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Does Money Explain Asset Returns? Theory and Empirical Analysis

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ABSTRACT

A cash-in-advance model of a monetary economy is used to derive a *money-based* CAPM (M-CAPM), which allows us to implement tests of asset pricing restrictions without consumption data. A test as in Fama and MacBeth of the model suggests that the money betas have some explanatory power for the cross-sectional variation of expected returns; however, the model is rejected using conditional information. Consistent with our predictions, estimates of the curvature parameter are lower than those of the consumption CAPM (C-CAPM) and pricing errors of the M-CAPM tend to be smaller than those of the C-CAPM.

WHILE THE RELATION BETWEEN asset prices and real macroeconomic variables such as aggregate consumption has been extensively investigated, less attention has been devoted to the relation between monetary aggregates and asset prices that arises from models which incorporate a monetary sector.[1] It is often argued that one possible reason for the rejection of the consumption-oriented CAPM (C-CAPM) is the absence of monetary considerations. This paper takes a step towards encompassing money into the C-CAPM and proposes a money-based CAPM (M-CAPM) that can be tested without using consumption data. This M-CAPM is interesting in the way it captures the time-series and cross-sectional variation in stock returns and it provides insights in the possible misspecification of existing C-CAPM models.

The setup is based on the cash-in-advance model of Lucas and Stokey (1987) in which some consumption goods are financed with outside money (i.e., cash) and some are financed with inside money (e.g., bank deposits). The pricing results of this paper differ from those of other monetary models such as those considered by Finn, Hoffman, and Schlagenauf (1990) or Singleton (1985), which focus on the role of outside money in affecting the marginal utility of consumption. Instead, we concentrate on inside money. In asset pricing, the focus on inside money is appealing because inside money reflects conditions in

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[1] Notable exceptions are the works of Singleton (1985) on the returns on three-month Treasury bills, Stulz (1986), Boyle and Young (1988), and Marshall (1991) on inflation and stock returns.

both the monetary and the real sectors of the economy. Thus, while outside money is controlled by the Fed, inside money is to a large extent endogenous and varies with the transactions requirements in the economy.

Intuitively, since it is used to finance consumption, inside money can be used to define a stochastic discount factor and it is this observation that allows us to implement tests of asset pricing that circumvent the use of consumption data altogether. Thus our model justifies the use of inside money as a proxy for consumption, which is appealing because of the known limitations of the consumption series due to possible errors in measurement (see, e.g., Wheatley (1988)) and time aggregation bias (Breeden, Gibbons, and Litzenberger (1989) and Grossman, Melino, and Shiller (1987)).

Besides having problems associated with consumption measurement, if the economy conforms to the M-CAPM model, the aggregate-consumption CAPM is misspecified. Use of aggregate consumption is likely to result in a stochastic discount factor which is less volatile and which cannot capture risk associated with monetary uncertainty. Thus we predict that the C-CAPM 1) requires a higher curvature parameter and 2) yields a more pronounced mispricing than the M-CAPM.

On a sample of twenty portfolios of stocks sorted by their capitalization value, the unconditional version of the model is not rejected. We test both a restricted and an unrestricted version of the model. The restricted version, based on the assumption of logarithmic utility, is subject to a test as in Fama and MacBeth (1973) that fails to reject it, suggesting that the money "betas" have some explanatory power for the cross-sectional variation of expected stock returns. The model is rejected when we use conditioning information in the form of "managed" portfolios (Cochrane (1994)). A direct comparison with the C-CAPM is also conducted. The misspecification test proposed by Hansen and Jagannathan (1994) suggests that the specification error of the stochastic discount factor implied by the M-CAPM is somewhat lower than that of the C-CAPM, but both are outperformed by the stochastic discount factor of the Rubinstein (1976) CAPM, based on the returns on a value weighted market portfolio. While the M-CAPM does not provide a solution to the "equity premium puzzle" of Mehra and Prescott (1985), we estimate a curvature parameter that is lower than that of the C-CAPM. Also, the pricing errors of the M-CAPM tend to be smaller than those of the C-CAPM. These findings on the curvature parameter and the pricing errors are consistent with the predictions of the model.

Section I develops a simple asset pricing model with money. Section II presents the empirical analysis. Section III concludes the paper.

I. A Money-Based CAPM

A. Inside and Outside Money

We consider a stylized monetary economy based on the cash-in-advance model of Lucas and Stokey (1987). Investors enjoy two different consumption goods, c_1 and c_2. These goods are different in the way they can be paid for: c_1 can be paid for only with cash set aside in advance, while c_2 can be bought on credit. Preferences are given by the lifetime utility function $\sum_{t=0}^{\infty} \delta^t U(c_{1t}, c_{2t})$, with $\delta < 1$, where

$$U(c_{1t}, c_{2t}) = U_1(c_{1t}) + c_{2t}^{1-\gamma}(1 - \gamma)^{-1}, \tag{1}$$

with the curvature parameter $\gamma > 0$. An "investor" can be thought of as a household comprised of four individuals: a shopper for c_1 goods ("shopper 1"), a shopper for c_2 goods ("shopper 2"), a financial intermediary (the "bank"), and a productive unit (the "firm"). This captures the idea that there exist different sectors in the economy, but retains the tractability of a representative individual setup. The firm's output cannot be stored and is sold in market 1 and market 2 for the same price, P. At the beginning of the period the household gives cash (*outside money*), C, to shopper 1, since c_1 goods can be purchased only with cash acquired in advance, i.e.,

$$c_{1t}P_t \leq C_{t-1}, \qquad \text{for all } t, \tag{2}$$

where C_{t-1} is the quantity of cash accumulated in the previous period and P_t is the price level at time t. On the other hand, shopper 2 is known to the financial intermediary, the bank, who is willing to finance shopper 2's purchases by extending him credit (*inside money*) M. While outside money represents non-interest bearing government debt and is part of the net wealth of the private sector, inside money (bank deposits) represents claims to the private sector and is not part of net wealth. At the end of the period the members of the household reconvene and pool cash, goods, and satisfaction.

The empirical study by Hodrick, Kocherlakota, and Lucas (1991) suggests that the constraint (2) is almost always binding. This is consistent with the intuition that, as it is easy to show with logarithmic utility for example, the constraint should be binding when cash grows over time (see Svensson (1985), page 929), which is the case for most of our sample. To simplify the analysis, we assume that the cash-in-advance constraint is always binding.

B. Asset Prices

We denote by R_i nominal returns and by r_i real returns on security i. Define $r_{i,t+1}$ as the real rate of return on asset i from time t to $t + 1$, and $r_{M,t+1} = (M_{t+1}/P_{t+1})(M_t/P_t)$ as the real rate of growth of inside money from time t to time $t + 1$. Let Ω_t represent the information at time t that the investor uses to set prices.

As in moneyless models of asset pricing, there exists a stochastic discount factor m_{t+1} such that $E(m_{t+1}\, r_{i,t+1} | \Omega_t) = 1$. The novelty of this paper for asset pricing is that it associates the stochastic discount factor, m_t, with the rate of growth of real inside money, r_{Mt}.

RESULT 1—MONEY-BASED CAPM: *Asset returns satisfy the stochastic pricing condition*

$$\mathrm{E}[\delta r_{M,t+1}^{-\gamma}\ r_{i,t+1}|\Omega_t] = 1, \qquad i = 1, 2, \ldots, n. \tag{3}$$

By equating the marginal utility of one dollar spent on asset i today to the marginal utility of its real return next period, $r_{i,t+1}$, we have the familiar stochastic pricing equation $\mathrm{E}[\delta(c_{2t}/c_{2,t+1})^{\gamma}\ r_{i,t+1}|\Omega_t] = 1$; since the cash-in-advance constraint is binding, $c_{2t} = M_t/P_t$, and $c_{2,t+1}/c_{2t} = r_{M,t+1}$.

Since credit goods consumption is financed with inside money and utility is separable, *inside* money can be used to proxy for the consumption aggregate appropriate for pricing. In this respect Result 1 is quite different from the pricing result of other monetary models such as those considered by Finn, Hoffman, and Schlagenauf (1990), which emphasize the role of *outside* money in affecting the marginal utility of consumption.

To develop a feel for how well a money-based ranking measure may explain the cross-sectional variation in expected returns, we derive here and test in Section II a simple "beta" pricing model directly comparable to the Rubinstein (1976) version of a market-based CAPM. Define $R_{i,t+1}$ as the nominal rate of return on asset i and $R_{M,t+1}$ as the nominal rate of growth of inside money: $R_{M,t+1} = M_{t+1}/M_t$. Define the money beta on security i, β_i, and the premiums, λ_0 and λ_1, as

$$\beta_i = \mathrm{cov}(R_{M,t+1}^{-1}, R_{i,t+1}|\Omega_0)/\mathrm{var}(R_{M,t+1}^{-1}|\Omega_0), \qquad i = 1, 2, \ldots, n, \tag{4}$$

$$\lambda_0 = [\delta\mathrm{E}(R_{M,t+1}^{-1}|\Omega_0)]^{-1},$$

$$\lambda_1 = -\mathrm{var}(R_{M,t+1}^{-1}|\Omega_0)/\mathrm{E}(R_{M,t+1}^{-1}|\Omega_0).$$

RESULT 2—NOMINAL MONEY-BASED CAPM: *Assume the investor has a logarithmic utility function* ($\gamma = 1$) *and that the joint distribution of* R_i, $i = 1, 2, \ldots, n$, *and* R_M *is stationary, with* $\mathrm{E}(R_M^{-1}|\Omega_0)$ *nonzero. Then the expected nominal return on asset i is linear in* β_i

$$\mathrm{E}(R_i|\Omega_0) = \lambda_0 + \lambda_1\beta_i, \qquad i = 1, 2, \ldots, n. \tag{5}$$

By stationarity $\mathrm{E}[\delta\ r_{M,t+1}^{-1}\ r_{i,t+1}|\Omega_t] = 1$ implies $\mathrm{E}[\delta\ r_{M,t+1}^{-1}\ r_{i,t+1}|\Omega_0] = 1$, and because of logarithmic utility $\mathrm{E}[\delta\ r_{M,t+1}^{-1}\ r_{i,t+1}|\Omega_0] = \mathrm{E}[\delta\ R_{M,t+1}^{-1}\ R_{i,t+1}|\Omega_0] = 1$. By the definition of covariance, $\mathrm{E}(R_{M,t+1}^{-1}\ R_{i,t+1}|\Omega_0) = \mathrm{cov}(R_{M,t+1}^{-1}, R_i|\Omega_0) + \mathrm{E}(R_{M,t+1}^{-1}|\Omega_0)E(R_{i,t+1}|\Omega_0)$: solving for $\mathrm{E}(R_{i,t+1}|\Omega_0)$ and using the definition of beta and lambda in (4) we obtain (5).

C. Implications of the M-CAPM

At a very basic level, this paper may be interpreted as a theoretically motivated attempt to use a broad monetary aggregate as a state variable in a parsimonious representation of the stochastic discount factor. In this respect, the motivation for this paper is akin to that of the production-based CAPM of

Cochrane (1991), which includes investment in the stochastic discount factor. While some papers have examined the information contained in outside money (monetary base) for asset valuation,[2] few have associated broader measures of liquidity, such as M2 or M3, with the stochastic discount factor.[3] The inclusion of inside money is intuitively appealing because inside money reflects conditions in both the monetary and the real sectors of the economy. Empirical implementation of pricing restrictions encompassing some measure of liquidity could help identify whether shortcomings of existing models may be due to overlooking these monetary factors.

Contrast the M-CAPM with the consumption-oriented CAPM. If the expenditure shares $c_i/(c_1 + c_2)$, $i = 1, 2$ were constant, then one could use indifferently c_1, c_2, or aggregate consumption in the stochastic discount factor m, and the C-CAPM would still hold. To see why this is not the case in our framework, and as a useful benchmark, assume investors have logarithmic preferences, $U(c_{1t}, c_{2t}) = \theta \ln c_{1t} + (1 - \theta) \ln c_{2t}$. The stochastic discount factor is

$$m_{t+1} = \delta \frac{c_{2,t}}{c_{2,t+1}} = \delta\left(\frac{D_t}{D_{t+1}}\right)\left(\frac{1 + A(C_{t-1}/C_t)}{1 + A(C_t/C_{t+1})}\right), \tag{6}$$

with $A = \delta\theta/(1 - \theta)$ and dividends $D_t = c_{1t} + c_{2t}$.[4]

Unless the supply of outside money is constant, $C_t = C$, the aggregate C-CAPM does not hold.

We expect a more reasonable curvature parameter from a M-CAPM than from a C-CAPM. Basically, a high curvature parameter γ is required to reconcile the observed equity premium with the low volatility of dividends or consumption. There are, however, reasons to expect m_{t+1} to be more volatile than m^*_{t+1}. Increases in C_t precede increases in D_{t+1} (and smaller increases in C_{t+1}) over the business cycle. The positive covariance between D_t/D_{t+1} and the second term in parenthesis in (6) makes m_{t+1} more volatile than m^*_{t+1}. Secondly, Breeden, Gibbons, and Litzenberger (1989) and Grossman, Melino, and Shiller (1987) point out that reported consumption is the integral of the

[2] See for example Singleton (1985), Finn, Hoffman, and Schlagenauf (1990), and Marshall (1992).

[3] Finn, Hoffman, and Schlagenauf (1990) and Marshall (1992) use M1, even if the monetary aggregate suggested by their theoretical analysis is the monetary base. The use of broader monetary aggregates is sometimes justified in terms of the more desirable statistical properties of these aggregates; for example, in their study of the velocity of circulation of money Hodrick, Kocherlakota, and Lucas (1991) use M2 mainly to circumvent the well-known problem of nonstationarity of the velocity of M1.

[4] Investors should be indifferent between increasing current consumption of c_2 and setting aside cash for consumption of c_1 next period

$$\frac{1 - \theta}{c_{2t}} = \delta \mathrm{E}\left(\frac{\theta}{c_{1,t+1}} \frac{P_t}{P_{t+1}} \middle| \Omega_t\right), \tag{12}$$

but $c_{1t} = C_{t-1}/P_t$, since the cash-in-advance constraint is binding, and the inflation level is $P_{t+1}/P_t = (c_{1t}C_t)/(c_{1,t+1}C_{t-1})$. Thus the ratio of cash to credit goods is $(c_{1t}/c_{2t}) = AC_{t-1}/C_t$, with $A = \theta\delta/(1 - \theta)$ and substituting $D = c_1 + c_2$, we find $c_{2t} = D_t/(1 + A(C_{t-1}/C_t))$.

consumption flow over the reporting period and argue that this may result in the estimated consumption variance being biased downward. Since money is a stock, using money to proxy for the correct consumption aggregate can avoid this bias. Finally, while consumption aggregates used in C-CAPM tests usually include nondurables and services only, variations in monetary aggregates that are used to carry over a wide range of transactions may also reflect variations in durables consumption over the business cycle.

Furthermore, the C-CAPM explains the cross-sectional variation in returns using only one factor, the dividend/consumption growth. However, equation (6) suggests that two factors—dividend/consumption growth and outside money growth—may be more appropriate. The M-CAPM stochastic discount factor encompasses both in the correct proportions. While the direction of the mispricing in the C-CAPM is difficult to predict a priori even in the logarithmic utility case as it depends on the covariance structure of factors and returns, unless the monetary factor does not affect the stochastic discount factor, the pattern of cross-sectional mispricing in the C-CAPM should be more pronounced than that in the M-CAPM.

In summary, we have two predictions. First, we expect to estimate a lower curvature parameter γ using the M-CAPM than using the C-CAPM. Second, we expect more mispricing for the C-CAPM than for the M-CAPM.

II. Empirical Analysis

What is a sensible measure of inside money? A narrow monetary aggregate such as the monetary base is inconsistent with the intuition captured by the model of the previous section in which outside money (currency) is used to finance cash-goods transactions while inside money is provided by the financial system to finance all other spending. The choice of M1, M2, or M3 minus currency is more appropriate. While M2 velocity has been roughly stable since the beginning of the century, M1 velocity constantly increased between 1945 and 1981 when NOW accounts were introduced and the opportunity cost of checking deposits dropped sharply. As anticipated, in the empirical analysis, M1 in fact does not perform as well as M2 or M3. Hence, we do not report results for M1.

A. Data

Financial data include nominal stock returns, one-month Treasury bill yields, term spreads, measured by the difference between the yield on 10-year Treasury bonds and one-month Treasury bills, and default spreads, measured by the difference between the yields on a portfolio of Baa corporate bonds and the 10-year Treasury. The nominal returns on the Treasury bonds and bills are from *Stock, Bonds, Bills and Inflation—1991 Yearbook* of Ibbotson Associates; nominal returns on the other securities are from the Center for Research in Security Prices (CRSP). The price index is the consumer price index from the U.S. Bureau of Labor Statistics. Per-capita inside money—*money*, for short—is measured by M2 (or M3) less currency, divided by total population. We consider both seasonally adjusted and seasonally unadjusted series. Data for monetary aggregates and population are obtained from Citibase. Per-capita consumption is measured as consumption of nondurables and services, available from the Commerce Department, divided by total population. The monthly consumption series is seasonally adjusted; seasonally unadjusted data are available only on a quarterly basis. Real quantities are obtained by deflating nominal data by the consumer price index. The sample period is 1959.01–1988.12.

The standard deviation of money growth (0.0072), while small compared to that of stock returns (0.0476), is almost two times bigger than that of consumption growth (0.0039). The correlation of the rate of growth of real (seasonally adjusted) money with the rate of growth of consumption (0.1906) is lower than that with stocks returns (0.2909), while the correlation of consumption with stock returns (0.1852) is even lower. Money as a proxy for consumption varies more than consumption aggregates and is more highly correlated with stock returns than consumption.

B. GMM Estimation

To test the model (3), we follow a panel of n portfolio returns for T periods and define the disturbance vector $\mathbf{e}_t$ associated with the pricing conditions (3) as

$$\mathbf{e}_t = [(\delta r_{M,t+1}^{-\gamma}\, r_{1,t+1} - 1)(\delta r_{M,t+1}^{-\gamma}\, r_{2,t+1} - 1)\ \ldots\ (\delta r_{M,t+1}^{-\gamma}\, r_{n,t+1} - 1)]', \tag{7}$$

with $t = 1, 2, \ldots, T$. Let $\mathbf{0}$ be an n-dimensional vector of zeros. Hansen's (1982) GMM provides a natural way to estimate the model without making specific assumptions about the stochastic processes governing the returns r_{it} and the rate of growth of real inside money, r_{Mt}, other than that of stationarity of their joint distribution. To implement this procedure, we form the sample counterpart of the moment conditions $\mathrm{E}(\mathbf{e}_t|\Omega_0) = \mathbf{0}$, that is

$$\mathbf{g}(\delta,\ \gamma) = \sum_{t=1}^{T} \mathbf{e}_t/T \tag{8}$$

and choose γ and δ to minimize the quadratic form $\mathbf{g}'\mathbf{w}\mathbf{g}$. We follow Hansen (1982) to form the optimal weighting matrix $\mathbf{w}$ that guarantees that the estimates are consistent and asymptotically normal. The weighting matrix is initially set to be an identity matrix and re-estimated with the parameter estimates through the iterations until convergence.

Table I shows the results of the estimation of the model on a sample of *twenty* portfolios of stocks sorted by their capitalization value using monthly data from 1959 to 1988 ($n = 20$, $T = 359$). For comparison with the C-CAPM for which we do not have seasonally unadjusted monthly observations, we also report results for the seasonally adjusted series. The standard errors of the

Table I

GMM Estimates and a Test of the Money-Based CAPM

Twenty Size Portfolios

We build the sample counterpart of the moment conditions $E(e_t|\Omega_0) = \mathbf{0}$, $\mathbf{g}(\delta, \gamma) = \sum_{t=1}^{T} e_t/T$, using the forecast errors for twenty size portfolios,

$$e_t = [(\delta r_{M,t+1}^{-\gamma} r_{1,t+1} - 1)(\delta r_{M,t+1}^{-\gamma} r_{2,t+1} - 1) \ldots (\delta_{M,t+1}^{-\gamma} r_{20,t+1} - 1)]', \tag{2}$$

where r_M is the rate of growth of inside money (M2C or M3C) in real terms, and $r_{j,t+1}$ denotes real returns on portfolio $j = 1, 2, \ldots, 20$. We chose γ and δ to minimize the quadratic form $\mathbf{g'wg}$, where $\mathbf{w}$ is Hansen's (1982) optimal weighting matrix. *SU* denotes seasonally unadjusted and *SA* denotes seasonally adjusted monetary aggregates. Both the returns and the rates of growth of money are deflated by the rate of growth of the consumer price index.

	γ (s.e.)	δ (s.e.)	χ^2 (d.f.)	p-Value
M2C (SU)	133.118	1.069	24.741	0.132
	(49.489)	(0.024)	(18)	
M2C (SA)	214.949	1.158	22.709	0.202
	(83.408)	(0.038)	(18)	
M3C (SU)	162.168	1.153	24.140	0.150
	(56.663)	(0.039)	(18)	
M3C (SA)	168.041	1.249	24.408	0.142
	(91.678)	(0.092)	(18)	

estimates in Table I are large for both δ and γ, but one can reject the restriction that $\gamma = 0$ (the p-value of the χ^2-test of the restriction is 0.0381). The χ^2-statistics in Table I suggest that the model cannot be rejected.

C. A Simple Beta Model

Result 2, which is based on the assumption of logarithmic utility, is cast in terms of "betas" and allows a direct comparison with traditional ranking measures based on the covariance of returns with the market portfolio. With logarithmic utility, disregarding monetary complications, $m = \delta r_{\text{cons}}^{-1}$ and a simple aggregate C-CAPM holds. In this case, aggregate consumption is proportional to wealth, W, and r_{cons} can be replaced by the rate of growth of the market portfolio, r_W, and we obtain a Rubinstein (1976) CAPM. By the same arguments in Result 2, we can test the restrictions implied by the log-utility C-CAPM and R-CAPM models in nominal terms. This comparison helps develop a feel for how reasonable it is to rank asset risk in terms of covariance of returns with a monetary aggregate.

We estimate each beta model using a two-pass procedure similar to that of Fama and MacBeth (1973) and Chan and Chen (1988) using nominal returns on the twenty size portfolios of the previous section. An estimate of beta is obtained from the time series regression of R_i on R_M (or the rate of growth of nominal consumption or the nominal rate of return on the market portfolio).

Table II

Test of the Restricted Model—Fama-MacBeth (1973)

Rates of return R_i are raw nominal rates of return for a sample of twenty size-sorted stock portfolios. A two-pass procedure is adopted to estimate the model $E(R_{i,t+1} \mid \Omega_0) = \lambda_0 + \hat{\beta}_i\hat{\lambda}_1$, where $\hat{\beta}_i$, or more precisely $-\hat{\beta}_i$, measures the sensitivity to the different benchmark pricing variables. To have a positive risk sensitivity and thus a positive risk premium $\hat{\lambda}_1$ $(= -\lambda_1)$, the sensitivities are computed with respect to the rate of growth of nominal inside money (R_M not R_M^{-1}), the rate of growth of nominal consumption, and the nominal rate of return on the market portfolio. Nominal consumption is measured as the seasonally adjusted monthly consumption of nondurables and services divided by total population. Nominal inside money is measured as M2 minus currency (M2C) and M3 minus currency (M3C), seasonally unadjusted, divided by total population. Rates of return on the market portfolio are raw nominal rates of return on the equally weighted (EW) or the value-weighted (VW) portfolio of NYSE stocks.

	λ_0 (s.e.)	$\hat{\lambda}_1$ (s.e.)
M2C	0.0005	0.0091
	(0.0032)	(0.0027)
M3C	0.0016	0.0071
	(0.0031)	(0.0019)
consumption	0.0000	0.0023
	(0.0332)	(0.0007)
EW	−0.0060	0.0188
	(0.0047)	(0.0057)
VW	−0.0295	0.0372
	(0.0112)	(0.0112)

The estimated beta is used as the independent variable in a cross-sectional regression of the average returns for the twenty portfolios. Finally, the mean and standard errors of the cross-sectional estimates of λ_0 and the risk premium $\hat{\lambda}_1$ $(= -\lambda_1)$ are calculated. The Hotelling T^2 (not reported in Table II) suggests that we cannot reject the null hypothesis that the cross-sectional residuals have means equal to zero, especially after incorporating the adjustment suggested by Shanken (1992) to correct for the error in the time series estimates of the sensitivities: p-values are 0.200 (without adjustment) and 0.961 (with Shanken's adjustment) for M2 and similar statistics are obtained for M3. The risk premium coefficient, λ_1, is more than two standard errors away from zero, even after the Shanken (1992) adjustment for estimation errors. We cannot reject the hypothesis that λ_0 is equal to the average Treasury bill rate for the sample: the difference between the intercept and the average interest rate is -0.0045 (t-statistic $= 1.40$) for M2 and -0.0035 (t-statistic $= 1.14$) for M3. The results are robust to changes in the estimation period: if we divide the sample into two equal subperiods, the money betas remain significant in both subsamples (tests not shown).

To help recognize the relative performance of the models, Figure 1 plots the predicted average returns against the historical average returns for the M-CAPM, the C-CAPM, and the R-CAPM. We use M2C for the M-CAPM and the value-weighted market portfolio for the R-CAPM. Figure 1 suggests that

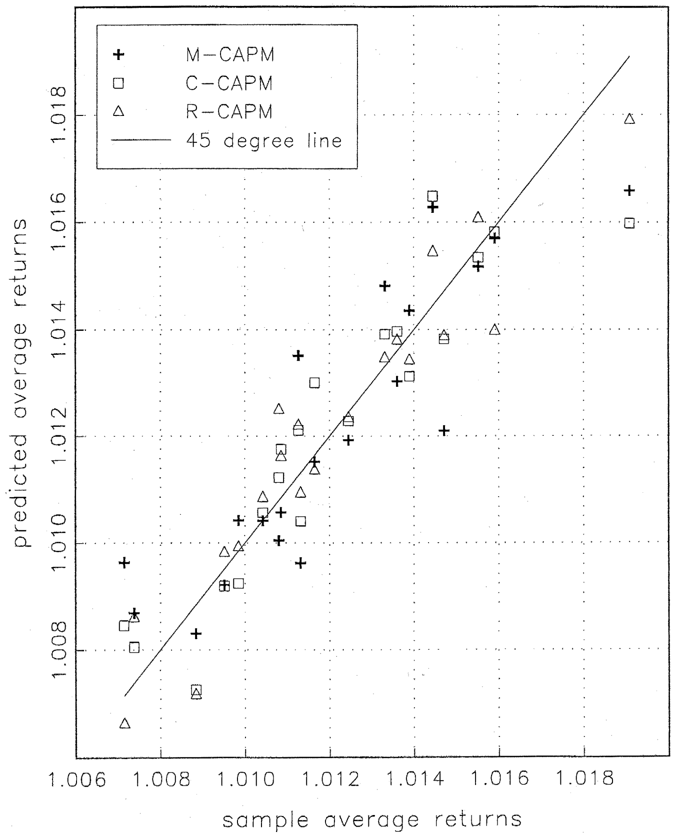

Figure 1. Predicted and historical average returns. Plot of predicted average returns against historical average returns for the M-CAPM, the C-CAPM, and the R-CAPM; it uses M2C for the M-CAPM and the value-weighted market portfolio for the R-CAPM.

the M-CAPM fares somewhat worse than the R-CAPM, but not worse than the C-CAPM. It is interesting that the pricing errors of the M-CAPM and the C-CAPM tend to be of opposite sign for the intermediate-size portfolios. A similar pattern of opposite mispricing for the intermediate-size portfolios emerges from the comparison of the M-CAPM with the R-CAPM; the cross-sectional correlation of the pricing errors of the M-CAPM and R-CAPM is 0.38. This suggests that a model of expected returns could probably benefit from including a monetary component.

In summary, the M-CAPM captures some aspects of the data even in the restrictive setting of Result 2, which assumes that the utility function is logarithmic.

Table III

GMM Estimates and a Test of the Money-Based CAPM

Fifteen Managed Portfolios

We build the sample counterpart of the moment conditions $E(e_t|\Omega_0) = \mathbf{0}$, $\mathbf{g}(\delta, \gamma) = \sum_{t=1}^{T} e_t/T$, using the forecast errors for fifteen managed portfolios,

$$e_t = [(\delta r_{M,t+1}^{-\gamma} r_{1,t+1}^{s} - 1)(\delta r_{M,t+1}^{-\gamma} r_{2,t+1}^{s} - 1) \ldots (\delta r_{M,t+1}^{-\gamma} r_{15,t+1}^{s} - 1)]', \qquad (14)$$

where r_M is the rate of growth of real inside money (M2C or M3C) and $r_{j,t+1}^{s}$ denotes scaled real returns on the managed portfolio $j = 1, 2, \ldots, 15$. We chose γ and δ to minimize the quadratic form $\mathbf{g'wg}$, where $\mathbf{w}$ is Hansen's (1982) optimal weighting matrix. *SU* denotes seasonally unadjusted and *SA* denotes seasonally adjusted monetary aggregates. Both the returns and the rates of growth of money are deflated by the rate of growth of the consumer price index.

	γ (s.e.)	δ (s.e.)	χ^2 (d.f.)	p-Value
M2C (SU)	94.697	1.068	33.057	0.002
	(57.452)	(0.025)	(13)	
M2C (SA)	91.157	1.093	35.096	0.001
	(75.679)	(0.069)	(13)	
M3C (SU)	125.140	1.130	31.157	0.003
	(62.049)	(0.045)	(13)	
M3C (SA)	182.700	1.233	31.454	0.003
	(77.610)	(0.084)	(13)	

D. Using Conditional Information

We turn now to fit the model on a smaller set of portfolios using conditioning information. One implication of (3) is that for any instrument z_t in Ω_t, $E(z_t|\Omega_0) = E[\delta\, r_{M,t+1}^{-\gamma}\, r_{i,t+1} z_t|\Omega_0]$,

$$1 = E\left[\delta r_{M,t+1}^{-\gamma} \frac{r_{i,t+1} z_t}{E(z_t|\Omega_0)} \middle| \Omega_0\right], \qquad i = 1, 2, \ldots, n. \qquad (9)$$

One way of interpreting z_t is as weight to form *scaled* returns, $r_{i,t+1}^{s} = r_{i,t+1} z_t / E(z_t|\Omega_0)$, in an *unconditional* test of (3) (see Cochrane, 1994). These scaled returns have a natural interpretation as returns on *managed portfolios*.

The length of our time series is not adequate to use *all* twenty size-sorted portfolios in conjunction with conditioning information, and in Table III we consider a test using returns on *fifteen managed portfolios* ($n = 15$, $T = 359$). We construct the managed portfolios as follows. First, from the twenty size portfolios, we form *five* equally-weighted stock portfolios sorted by size. Then, as in Hansen and Jagannathan (1994), we form managed portfolios which invest $(1 - z_t)$ dollars in Treasury bills and z_t dollars in stocks. For each of the five size portfolios, we use three instruments as the portfolio weight z for a total of fifteen ($= 5 \times 3$) managed portfolios.

Table IV

GMM Estimates and a Test of the Consumption-Based CAPM

We build the sample counterpart of the moment conditions $E(e_t|\Omega_0) = \mathbf{0}$, $\mathbf{g}(\delta, \gamma) = \sum_{t=1}^{T} e_t/T$, using the forecast errors,

$$e_t = [(\delta r_{cons,t+1}^{-\gamma} r_{1,t+1}^{s} - 1)(\delta r_{cons,t+1}^{-\gamma} r_{2,t+1}^{s} - 1) \ldots (\delta r_{cons,t+1}^{-\gamma} r_{n,t+1}^{s} - 1)]', \tag{15}$$

where $r_{cons,t+1}$ is the rate of growth of consumption in real terms and $r_{j,t+1}^{s}$ denotes (possibly scaled) real returns on portfolio $j = 1, 2, \ldots, n$. We chose γ and δ to minimize the quadratic form $\mathbf{g'wg}$, where $\mathbf{w}$ is Hansen's (1982) optimal weighting matrix. Returns are deflated by the rate of growth of the consumer price index.

	γ (s.e.)	δ (s.e.)	χ^2 (d.f.)	p-Value
Twenty size portfolios (not scaled)	185.986	1.008	28.452	0.055
	(135.034)	(0.089)	(18)	
Fifteen managed portfolios	196.809	1.052	35.462	0.001
	(103.938)	(0.022)	(13)	

The instruments are a term spread, a default spread, and a constant, 1. The term spread is measured by the difference between the yield on 10-year Treasury bonds and one-month Treasury bills expressed in percent per annum. The default spread is measured by the difference between the yields on a portfolio of Baa corporate bonds and the 10-year Treasury expressed in percent per annum. The instruments were selected from the previous time period so that they were in the conditioning information set of the investor at the time of the investment.

The χ^2-test statistic shows that the M-CAPM is rejected when using conditioning information. The poor performance is mainly due to the limited ability of the model to capture variations in short term interest rates which affect managed portfolio returns. The M-CAPM model predicts a positive correlation between *inside money* and short term risk-free interest rates R, as is easily seen, for example, when utility is logarithmic and $E[\delta(R_{t+1}/R_{M,t+1})|\Omega_t] = 1$. Contrary to this prediction, however, the correlation between M2 and Treasury bill rates in the sample is negative (-0.0451). On the other hand, the correlation between M3 and Treasury bill rates is positive (0.0372), but not as high as the model would require.

E. Comparing the M-CAPM with the C-CAPM

The M-CAPM findings should be interpreted in light of evidence on the C-CAPM. Table IV reports results for the C-CAPM of GMM tests similar to those performed on the M-CAPM in Table I and Table III. The estimates of γ are imprecise, but one can reject the restriction that $\gamma = 0$. As for the M-CAPM, the C-CAPM model cannot be rejected when fitted to the twenty size portfolios, but it is rejected using the conditioning information embedded in the fifteen managed portfolios.

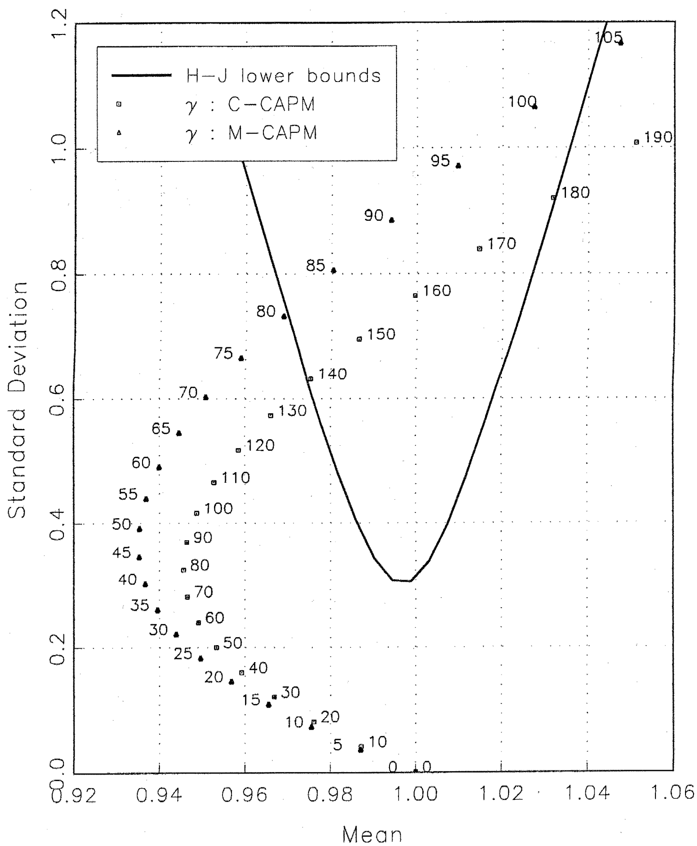

Figure 2. Hansen-Jagannathan (1991) bounds. Let $\mathbf{r}_{t+1} = [r^s_{1,t+1}\ r^s_{2,t+1}\ r^s_{3,t+1}\ r^s_{4,t+1} \cdots r^s_{15,t+1}]'$ be the vector of real returns of the fifteen managed portfolios from time t to $t+1$ and $\mathbf{1}$ a 15×1 unit vector. Dropping time subscripts and references to the conditioning information set, the mean and standard deviation of the stochastic discount factor, $\mathrm{E}(m)$ and $\sigma(m)$, satisfy the inequality restriction $\sigma^2(m) \geq [\mathbf{1} - \mathrm{E}(m)\mathrm{E}(\mathbf{r})]'[\mathrm{var}(\mathbf{r})]^{-1}[\mathbf{1} - \mathrm{E}(m)\mathrm{E}(\mathbf{r})]$ for given unconditional mean, $\mathrm{E}(\mathbf{r})$, and variance-covariance, $\mathrm{var}(\mathbf{r})$, of the fifteen managed portfolio returns.

We now turn to the economic significance of the parameter estimates and to the pricing performance of the M-CAPM, bearing in mind the predictions derived in Section I. If the economy conforms to that of the M-CAPM, we expect to estimate a lower curvature parameter γ using the M-CAPM than using the C-CAPM. Second, we expect more mispricing for the C-CAPM than for the M-CAPM.

Figure 2 shows the Hansen and Jagannathan (1991) bounds based on the returns of the fifteen managed portfolios. The solid line in Figure 2 represents the lower bound for the unconditional mean and standard deviation pairs of the stochastic discount factor m. The triangles to the left represent the mean and standard deviation pairs of the stochastic discount factor of the M-CAPM based on M2C calculated for values of γ ranging from 0 to 105. The squares represent the mean and standard deviation pairs of the stochastic discount factor of the C-CAPM calculated for values of γ ranging from 0 to 190. Whereas the M-CAPM requires $\gamma > 81$ to give a mean and variance pair within the admissible region, the C-CAPM requires $\gamma > 139$. These values are consistent with the GMM estimates of γ using the fifteen managed portfolios in Table III (M-CAPM, M2) and Table IV (C-CAPM).

The M-CAPM obviously does not provide a solution to the "equity premium puzzle" of Mehra and Prescott (1985), but the lower curvature for the M-CAPM is consistent with the first prediction of our model.

We now consider the pricing errors of the C-CAPM and the M-CAPM. Departures of the returns conditions $\mathbf{g}$ from zero measure how well the model explains the mean return of each portfolio. Dropping subscripts and references to the conditioning information set for simplicity, the standard pricing condition implies

$$\begin{aligned} \mathrm{E}(mr) - 1 &= \mathrm{E}(m)\left(\mathrm{E}(r) - \frac{1}{\mathrm{E}(m)} + \frac{\operatorname{cov}(m, r)}{\mathrm{E}(m)}\right) \\ &= \mathrm{E}(m)[\text{mean}(r) - \text{predicted mean}(r)] \end{aligned} \tag{10}$$

The sample counterpart of this condition, $\mathbf{g}_i$ in equation (8), can be viewed as the percentage point error in expected returns relative to the shadow risk free rate, $1/\mathrm{E}(m)$ (Cochrane, 1994).

Figure 3 shows the pricing errors of the M-CAPM, the C-CAPM, and the R-CAPM for the twenty size-sorted portfolios (top three panels) and for the fifteen managed portfolios (bottom three panels). The pricing errors for the M-CAPM are based on M2C seasonally unadjusted; qualitatively similar results were obtained using the seasonally adjusted series (not shown). The M-CAPM fares worse than the R-CAPM or the C-CAPM for the large size (low beta) portfolios 18–20 but does better than the C-CAPM for small-size portfolios, which is consistent with the results for the Fama-MacBeth exercise shown in Figure 1. The estimated pricing errors of the M-CAPM are smaller than those of the C-CAPM for all the remaining size portfolios and for most of the fifteen managed portfolios, a finding that is broadly consistent with the *second* prediction of our model. The associated standard errors are very large, though, making these differences statistically insignificant.

Comparing the degree of misspecification of the different models by looking at their χ^2-statistics is not possible because of the different volatilities of the proxies used for the stochastic discount factor. Hansen and Jagannathan (1994) propose a measure of model misspecification which does not reward the volatility of the proxy for the stochastic discount factor. This statistic is a least-square distance between the proxy used as a stochastic discount factor, $\hat{m}$, and the set of admissible stochastic discount factors. Hansen and Jagannathan (1994) show how to estimate this distance measure Δ by solving the problem

$$\Delta^2 = \max_{\Lambda} \mathrm{E}(\hat{m}_{t+1}^2) - \mathrm{E}[(\hat{m}_{t+1} - \boldsymbol{\Lambda}'\mathbf{r}_{t+1})^2] - 2\boldsymbol{\Lambda}'\mathbf{1}; \tag{11}$$

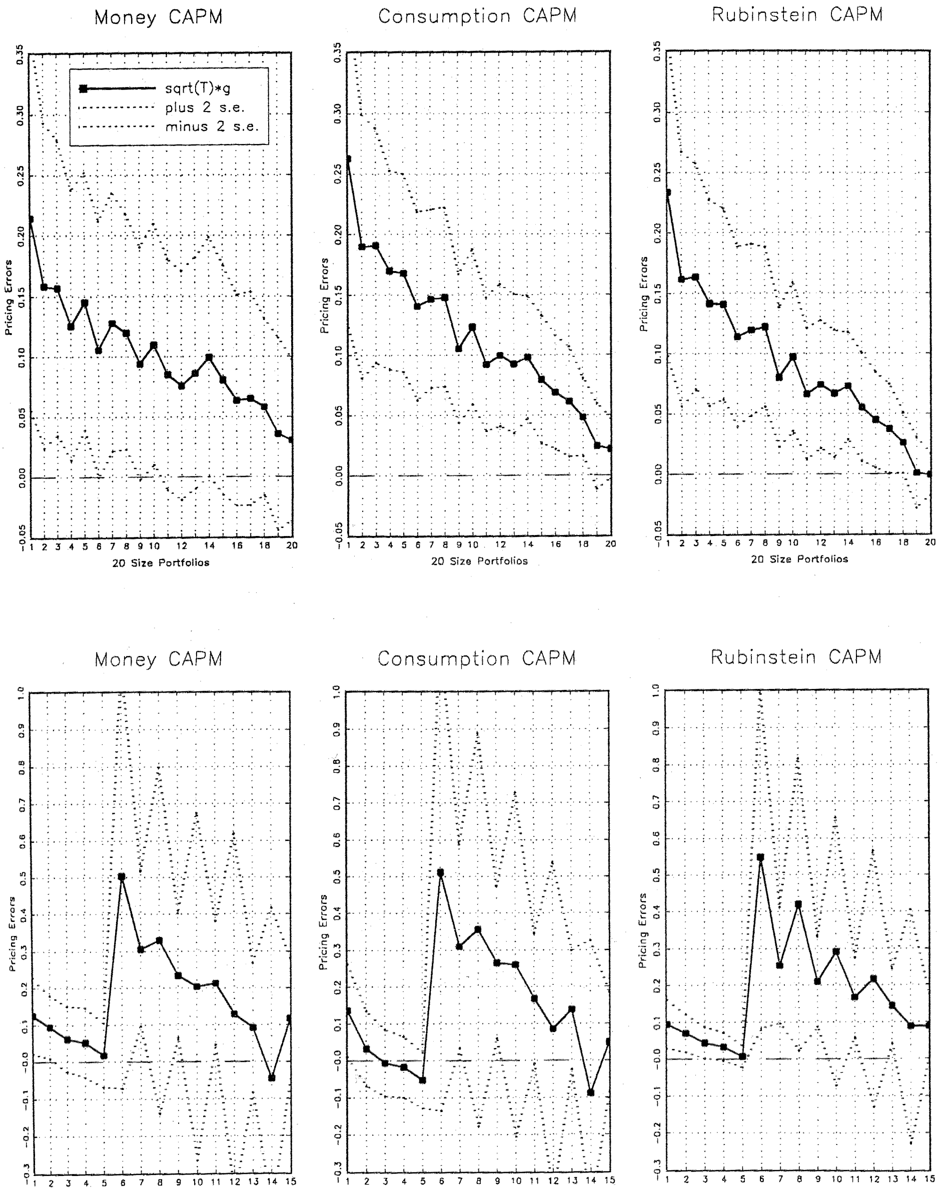

Figure 3. Pricing errors. The pricing error is measured by $\sqrt{T}\,\mathbf{g}_i(\hat{\delta}, \hat{\gamma})$, where $\mathbf{g}_i$ is evaluated based on the values $\{\hat{\delta}, \hat{\gamma}\}$ (= $\text{argmin}_{\delta,\gamma}\, \mathbf{g}'\mathbf{w}\mathbf{g}$) reported in Tables I and III (M2C) and Table IV for the M-CAPM and the C-CAPM, respectively. Hansen (1982) gives conditions for $\sqrt{T}\,\mathbf{g}(\hat{\delta}, \hat{\gamma})$ to converge to a normally distributed random vector with mean zero; a consistent estimator of the covariance matrix of $\sqrt{T}\,\mathbf{g}(\hat{\delta}, \hat{\gamma})$ is $\Xi_0 = T(\mathbf{w}^{-1} - \mathbf{d}(\mathbf{d}'\mathbf{w}\mathbf{d})^{-1}\mathbf{d}')$, where $\mathbf{w}$ is Hansen's (1982) optimal weighting matrix and $\mathbf{d}$ denotes the gradient of $\mathbf{g}$ evaluated at $(\hat{\delta}, \hat{\gamma})$. The standard error for each element of $\sqrt{T}\,\mathbf{g}_i(\hat{\delta}, \hat{\gamma})$ is the square root of the corresponding diagonal element of Ξ_0. No estimation is involved for the R-CAPM, for which we report the empirical mean and standard errors of the forecast errors, $e_{j,t} = (r_{W,t+1}^{-1}\, r_{j,t+1} - 1)$, where r_W is the rate of return of the market portfolio measured by the value-weighted portfolio of NYSE stocks. The top panels plot the pricing errors for twenty size-sorted portfolios using estimates from Table I (M2C) and Table IV. The bottom panels plot the pricing errors for the fifteen managed portfolios (Table III and Table IV).

Table V

Hansen-Jagannathan (1994) Distance

The stochastic discount factor proxies for the M-CAPM and C-CAPM are constructed using γ and δ estimates from Table III (M2C) and Table IV for the fifteen managed portfolios. *SU* denotes seasonally unadjusted and *SA* denotes seasonally adjusted monetary aggregates.

	Δ	s.e.
M2C (SU)	0.439	0.086
M2C (SA)	0.431	0.081
M3C (SU)	0.432	0.087
M3C (SA)	0.471	0.117
Consumption	0.445	0.077
Market	0.383	0.050
Constant	0.385	0.049

in our case $\mathbf{r}_{t+1} = [r^s_{1,t+1}\ r^s_{2,t+1}\ r^s_{3,t+1}\ r^s_{4,t+1} \cdots r^s_{15,t+1}]'$ is the vector of real returns of the fifteen managed portfolios from time t to time $t + 1$ and Λ is a constant vector of comparable dimension. Hansen, Heaton, and Luttmer (1994) discuss how to obtain estimates of the asymptotic standard error of the estimated distance. We follow the procedure highlighted in Hansen and Jagannathan (1994): that is, we take the stochastic discount factor parameters $\{\hat{\delta}, \hat{\gamma}\}$ ($= \text{argmin}_{\delta,\gamma}\ \mathbf{g}'\mathbf{w}\mathbf{g}$) as given (Table III (M2C) and Table IV for the M-CAPM and the C-CAPM, respectively) and do not attempt to minimize Δ by choosing different values of the curvature parameter γ. We compute standard errors as suggested by Hansen and Jagannathan (1994) using the method described by Newey and West (1987) with twenty four lags. Table V reports the estimated Δ with associated standard errors for the M-CAPM, the C-CAPM, the R-CAPM, and for a constant discount factor. The specification error Δ of the R-CAPM and the constant discount factor are the smallest; with the exception of M3C (SA), the specification error of the M-CAPM is somewhat lower than that of the C-CAPM, but the difference across Δs are small compared with their estimated standard errors.

III. Conclusions

In a monetary economy consumption is financed at least in part with inside money. This observation motivates using broad monetary aggregates as a proxy for consumption in tests of consumption-based asset pricing models. We argue that using monetary aggregates should yield lower estimates of the curvature parameter and smaller pricing errors than using consumption data. While the model is rejected, the empirical results are broadly consistent with these predictions, suggesting that asset pricing models could probably benefit from including a monetary aggregate in a parsimonious representation of stochastic discount factors.

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