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The Errors in the Variables Problem in the Cross-Section of Expected Stock Returns

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ABSTRACT

Recent research has documented the failure of market beta to capture the cross-section of expected returns within the context of a two-pass estimation methodology. However, the two-pass methodology suffers from the errors-in-variables (EIV) problem that could attenuate the apparent significance of market beta. This article provides a new correction for the EIV problem that is robust to conditional heteroscedasticity. After the correction, I find more support for the role of market beta and less support for the role of firm size in explaining the cross-section of expected returns. While the EIV correction leads to a diminished role of firm size, the size variable remains a significant force in explaining the cross-section of expected returns.

RECENT FINDINGS BY FAMA and French (1992) suggest that market beta has very limited ability to explain the cross-section of average stock returns, while firm size and the book-to-market equity ratio have considerable power. These findings cast doubt on the validity of the capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), and Black (1972). However, the Fama and French findings are subject to the errors-in-variables (EIV) problem of the traditional two-pass estimation methodology: In the first-pass, beta estimates are obtained from separate time-series regressions for each asset, and in the second-pass, gammas are estimated cross-sectionally by regressing asset returns on the estimated betas. Therefore, the explanatory variable in the cross-sectional regression (CSR) is measured with error. The EIV problem results in an underestimation of the price of beta risk and an overestimation of the other CSR coefficients associated with variables observed without error (such as firm size or the book-to-market equity ratio). Hence, it is very important to determine whether the weak relation between average stock returns and market beta is the result of the misspecification of the asset pricing model or is simply a consequence of the EIV problem.

Several methods have been proposed to address the EIV problem. Litzenberger and Ramaswamy (1979) suggest a correction that involves a weighted least squares version of the CSR estimator with a modified design matrix.

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However, their corrected estimator can be obtained only when security residual variances are exactly known. Shanken (1992) modifies the traditional two-pass procedure and derives an asymptotic distribution of the CSR estimator within a multifactor framework in which asset returns are generated by portfolio returns and prespecified factors. He presents some support for the use of the modified two-pass estimation as an alternative to the traditional two-pass estimation. Shanken also provides an adjustment for the standard errors of the CSR estimators with a multifactor interpretation. Instead of employing the two-pass procedure, Gibbons (1982) uses the maximum likelihood estimation (MLE) approach to eliminate the EIV problem by simultaneously estimating betas and beta risk prices. As Shanken mentions, however, Gibbons' Gauss-Newton estimator is identical to the second-pass generalized least squares (GLS) estimator, and the advantage of the simultaneous estimation is, indeed, lost in linearizing the nonlinear constraint. Gibbons' estimator is thus still subject to an EIV bias.

The purpose of this article is to propose a new correction for the EIV problem and, subsequently, to reexamine the explanatory power of market beta and firm size for average stock returns in the context of the new correction. Without additional information about the measurement error of the beta variable in the CSR model, the correction for the EIV problem is not feasible. It is known, however, that the measurement error of the beta variable is the same as the estimation error of the beta estimated in the first-pass regression. Specifically, conditional on the market return, the measurement error can be linearly represented by the idiosyncratic error terms prior to each CSR. Based on this fact, the necessary information about the relationship between the measurement error variance and the idiosyncratic error variance can be extracted. By utilizing the extracted information, I derive a new correction that is performed through a maximum likelihood estimation under either homoscedasticity or heteroscedasticity of the disturbance terms of the market model. Since this EIV-correction estimation is N-consistent (i.e., consistent when the time-series sample size, T, is fixed and the number of assets, N, is allowed to increase without bound), it is most useful when all available individual stocks (without forming portfolios) are used in the CSR estimation. In fact, the formation of portfolios for the CSR estimation might cause a loss of valuable information about cross-sectional behavior among individual assets, since the cross-sectional variations would be smoothed out.

The results show that after correcting for the EIV problem, market beta has economically and statistically significant explanatory power for average stock returns both in the presence and in the absence of the firm size variable in the regression, and that the firm size factor, while significant, becomes much less important. In other words, my results suggest that Fama and French's (1992) findings may have been exaggerated because of their failure to correct for the EIV problem. Nevertheless, my results indicate that there is a misspecification in the CAPM related to firm size.

The rest of the article is organized as follows: Section I discusses the EIV problem in the traditional least squares estimation of risk premia. Section II derives the EIV-corrected estimation and the EIV-correction factor for the traditional least squares estimator. Section III presents empirical results, and Section IV concludes.

I. The Errors-In-Variables Problem

The Fama and MacBeth (1973) second-pass CSR model of estimating the return-risk relation at a specific time t is

$$\mathbf{R}_t = \gamma_{0t} + \gamma_{1t}\boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\mathbf{R}_t = (R_{1t}, \cdots, R_{Nt})'$ is the return vector in excess of the riskless return or the return on a zero-beta portfolio, $\boldsymbol{\beta}_t = (\beta_{1t}, \cdots, \beta_{Nt})'$ is the market beta vector, $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \cdots, \varepsilon_{Nt})'$ is the idiosyncratic error vector, and N is the number of assets. Since β_t is unobservable, the estimated market beta, $\hat{\boldsymbol{\beta}}_{t-1}$, is used as a proxy for the unknown β_t. The independent variable, therefore, is observed with error

$$\hat{\boldsymbol{\beta}}_{t-1} = \boldsymbol{\beta}_t + \boldsymbol{\xi}_{t-1}, \tag{2}$$

where $\hat{\boldsymbol{\beta}}_{t-1} = (\hat{\beta}_{1t-1}, \cdots, \hat{\beta}_{Nt-1})'$ is the market beta vector estimated from the first-pass time-series regression model (the market model) using T time-series data available up to $t - 1$, and $\boldsymbol{\xi}_{t-1} = (\xi_{1t-1}, \cdots, \xi_{Nt-1})'$ is the measurement error vector. In the Fama and MacBeth risk premia estimation procedure, the estimates $\hat{\gamma}_{1t}$ are taken to be the independent and identical sampled values of the price of beta risk, γ_1. The time-series sample mean($\bar{\hat{\gamma}}_1$) of the estimates $\hat{\gamma}_{1t}$ is regarded as the final estimate of γ_1, and is used for testing whether the price of beta risk is positive and significantly different from zero.[1] The time-series sample mean ($\bar{\hat{\gamma}}_0$) of the estimates $\hat{\gamma}_{0t}$ is used for testing whether γ_0 is zero; γ_0 should not be significantly different from zero for the CAPM to be valid.

As Shanken (1992) points out, use of the predictive beta ($\hat{\boldsymbol{\beta}}_{t-1}$) in the CSR estimation has some advantages. First, the problem of a spurious cross-sectional relation arising from statistical correlation between returns and estimated market betas can be avoided. Second, the independence between the explanatory variable ($\hat{\boldsymbol{\beta}}_{t-1}$) and the regression error term ($\boldsymbol{\varepsilon}_t$) in the CSR can be maintained. Furthermore, the measurement error of the estimated beta could be assumed to be independent of the idiosyncratic error term, because the measurement error is a function of past idiosyncratic error terms prior to each CSR.

It is well known that the traditional second-pass least squares estimate of γ_{1t} is not N-consistent. Assuming that the measurement error and the idiosyncratic

[1] Instead of using the time-series average, Amihud, Christensen, and Mendelson (1993) propose a one-time γ_1 estimate from the joint-pooled time-series and cross-sectional model. They analytically show that the joint-pooled estimator of γ_1 is more efficient than the time-series average of the least squares estimates of γ_{1t} obtained by Fama and MacBeth's two-pass procedure, since its variance is less than that of the time-series average. However, the joint-pooled estimator uses the beta variable measured with error and thus is still subject to an EIV bias.

error are independent, Richardson and Wu (1970) show that the traditional ordinary least squares (OLS) estimator, $\hat{\gamma}_{1t}^{\mathrm{OLS}}$, is negatively (downward) biased:

$$E(\hat{\gamma}_{1t}^{\mathrm{OLS}}) - \gamma_{1t} = -\frac{1}{1+\tau}\left(1 - \frac{2}{N-1}\frac{\tau^2}{(1+\tau)^2}\right)\gamma_{1t} + O(N^{-2}), \tag{3}$$

where $\tau = \sigma^2_{\beta_{t-1}}/\sigma^2_{\xi_{t-1}}$, $\sigma^2_{\xi_{t-1}}$ is the measurement error variance, and $O(N^{-2})$ is the term containing all remaining terms with order less than or equal to N^{-2}. The term $\sigma^2_{\beta_{t-1}}$ is used to designate $\sum_{i=1}^{N}(\beta_{it-1} - \bar{\beta}_{t-1})^2/(N-1)$. Hence, γ_{1t} is underestimated under the traditional least squares estimation and so is the price of beta risk. The underestimation of the price of beta risk leads to an overestimation of the intercept γ_0. The misspecification of the CAPM is, therefore, exaggerated in the traditional least squares estimation.

The firm size effect (Reinganum (1981), Banz (1981)) is frequently cited as evidence of the misspecification of the CAPM. To examine whether firm size has the power to explain the cross-section of stock returns, the firm size variable is added into equation (1):

$$\mathbf{R}_t = \gamma_{0t} + \gamma_{1t}\boldsymbol{\beta}_t + \gamma_{2t}\mathbf{V}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{4}$$

where $\mathbf{V}_{t-1} = (V_{1t-1}, \cdots, V_{Nt-1})'$ is the vector of the logarithm of market values measured without error at $t-1$. The CSR model of equation (4) has one explanatory variable measured with error and another explanatory variable measured without error. The time-series sample mean ($\bar{\hat{\gamma}}_2$) of the estimates $\hat{\gamma}_{2t}$ is used for testing the explanatory power of firm size. Assuming that the measurement error and the idiosyncratic error are independent, McCallum (1972) and Aigner (1974) show that the biases of the traditional OLS estimator of γ_{1t} and γ_{2t} in equation (4) are

$$\mathrm{E}(\hat{\gamma}_{1t}^{\mathrm{OLS}}) - \gamma_{1t} = -\left(\frac{1}{1+\tau(1-\rho^2_{\hat{\beta}V})}\right)\gamma_{1t} + O(N^{-1}) \tag{5}$$

$$\mathrm{E}(\hat{\gamma}_{2t}^{\mathrm{OLS}}) - \gamma_{2t} = \frac{\sigma_{\hat{\beta}V}}{\sigma^2_V}\left(\frac{1}{1+\tau(1-\rho^2_{\hat{\beta}V})}\right)\gamma_{1t} + O(N^{-1})$$

$$= -\frac{\sigma_{\hat{\beta}V}}{\sigma^2_V}(\mathrm{E}(\hat{\gamma}_{1t}^{\mathrm{OLS}}) - \gamma_{1t}) + O(N^{-1}), \tag{6}$$

where $\sigma_{\hat{\beta}V}$ ($\rho_{\hat{\beta}V}$) is the cross-sectional covariance (correlation coefficient) between the estimated beta and the firm size variable, σ^2_V is the cross-sectional variance of the firm size variable, and $O(N^{-1})$ is the term containing all remaining terms with order less than or equal to N^{-1}.

Equation (3) shows that the OLS estimator of γ_{1t}, $\hat{\gamma}_{1t}^{\mathrm{OLS}}$, is negatively biased. Equation (5) now shows that after including the firm size variable, γ_{1t} becomes even more underestimated, because the magnitude of negative bias in equation (5) is greater than that in equation (3).[2] At the same time, equation (6) shows that the traditional least squares coefficient estimate associated with the firm size variable, $\hat{\gamma}_{2t}^{\mathrm{OLS}}$, is negatively biased, as long as the estimated beta and the firm size are negatively correlated. In other words, the CSR coefficient associated with the firm size variable is on average estimated more negatively than its true value under the traditional least squares estimation procedure. It is, therefore, possible that the explanatory power of the firm size variable has been overestimated in previous empirical studies, since the negative correlation between these two variables is empirically well-known. The extent of the overestimation depends on the magnitude of the correlation coefficient between these two variables. The greater the negative correlation, the more severe the overestimation of the coefficient associated with firm size.

Equation (6) indicates that if the estimated beta and the exactly measured second explanatory variable are *positively* correlated, the traditional least squares coefficient estimate associated with the second variable is *positively* biased. An example of this case is the book-to-market equity ratio. This variable is positively correlated with the estimated beta. When the book-to-market equity ratio is included as the second variable in equation (4), the least squares estimate of γ_{2t} is positively (upward) biased; that is, the average value of the least squares estimates of γ_{2t} is greater than the true value. Therefore, the explanatory power of the book-to-market equity ratio for average stock returns reported by Fama and French (1992) is also exaggerated under the traditional least squares estimation procedure.

The biases of the traditional least squares estimators could be eliminated if T-consistent beta estimates are used ($\tau \to \infty$ as $T \to \infty$), or if a finite set of well-diversified portfolios can be constructed ($\tau \to \infty$ as $\sigma_{\xi_t}^2 \to 0$). However, concerns about changing risk premia parameters and beta nonstationarity limit the time-series length T that can be used. Similarly, the construction of a finite set of well-diversified portfolios is difficult in practice. Under these circumstances, an N-consistent estimation is necessary to asymptotically remove the biases. The N-consistent estimation could also resolve two other empirical issues: the optimal number of portfolios in the two-pass estimation that minimize the mean squared error of risk premia estimates and the best way to form portfolios.[3]

[2] In terms of $O(N^{-1})$, the negative bias of $\hat{\gamma}_{1t}^{\mathrm{OLS}}$ is apparently greater when the size variable is included in the model than when the beta variable alone is in the model, since $\rho_{\beta V}^2$ is always less than one. The greater the correlation coefficient between the beta variable and the second variable (firm size or book-to-market equity), the greater the negative bias of $\hat{\gamma}_{1t}^{\mathrm{OLS}}$.

[3] Portfolio grouping is widely used to decrease the biases of the traditional least squares CSR estimates ($\hat{\gamma}_{\cdot t}^{\mathrm{OLS}}$). However, there is a trade-off between the bias and the variance of the estimate according to the number of portfolios. As the number of portfolios (N) is increased (and hence, there is a smaller number of individual securities within a portfolio), the magnitude of the bias becomes greater, while the variance of the estimate becomes smaller, and vice versa. Hence, there is an optimal number of portfolios with which a minimum mean squared error can be achieved. However, when risk premium is estimated by the time-series mean of the CSR estimates, the mean squared error of the risk premium estimate ($\bar{\hat{\gamma}}$) would be dominated by its bias, since its variance would monotonically decrease as the testing period becomes longer.

II. A Correction for the Errors-In-Variables Bias

This section presents an N-consistent estimation of the CSR coefficients (γ_{0t}, γ_{1t}, γ_{2t}) that asymptotically removes the biases of the Fama and MacBeth's (1973) two-pass procedure. This N-consistent estimation is feasible when additional information about the relation between the measurement error variance and the idiosyncratic error variance is extracted and this information is incorporated into the maximum likelihood estimation. I assume both homoscedasticity and heteroscedasticity of the disturbance term of the market model. In particular, under homoscedasticity of the disturbance term of the market model, I provide direct EIV-correction factors for the traditional least squares estimators of the CSR coefficients. Rather than forming portfolios or increasing the time-series sample size, the proposed methodology increases the cross-sectional sample size (N) to its maximum. The N-consistent estimation is most useful when all available individual stocks (i.e., maximum cross-sectional sample size) are used in the CSR estimation. In fact, use of all available individual stocks ensures the full utilization of information about the cross-sectional behavior of individual stocks, which might otherwise be lost in forming portfolios.

In order to derive the EIV-corrected N-consistent estimators of the CSR coefficients when one explanatory variable is measured with error and the other variable is measured without error in the model (see Appendix for the case of K variables measured without error), I define

$$\boldsymbol{\eta}_t = \begin{pmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\xi}_{t-1} \end{pmatrix} = \begin{pmatrix} \mathbf{R}_t - \gamma_{0t} - \gamma_{1t}\boldsymbol{\beta}_t - \gamma_{2t}\mathbf{V}_{t-1} \\ \hat{\boldsymbol{\beta}}_{t-1} - \boldsymbol{\beta}_t \end{pmatrix} \sim N(\mathbf{O}, \Omega), \quad \text{where} \quad \Omega = \begin{pmatrix} \Sigma_\varepsilon & \Sigma_{\varepsilon\xi} \\ \Sigma_{\varepsilon\xi} & \Sigma_\xi \end{pmatrix}$$

(rank $2N$), and $\hat{\boldsymbol{\beta}}_{t-1}$ is the $(N \times 1)$ beta vector estimated through the market model by using T time-series return observations from $t - T$ through $t - 1$ (prior to each CSR). Then I assume that

$$\mathrm{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}'_t \mid \mathbf{R}_m) = \begin{cases} \Sigma_\varepsilon & \text{for} \quad t = t' \\ \mathbf{0} & \text{for} \quad t \neq t' \end{cases} \tag{A1}$$

$$\mathrm{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\xi}_{t-1} \mid \mathbf{R}_m) \equiv \Sigma_{\varepsilon\xi} = \mathbf{0}, \tag{A2}$$

where $\mathbf{R}_m$ is the set of T market returns used to estimate betas prior to each CSR. The assumption (A1) indicates that the idiosyncratic error terms $\boldsymbol{\varepsilon}_t$ are intertemporally independent and cross-sectionally heteroscedastic. Without loss of generality, assumption (A2) can be maintained within the two-pass estimation framework, since conditional on the market return, $\hat{\boldsymbol{\beta}}_{t-1}$ is the best linear unbiased estimate by the classical regression theory and thus $\boldsymbol{\xi}_{t-1}$ is a *linear* function of past idiosyncratic error terms $\boldsymbol{\varepsilon}_s$ prior to the CSR at time t ($s \leq t - 1$). Put more specifically, the measurement error term of an asset i's beta variable in the CSR at time t is represented by

$$\xi_{it-1} = \sum_{s=t-T}^{t-1} a_{is}\varepsilon_{is}, \tag{7}$$

where $a_{is} = h_{is}(R_{ms} - \tilde{R}_m)/\sum_{s=t-T}^{t-1} h_{is}(R_{ms} - \tilde{R}_m)^2$, $h_{is} = 1/\mathrm{Var}(\varepsilon_{is})$, and $\tilde{R}_m = \sum_{s=t-T}^{t-1} h_{is}\, R_{ms}/\sum_{s=t-T}^{t-1} h_{is}$ is the weighted average of the market returns. Conditional on the market return, therefore, the measurement error term of the beta variable in each CSR (ξ_{it-1}) is independent of the idiosyncratic error term in the CSR (ε_{it}). This is a consequence of the use of the predictive beta.[4]

Without additional information about the error terms, $\boldsymbol{\varepsilon}_t$ and $\boldsymbol{\xi}_{t-1}$, the MLE of the unknown parameter vector $\boldsymbol{\theta} = (\gamma_{0t}, \gamma_{1t}, \gamma_{2t}, \beta_t)$ that minimizes the quadratic function (likelihood function) $L = \boldsymbol{\eta}'_t\Omega^{-1}\boldsymbol{\eta}_t$ does not exist. However, based on the fact that the measurement error is linearly related to the idiosyncratic error terms prior to each CSR (equation (7)), additional information about the relationship between the variances of the error terms can be obtained. That is, conditional on the market return, the ratio of the idiosyncratic error variance matrix to the measurement error variance matrix is $\Sigma_\varepsilon\Sigma_\xi^{-1} = \mathbf{D}$, where $\mathbf{D} = \mathrm{diag}(\delta_{1t}, \cdots, \delta_{Nt})$, and

$$\delta_{it} = \sum_{s=t-T}^{t-1} (R_{ms} - \tilde{R}_m)^2(\mathrm{Var}(\varepsilon_{it})/\mathrm{Var}(\varepsilon_{is})). \tag{8}$$

With this additional information, the N-consistent MLE of γ_{0t}, γ_{1t}, and γ_{2t} can be obtained by (1) setting the derivative of L with respect to $\boldsymbol{\beta}_{t-1}$ to zero, (2) substituting the unknown $\boldsymbol{\beta}_{t-1}$ in L with its MLE (from the equation $\partial L/\partial\boldsymbol{\beta}_{t-1} = 0$), and (3) setting the derivatives of L with respect to γ_{0t}, γ_{1t}, and γ_{2t} to zero; that is,

$$\frac{\partial L}{\partial \gamma_{1t}} = \hat{\boldsymbol{\varepsilon}}'_t\mathbf{D}(\gamma_{1t}^2 I_N + \mathbf{D})^{-1}\hat{\Sigma}_\varepsilon^{-1}\hat{\boldsymbol{\beta}}_{t-1} + \gamma_{1t}\hat{\boldsymbol{\varepsilon}}'_t\mathbf{D}(\gamma_{1t}^2 I_N + \mathbf{D})^{-2}\hat{\Sigma}_\varepsilon^{-1}\,\hat{\boldsymbol{\varepsilon}}_t = 0 \tag{9}$$

$$\frac{\partial L}{\partial \gamma_{2t}} = \hat{\boldsymbol{\varepsilon}}'_t\mathbf{D}(\gamma_{1t}^2 I_N + \mathbf{D})^{-1}\hat{\Sigma}_\varepsilon^{-1}\mathbf{V}_{t-1} = 0 \tag{10}$$

$$\frac{\partial L}{\partial \gamma_{0t}} = \hat{\boldsymbol{\varepsilon}}'_t\hat{\Sigma}_\varepsilon^{-1}\boldsymbol{\iota} = 0, \tag{11}$$

where $\hat{\boldsymbol{\varepsilon}}_t = \mathbf{R}_t - \gamma_{0t} - \gamma_{1t}\hat{\boldsymbol{\beta}}_{t-1} - \gamma_{2t}\mathbf{V}_{t-1}$, and $\boldsymbol{\iota}$ is a vector of ones. The solution of equations (9), (10), and (11) is the feasible GLS version of the MLEs of the CSR coefficients (γ_{0t}, γ_{1t}, γ_{2t}), since the unknown error covariance matrix Σ_ε is substituted with its consistent estimator, $\hat{\Sigma}_\varepsilon$, which is estimated from the

[4] When betas estimated by using the full-period sample are used in the CSR as in Fama and French (1992), the measurement error term is a linear function of all idiosyncratic error terms over the testing period, not of only past idiosyncratic error terms. In each CSR, therefore, one of the idiosyncratic error terms contained in the measurement error term is overlapped with the idiosyncratic error term of the CSR model. In this case, the independence between the idiosyncratic error term of the CSR model and the measurement error term can not be maintained.

market model residuals.[5] Regardless of whether Σ_ε or $\hat{\Sigma}_\varepsilon$ is used, the solution has the same asymptotic distribution under certain conditions (see Zellner (1962), Schmidt (1976), and Fomby, Hill, and Johnson (1984)). Obviously, the weighted least squares (WLS) or OLS version of the MLEs can also be obtained if Σ_ε is a diagonal or a scalar matrix, respectively.

In deriving the N-consistent estimators of the CSR coefficients at time t, the knowledge of the ratio, δ_{it}, is crucial. When the disturbance term of the market model is homoscedastic (i.e., two idiosyncratic error variance terms in equation (8), $\mathrm{Var}(\varepsilon_{is})$ and $\mathrm{Var}(\varepsilon_{it})$, are equal), the ratio is known conditionally on the market return and is the same as the sum of the squared mean-adjusted market returns for all assets because two idiosyncratic error variance terms in equation (8) are canceled out. On the other hand, when the disturbance term of the market model is heteroscedastic, the ratio is different across assets, and the idiosyncratic error variance terms contained in the ratio should be estimated. I first discuss the case of homoscedasticity and then the case of heteroscedasticity.

A. Homoscedastic Market Model Disturbances

When the disturbance term of the market model is intertemporally homoscedastic, the ratio of the idiosyncratic error variance to the measurement error variance, δ_{it}, is the same for all assets; that is,

$$\delta_{it} = \delta_t = T\hat{\sigma}^2_{m,t-1},$$

where $\hat{\sigma}^2_{m,t-1}$ is the MLE of the market variance obtained by using the market returns from $t - T$ through $t - 1$. The closed form solution of equations (9), (10), and (11) is then available. By solving these equations, the following EIV-corrected MLEs of the CSR coefficients at time t are obtained:

$$\hat{\gamma}_{1t} = \frac{M + [M^2 + 4\delta_t m^2_{R\hat{\beta}}(1 - (\hat{\rho}_{RV}\,\hat{\rho}_{\hat{\beta}V}/\hat{\rho}_{R\hat{\beta}}))^2]^{1/2}}{2m_{R\hat{\beta}}(1 - (\hat{\rho}_{RV}\hat{\rho}_{\hat{\beta}V}/\hat{\rho}_{R\hat{\beta}}))} \tag{12}$$

$$\hat{\gamma}_{2t} = (m_{RV} - \hat{\gamma}_{1t}\, m_{\hat{\beta}V})/m_{VV} \tag{13}$$

$$\hat{\gamma}_{0t} = \bar{R}_t - \hat{\gamma}_{1t}\tilde{\bar{\beta}}_{t-1} - \hat{\gamma}_{2t}\tilde{V}_{t-1}, \tag{14}$$

where

$$M = m_{RR}(1 - \hat{\rho}^2_{RV}) - \delta_t m_{\hat{\beta}\hat{\beta}}(1 - \hat{\rho}^2_{\hat{\beta}V}),$$

[5] These MLEs under the normality condition are identical to the homoscedastic version of the method of moments estimators without any distributional assumption of the error terms, since the system of moment conditions is exactly identified and the differentiating equations for the MLE are equivalent to the equations defining the method of moments estimators. See also Judge *et al.* (1980), Johnston (1984), and Fuller (1987) for deriving the method of moments estimation for the case that one explanatory variable measured with error is alone in the model and $\Sigma_\varepsilon = I_N$.

$m_{xy} = (1/N)(\mathbf{x} - \iota\tilde{x})'\hat{\Sigma}_\varepsilon^{-1}(\mathbf{y} - \iota\tilde{y}) = (1/N)\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}(x_i - \tilde{x})(y_j - \tilde{y})$ is the weighted and centered cross-sectional second sample co-moment about the weighted average between variables $\mathbf{x}$ and $\mathbf{y}$, $\tilde{x} = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}\, x_i / \sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}$ is the cross-sectional weighted average of a variable $\mathbf{x}$, w_{ij} is the (i, j)-th element of $\hat{\Sigma}_\varepsilon^{-1}$, and $\hat{\rho}_{xy} = m_{xy}/(m_{xx}m_{yy})^{1/2}$ is the estimated cross-sectional correlation coefficient between variables $\mathbf{x}$ and $\mathbf{y}$.[6] The subscripts R, $\hat{\beta}$, and V correspond to the variables $\mathbf{R}_t$, $\hat{\boldsymbol{\beta}}_{t-1}$, and $\mathbf{V}_{t-1}$, respectively. The time subscript t is dropped in the expression of the cross-sectional sample co-moments (m_{xy}) for convenience. The above MLEs are N-consistent and asymptotically normal. The derivation of the MLEs and the proof of the consistency are in the Appendix. As mentioned earlier, the final EIV-corrected estimates of γ_0, γ_1, and γ_2 are the time-series averages ($\bar{\hat{\gamma}}_0$, $\bar{\hat{\gamma}}_1$, $\bar{\hat{\gamma}}_2$) of the estimates $\hat{\gamma}_{0t}$, $\hat{\gamma}_{1t}$, and $\hat{\gamma}_{2t}$, respectively.

By comparing the above EIV-corrected MLEs with the traditional least squares estimators that are subject to the EIV problem, the EIV-correction factors for the least squares estimators can be derived. After some algebra, the following relationships between the EIV-corrected MLEs and the traditional least squares (LS) estimates of the CSR coefficients at time t are obtained;

$$\hat{\gamma}_{1t} = C_{1t}\hat{\gamma}_{1t}^{LS} \tag{15a}$$

$$\hat{\gamma}_{2t} = C_{2t}\hat{\gamma}_{2t}^{LS} \tag{16a}$$

$$\hat{\gamma}_{0t} = \hat{\gamma}_{0t}^{LS} - (C_{1t} - 1)\hat{\gamma}_{1t}^{LS}\tilde{\hat{\beta}}_{t-1} - (C_{2t} - 1)\hat{\gamma}_{2t}^{LS}\tilde{V}_{t-1}, \tag{17a}$$

where

$$C_{1t} = 1 \Bigg/ \left[1 - \frac{1}{2}\left(\frac{m_{RR}(1 - \hat{\rho}_{RV}^2)}{\delta_t m_{\hat{\beta}\hat{\beta}}(1 - \hat{\rho}_{\hat{\beta}V}^2)} + 1\right)(1 - (1 - Q)^{1/2})\right] \geq 1$$

$$Q = (1 - \hat{\rho}_{R\hat{\beta}\cdot V}^2) \Bigg/ \left[\frac{1}{2} + \frac{1}{4}\left(\frac{\delta_t m_{\hat{\beta}\hat{\beta}}(1 - \hat{\rho}_{\hat{\beta}V}^2)}{m_{RR}(1 - \hat{\rho}_{RV}^2)} + \frac{m_{RR}(1 - \hat{\rho}_{RV}^2)}{\delta_t\, m_{\hat{\beta}\hat{\beta}}(1 - \hat{\rho}_{\hat{\beta}V}^2)}\right)\right]$$

$$C_{2t} = 1 - \frac{(C_{1t} - 1)\{(\hat{\rho}_{R\hat{\beta}}\,\hat{\rho}_{\hat{\beta}V}/\hat{\rho}_{RV}) - \hat{\rho}_{\hat{\beta}V}^2\}}{1 - (\hat{\rho}_{R\hat{\beta}}\,\hat{\rho}_{\hat{\beta}V}/\hat{\rho}_{RV})},$$

and $\hat{\rho}_{R\hat{\beta}\cdot V}^2 = (\hat{\rho}_{R\hat{\beta}} - \hat{\rho}_{\hat{\beta}V}\hat{\rho}_{RV})^2/\{(1 - \hat{\rho}_{\hat{\beta}V}^2)(1 - \hat{\rho}_{RV}^2)\}$ is the estimated cross-sectional squared partial correlation coefficient between the variables $\mathbf{R}_t$ and $\hat{\boldsymbol{\beta}}_{t-1}$, given $\mathbf{V}_{t-1}$. Here C_{1t} and C_{2t} are the direct EIV-correction factors for the least squares estimators of γ_{1t} and γ_{2t}, respectively. When $\hat{\Sigma}_\varepsilon$ is a diagonal (scalar) matrix, equations (15a), (16a), and (17a) represent a relationship between the

[6] For example, the sample co-moment between $\mathbf{R}_t$ and $\hat{\boldsymbol{\beta}}_{t-1}$ is $m_{R\hat{\beta}} = (1/N)\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}(R_{it} - \tilde{R}_t)(\hat{\beta}_{jt-1} - \tilde{\hat{\beta}}_{t-1})$ for the GLS version, $m_{R\hat{\beta}} = (1/N)\sum_{i=1}^{N} w_i(R_{it} - \tilde{R}_t)(\hat{\beta}_{it-1} - \tilde{\hat{\beta}}_{t-1})$ for the WLS version when w_i is the i-th element of the inverse of the diagonal error variance matrix, and $m_{R\hat{\beta}} = (1/N)\sum_{i=1}^{N}(R_{it} - \bar{R}_t)(\hat{\beta}_{it-1} - \bar{\hat{\beta}}_{t-1})$ for the OLS version.

WLS (OLS) version of the EIV-corrected MLEs and the traditional WLS (OLS) estimates. The above relationship also hold for the GLS version of estimates when $\hat{\Sigma}_\varepsilon$ is a full matrix.

In the widely-used special case when the beta variable alone is in the model, the EIV-corrected MLEs of the CSR coefficients (γ_{0t}, γ_{1t}) in equations (1) and (2) and the corresponding EIV-correction factor can also be obtained from the above equations by setting $\hat{\rho}_{RV} = \hat{\rho}_{\hat{\beta}V} = 0$:

$$\hat{\gamma}_{1t} = C^*_{1t}\hat{\gamma}^{\mathrm{LS}}_{1t} \tag{15b}$$

$$\hat{\gamma}_{0t} = \hat{\gamma}^{\mathrm{LS}}_{0t} - (C^*_{1t} - 1)\hat{\gamma}^{\mathrm{LS}}_{1t}\bar{\hat{\beta}}_{t-1}, \tag{17b}$$

where

$$C^*_{1t} = 1\Bigg/\left[1 - \frac{1}{2}\left(\frac{m_{RR}}{\delta_t m_{\hat{\beta}\hat{\beta}}} + 1\right)(1 - (1 - Q^*)^{1/2})\right] \geq 1$$

$$Q^* = (1 - \hat{\rho}^2_{R\hat{\beta}})\Bigg/\left[\frac{1}{2} + \frac{1}{4}\left(\frac{\delta_t m_{\hat{\beta}\hat{\beta}}}{m_{RR}} + \frac{m_{RR}}{\delta_t m_{\hat{\beta}\hat{\beta}}}\right)\right].$$

The EIV-correction factor C_{1t} for the traditional least squares estimator of γ_{1t} is always greater than or equal to one, regardless of the presence of the firm size variable in the cross-sectional regressions.[7] Hence, the EIV-corrected MLE of γ_{1t}, $\hat{\gamma}_{1t}$, has a greater positive value than the traditional least squares estimator, $\hat{\gamma}^{\mathrm{LS}}_{1t}$, if the least squares estimator is positive, and vice versa. Since many empirical findings show that the traditional least squares estimate of the price of beta risk is positive, the EIV-corrected estimate of the price of beta risk is greater than the traditional positive least squares estimate. Put differently, the MLE of the price of beta risk corrects the downward bias of the traditional least squares estimate.

The EIV-correction factor C_{2t} for the least squares estimator of the coefficient associated with the firm size variable would be between zero and one in most cases. Therefore, the EIV-corrected coefficient estimate of the firm size variable, $\hat{\gamma}_{2t}$, is less negative than its least squares estimate, $\hat{\gamma}^{\mathrm{LS}}_{2t}$, when the least squares estimate is itself negative as in most cases. As a result, the ultimate EIV-corrected estimate of the coefficient associated with the firm size ($\bar{\hat{\gamma}}_2$) would be less negative than its traditional least squares estimate ($\bar{\hat{\gamma}}^{\mathrm{LS}}_2$).

[7] Since the denominator of Q is greater than 1, $0 \leq Q \leq 1$. So, $(1 - Q)^{1/2} = 1 - \tfrac{1}{2}Q - \tfrac{1}{8}Q^2 - \tfrac{1}{16}Q^3 - \tfrac{5}{128}Q^4 - \cdots < 1 - \tfrac{1}{2}Q$. Now, the EIV-correction factor C_{1t} is

$$C_{1t} \geq \frac{1}{1 - \frac{1}{4}Q_1\left(\frac{m_{RR}(1 - \hat{\rho}^2_{RV})}{\delta_t\, m_{\hat{\beta}\hat{\beta}}(1 - \hat{\rho}^2_{\hat{\beta}V})} + 1\right)} = \frac{1}{1 - (1 - \hat{\rho}^2_{R\hat{\beta}\cdot V})/(1 + \delta_t m_{\hat{\beta}\hat{\beta}}(1 - \hat{\rho}^2_{\hat{\beta}V})/m_{RR}(1 - \hat{\beta}^2_{RV}))} \geq 1.$$

C_{1t} equals one when $T \to \infty$ and thus $\delta_t \to \infty$; that is, when the T-consistent market beta estimates are used, there is no EIV-correction needed.

It is clear from equation (17b) that when the beta variable alone is in the model, the correction of the downward bias of the traditional least squares estimator of the price of beta risk induces a correction of the upward bias of the traditional least squares intercept estimator. More specifically, the EIV-corrected intercept estimator, $\hat{\gamma}_{0t}$, is smaller than the traditional least squares intercept estimator, $\hat{\gamma}_{0t}^{\mathrm{LS}}$. Moreover, in the more general case when the firm size variable is included in the cross-sectional regression, equation (17a) shows that the EIV-corrected intercept estimator is also smaller than the traditional least squares intercept estimator. Therefore, the ultimate intercept estimate by the EIV-correction method ($\bar{\hat{\gamma}}_0$) is smaller than that by the traditional least squares method ($\bar{\hat{\gamma}}_0^{\mathrm{LS}}$).

In summary, the EIV correction implies a greater (more positive) estimate of the price of beta risk, a smaller (less negative) estimate of the coefficient associated with firm size, and a smaller intercept estimate.

B. Heteroscedastic Market Model Disturbances

Miller and Scholes (1972), Brown (1977), Belkaoui (1977), Bey and Pinches (1980), and Barone-Adesi and Talwar (1983) report some evidence of heteroscedasticity of residual asset returns. Heteroscedasticity of residual asset returns reduces the efficiency of empirical tests in which homoscedasticity is assumed. It is worthwhile, therefore, to perform the EIV-corrected estimation under heteroscedasticity and compare it with the estimation under homoscedasticity of the disturbance term of the market model.

Under heteroscedasticity of the disturbance term of the market model, the ratio of the idiosyncratic error variance to the measurement error variance of equation (8), δ_{it}, is different across assets. The idiosyncratic error variance terms contained in the ratio δ_{it} can be estimated through a functional form of conditional heteroscedasticity on the market return, which can capture a time-varying behavior of idiosyncratic error variances. A plausible form of the conditional heteroscedasticity in the disturbance term is that the square of the disturbance term covaries with the contemporaneous market return and the square of contemporaneous market return, that is,

$$\hat{\varepsilon}_{is}^2 = \alpha_{0i} + \alpha_{1i} R_{ms} + \alpha_{2i} R_{ms}^2 + \text{error term}, \qquad s = t - T, \cdots, t - 1, \tag{18}$$

where $\hat{\varepsilon}_{is}^2$ is the squared residual return from the market model estimated using T observations prior to each CSR. In fact, this model is equivalent to White's (1980) test for heteroscedasticity in the disturbance term of the market model. By substituting the idiosyncratic error variance terms contained in the ratio δ_{it} of equation (8) with the estimated conditional residual variances from equation (18), the value of the ratio is obtained for each firm. As long as the estimated coefficients of equation (18) are different across assets, the ratios are also all different across assets.

When the ratio δ_{it} is different across assets, the solution of equations (9), (10), and (11) no longer has a closed form and requires the use of an iterative method. Unlike the earlier closed form MLE, it now is hard to analyze the consistency of the iterative MLE. It is worthwhile, therefore, to perform a Monte Carlo simulation to examine the consistency of the iterative MLE and to compare it with that of the closed form MLE.

C. Simulation Evidence

In order to examine the estimation efficiency of the closed form MLE under homoscedasticity of the disturbance term of the market model, the iterative MLE under conditional heteroscedasticity, and the traditional least squares estimate under both homoscedasticity and conditional heteroscedasticity, a Monte Carlo simulation is used. The simulation draws from a finite universe of stock returns, whose behavior is governed by the capital asset pricing model (CAPM). To simulate stock returns, I collect necessary parameters from actual stock return data. That is, betas and residual variances are taken from an estimation of the market model of 2199 firms listed on the Center for Research in Security Prices (CRSP) monthly return file over the sample period from January 1976 through December 1980. The coefficients (α_{0i}, α_{1i}, α_{2i}) of the conditional heteroscedasticity equation (18) are taken from a regression of the squared market model residuals on the contemporaneous market returns and the square of the contemporaneous market returns. The average market value of each firm over the period from January 1976 to December 1980 is also obtained in order to form portfolios later in the simulation.

Based on these collected parameter values, I generate 180 monthly returns for each firm using the following market model:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}, \qquad t = 1, \cdots, 180,$$

where R_{mt} is the CRSP equally-weighted market returns from January 1971 through December 1985. The mean return (E_i) of each firm's simulated monthly returns is determined by the following CAPM equation: $E_i = 0.004 + 0.010768\ \beta_i$. Therefore, α_i is determined by $\alpha_i = E_i - \beta_i \bar{R}_m$. *Under homoscedasticity, the error terms,* ε_{it}, are randomly generated from a normal distribution that has the same variance as the previously collected residual variance. Under conditional heteroscedasticity, the error terms are also randomly generated with a time-varying variance, whose condition is determined by the coefficients of the conditional heteroscedasticity equation (18). These coefficients were taken earlier from the actual stock return data. In this manner, a series of 180 monthly return data are generated for each of the 2199 firms.

Size-β portfolios are then formed; that is, portfolios are first formed based on firm size (the average market value), and each size-sorted portfolio is then subdivided on the basis of each firm's preranking beta on December of each year. The preranking beta is estimated using 60 monthly observations over the previous five years by rolling over every month. The composition of the portfolio is held constant for the following year. The postranking returns are calculated with equal weights, and the postranking betas are estimated by rolling over the previous five-year postranking returns. Therefore, the testing period is from $t = 121$ to $t = 180$ (or equivalently, from January 1981 to December 1985). Notice that the given price of beta risk of $\gamma_1 = 0.010768$ is equal to the average market return from January 1981 to December 1985 minus the given riskless rate of $\gamma_0 = 0.004$. The number of portfolios formed are $N = 9, 16, 25, 49, 64, 100, 196, 289, 400$, and 2199 (all individual firms).

The true value of γ_1 is estimated by the time-series average ($\bar{\hat{\gamma}}_1$) of the sixty estimated values of γ_{1t} over the testing period for each replication. The total number of replications is 200. Table I presents the biases and the square-root of the mean squared errors (MSEs) of the (WLS version) EIV-corrected MLE and the traditional WLS estimate of γ_1. When the true disturbance term of the market model is homoscedastic, the bias of the MLE decreases and approaches zero as the number of portfolios, N, increases, while that of the WLS estimate is magnified. The MSEs of the two estimates behave in a similar fashion. When the true disturbance term of the market model is governed by the conditional heteroscedasticity of equation (18), two different MLEs are computed: the first MLE is derived by assuming there is conditional heteroscedasticity, while the second MLE is derived by assuming there is homoscedasticity. The bias of the first MLE (estimated under the right assumption of the conditional heteroscedasticity) also decreases and approaches zero as N increases. However, the first MLE converges more slowly to the true value than does the MLE derived under the circumstance that the true disturbance term is homoscedastic. The bias of the second MLE (estimated under the incorrect assumption of homoscedasticity) decreases with N, but increases at the large values of N. As expected, the bias and MSE of the second MLE are greater than those of the first MLE. The bias and MSE of the WLS estimate in the case of conditional heteroscedasticity increase with N and are worse than in the case of homoscedasticity.

In sum, when the true disturbance term of the market model is homoscedastic, as the number of portfolios, N, increases, the (closed form) MLE of γ_1 converges to the true value. When the true disturbance term of the market model is conditionally heteroscedastic, the (iterative) MLE of γ_1 also converges to the true value as long as it is estimated under the assumption of conditional heteroscedasticity. However, the convergence of the (closed form) MLE estimated under the assumption of homoscedasticity is not guaranteed when the true disturbance term of the market model is conditionally heteroscedastic.

III. Results

A. Data

All NYSE and AMEX stocks listed on the CRSP monthly return file for at least two years during the period 1926–1991 are used for the CSR estimation with portfolio groupings. Among them, all stocks listed for at least five years are used for the CSR estimation without portfolio groupings (i.e., with individual stocks). All stocks listed on the monthly Nasdaq return file for at least five years during the period 1973–1991 are added to the NYSE and AMEX

Table I

Simulation Evidence: Performance of the EIV-Corrected and Traditional Estimates of the Risk Premium

The data are simulated by the following capital asset pricing model: $E_i = 0.004 + 0.010768\ \beta_i$, where E_i and β_i are the expected return on an asset i and the beta of the asset, respectively. The price of beta risk (γ_1) is estimated by the time-series average of the month-by-month cross-sectional regression (CSR) coefficient estimates γ_{1t} for each replication. The total number of replications is 200.

When the true disturbance term of the market model is conditionally homoscedastic, the CSR coefficient γ_{1t} is estimated by the errors in variables (EIV)-corrected method (the weighted least squares (WLS) version of the maximum likelihood estimation (MLE)) and the uncorrected method (the traditional WLS) under the assumption of conditional homoscedasticity. When the true disturbance term of the market model is conditionally heteroscedastic, two different EIV-corrected MLEs are computed: the first (iterative MLE) is computed under the assumption of conditional heteroscedasticity, while the second (closed-form MLE) is computed under the assumption of conditional homoscedasticity. The conditional heteroscedasticity is modeled by $\varepsilon_{is}^2 = \alpha_{0i} + \alpha_{1i}R_{ms} + \alpha_{2i}R_{ms}^2$, where ε_{is} and R_{ms} are the disturbance term of the market model for an asset i at time s and the market return at time s, respectively.

	Simulated Under Conditional Homoscedasticity		Simulated Under Conditional Heteroscedasticity		
Portfolio Size (N)	EIV-Corrected	Uncorrected	EIV-Corrected with Heteroscedasticity	EIV-Corrected with Homoscedasticity	Uncorrected
			Panel A: Bias ($\times 10^2$)		
9	−0.0229	−0.0245	−0.0280	−0.0391	−0.0416
16	−0.0220	−0.0249	−0.0273	−0.0377	−0.0421
25	−0.0213	−0.0256	−0.0237	−0.0320	−0.0389
49	−0.0210	−0.0294	−0.0219	−0.0307	−0.0440
64	−0.0178	−0.0286	−0.0202	−0.0284	−0.0458
100	−0.0174	−0.0344	−0.0175	−0.0209	−0.0481
196	−0.0129	−0.0457	−0.0139	−0.0074	−0.0597
289	−0.0066	−0.0543	−0.0106	−0.0029	−0.0780
400	−0.0047	−0.0711	−0.0074	0.0036	−0.1007
2199	−0.0019	−0.1668	−0.0033	0.0104	−0.2075
			Panel B: Square-Root of the Mean Squared Error ($\times 10^2$)		
9	0.0249	0.0264	0.0288	0.0395	0.0420
16	0.0237	0.0264	0.0281	0.0381	0.0425
25	0.0228	0.0270	0.0245	0.0325	0.0393
49	0.0226	0.0305	0.0228	0.0311	0.0443
64	0.0197	0.0298	0.0213	0.0289	0.0461
100	0.0193	0.0353	0.0186	0.0215	0.0484
196	0.0153	0.0464	0.0154	0.0093	0.0600
289	0.0108	0.0549	0.0124	0.0061	0.0781
400	0.0082	0.0721	0.0108	0.0067	0.1008
2199	0.0048	0.1669	0.0069	0.0109	0.2076

stocks for the CSR estimation for individual stocks. The restriction of at least five-year survival for the CSR estimation for individual stocks is imposed in order to maintain the same value of the ratio δ_t for all stocks in the EIV-corrected maximum likelihood estimation. Each firm's monthly returns and market value data (stock price per share times number of shares outstanding) are obtained from the CRSP monthly return tape. The one-month Treasury bill returns are taken from Ibbotson (1992) as the riskless returns.

For the CSR estimation for portfolios, size-β portfolios are formed using the same grouping method as Fama and French (1992). That is, portfolios are first formed based on each firm's market value in June of year y. Each size-sorted portfolio is then subdivided on the basis of each firm's pre-ranking beta in June of year y. Each firm's preranking beta for month t is estimated by using at least 24 monthly return observations over the previous five-years up to month $t - 1$. All individual stocks are assigned to one of the size-β portfolios in order of increasing firm size and then preranking beta magnitude. The composition of the portfolio is held constant for the following year from July of year y to June of year $y + 1$, and the equal-weighted returns on the portfolios are calculated for this period. The portfolio's market value is also calculated with equal weights. In the end, I have postranking returns for July 1931 to December 1991 on the portfolios formed on size and preranking betas. Each portfolio's postranking beta for month t is then estimated by using the previous five-year postranking returns up to month $t - 1$. Since the first postranking beta is estimated using postranking returns from July 1931 through June 1936, the whole testing period (i.e., the estimation period of gammas) is from July 1936 to December 1991. The postranking beta estimation method differs from the Fama and French method. They estimate postranking betas by using the full-period sample of postranking returns on each of 100 size-β portfolios formed with nonfinancial firms. Therefore, five more years' postranking returns prior to the first testing month are needed in my case than in the Fama and French case. The numbers of portfolios to be formed are $N = 9$ (3×3), 16 (4×4), 25 (5×5), 49 (7×7), 64 (8×8), 100 (10×10), 196 (14×14), 289 (17×17), and 400 (20×20).

For the CSR estimation for individual stocks (all NYSE/AMEX and all NYSE/AMEX/OTC), each firm's beta for month t is also estimated by using the previous five-year return observations up to month $t - 1$. In order to be consistent with the case of the CSR estimation for portfolios, returns from July 1931 through June 1936 are used to estimate the first beta of each firm. In both cases of the CSR estimation for portfolios and for individual stocks, the whole testing period is from July of 1936 through December of 1991.

B. Cross-Sectional Dependence Between Residuals

In estimating the risk prices, concerns of changing risk premia parameters and beta nonstationarity frequently limit the available time-series length T. In these cases, the GLS version of our EIV-corrected estimator may be infeasible because the inverse of the estimated error covariance matrix does not exist for large N, unless the error covariance matrix is diagonal. It is necessary, therefore, to examine whether the error covariance matrix is diagonal, that is, if the cross-sectional dependence between residual asset returns is sufficiently weak that—especially when N is large—the WLS version estimator can be used.

In order to test the cross-sectional dependence between residuals, the market model is estimated for each portfolio in overlapping five-year periods by rolling over every month, and then all possible pairwise correlation t-tests are performed with the five-year residuals. The testing results are presented in Table II. As the number of portfolios increases, the rejection rate of the null hypothesis that there is no correlation between the residuals decreases monotonically, and the magnitude of the average absolute correlation coefficient between the residuals also decreases.[8] In particular, the rejection rates of the null hypothesis for the case of all individual stocks are quite low. They are only about 0.076 for the NYSE/AMEX stocks and 0.065 for the over the counter (OTC) stocks at the five percent significance level, when the CRSP equally-weighted market returns are used.[9] When the value-weighted market returns are used, the results are similar. The low rejection rates of the null hypothesis, which could easily be due to Type I error, are quite supportive of the assumption of weak cross-sectional dependence between individual stock residuals. The small average magnitude of the absolute correlation coefficients is also evidence supporting the assumption of weak cross-sectional dependence in the case when the number of assets is large. When the equally-weighted market returns are used, the average absolute correlation coefficients for the NYSE/AMEX and for the OTC individual stocks are only 0.083 and 0.065 respectively. Moreover, note that when the time-series length is 60, the marginal magnitude of the correlation coefficient, which is required to reject the null hypothesis at the five percent significance level, is approximately 0.254.[10] Based on these results, the WLS version of the estimation is quite acceptable, especially when all individual assets are used. Therefore, the risk premia will be estimated using the WLS version of the maximum likelihood estimation and the traditional WLS estimation.

[8] These empirical results are consistent with the following theoretical arguments. When the number of assets in the portfolios is n, the correlation coefficient between two equally-weighted portfolios' returns, ρ_p, is given by

$$\rho_p = \mathrm{Corr}\left(\frac{1}{n}\sum_{i=1}^{n} R_{it}, \frac{1}{n}\sum_{j=1}^{n} R_{jt}\right) \simeq \frac{\bar{\rho}}{\bar{\rho} + (1-\bar{\rho})/n},$$

where $\bar{\rho}$ is the average correlation coefficient between two individual assets' returns. As n increases, ρ_p goes to one. Conversely, as n decreases, that is, as the number of portfolios (N) increases, the correlation coefficient between portfolio returns decreases. These arguments also hold for portfolios' residual returns.

[9] Among all NYSE/AMEX stocks, 1200 stocks are randomly chosen to perform the correlation t-test. Another 1200 stocks are also randomly chosen among all OTC stocks.

[10] The marginal correlation coefficient of 0.254 is derived as follows: Based on the fact that $\hat{\rho}/\sqrt{(1-\hat{\rho}^2)/(T-2)}$ is approximately t-statistic with $(T-2)$ degrees of freedom, $\hat{\rho}$ of 0.254 is obtained when T is 60 and the critical value at the five percent significance level is $t_{0.025}$ (58) = 2.0017.

Table II

Cross-Sectional Dependence Between Residuals from the Market Model

The average absolute correlation coefficient is the average of all pairwise absolute correlation coefficients estimated using overlapping five-year monthly residuals from the market model from July 1936 through December 1991. The rejection rate is the percentage of rejecting the null that there is no correlation between residuals, when the correlation t-test is performed for all computed correlation coefficients. For New York Stock Exchange/American Stock Exchange (NYSE/AMEX), 1200 randomly chosen individual stocks are used to estimate all possible pairwise correlation coefficients between residuals. For over-the-counter (OTC), another 1200 stocks are randomly chosen among all OTC stocks.

Portfolio Size (N)	Average Absolute Correlation Coefficient (Standard Error)	Rejection Rate at 5%	Average Absolute Correlation Coefficient (Standard Error)	Rejection Rate at 5%
	Residuals from the Market Model with Equal-Weighted Returns		Residuals from the Market Model with Value-Weighted Returns	
9	0.399 (0.005)	0.656	0.485 (0.006)	0.725
16	0.341 (0.004)	0.578	0.450 (0.006)	0.684
25	0.308 (0.004)	0.545	0.421 (0.006)	0.663
49	0.244 (0.003)	0.415	0.359 (0.005)	0.595
64	0.222 (0.003)	0.374	0.334 (0.005)	0.567
100	0.189 (0.003)	0.298	0.288 (0.005)	0.496
196	0.152 (0.002)	0.202	0.221 (0.004)	0.363
289	0.136 (0.002)	0.146	0.188 (0.004)	0.268
400	0.108 (0.002)	0.095	0.151 (0.003)	0.194
NYSE/AMEX	0.083 (0.001)	0.076	0.123 (0.001)	0.108
OTC only	0.075 (0.001)	0.065	0.110 (0.001)	0.084

C. Beta Risk Price Estimates After the EIV Correction

Table III presents the time-series averages (t-statistics in parentheses) of the CSR coefficients estimated by the (WLS version) EIV-corrected MLE method and by the traditional WLS method under the assumption of homoscedasticity of the disturbance term of the market model, when the CRSP equally-weighted market return is used as a proxy for the return on the market portfolio. Panel A contains the results by all firms for the whole period from July 1936 through December 1991 (666 months), and Panel B contains the results by nonfinancial firms for the subperiod from July 1963 through December 1990 (330 months) that is the same period used by Fama and French (1992).

Table III shows that as the number of portfolios (N) increases, the relation between average stock returns and market beta becomes stronger and the intercept estimate becomes weaker both for the whole period and for the subperiod after the EIV correction. For the whole period, when the number of portfolios exceeds 49 and the beta variable alone is in the model, all t-statistics of the EIV-corrected estimates of the price of beta risk (γ_1) exceed two. Especially, when all NYSE and AMEX individual stocks are used, the estimate of the price of beta risk is 0.766 percent per month, with t-statistic of 2.59. When all the post-January 1973 OTC individual stocks are added to the NYSE and AMEX stocks, the estimate of the price of beta risk is increased to 0.809 percent per month, with t-statistic of 2.63. For $N \geq 400$, the intercept (γ_0) is insignificantly estimated when the EIV bias is corrected, a desirable feature for the CAPM. When the firm size variable is included in the model and all

Table III

Risk Price Estimates (t-Statistics) Using the CRSP Equally-Weighted Market Return and Assuming Conditional Homoscedasticity.

The risk price estimates ($\bar{\bar{\gamma}}_0$, $\bar{\bar{\gamma}}_1$, $\bar{\bar{\gamma}}_2$) are the time-series averages of the month-by-month cross-sectional regression (CSR) coefficient estimates. The monthly CSR coefficients are estimated through the errors in variables (EIV)-corrected (WLS) version of the maximum likelihood estimation (MLE) and the traditional uncorrected WLS estimation by assuming conditional homoscedasticity of the disturbance term of the market model. For the whole period from July 1936 to December 1991, both financial and nonfinancial firms are used, while for the subperiod from July 1963 to December 1990, only nonfinancial firms are used.

The portfolios are size-β portfolios; they are first formed based on each firm's market value in June of year y, and each size-sorted portfolio is then subdivided based on each firm's pre-ranking β in June of year y for July of year y through June of year $y + 1$. All firms traded for at least two years on the NYSE and AMEX are used to form the portfolios.

NYSE/AMEX (NYSE/AMEX/OTC) indicates individual stocks traded for at least five years on these exchanges. For the whole period (for the subperiod), on average, there are 1208 and 1579 stocks (1407 and 2083 nonfinancial stocks) in the monthly regressions for NYSE/AMEX and NYSE/AMEX/OTC, respectively.

Portfolio Size (N)	$R_{it} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{it-1} + \gamma_{2t}\log(V_{it-1}) + \omega_{it}$					
	EIV-Corrected WLS (MLE)			Uncorrected WLS		
	$\bar{\bar{\gamma}}_0$	$\bar{\bar{\gamma}}_1$	$\bar{\bar{\gamma}}_2$	$\bar{\bar{\gamma}}_0$	$\bar{\bar{\gamma}}_1$	$\bar{\bar{\gamma}}_2$
	Panel A: July 1936 through December 1991					
9	0.452 (3.29)	0.530 (1.92)		0.503 (3.82)	0.476 (1.78)	
16	0.471 (3.48)	0.506 (1.86)		0.523 (4.03)	0.450 (1.71)	
25	0.462 (3.44)	0.515 (1.89)		0.522 (4.08)	0.449 (1.72)	
49	0.418 (3.06)	0.552 (2.01)		0.492 (3.82)	0.471 (1.82)	
64	0.407 (3.00)	0.564 (2.07)		0.487 (3.83)	0.475 (1.85)	
100	0.394 (2.82)	0.573 (2.08)		0.498 (3.89)	0.456 (1.80)	
196	0.360 (2.47)	0.599 (2.14)		0.502 (3.95)	0.438 (1.78)	
289	0.299 (1.99)	0.671 (2.33)		0.512 (4.11)	0.425 (1.79)	
400	0.231 (1.43)	0.752 (2.52)		0.525 (4.26)	0.413 (1.80)	
NYSE/AMEX	0.230 (1.42)	0.766 (2.59)		0.564 (5.06)	0.375 (1.76)	
NYSE/AMEX/OTC	0.207 (1.40)	0.809 (2.63)		0.578 (5.22)	0.368 (1.76)	
9	1.097 (4.98)	0.190 (0.65)	−0.086 (−2.77)	1.134 (5.25)	0.160 (0.57)	−0.089 (−2.87)
16	1.017 (4.92)	0.212 (0.75)	−0.074 (−2.48)	1.062 (5.26)	0.176 (0.64)	−0.078 (−2.62)
25	1.076 (5.27)	0.188 (0.67)	−0.084 (−2.88)	1.133 (5.71)	0.142 (0.52)	−0.089 (−3.05)
49	1.001 (4.89)	0.240 (0.85)	−0.079 (−2.72)	1.089 (5.54)	0.169 (0.64)	−0.086 (−2.99)
64	1.023 (5.06)	0.228 (0.81)	−0.083 (−2.85)	1.118 (5.77)	0.151 (0.57)	−0.091 (−3.14)
100	0.988 (4.37)	0.252 (0.91)	−0.080 (−2.76)	1.115 (5.83)	0.147 (0.58)	−0.089 (−3.10)
196	0.901 (4.37)	0.300 (1.06)	−0.070 (−2.47)	1.104 (5.90)	0.133 (0.55)	−0.087 (−3.07)
289	0.833 (3.90)	0.337 (1.29)	−0.068 (−2.39)	1.167 (6.34)	0.100 (0.44)	−0.095 (−3.38)
400	0.683 (2.86)	0.515 (1.64)	−0.059 (−2.28)	1.188 (6.38)	0.094 (0.44)	−0.100 (−3.52)
NYSE/AMEX	0.609 (2.58)	0.644 (1.93)	−0.057 (−2.16)	1.132 (6.70)	0.089 (0.90)	−0.105 (−3.95)
NYSE/AMEX/OTC	0.566 (2.52)	0.704 (2.14)	−0.054 (−2.03)	1.090 (6.92)	0.091 (1.09)	−0.103 (−3.92)

Table III—*Continued*

Portfolio Size (N)	$R_{it} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{it-1} + \gamma_{2t}\log(V_{it-1}) + \omega_{it}$					
	EIV-Corrected WLS (MLE)			Uncorrected WLS		
	$\bar{\hat{\gamma}}_0$	$\bar{\hat{\gamma}}_1$	$\bar{\hat{\gamma}}_2$	$\bar{\hat{\gamma}}_0$	$\bar{\hat{\gamma}}_1$	$\bar{\hat{\gamma}}_2$
	Panel B: July 1963 through December 1990					
9	0.425 (1.98)	0.353 (0.88)		0.479 (2.30)	0.297 (0.76)	
16	0.413 (1.95)	0.363 (0.92)		0.467 (2.29)	0.307 (0.80)	
25	0.422 (1.98)	0.352 (0.88)		0.477 (2.33)	0.293 (0.77)	
49	0.376 (1.71)	0.393 (0.98)		0.448 (2.14)	0.315 (0.83)	
64	0.335 (1.53)	0.436 (1.08)		0.414 (2.00)	0.349 (0.93)	
100	0.359 (1.61)	0.406 (1.00)		0.444 (2.15)	0.311 (0.84)	
196	(0.242 (1.07)	0.522 (1.27)		0.379 (1.87)	0.366 (1.01)	
289	0.190 (0.82)	0.574 (1.37)		0.386 (1.93)	0.350 (0.99)	
400	0.141 (0.60)	0.636 (1.50)		0.388 (1.96)	0.350 (1.03)	
NYSE/AMEX	0.111 (0.58)	0.699 (1.73)		0.410 (2.37)	0.313 (1.11)	
NYSE/AMEX/OTC	0.072 (0.30)	0.741 (1.95)		0.408 (2.34)	0.316 (1.34)	
9	1.354 (3.72)	−0.147 (−0.36)	−0.106 (−2.27)	1.428 (4.03)	−0.174 (−0.044)	−0.117 (−2.55)
16	1.343 (3.56)	−0.149 (−0.32)	−0.102 (−2.10)	1.395 (4.09)	−0.169 (−0.44)	−0.116 (−2.54)
25	1.396 (3.98)	−0.169 (−0.48)	−0.114 (−2.48)	1.448 (4.36)	−0.207 (−0.55)	−0.120 (−2.66)
49	1.379 (3.95)	−0.161 (−0.40)	−0.113 (−2.48)	1.448 (4.36)	−0.207 (−0.55)	−0.120 (−2.66)
64	1.309 (3.76)	−0.119 (−0.30)	−0.108 (−2.35)	1.406 (4.27)	−0.173 (−0.47)	−0.120 (−2.64)
100	1.345 (3.96)	−0.143 (−0.36)	−0.111 (−2.45)	1.443 (4.52)	−0.203 (−0.56)	−0.122 (−2.72)
196	1.241 (3.58)	−0.031 (−0.08)	−0.112 (−2.44)	1.397 (4.39)	−0.156 (−0.45)	−0.124 (−2.74)
289	1.102 (3.14)	0.051 (0.12)	−0.101 (−2.22)	1.390 (4.47)	−0.162 (−0.49)	−0.123 (−2.77)
400	1.121 (3.21)	0.080 (0.19)	−0.107 (−2.31)	1.451 (4.69)	−0.182 (−0.59)	−0.132 (−2.95)
NYSE/AMEX	0.688 (2.18)	0.484 (1.18)	−0.081 (−2.00)	1.197 (4.19)	0.061 (0.23)	−0.119 (−2.96)
NYSE/AMEX/OTC	0.617 (2.07)	0.580 (1.50)	−0.078 (−1.81)	1.072 (4.10)	0.070 (0.43)	−0.107 (−2.71)

NYSE, AMEX, and OTC individual stocks are used, the EIV-corrected estimate of the price of beta risk continues to be statistically significant: It equals 0.704 percent per month and has a t-statistic of 2.14. On the other hand, when the traditional WLS estimation is performed, the estimate of the price of beta risk is statistically insignificant and its magnitude decreases as N increases. At the same time, as N increases, the intercept estimates grow larger and more significant, which results from the underestimation of γ_1. Put differently, as N increases, the estimation results from the traditional least squares method become more adverse to the CAPM due to the more severe EIV problem.

Table III also presents the time-series averages of the estimated coefficients associated with the firm size variable under the assumption of homoscedasticity of the disturbance term of the market model. The natural logarithm of the portfolio's average market value is used for the firm size variable.[11] As the number of portfolios increases, the size of the traditional negative WLS coefficient estimate of the firm size variable increases slightly, and its statistical significance becomes stronger. After correcting for the EIV problem, however, as the number of portfolios increases, the coefficient estimate associated with

[11] The other measures of the firm size variable such as the square root of the average market value, the reciprocal of the average market value, and the reciprocal of the square root of the average market value itself were also tried. The results were similar.

Table IV

Risk Price Estimates (*t*-Statistics) Using the CRSP Value-Weighted Market Return and Assuming Conditional Homoscedasticity

The risk price estimates ($\bar{\gamma}_0$, $\bar{\gamma}_1$, $\bar{\gamma}_2$) are the time-series averages of the month-by-month cross-sectional regression (CSR) coefficient estimates. The monthly CSR coefficients are estimated through the errors in variables (EIV)-corrected WLS version of the maximum likelihood estimation (MLE) and the traditional uncorrected WLS estimation by assuming conditional homoscedasticity of the disturbance term of the market model. For the whole period from July 1936 to December 1991, both financial and nonfinancial firms are used, while for the subperiod from July 1963 to December 1990, only nonfinancial firms are used.

The portfolios are size-β portfolios; they are first formed based on each firm's market value in June of year y, and each size-sorted portfolio is then subdivided based on each firm's pre-ranking β in June of year y for July of year $y + 1$. All firms traded for at least two years on the NYSE and AMEX are used to form the portfolios.

NYSE/AMEX (NYSE/AMEX/OTC) indicates individual stocks traded for at least five years on these exchanges. For the whole period (for the subperiod), on average, there are 1208 and 1579 stocks (1407 and 2083 nonfinancial stocks) in the monthly regressions for NYSE/AMEX and NYSE/AMEX/OTC, respectively.

Portfolio Size (*N*)	$R_{it} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{it-1} + \gamma_{2t}\log(V_{it-1}) + \omega_{it}$					
	EIV-Corrected WLS (MLE)			Uncorrected WLS		
	$\bar{\gamma}_0$	$\bar{\gamma}_1$	$\bar{\gamma}_2$	$\bar{\gamma}_0$	$\bar{\gamma}_1$	$\bar{\gamma}_2$
Panel A: July 1936 through December 1991						
9	0.306 (1.82)	0.459 (1.80)		0.407 (2.65)	0.362 (1.53)	
16	0.301 (1.80)	0.454 (1.80)		0.414 (2.75)	0.346 (1.50)	
25	0.260 (1.53)	0.494 (1.94)		0.394 (2.65)	0.366 (1.60)	
49	0.195 (1.14)	0.546 (2.13)		0.365 (2.52)	0.386 (1.73)	
64	0.210 (1.24)	0.538 (2.11)		0.385 (2.73)	0.373 (1.70)	
100	0.186 (1.06)	0.563 (2.17)		0.403 (2.89)	0.360 (1.66)	
196	0.145 (0.82)	0.604 (2.29)		0.443 (3.40)	0.328 (1.61)	
289	0.132 (0.68)	0.626 (2.25)		0.526 (4.17)	0.263 (1.35)	
400	0.123 (0.58)	0.641 (2.36)		0.540 (4.58)	0.249 (1.37)	
NYSE/AMEX	0.120 (0.56)	0.650 (2.54)		0.576 (5.29)	0.242 (1.49)	
NYSE/AMEX/OTC	0.104 (0.47)	0.684 (2.75)		0.572 (5.51)	0.254 (1.61)	
9	1.097 (4.53)	0.071 (0.30)	−0.073 (−2.53)	1.116 (4.67)	0.058 (0.25)	−0.074 (−2.57)
16	1.073 (4.94)	0.072 (0.31)	−0.072 (−2.68)	1.103 (5.16)	0.051 (0.23)	−0.074 (−2.73)
25	1.040 (4.91)	0.105 (0.46)	−0.074 (−2.79)	1.080 (5.19)	0.076 (0.34)	−0.076 (−2.86)
49	1.023 (4.88)	0.128 (0.56)	−0.079 (−2.96)	1.088 (5.32)	0.081 (0.38)	−0.082 (−3.14)
64	1.026 (4.96)	0.120 (0.53)	−0.078 (−2.92)	1.103 (5.49)	0.065 (0.31)	−0.082 (−3.06)
100	1.024 (4.93)	0.140 (0.61)	−0.083 (−3.02)	1.133 (5.68)	0.060 (0.29)	−0.088 (−3.21)
196	0.916 (4.40)	0.215 (0.93)	−0.078 (−2.79)	1.103 (5.61)	0.077 (0.40)	−0.087 (−3.09)
289	0.867 (3.83)	0.257 (1.05)	−0.074 (−2.63)	1.182 (5.95)	0.045 (0.29)	−0.092 (−3.21)
400	0.720 (3.11)	0.387 (1.51)	−0.074 (−2.58)	1.178 (5.90)	0.038 (0.22)	−0.095 (−3.20)
NYSE/AMEX	0.575 (2.49)	0.564 (2.10)	−0.073 (−2.75)	1.121 (6.28)	0.036 (0.39)	−0.103 (−3.74)
NYSE/AMEX/OTC	0.528 (2.40)	0.634 (2.39)	−0.074 (−2.86)	1.079 (6.42)	0.038 (0.51)	−0.100 (−3.71)

the firm size variable decreases in absolute magnitude and its statistical significance becomes weaker both for the whole period and for the subperiod. The *t*-statistic of the EIV-corrected coefficient estimate associated with the firm size variable is much smaller than that of the traditional WLS estimate.

Table IV presents results analogous to Table III in the case when the CRSP value-weighted market return is used. The results are quite similar, except

Table IV—*Continued*

Portfolio Size (N)	$R_{it} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{it-1} + \gamma_{2t}\log(V_{it-1}) + \omega_{it}$					
	EIV-Corrected WLS (MLE)			Uncorrected WLS		
	$\bar{\hat{\gamma}}_0$	$\bar{\hat{\gamma}}_1$	$\bar{\hat{\gamma}}_2$	$\bar{\hat{\gamma}}_0$	$\bar{\hat{\gamma}}_1$	$\bar{\hat{\gamma}}_2$
	Panel B: July 1963 through December 1990					
9	−0.039 (−0.13)	0.573 (1.34)		0.109 (0.40)	0.431 (1.12)	
16	−0.023 (−0.08)	0.554 (1.33)		0.144 (0.56)	0.395 (1.08)	
25	−0.044 (−0.15)	0.567 (1.36)		0.127 (0.50)	0.404 (1.12)	
49	−0.071 (−0.24)	0.589 (1.40)		0.145 (0.59)	0.385 (1.09)	
64	−0.080 (−0.27)	0.600 (1.43)		0.167 (0.70)	0.366 (1.06)	
100	−0.020 (−0.07)	0.550 (1.30)		0.248 (1.08)	0.300 (0.90)	
196	−0.108 (−0.36)	0.641 (1.49)		0.277 (1.30)	0.282 (0.90)	
289	−0.157 (−0.50)	0.697 (1.57)		0.337 (1.65)	0.238 (0.79)	
400	−0.178 (−0.64)	0.722 (1.67)		0.391 (2.02)	0.198 (0.70)	
NYSE/AMEX	−0.142 (−0.50)	0.732 (1.73)		0.415 (2.46)	0.183 (0.64)	
NYSE/AMEX/OTC	−0.163 (−0.59)	0.762 (1.98)		0.422 (2.79)	0.178 (0.69)	
9	1.198 (3.00)	−0.076 (−0.21)	−0.094 (−2.13)	1.226 (3.18)	−0.085 (−0.24)	−0.097 (−2.20)
16	1.265 (3.48)	−0.073 (−0.19)	−0.108 (−2.32)	1.251 (3.52)	−0.137 (−0.40)	−0.095 (−2.23)
25	1.307 (3.48)	−0.084 (−0.23)	−0.166 (−2.34)	1.212 (3.53)	−0.122 (−0.37)	−0.092 (−2.22)
49	1.274 (3.31)	−0.060 (−0.16)	−0.109 (−2.35)	1.246 (3.74)	−0.125 (−0.38)	−0.100 (−2.36)
64	1.234 (3.21)	−0.0546 (−0.15)	−0.114 (−2.32)	1.259 (3.85)	−0.126 (−0.40)	−0.104 (−2.44)
100	1.256 (3.26)	−0.052 (−0.15)	−0.116 (−2.26)	1.274 (3.93)	−0.132 (−0.43)	−0.105 (−2.45)
196	1.073 (3.11)	0.040 (0.11)	−0.105 (−2.24)	1.252 (3.91)	−0.094 (−0.33)	−0.111 (−2.48)
289	0.985 (2.67)	0.135 (0.37)	−0.106 (−2.20)	1.275 (4.03)	−0.110 (−0.40)	−0.112 (−2.49)
400	0.793 (2.26)	0.185 (0.50)	−0.098 (−2.08)	1.316 (4.12)	−0.117 (−0.46)	−0.118 (−2.57)
NYSE/AMEX	0.611 (1.88)	0.496 (1.32)	−0.100 (−2.13)	1.204 (3.88)	0.178 (0.08)	−0.121 (−2.80)
NYSE/AMEX/OTC	0.463 (1.70)	0.586 (1.65)	−0.089 (−2.15)	1.128 (3.77)	0.241 (0.19)	−0.114 (−2.63)

that the impact of EIV-correction is more profound for the WLS estimate of the beta risk price and less profound for the WLS estimate of the coefficient associated with the size variable.[12] In particular, the estimation results in Panel B for the subperiod from July 1963 through December 1990 is comparable to those presented in Table III of Fama and French (1992), since they use the same portfolio grouping method (100 size-β portfolios) by using nonfinancial firms and also use the CRSP value-weighted market return. However, remember that their beta estimation method is different in that they estimate

[12] These empirical findings can be explained by the following arguments. The EIV correction factors are a function of the ratio $\delta_t = T\hat{\sigma}_{m,t-1}$. It can be easily proven that as the ratio increases, the EIV correction factors C_{1t} and C_{1t}^* for $\hat{\gamma}_{1t}^{LS}$ of equations (15a) and (15b) monotonically decrease as long as δ_t is greater than $m_{RR}/m_{\hat{\beta}\hat{\beta}}$, assuming the other terms contained in the EIV correction factors remain fixed. However, it can be empirically confirmed that the ratio δ_t is always greater than $m_{RR}/m_{\hat{\beta}\hat{\beta}}$. Consequently, as the ratio increases, the EIV correction factor C_{2t} for $\hat{\gamma}_{2t}^{LS}$ of equation (16a) increases. Since the variance of the equally-weighted market return (0.003815 over the whole period) is greater than that of the value-weighted market return (0.002114 over the whole period), the ratio is greater when the equally-weighted market return is used than when the value-weighted market return is used. Put more specifically, when the value-weighted market return is used, the EIV correction for $\hat{\gamma}_{1t}^{LS}$ is more profound while the EIV correction for $\hat{\gamma}_{2t}^{LS}$ is less profound than when the equally-weighted market return is used.

post-ranking betas by using the full-period sample of post-ranking returns on each of the size-β portfolios and allocate the post-ranking beta of a size-β portfolio to each stock in the portfolio. These allocated betas are then used in the CSR estimation for individual stocks. Even given this different method, Fama and French's results are broadly similar to those reported in Panel B. Both results, which are contaminated by the EIV problem, show that beta has little power to explain average stock returns. After correcting for the EIV problem, however, beta shows more explanatory power for average stock returns; that is, the estimate of the price of beta risk for this subperiod in the CSR with beta alone is 0.762 percent per month, with t-statistic of 1.98. However, when the size variable is included in the model, the EIV-corrected estimate of the price of beta risk is not as significant as that obtained for the whole period in Panel A, partly because the time-series sample size of the CSR coefficient estimates is smaller. It is 0.634 percent per month, with t-statistic of 2.39, for the whole period, while it is 0.586 percent per month, with t-statistic of 1.65, for the subperiod.

Table V presents the EIV-corrected estimation results for the whole period by the iterative MLE when the conditional heteroscedasticity of the disturbance term of the market model of equation (18) is assumed.[13] The results are quite similar to those obtained with the assumption of homoscedasticity.[14] The similarity may be explained by the fact that when all NYSE, AMEX, and OTC stocks are examined, the estimated intercept term ($\hat{\alpha}_{0i}$) in the proposed conditional heteroscedasticity equation (18) accounts on average for ninety percent of the total residual variance ($\hat{\varepsilon}_i^2$), while the time-varying part ($\hat{\alpha}_{1i}R_{ms} + \hat{\alpha}_{2i}R_{ms}^2$) accounts for only ten percent. In other words, although there is a heteroscedastic component in the disturbance term of the market model, the homoscedastic component (intercept) overwhelmingly dominates the heteroscedastic component. Moreover, with five years of return data on individual stocks, the rejection rate of homoscedasticity by the White's (1980) test is only 6.4 percent at the five percent significance level. It could be asserted, therefore, that the heteroscedasticity of the market model disturbance term is not a serious concern in the EIV-correction estimation of risk premia when individual stocks are used. This view is also supported by the simulation results of the previous section.

In summary, after correcting for the EIV problem, market beta has an economically and statistically significant explanatory power for average stock returns for the whole period from July 1936 to December 1991, regardless of whether or not the firm size variable is included in the cross-sectional regressions and regardless of whether or not one adjusts for heteroscedasticity of the disturbance term of the market model. It is worth-

[13] By using the traditional least squares estimates or the closed form MLEs as initial values, Newton's iterative method converges quickly.

[14] Different forms of conditional heteroscedasticity have also been considered; $\hat{\varepsilon}_{it}^2 = \alpha_{0i} + \alpha_{1i}R_{mt}^2$ and $\hat{\varepsilon}_{it}^2 = \alpha_{0i} + \alpha_{1i}(R_{mt} - \bar{R}_m)^2$. However, the estimation results of the price of beta risk (not reported) are quite similar.

Table V

Risk Price Estimates (*t*-Statistics) Assuming Conditional Heteroscedasticity

The risk price estimates ($\bar{\bar{\gamma}}_0$, $\bar{\bar{\gamma}}_1$, $\bar{\bar{\gamma}}_2$) are the time-series averages of the month-by-month cross-sectional regression (CSR) coefficient estimates for the whole period from July 1936 to December 1991. The monthly CSR coefficients are estimated through the errors in variables (EIV)-corrected weighted least squares (WLS) version of the (iterative) maximum likelihood estimation (MLE) by assuming conditional heteroscedasticity of the disturbance term of the market model. The conditional heteroscedasticity is modeled by $\varepsilon_{is}^2 = \alpha_{0i} + \alpha_{1i}R_{ms} + \alpha_{2i}R_{ms}^2$, where ε_{is} and R_{ms} are the disturbance term of the market model for an asset i at time s and the market return at time s, respectively.

The portfolios are size-β portfolios; they are first formed based on each firm's market value in June of year t, and each size-sorted portfolio is then subdivided based on each firm's pre-ranking β in June of year t for July of year t through June of year $t + 1$. All firms traded for at least two years on the NYSE and AMEX are used to form the portfolios.

NYSE/AMEX (NYSE/AMEX/OTC) indicates individual stocks traded for at least five years on these exchanges. For the whole period, on average, there are 1208 and 1579 stocks in the monthly regressions for NYSE/AMEX and NYSE/AMEX/OTC, respectively.

	$R_{it} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_{it-1} + \gamma_{2t}\log(V_{it-1}) + \omega_{it}$					
Portfolio Size (N)	$\bar{\bar{\gamma}}_0$	$\bar{\bar{\gamma}}_1$	$\bar{\bar{\gamma}}_2$	$\bar{\bar{\gamma}}_0$	$\bar{\bar{\gamma}}_1$	$\bar{\bar{\gamma}}_2$
	Equally-Weighted Market Returns			Value-Weighted Market Returns		
9	0.463 (3.33)	0.519 (1.89)		0.315 (1.91)	0.451 (1.76)	
16	0.471 (3.45)	0.506 (1.87)		0.302 (1.86)	0.453 (1.77)	
25	0.466 (3.44)	0.511 (1.90)		0.264 (1.58)	0.490 (1.91)	
49	0.418 (3.05)	0.552 (2.01)		0.198 (1.27)	0.543 (2.11)	
64	0.408 (2.99)	0.563 (2.06)		0.208 (1.33)	0.540 (2.09)	
100	0.395 (2.83)	0.572 (2.11)		0.180 (1.12)	0.560 (2.16)	
196	0.351 (2.41)	0.607 (2.22)		0.138 (0.81)	0.611 (2.33)	
289	0.294 (1.83)	0.676 (2.39)		0.121 (0.63)	0.637 (2.35)	
400	0.225 (1.35)	0.758 (2.53)		0.114 (0.50)	0.650 (2.43)	
NYSE/AMEX	0.225 (1.33)	0.771 (2.61)		0.109 (0.47)	0.661 (2.57)	
NYSE/AMEX/OTC	0.203 (1.30)	0.813 (2.66)		0.095 (0.42)	0.693 (2.78)	
9	1.102 (5.12)	0.186 (0.64)	−0.088 (−2.82)	1.102 (4.55)	0.068 (0.27)	−0.075 (−2.58)
16	1.020 (5.01)	0.210 (0.75)	−0.075 (−2.51)	1.076 (4.89)	0.071 (0.31)	−0.074 (−2.68)
25	1.079 (5.23)	0.184 (0.63)	−0.085 (−2.86)	1.044 (4.93)	0.103 (0.43)	−0.076 (−2.77)
49	0.999 (4.83)	0.242 (0.89)	−0.079 (−2.73)	1.020 (4.82)	0.129 (0.57)	−0.077 (−2.86)
64	1.040 (5.11)	0.216 (0.82)	−0.088 (−2.83)	1.026 (4.95)	0.121 (0.55)	−0.079 (−2.87)
100	0.995 (4.93)	0.247 (0.92)	−0.082 (−2.77)	1.028 (4.97)	0.137 (0.58)	−0.084 (−2.98)
196	0.897 (4.21)	0.305 (1.07)	−0.071 (−2.46)	0.914 (4.37)	0.217 (0.96)	−0.078 (−2.84)
289	0.827 (3.59)	0.381 (1.32)	−0.066 (−2.35)	0.864 (3.81)	0.259 (1.08)	−0.073 (−2.60)
400	0.682 (2.83)	0.517 (1.65)	−0.060 (−2.23)	0.714 (3.06)	0.390 (1.54)	−0.071 (−2.55)
NYSE/AMEX	0.604 (2.47)	0.647 (1.96)	−0.055 (−2.10)	0.714 (2.44)	0.567 (2.11)	−0.071 (−2.70)
NYSE/AMEX/OTC	0.558 (2.41)	0.710 (2.17)	−0.052 (−2.01)	0.519 (2.37)	0.638 (2.40)	−0.069 (−2.77)

while, however, to notice that the results are sensitive to the choice of the testing period. For the subperiod from July 1963 to December 1990, for example, when the size variable is included in the model, the explanatory power of market beta is weaker than that obtained for the whole period. However, it is more significant than that obtained without correcting for the EIV bias. The EIV problem, if not corrected, can lead to misleading conclusions. For example, the EIV problem caused Fama and French (1992) to underestimate the importance of market beta and to overestimate the importance of idiosyncratic factors. Nevertheless, my results indicate that firm size is still significant in explaining the cross-section of average returns, which is consistent with a misspecification in the CAPM.

D. Estimation Efficiency of the Beta Risk Price Estimates

While the traditional least squares estimate of the price of beta risk is biased, its asymptotic variance is smaller than that of the EIV-corrected MLE.[15] It is interesting, therefore, to compare the MLE and traditional WLS estimator in terms of both bias and MSE as the number of portfolios increases. To do this, I assume that the average of the CRSP monthly market returns minus the one month Treasury Bill returns is the true price of beta risk. I report only the results obtained under the assumption of homoscedasticity of the disturbance term of the market model, since the results obtained under the assumption of heteroscedasticity are similar. Figure 1 presents the movements of the relative bias $((\bar{\hat{\gamma}}_1 - \gamma_1)/\gamma_1)$ and the square-root of the relative MSE $(\sqrt{\mathrm{MSE}(\bar{\hat{\gamma}}_1)/\gamma_1^2})$ of the estimates of the price of beta risk for the whole period, when beta is the only explanatory variable in the model. The relative bias of the MLE of γ_1 decreases almost monotonically and approaches zero as the number of portfolios increases. The relative MSE of the MLE of γ_1 also decreases with the number of portfolios. On the other hand, the bias of the traditional WLS estimate of γ_1 becomes worse as the number of portfolios increases, a result that is expected from our earlier analysis. The MSE of the WLS estimate also increases with the portfolio size, since the increase of the bias dominates the decrease of the variance.[16] The relative bias of the MLE of γ_1 is smaller when the value-weighted market returns are used than when the equally-weighted market returns are used, while the relative MSE is not very different.

[15] After manipulating the variance of the EIV-corrected MLE of the CSR coefficient γ_{1t}, $\mathrm{Var}(\hat{\gamma}_{1t})$, of equation (A5) in Appendix, the following relation is obtained:

$$\mathrm{Var}(\hat{\gamma}_{1t}) = \frac{\mathrm{Var}(\hat{\gamma}_{1t}^{\mathrm{LS}})}{1-R} + \frac{1}{N}\frac{\delta_t}{(1-\rho_{\hat{\beta}V}^2)^2}\left(\frac{R}{1-R}\right)^2,$$

where $R = \mathrm{Var}(\xi_{it-1})/m_{\hat{\beta}\hat{\beta}}$. Since $m_{\hat{\beta}\hat{\beta}}$ is greater than $\mathrm{Var}(\xi_{it-1})$ (from equation (2)), $0 \leq R \leq 1$. Therefore, the variance of the traditional least squares estimate of γ_{1t}, $\mathrm{Var}(\hat{\gamma}_{1t}^{\mathrm{LS}})$, is smaller than the variance of its EIV-corrected MLE. Since the estimate of the price of beta risk is the time-series average of the CSR estimates of γ_{1t}, $\mathrm{Var}(\bar{\hat{\gamma}}_1) = (1/T)\mathrm{Var}(\hat{\gamma}_{1t})$ and $\mathrm{Var}(\bar{\hat{\gamma}}_1^{\mathrm{LS}}) = (1/T)\mathrm{Var}(\hat{\gamma}_{1t}^{\mathrm{LS}})$, where T is the time-series sample size. Therefore, the variance of the least squares estimate of the price of beta risk is smaller than the variance of its EIV-corrected MLE.

[16] As the CSR sample size (N) and/or the time-series sample size (T) increase, the variance of the price of beta risk estimate decreases. However, the bias of the traditional least squares estimate $\bar{\hat{\gamma}}_1^{\mathrm{LS}}$ is not eliminated but increased, although both N and T increase. The MSE of $\bar{\hat{\gamma}}_1^{\mathrm{LS}}$ is therefore dominated by its bias as N and T increase. On the other hand, both the variance and the bias of the MLE $\bar{\hat{\gamma}}_1$ decrease as N and T increase and thus the MSE of $\bar{\hat{\gamma}}_1$ decreases.

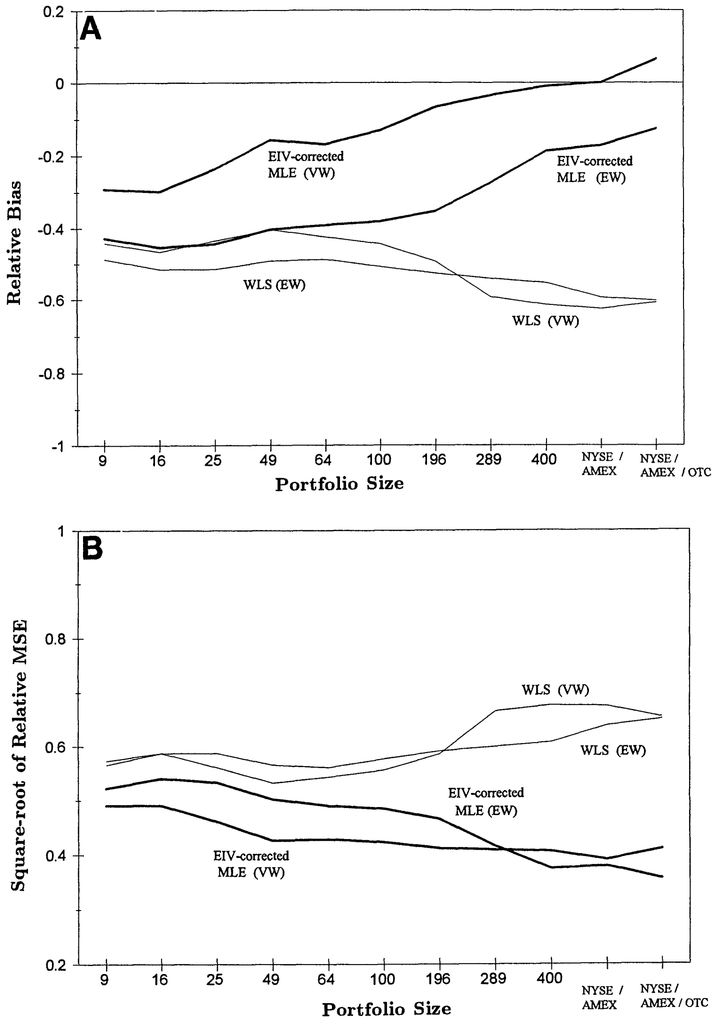

Figure 1. Estimation efficiency of the price of beta risk estimate when the beta variable alone is in the model. MLE (EW) and MLE (VW) indicate the EIV-corrected MLE estimates using the equally-weighted and value-weighted market returns, respectively, and WLS (EW) and WLS (VW) are accordingly the traditional WLS estimates. The relative bias is calculated as $(\bar{\hat{\gamma}}_1 - \gamma_1)/\gamma_1$ and the square-root of the relative MSE is calculated as $\sqrt{\mathrm{MSE}(\hat{\gamma}_1)/\gamma_1^2}$, where γ_1 is the true value of the price of beta risk and $\bar{\hat{\gamma}}_1$ is its estimate. The estimation period is from July 1936 to December 1991. (A) Relative bias of the price of beta risk estimate. (B) Square-root of the relative mean squared error (MSE) of the price of beta risk estimate.

Figure 2 presents the relative bias and the square-root of the relative MSE of the estimates of the price of beta risk for the whole period, when both beta and the firm size variables are in the model. Figure 2 is similar to Figure 1, except that the relative bias and MSE of the MLE of the beta risk price decrease more drastically as the number of portfolios increases.

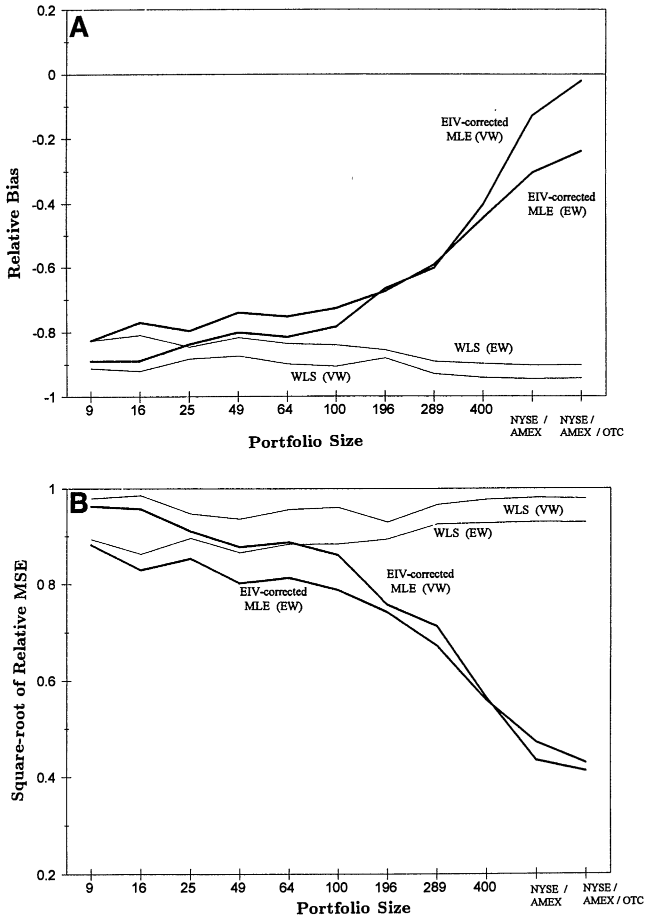

Figure 2. Estimation efficiency of the price of beta risk estimate when the beta and size variables are in the model. MLE (EW) and MLE (VW) indicate the EIV-corrected MLE estimates using the equally-weighted and value-weighted market returns, respectively, and WLS (EW) and WLS (VW) are accordingly the traditional WLS estimates. The relative bias is calculated as $(\bar{\hat{\gamma}}_1 - \gamma_1)/\gamma_1$ and the square-root of the relative MSE is calculated as $\sqrt{\text{MSE}(\bar{\hat{\gamma}}_1)/\gamma_1^2}$, where γ_1 is the true value of the price of beta risk and $\bar{\hat{\gamma}}_1$ is its estimate. The estimation period is from July 1936 to December 1991. (A) Relative bias of the price of beta risk estimate. (B) Square-root of the relative mean squared error (MSE) of the price of beta risk estimate.

Overall, the EIV-corrected MLE method dominates the traditional WLS estimation method both with respect to bias and MSE and for any considered portfolio size.[17]

[17] In a Monte Carlo simulation study (not reported), the MLE also outperforms the traditional WLS estimate in any considered portfolio formation method (portfolios by size, β, size-β, half in size and half in β, and independent size-β) with respect to both bias and MSE. The size-β portfolio formation gives better estimation results than the other portfolio formation methods.

IV. Conclusion

This article develops a new method of correcting the EIV problem in the estimation of the price of beta risk within the two-pass estimation framework. I perform the correction for the EIV problem by incorporating the extracted information about the relationship between the measurement error variance and the idiosyncratic error variance into the maximum likelihood estimation under either homoscedasticity or heteroscedasticity of the disturbance term of the market model.

This article also reexamines the explanatory power of beta and firm size for average stock returns using the proposed correction. If no correction for the EIV problem is implemented, then, consistent with recent empirical studies, there is no significant relation between beta and expected returns—especially when firm size is included as an additional explanatory variable. With the EIV correction, both the price and significance of betas increase. This is most evident when the full sample from July 1936 to December 1991 is used. As a result of the EIV correction, the intercept (which should be zero under the null hypothesis) is insignificant from zero. Finally, the EIV correction diminishes the impact of the firm size variable. Nevertheless, firm size remains a significant force in explaining the cross-section of average returns. While my results suggest a more important role for market beta, the evidence is still consistent with a misspecification in the capital asset pricing model.

Appendix

When there is only one explanatory variable measured with error in the model, the MLE is derived by Judge *et al.* (1980), Johnston (1984), Fuller (1987) among others assuming that $\Sigma_\varepsilon = I_N$. Here I extend their results to the case that K variables measured without error are added in the model. I also derive the variance matrix of the MLEs.

When there is one explanatory variable measured with error and the other K variables are measured without error in the model, the CSR model to be estimated at time t is

$$\mathbf{R}_t = \gamma_{0t} + \boldsymbol{\beta}_t\gamma_{1t} + \mathbf{B}_{2t-1}\Gamma_{2t} + \boldsymbol{\varepsilon}_t$$

$$\hat{\boldsymbol{\beta}}_{t-1} = \boldsymbol{\beta}_t + \xi_{t-1},$$

where $\mathbf{B}_{2t-1}$ $(N \times K)$ is a set of the explanatory variables measured without error at time $t-1$ ($\mathbf{V}_{t-1}$ is replaced with $\mathbf{B}_{2t-1}$), and Γ_{2t} is the $(K \times 1)$ second coefficient vector. Based on the assumptions (A1) and (A2) and $\Sigma_\varepsilon\Sigma_\xi^{-1} = \mathbf{D}I_N$, the following likelihood function is obtained:

$$L = (\mathbf{R}_t - \gamma_{0t} - \hat{\boldsymbol{\beta}}_{t-1}\gamma_{1t} - \mathbf{B}_{2t-1}\Gamma_{2t})'\mathbf{D}(\gamma_{1t}^2 I_N + \mathbf{D})$$

$$- 1\ \hat{\Sigma}_\varepsilon^{-1}(\mathbf{R}_t - \gamma_{0t} - \hat{\boldsymbol{\beta}}_{t-1}\gamma_{1t} - \mathbf{B}_{2t-1}\Gamma_{2t}).$$

By equating the derivatives of L with respect to the unknown parameter vector $\boldsymbol{\theta}$ to zero, equations (9), (10), and (11) are obtained.

Arranging equations (9), (10), and (11) gives a following quadratic equation of γ_{1t};

$$(m_{R\hat{\beta}} - \mathbf{M}'_{\hat{\beta}2}\mathbf{M}_{22}^{-1}\mathbf{M}'_{R2})\gamma_{1t}^2 - (m_{RR} - \mathbf{M}'_{R2}\mathbf{M}_{22}^{-1}\mathbf{M}_{R2} - \delta_t(m_{\hat{\beta}\hat{\beta}} - \mathbf{M}'_{\hat{\beta}2}\mathbf{M}_{22}^{-1}\mathbf{M}_{\hat{\beta}2}))\gamma_{1t} - \delta_t(m_{R\hat{\beta}} - \mathbf{M}'_{\hat{\beta}2}\mathbf{M}_{22}^{-1}\mathbf{M}_{R2}) = 0, \quad \text{(A1)}$$

where m_{xy} and $\mathbf{M}_{xy}$ are the cross-sectional centered weighted second sample co-moment between variables $\mathbf{x}$ and $\mathbf{y}$ (the boldface represents a vector or a matrix). The subscript "2" corresponds to the variable $\mathbf{B}_{2t-1}$. Among two roots of equation (A1), the solution $\hat{\gamma}_{1t}$ which has the same sign as $m_{R\hat{\beta}}$ is chosen. This sign restriction can also be applied to choose the unique solution in the iterative algorithm for the case of heteroscedasticity of the disturbance term of the market model. Then the MLE of γ_{1t} is given by

$$\hat{\gamma}_{1t} = \frac{S + [S^2 + 4\delta_t(m_{R\hat{\beta}} - \mathbf{M}'_{\hat{\beta}2}\mathbf{M}_{22}^{-1}\mathbf{M}_{R2})^2]^{1/2}}{2(m_{R\hat{\beta}} - \mathbf{M}'_{\hat{\beta}2}\mathbf{M}_{22}^{-1}\mathbf{M}_{R2})}, \quad \text{(A2)}$$

where

$$S = m_{RR} - \mathbf{M}'_{R2}\mathbf{M}_{22}^{-1}\mathbf{M}_{R2} - \delta_t(m_{\hat{\beta}\hat{\beta}} - \mathbf{M}'_{\hat{\beta}2}\mathbf{M}_{22}^{-1}\mathbf{M}_{\hat{\beta}2}).$$

From equations (10) and (11), the MLE's of Γ_{2t} and γ_{0t} are also obtained by

$$\hat{\Gamma}_{2t} = \mathbf{M}_{22}^{-1}\mathbf{M}'_{R2} - \mathbf{M}_{22}^{-1}\mathbf{M}'_{\hat{\beta}2}\hat{\gamma}_{1t} \quad \text{(A3)}$$

$$\hat{\gamma}_{0t} = \tilde{R}_t - \hat{\gamma}_{1t}\tilde{\hat{\boldsymbol{\beta}}}_{t-1} - \tilde{\mathbf{B}}_{2t-1}\hat{\Gamma}_{2t}, \quad \text{(A4)}$$

where $\tilde{\mathbf{B}}_{2t-1}$ is the $(1 \times K)$ cross-sectional weighted average vector of $\mathbf{B}_{2t-1}$.

In order to prove the consistency of the MLEs, I first show the existence of the limits of equations (A1), (A3), and (A4). Based on the standard assumptions for the classical GLS regression analysis (see Fomby, Hill, and Johnson (1984)), the limit values of the cross-sectional sample co-moments in these equations exist and thus the coefficients of these equations converge to limits as N goes to infinity. The limiting equations of those equations also exist.

I now prove the convergence of the MLEs to the corresponding roots of the limiting equations. If the solutions of equations (A1), (A3), and (A4) converge to those of their limiting equations as N goes to infinity, that is, if $\hat{\Gamma}_t = (\hat{\gamma}_{0t}, \hat{\gamma}_{1t}, \hat{\Gamma}_{2t}) \xrightarrow{L} \Gamma_t = (\gamma_{0t}, \gamma_{1t}, \Gamma_{2t})$, then $\hat{\Gamma}_t$ is an N-consistent estimator for Γ_t. In order to show this convergence and to derive the asymptotic variance of $\hat{\Gamma}_t$, I extend Theorem (1.3.1) and (1.A.1) of Fuller (1987) to the more general case. Notice that Theorem (1.3.1) and (1.A.1) of Fuller are for the case when one explanatory variable measured with error is alone in the model and $\Sigma_\varepsilon = I_N$. Then I obtain the asymptotic distribution of $\hat{\Gamma}_t$ given by

$$\sqrt{N}(\hat{\Gamma}_t - \Gamma_t) \overset{L}{\to} N(\mathbf{0}, \Sigma_{\hat{\Gamma}_t}),$$

where

$$\Sigma_{\hat{\Gamma}_t} = \begin{pmatrix} (1 + \boldsymbol{\mu}_2' \Sigma_{22}^{-1} \boldsymbol{\mu}_2)\sigma_\omega^2 + \phi^2 \sigma_{\hat{\gamma}_{1t}}^2 & -\phi\, \sigma_{\hat{\gamma}_{1t}}^2 & -\sigma_\omega^2 \Sigma_{22}^{-1}\, \boldsymbol{\mu}_2 + \phi \Sigma_{22}^{-1}\, \Sigma_{\beta 2}\, \sigma_{\hat{\gamma}_{1t}}^2 \\ -\phi \sigma_{\hat{\gamma}_{1t}}^2 & \sigma_{\hat{\gamma}_{1t}}^2 & -\Sigma_{\beta 2}'\, \Sigma_{22}^{-1}\, \sigma_{\hat{\gamma}_{1t}}^2 \\ -\sigma_\omega^2\, \Sigma_{22}^{-1} \boldsymbol{\mu}_2 + \phi\, \Sigma_{22}^{-1} \Sigma_{\beta 2} \sigma_{\hat{\gamma}_{1t}}^2 & -\Sigma_{22}^{-1} \Sigma_{\beta 2}\, \sigma_{\hat{\gamma}_{1t}}^2 & \sigma_\omega^2\, \Sigma_{22}^{-1} + \Sigma_{22}^{-1}\, \Sigma_{\beta 2}\, \Sigma_{\beta 2}'\, \Sigma_{22}^{-1}\, \sigma_{\hat{\gamma}_{1t}}^2 \end{pmatrix},$$

$$\sigma_{\hat{\gamma}_{1t}}^2 \equiv \mathrm{Var}(\hat{\gamma}_{1t}) = (\sigma_\beta^2 - \Sigma_{\beta 2}' \Sigma_{22}^{-1} \Sigma_{\beta 2})^{-2} [(\sigma_\beta^2 - \Sigma_{\beta 2}' \Sigma_{22}^{-1} \Sigma_{\beta 2})\, \sigma_\omega^2 + (1/\delta_t)\sigma_\varepsilon^4],$$

$$\sigma_\omega^2 = (1 + \gamma_{1t}^2/\delta_t)\sigma_\varepsilon^2, \qquad \phi = \mu_\beta - \boldsymbol{\mu}_2' \Sigma_{22}^{-1} \Sigma_{\beta 2}, \tag{A5}$$

$$\text{and} \quad (\mu_\beta, \boldsymbol{\mu}_2) \quad \text{and} \quad \begin{pmatrix} \sigma_\beta^2 & \Sigma_{\beta 2}' \\ \Sigma_{\beta 2} & \Sigma_{22} \end{pmatrix}$$

(rank $K + 1$) are the (cross-sectional) mean vector and variance matrix of the first unobservable explanatory variable ($\boldsymbol{\beta}_t$) and the second K explanatory variables ($\mathbf{B}_{2t-1}$), respectively, and σ_ε^2 is the variance of a component of the transformed error vector $\mathbf{P}\boldsymbol{\varepsilon}_t$ such as $\mathbf{PP}' = \hat{\Sigma}_\varepsilon^{-1}$. The proof of the N-consistency of the WLS and OLS version of the MLE within a multifactor framework is also presented in Shanken (1992).

In the Fama and MacBeth (1973) two-pass methodology, the above variance matrix of $\hat{\Gamma}_t$, $\Sigma_{\hat{\Gamma}_t}$, is not directly used to compute the statistical significance of risk premia estimates, since their standard errors are calculated by using time-series data of the CSR coefficient estimates, $\hat{\Gamma}_t$. Under the circumstance that only one-time CSR is performed, however, this variance matrix can be used to compute the standard errors. The variance matrix, $\Sigma_{\hat{\Gamma}_t}$, is estimated by replacing the unknown parameters with their sample estimates. In particular, the variance of the first unobservable explanatory variable can be estimated by $\hat{\sigma}_\beta^2 = (2\delta_t)^{-1}\{S^2 + 4\delta_t(m_{R\hat{\beta}} - \mathbf{M}_{\hat{\beta}2}' \mathbf{M}_{22}^{-1} \mathbf{M}_{R2})^2]^{1/2} - S\}$, and $\hat{\sigma}_\omega^2 = (\mathbf{R}_t'\mathbf{R}_t - \hat{\Gamma}_t' \mathbf{X}_t' \mathbf{X}_t \hat{\Gamma}_t)/(N - k - 2)$, where $\mathbf{X}_t$ is the set of the explanatory variables. Notice also that the variance of each component of $\hat{\Gamma}_t$ is greater than that of the corresponding least squares estimate.

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