)	0.0223 (0.83)	-0.0310 (-1.47)	-0.0168 (-0.93)
ΔF_2	-0.0238 (-0.54)	0.0400 (1.25)	0.0116 (0.45)	0.0241 (1.08)
ΔF_3	0.0100 (0.27)	0.0578 (1.93)	0.0251 (1.08)	0.0268 (1.31)
8204-8706				
ΔF_1	-0.0654 (-1.24)	0.0373 (1.00)	-0.0337 (-1.15)	-0.0289 (-1.18)
ΔF_2	0.0006 (0.01)	0.0597 (1.27)	0.0296 (0.77)	0.0266 (0.81)
ΔF_3	0.0128 (0.24)	0.0853 (1.94)	0.0427 (1.29)	0.0277 (0.95)
8707-9212				
ΔF_1	-0.0319 (-0.58)	-0.0301 (-0.75)	-0.0298 (-0.92)	0.0158 (0.56)
ΔF_2	-0.0650 (-1.18)	0.0002 (0.00)	-0.0215 (-0.64)	0.0180 (0.61)
ΔF_3	0.0025 (0.05)	0.0077 (0.19)	-0.0120 (-0.37)	0.0250 (0.86)

of information from the forward market to the futures market, but (from Table IV) there is a delay of information flow from the futures to the forward market.

V. Factors that Limit Arbitrage

The results so far suggest that the most likely explanation for the futures-forward yield difference in the early part of our sample period is mispricing.

These findings raise the question of why the futures contracts were mispriced relative to the forward rates by as much as 100 basis points in 1982, the year the contract was first traded, and why the mispricing lasted so long. In this section, we explore various factors that may explain why arbitrage of the mispricing may have been limited, and thus not sufficient to bring forward and futures prices together.

First, it seems unlikely that direct transactions costs were sufficiently large to be consistent with the level of mispricing found in the first subperiod. An arbitrage strategy between the futures and forward market will involve taking a long position in the CME futures contract, borrowing long at LIBOR, and investing short at LIBID (London Interbank Bid Rate). Today, the spread between the LIBOR and LIBID is less than 1/16, approximately six basis points. The futures contract spread is about 2 basis points. So the total transaction cost is less than 10 basis points. In the early 1980s, even if the transaction cost had been four times this magnitude, the pricing error was outside the transaction cost bound.

One complication with the arbitrage strategy that may have to be considered is that the maturities of the futures and forward contracts do not coincide except when the time to maturity of the nearest futures contract is exactly three months. However, we find that the futures-forward yield differences when the maturity dates of these contracts were less than 10 days apart are essentially the same as those for longer maturity differences. In addition, the transaction costs of the arbitrage are largely irrelevant to LIBOR lending banks. These banks lent at LIBOR for maturities of 9 and 12 months when they could have shortened their loan maturities to six months or less while more profitably matching the desired asset maturity of the 9 and 12 month loans by simultaneously acquiring futures positions.

There may also have been additional costs for implementing an arbitrage strategy if market depth (presumably in the second, third, and fourth nearest futures contracts) was low. However, virtually all of the slope coefficients in unreported regressions, of the form

$$\Delta_{t+1}F_i \times \Delta_t F_i = a_i + b_i \text{Unexpected Volume}_{i,t} + e_{i,t} \quad i = 1, \dots, 4,^{18}$$

are close to zero. Hence, it appears that large trades could have been executed without adverse price impact, even in the early years of the futures contract.

It is also possible that not enough investors knew about or took advantage of the arbitrage in the contract's early years. A number of sources confirmed that the money center banks, who were big players in the LIBOR spot market, largely ignored the Eurodollar futures market in the early 1980s. This was either because they were unaware of the arbitrage opportunities or because

¹⁸ $\Delta_t F_i$ is the change in the futures price from the i th futures contract on day t and $\text{Unexpected Volume}_{i,t}$ is the innovation in an AR(5) daily volume regression for the i th nearest contract on day t . $\Delta_{t+1}F_i \times \Delta_t F_i$, the product of consecutive futures price changes, is a measure of the temporary component of the price change at date t that was reversed at date $t + 1$.

they were simply not tooled up to trade futures.¹⁹ Also, there seems to have been some informal constraint on bank participation in the futures market. According to Parkinson and Spindt (1986), "...regulatory guidelines for futures activity by domestic banks stipulate that banks must use futures to hedge. . .rather than to speculate." As interest rate swaps became more standard as hedging tools for banks, participation in futures gradually became more acceptable. By the end of 1985, domestic bank positions in futures exceeded 65 billion dollars, with foreign banks having positions of approximately 25 billion dollars.

Investment banks faced a different impediment. They were not able to borrow at LIBOR rates in the early 80s and thus could not directly form the LIBOR forward hedge to the futures position. Some investment bank arbitrageurs were able to get around this impediment by shorting Treasuries to create synthetic forwards. Such short positions require the investment bank to offer a loan in the Treasury repurchase agreement market in order to borrow the securities necessary for the short positions. The additional transaction costs of this more complicated set of transactions as well as the interest rate risk engendered by rolling over the repo positions may have limited the extent of the arbitrage.

VI. Conclusion

This article has examined the relative pricing of Eurodollar futures and forward contracts from 1982–1992. While the differences between the futures and forward yields are large in the early part of the sample period, which is consistent with prior studies, the yield differences are close to zero toward the end of the sample period. We evaluate a number of hypotheses that could potentially explain both the futures-forward yield differences and the changes in these differences over time.

From a theoretical perspective, we show that the yield differences due to the mark-to-market feature of the futures contract should be small for interest rate processes that are consistent with observed time series. Differential default risk, Kamara's differential liquidity hypothesis, lack of market depth, taxation, and the structures of the futures and forward markets are also not satisfactory explanations, since they are either inconsistent with the sign and magnitude of the observed futures-forward yield difference or theoretically should have no effect. We also find that the information flow between the futures and forward markets is not instantaneous in the early part of our sample period.

Our findings lead to the conclusion that the relative mispricing of the futures contract in the early part of our sample period is attributable to a lack of

¹⁹ In a somewhat prescient statement, Marcia Stigum (1983, p. 569) wrote, "Some banks have used Euro futures to arbitrage against forward positions in the interbank Eurodeposit market. In 1982, when the Euro contract was new and few people were watching this trade, bank traders who did it were at times able to take out a 50 to 100 basis-point profit. Presumably, as volume in Euro futures grows, rates in this market and rates on forward forwards in the interbank market will move into closer synchronization."

arbitrage activity, particularly on the part of banks participating in the spot LIBOR market. The finding that mispricing persisted for many years is surprising in that the conventional wisdom is that the market quickly arbitrages away any mispricing. It appears that market frictions and ignorance may limit the arbitrage activity and prolong the price adjustment process.

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