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The Capital Budgeting Process: Incentives and Information

MILTON HARRIS and ARTUR RAVIV*

ABSTRACT

We study the capital allocation process within firms. Observed budgeting processes are explained as a response to decentralized information and incentive problems. It is shown that these imperfections can result in underinvestment when capital productivity is high and overinvestment when it is low. We also investigate how the budgeting process may be expected to vary with firm or division characteristics such as investment opportunities and the technology for information transfer.

THERE IS GENERAL AGREEMENT that investment decisions are the most important decisions made by corporations. The choice of projects and the level of investment is critical not just for stakeholders of the firm but also for the economic well-being of society as a whole. It has been alleged that U.S. firms invest too little and tend to overemphasize short-term results. For example, Porter (1992), citing Poterba and Summers (1992), asserts that U.S. firms use hurdle rates to evaluate investment projects that are higher than their estimated costs of capital.

To understand the investment behavior of firms, one must consider both the process by which external capital is made available to firms and the process by which internally and externally raised capital is allocated to investment projects within the firm. Considerable attention has been paid to how firms raise external capital. The fundamental insight in this area is the celebrated result of Modigliani and Miller (MM) that capital structure is irrelevant in the absence of capital market frictions. Accordingly, in the three decades since their contribution, research on how external capital is raised has focused on market frictions, including information and incentive problems.

Relatively little research has focused on the internal allocation process and its relationship to the organization of the firm. Modern finance theory prescribes allocating capital according to the net present value (NPV) rule. Just as the MM result itself provides no theory of the arrangements under which

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external capital is raised, the NPV rule provides no role for details of the internal allocation process. Just as modern theories of *external* financing are built upon explicit consideration of capital market frictions, a theory of *internal* capital allocation processes must account explicitly for decentralized information and incentive problems.[1] This article provides a step in that direction.

In practice, details of the capital budgeting procedure seem to be critical in determining actual capital allocations. Decisions are often delegated throughout an organization. The capital budgeting process governs the way in which managers at various levels produce and share information about proposed investments and determines which decisions are delegated, to whom, and under what constraints. For example, in many firms, individual managers or divisions are assigned capital spending limits, perhaps based on proposals generated by the managers. These limits may result in passing up projects with positive NPV, an outcome that is clearly inconsistent with the NPV rule. The purpose of this article is to rationalize capital budgeting procedures and the resulting capital spending limits by relaxing the assumptions implicit in the NPV rule that there are no information or incentive problems. In particular, we explain how the process may be expected to vary with firm or division characteristics such as investment opportunities and the technology for information generation and transfer.

Our approach is to start with the simplest environment in which capital budgeting processes are necessary. The model consists of headquarters and a single division. The division manager must obtain capital from headquarters.[2] The reason for decentralization in our model is that the division manager is assumed to have information, that is not freely available to headquarters, about his division's production technology. Even if information is decentralized, however, decisions can still be centralized if managers have no incentive to withhold or misrepresent their information. Consequently, we must include a divergence of preferences between headquarters and the division manager. The specific agency problem we postulate is that headquarters allocates capital so as to maximize the value of the firm, but the division manager prefers larger capital allocations to smaller ones, other things equal. This preference could reflect managerial utility for being in charge of larger enterprises (i.e., a preference for a larger "empire") or the fact that larger capital allocations result in greater managerial perquisite consumption. Headquarters can discover the manager's information precisely at some cost through a detailed audit. Headquarters seeks to design an incentive compatible capital allocation

[1] George Pinches, in his presidential address to the Financial Management Association (Pinches (1982, p. 16)), accuses academicians of focusing too much of their attention on the selection of projects and too little on the other aspects of the capital budgeting process, especially the information requirements. He states, "This overemphasis on the selection phase has resulted from a myopic view of what the capital budgeting process is all about in practice."

[2] We do not address the issue of whether the division would be better off as a stand-alone entity. Two recent articles that provide a rationale for including various divisions within a single corporate structure are Stein (1995) and Thakor (1990).

scheme and managerial salary to minimize the distortion due to decentralized information and managerial preference for empire. In particular headquarters chooses an audit strategy, capital allocations, and salary as functions of the manager's request for capital to maximize the value of the residual claim, given the constraints implied by private information and the manager's preference for empire.

We show that an optimal scheme involves an initial capital spending limit. The manager can either accept this amount or request additional capital above the spending limit. If he requests a larger amount, headquarters may either allocate a compromise level of capital (between the initial spending limit and the amount requested), or it may audit the proposed project and discover the true productivity of capital. If the audit reveals that the project's productivity justifies the higher level of capital requested, this amount is allocated. Otherwise, no capital is allocated to this project.[3] There is a critical amount of capital such that if the requested level is below the critical amount, headquarters will not investigate, and if it is above the critical amount, headquarters will investigate with positive probability that increases with the requested amount. Moreover, the compromise capital allocation increases with the requested allocation. This process could result in underinvestment, i.e., a capital allocation that is below the first-best level, or overinvestment.

The intuition for these results follows from the fact that headquarters' problem is to prevent overinvestment due to exaggerated claims about the productivity of capital. This leads to a capital spending limit. A completely rigid limit, however, fails to take advantage of the manager's private information. Therefore, when audit costs are not "too high," it is efficient for headquarters to allow managers to request larger allocations. Headquarters employs several devices to "keep the manager honest." First, it audits the division manager's capital request and penalizes him if his request is unjustified. Auditing occurs with a probability that increases with the manager's capital request, but, to save audit costs, the probability is always less than one. Second, the initial spending limit is "generous" relative to the first-best level of investment for the least productive technology. This encourages the manager not to request more capital when in fact the technology is the least productive. Thus, when the least productive technology obtains, the optimal capital budgeting procedure results in *over*investment. Third, the compromise allocation for an unaudited request of capital in excess of the initial spending limit is "stingy" relative to first-best. Thus, for high productivities, in the absence of an audit, the optimal procedure results in *under*investment.

We also characterize how the optimal capital budgeting scheme is affected by changes in the cost of audit, the investment opportunity set, and the probability distribution of capital productivity. When it is very difficult for

[3] This helps ensure that the manager never "cheats," i.e., requests more capital than the project justifies. In fact, we need not assume that the project is passed up in this case. All that is required is that this particular manager is not in charge of the project and, hence, obtains no utility from the capital allocation.

headquarters to discover the true productivity of capital, the capital budgeting scheme is rigid in the sense that requests for allocations above the initial capital spending limit are simply ignored. As the cost of audit declines, initial spending limits decline, but the scheme becomes less rigid, compromise allocations increase, and approval of requests is more likely. With respect to the investment opportunity set, as investment opportunities become more valuable, initial spending limits decline, but approval of requests becomes more likely and compromise allocations increase. As low productivity becomes more likely (and high productivity less likely), initial spending limits decline, the scheme becomes less rigid, and approval becomes more likely, but compromise allocations are unaffected. These comparative statics results are used to generate testable implications regarding the behavior of the various features of the capital budgeting scheme with respect to changes in observable features of the economic environment. We also derive implications regarding the covariances between the endogenous aspects of capital budgeting procedures. For example, the model predicts that higher initial spending limits should, on average, be associated with lower approval rates for requested capital increases.

The remainder of the article is organized as follows. In the next section, we review the literature on capital budgeting processes. We present our model in Section II, our results in Section III, and the empirical implications of the results in Section IV. Section V contains a summary of the research and suggestions for further work in this area.

I. Literature Review

Most of the literature on this topic is empirical, and most of this is survey research. These surveys are concerned with identifying the goals of top management, discovering the extent to which various tools such as NPV, internal rate of return, payback, etc. are used, how management calculates the cost of capital, and how risk is accounted for (Mao (1970), Gittman and Forrester (1977), Schall, Sundem, and Geijsbeek (1978), Scott and Petty (1984), Stanley and Block (1984), and Ross (1986)). The "typical" capital budgeting procedure that emerges from these studies and articles by corporate executives is described in Taggart (1987). Top management starts the process by assigning preliminary budgets for its divisions. Projects are generally initiated from the bottom up, suggesting that the information of those working most closely with the technology is important, and, although project initiation is decentralized, capital allocation is generally centralized (Stanley and Block (1984)). Within the initial budgets, some projects are approved at the division level. Which projects require higher approval may be determined by size (those requiring more than a certain expenditure) or by nature of the project (e.g., new product lines). Projects that must be submitted to headquarters require formal proposals including detailed cash flow projections. Indeed, Ross (1986) finds that larger investments require greater documentation and that the level at which decisions are made increases with investment size. In his sample of twelve firms, six used capital rationing in which projects compete for a fixed budget.

He finds that divisions can increase their capital allocations, but that this typically requires a major convincing effort and uses up "credit" with corporate headquarters. This is consistent with our view of costly monitoring as the means by which capital spending limits are exceeded. Ross also documents that in eight of the twelve firms, smaller projects faced higher de facto hurdle rates.

Surprisingly, we found little theoretical literature on capital budgeting *processes*. Most of the theoretical work does not explicitly consider capital rationing, but instead focuses on the issue of when one would expect to observe communication of private information in such processes (Baiman and Evans (1983), Magee (1980), Melumad and Reichelstein (1989)). Lambert (1986) examines incentives for information gathering and investment choice in a principal-agent setting. Baiman and Rajan (1994) examine the issue of centralized (owner) versus decentralized (manager) investment choice. The degree of centralization is not driven by superior information. Instead, it is based on the inability of the owner to commit to an investment choice function when he chooses the manager's compensation scheme. Gertner, Scharfstein, and Stein (1994) are primarily concerned with the comparison between internal and external finance. As a result, most of the features of the internal financing process in which we are interested are exogenous in their model.

Harris, Kriebel, and Raviv (1982), hereafter HKR, are the first to analyze intrafirm resource allocation in the presence of both information and incentive problems. They criticize previous analyses based purely on asymmetric information on grounds that, without incentive problems, the elaborate schemes proposed in those models were unnecessary. HKR focuses on the role of transfer prices in allocating a common resource to divisions with private information about the productivity of the resource in the division. Division managers also have a preference for larger allocations because the resource could be substituted for costly managerial effort. HKR show that allowing division managers to choose price-quantity combinations from a menu of choices is optimal in their setting.

Antle and Eppen (1985) attempt to explain "organizational slack" (allocation of more capital than is actually used in a project), capital rationing, and underinvestment. In their model, as in HKR, managers have private information about the production technology and prefer larger capital allocations (indeed, their managers consume any capital not used for production). Antle and Eppen show that an optimal scheme in their model is similar to that in HKR, i.e., division managers choose from a menu of price-capital allocation pairs. In Antle and Eppen, however, there are only two choices, even though there may be more than two possible levels of productivity. One choice involves no capital (and therefore no production) and no payment. The other involves a price above the market price and a positive capital allocation. Managers whose productivity is below a cutoff level choose the first option and do not produce. Since the transfer price is above the market price of capital, some divisions for which NPV is positive do not produce. Managers with productivity at or above the cutoff level choose the second option. Under this scheme, divisions with productivities strictly above the cutoff receive more capital than is needed to produce output sufficient to pay for the capital. The manager consumes this excess capital (slack). Thus the model exhibits both underinvestment and organizational slack.

The current article follows in the tradition of HKR and Antle and Eppen by assuming asymmetric information and preference for capital on the part of division managers. We abstract from transfer pricing to introduce auditing by headquarters. Our results provide a more detailed explanation of the capital budgeting process including an initial spending limit, the possibility of requests for additional capital, stochastic auditing of requests, and capital allocation that depends on the amount requested and the audit results.

Finally, Holmstrom and Ricart i Costa (1986) derive capital rationing as a response to managerial overinvestment, as in our approach. In their model, however, overinvestment results from imperfect, but symmetric, information about managerial ability. An optimal wage contract insures the manager against the possibility that an investment will reveal him to be of low ability, but allows the manager to capture the gains in the opposite situation. That is, the wage contract provides the manager an option on the value of his human capital. This contract leads the manager to take on "too much" risk through overinvesting in unprofitable projects. Capital rationing curtails this overinvestment. Except for capital rationing per se, this model does not address the features of the capital budgeting process in which we are interested.

II. Model

Our model consists of two entities. Headquarters is risk-neutral and represents the interests of the stockholders. It has access to capital but cannot determine with certainty the true available production technology. This information is critical in determining the efficient allocation of capital. Headquarters can, however, investigate to discover the true technology. This audit costs q and reveals the true technology with certainty.[4] To avoid paying the audit costs in all cases, headquarters engages a manager who knows the production technology. The manager is risk-neutral and has no access to capital on his own.[5]

More specifically, we assume headquarters knows that there are three possible technologies with different capital productivities. The technologies are

[4] Townsend (1979) was one of the first models to use the idea of costly verification. A model more similar in spirit to ours is that of Border and Sobel (1987).

[5] The model can also be interpreted as describing the relationship between a venture capitalist (playing the role we call headquarters) and an entrepreneur (division manager). Another interpretation results from replacing "headquarters" with shareholders and "divisional manager" with CEO. The main requirements for casting these roles are that "headquarters" be the residual claimant (or represent the residual claimant without any agency or information problems) and has the power to design, implement, and enforce a capital budgeting process, and the "divisional manager" must have private information about the production process and a desire for empire.

indexed by $t \in T = \{1, 2, 3\}$.[6] Headquarters views these productivities as having probabilities π_t, for $t \in T$ (with $\pi_t \in (0, 1)$ for each t and $\pi_1 + \pi_2 + \pi_3 = 1$). Technology t produces net present value $V_t(k)$ if the capital input is k.[7] We assume that $V_t(0) = 0$ for all t and that the productivities are ordered such that $0 < V_1'(k) < V_2'(k) < V_3'(k)$ for all k. To avoid corner solutions, we assume that marginal values, V_t', are infinite at zero input for all technologies. Technology t is assumed to have strictly concave value and to have a unique, value-maximizing capital input, k_t^*, that maximizes $V_t(k)$. k_t^* is the level of capital that headquarters would choose if it were informed that the capital productivity is t and the manager had no desire for empire. The ordering of marginal productivities implies that $k_1^* < k_2^* < k_3^*$.

As mentioned in the introduction, we must assume that a conflict of interest exists between the manager and headquarters. One way to model such a conflict is to assume that the manager derives utility both from monetary rewards and from managing the division. The utility derived from managing the division may be thought of as resulting from exogenous, unmodeled reputational effects. It is natural to assume that the reputation of a manager improves with the division's size (capital invested) and its productivity of capital.[8] To model these effects, we assume that, when productivity is t, the manager's utility function is $S + b_t k$, where S is his salary, k is the amount of capital he is allocated, and $0 < b_1 < b_2 < b_3$ are parameters that reflect the marginal reputational effect of additional capital. The manager's direct benefits from controlling more capital provide the required conflict of interest between him and headquarters.[9]

Another consequence of the manager's utility from capital is that he can be compensated not only with cash but also with additional capital allocation. Because of the assumed diseconomies of scale (concavity of V_t), for a sufficiently large capital allocation, additional compensation is less costly to head-

[6] We believe that our results will hold for any finite number of possible productivities; however, to keep the proofs simple, we restrict the model to three. As will be shown below, headquarters never audits when the manager claims the least productive technology. To compare across cases in which the audit probabilities are positive, therefore, we must assume at least three possible productivities.

[7] We use the phrase "net present value" (NPV) here to mean the net present value of the project if there were no incentive or information problems, i.e., what would be called NPV in any corporate finance textbook.

[8] Obviously, for profitability to have an effect on the manager's reputation, it must be observable to the manager's potential employers. It does not follow, however, that the project's technology is known precisely ex post. First, it may be the case that more productive technology only guarantees greater profit *on average*. Second, the greater profit of a more productive technology may accrue only over the long run, rendering immediate use of the information contained in profit difficult.

[9] Another possible interpretation of our assumptions is that the benefits of divisional size are derived from perquisite consumption. Our assumption is that more perquisites can be consumed in larger divisions and in more profitable divisions of a given size (this is similar to Antle and Eppen (1985)). In this interpretation, direct losses to the stockholders from perquisite consumption must be assumed to be negligible, although the distortions resulting from the necessity to provide incentives for truth-telling may be significant.

quarters if paid in cash instead of capital. To simplify the problem, we assume that diseconomies of scale are sufficiently severe that, for technology t, this point has been reached at k^*_{t+1} (for $t = 1, 2$).

Formally, we assume that

$$b_t + \pi_t V_t'(k^*_{t+1}) < 0, \quad \text{for } t < 3. \tag{1}$$

In this equation, b_t is the benefit to headquarters of increasing the manager's capital allocation when capital productivity is t, since an increase in capital increases the manager's utility by b_t, thus saving headquarters b_t in salary. The cost of such an increase is the reduction in NPV if productivity t obtains times the probability that productivity t obtains. This cost is, therefore, $\pi_t V_t'(k^*_{t+1})$ for a change in the capital allocation from k^*_{t+1}. Thus the assumption is that the net benefit is negative. Note that, by concavity of V_t, if equation (1) holds at k, it holds for all $k' > k$.

In this model, headquarters is assumed to be the residual claimant to the output of the firm. Its problem is to design a capital budgeting process that maximizes its expected return. We apply the Revelation Principle to assume, without loss of generality, that the process takes the form of a revelation game in which the manager reports the productivity of capital, t, and is given incentives to report truthfully. Headquarters' choice variables are the manager's salary, S, the probability of auditing, α_t, the amount of capital allocated if there is no audit, k_{0t}, and the amount of capital allocated if there is an audit, as functions of the reported technology t. The capital allocated if there is an audit can also depend on the results of the audit. We denote by k_{1t} the capital allocated after an audit if the manager reports t and the true productivity is t. We do not need notation for the capital allocated after an audit when the report is not truthful, since it is easy to show that in any optimal solution, this capital allocation is zero (or, equivalently, the project is assigned to a different manager). Intuitively, this allocation never occurs in equilibrium. Therefore, there is no cost to headquarters of inflicting the maximum punishment on the manager for lying. Because of the nonnegativity constraints, the maximum punishment in this case is to allocate no capital (or replace the manager).[10] Headquarters' problem is then to choose S, $\{\alpha_t\}$, $\{k_{0t}\}$, and $\{k_{1t}\}$ to

$$\max -S + \sum_{t \in T} \pi_t\{(1 - \alpha_t)V_t(k_{0t}) + \alpha_t[V_t(k_{1t}) - q]\} \tag{2}$$

[10] As is common in models like ours, we assume that headquarters can commit to allocating zero capital in the out-of-equilibrium event that the manager lies. Similarly, we assume that headquarters can commit to auditing with the chosen probability, even though this is costly and headquarters knows that the manager is telling the truth. Such a commitment can stem from reputational considerations that we do not model here.

subject to

$$S + (1-\alpha_t)b_t k_{0t} + \alpha_t b_t k_{1t} \geq S + (1-\alpha_m)b_t k_{0m}, \qquad \text{for } t, m \in T, m \neq t, \quad (3)$$

$$S + (1-\alpha_t)b_t k_{0t} + \alpha_t b_t k_{1t} \geq \bar{u}, \qquad \text{for all } t \in T, \quad (4)$$

$$S \geq 0, \qquad 1 \geq \alpha_t \geq 0, \qquad k_{0t} \geq 0, \qquad k_{1t} \geq 0, \qquad \text{for all } t \in T. \quad (5)$$

The objective function, equation (2), in this problem is simply the NPV of the firm net of audit costs and the manager's salary. The first set of constraints, equation (3), are the familiar incentive compatibility conditions that guarantee that the manager has no incentive to report falsely. The second set of constraints, equation (4), are the participation (or individual rationality) conditions that ensure that the manager is willing to participate no matter which productivity he observes. Here $\bar{u}$ is interpreted as the manager's opportunity cost of managing in this firm. The constraints in (5) ensure that α_t is a valid probability and capital allocations are not negative. The constraint also reflects the assumption that the manager cannot be made to pay for the privilege of being the manager ($S \geq 0$). Given risk neutrality of the manager, the requirement that $S \geq 0$ prevents trivial resolution of the agency problem by simply selling him the firm.[11]

An important simplifying assumption is that we do not allow the manager's salary, S, to depend on reports, audit results, capital allocated, or output, i.e., managerial compensation must be straight salary. That is, the only incentive device we consider is the capital allocation process. It can be shown that, even if one allows managerial compensation to depend on these other factors, one cannot achieve first-best without a capital budgeting process that includes the possibility of auditing managerial reports. That is, there is a role for the capital allocation process in addition to compensation schemes. Of course, if salary can depend on reports, audit results, etc., the allocation of capital will bear less of the burden of creating incentives for truthtelling. This will result in less distortion of investment relative to first-best. Nevertheless, we believe the essential nature of the optimal capital budgeting process would remain. Given the obvious difficulty of analyzing both capital allocation and compensation in one model and the relative scarcity of models of the former, we have chosen to focus on capital allocation (see the concluding section for further discussion of this issue).[12]

Before analyzing headquarters' problem, we consider, as a benchmark, the first-best allocation of capital and salary. Later, we will compare the first-best level of investment with the NPV maximizing investment (k_t^*) and the allocations resulting from solving headquarters' problem. To be directly comparable

[11] For similar results in different contexts, see Demski, Sappington, and Spiller (1988) and Fellingham and Young (1990).

[12] For an analysis of a situation involving project selection and managerial compensation that focuses exclusively on the role of compensation in creating incentives, see Hirshleifer and Suh (1992).

to the solution of headquarters' problem derived below, we define the first-best to be the solution that would be obtained if there were no information asymmetry, i.e., the solution to headquarters' problem after removing constraints (3). Note that this is equivalent to solving headquarters' problem under the assumption that auditing is free ($q = 0$). Not surprisingly, to exploit the (symmetric) information about capital productivity, first-best investment is different for different technologies.[13] It follows that the first-best allocation cannot be implemented without auditing when information is asymmetric. Since the manager's salary is the same regardless of reported productivity, the manager will always claim to have the productivity that results in the largest capital allocation.

Finally, we compare the first-best capital allocation with the NPV maximizing allocation. It is easy to check that the first-best capital allocation for technology t satisfies the marginal condition $V_t'(k) + \gamma_t b_t = 0$, where $\gamma_t \geq 0$. The NPV maximizing investment satisfies the condition $V_t'(k) = 0$. Thus, unlike the NPV maximizing investment, the first-best investment takes into consideration the fact that the manager obtains utility from capital (b_t) and is therefore larger for each technology than NPV maximizing investment levels, k_t^* (mathematically, this follows from the fact that V_t' is decreasing).

Table I presents numerical solutions to headquarters' problem for various values of audit costs, q. Notice that the audit probability when the manager reports the lowest productivity, α_1, is zero for all values of q. Also, $\alpha_2 \leq \alpha_3$ for all values of q and both decline to zero as q increases. The capital allocation when the manager reports the lowest productivity and there is no audit increases as q increases. The corresponding allocation when the manager reports the middle productivity, $k_{02} \geq k_{01}$ and declines with q until the two become equal at the value of q at which α_2 becomes zero ($q = 0.045$). Similarly, $k_{03} \geq k_{02}$ and declines with q until it becomes equal to k_{01} and k_{02} at the value of q at which α_3 becomes zero ($q = 0.105$). Finally, the capital allocations when the manager's report (and the true productivity) is $t = 2$ or 3 and the report is audited are the NPV maximizing levels, i.e., $k_{12} = k_2^*$ and $k_{13} = k_3^*$. The value of k_{11} is irrelevant, since there is no probability of audit if the manager reports $t = 1$. The first-best solution is also indicated. Notice that, in the first-best solution, $k_{11} > k_1^*$. In the next section, we show that these properties hold in general and provide the intuition for these results.[14]

[13] If the capital allocations were the same for all technologies, the participation constraints for technologies two and three would be slack. Moreover, at least one of V_2' and V_3' cannot be zero at the common capital allocation. Since the two participation constraints are slack, the objective function can be increased without violating any constraints by changing the capital allocated when one (or both) of technologies two and three obtains.

[14] Extremely careful readers may notice that the NPV functions used in this example do not satisfy the monotonicity assumptions with respect to the technology type. Nevertheless, the numerical solutions exemplify the results in our propositions (i.e., the violated assumptions are sufficient but not necessary).

Table I

Numerical Example

This table presents numerical solutions to headquarters' problem for various values of audit cost, q. Each column shows, for a particular audit cost, the audit probabilities, α_1, α_2, and α_3, the capital allocations when no audit occurs, k_{01}, k_{02}, and k_{03}, and the manager's salary, S. For these examples, the assumed production technologies are

$$V_t(k) = 20(k^{\theta_t} - k),$$

for $t = 1, 2, 3$, where $\theta_1 = 0.5$, $\theta_2 = 0.6$, and $\theta_3 = 0.7$. The other parameters are $b_1 = 0.002$, $b_2 = 0.003$, $b_3 = 1$, $\pi_1 = 0.1$, $\pi_2 = 0.3$, $\pi_3 = 0.6$, $\bar{u} = 0.0006$.

For these parameter values, the NPV maximizing investment levels are $k_1^* = 0.25$, $k_2^* = 0.27885$, and $k_3^* = 0.30455$. Finally, k_{1t} coincides with k_t^* for $t = 2$ and 3, for all values of q (the value of k_{11} is irrelevant since $\alpha_1 \equiv 0$). For the first-best solution, $\alpha_t = 1$ and k_{0t} is irrelevant for all t, $k_{11} = 0.25050$, and k_{1t} coincides with k_t^* for $t = 2$ and 3.

q	0.00500	0.02500	0.04500	0.06500	0.08500	0.10500	0.12500
α_1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
α_2	0.08576	0.01760	0.00000	0.00000	0.00000	0.00000	0.00000
α_3	0.16248	0.09823	0.06541	0.04335	0.02049	0.00000	0.00000
k_{01}	0.25437	0.27088	0.27765	0.28104	0.28453	0.28764	0.28764
k_{02}	0.27823	0.27574	0.27765	0.28104	0.28453	0.28764	0.28764
k_{03}	0.30372	0.30039	0.29708	0.29378	0.29048	0.28764	0.28764
S	0.00009	0.00006	0.00004	0.00004	0.00003	0.00002	0.00002

III. Results

This section fully characterizes the solution to headquarters' problem for various values of the parameters of the problem. Intuition for the results is also provided (formal proofs are in the Appendix). The interpretation of the results in terms of observed phenomena is postponed to the next section.

First consider the optimal audit probabilities, α_t. Since the purpose of an audit by headquarters is to prevent the manager from overstating the productivity of capital, it is not surprising that, if the manager claims to have the lowest productivity ($t = 1$), headquarters will not pursue a costly audit, i.e., $\alpha_1 = 0$. Whether headquarters audits when the manager claims that productivity is not the lowest depends on the reported productivity and the cost of auditing. If audit costs, q, are low (below a critical level denoted q_2), headquarters will audit with positive probability α_2, even when the manager claims intermediate productivity ($t = 2$). It audits with higher probability $\alpha_3 > \alpha_2$ if the manager claims high productivity ($t = 3$). As audit costs increase, these probabilities, α_2 and α_3, decline until α_2 becomes zero at $q = q_2$. As audit costs increase above q_2, headquarters will audit with positive probability if and only if the manager claims to have high productivity. This probability, α_3, declines to zero as audit costs increase to a critical level denoted q_3. For audit costs above q_3, headquarters finds it too costly to audit even if the manager claims to have high productivity. Finally, as long as audit is costly, it is never optimal to audit with certainty. The reason for this is that the manager can be

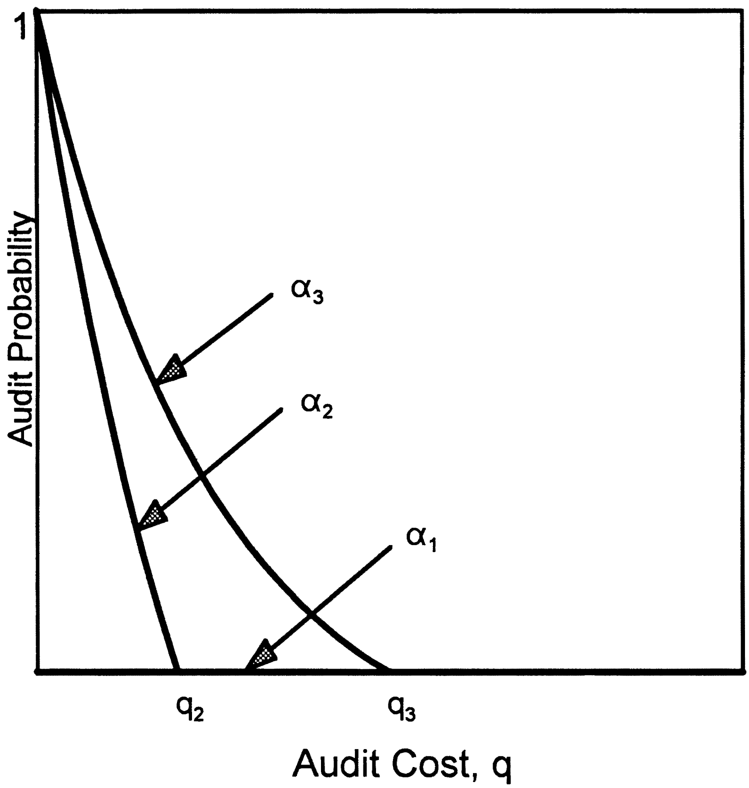

Figure 1. Audit probabilities as functions of audit cost. For each level of reported productivity, $t \in \{1, 2, 3\}$, the probability of an audit, α_t, is shown as a function of audit cost, q. The critical levels of audit costs, q_2 and q_3, at which α_2 and α_3 become zero, respectively, are also indicated.

motivated to tell the truth using the same allocation as when $\alpha_t = 1$, even if headquarters audits with slightly lower probability. This saves expected audit costs while not affecting the capital allocation. These results are summarized in

PROPOSITION 1. *The optimal audit probabilities, α_t, satisfy:*

- $0 = \alpha_1 \leq \alpha_2 \leq \alpha_3 < 1$ *for all* $q > 0$,
- α_t *is nonincreasing in* q, *and*
- *there exist audit costs* $0 < q_2 < q_3$, *such that for* $q \in (0, q_t)$, $\alpha_t > 0$ *and strictly decreasing and for* $q > q_t$, $\alpha_t \equiv 0$.

The audit probabilities as functions of audit cost are depicted in Figure 1. The proof of Proposition 1 is contained in Theorems 1 and 2 in the Appendix.

The next proposition characterizes the optimal capital allocation.

PROPOSITION 2. *If an audit occurs, headquarters allocates the NPV maximizing level of capital,* $k_{1t} = k_t^*$.[15] *If no audit occurs,* $k_1^* < k_{01} \leq k_{02} \leq k_{03} < k_3^*$, *for* $q > 0$, k_{01} *is nondecreasing and* k_{03} *is nonincreasing in* q. *Furthermore,*

- *for* $q < q_2$, $k_{01} < k_{02} < k_{03}$, k_{01} *increases strictly and* k_{02} *and* k_{03} *decrease strictly with* q, *and* k_{01} *and* k_{02} *converge to a common value at* $q = q_2$,[16]
- *for* $q_2 \leq q < q_3$, $k_{01} \equiv k_{02} < k_{03}$, $k_{01} = k_{02}$ *increases strictly and* k_{03} *decreases strictly with* q, *and* k_{03} *converges to* $k_{01} = k_{02}$ *at* $q = q_3$,
- *for* $q \geq q_3$, $k_{01} \equiv k_{02} \equiv k_{03} \equiv k^{***}$, *a constant, independent of* q.

The manager's salary, S, *is chosen to provide him utility of* $\bar{u}$ *for the lowest productivity, i.e.,* $S = \max\{0, \bar{u} - b_1 k_{01}\}$.

The capital allocated as a function of audit cost is depicted in Figure 2. The proof of Proposition 2 is contained in Theorems 1 and 2 in the Appendix. We discuss the intuition for this result in the next paragraphs.

Consider the capital allocated following an audit. As mentioned above, if the audit reveals a false report by the manager, no capital is allocated. If the audit reveals that a report is truthful, it is optimal to allocate the value maximizing amount of capital, k_t^*, for that productivity.[17]

Next, consider the capital allocated when no audit takes place. The incentive compatible allocation depends on which reports are audited with positive probability. If headquarters is certain not to audit either of two different reports, then the capital allocated must be the same for those two reports (otherwise the manager would always report the productivity that results in the larger capital allocation). Since, from Proposition 1, for high audit costs ($q \geq q_3$) no report is audited with positive probability, it follows that for $q \geq q_3$, the capital allocation must be the same for all reports, i.e., $k_{01} = k_{02} = k_{03}$. The common value is independent of q over this range, and we denote it by k^{***}. This is the allocation that maximizes headquarters' expected return subject to the participation constraints, given that reports are not audited for sure, and the same amount is allocated regardless of reported

[15] Recall that, in equilibrium, the audit always reveals that the manager's report is truthful.

[16] As q approaches zero, k_{02} and k_{03} approach k_2^* and k_3^*, respectively (as shown in Fig. 2). When the participation constraint for the least productive technology, i.e., constraint (4) for $t = 1$ is binding, then k_{01} approaches an amount that is greater than k_1^* as q approaches zero. This is the case shown in Figure 2. If this constraint is not binding, then k_{01} approaches k_1^* as q approaches zero. This latter case would generally not be observed in practice as it implies that the manager's salary is zero.

[17] This would be clear if there were no incentive or participation constraints. The four relevant incentive constraints, however, are slack. Three of these are slack because they ensure that the manager does not claim that productivity is lower than it actually is. Given the manager's preference for more capital, preventing understatement of the true productivity is not a problem. The fourth incentive constraint is implied by two other incentive constraints. Finally, since higher productivity is associated with greater managerial preference for capital ($b_1 < b_2 < b_3$), satisfying the participation constraint when the manager reports low productivity ensures that this constraint will be satisfied for higher reports. Thus, none of the relevant constraints is binding, so the optimal allocation after an audit is NPV maximizing.

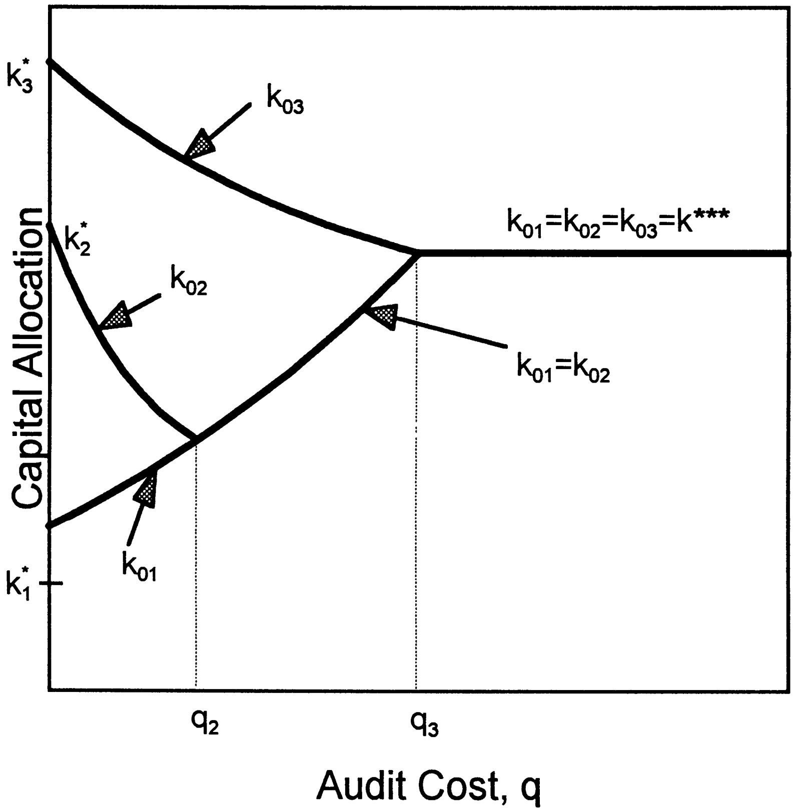

Figure 2. Capital allocations as functions of audit cost. For each level of reported productivity, $t \in \{1, 2, 3\}$, the capital allocated if no audit occurs, k_{0t}, is shown as a function of audit cost, q. The NPV maximizing level of investment, k_t^*, for each t is shown. The critical levels of audit costs, q_2 and q_3, at which α_2 and α_3 become zero, respectively, are also indicated.

productivity (see equation (A11) in the Appendix for a formal statement). Similarly, for intermediate audit cost levels ($q_3 > q \geq q_2$), headquarters never audits if the manager claims low or intermediate productivity ($\alpha_2 = \alpha_1 = 0$). Therefore, capital allocations for these reports must be the same ($k_{01} = k_{02}$ over this range of audit costs). Since productivity of capital is greater in equilibrium when the manager reports high productivity ($t = 3$), over this range, a claim of higher productivity is rewarded with a larger capital allocation ($k_{03} > k_{02} = k_{01}$). As q increases from q_2 to q_3, the probability of auditing a report of high productivity declines (see Proposition 1), so to prevent a manager who observes low or intermediate productivity from reporting high productivity, the difference between k_{03} and the common value of k_{01} and k_{02} must shrink (to zero at $q = q_3$). Consequently, k_{03} declines to k^{***}, and k_{01} and k_{02} increase to k^{***} as q increases from q_2 to q_3. For low audit costs ($q < q_2$), since $\alpha_2 > 0$ and $\alpha_3 > 0$ and decreasing with q (Proposition 1), a similar argument applies to k_{01}, k_{02}, and k_{03}, i.e.,

$k_{03} > k_{02} > k_{01}$, k_{01} increases and k_{02} and k_{03} decrease with q. k_{01} and k_{02} reach their common value at $q = q_2$.

Finally, since higher productivity is associated with greater managerial preference for capital ($b_1 < b_2 < b_3$), satisfying the participation constraint when the manager reports low productivity ensures that this constraint will be satisfied for higher reports. Salary is set to ensure that this constraint is satisfied as an equality (given $\alpha_1 = 0$), unless no salary is required to satisfy it.

Proposition 2 shows that, when no audit occurs, the capital allocated when productivity is low (high) is greater (less) than the NPV maximizing amount, $k_{01} > k_1^*$ ($k_{03} < k_3^*$). The next result shows that this result extends to the first-best level of investment (see the end of Section II for a discussion of the first-best).

COROLLARY 1. *Relative to the first-best and NPV maximizing levels of investment, the optimal capital budgeting procedure exhibits underinvestment when realized capital productivity is high and overinvestment when realized capital productivity is low.*[18]

Propositions 1 and 2 characterize the solution as a function of audit costs. Our next result characterizes the solution as a function of the investment opportunity set. We model an improvement in the investment opportunity set as a shift in the production technologies in which, for any given level of investment and for any technology, NPV increases by the same percent. Formally, we introduce a scale parameter, s, such that NPV of technology t with scale parameter s is given by $V_t(k; s) = sV_t(k; 1) \equiv sV_t(k)$. We refer to changes in s as changes in the *scale of investment opportunities*.

PROPOSITION 3. *Suppose s increases from s_0 to s_1. This results in an increase in q_2, q_3, $\alpha_t(q)$ and a decrease in k_{01}. Also, there exists $q^t \in (q_t(s_0), q_t(s_1))$ such that k_{0t} increases for every $q \in (0, q^t)$ and decreases for every $q \geq q^t$, for $t = 2, 3$. Finally, k_{1t} is independent of s.*

The behavior of q_t, k_{0t}, and α_t with respect to s is shown in Figures 3 and 4. This result is proved as Theorem 3 in the Appendix. The intuition can be understood most easily by noting that an increase in the scale of investment opportunities does not change the NPV maximizing capital input, k_t^*, but does increase the cost (in terms of NPV) of deviating from this level. Thus, after an increase in s, it is optimal to audit with higher probability (recall audit always results in allocating k_t^*) and to allocate capital that is closer to k_t^* when not auditing. Since $k_{01} > k_1^*$, the increase in s results in a reduction in k_{01}.

[18] The proof is straightforward: for $q = 0$, it is obvious that k_{0t} is first-best. As q increases from zero, by Proposition 2, k_{01} increases and k_{03} decreases, thus becoming strictly greater than (respectively less than) first-best. The result for the intermediate productivity depends on audit costs. When $q < q_2$, there is underinvestment, $k_{02} < k_2^*$. For $q \geq q_2$, when $t = 2$, there could be either over- or underinvestment depending on the production functions.

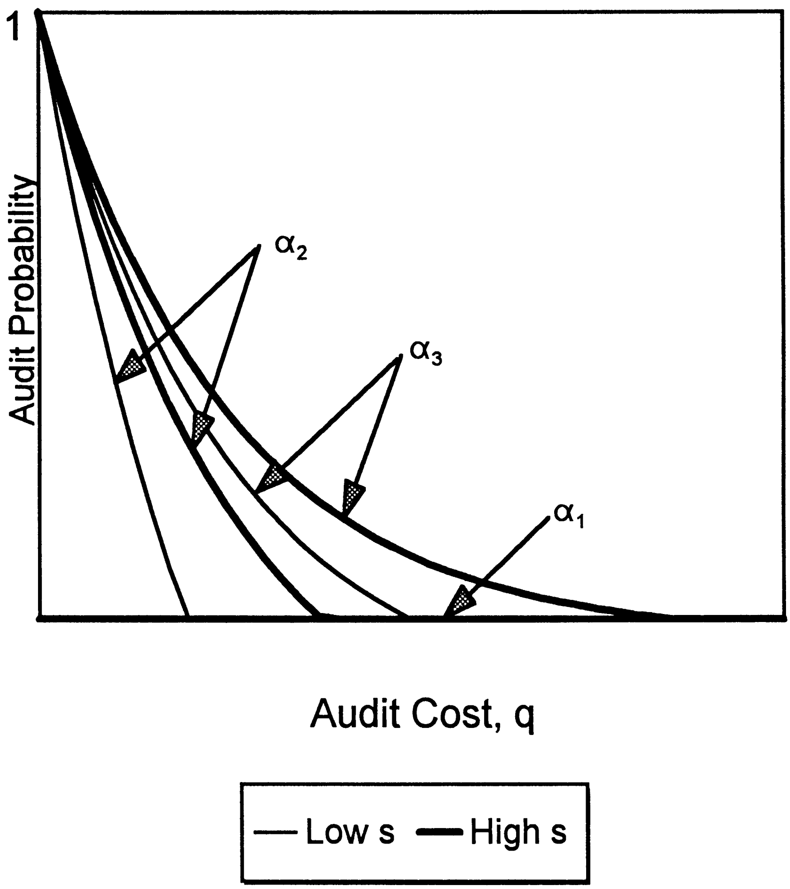

Figure 3. Effect of scale on audit probabilities. For each level of reported productivity, $t \in \{1, 2, 3\}$, the probability of an audit, α_t, is shown as a function of audit cost, q, for two different levels of the scale parameter, s.

Similarly, k^{***} (the capital allocated when there is no probability of audit) exceeds the level of capital that maximizes average NPV, so k^{***} decreases with s.

We next investigate how the optimal capital budgeting procedure depends on headquarters' prior beliefs about the division's technology. In particular, we show what happens to audit probabilities and capital allocations when the distribution of investment opportunities shifts to the left (in the sense of first-order stochastic dominance). Specifically, we shift prior probability between the two extreme productivities, 1 and 3, holding constant the probability of the intermediate productivity. This parameterization admits easy analysis. Later we interpret high probability of the least productive technology (high π_1 and low π_3) as indicative of bad times, e.g., recessions, and low π_1 as indicative of good times, e.g., booms.

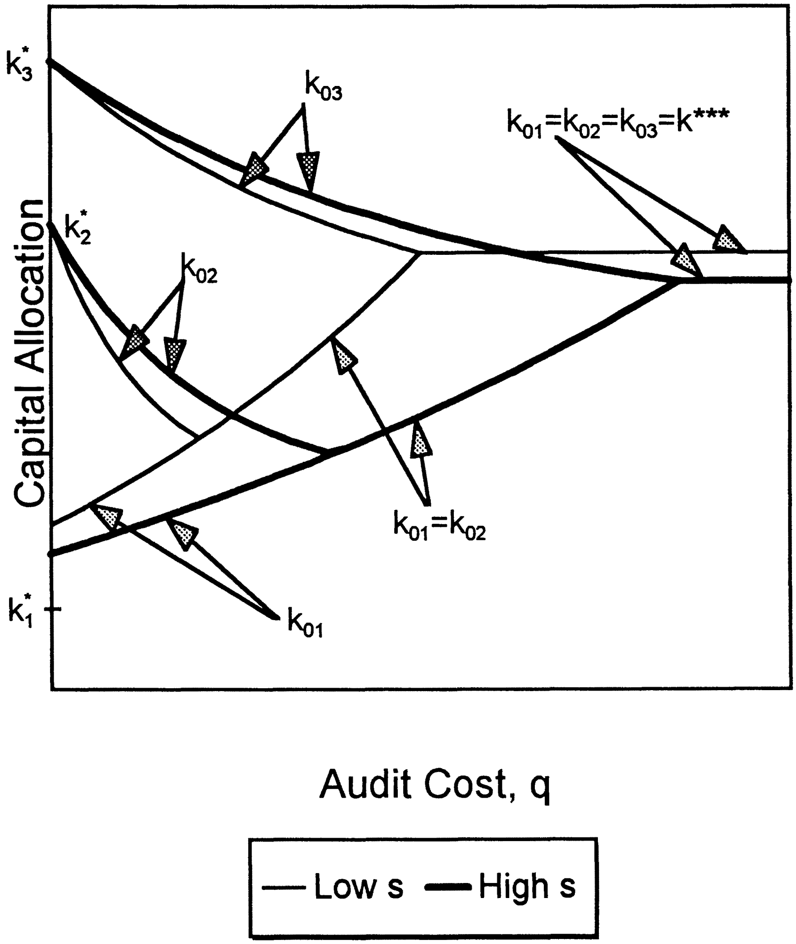

Figure 4. Effect of scale on capital allocations. For each level of reported productivity, $t \in \{1, 2, 3\}$, the capital allocated if no audit occurs, k_{0t}, is shown as a function of audit cost, q, for two different levels of the scale parameter, s.

PROPOSITION 4. *Consider an increase in the probability of low productivity, π_1, holding π_2 constant (and reducing π_3 accordingly). This results in an increase in α_t and a decrease in k_{01} for each q. Also, q_t increases; k_{0t} does not change for every $q \in (0, q_t]$ and decreases for every $q > q_t$, for $t = 2, 3$. Finally, $k_{1t} = k_t^*$, independent of π_1.*

The behavior of the capital allocations, k_{0t}, is shown in Figure 5. This result is proved as Theorem 4 in the Appendix. Intuitively, as the probability of the lowest productivity increases, it is optimal to make the capital allocated when productivity is lowest closer to its NPV maximizing level. Since the capital allocated when productivity is lowest is above its NPV maximizing level ($k_{01} > k_1^*$), an increase in π_1 results in a reduction in k_{01}. For technologies, t, in

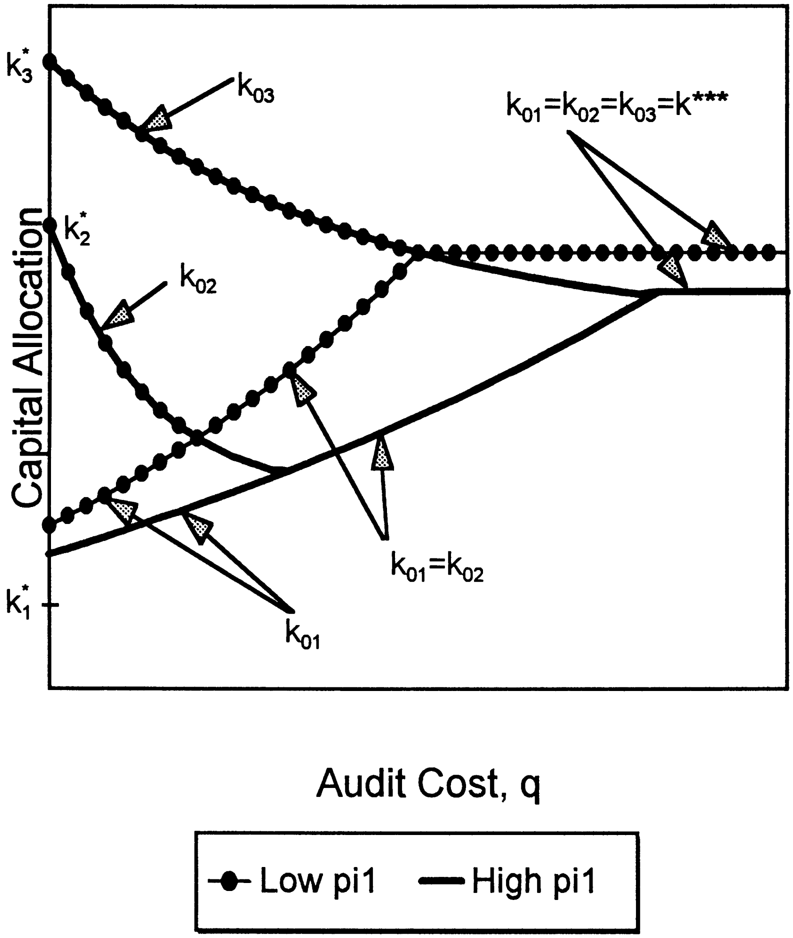

Figure 5. Effect of prior beliefs on capital allocations. For each level of reported productivity, $t \in \{1, 2, 3\}$, the capital allocated if no audit occurs, k_{0t}, is shown as a function of audit cost, q, for two different levels of the prior probability of low productivity, π_1.

which $\alpha_t > 0$, k_{0t} is chosen to balance the cost and benefit of audit. The cost is simply q. The benefit is in allocating the NPV maximizing capital, k_t^*, instead of the nonmaximizing capital k_{0t}. Since the probabilities, π_t, do not affect q and k_t^*, they do not affect the optimal choice of k_{0t} when $\alpha_t > 0$. Since k_{01} decreases when π_1 increases and k_{0t} stays constant (for $t = 2,3$), α_t must increase to ensure that the division manager continues to report truthfully.

IV. Empirical Implications

In this section, we demonstrate the relevance of our results for the understanding of observed capital budgeting procedures. The previous two sections show that an optimal capital budgeting procedure consists of a revelation game whose allocation rule was derived in Propositions 1–4. We start by reinterpreting the procedure to correspond more closely with observed practice, which involves capital rationing but includes a mechanism for exceeding the spending limit. The implication is that observed capital budgeting schemes can be rationalized as optimal responses to private information and incentive problems. We also use the results from Section III to obtain additional empirical implications.

The optimal revelation mechanism derived in the previous sections is equivalent to the following procedure in that the two schemes result in exactly the same equilibrium allocations. In the reinterpreted capital budgeting procedure, first headquarters provides an *initial spending limit*, k_{01}, that, unless the manager requests more, will be allocated without further audit of the manager's proposed project. Second, the manager may simply accept the initial spending limit or request a *capital spending increase* to either k_2^* or k_3^*. Headquarters may or may not consider a requested increase. In particular, headquarters specifies a *minimum considered request* and ignores requests below this level. The minimum considered request is the smallest request for which there is any positive probability of audit. In response to a request (of at least the minimum), headquarters may grant *a compromise increase*, k_{0t}, if the manager requested k_t^*, without auditing (probability $1-\alpha_t$). The compromise increase is between the initial spending limit and the requested amount. Alternatively, headquarters may audit the division's project (with probability α_t) and, assuming the request is justified, allocate the manager's requested increase. It should be clear that the reinterpreted capital budgeting procedure results in the same equilibrium outcome as the revelation scheme in the previous section and is therefore optimal.

In the remainder of this section, we apply the comparative statics results in Propositions 1–4 to the reinterpreted capital budgeting procedure. There are four exogenous variables of interest: audit costs, q, the scale of investment opportunities as measured by the scale parameter, s, the prior probability that capital productivity is low, π_1, and the size of the division's requested capital spending increase. Empirically, one might identify R & D divisions, divisions within a conglomerate, and new product divisions as more costly (higher q) than other types of divisions for headquarters to audit. Similarly, we expect divisions in less competitive industries to have greater scale of investment opportunities (higher s). Finally, we think of recessions as periods when low productivity is more likely (higher π_1) and booms as periods when low productivity is less likely (lower π_1). One can use these observable proxies to test the empirical implications of the model.

The five endogenous variables are the rigidity of the capital budgeting system, the initial spending limit, the probability that the division's request for a capital spending increase is approved (referred to as the *approval probability*), the compromise increases, and the manager's salary.[19] *Rigidity* is mea-

[19] The features other than the approval probability are properties of the *procedure*, but the approval probability is a property of the *equilibrium*. This probability corresponds exactly to the audit probability (a property of the procedure), since, in equilibrium, a request is granted if and only if it is audited.

Table II
Direction of Change in Various Endogenous Variables Resulting from an Increase in the Given Exogenous Variables

	Exogenous Variables			
Endogenous Variables	Audit cost, q	Scale, s	Pr(low productivity), π_1	Request, k_t^*
Rigidity (minimum considered request)	+	−	−	
Initial spending limit	+	−	−	
Approval probabilities	−	+	+	+
Compromise increases	−	+	0	+
Salary	−	+	+	

Table III
Covariances Between Pairs of Endogenous Variables

	Rigidity (Minimum Considered Request)	Initial Spending Limit	Approval Probabilities	Compromise Increases
Initial spending limit	+			
Approval probabilities	−	−		
Compromise increases	−	−	+	
Salary	−	−	+	+

sured by headquarters' minimum considered request. By the term rigidity, we have in mind the strictness with which the initial spending limit is enforced. In particular, the system is considered more rigid the more headquarters limits the set of requests it will consider. The larger is the minimum considered request, the fewer requests headquarters will entertain, resulting in a more rigid system. In a completely rigid system, for example, headquarters refuses to increase the initial spending limit under any circumstances.

Comparative statics are summarized in Tables II and III. Table II traces the effect on each endogenous variable of an increase in the various exogenous variables of interest. These implications follow immediately from Propositions 1–4. Given the empirical proxies mentioned above, we expect R & D divisions, divisions within a conglomerate, and new product divisions to face more rigid capital budgeting procedures, have higher initial spending limits, lower probabilities of approval for a given request, lower compromise increases for a given request, and lower managerial salary. We also expect more monopolistic divisions to face less rigid capital budgeting systems, lower initial spending limits, to have greater probabilities of approval of spending increases, larger compromise increases, and higher managerial salaries. Our results further suggest that capital budgeting procedures should be less rigid, initial spending limits should be lower, approval probabilities should be higher, and salary should be higher in recessions.[20] Finally, in a time series of divisional requests and headquarters' responses, approvals should be more frequent for larger requests.

Table III describes the covariation between various pairs of endogenous variables even if all the exogenous variables are allowed to vary at once, providing they are statistically independent. This result follows from Table II and some results on associated random variables (see, e.g., Holmstrom and Milgrom (1994)). A detailed proof is given in the Appendix, Lemma 19. The relationships described in Table III provide another avenue for testing the model. Using this result, in a sample of capital budgeting procedures and outcomes, one would expect to observe high initial spending limits associated with less receptivity to divisional requests for increases (i.e., high rigidity), low frequency of approval of divisional requests, and smaller spending increases on average.[21]

V. Summary and Conclusions

In this article we have taken a step toward reconciling investment theory with actual practice. In our view, the essential features of the environment that give rise to the type of capital budgeting processes observed in practice are managerial incentive problems and asymmetric information. We construct a simple model that incorporates these two features and show that an optimal capital budgeting process in this environment exhibits many of the characteristics of actual allocation schemes. In particular, we show that in an optimal process, headquarters specifies an initial capital spending limit. Managers may request additional capital, and when audit costs are not too high, this may result in either an audit and fulfillment of the request or a compromise increase in the allocation. This procedure obviously deviates from the NPV rule and can result in underinvestment for high productivity projects and overinvestment for low productivity projects. We also establish a number of empirical implications relating the initial spending limit, compromise allocations, approval probabilities, salary, and the rigidity of the system to each other and to audit costs, the scale of investment opportunities, the prior distribution of capital productivities, and the size of divisional requests for spending increases.

There are a number of issues that this model does not address. For example, at what level would one expect capital spending limits to apply: at the project

[20] Note, however, that if the probability of low productivity is high in recessions, one would expect the manager's opportunity cost of working for this firm, $\bar{u}$, to be low whenever the probability of low productivity is high. Thus, in any empirical test, the manager's opportunity cost must be controlled for.

[21] Although we have relaxed some of the ceteris paribus requirements, one should be careful in applying these results to note that exogenous variables other than q, s, and π_1 must be controlled for. Thus, for example, one must control for parameters of the technology other than s, as well as b_1 and $\bar{u}$.

level or at the level of a business unit? This issue can be addressed by extending the current model to allow multiple projects.

Another set of important issues concerns the interaction of the capital allocation process and managerial compensation schemes. Obviously, these schemes affect managers' incentives to communicate truthfully and to request realistic capital allocations.[22] Should compensation depend on budget requests, allocated budgets, both, or neither? Would one expect a manager's compensation to depend on final outcomes? If multiple projects are involved, should compensation depend on aggregate performance or on project-by-project results?

Consideration of the dynamics of the capital allocation process raises further interesting questions. Should budgets take account of past expenditures relative to past spending limits, e.g., should managers be allowed to roll over unspent allocations? To what extent should current budgets depend on past performance? How do the characteristics of the capital allocation process, including managerial compensation schemes, affect the choice between long term and short term projects? Clearly, addressing these issues requires a multiperiod framework.[23]

Finally, it would be interesting to explore an environment in which the division manager has some control over the type of project available to the division and its audit cost. In this case, the capital budgeting process will affect the manager's project choice, and the manager's ability to manipulate the project technology and information cost will affect the capital budgeting process.

Appendix

In what follows we rewrite constraint (3) by canceling S from both sides, then dividing through by b_t. From now on, references to "headquarters' problem" or constraint (3) are taken to mean the problem in the text with constraint (3) modified as just described or the modified constraint. We will also refer to the incentive compatibility constraint in equation (3) that has the subscript t on the left side and m on the right ($t, m \in T, m \neq t$) as $(3)_{tm}$.

Define $\mu_{tm} \geq 0$ to be the Lagrange multiplier for $(3)_{tm}$. Also, let $\lambda_t \geq 0$ be the Lagrange multiplier for the individual rationality constraint in equation

[22] Pinches (1982, p. 16) criticizes business practitioners for ignoring this interaction: ". . . one of the most crucial weaknesses in practice is the failure to design the evaluation/reward/punishment system in such a manner that it complements the firm's capital budgeting operation."

[23] Two articles that address intrafirm resource allocation in a multiperiod framework are Antle and Fellingham (1990) and Fellingham and Young (1990). These models are multi-period versions of Antle and Eppen (1985) and show that, even though no one learns about next period's project from this period's, dependence of future investment decisions on past decisions can improve resource allocation. Adding periods relaxes incentive compatibility constraints by giving headquarters an additional tool to discipline the manager in earlier periods.

(4) that has the subscript t (hereafter referred to as $(4)_t$). Using this notation, the first-order conditions for an interior solution for k_0 and k_1 are:

$$\pi_1 V_1'(k_{01}) = \mu_{21} + \mu_{31} - \mu_{12} - \mu_{13} - \lambda_1 b_1 \quad (\text{if } \alpha_1 < 1), \tag{A1}$$

$$\pi_1 V_1'(k_{11}) = -\mu_{12} - \mu_{13} - \lambda_1 b_1 \quad (\text{if } \alpha_1 > 0), \tag{A2}$$

$$\pi_2 V_2'(k_{02}) = \mu_{12} + \mu_{32} - \mu_{21} - \mu_{23} - \lambda_2 b_2 \quad (\text{if } \alpha_2 < 1), \tag{A3}$$

$$\pi_2 V_2'(k_{12}) = -\mu_{21} - \mu_{23} - \lambda_2 b_2 \quad (\text{if } \alpha_2 > 0), \tag{A4}$$

$$\pi_3 V_3'(k_{03}) = \mu_{13} + \mu_{23} - \mu_{31} - \mu_{32} - \lambda_3 b_3 \quad (\text{if } \alpha_3 < 1), \tag{A5}$$

$$\pi_3 V_3'(k_{13}) = -\mu_{31} - \mu_{32} - \lambda_3 b_3 \quad (\text{if } \alpha_3 > 0), \tag{A6}$$

Note that, since $V_t'(0) = \infty$, $k_{0t} > 0$ if $\alpha_t < 1$, and $k_{1t} > 0$ if $\alpha_t > 0$, so the above conditions hold as stated. The first-order condition for S is

$$\sum_{t \in T} \lambda_t \leq 1 \qquad (\text{with equality if } S > 0). \tag{A7}$$

The first-order conditions for α are

$$\begin{aligned} &\text{if } \alpha_t = 0, && \text{then } C_t \leq 0; \\ &\text{if } \alpha_t = 1, && \text{then } C_t \geq 0; \\ &\text{if } \alpha_t \in (0,\ 1), && \text{then } C_t = 0, \end{aligned} \tag{A8}$$

where

$$C_t = \pi_t[V_t(k_{1t}) - q] + \left(\lambda_t b_t + \sum_{m \neq t} \mu_{tm}\right)(k_{1t} - k_{0t}) + \sum_{m \neq t} \mu_{mt} k_{0t}. \tag{A9}$$

Summing conditions (A1), (A3), and (A5) gives

$$\sum_{t \in T} [\pi_t V_t'(k_{0t}) + \lambda_t b_t] = 0 \qquad (\text{if } \alpha_t < 1\ \forall t). \tag{A10}$$

For the results that follow, $(S,\ \alpha,\ k_0,\ k_1,\ \mu,\ \lambda)$ is an optimal solution of headquarters' problem.

LEMMA 0: $\alpha_t < 1$ *for all* t.

Proof: Suppose $\alpha_t = 1$ for some t. Consider the alternative solution $(\tilde{S},\ \tilde{\alpha},\ \tilde{k}_0,\ \tilde{k}_1)$ in which $\tilde{S} = S$; $\tilde{\alpha}_m = \alpha_m$, $\tilde{k}_{0m} = k_{0m}$ and $\tilde{k}_{1m} = k_{1m}$ for $m \neq t$; $\tilde{\alpha}_t = 1 - \varepsilon$, $\tilde{k}_{0t} = \tilde{k}_{1t} = k_{1t}$, where $\varepsilon > 0$ will be determined below. The objective function, equation (2), evaluated at the alternative solution is larger than at the initial solution by $\pi_t \varepsilon q$. Also both sides of all constraints except the right sides of $(3)_{mt}$ for $m \neq t$ are the same as in the initial solution. Since $k_{0m} > 0$ if $\alpha_m < 1$, and $k_{1m} > 0$ if $\alpha_m > 0$, the left sides of $(3)_{mt}$ for $m \neq t$ are strictly positive (for both the initial and alternative solutions). The right sides of $(3)_{mt}$ for $m \neq t$ for the alternative solution are given by εk_{1t}. For ε sufficiently small, the alternative solution satisfies these constraints. Therefore, the alternative solution is feasible and achieves a higher value of the objective function than the initial solution contradicting its optimality. Q.E.D.

LEMMA 1: *If*

$$\sum_{m \neq t} \mu_{mt} = 0,$$

or $k_{0t} = k_{1t}$ *for some* t, *then* $\alpha_t = 0$.

Proof: Suppose

$$\sum_{m \neq t} \mu_{mt} = 0.$$

Therefore, $\mu_{mt} = 0$ for all $m \neq t$. If $\alpha_t \in (0,1)$, then the first-order conditions for k_{0t} and k_{1t} imply that $k_{0t} = k_{1t}$. It follows that $C_t = -\pi_t q < 0$, which contradicts (A8). Therefore, we must have $\alpha_t \in \{0,1\}$, and by Lemma 0, $\alpha_t = 0$. Now suppose $k_{0t} = k_{1t}$ and $\alpha_t \in (0,1)$. Then, from the first-order conditions for k_{0t} and k_{1t},

$$\sum_{m \neq t} \mu_{mt} = 0.$$

The result now follows as before. Q.E.D.

LEMMA 2: *If headquarters' problem has a solution, then there is an optimal solution in which* $k_{1t} \geq k_{0t}$ *for all* t.

Proof: Since equation (2) is concave, if the problem has a solution, it has an optimal solution. Suppose $(S, \alpha, k_0, k_1, \mu, \lambda)$ is an optimal solution. If, for some t, $\alpha_t \in (0,1)$, then the conclusion for this t is obvious from the first-order conditions and the nonnegativity of the Lagrange multipliers. If $\alpha_t = 0$, then k_{1t} is irrelevant and can be assumed to satisfy the inequality. Q.E.D.

In what follows, we consider only solutions in which $k_{0t} \leq k_{1t}$.

LEMMA 3: $\mu_{12} + \mu_{13} + \mu_{23} > 0,$
$\mu_{21} + \mu_{13} + \mu_{23} > 0,$
$\mu_{13} + \mu_{12} + \mu_{32} > 0.$

Proof: We prove only the first inequality; the others are proved similarly. Suppose the first inequality does not hold. Then $\mu_{12} = \mu_{13} = \mu_{23} = 0$. It then follows from Lemma 1 that $\alpha_3 = 0$. Also, from (A5), $V_3'(k_{03}) \leq 0$, so, by concavity of V_3 and the definition of k_3^*, $k_{03} \geq k_3^*$. Using equations (A1) and (A2), we have $\pi_1 V_1'(k_{01}) \geq -\lambda_1 b_1$ and $\pi_1 V_1'(k_{11}) = -\lambda_1 b_1$ (if $\alpha_1 > 0$). But, by (A7), $\lambda_1 \leq 1$, so $\pi_1 V_1'(k_{01}) + b_1 \geq 0$ and $\pi_1 V_1'(k_{11}) + b_1 \geq 0$ (if $\alpha_1 > 0$). Thus, using, (1), $k_{01} < k_2^*$ and $k_{11} < k_2^*$ if $\alpha_1 > 0$. Therefore,

$$(1 - \alpha_1)k_{01} + \alpha_1 k_{11} < k_2^* < k_3^* \leq k_{03} = (1 - \alpha_3)k_{03}.$$

This, however, contradicts equation $(3)_{13}$. Q.E.D.

LEMMA 4: $\mu_{12} + \mu_{13} > 0$ *and* $\mu_{13} + \mu_{23} > 0$.

Proof: First suppose $\mu_{12} + \mu_{13} = 0$. Then, by Lemma 3, $\mu_{23} > 0$ and $\mu_{32} > 0$, and hence $(3)_{23}$ and $(3)_{32}$ are binding. This implies

$$\alpha_2 k_{12} + \alpha_3 k_{13} = 0.$$

Moreover, from equations (A4) and (A6), $k_2^* < k_{12}$ if $\alpha_2 > 0$, and $k_3^* < k_{13}$ if $\alpha_3 > 0$. It follows that $\alpha_2 = \alpha_3 = 0$ and $k_{02} = k_{03}$. As in the proof of Lemma 3, we have $k_{01} < k_2^*$ and $k_{11} < k_2^*$ if $\alpha_1 > 0$. Using $(3)_{12}$, it follows that

$$k_3^* > k_2^* \geq (1 - \alpha_1)k_{01} + \alpha_1 k_{11} \geq k_{02} = k_{03}.$$

Therefore, for $t = 2,3$, $\pi_t V_t'(k_{0t}) + \lambda_t b_t \geq 0$, with strict inequality at least for $t = 3$. For $t = 1$, using equation (A1), this inequality holds weakly. Therefore, these inequalities contradict (A10) and show that $\mu_{12} + \mu_{13} > 0$.

Now suppose $\mu_{13} + \mu_{23} = 0$. Then, from Lemma 3, $\mu_{12} > 0$ and $\mu_{21} > 0$, so $(3)_{12}$ and $(3)_{21}$ must be binding. Also, from Lemma 1, $\alpha_3 = 0$, and, as in the proof of Lemma 3, $k_{03} \geq k_3^*$. The fact that $(3)_{12}$ and $(3)_{21}$ must be binding implies

$$\alpha_1 k_{11} + \alpha_2 k_{12} = 0.$$

Thus $\alpha_1 = \alpha_2 = 0$ and $k_{01} = k_{02}$ as in the proof of the previous case. Similarly, $(3)_{13}$ and $(3)_{31}$ and the fact that $\alpha_1 = \alpha_3 = 0$ imply that $k_{01} = k_{03}$. Thus, we have $\alpha_t = 0$ for all t, and k_{0t} are the same for all t, say $k_{0t} = k_0$, with $k_0 \geq k_3^* > k_2^* > k_1^*$.

Now, since $k_{0t} = k_0$ and $\alpha_t = 0$ for all t, examination of constraint (4) shows that, at most, this constraint is binding for $t = 1$. Therefore from (A7), $\lambda_2 = \lambda_3 = 0$ and $\lambda_1 \leq 1$. Now, $k_0 \geq k_3^*$ implies $\pi_t V_t'(k_0) \leq 0$ for $t = 2,3$ and

$$\pi_1 V_1'(k_0) + \lambda_1 b_1 \leq \pi_1 V_1'(k_2^*) + b_1 < 0$$

using equation (1). This along with $\pi_t V_t'(k_0) \leq 0$ for $t = 2,3$, however, violates equation (A10). The contradiction establishes $\mu_{13} + \mu_{23} > 0$. Q.E.D.

LEMMA 5: *Suppose* $(3)_{kt}$ *is binding. Then* $(3)_{tk}$ *is redundant,* $(3)_{km}$ *implies* $(3)_{tm}$ *for all* $m \neq t$ *or* k, *and* $(3)_{mt}$ *implies* $(3)_{mk}$ *for all* $m \neq t$ *or* k.

Proof: Assume $(3)_{kt}$ is binding, i.e.,

$$(1 - \alpha_k)k_{0k} + \alpha_k k_{1k} = (1 - \alpha_t)k_{0t}.$$

Then,

$$(1 - \alpha_t)k_{0t} + \alpha_t k_{1t} \geq (1 - \alpha_t)k_{0t} \geq (1 - \alpha_k)k_{0k},$$

so $(3)_{tk}$ is redundant. Also, for $m \neq k$ or t, $(3)_{km}$ implies

$$(1 - \alpha_t)k_{0t} + \alpha_t k_{1t} \geq (1 - \alpha_t)k_{0t} = (1 - \alpha_k)k_{0k} + \alpha_k k_{1k} \geq (1 - \alpha_m)k_{0m},$$

which implies $(3)_{tm}$. Finally, for $m \neq k,t$, $(3)_{mt}$ implies

$$(1 - \alpha_m)k_{0m} + \alpha_m k_{1m} \geq (1 - \alpha_t)k_{0t} = (1 - \alpha_k)k_{0k} + \alpha_k k_{1k} \geq (1 - \alpha_k)k_{0k},$$

which implies $(3)_{mt}$.

LEMMA 6: $(3)_{13}$ *is binding.*

Proof: Suppose not. Then $\mu_{13} = 0$, and, by Lemma 4, $\mu_{12} > 0$. Therefore, we can assume that $(3)_{12}$ is binding. Lemma 5 then implies that $\mu_{23} = 0$. But this contradicts Lemma 4. Q.E.D.

Using Lemmas 5 and 6, we can assume without loss of generality that $(3)_{13}$ is binding and (by Lemma 5) $\mu_{31} = \mu_{32} = \mu_{21} = 0$.

LEMMA 7: $\alpha_1 = 0$, $\lambda_3 = 0$, *and, if* $\alpha_3 > 0$, $k_{13} = k_3^*$.

Proof: $\alpha_1 = 0$ follows from $\mu_{21} = \mu_{31} = 0$ using Lemma 1. If $\alpha_3 > 0$, then, by (A6), $\pi_3 V_3'(k_{13}) + \lambda_3 b_3 = 0$. Thus $k_{13} = k_3^*$ follows if $\lambda_3 = 0$. The fact that $(3)_{13}$ is binding and $\alpha_1 = 0$, however, implies that $k_{01} = (1 - \alpha_3)k_{03}$. Using $\alpha_3 < 1$, $k_{03} > 0$, and $b_3 > b_1$, we have

$$0 \leq S + b_1 k_{01} < S + b_3(1 - \alpha_3)k_{03} \leq S + b_3[(1 - \alpha_3)k_{03} + \alpha_3 k_{13}].$$

Therefore, $(4)_3$ is slack, so $\lambda_3 = 0$. Q.E.D.

LEMMA 8: *If* $\alpha_2 > 0$, *then* $\mu_{12} > 0$, $(3)_{12}$ *is binding,* $(3)_{23}$ *and* $(4)_2$ *are slack (so* $\lambda_2 = 0$*),* $k_{01} = (1 - \alpha_2)k_{02} = (1 - \alpha_3)k_{03}$, *and* $k_{02} < k_2^* = k_{12}$.

Proof: First suppose $\mu_{12} = 0$. Then, from (A3) and (A4), $k_{02} = k_{12}$. Lemma 1 then implies that $\alpha_2 = 0$. Therefore, $\mu_{12} > 0$, and $(3)_{12}$ is binding. That $k_{01} = (1 - \alpha_2)k_{02} = (1 - \alpha_3)k_{03}$ follows from the fact that $(3)_{12}$ and $(3)_{13}$ are binding and $\alpha_1 = 0$ (see Lemma 7). Since $\alpha_2 k_{12} > 0$, this implies that $(3)_{23}$ is slack. Therefore, $\mu_{23} = 0$, so from (A3), $\pi_2 V_2'(k_{02}) = \mu_{12} - \lambda_2 b_2$, and from (A4), $\pi_2 V_2'(k_{12}) = -\lambda_2 b_2$. Since $\mu_{12} > 0$, the remaining conclusions follow if $\lambda_2 = 0$. But, using $k_{02} > 0$, and $b_2 > b_1$, we have

$$0 \leq S + b_1 k_{01} < S + b_2(1 - \alpha_2)k_{02} \leq S + b_2[(1 - \alpha_2)k_{02} + \alpha_2 k_{12}].$$

Therefore, $(4)_2$ is slack, so $\lambda_2 = 0$. Q.E.D.

LEMMA 9: *If* $\alpha_2 > 0$, *then* $\alpha_2 < \alpha_3$ *and* $k_{13} = k_3^* > k_{03} > k_{02} > k_{01} > k_1^*$.

Proof: Since $\mu_{12} > 0$ (Lemma 8) and $\mu_{21} = \mu_{31} = 0$, (A1) implies that $k_{01} > k_1^*$. Also, $\alpha_2 > 0$ implies that $k_{02} > k_{01}$ (using $k_{01} = (1 - \alpha_2)k_{02}$ from Lemma 8). Next, we show that $k_{03} > k_{02}$.

Suppose that $k_{02} \geq k_{03}$. First, note that, since $V_2' < V_3'$, $V_2(k') - V_2(k) \leq V_3(k') - V_3(k)$ whenever $k' \geq k$. Also, from Lemma 8 and equations (A3) and (A5), $\pi_t V_t'(k_{0t}) = \mu_{1t}$ for $t = 2,3$. Finally, due to concavity of V_t, the function $V_t(k) - V_t'(k)k$ is increasing. Now, since $\alpha_2 \in (0,1)$, $C_2 = 0$. Therefore, from equation (A9),

$$
\begin{aligned}
0 = \frac{C_2}{\pi_2} &= V_2(k_2^*) - V_2(k_{02}) + V_2'(k_{02})k_{02} - q \\
&< V_3(k_2^*) - V_3(k_{02}) + V_3'(k_{02})k_{02} - q \\
&< V_3(k_3^*) - V_3(k_{03}) + V_3'(k_{03})k_{03} - q \\
&= \frac{C_3}{\pi_3}.
\end{aligned}
$$

This implies $\alpha_3 = 1$ contradicting Lemma 0. This proves that $k_{03} > k_{02}$. It follows from $(1 - \alpha_2)k_{02} = (1 - \alpha_3)k_{03}$ that $\alpha_3 > \alpha_2 > 0$. Moreover, $\alpha_3 \in (0, 1)$ implies that $k_{03} \neq k_{13}$ from Lemma 1. Therefore, from Lemmas 2 and 7, $k_{03} < k_{13} = k_3^*$. Q.E.D.

LEMMA 10: *Suppose $\alpha_3 = 0$. Then $\alpha_2 = 0$ and $k_{01} = k_{02} = k_{03} \equiv k^{***}$, where k^{***} solves*

$$\max_{k\geq 0,\, S\geq 0} -S + F^{***}(k) \tag{A11}$$

$$\textit{subject to } S + b_1 k \geq \bar{u},$$

where

$$F^{***}(k) = \sum_{t=1}^{3} \pi_t V_t(k). \tag{A12}$$

Also, $k^{***} < k_3^*$, $S = \max\{0, \bar{u} - b_1 k^{***}\}$. Finally, if $\bar{u} - b_1 k^{***} > 0$, the Lagrange multiplier associated with the constraint, $\lambda = 1$, and if $\bar{u} - b_1 k^{***} < 0$, $\lambda = 0$.

Proof: First, $\alpha_2 = 0$ since otherwise, by Lemma 9, $\alpha_3 > \alpha_2 > 0$ which contradicts $\alpha_3 = 0$. Now, $\alpha_3 = \alpha_2 = \alpha_1 = 0$ and (3) imply $k_{01} = k_{02} = k_{03}$. This and $b_1 < b_2 < b_3$ then imply that $(4)_2$ is slack. The rest, except for $k^{***} < k_3^*$, follows from using these results to simplify headquarters' problem. Finally, suppose $k^{***} \geq k_3^*$. Then (1) and $\lambda \leq 1$ imply that

$$\lambda b_1 + \sum_{t \in T} \pi_t V_t'(k^*) < 0,$$

which contradicts the first order condition for equation (A11). Q.E.D.

Define the functions $g_t(k) = V_t(k) - V_t'(k)k$. Note that, since V_t is concave, $g_t \geq 0$ and increasing in k. Using equations (A1)–(A6),

$$C_t = \pi_t[g_t(k_{1t}) - g_t(k_{0t}) - q] \qquad (\text{if } \alpha_t \in (0,1)). \tag{A13}$$

Also, for $t = 2,3$, if $\alpha_t > 0$, then $C_t = 0$ [by (A8)] and $k_{1t} = k_t^*$ [by Lemmas 7 and 8], so k_{0t} must satisfy

$$g_t(k_{0t}) = g_t(k_t^*) - q = V_t(k_t^*) - q, \tag{A14}$$

where the second equality follows from the fact that $V_t'(k_t^*) = 0$.

LEMMA 11: *Suppose* $\alpha_3 > 0$ *and* $\alpha_2 = 0$. *Then* $k_{01} = k_{02} = (1 - \alpha_3)k_{03} = k^{**}$, k_{03} *satisfies* (A14)$_3$, *and* $k_3^* > k_{03} > k_{02} = k_{01} > k_1^*$, *where* k^{**} *solves*

$$\max_{k \geq 0,\, S \geq 0} -S + F^{**}(k, k_{03}) \tag{A15}$$

$$\textit{subject to } S + b_1 k \geq \bar{u},$$

where

$$F^{**}(k, k_{03}) = \sum_{t=1}^{2} \pi_t V_t(k) + p_3 V_3'(k_{03})k. \tag{A16}$$

Proof: That k_{03} satisfies (A14)$_3$ follows from $\alpha_3 \in (0, 1)$. That $k_{01} = k_{02} = (1 - \alpha_3)k_{03}$ follows from (3) and $\alpha_2 = \alpha_1 = 0$. Since $\alpha_3 > 0$, this implies that $k_{03} > k_{02}$. That $k_{03} < k_3^*$ follows from (A14)$_3$ and the fact that $q > 0$ and g_3 is increasing. Using (A10) and $k_{03} < k_3^*$, at least one of $k_{01} > k_1^*$ and $k_{02} > k_2^*$ must hold. But, $k_{02} = k_{01}$, so we must have $k_{01} > k_1^*$. Finally, k^{**} must solve headquarters' problem after fixing k_{03}, k_{13}, and α_t at their optimal values and imposing the facts that $k_{01} = k_{02} = (1 - \alpha_3)k_{03}$ and (4)$_2$ and (4)$_3$ are slack. Letting $k_{01} = k_{02} = k$, $1 - \alpha_3 = k/k_{03}$ and using equation (A14) results in equation (A15). Q.E.D.

LEMMA 12: *Suppose* $\alpha_3 > \alpha_2 > 0$. *Then* k_{02} *and* k_{03} *satisfy* (A14)$_2$ *and* (A14)$_3$, *respectively, and* $k_{01} = k^*$, *where* k^* *solves*

$$\max_{k \geq 0,\, S \geq 0} -S + F^*(k, k_{02}, k_{03}) \tag{A17}$$

$$\textit{subject to } S + b_1 k \geq \bar{u},$$

where

$$F^*(k,\ k_{02},\ k_{03}) = \pi_1 V_1(k) + \sum_{t=2}^{3} \pi_t V_t'(k_{0t})k. \tag{A18}$$

Proof: That k_{02} and k_{03} satisfy equations $(A14)_2$ and $(A14)_3$, respectively, follows from α_2 and $\alpha_3 \in (0,1)$. k^* must solve headquarters' problem after fixing k_{02}, k_{03}, k_{12}, k_{13}, and α_t at their optimal values and imposing the facts that $k_{01} = (1 - \alpha_2)k_{02} = (1 - \alpha_3)k_{03}$ and $(4)_2$ and $(4)_3$ are slack. Letting $k_{01} = k$, $1 - \alpha_2 = k/k_{02}$, $1 - \alpha_3 = k/k_{03}$ and using (A14) results in (A17). Q.E.D.

We combine our results thus far in

THEOREM 1:

a) $0 = \alpha_1 \leq \alpha_2 \leq \alpha_3 < 1$, *constraints* $(4)_2$ *and* $(4)_3$ *are slack* (*so* $\lambda_2 = \lambda_3 = 0$), *and* $S = max\{0,\ \bar{u} - b_1 k_{01}\}$.
b) *If* $\alpha_3 > 0$, *then* $k_{13} = k_3^*$, k_{03} *satisfies* $(A14)_3$, $\alpha_2 < \alpha_3$, $k_{01} = (1 - \alpha_2)k_{02} = (1 - \alpha_3)k_{03}$, $k_3^* > k_{03} > k_{02} \geq k_{01} > k_1^*$. *If* $\alpha_2 > 0$, k_{02} *satisfies* $(A14)_2$ *and* $k^* = k_{01} < k_{02} < k_2^* = k_{12}$ (*where* k^* *solves* (A17)). *If* $\alpha_2 = 0$, $k_{02} = k_{01} = k^{**}$ (*where* k^{**} *solves (A15)).*
c) *If* $\alpha_3 = 0$, *then* $\alpha_1 = \alpha_2 = 0$, $k_{01} = k_{02} = k_{03} = k^{***}$ (*where* k^{***} *solves* (A11)), $k^{***} < k_3^*$.

Proof: Everything has been proven in the previous lemmas and discussion except for the formula for S and that $(4)_2$ is slack when $\alpha_2 = 0$. When $\alpha_2 = 0$, however, $k_{01} = k_{02}$. This and $b_1 < b_2$ imply that $(4)_2$ is slack. Since both $(4)_2$ and $(4)_3$ are slack and $\alpha_1 = 0$, $(4)_1$ implies that $S = \max\{0, \bar{u} - b_1 k_{01}\}$. Q.E.D.

LEMMA 13: *For* $n = 1,\ 2$, *define the problem* P_n *by*

$$\max_{k \geq 0,\ S \geq 0} \ -S + F_n(k)$$

$$\textit{subject to } S + b_1 k \geq \bar{u},$$

where F_n *is strictly increasing, strictly concave, and* $F_n'(0) = \infty$. *Suppose* $(\bar{k}_n, \bar{S}_n, \bar{\gamma}_n)$ *solves* P_n, *where* $\bar{\gamma}_n$ *is the optimal Lagrange multiplier for the constraint. If* $F_1'(\bar{k}_1) > F_2'(\bar{k}_1)$, *then* $\bar{k}_1 > \bar{k}_2$.[24]

Proof: First note that, from the first-order condition for S, $\bar{\gamma}_n \leq 1$ with equality if $\bar{S}_n > 0$. Also $\bar{S}_n = \max\{0, \bar{u} - b_1\bar{k}_n\}$ and $\bar{\gamma}_n = 0$ if $\bar{u} - b_1\bar{k}_n < 0$. Finally, note that, since $F_n'(0) = \infty$, $\bar{k}_n > 0$, so the first-order condition for k holds with equality. Now suppose $\bar{k}_1 \leq \bar{k}_2$. Then $\bar{u} - b_1\bar{k}_1 \geq \bar{u} - b_1\bar{k}_2$. There-

[24] Strictly speaking, the inequality in this conclusion may be weak if $\bar{k}_1 = \bar{u}/b_1$. Therefore, in some of the results that follow, certain inequalities stated as strict inequalities may actually be weak in this special case. For ease of exposition, we ignore this subtlety.

fore if $\bar{\gamma}_2 > 0$, $\bar{u} - b_1\bar{k}_2 > 0$, so $\bar{\gamma}_1 = 1$. Thus $\bar{\gamma}_1 \geq \bar{\gamma}_2$. Now, using the first-order condition for k from P_1 and concavity of F_2, we have

$$0 = F'_1(\bar{k}_1) + \bar{\gamma}_1 b_1 > F'_2(\bar{k}_1) + \bar{\gamma}_2 b_1 \geq F'_2(\bar{k}_2) + \bar{\gamma}_2 b_1,$$

which contradicts the fact that $\bar{k}_2$ must satisfy the first-order condition for k in P_2. Q.E.D.

LEMMA 14: $\alpha_3 = 0$ *if and only if* $q \geq q_3$, *where*

$$q_3 = g_3(k_3^*) - g_3(k^{***}). \tag{A19}$$

Proof: First suppose $q \geq q_3$ and $\alpha_3 > 0$. Then k_{03} satisfies $(\text{A14})_3$, so, using (A19),

$$g_3(k_{03}) = g_3(k_3^*) - q \leq g_3(k_3^*) - q_3 = g_3(k^{***}).$$

Therefore, $k_{03} \leq k^{***}$. Now by Theorem 1(b), $k_{01} < k_{03}$ and $k_{02} < k_{03}$. It follows that $V'_t(k_{0t}) \geq V'_t(k^{***})$ with strict inequality for $t = 1,2$. Let λ^{***} be the optimal Lagrange multiplier for the constraint in equation (A11). Then, if $\lambda^{***} > 0$, $\bar{u} - b_1 k^{***} \geq 0$. This implies that $\bar{u} - b_1 k_{01} > 0$, so $S > 0$ and $\lambda_1 = 1$. Therefore $\lambda_1 \geq \lambda^{***}$. This and $V'_t(k_{0t}) \geq V'_t(k^{***})$ with strict inequality for $t = 1,2$ implies that

$$\lambda_1 b_1 + \sum_{t \in T} \pi_t V'_t(k_{0t}) > \lambda^{***} b_1 + \sum_{t \in T} \pi_t V'_t(k^{***}) = 0,$$

where the equality follows from the definitions of k^{***} and λ^{***}. This, however, contradicts equation (A10), since $\lambda_2 = \lambda_3 = 0$.

Now suppose that $q < q_3$ and $\alpha_3 = 0$. Let $\delta = q_3 - q > 0$. Then, by Theorem 1, $\alpha_t = 0$ and $k_{0t} = k^{***} < k_3^*$ for all t. Consider the alternative solution $(\tilde{S}, \tilde{\alpha}, \tilde{k}_0, \tilde{k}_1)$ in which $\tilde{S} = S$; $\tilde{\alpha}_t = \alpha_t = 0$, $\tilde{k}_{0t} = k_{0t}$ for $t = 1, 2$, $\alpha_3 = \varepsilon$, $k_{03} = k^{***}/(1 - \varepsilon)$, and $\tilde{k}_{13} = k_3^*$, where $\varepsilon > 0$ will be determined below. Both sides of all constraints except the left sides of $(3)_{3t}$ and $(4)_3$ are the same as in the initial solution. The left sides of $(3)_{3t}$ and $(4)_3$ are larger in the alternative solution since they are a weighted average of $k^{***}/(1 - \varepsilon)$ and k_3^*, both of which exceed k^{***}, the left sides of $(3)_{3t}$ and $(4)_3$ in the initial solution. Therefore the alternative solution satisfies all constraints. It remains to show that the objective function, equation (2), evaluated at the alternative solution is larger than at the initial solution. Using equation (A19), the change in equation (2) is proportional to

$$f(\varepsilon) \equiv (1 - \varepsilon) V_3\left(\frac{k^{***}}{1 - \varepsilon}\right) + \varepsilon V_3(k_3^*) - V_3(k^{***}) - \varepsilon q$$

$$= \delta\varepsilon + (1 - \varepsilon)\left[V_3\left(\frac{k^{***}}{1 - \varepsilon}\right) - V_3(k^{***}) \right] - \varepsilon V'_3(k^{***}) k^{***}.$$

It is easy to check that $f(0) = 0$ and $f'(0) = \delta > 0$. Therefore, for ε sufficiently small, (2), evaluated at the alternative solution is larger than at the initial solution. Therefore, $\alpha_3 = 0$ cannot be optimal when $q < q_3$. Q.E.D.

LEMMA 15: *Let $k_{03}(q)$ be given by* $(A14)_3$ *for* $0 \le q < q_3$ *and by* k^{***} *for* $q \ge q_3$*; let* $k^{**}(q)$ *solve (A15) for* $k_{03} = k_{03}(q)$. *Then* $k_{03}(q)$ *is continuous on* $q \ge 0$ *and strictly decreasing for* $q < q_3$. $k^{**}(q)$ *is continuous and increasing on* $q \ge 0$. *Moreover,* $k^{**}(0) < k_2^*$ *and* $k^{**}(q) = k^{***}$ *for* $q \ge q_3$.

Proof: The first part is obvious from $(A14)_3$ and (A19), since g_3 is strictly increasing and continuous. Continuity of $k^{**}(q)$ follows from continuity of $k_{03}(q)$ and the V_t and strict concavity of the V_t, using, e.g., Theorem 1.1, p. 11 of Harris (1987). Suppose $q^1 > q^2$ and let $F_n(k) = F^{**}(k, k_{03}(q^n))$, for $n = 1, 2$. Since $k_{03}(q)$ is decreasing in q and V_3 is concave, $F_1' > F_2'$. Therefore, by Lemma 13, $k^{**}(q^1) > k^{**}(q^2)$.

Now $F^{**}(k, k_{03}(0)) = \pi_1 V_1(k) + \pi_2 V_2(k)$, since $k_{03}(0) = k_3^*$. Suppose $k_2^* \le k^{**}(0)$. Then $F_1^{**}(k^{**}(0), k_{03}(0)) + \lambda^{**}(0)b_1 \le \pi_1 V_1'(k_2^*) + b_1 < 0$ by (1), where the subscript on F^{**} refers to the partial derivative with respect to the first argument. This contradicts the optimality of k^{**} for equation (A15).

Finally, the fact that $k_{03}(q) = k^{***}$ for $q \ge q_3$ implies that k^{***} and λ^{***} satisfy the first-order condition for equation (A15) when $q \ge q_3$, using the definition of k^{***}. Since this problem is concave, the first-order condition is sufficient, so k^{***} solves equation (A15) and $k^{**}(q) = k^{***}$ for $q \ge q_3$. Q.E.D.

LEMMA 16: *Define the function* $f(q) = g_2(k_2^*) - g_2(k^{**}(q))$. *Then* f *has a unique fixed point,* $q_2 \in (0, q_3)$, *such that*

$$q_2 = g_2(k_2^*) - g_2(k^{**}(q_2)) = V_2(k_2^*) - g_2(k^{**}(q_2)). \qquad \text{(A20)}$$

Proof: By Lemma 15 and the fact that g_2 is increasing, $f(0) > 0$, and f is decreasing and continuous. The result follows if $f(q_3) < q_3$. From (A19), if $k^{***} \ge k_2^*$, then this is obvious since, in that case $f(q_3) \le 0$ while $q_3 > 0$. Suppose $k^{***} < k_2^*$. Using the definitions of g_t, k_t^*, and q_3, we must show that

$$V_2(k_2^*) - V_2(k^{***}) < V_3(k_3^*) - V_3(k^{***}) + [V_3'(k^{***}) - V_2'(k^{***})]k^{***}.$$

But, $V_3' > V_2'$, implies that the term in brackets above is positive and that $V_2(k_2^*) - V_2(k^{***}) < V_3(k_3^*) - V_3(k^{***})$ if $k^{***} < k_2^*$. Q.E.D.

LEMMA 17: $\alpha_2 = 0$ *if and only if* $q \ge q_2$.

Proof. First suppose $q \ge q_2$ and $\alpha_2 > 0$. Then k_{02} satisfies $(A14)_2$, so, using equation (A20),

$$g_2(k_{02}) = g_2(k_2^*) - q \le g_2(k_2^*) - q_2 = g_2(k^{**}(q_2)).$$

Therefore, $k_{02} \le k^{**}(q_2)$. Also, by Lemma 15, $k_{03}(q) \le k_{03}(q_2)$. Therefore, $k_{03} \le k_{03}(q_2)$ since $k_{03} = k_{03}(q)$ if $q < q_3$, and $k_{03} = k^{***} = k_{03}(q_3) \le k_{03}(q_2)$ (by Lemma 15) if $q \ge q_3$. Now by Theorem 1(b), $k_{01} < k_{02}$. It follows that $V_t'(k_{0t}) \geq V_t'(k^{**}(q_2))$ for $t = 1, 2$, with strict inequality for $t = 1$ and $V_3'(k_{03}) \geq V_3'(k_{03}(q_2))$. Let λ^{**} be the optimal Lagrange multiplier for the constraint in equation (A15). Then, if $\lambda^{**} > 0$, $\bar{u} - b_1 k^{**}(q_2) \geq 0$. This implies that $\bar{u} - b_1 k_{01} > 0$, so $S > 0$ and $\lambda_1 = 1$. Therefore $\lambda_1 \geq \lambda^{**}$. This, together with the above inequalities, implies that

$$\lambda_1 b_1 + \sum_{t \in T} \pi_t V_t'(k_{0t}) > \lambda^{**} b_1 + \sum_{t=1}^{2} \pi_t V_t'(k^{**}(q_2)) + \pi_3 V_3'(k_{03}(q_2)) = 0,$$

where the equality follows from the definitions of $k^{**}(q_2)$ and λ^{**}. This, however, contradicts equation (A10), since $\lambda_2 = \lambda_3 = 0$.

Now suppose $q < q_2$ and $\alpha_2 = 0$. Let $\delta = q_2 - q > 0$. Then, by Theorem 1, $k_{0t} = k^{**}(q) < k_3^*$ for $t = 1,2$. Consider the alternative solution ($\tilde{S}$, $\tilde{\alpha}$, $\tilde{k}_0$, $\tilde{k}_1$) in which $\tilde{S} = S$; $\tilde{\alpha}_t = \alpha_t = 0$, $\tilde{k}_{0t} = k_{0t}$ for $t = 1,3$, $\tilde{k}_{13} = k_{13} = k_3^*$, $\tilde{\alpha}_2 = \varepsilon$, $k_{02} = k^{**}(q)/(1 - \varepsilon)$, and $\tilde{k}_{12} = k_2^*$, where $\varepsilon > 0$ will be determined below. Both sides of all constraints except the left sides of $(3)_{2t}$ and $(4)_2$ are the same as in the initial solution. The left sides of $(3)_{2t}$ and $(4)_2$ are larger in the alternative solution since they are a weighted average of $k^{**}(q)/(1 - \varepsilon)$ and k_2^*, both of which exceed $k^{**}(q)$, the left sides of $(3)_{2t}$ and $(4)_2$ in the initial solution. Therefore the alternative solution satisfies all constraints. It remains to show that the objective function, equation (2), evaluated at the alternative solution is larger than at the initial solution. Using the definition of q_2, the change in equation (2) is proportional to

$$\begin{aligned} f(\varepsilon) &\equiv (1 - \varepsilon) V_2\left(\frac{k^{**}(q)}{1 - \varepsilon}\right) + \varepsilon V_2(k_2^*) - V_2(k^{**}(q)) - \varepsilon q \\ &= \delta\varepsilon + (1 - \varepsilon)\left[V_2\left(\frac{k^{**}(q)}{1 - \varepsilon}\right) - V_2(k^{**}(q))\right] \\ &\quad - \varepsilon V_2'(k^{**}(q)) k^{**}(q). \end{aligned}$$

It is easy to check that $f(0) = 0$ and $f'(0) = \delta > 0$. Therefore, for ε sufficiently small, (2), evaluated at the alternative solution is larger than at the initial solution. Therefore, $\alpha_2 = 0$ cannot be optimal when $q < q_2$. Q.E.D.

LEMMA 18: *Let $k_{02}(q)$ be given by $(A14)_2$ for $0 \leq q < q_2$ and by $k^{**}(q)$ for $q \geq q_2$; let $k^*(q)$ solve $(A17)$ for $k_{02} = k_{02}(q)$ and $k_{03} = k_{03}(q)$. Then $k_{02}(q)$ is continuous on $q \geq 0$ and strictly decreasing for $q < q_2$. $k^*(q)$ is continuous and increasing on $[0, q_2)$.*

Proof: The first part is obvious from $(A14)_2$ and $(A20)$, since g_2 is strictly increasing and continuous. Continuity of $k^*(q)$ follows from continuity of $k_{02}(q)$, $k_{03}(q)$ and the V_t and strict concavity of the V_t as in Lemma 15. Suppose $q^1 > q^2$ and let $F_n(k) = F^*(k, k_{02}(q^n), k_{03}(q^n))$, for $n = 1,2$. Since $k_{0t}(q)$, for $t = 2, 3$, is decreasing over $[0, q_2)$ and V_t, for $t = 2, 3$, is concave, $F_1' > F_2'$. Therefore, by Lemma 13, $k^*(q^1) > k^*(q^2)$. Q.E.D.

Using Lemmas 14 and 17, $k_{0t} = k_{0t}(q)$, where $k_{02}(q)$ and $k_{03}(q)$ are defined above, and

$$k_{01}(q) = \begin{cases} k^*(q), & \text{for } q \in [0, q_2); \\ k^{**}(q), & \text{for } q \in [q_2, q_3); \\ k^{***}, & \text{for } q \geq q_3. \end{cases} \tag{A21}$$

Also, from Theorem 1, $\alpha_t = \alpha_t(q)$, for $t = 2, 3$, where

$$\alpha_t(q) = \begin{cases} 1 - \dfrac{k_{01}(q)}{k_{0t}(q)}, & \text{for } q \in (0, q_t); \\ 0, & \text{for } q \geq q_t. \end{cases} \tag{A22}$$

We may now prove the following theorem.

THEOREM 2: *The nature of the solution to headquarters' problem as a function of q is as depicted in Figures 1 and 2, i.e., $\alpha_1 \equiv 0$, α_t is continuous for $q \geq 0$, strictly decreasing on $(0, q_t)$, and zero for $q \geq q_t$ ($t = 2, 3$); k_{03} is continuous for $q \geq 0$, strictly decreasing on $[0, q_3)$, and constant at k^{***} for $q \geq q_3$; k_{02} is continuous for $q \geq 0$, strictly decreasing on $[0, q_2)$, increasing on $[q_2, q_3)$, and constant at k^{***} for $q \geq q_3$; and k_{01} is continuous and increasing on $[0, q_3)$ and constant at k^{***} for $q \geq q_3$.*

Proof: Everything except the continuity and monotonicity of α_t for $t = 2,3$, has already been shown. But this is implied by the continuity and monotonicity of k_{0t} (over $[0, q_2)$ for $t = 2$). Q.E.D.

Next we consider comparative statics with respect to scale of the enterprise (firm or division). Note that $g_t(s, k) = sg_t(k)$. We can therefore rewrite equation (A14), dividing by s, to obtain

$$g_t(k_{0t}) = g_t(k_t^*) - \frac{q}{s} = V_t(k_t^*) - \frac{q}{s}. \tag{A23}$$

THEOREM 3: *The behavior of q_t, k_{0t}, and α_t with respect to s is as shown in Figures 3 and 4, and k_{1t} is independent of s. That is, suppose s increases from s_0 to s_1. This results in an increase in q_2, q_3, $\alpha_t(q)$ and a decrease in k_{01}. Also, there exists $q^t \in (q_t(s_0), q_t(s_1))$ such that k_{0t} increases for every $q \in (0, q^t)$ and decreases for every $q \geq q^t$, for $t = 2, 3$.*

Proof: First, it is clear that $k_{1t} = k_t^*$ is independent of s for all t. Consider k^{***}. An increase in s increases the absolute value of the slope of F^{***}. Therefore, since this slope is nonpositive at k^{***}, by Lemma 13, k^{***} is decreasing in s. Now, it is easy to check from (A19) that $q_3(s_1) > q_3(s_0)$. For $q < q_3(s_1)$ the function $k_{03}(q, s)$ defined by (A23) is increasing in s. Therefore, k_{03} behaves as shown in Figure 4. It is also easy to check that an increase in s decreases the slope of F^{**} at k^{**} (taking into account the effect of s on k_{03}), so by Lemma 13, k^{**} is decreasing in s. It follows that the function $g_2(k_2^*(s)) - g_2(k^{**}(q, s))$ increases with s. Consequently, q_2 increases with s. For $q < q_2(s_1)$ the function $k_{02}(q, s)$ defined by equation (A23) is increasing in s. Therefore, k_{02} behaves as shown in Figure 4. Finally, it is easily shown that an increase in s decreases the slope of F^* at k^* (taking into account the effect of s on k_{02} and k_{03}), so by Lemma 13, k^* is decreasing in s. Therefore, k_{01} behaves as shown in Figure 4. Behavior of the α_t for $t =$ 2,3, follows immediately from the behavior of k_{0t}, using (A22). Q.E.D.

THEOREM 4: *Consider changes in π_1 holding π_2 constant (and $\pi_3 = 1 - \pi_1 - \pi_2$). The behavior of α_t is the same as in Figure 3 with π_1 substituted for s. The behavior of q_t and k_{0t} with respect to π_1 is as shown in Figure 5, and k_{1t} is independent of π_1. That is, suppose π_1 increases from π_{10} to π_{11}. This results in an increase in q_2, q_3, $\alpha_t(q)$ and a decrease in k_{01}. Also, k_{0t} does not change for every $q \in (0, q_t(\pi_{10})]$ and decreases for every $q > q_t(\pi_{10})$, for $t = 2,3$.*

Proof: First, it is clear that $k_{1t} = k_t^*$ is independent of π_1 for all t. Consider k^{***}. An increase in π_1 (holding π_2 constant) shifts weight from V_3 to V_1. Since $k^{***} \in (k_1^*, k_3^*)$, this decreases the slope of F^{***}. Therefore, since this slope is nonpositive at k^{***}, by Lemma 13, k^{***} is decreasing in π_1. Now, it is easy to check from (A19) that $q_3(\pi_{11}) > q_3(\pi_{10})$. For $q < q_3(\pi_{11})$ the function $k_{03}(q)$ defined by (A14) is independent of π_t. Therefore, k_{03} behaves as shown in Figure 5. It is also easy to check, using the same argument as for F^{***}, that an increase in π_1 (and offsetting decrease in π_3) decreases the slope of F^{**} at k^{**} (since k_{03} is independent of π_t), so by Lemma 13, k^{**} is decreasing in π_1. It follows that the function $g_2(k_2^*) - g_2(k^{**}(q, \pi_1))$ increases with π_1. Consequently, q_2 increases with π_1. For $q < q_2(\pi_{11})$ the function $k_{02}(q)$ defined by (A14) is independent of π_t. Therefore, k_{02} behaves as shown in Figure 5. Finally, it is easily shown as before that an increase in π_1 decreases the slope of F^* at k^*, so by Lemma 13, k^* is decreasing in π_1. Therefore, k_{01} behaves as shown in Figure 5. Behavior of the α_t for $t = 2,3$, follows immediately from the behavior of k_{0t}, using equation (A22). Q.E.D.

LEMMA 19: *Suppose $\pi = (\pi_1, \pi_2, \pi_3)$ is a vector of independent random variables. Let $w(\pi)$ and $y(\pi)$ be two real-valued functions such that, for each $t \in \{1, 2, 3\}$, either w and y are monotone increasing in π_t or w and y are monotone decreasing in π_t. Then,* $\mathrm{Cov}(w, y) \geq 0$. *Also, if, for each t, w and y are monotone in the opposite direction, then* $\mathrm{Cov}(w, y) \leq 0$.

Proof: Define γ_t to be π_t if w and y are increasing in π_t and to be $-\pi_t$ if w and y are decreasing in π_t. Then $\gamma = (\gamma_1, \gamma_2, \gamma_3)$ is a vector of independent, hence associated (see Holmstrom and Milgrom (1994), Theorem 2,iii), random variables. Moreover $w(\gamma)$ and $y(\gamma)$ are monotone increasing functions. Therefore, by Holmstrom and Milgrom (1994), Theorem 2, $\mathrm{Cov}(w, y) \geq 0$. The last part follows by replacing w with $-w$ (or y with $-y$) and using the first part to conclude that $\mathrm{Cov}(-w, y) = -\mathrm{Cov}(w, y) \geq 0$. Q.E.D.

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