THE JOURNAL OF FINANCE • VOL. L, NO. 5 • DECEMBER 1995

Initial Shareholdings and Overbidding in Takeover Contests

MIKE BURKART*

ABSTRACT

Within the context of takeovers, this paper shows that in private-value auctions the optimal individually rational strategy for a bidder with partial ownership of the item is to overbid, i.e., to bid more than his valuation. This strategy, however, can lead to *i*) an inefficient outcome, and *ii*) the winning bidder making a net loss. Further, the overbidding result implies that the presence of a large shareholder increases the bid premium in single-bidder takeovers at the expense of reducing the probability of the takeover actually occurring.

THE PRESENCE, OR THE likely existence, of competing bids for a target is an important feature of the takeover process. An immediate consequence of competition is that an acquiring party cannot assume that he will be able to take over the target at a cheap price. On the contrary, he must be prepared to accept that a substantial share of the gains will go to the target's shareholders. Bidding and counter-bidding or the endeavor to deter rivals from entering the contest by means of a high initial bid all result in high takeover premia. In fact, stock market studies conducted mainly with U.S. data find that the average takeover premium is 30 percent, and peak values reach 100 percent.[1] One implication of the unequal division of gains in tender offers is the importance of initial stakes, i.e., shares acquired prior to the tender offer, as these are a primary source of profit for the acquiring party.

This paper shows how bidders' initial stakes, often referred to as toeholds, affect competition. In particular, the presence of toeholds in bidding contests, modelled as English auctions or second-price sealed-bid auctions, can lead to

* Financial Markets Group, London School of Economics, and Department of Finance, Stockholm School of Economics. An earlier version of this paper appeared as LSE Financial Markets Group Discussion Paper No. 180 under the title "Overbidding in Takeover Contests." I would like to thank Margaret Bray, Maria De Liseo, Jan Ericsson, Julian Franks, Ron Giammarino, Denis Gromb, Oliver Hart, Martin Hellwig, Alison Hole, Godfrey Keller, Reinout Koopmans, Thomas Piketty, Ailsa Röell, Bengt Rosén, Kristian Rydqvist, René Stulz (the editor), Jean Tirole, anonymous referees as well as seminar participants at the Econometric Society European Meeting (Uppsala 93), the Eighth Annual Congress of the European Economic Association (Helsinki 93), and the Department of Finance at the Stockholm School of Economics, for their helpful comments and suggestions. All remaining errors are mine. Part of this paper was written while I was visiting the Department of Finance at the Stockholm School of Economics. Financial support from the Financial Markets Group (LSE), the Swiss National Science Foundation, Max-Geldner Stiftung, Nikolaus und Bertha Burckhardt-Bürgin Stiftung and Bankforskningsinstitutet is gratefully acknowledged.

[1] Cf. Jarrell et al. (1988).

inefficient outcomes. This arises because a buyer who owns a share in the item being sold has an incentive to overbid, i.e., to submit a bid in excess of his valuation of the item. In contrast to the standard English auction and second-price sealed-bid auction, a losing bidder sells his initial stake to the winner, and is therefore interested in a high price. One way to ensure a high price is to submit a higher bid with the aim of provoking a counterbid. However, by adopting such a price-raising strategy, he incurs the risk of winning the contest instead of merely improving the selling price. This unwanted outcome results in an inefficient allocation of the object.

The paper examines the specific example of a takeover contest between two competing bidders. In this context, overbidding will be defined as offering a price per share that is higher than its post-takeover value, once the bidder has gained control. Whether this also implies offering more for the target than it is worth depends upon the size of the bidder's initial stake and upon the extent to which he overbids per share. While the analysis deals with a two bidder private-value auction, overbidding is a general feature of auctions where bidders own a stake prior to the bidding. Furthermore, overbidding as an economic phenomenon is by no means restricted to bidding contests between two outsiders. Overbidding can also occur, for example, in control contests between an incumbent management with equity ownership and an outside investor, or in liquidations of failed businesses as prescribed by Chapter 7 of the U.S. Bankruptcy Code.

The overbidding theory presented in this paper provides an explanation for the observed tendency of winning bidders to overvalue the target.[2] Unlike the hubris hypothesis proposed by Roll (1986), this theory does not rely on an assumption about managerial behavior, but originates from individually rational, i.e., profit-maximizing, behavior on the part of the bidders. Another possible explanation for overbidding lies in the winner's curse. When each bidder offers his estimate of the object in a common-value auction, the winner will find that (on average) he has overestimated its value. However, such bidding behavior is not fully rational. This paper establishes that fully rational bidders with initial stakes may also fall victim to a winner's curse, even though each bidder knows his valuation of the object. Overbidding is not dependent upon the private-value assumption, however. The necessary ingredient for overbidding is that bidders have an initial stake in the object being sold. For example, toeholds would also cause overbidding in a common-value setting where each bidder observes a noisy signal of the target's common value.

In the context of takeovers, there is empirical support for introducing toeholds into the existing models of bidding contests. It is not unusual for acquiring parties to own shares prior to the acquisition. For example, in Jennings and Mazzeo's (1993) sample of 647 observations of proposed tender offers and mergers, the mean toehold is 3 percent. However, the distribution is skewed, and only around 15 percent of the bidders own an initial stake. In the sub-

[2] Cf. e.g. Bradley et al. (1988).

sample of multiple-bidder contests, the average is slightly higher at around 18 percent.[3]

Although many models deal with the interplay among multiple bidders,[4] few papers consider the effect which initial stakes have on the bidding competition.[5] Dewatripont (1991) and Ravid and Spiegel (1992) show that a toehold can be effective in deterring rival bidders. When modeling the competition among the bidders, both papers place an exogeneous restriction on the bidding strategies which bidders can adopt. Despite one bidder owning a toehold, he is never permitted to bid above the price level where he is indifferent about winning and losing, i.e., above his valuation of the target. This restriction immediately implies that the bidder with the higher valuation always wins, ensuring an efficient allocation. This paper, however, shows that these results change substantially if bidders' strategies are no longer restricted in this way.

Finally, this paper is closely related to two recent works by Chowdhry and Nanda (1993) and Singh (1993). Chowdhry and Nanda also identify an incentive to overbid although within an entirely different setting. In their model, debt serves as a commitment to bid aggressively in a subsequent contest. While this might deter potential rivals, the drawback is that the bidder might have to pay a price above his valuation if a rival appears. However, as the takeover is exclusively financed by new debt of equal seniority, the shareholders pass on part of the loss to all existing debtholders.

Independent of this research, Singh (1993) also identifies partial ownership as a cause of overbidding. He develops the overbidding result in a one-sided asymmetric information setting, where it is publicly known that the blockholder is a passive investor. He then goes on to allow the blockholder to be either an outsider capable of generating synergies, or to be the incumbent management. This approach allows a richer interpretation, although the different cases are essentially encompassed in the model presented here with two outside bidders and two-sided asymmetric information. In addition, Singh discusses a variety of sales procedures and considers the links between overbidding and other defensive measures.

The present paper is organized as follows. Section I outlines the model. Section II shows that bidders with initial stakes always overbid. Furthermore, explicit solutions for the bidders' optimal strategies are derived for the one-toehold case, and the robustness of the overbidding result with respect to bid form and bidding costs is discussed. In Sections III and IV, applications are

[3] Bradley et al. (1988) find that more than half of the acquiring firms have a toehold. The average size of the initial stake in their sample is 9.8 percent. In contrast to Jennings and Mazzeo (1993), Bradley et al. analyze only successful tender offers but do not exclude bidders with over a 50 percent stake from their sample.

[4] See Spatt (1989) for a survey.

[5] The role of toeholds within a single-bidder setting has been analysed by Shleifer and Vishny (1986) and Chowdhry and Jegadeesh (1989). The former show that toeholds are a way of overcoming the free-rider problem of Grossman and Hart (1980). The latter also use the toehold to circumvent the free-rider behavior, but they endogenize the size of the toehold and use it as a signalling device.

proposed and the main results are linked to empirical findings in the literature. The conclusions are presented in Section V.

I. A Model of Bidding Contests

A. Model

Bidder 1 and 2 are potential acquirers of a target firm. They are both risk-neutral and hold an initial fraction α_1 and α_2 of the N target shares, where $\alpha_i < 1/2$, $i = 1, 2$. The remaining $N(1 - \alpha_1 - \alpha_2)$ shares are widely held. Other claims held against the target firm are not considered.

Let ε_i denote the value of one share once the target is under bidder i's control. The valuations ε_1 and ε_2 are independently distributed on [0, 1] according to a nonatomic distribution function G_i. The function G_i is continuous and strictly increasing for all $\varepsilon_i \in [0, 1]$, and $G_i(0) = 0$. Each bidder knows his private valuation, while the realization of the opponent's valuation remains unknown to him. The distribution functions and the size of the toeholds are common knowledge. For simplicity, the value of the target firm under the incumbent management is normalized to zero and is known by all parties.

Competition between the two bidders for the target company is modeled as the Japanese version of the common English auction.[6] In this game, at any one moment, all players in the auction offer the same price. This price rises continuously and players choose whether or not to remain in the auction. Once a player has withdrawn, he cannot re-enter the bidding. The last remaining player in the auction wins and has to pay the price at which he withdrew. By assumption, participation in the auction involves no costs. (The effects of introducing bidding costs are discussed in section 3.5.) Further, the bids must be non-negative, i.e., $p_i \geq 0$, where $i = 1, 2$.

Tender offers are the only admissible mode of takeover and negotiations between a bidder and the incumbent management are ruled out. Furthermore, neither collusion among the bidders nor retrading after the auction are allowed. The tender offers are binding, conditional, and unrestricted. That is, the bid can only be superseded by a higher (competing) bid, the bid is contingent upon gaining control, and the winning bidder is obliged to accept all tendered shares, including those of the rival. The target shareholders are assumed to act as price-takers. They accept the highest offer if the offered price is at least as high as the value of the firm under the incumbent management, i.e., $p \geq 0$.[7]

[6] There are many variants of the English auction each corresponding to a different game. The use of the Japanese or "ascending clock" variant in the literature, e.g., Milgrom and Weber (1982), is widespread for two reasons: it can be solved by eliminating dominated strategies, and its equilibrium outcome is the same as in the second-price sealed-bid auction. As Kamecke (1993) shows, most other English auctions have signaling equilibria, in addition to the standard competitive bidding outcome. These are robust to the application of the dominance criterion. The term English auction will subsequently refer to its Japanese version.

[7] If a more favorable bid is offered, shareholders must have the right to withdraw tendered shares in order to accept the best offer. The U.K. City Code (Rule 32.3), the proposed E.C. Directive (Rule 20.5) as well as the U.S. takeover legislation grant this right. Cf. De Mott (1988).

The price-taking assumption together with the unrestricted nature of the bids imply that the winning bidder acquires all $(1 - \alpha_i)N$ remaining shares. (The robustness of the results with respect to alternative types of tender offers is discussed in section 3.5.)

B. Background to the Model

Before presenting an analysis of the model, some explanations for the choice of the framework may be in order. Since the focus of the paper is on competition between two bidders, neither the acquisition process nor the size of the toeholds is explained. Toeholds should be seen as falling in the range of the legal disclosure threshold. Open-market acquisitions beyond this level are difficult or very costly immediately prior to a tender offer. The U.K. City Code requires disclosure at 3 percent, the proposed E.C. Directive at 10 percent, and the U.S. regulation at 5 percent with a ten-day window allowing for further acquisitions. Since takeover regulations require bidding parties to disclose their toeholds, α_i is known to the rival.

The assumption that the bidders' valuations are independently distributed is tantamount to asserting that the values which bidders attribute to a target depend upon idiosyncratic factors, such as possible synergy gains. With respect to takeovers, this choice is controversial, and the common-value assumption is sometimes claimed to be more suitable. In reality, the value of a target is likely to have a common and as well as a bidder-specific component. If these components are assumed to be additively separable, the private-value auction is merely a special case of a more general formulation, where the common component is known by all bidders and normalized to zero.

The above distinction between common and bidder-specific components is not to be confused with the distinction, common in the corporate control literature, between security benefits and private benefits of control. Since the focus here is on the competition between bidders, this latter distinction is not relevant. What is relevant is the target firm's value to the bidder, i.e., his willingness-to-pay, irrespective of whether it stems from large security benefits or from large private benefits of control. However, the distinction becomes essential when comparing the model's predictions with the empirical evidence. To generate clear empirical implications, ε_i will from now on refer exclusively to security benefits under bidder i's control.

In a typical bidding contest, a potential acquirer will initially submit a tender offer. A second party will then enter the contest and offer a higher price. The first bidder may then counter this bid by offering an even higher price, and so on until one party makes an offer which is not improved upon by the rival party. Hence, competition between bidders is best captured by an open ascending, or common English, auction. For tractability, this is approximated by its Japanese version. Furthermore, since contests with more than two bidders are rare, the model is restricted to two bidders. For example, in the study by Bradley *et al.* (1988), the vast majority of the multiple-bidder contests (65 out of 73) involved only two bidders.

The model does not allow the parties to bargain if the outcome of the bidding contest is inefficient. Inefficient outcomes are inherent in a world characterized by private information, and there are no mechanisms which guarantee the transition to an efficient allocation. Therefore, bargaining is not as obvious a consequence of an inefficient outcome as it would be under perfect information. Moreover, the introduction of post-auction bargaining would substantially alter the nature of the game. In such a setting, additional factors need to be taken into account when deriving bidding strategies. In particular, offers made during the bidding contest might act as a signal in any subsequent bargaining, inducing bidders to act cautiously so as not to reveal a high private value. Another possible effect might be that bidders with toeholds bid more aggressively, as the post-auction bargaining enables them to limit their losses should they win the bidding contest.

The model does not permit bidders to withdraw their offers. The assumption of binding bids is common and is supported by the legal framework in which tender offers take place. The U.K. City Code and the proposed E.C. Directive effectively prevent a bidder from pulling out should he win a takeover contest.[8] While the Williams Act does not restrict the bidder's ability to condition his offer to the same extent, withdrawal is not an option in the United States either, as the winning bidder is likely to face litigation.

In contrast to Grossman and Hart's model (1980), the target's shareholders do not use their ability to free-ride on a bidder's value improvement but accept the highest offer. The price-taking assumption facilitates the analysis of the bidding competition. Alternatively, the free-rider problem could be eliminated by assuming that the successful bidder can fully dilute the value of minority shares.

II. Overbidding in Equilibrium

A. *Equivalence of Auction Forms and Nonexistence of Dominant Strategies*

Takeover contests have been described in terms of an English auction. A bidder's strategy in the English auction does not only depend on his valuation, but is also a function of the previous bidding activities. In contrast, the second-price sealed-bid auction is a static simultaneous move game with no history on which the players might condition their strategies. Hence, for analytical convenience the second-price sealed-bid auction is adopted.

LEMMA 1: *In the case of two bidders with initial stakes, the English and the second-price sealed-bid auctions are strategically equivalent.*

[8] U.K. City Code (Rule 13) and the proposed E.C. Directive (Rule 10) prohibit offers subject to conditions whose fulfilment is in the bidder's hands. Additionally, the City Code demands that bidders make sure before the acquisition that they are able to implement the offer (General Principle 10), and that the availability of the financing is confirmed by a financial adviser (Rule 2.5). Thus, the condition that an offer is subject to obtaining satisfactory financing is not acceptable.

Proof: The player's strategy in the second-price sealed-bid auction is to choose a price. In the English auction with two players, each player determines a price ceiling at which he will withdraw from the auction, given that his rival is still actively bidding. If the rival withdraws before the price ceiling is reached, the bidder clearly benefits from withdrawing immediately thereafter. Thus, after eliminating the dominated strategies of remaining in the auction after the rival's withdrawal, the strategy sets are equivalent in the English and the second-price sealed-bid auction. A strategy consists of choosing one price. Furthermore, for any two prices chosen by the bidders, the winner and the purchasing price are determined in the same way in both auctions. The higher bidder wins and has to pay a price equal to the lower bid. Hence, the equivalence holds. Q.E.D.

The strategic equivalence of the English auction and the second-price sealed-bid auction with toeholds implies that the equilibrium outcomes of both auction forms must coincide. It should be noted, however, that the equivalence does not hold in the case of more than two bidders. In the English auction, each bidder's price ceiling will be contingent upon the observed exit of other bidders. Hence, the elimination of the dominated strategies, as outlined in the proof above, does not reduce the strategy set to that of the second-price sealed-bid auction. With only two buyers, the rival's exit induces the opponent to withdraw immediately thereafter and brings the game to an end.

In the standard setting with no toeholds and many buyers, the English auction and the second-price sealed-bid auction are equivalent, although in a weaker sense than in Lemma 1. Each auction admits a unique dominant strategy equilibrium, and both have the same outcome. However, as will be shown, this does not apply to auctions with toeholds. Consequently, the equivalence needs to be established for the purpose at hand. The following analysis concerns the second-price sealed-bid auction where one or both bidders own toeholds, and begins by ruling out dominated strategies.

LEMMA 2: *It is a dominated strategy for any bidder to bid below his own valuation.*

LEMMA 3: *It is dominated strategy for a bidder with no initial stake to bid above his own valuation.*

Lemmata 2 and 3 merely restate the familiar result that for a bidder without a toehold, bidding his valuation dominates all other strategies. One aspect of Lemma 2, however, is particularly noteworthy. The potential loss associated with underbidding is exacerbated for a bidder with a toehold. When bidding $p_1 < \varepsilon_1$, bidder 1 without a toehold forgoes some profit only if $p_1 < p_2 < \varepsilon_1$. In contrast, when bidding $p_1 < \varepsilon_1$, bidder 1 with an initial stake always forgoes some profit when he loses the auction, i.e., whether $p_1 < p_2 < \varepsilon_1$ or $p_1 < \varepsilon_1 \leq p_2$. In the latter case, a higher bid would increase the selling price of his initial stake.

LEMMA 4: *A bidder with a toehold does not have a dominant strategy.*

Since a losing bidder with a toehold has an incentive to increase his bid, neither bidding his valuation nor overbidding is a dominant strategy. To illustrate this point, consider bidder 1's decision problem. If $p_1 < p_2$ holds, a higher bid would increase bidder 1's return on the sale of his stake. To ensure the existence of a best response for bidder 1, assume that in case of a tie, i.e., $p_1 = p_2$, bidder 2 is the winner. Bidder 1's best response would then be to match bidder 2's offer exactly if $p_2 \geq \varepsilon_1$, and to bid any $p_1 > p_2$ if $p_2 < \varepsilon_1$. Hence, an auction where bidders have partial ownership cannot be solved by the dominance criterion.

B. General Case

This section shows that a rational bidder with a toehold inevitably submits a bid in excess of his valuation.

PROPOSITION 1: *A bidder with an initial stake overbids with probability 1, i.e., for all $\alpha_i > 0$, $p_i(\varepsilon_i) > \varepsilon_i$ for almost all $\varepsilon_i \in [0, 1)$.*

The proof is given in the appendix.[9] Compare bidder i's expected return when overbidding by δ with his expected return when bidding his valuation ε_i. First, it should be noted that both bids yield the same return when the opponent's bid is lower than ε_i. Thus, outcomes need only be considered where the opponent bids at least bidder i's valuation ε_i. On the one hand, overbidding by δ gives bidder i an additional gain of $\delta\alpha_i$ when the opponent's bid is higher than his own. On the other hand, bidder i incurs a (maximum) loss of $\delta(1 - \alpha_i)$, when the opponent's bid is lower than $\varepsilon_i + \delta$, but higher than ε_i. The probability that overbidding by δ will reverse the outcome of the contest, making bidder i the winner, is of the order δ. Consequently, the loss associated with overbidding is of the order δ^2, while the expected gain is of the order δ. Hence, for small δ, overbidding yields a higher expected return than bidding his valuation to a bidder with a toehold.

Since the types' distribution functions are only required to be nonatomic and have a common support, the overbidding result is very robust. It neither requires that G_1 and G_2 be identical nor that they satisfy further criteria, such as the monotone hazard rate condition. Proposition 1 implies that in any equilibrium of the bidding contest, bidders with initial stakes will bid a price which is higher than their valuation. However, due to the asymmetry between the bidders, both in ε and α, it is very difficult to derive an analytical solution. Moreover, an equilibrium may not exist as it is an infinite strategic-form game.[10] In the next section, an analytical solution of the auction problem will

[9] I am indebted to an anonymous referee for suggesting this proof. Extending the overbidding result to N-bidders requires additional restrictions. These arise because bidders with low valuations, and hence little prospect of either winning the auction or determining the purchasing price, are largely indifferent as regards their bids.

[10] One way to solve the existence problem is to convert the continuous auction into a discrete auction by restricting the bidding space and the type space to a finite number of values.

be derived for the simpler setting where only one bidder owns a stake prior to the contest.

C. One-toehold Case

Although the one-toehold case is less general, it provides further insights into the effects which toeholds have on bidding behavior. In particular, its tractability allows comparative-static results to be derived with respect to the optimal bid and the expected profits as a function of the toehold size. Situations where one bidder has a first-mover advantage due to his toehold are of interest in their own right, since they are the most likely setting in which a takeover contest takes place. Once a first tender offer has been made, it becomes difficult for a rival to acquire a substantial number of shares on the open market.[11]

To ensure the existence of a unique closed-form solution, the additional assumption is made that the bidders' types are i.i.d. and that $G(\cdot)$ is twice continuously differentiable and satisfies the monotone hazard rate condition, i.e.,

$$\frac{d}{d\varepsilon_i}\frac{g(\varepsilon_i)}{[1 - G(\varepsilon_i)]} \geq 0.$$

Consider the case where only bidder 1 owns an initial stake, i.e., $\alpha_2 = 0$. By Proposition 1, bidder 1 overbids. The degree to which bidder 1 should overbid is discussed in the subsequent propositions.

PROPOSITION 2: *In the one toehold case with $\alpha_1 > 0$, bidder 1's equilibrium bid $p_1^*(\varepsilon_1)$ is the solution to*

$$p_1^* = \varepsilon_1 + \alpha_1 \frac{[1 - G_2(p_1^*)]}{g_2(p_1^*)} \tag{1}$$

and bidder 2's equilibrium bid is $p_2^* = \varepsilon_2$.

Proof: Let $F_2(p_2)$ be the c.d.f. of bidder 2's bid and $f_2(\cdot)$ the corresponding density function. Bidder 1's maximization problem is then

$$\max_{p_1} \alpha_1 p_1[1 - F_2(p_1)] + \varepsilon_1 F_2(p_1) - (1 - \alpha_1)\int_0^{p_1} p_2 \, dF_2(p_2).$$

Lemmata 2 and 3 imply $p_2 = \varepsilon_2$. Hence $G_2 = F_2$, and $F_2(\cdot)$ is twice continuously differentiable and satisfies the monotone hazard rate condition. Differentiat-

[11] Some support for this view is provided by Zwiebel (1991) and Jennings and Mazzeo (1993). The former reports that in firms with large blockholders, there is typically a single large blockholder. In the multiple-bidder contests included in the study of Jennings and Mazzeo, a minority of bidders (18 percent) have an initial stake.

ing bidder 1's objective function with respect to p_1 yields after substituting and rearranging:

$$\varepsilon_1 g_2(p_1) - p_1 g_2(p_1) + \alpha_1[1 - G_2(p_1)].$$

Given that $\alpha_1 < ½$, the monotone hazard rate condition ensures that bidder 1's maximization problem is concave. The boundary conditions are not binding. For $p_1 = 0$, the expression reads $\varepsilon_1 g_2(0) + \alpha_1 > 0$; for $p_1 \geq 1$ and $\varepsilon_1 \in [0, 1)$, it reads $\varepsilon_1 g_2(p_1) - p_1 g_2(p_1) < 0$. (Since $p_2 = \varepsilon_2$, $G_2(1) = 1$ and the upper boundary condition is equal to zero for $\varepsilon_1 = 1$.) Hence, bidder 1's price p_1 is given by the first-order condition. Q.E.D.

As $[1 - G_2(p_1^*)]/g_2(p_1^*)$ is positive for $\varepsilon_1 \in [0, 1)$, bidder 1 overbids for all values of $\varepsilon_1 \in [0, 1)$. For $\varepsilon_1 = 1$, which occurs with probability zero, there is no overbidding. This is because bidder 1 cannot hope to be competing against a bidder 2 type who might bid even higher, as $p_2 = \varepsilon_2 \in [0, 1]$.

PROPOSITION 3: *Bidder 1's equilibrium bid p_1^* increases with his valuation, but the extent to which he overbids decreases with his valuation.*

PROPOSITION 4: *Bidder 1's equilibrium bid p_1^* increases with the size of the initial stake.*

The proofs are given in the appendix. If bidder 1 has a low valuation, he has a small chance of winning the contest. Consequently, when he overbids, the risk of having to pay a price that is in excess of ε_1 is relatively small, compared to the expected higher return on the sale of his toehold. As the bidder's valuation increases, the risk of winning also increases, inducing him to overbid to a smaller extent. Consider next how the size of bidder 1's toehold affects his incentives to overbid. Increasing the bid from $p_1 \geq \varepsilon_1$ to $p_1' > p_1$ gives bidder 1 an additional return of $\alpha_1(p_1' - p_1)$, when he loses the bidding contest. This gain increases with the size of the toehold. The risk associated with bidding p_1' instead of p_1 is that bidder 2's valuation is lower than p_1', but higher than p_1. The outcome of the bidding contests is then reversed. Relative to bidding p_1, bidder 1 makes a loss equal to $\alpha_1 p_1 - [\varepsilon_1 - (1 - \alpha_1)\varepsilon_2] = \varepsilon_2 - \varepsilon_1 - \alpha_1(\varepsilon_2 - p_1)$. This loss decreases with the size of the toehold, since the remaining shares which he needs to acquire also decrease. Thus, a larger initial stake increases the gains and reduces the losses associated with submitting a higher bid, encouraging bidder 1 to bid more aggressively.

COROLLARY 1: *Conditional on winning, bidder 1's expected profit per share on the tendered shares decreases with the size of his toehold.*

The proof is given in the appendix. Corollary 1 follows from Proposition 4. For a given ε_1, bidder 1's optimal bid increases with the size of his toehold. As his bid forms the upper boundary of the integral that defines the expected price, which he has to pay as the winner, the expected purchase price also increases. Hence, the average differences between the bidder's valuation and the purchase price decreases with the size of his initial stake. It should be noted that the expected profit per share on the tendered shares is not bounded by zero, but can in fact be negative (Corollary 2). Furthermore, the expected loss on all tendered shares may even exceed the profits on the toehold. Conditional on winning the contest, bidder 1 makes an expected overall loss (Corollary 3).

COROLLARY 2: *An inefficient allocation occurs with positive probability.*

For $\varepsilon_1 < \varepsilon_2 < \varepsilon_1 + \alpha_1[1 - G_2(p_1^*)]/g_2(p_1^*)$, bidder 1 wins the auction and pays a price in excess of his valuation. Since $\varepsilon_1 < \varepsilon_2$, this outcome invalidates the common claim in the takeover literature that auctions lead to the highest-value use of the target's resources. The inefficiency of the auction mechanism in the presence of partial ownership is related to a result from the optimal auction literature.[12] The strategy which maximizes a seller's expected return is to announce a reservation price which is greater than his personal valuation. Obviously, this strategy can also lead to an inefficient allocation. If all the buyers' valuations are below the reservation price, the seller keeps the item even though some bidders' valuations exceed his own valuation.

COROLLARY 3: *Bidder 1's expected profits, conditional on acquiring the target, can be negative.*

Consider bidder 1 with a valuation $\varepsilon_1 = 0$. By Proposition 2 he overbids. When $\varepsilon_2 \in (0, \alpha_1[1 - G_2(p_1^*)]/g_2(p_1^*))$, he wins the bidding contests and makes a loss. Hence, his expected profits conditional on winning the contest are negative. By continuity, he also makes an expected loss if ε_1 is close to zero, when he wins. Imagine that the bids are submitted simultaneously but bidder 1's bid is revealed before bidder 2's bid. Consider the market reaction when the second bid is revealed in those cases where ε_1 is low. For the shareholders of bidder 1, it would be good news when bidder 1 loses the contest (positive average price reaction) and bad news when he wins (negative average price reaction), even though the bid maximizes the shareholders' expected payoff. However, the expected profits conditional on winning the auction are only negative if bidder 1's valuation is low. For bidder 1 with a high ε_1, the profits on the initial stake cover potential losses on the tendered shares, when he wins the auction.

It should be noted that Corollaries 2 and 3 describe possible rather than necessary outcomes. In all realizations other than $\varepsilon_1 < \varepsilon_2 < \varepsilon_1 + \alpha_1[1 - G_2(p_1^*)]/g_2(p_1^*)$, the bidding contest neither leads to an inefficient outcome nor would bidder 1 be better off selling his toehold, in the event of his winning.

PROPOSITION 5: *Bidder 1's expected profits increase with the size of his toehold.*

The result follows immediately from a comparison of two bidders A and B with toeholds α_A and α_B, respectively. Assume that $\alpha_A < \alpha_B$, and assume further that each bidder competes against an identical third party in two separate bidding contests. If bidder B adopts bidder A's optimal strategy, $p^*(\alpha_A)$, then his expected return will be higher than A's. He either wins and needs to buy

[12] Cf. Riley and Samuelson (1981).

fewer shares at the same expected price, or he loses and sells more shares at $p^*(\alpha_A)$. Since his expected return from his optimal bid $p^*(\alpha_B)$ must be at least as large as from bidding $p^*(\alpha_A)$, bidder B's expected return is greater than A's.

The papers by Dewatripont (1991) and Ravid and Spiegel (1992) consider an identical one-toehold setting, although under full information. In contrast to this paper, they exogeneously rule out the possibility of overbidding. In the full information setting, this is tantamount to selecting, somewhat arbitrarily, one of a continuum of Nash equilibria. By way of demonstration, consider a second-price sealed-bid auction under full information where bidder 1 owns a toehold and $\varepsilon_1 < \varepsilon_2$. Next, assume again that bidder 2 wins if there is a tie, i.e. $p_1 = p_2$. The discussion of Lemma 4 implies that there is a continuum of Nash equilibria, where bidder 2 wins the auction by offering any price in the range $[\varepsilon_1, \varepsilon_2]$ and bidder 1 matches his offer. For example, if bidder 1 believes that bidder 2 will not bid more than $(\varepsilon_1 + \varepsilon_2)/2$ his best response is to offer $(\varepsilon_1 + \varepsilon_2)/2$ as well. Hence, in all but one equilibrium, bidder 1 overbids. Among these equilibria is a unique one that satisfies the trembling-hand refinement criterion: the requirement that no player chooses a weakly dominated strategy fixes bidder 2's bid to his valuation. Bidder 1's best response is also to bid ε_2. Although bidder 2 wins the auction, all the surplus goes to the tendering shareholders due to bidder 1's aggressive overbidding strategy. The full surplus extraction outcome is the only trembling-hand perfect equilibrium, further illustrating the robustness of overbidding.[13]

D. An Example

To illustrate the results of the one-toehold case, consider a specific distribution that allows us to derive an explicit expression for p_1^*. For simplicity, ε_i, $i = 1, 2$, is independently and uniformly distributed on the [0, 1] interval so that $F_2(p_1) = G_2(p_1) = p_1$ and $g_2(\cdot) = 1$. Equation (1) can be rewritten as

$$p_1^* = \varepsilon_1 + \frac{\alpha_1}{1 + \alpha_1}(1 - \varepsilon_1)$$

Inspection of the explicit expression for p_1^* illustrates the previous results. The optimal bid price exceeds the bidder's valuation and increases with the size of the initial stake, while the extent of overbidding decreases with the bidder's valuation. Figure 1 gives a comprehensive picture of the extent of overbidding by plotting $p_1^* - \varepsilon_1$ as a function of $\varepsilon_1 \in [0, 1]$ and $\alpha_1 \in [0, 1/2]$.

In the example with the uniform distribution functions, an explicit solution can be derived for the special case where both bidders have identical toeholds, i.e., $\alpha_1 = \alpha_2 \neq 0$. The optimal bid p_i^*, $i = 1, 2$, is derived (see Appendix) under

[13] If one models the bidding contest as a common English auction, the full surplus extraction outcome is the only equilibrium. The uniqueness is due to the fact that the sequential bidding process leaves bidder 2 unable to credibly commit to withdrawing from the auction while the price is below ε_2.

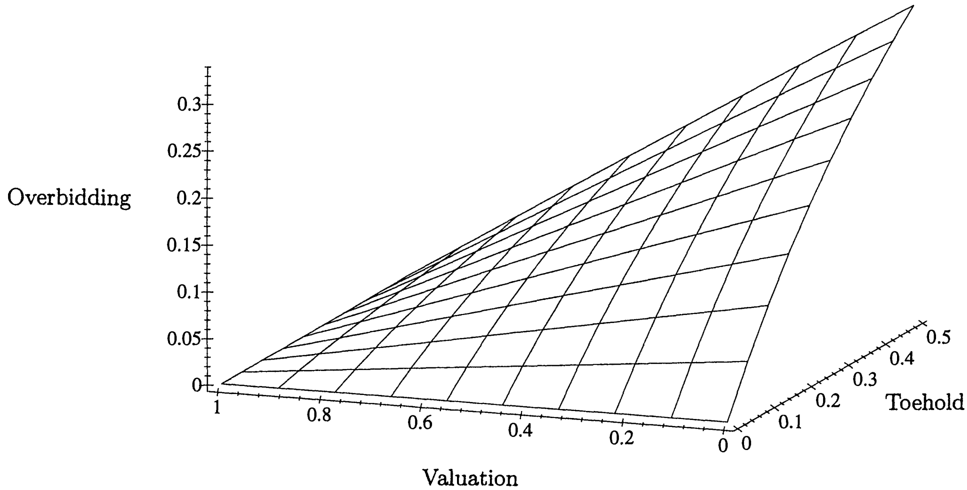

Figure 1. Overbidding in the one-toehold case with uniform distributions. Overbidding by bidder 1, measured by the difference between his optimal bid p_1^* and his valuation ε_1, is plotted as a function of his valuation $\varepsilon_1 \in [0, 1]$ and his initial stake $\alpha_1 \in [0, ½]$.

the assumption that both bidders adopt a common equilibrium strategy, and equilibrium bids increase with the valuation.

$$p_i^* = \varepsilon_i + \frac{\alpha_i}{(1 + \alpha_i)}(1 - \varepsilon_i) + \frac{K}{\alpha_i}\frac{1}{(1 - \varepsilon_i)^{1/\alpha_i}}$$

The imposed monotonicity restricts the constant of integration K to nonnegative values. Hence, both bidders overbid in all equilibria of the symmetric auction. Although overbidding always takes place in the symmetric case, the possibility of an inefficient outcome ceases to exist. When both parties own an identical toehold and bid according to the same strategy, the bidder with the higher valuation always wins.

By requiring the bids to be in the range $[0, \infty]$ for all $\varepsilon_i \in [0, 1]$, a unique equilibrium can be selected where $K = 0$. In this case, both parties adopt the same overbidding strategy as bidder 1 in the one-toehold example. This surprising result is due to the fact that the presence of bidder 2's identical toehold increases bidder 1's expected marginal gain and his expected marginal loss by a factor $(1 + \alpha)$, which leaves the ratio between them unchanged. When bidder 2 has the same toehold as bidder 1, he bids according to a truncated uniform distribution function $F(x) = (1 + \alpha)x - \alpha$ with support $[\alpha/(1 + \alpha), 1]$. The truncation of bidder 2's bid distribution gives rise to two countervailing effects. On the one hand, it induces bidder 2 to overbid. For a given p_1, this increases the probability that bidder 1 will be the seller from $1 - p_1$ to $(1 + \alpha)(1 - p_1)$. On the other hand, the risk from overbidding also increases. When bidder 1 overbids by Δ, the probability that the opponent's bid will be lower than $\varepsilon_1 + \Delta$ but higher than ε_1, increases from Δ to $\Delta\ (1 + \alpha)$. Due to the linear relationship between bid and valuation, which is specific to the uniform distribution, these two effects cancel each other out, leaving the optimal bid unchanged. Finally, it should be noted that $p_i = \varepsilon_i + [\alpha_i/(1 + \alpha_i)](1 - \varepsilon_i)$, $i = 1, 2$ is optimal only if both bidders have the same initial stake. For different toeholds, e.g., $\alpha_1 > \alpha_2 > 0$, the formula would imply that for low realizations of ε_2, bidder 2's bid is lower than bidder 1's lowest bid, which clearly is not an equilibrium.

E. Partial Offers and Bidding Costs

This section considers the robustness of the overbidding results with respect to the chosen bid form and the introduction of bidding costs. An alternative to an unrestricted offer is a partial bid for ($\frac{1}{2} - \alpha_i$) of the shares, which also excludes the rival from participating in the offer.[14] In such a partial-bid contest, none of the bidders has an interest in increasing the price unless winning is the aim. Thus, competition with exclusionary bids is essentially reduced to an auction without toeholds. However, none of the major takeover regulations permits discriminatory offers. In 1986, the Security and Exchange Commission (SEC) banned exclusionary tender offers,[15] and partial bids now come with the obligation to accept shares on a prorated basis from all tendering shareholders. Therefore, losing bidders are able to sell part of their initial shareholdings and consequently benefit from their own overbidding. Compared to unrestricted offers, a switch to partial offers gives rise to two opposite effects. On the one hand, overbidding becomes less attractive, since the losing bidder is only able to sell half of his initial shareholdings. On the other hand, overbidding is less risky since bidder i needs to acquire only the fraction ($\frac{1}{2} - \alpha_i$), thereby reducing the costs of winning if he overbids. Which of these two effects will prevail depends upon the choice of parameters.

Compared to the costs involved in preparing an initial offer, the cost of revising a bid seems negligible, and therefore zero-bidding costs have been assumed. However, some auction forms are known to be highly sensitive to the introduction of small bidding costs. For example, Daniel and Hirshleifer (1993) show that the competitive bidding solution in the common English auction relies on the extreme singularity of zero-bidding costs.[16] In contrast, the overbidding effect caused by initial shareholdings is robust with respect to bidding costs.

[14] Partial offers are legal in the United States except in Pennsylvania and Maine. Cf. Karpoff and Malatesta (1989). The U.K. City Code (Rule 9) requires parties who purchase more than 30 percent of the voting securities to extend the offer to the remaining shareholders. The proposed E.C. threshold (Rule 4) for mandatory bids is 33.3 percent. The imposition of this buy-out requirement effectively prevents partial bids aimed at obtaining control.

[15] This amendment of the SEC rules was a reaction to the Delaware Supreme Court's decision to uphold the legality of discriminatory offers to repurchase shares by Unocal, Great Lakes International and Revlon. Cf. Kamma et al. (1988). The requirement to treat all shareholders of the same class equally in the U.K. City Code (General Principle 1) and the proposed E.C. Directive (Rule 6a) is a rule to the same effect.

[16] In the context of takeovers, Hirshleifer and Png (1989) demonstrate how the introduction of bidding costs can reverse the result in a framework where bidders have to incur costs to learn their valuation prior to the bidding.

For example, consider a bidding contest between two fully informed bidders where $\varepsilon_2 > \varepsilon_1$. In the absence of toeholds and bidding costs, the familiar dominant-strategy solution is that bidder 2 wins and pays a price equal to ε_1. In addition, there is a multiplicity of Nash equilibria in which bidder 2 pays less than ε_1. The multiplicity stems from bidder 1's indifference about whether he loses at a low or high price. When bidding is costly, however, bidder 1 strictly prefers not to participate at all, since he is certain to lose. Hence, bidder 2 wins the auction at a zero price.

The introduction of small bidding costs in an auction with toeholds does not cause the selling price to drop to zero. The losing bidder with the toehold is no longer indifferent about the selling price. Although he is certain to lose, bidder 1 can earn a positive return by ensuring that the price reaches at least ε_1. Staying out of the auction is costly, even if ε_1 is very small, since this would mean selling his stake at a zero price. Bidding costs do not affect these return prospects, but merely add a lump-sum cost. Provided that the bidding costs are lower than the return on the sale of the toehold, they do not prevent bidder 1 from participating. Thus, arbitrarily small bidding costs have a large impact on the equilibrium price in a common auction, but not in a contest with toeholds. Toeholds retain this effect in an asymmetric information setting, although bidders will overbid more cautiously because of the risk of winning the contest instead of merely increasing the price.

III. Further Applications

A. Large Shareholders

An alternative interpretation of the overbidding result involves a firm owned by one large shareholder and many small ones which attracts a single outside raider. Once the firm is under raider's control, he receives private benefits $N\varepsilon$, while the security benefits are zero. This corresponds to the special case where bidder 1 with a toehold has a valuation $\varepsilon_1 = 0$, which is common knowledge. When the raider makes his move, the large shareholder will challenge him in order to induce a counteroffer with the aim of raising the selling price. Consequently, the successful outside raider is forced to share part of his private benefits with the shareholders. This does not occur when the target's ownership structure is completely dispersed. Since atomistic shareholders tender at any price above the post-takeover minority share value, the outsider only needs to offer marginally more than zero. Thus, a large shareholder mitigates the free-rider problem of extracting a single bidder's private gains. It should be noted that this free-rider problem differs from the one discussed by Grossman and Hart (1980), although Shleifer and Vishny (1986) show that a large party which internalizes the takeover costs also solves their free-rider problem.

Providing the takeover eventuates, small shareholders benefit from the large shareholder, since the latter extracts private benefits from the bidder. However, the large shareholder's presence has also the negative effect of preventing some takeovers. The prospect of the large shareholder counterbidding and thereby raising the purchasing price renders some value-increasing bids unprofitable. Only an outsider with a high valuation which ensures that a takeover will be profitable despite the counterbidding, will attempt to acquire the company.[17] Thus, the presence of a large shareholder involves a trade-off between the takeover premium and the probability of an attempted takeover. The premium increases and the probability decreases with the size of the large shareholder's stake. This suggests that the size of the toehold which maximizes the firm value is neither zero nor one, but lies strictly between zero and one. This implication parallels the result by Stulz (1988).

Instead of being a passive investor, the large shareholder might be the incumbent management which may also derive some private benefits from being in control. In the control contest, the incumbent management adopts the overbidding strategy,[18] making the takeover more costly to the outside raider. Hence, equity-ownership facilitates management entrenchment not only by directly controlling votes but also by providing incentives to overbid. It should be noted that in this simple framework, the shareholders unambiguously benefit from management entrenchment. If the incumbent management were not to counterbid, the bidder would not need to offer a premium, and the shareholders would always receive zero return. Although the premium offered in a takeover increases with the incumbent's equity-ownership (and private benefits of control), the probability of a takeover contest decreases. Thus, management entrenchment may reach levels beyond the shareholders' interests.

B. Liquidation Procedures

A further context in which overbidding has some bearing is the liquidation process of Chapter 7 of the U.S. Bankruptcy Reform Act of 1978. When a firm files to liquidate under Chapter 7, a court-appointed trustee sells the firm's assets and distributes the proceeds to the creditors on the basis of the Absolute Priority Rule. The trustee organizes the sale by means of soliciting cash bids for the firm's assets and anyone is free to participate in the auction. Auctions in bankruptcy proceedings are advocated on the grounds that they lead to efficient outcomes, i.e., that the firm's assets are sold to the bidders with the highest-value use. However, Aghion *et al.* (1992) point out that problems with financing and the lack of competition are likely to prevent auctions from being efficient in bankruptcy procedures. This paper provides further support for this claim. When a bankrupt firm's assets are auctioned off, the firm's creditors are in a similar position to that of a large shareholder. Liquidation proceeds and hence the extent to which they can recover their loans depend upon the selling price. As net sellers, therefore, they have a strong interest in adopting

[17] To ensure that takeovers are feasible in this setting, one needs to assume that the outsider owns a stake prior to the tender offer. Cf. Shleifer and Vishny (1986).

[18] Incumbent managers have various means at their disposal to resist a hostile tender, such as poison pills or sale of crown jewels. Thus, launching a MBO is only one among many defensive measures incumbent managers can adopt. Nevertheless, MBOs are often a response to a hostile takeover threat. See Shleifer and Vishny (1988).

a price-raising strategy, i.e., in submitting a bid with the sole purpose of raising the purchasing price. This is particularly true of junior or unsecured creditors as they cannot rely on recovering their initial claims in full.

C. An Extension with Search Costs

When considering a takeover, a potential buyer typically invests resources in searching for an appropriate target and preparing the actual bid. He incurs most of these costs before knowing how efficiently he can redeploy the target's assets. To take this into account, the one-toehold model with uniform distributions (example II.*D*.) is extended by a search decision prior to the bidding. Each bidder must incur a cost in order to learn his valuation of the target and can only bid if he has done so.[19] The extended game, which has been borrowed from Fishman (1988), unfolds as follows. First, bidder 1 can pay the search cost and observe the realization of ε_1. If he decides to make a takeover bid, he buys a fixed amount α_1 of the target's shares at a price 0 on the open market and then makes an initial offer p_1. After observing p_1, bidder 2 decides whether to learn his valuation ε_2. Finally, if bidder 2 joins the takeover contest, an English auction takes place.

The sequential nature of the search process may lead bidder 1 to preempt bidder 2. By submitting a high initial price p_1, he reduces bidder 2's expectation of winning the auction and thereby deters him from taking part in the contest. However, preemption is only profitable for bidder 1 if he has a high valuation. Thus, the extended game has two possible outcomes, preemption or a bidding contest. Preemption occurs if bidder 1's valuation ε_1 lies above the critical value $\varepsilon_1^{\mathrm{p}}$. In this case, bidder 1 submits a high initial offer p_1^{p}. Bidder 2 observes this high offer, updates his beliefs about ε_1 and finds that his expected profits fall short of the search costs. Consequently, he abstains from bidding and bidder 1 takes over the target at the preemptive bid price p_1^{p}. Alternatively, a bidding contest takes place when ε_1 is less than the critical value $\varepsilon_1^{\mathrm{p}}$. Bidder 1 offers a low initial price which encourages bidder 2 to incur the search cost, and an auction ensues. Given that the bidders' valuations are uniformly distributed on [0, 1], the auction has the following outcomes: for $\varepsilon_2 > p_1^* = (\alpha_1 + \varepsilon_1)/(1 + \alpha_1)$, bidder 2 wins the bidding contest and takes over the target at p_1^*. If $p_1^* > \varepsilon_2$, bidder 1 wins the auction. For $\varepsilon_1 < \varepsilon_2 < p_1^*$, bidder 1's overbidding strategy leads to an inefficient allocation.

Thus, overbidding also occurs with positive probability in a framework in which the sequential search process introduces the possibility of preemption. Since only bidder 1 with a low valuation is challenged, bidder 1 with a high valuation does not incur the risk of overbidding. In comparison with the model presented in Section II.*D*, this implies that bidder 2 will win the auction more often, provided it takes place. Furthermore, the qualitative results are robust with respect to bidder 2 owning the toehold, or both parties owning an identical toehold.

[19] Given that the search costs are not excessively high, this follows endogeneously. That is, both bidders prefer to incur the search cost rather than to submit a bid without knowing their valuation. For the full analysis of the extended model, see Burkart (1993).

This extension of the model also illustrates how toeholds can cause inefficiencies, apart from inefficient auction outcomes. In fact, inefficiencies may arise due to toeholds facilitating preemption. Bidder 1's more aggressive bidding reduces bidder 2's expected gain since the latter has to pay a higher price if he wins the contest. Bidder 2's diminished prospects of making profits are reflected by the negative relationship between the preemptive price p_1^p and the toehold α_1. Hence, the larger bidder 1's toehold, the lower his valuation needs to be in order to deter potential rivals. This in turn reduces the frequency of bidding competitions which consequently reduces the likelihood of the target being taken over by the party with the higher valuation.

IV. Empirical Evidence

The paper shows that the ownership of an initial stake induces bidders to overbid. This has several empirical implications for the bid premium in contested takeovers, the expected gains of successful bidders, and the likelihood of a competing bid. First, the purchasing price of a target, i.e., the bid premium, is on average higher in takeover contests where at least one bidder owns a toehold. Second, conditional on winning, the successful bidder's average profit per share on the tendered shares decreases with the size of his toehold (Corollary 1). Third, successful bidders who own an initial stake are more likely to realize a net loss in multiple-bidder contests than successful bidders without a toehold (Corollary 3). Fourth, the level of the initial bid necessary to preempt rival bidders decreases with the size of the toehold (Section III.*C*.). Fifth, counterbids from the incumbent management teams increase with the size of their equity-ownership (Section III.*A*.). Despite the abundance of empirical papers on takeovers, there appears to be no direct test of any of the above predictions. Consequently, when examining the empirical validity of the overbidding proposition, one has to rely on indirect evidence. In the following paragraphs, each prediction will be discussed and related to the existing empirical evidence.

The higher returns earned by target shareholders in multiple-bidder contests are well documented, e.g., Bradley *et al.* (1988). However, studies typically only distinguish between single-bidder and multiple-bidder contests, without specifying in the latter case the size of any initial stakes. In their extensive study of U.K. acquisitions, Franks and Harris (1989) provide some evidence for the prediction that bidding contests with initial stakes result in higher premia. For the subsample of single-bidder takeovers, they find that toeholds lead to higher premia provided that they are below 30 percent. This toehold effect is smaller for the subsample of single-bidders than for the whole sample, implying that it is more pronounced in multiple-bidder subsample.[20]

[20] This conflicts with the findings of Stulz et al. (1990) who report that the bidder's pre-tender offer holdings have a negative effect on the target's gains. However, the negative effect is neither significant for the single-bidder subsample nor for the multiple-bidder subsample. The authors suggest that the negative impact in the whole sample may be caused by the systematic difference in toeholds and target gains across subsamples. Single-bidders have significantly higher pre-tender offer holdings while average target gains are lower in single-bidder contests.

To test the hypothesis that the profit per share on tendered shares decreases with the size of the winner's toehold, it is necessary to observe the value of the target firm under the winning bidder's control. Unfortunately, this is usually not possible for full bids as the target firms are delisted. An alternative would be to look at bidding contests which only involve partial bids, where some target shares continue to be traded on the market, thereby providing information about the target's value under the new owner.

The possibility that successful bidders earn negative stock returns is an empirical phenomenon which has been well documented, although it has not yet been related to the existence of toeholds. For the period 1968–86, Stulz *et al.* (1990) find an average abnormal return on the acquiring firm's stock of +1.3 percent for a single-bidder takeover but an average abnormal return of −4.2 percent for multiple-bidder contests. However, neither is significantly different from zero. Bradley *et al.* (1988) report similar results in their study of the period 1963–84. For the subperiod 1981–84, the average abnormal return for acquiring firms is negative for both single- and multiple-bidder contests, though only significant for the latter: The significant losses that winners of multiple-bidder contests suffered during the 1980s have been attributed to an increase in competition among bidding firms. However, fierce competition alone cannot explain losses but only account for a reduction in takeover profits. Although the present model does not predict average losses for acquiring firms, it provides an explanation for the empirical phenomenon that successful bidders earn negative returns in some instances. Given that overbidding does not necessarily lead to negative returns, the most appropriate test would involve case studies that might reveal whether the aim of overbidding was to provoke a higher counteroffer. Kaplan (1989) analyzes Campeau's takeover of Federated. Campeau owned an initial stake in Federated and its final offer exceeded, according to Kaplan's estimates, the value of the assets under its control by 25 percent. To overbid by such a large extent may be perfectly rational within the present model. For a bidder with a large initial stake and a low valuation, the ex-ante optimal offer entails substantial overbidding. However, the 40 percent increase in Federated's value under Campeau's control indicates that Campeau's valuation was relatively high, while its initial stake was only .5 percent. For this reason, Campeau's overbidding is difficult to reconcile with the model's optimal bidding behavior.

The fourth prediction shows that initial stakes facilitate preemption. Empirically, for a given initial bid, a negative correlation would be expected between the likelihood of a competing bid and the size of the initial bidder's toehold. Although the link between initial bid and competition has been analyzed,[21] the effect which toeholds have on the likelihood of a competing bid has not been tested. However, Stulz *et al.* (1990) provide some tentative support for the hypothesis that toeholds effectively deter rivals. Their findings show that single bidders have significantly larger initial stakes and secure a

[21] For example, Jennings and Mazzeo (1993) find that high initial premia offers are associated with a lower likelihood of competing offers.

larger share of the total takeover gains which is consistent with this interpretation. The evidence from the United Kingdom mentioned above, however, does not support this view. Franks and Harris (1989) find a positive relationship between bid premia and toeholds in single-bidder takeovers.

In their study of large management buyouts, Kaplan and Stein (1993) find support for the view that large initial stakes lead incumbent management teams to bid more aggressively. They report an increase during the 1980s in the offered buyout prices and the initial stakes owned by management. As the share of buyouts subject to hostile pressure remained constant over time, the price rise cannot be explained by increased competition. Rather, the positive correlation between takeover prices and toeholds can be interpreted as overbidding due to the presence of toeholds. Kaplan and Stein do not consider this explanation but attribute the price rise to the increasing amount that managers cashed out at the time of the buyout.

V. Conclusions

The paper shows that the introduction of toeholds into a bidding contest leads to overbidding by a party owning an initial stake. As a result of overbidding, the bidder with the lower valuation buys the target with positive probability. Furthermore, overbidding also leads to the likelihood of the winning bidder making a loss. As shown, overbidding is neither specific to the one-toehold case nor bidders having incomplete information. Moreover, the phenomenon can arise in settings other than multiple-bidder takeover contests, e.g., in the liquidation of failed businesses. Finally, overbidding also occurs with positive probability in an extended model where, prior to the auction, both parties incur costs to learn their valuations.

The overbidding result invalidates the common claim in the takeover literature that auctions ensure the highest-value use of the resources of the acquired firms. Consequently, overbidding does not merely lead to redistribution in favor of the target shareholders but also has implications for efficiency. The regulatory implication is that overbidding can provide a further rationale for tight disclosure rules. A low disclosure threshold limits the initial stake that a bidder can acquire prior to submitting a tender offer. This restricts not only the ease with which he can preempt, but also his tendency to overbid. However, it should be noted that disclosure rules also have a negative impact on the efficiency of an economy. By limiting bidders' profits, tighter disclosure rules curb incentives to search for suitable targets, reducing the frequency of efficiency-enhancing takeovers. The potential for overbidding is only one of many arguments which must be considered when formulating disclosure rules.

Appendix

A. Proof of Proposition 1

Consider bidder 1 and regard $\varepsilon_1 \in [0, 1)$ and $p_1 = p_1(\varepsilon_1)$ as given numbers, while ε_2 and $p_2 = p_2(\varepsilon_2)$ are viewed as random variables with respective c.d.f.'s $G_2(x)$, $0 \leq x \leq 1$, and $F_2(\cdot)$. The following two relations hold. *i)* $F_2(\varepsilon_1 + \delta) - F_2(\varepsilon_1) \to 0$ as $\delta \downarrow 0$, *ii)* $\exists\ \delta_0 > 0$ such that $[1 - F_2(\varepsilon_1)]\alpha_1 - [F_2(\varepsilon_1 + \delta) - F_2(\varepsilon_1)] > 0$ for $0 < \delta \leq \delta_0$. The claim in *i)* is a consequence of the fact that a c.d.f. is continuous to the right. The claim in *ii)* is proved as follows. Since $p_2 \geq \varepsilon_2$, $F_2(x) \leq G_2(x)$, $0 \leq x < 1$, which yields $1 - F_2(x) \geq 1 - G_2(x)$. By assumption $1 - G_2(x) > 0$ for all $x < 1$. This in combination with $\alpha_1 > 0$ and *i)* implies that *ii)* holds.

Lemma 2 implies that $p_1 \geq \varepsilon_1$. It will be shown that there is a bid $p_1 = \varepsilon_1 + \delta$, where $\delta > 0$, which is strictly better than $p_1 = \varepsilon_1$. To this end, compare bidder 1's expected payoff $E\Pi_{p_1=\varepsilon_1+\delta}$ when overbidding by δ, where δ is presumed to lie on the interval $0 < \delta \leq \delta_0$, with his expected payoff $E\Pi_{p_1=\varepsilon_1}$ when bidding according to his valuation. To avoid ties, it is assumed that $F_2(\cdot)$ is continuous at ε_1 as well as at $\varepsilon_1 + \delta$. This can be justified by the following argument. Since $F_2(\cdot)$ is nondecreasing, at most it has countably many discontinuity points. Therefore, the probability is zero that ε_1, with a nonatomic distribution on [0, 1], should coincide with a discontinuity point for $F_2(\cdot)$. Moreover, on a δ-set which is dense on $0 < \delta \leq \delta_0$, $\varepsilon_1 + \delta$ keeps away from $F_2(\cdot)$'s discontinuity points.

If bidder 1 wins the auction, his return equals the value of the target firm under his control minus the price he has to pay for the $(1 - \alpha_1)N$ shares, i.e., $N[\varepsilon_1 - (1 - \alpha_1)p_2]$. If he loses the contest, his return is equal to $\alpha_1 N p_1$. Hence, the expected return from bidding $\varepsilon_1 + \delta$, normalized by $1/N$ is equal to

$$
\begin{aligned}
&E\Pi_{p_1=\varepsilon_1+\delta} \\
&= [1 - F_2(\varepsilon_1 + \delta)]\alpha_1(\varepsilon_1 + \delta) + F_2(\varepsilon_1 + \delta)\varepsilon_1 - (1 - \alpha_1)\int_0^{\varepsilon_1+\delta} p_2\, dF_2(p_2) \\
&\geq [1 - F_2(\varepsilon_1 + \delta)]\alpha_1(\varepsilon_1 + \delta) + F_2(\varepsilon_1 + \delta)\varepsilon_1 - (1 - \alpha_1)\int_0^{\varepsilon_1} p_2\, dF_2(p_2) \\
&\quad - (1 - \alpha_1)(\varepsilon_1 + \delta)[F_2(\varepsilon_1 + \delta) - F_2(\varepsilon_1)] \\
&= [1 - F_2(\varepsilon_1)]\alpha_1\varepsilon_1 + F_2(\varepsilon_1)\varepsilon_1 - (1 - \alpha_1)\int_0^{\varepsilon_1} p_2\, dF_2(p_2) \\
&\quad + \delta\{[1 - F_2(\varepsilon_1)]\alpha_1 - [F_2(\varepsilon_1 + \delta) - F_2(\varepsilon_1)]\} \\
&> [1 - F_2(\varepsilon_1)]\alpha_1\varepsilon_1 + F_2(\varepsilon_1)\varepsilon_1 - (1 - \alpha_1)\int_0^{\varepsilon_1} p_2\, dF_2(p_2) = E\Pi_{p_1=\varepsilon_1}
\end{aligned}
$$

The last strict inequality follows from the choice of δ. Q.E.D.

B. Proof of Proposition 3

i) As ε_1 and ε_2 are i.i.d. and $p_2 = \varepsilon_2$, G_2 does not depend on ε_1. Rewrite equation 1 as

$$\varepsilon_1 = p_1^* - \alpha_1 \frac{[1 - G_2(p_1^*)]}{g_2(p_1^*)}.$$

Both terms on the RHS increase with p_1^*. *ii)* The extent of overbidding, $p_1^* - \varepsilon_1$, is equal to

$$p_1^* - \varepsilon_1 = \alpha_1 \frac{[1 - G_2(p_1^*)]}{g_2(p_1^*)}.$$

From *i)* it follows that as ε_1 increases, so does p_1, while the RHS decreases with p_1^*. Q.E.D.

C. Proof of Proposition 4

As ε_1 and ε_2 are i.i.d. and $p_2 = \varepsilon_2$, G_2 does not depend on α_1. Rewrite Equation 1 as

$$\alpha_1 = (p_1^* - \varepsilon_1) \frac{g_2(p_1^*)}{[1 - G_2(p_1^*)]}.$$

Each factor of the RHS is positive and increasing in p_1^*. Q.E.D.

D. Proof of Corollary 1

The expected profit per share on the tendered shares for a successful bidder 1 is given by

$$\varepsilon_1 - \frac{\int_0^{p_1(\alpha_1)} \varepsilon_2 \, dG_2(\varepsilon_2)}{G_2(p_1)}.$$

By Proposition 4 $\partial p_1/\partial \alpha_1 > 0$, and it is only necessary to show that

$$\frac{\int_0^{p_1} \varepsilon_2 g_2(\varepsilon_2) \, d\varepsilon_2}{G_2(p_1)}$$

increases with p_1. Differentiating with respect to p_1

$$= \frac{p_1 g_2(p_1) G_2(p_1) - g_2(p_1) \int_0^{p_1} \varepsilon_2 g_2(\varepsilon_2) d\varepsilon_2}{[G_2(p_1)]^2}$$

$$= \frac{g_2(p_1)}{[G_2(p_1)]^2} \left[p_1 G_2(p_1) - \int_0^{p_1} \varepsilon_2 g_2(\varepsilon_2)\, d\varepsilon_2 \right]$$

$$= \frac{g_2(p_1)}{[G_2(p_1)]^2} \int_0^{p_1} (p_1 - \varepsilon_2)\, dG_2(\varepsilon_2) > 0.$$

E. Example in the symmetric case

Given that ε_i, $i = 1, 2$, is uniformly distributed on the unit interval, and that $\alpha_1 = \alpha_2 \neq 0$, bidder i's maximization problem is:

$$\max_{p_i} \alpha_i p_i [1 - F_j(p_i)] + \left(\alpha_i \varepsilon_i + (1 - \alpha_i) \left(\varepsilon_i - \frac{\int_a^{p_i} x f_j(x)\, dx}{F_j(p_i)} \right) \right) F_j(p_i).$$

Differentiating with respect to p_i yields the first order condition:

$$p_i^* = \varepsilon_i + \frac{\alpha_i [1 - F_j(p_i)]}{f_j(p_i)}.$$

Looking for symmetric equilibria in strategies which increase with the valuation ε_i, $p'(\varepsilon_i) > 0$, implies: $p_i = h_i(\varepsilon_i) \Leftrightarrow h_i^{-1}(p_i) = \varepsilon_i$. The monotonicity of the bidding strategy and the assumed symmetric behavior by the bidders yields:

$$F_j[p_i(\varepsilon_i)] = F_j[h_i(\varepsilon_i)] = G_j[h_i^{-1}(p_i)] = G_j(\varepsilon_i) = \varepsilon_i = G_i(\varepsilon_i) = F_i[p_i(\varepsilon_i)]$$

and

$$\frac{\partial F_j[p_i(\varepsilon_i)]}{\partial \varepsilon_i} = f_j[p_i(\varepsilon_i)] p_i'(\varepsilon_i) = 1 \Rightarrow f_j[p_i(\varepsilon_i)] = \frac{1}{p_i'(\varepsilon_i)}.$$

Substituting the uniform distribution function and rearranging yields:

$$p_i'(\varepsilon_i) \alpha_i (1 - \varepsilon_i) - p_i = -\varepsilon_i$$

Multiplying both sides by $(1 - \varepsilon_i)^{(1-\alpha_i)/\alpha_i}$ makes this an exact differential equation of the form

$$(\varepsilon_i - p_i)(1 - \varepsilon_i)^{(1-\alpha_i)/\alpha_i} d\varepsilon_i + \alpha_i (1 - \varepsilon_i)^{1/\alpha_i}\, dp_i = 0.$$

The solution to this exact differential equation is

$$F(\varepsilon_i, p_i) = -\alpha_i(\varepsilon_i - p_i)(1 - \varepsilon_i)^{1/\alpha_i} - \frac{\alpha_i^2}{(1 + \alpha_i)} (1 - \varepsilon_i)^{\frac{(1+\alpha_i)}{\alpha_i}} + K_1 = K_2.$$

Denote $K = K_2 - K_1$ and solve for p_i

$$p_i = \frac{\varepsilon_i}{(1 + \alpha_i)} + \frac{\alpha_i}{(1 + \alpha_i)} + \frac{K}{\alpha_i} \frac{1}{(1 - \varepsilon_i)^{1/\alpha_i}}.$$

The assumption that the bids strictly increase with the bidders' valuations requires that

$$\frac{\partial p_i}{\partial \varepsilon_i} = \frac{1}{(1 + \alpha_i)} + \frac{K}{\alpha_i^2} \frac{1}{(1 - \varepsilon_i)^{\frac{(1+\alpha_i)}{\alpha_i}}} > 0 \qquad \forall\, \varepsilon_i.$$

As the second term tends to infinity when ε_i tends to 1, $K \geq 0$ in order to be consistent with the assumption of monotonic bids. The additional assumption that p_i should be finite over the whole range of $\varepsilon_i \in [0, 1]$ yields a unique equilibrium strategy with $K = 0$.

REFERENCES

Aghion, P., O. D. Hart, and J. Moore, 1992, The economics of bankruptcy reform, *Journal of Law, Economics, and Organization* 8, 523–546.

Bradley, M., A. Desai, and H. E. Kim, 1988, Synergistic gains from corporate acquisitions and their division between the stockholders of target and acquiring firms, *Journal of Financial Economics* 21, 3–40.

Burkart, M., 1993, Overbidding in takeover contests, Financial Markets Group Discussion Paper No. 180, London School of Economics.

Chowdhry, B., and V. Nanda, 1993, The strategic role of debt in takeover contests, *Journal of Finance* 48, 731–745.

De Mott, D. A., 1988, Comparative dimensions of takeover regulation, in J.C. Coffee, Jr., L. Lowenstein, and S. Rose-Ackerman, Eds.: *Knights, Raiders and Targets* (Oxford University Press, New York), 398–435.

Daniel, K., and D. Hirshleifer, 1993, A theory of costly sequential bidding, Working paper, University of Chicago.

Dewatripont, M., 1991, On takeover contests and stock price dynamics, Institut D'Analisi Economica Working Paper No. 91–4, Universitat Autonoma de Barcelona.

European Communities, Eds., 1988, Council Directive of 12 December 1988 on the information to be published when a major holding in a listed company is acquired or disposed of (88/627/EEC), *Official Journal of the European Communities* L 348, 62–65.

European Communities, Eds., 1990, Amended Proposal for a Thirteenth Council Directive on company law, concerning takeover and other general bids, *Official Journal of the European Communities* C 240, 7–30.

Fishman, M. J., 1988, A theory of preemptive takeover bidding, *Rand Journal of Economics* 19, 88–101.

Franks, J. R., and R. S. Harris, 1989, Shareholder wealth effects of corporate takeovers. The U.K. experience 1955–1985, *Journal of Financial Economics* 23, 225–249.

Grossman, S. J., and O. D. Hart, 1980, Takeover bids, the free-rider problem, and the theory of the corporation, *Bell Journal of Economics* 11, 42–64.

Hirshleifer, D., and I. P. L. Png, 1989, Facilitation of competing bids and the price of a takeover target, *Review of Financial Studies* 2, 587–606.

Jarrell, G. A., J. A. Brickley, and J. M. Netter, 1988, The market for corporate control: The empirical evidence since 1980, *Journal of Economic Perspectives* 2, 49–68.

Jegadeesh, N., and B. Chowdhry, 1989, Optimal pre-tender offer share acquisition strategy in takeovers, Working paper, Anderson Graduate Management School, University of California Los Angeles.

Jennings, R. H., and M. A. Mazzeo, 1993, Competing bids, target management resistance, and the structure of takeover bids, *Review of Financial Studies* 6, 883–909.

Kamecke, U., 1993, Dominance or maximin: How to solve an English auction, Discussion paper No. A-418, University of Bonn.

Kamma, S., J. Weintrop, and P. Wier, 1988, Investors' perceptions of the Delaware Supreme Court decision in Unocal v. Mesa, *Journal of Financial Economics* 20, 419–430.

Kaplan, S. N., 1989, Campeau's acquisition of Federated. Value destroyed or value added, *Journal of Financial Economics* 25, 191–212.

Kaplan, S. N., and J. C. Stein, 1993, The evolution of buyout pricing and financial structure in the 1980s, *Quarterly Journal of Economics* 108, 313–358.

Karpoff, J. M., and P. H. Malatesta, 1989, The wealth effects of second-generation state takeover legislation, *Journal of Financial Economics* 25, 291–322.

Milgrom, P. R., and R. J. Weber, 1982, A theory of auctions and competitive bidding, *Econometrica* 50, 1089–1122.

Panel on Takeovers and Mergers, eds., 1990, *The City Code on Takeovers and Mergers and The Rules Governing Substantial Acquisitions of Shares*, 3rd ed., (London).

Ravid, S. A., and M. Spiegel, 1992, On toeholds and bidding contests, Working paper, Columbia University.

Riley, J. G., and W. F. Samuelson, 1981, Optimal auctions, *American Economic Review* 71, 381–392.

Roll, R., 1986, The hubris hypothesis on corporate takeovers, *Journal of Business* 59, 176–216.

Singh, R., 1993, Takeover bidding with toeholds: The case of the owner's curse, Working paper, Graduate School of Industrial Administration, Carnegie Mellon University.

Shleifer, A., and R. W. Vishny, 1986, Large shareholders and corporate control, *Journal of Political Economy* 94, 461–488.

Shleifer, A., and R. W. Vishny, 1988, Management buyouts as a response to market pressure, in A. J. Auerbach, Ed.: *Mergers and Acquisitions* (University of Chicago Press, Chicago), 87–102.

Spatt, C., 1989, Strategic analysis of takeover bids, in S. Bhattacharya, and G. Constantinides, Eds.: *Financial Markets and Incomplete Information* (Rowan and Littlefield, Savage, Maryland), 106–121.

Stulz, R., 1988, Managerial control of voting rights. Financing policies and the market for corporate control, *Journal of Financial Economics* 20, 25–54.

Stulz, R., R. A. Walkling, and M. H. Song, 1990, The distribution of target ownership and the division of gains in successful takeovers, *Journal of Finance* 45, 817–833.

Zweibel, J., 1991, Block investment and partial benefits of corporate control, Working paper, Stanford University.