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The Exchange Rate in the Presence of Transaction Costs: Implications for Tests of Purchasing Power Parity

PIET SERCU, RAMAN UPPAL, AND CYNTHIA VAN HULLE*

ABSTRACT

With transaction costs for trading goods, the nominal exchange rate moves within a band around the nominal purchasing power parity (PPP) value. We model the behavior of the band and of the exchange rate within the band. The model explains why there are below-unity slope coefficients in regression tests of PPP, and why these increase toward unity under hyperinflation or with low-frequency data. Our results are independent of the presence of nontraded goods in the economy.

THE OBJECTIVE OF THIS article is to develop a model of the exchange rate that explains the following two stylized empirical facts about deviations from relative purchasing power parity (PPP). First, in a simple regression of changes in (nominal) exchange rates on inflation differentials, the slope coefficient is typically below unity. Second, the slope coefficient increases with the length of the observation interval, and the PPP link is also stronger under hyperinflation than under modest inflation.¹ The low coefficients in the relative PPP regression, especially when tested over short periods of time, are often ascribed to errors-in-variables biases that arise from, for example, infrequent price sampling for some items, nonsynchronicity among the prices composing the index, relative price effects in price indices that are weighted differently across countries, and the presence of nontradable goods.² In this article we present a complementary explanation for the stylized empirical facts, and show that below-unity regression coefficients are expected even when none of these errors in variables are present.

We focus on the effect of costs for trading goods—such as shipping and insurance costs, tariffs, and information costs—on the nominal exchange rate.

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¹ See, for example, McNown and Wallace (1989).

² For a review of estimations that take into account errors-in-variables, see, for instance, Apte, Kane, and Sercu (1994) or Betton, Levi, and Uppal (1994). Stulz (1987) provides a theoretical analysis of the effect of nontraded goods on the real exchange rate, and Kravis *et al.* (1975) offer empirical evidence.

As is well known, when trading is costly, the nominal exchange rate can deviate from the nominal price-parity value, but by no more than the transactions cost. Thus, friction introduces (bounded) “noise” in the price-parity relationship. In order to understand the effect of this noise on the regression coefficient in tests of relative PPP, we develop a theory about the movements of the bounds and the behavior of the exchange rate within the bounds. Our results extend earlier work by Dumas (1992), Lucas (1982), and Stulz (1987). We show that in our model the elasticity of the spot rate relative to the ratio of the two prices of tradable goods is below unity. Thus, in our view, the below-unity coefficients found in tests of relative PPP over short periods of time partly reflect a true phenomenon rather than just an errors-in-variables bias. One-to-one links between relative price changes and exchange rate movements are observed only if the exchange rate and the boundary are all changing by the same factor. We argue that hyperinflation episodes are close to such situations. Thus, our model offers at least a partial explanation for the stylized facts in the empirical literature on PPP.

The rest of the article is organized as follows. We describe the model and its solution in Section I. In Section II, we derive the nominal exchange rate and describe its implications for tests of PPP. We summarize our conclusions in Section III.

I. The Model

In this section, we present a model of two countries and one good. The home and foreign country ($j = 1$ and 2 , respectively) are similar in three respects. First, they are assumed to be populated by a large and equal number of infinitely-lived consumers with identical isoelastic utility functions. Second, their financial markets are perfectly integrated, complete, and frictionless. Third, the countries are assumed to have started out with identical endowments, and the parameters of the stochastic processes that determine later endowments are identical. The factors that distinguish one economy from another are the following. First, after the initial date the outputs of the good generally differ across countries, and costs for shipping goods across countries make the commodity markets less than perfectly integrated. Second, each country has its own money, and the nominal stocks of money generally differ across countries.

Every period, each country is endowed with a quantity of a single good that is homogeneous across countries, imperfectly tradable, and nonstorable. In each country, the available quantity of the good, $q_j(t)$, is assumed to be characterized by discrete-time, exogenous stochastic processes, as in Lucas (1982). There is a cost to shipping the good internationally. This cost is modeled, following Dumas (1992), as a waste of resources: if one unit is shipped, only $1/(1+\tau)$ units actually arrive.

We introduce money in the model using the cash-in-advance constraint³ and the timing convention used in Lucas (1982).⁴ Define the money supply in country j to be equal to $m_j(t)$ and let the evolution of $m_j(t)$ be given by a discrete-time deterministic or stochastic process. We assume that domestic (foreign) money is used to purchase goods consumed in the home (foreign) country. The domestic and foreign goods' prices referred to in this article are expressed in units of the corresponding country's currency. These prices may change over time because the local money supply changes, or because quantities available for consumption change.

We now derive the implications for trade. Let $c_j(t)$ be the consumption quantity of the good in country j , and let $x_j(t)$ denote the exports from country j (before transaction costs). As it is costly to transfer the good from one country to another, it will be optimal to trade only when the price of the tradable good at home is sufficiently different from the price abroad. Given our assumption of complete and frictionless financial markets, the decentralized solution is identical to that of the central planner because individuals can implement the optimal solution of the central planner through trading in financial claims. Assuming that the utility function in each country, u_j , is given isoelastic, the central planner's objective is to choose the amounts of trade so as to maximize the aggregate utility:⁵

$$\text{Max } E \left[\sum_{t=0}^{\infty} \rho^{-t} \frac{c_1(t)^{1-\eta}}{1-\eta} + \sum_{t=0}^{\infty} \rho^{-t} \frac{c_2(t)^{1-\eta}}{1-\eta} \right] \quad (1a)$$

subject to:

$$c_1(t) = q_1(t) - x_1(t) + \frac{x_2(t)}{1 + \tau} \quad (1b)$$

$$c_2(t) = q_2(t) - x_2(t) + \frac{x_1(t)}{1 + \tau} \quad (1c)$$

$$x_1(t) \geq 0, \quad x_2(t) \geq 0, \quad (1d)$$

³ The addition of a nontraded good or of money in the utility function, as in Stulz (1987) (rather than through a Lucas (1982) cash-in-advance constraint), does not affect our conclusions. Proofs are available upon request. Although the formal model presented here is of an endowment economy as considered in Lucas (1982), the key results also hold for the production economies considered by Stulz (1987) and Dumas (1992). See also Footnotes 6 and 7.

⁴ That is, we assume that investors can purchase goods only with currency. The timing convention is such that investors learn about the current state of the world, including monetary shocks, before trading in the markets for securities and consumption goods. Given that investors can hold riskless securities with a positive nominal return, money will be held only within a period (to buy the known amount of current period consumption), and not across periods.

⁵ The absence of weights preceding the utilities reflects the assumption that the two countries have started out with identical amounts of the good and that the endowment processes have identical parameters across countries. In the decentralized solution, the initial endowments determine the initial relative wealth of the two countries; and this, in complete markets, then determines how the claims on future consumption are distributed among the countries.

where ρ is the discount factor and $\eta > 0$ is the degree of relative risk aversion. Note that in constraints (1b) and (1c), a shipment of $x_1(t)$ from the home country leads to an increase in foreign consumption of only $x_1(t)/(1 + \tau)$, and likewise for shipments in the other direction. In the absence of shipping costs, the central planner would set the optimal relative amount of consumption equal to unity, and correct any deviation of $q_2(t)/q_1(t)$ from unity by shipping goods. But in the presence of shipping costs, there will be a no-trade zone within which deviations of $c_2(t)/c_1(t)$ from unity will be left uncorrected. Only if $q_2(t)/q_1(t)$ is outside the no-trade region will goods be exported from one country to another, and the shipped amounts will be such that $c_2(t)/c_1(t)$ remains on the nearest boundary of the no-trade region. To solve problem (1), note that the utility function in (1a) is time-separable, trade affects consumption only within the same period, and the constraints apply period-by-period. It follows that we can rewrite the intertemporal problem as a static problem,

$$\text{Max } \frac{c_1(t)^{1-\eta}}{1-\eta} + \frac{c_2(t)^{1-\eta}}{1-\eta} \quad \text{subject to (1b)–(1d).}$$

From the first-order conditions of this static problem, we immediately obtain the condition that indicates when to ship goods across countries.

PROPOSITION 1: *The ratio of the marginal utilities of the home and foreign investor are bounded above and below by the transaction cost:*

$$\frac{1}{1 + \tau} \leq \frac{\partial u_2[\cdot]/\partial c_2}{\partial u_1[\cdot]/\partial c_1} = \frac{c_2(t)^{-\eta}}{c_1(t)^{-\eta}} \leq 1 + \tau. \quad (2)$$

Equivalently, relative consumption is bounded by:

$$\left[\frac{1}{1 + \tau} \right]^{1/\eta} \leq \frac{c_2(t)}{c_1(t)} \leq [1 + \tau]^{1/\eta} \equiv \kappa. \quad (3)$$

Thus, when initial endowments are equal, we obtain a no-trade region that is symmetric around the 45-degree line.⁶ Figure 1 illustrates this: for $q_2(t)/q_1(t) > \kappa$ there will be exports from the foreign country; for $q_2(t)/q_1(t) < 1/\kappa$ there will be exports from the home country; and there is no trade when $1/\kappa \leq q_2(t)/q_1(t) \leq \kappa$. Having identified the region of no shipping and the sharing rule for consumption, we now determine the real and nominal exchange rates.

⁶ With unequal weights in the objective function (1), the region of no shipping would not be symmetric around the 45-degree line. Also, if a nontradable good is introduced, the bounds on the ratio of tradable good consumption depend on the relative output of the nontraded good at home and abroad. All this, however, would not affect the conclusion that the ratio of marginal utilities can deviate from unity, nor would it affect the inferences made from this in Section II.B. Finally, bounds (2) and (3) also apply in the model of Dumas (1992), which implies that our conclusions also hold for his model.

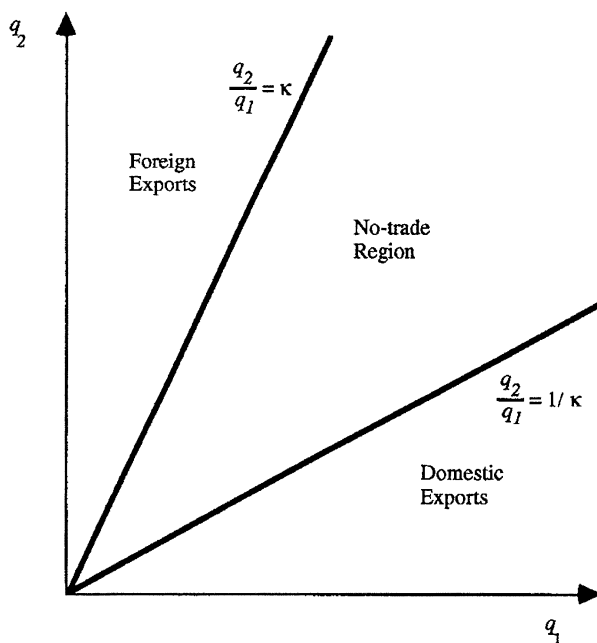


Figure 1. The region of no trade. When the weights assigned by the central planner to the two countries are the same, then the critical loci that separate the no-trade domain from the region with trade are symmetric around the 45-degree line: if the ratio of the output abroad to that at home, q_2/q_1 , is greater than κ , then goods are exported from the foreign to the home country, and if q_2/q_1 is less than $1/\kappa$, goods are exported from the home to the foreign country.

II. The Exchange Rate

In this section, we determine the real and nominal exchange rates in an economy with only one good, tradable at a cost τ . We derive the bounds these exchange rates must satisfy and discuss the implications of our exchange rate model for tests of PPP. The analysis for the case of multiple goods, some of them possibly nontradable, is available upon request.

A. Determination of Real and Nominal Exchange Rates

In an economy with only tradable goods, the real exchange rate is given by the ratio of the marginal utility of consumption of the tradable good abroad to that at home. In an economy with one good and no transactions costs ($\tau = 0$), such as Lucas (1982), the real exchange rate is always equal to unity. That is, the central planner's trade decision would always equalize marginal utilities of consumption across countries. In the presence of transaction costs, however, it is no longer optimal to correct every imbalance in the marginal utility of consumption at home and abroad at each date. The tradable good will be transferred across countries only when the imbalance in the marginal utilities of consumption is large enough to compensate for the cost of shipping goods.

Thus, from Proposition 1, the real exchange rate, $Z(t)$, is bounded by the transactions costs:⁷

$$\frac{1}{1 + \tau} \leq Z(t) \equiv \frac{\partial u_2[c_2(t)]/\partial c_2}{\partial u_1[c_1(t)]/\partial c_1} = \left[\frac{c_2(t)}{c_1(t)} \right]^{-\eta} \leq 1 + \tau, \quad (4)$$

Let $S(t) \equiv Z(t)[p_1(t)/p_2(t)]$ denote the nominal spot exchange rate, in terms of domestic currency per unit of foreign currency. Using the cash-in-advance constraint, $m_j(t) = c_j(t)p_j(t)$, and equation (4), the nominal exchange rate can be specified as follows.

PROPOSITION 2: *Given perfect capital markets and isoelastic utility, the nominal exchange rate satisfies:*

$$S(t) \equiv Z(t) \frac{p_1(t)}{p_2(t)} = \left[\frac{m_1(t)/p_1(t)}{m_2(t)/p_2(t)} \right]^\eta \frac{p_1(t)}{p_2(t)} = \left[\frac{m_1(t)}{m_2(t)} \right]^\eta \left[\frac{p_1(t)}{p_2(t)} \right]^{1-\eta}, \quad (5a)$$

$$\frac{1}{1 + \tau} \frac{p_1(t)}{p_2(t)} \leq S(t) \leq (1 + \tau) \frac{p_1(t)}{p_2(t)}. \quad (5b)$$

The inequalities in equation (5b) can be interpreted as bounds that the nominal exchange rate must satisfy to preclude arbitrage opportunities in the goods market.⁸ To see this, note that in order to import one (net) unit of the tradable good from abroad into the home country, the importer has to buy $(1 + \tau)$ units of this good at the foreign price $p_2(t)$ per unit. In the absence of arbitrage opportunities, the all-in cost of the foreign good must be no lower than the domestic price $p_1(t)$: $S(t)p_2(t)(1 + \tau) \geq p_1(t)$. A symmetric relation for exports gives us the other bound in equation (5b).

We can compare our results to the monetary model of exchange rates, which is based on a money demand equation for each country and the assumption of PPP. The money demand equations implied by the cash-in-advance constraint are a special case of the money demand equations of the monetary approach to the exchange rate, in the sense that we restrict the demand elasticities and the velocities of money to be equal to unity in each country. But the main reason

⁷ The same bounds apply in the Dumas (1992) model. To link this model with Dumas (1992) and Stulz (1987) where the variables are capital stocks rather than quantities of a consumption good, it is helpful to think of each output, $q_j(t)$, as being proportional to a (stochastic) capital stock, $Q_j(t)$. Thus, in our model the output ratio is proportional to the ratio capital stocks in the two countries. In the absence of shipping the real exchange rate, $Z(t) = [c_2(t)/c_1(t)]^{-\eta}$, is equal to $[Q_2(t)/Q_1(t)]^{-\eta}$ in our model, while in the model of Dumas the real exchange rate lies between $[Q_2(t)/Q_1(t)]^{-\eta}$ and unity. This is because, in Dumas' model, the good is also an asset and can be reinvested; as a result, consumption of this good is a varying fraction of the stock of the good rather than a constant proportion.

⁸ Benninga and Protopapadakis (1988) derive one-sided "forward" bounds when trading takes time. They also express the exchange rate in terms of general state prices, but do not have a closed-form solution.

why our model differs from the monetary model is that we allow for the possibility of no trade and for deviations from PPP. To see the similarities and the differences between both models, note that in a one-good economy $c_j(t)$ corresponds to real transaction volume in the standard monetary model. With unit elasticities and velocities, the monetary model then simplifies to

$$S(t) = \frac{m_1(t)}{m_2(t)} \frac{c_2(t)}{c_1(t)}. \quad (6)$$

It is easily shown that, for periods when there is trade, the predictions of models (5) and (6) are similar. If there are imports, the first relationship in equation (5b) holds as an equality, while the second relationship holds as an equality if there are exports. After substituting the cash-in-advance constraint, $c_j(t)p_j(t) = m_j(t)$, $j = \{1, 2\}$, into these equalities and into equation (5a), our model simplifies to

$$S(t) = \begin{cases} (1 + \tau) \frac{c_{T2}(t)}{c_{T1}(t)} \frac{m_1(t)}{m_2(t)} & \text{when there are exports,} \\ \frac{1}{1 + \tau} \frac{c_{T2}(t)}{c_{T1}(t)} \frac{m_1(t)}{m_2(t)} & \text{when there are imports.} \end{cases}$$

Thus, when trade is in the same direction for two or more consecutive periods, models (5) and (6) have identical implications for changes in the exchange rate—the reason being that, under those circumstances, relative PPP holds. But in the no-trade region the exchange rate in our model behaves very differently from that in the monetary model. Given that most estimates of η exceed unity, the exchange rate in our model, $S(t) = (c_{T2}(t)/c_{T1}(t))^{1-\eta}(m_1(t)/m_2(t))$, is decreasing in relative consumption rather than being directly proportional to it. This difference arises because, in the monetary approach, the real exchange rate is fixed at unity, while in our model the real exchange rate equals $(c_{T2}(t)/c_{T1}(t))^{-\eta}$.

B. Implications for Tests of Purchasing Power Parity

We now discuss the implications of our model for regression tests of relative PPP. The traditional test equation for relative PPP is:

$$\Delta \ln S_t(t) = \gamma_0 + \gamma_1 \Delta \ln \frac{p_1(t)}{p_2(t)} + e(t). \quad (7)$$

According to our model for the spot rate in equation (5), the true equation for $\Delta \ln S_t(t)$ is:

$$\Delta \ln S(t) = \eta \Delta \ln \frac{m_1(t)}{m_2(t)} + (1 - \eta) \Delta \ln \frac{p_1(t)}{p_2(t)}. \quad (8)$$

Thus, our model suggests that the traditional regression tests of relative PPP have a missing variable: the relative money stocks in the two countries.

PROPOSITION 3: *Controlling for the missing variable, $m_1(t)/m_2(t)$, according to our model the slope coefficient in equation (7) is equal to $(1 - \eta)$, and therefore, below unity. Moreover, in periods where there is positive association between the missing variable and $p_1(t)/p_2(t)$, estimates of γ_1 will be biased toward unity.*

The first part of Proposition 3 follows from equation (8) and says that, when monetary growth rates are similar across countries, one could get estimates of γ_1 that are less than unity even if all the goods in an economy were tradable.⁹ The second part of Proposition 3 follows from a standard analysis of the bias arising from the omitted variable. The bias can be stated as

$$\gamma_1 = \eta\delta + (1 - \eta), \quad (9)$$

where δ is the slope coefficient of the regression of $\Delta \ln[m_1(t)/m_2(t)]$ on $\Delta \ln[p_1(t)/p_2(t)]$. The coefficient δ is likely to be positive. We now show that δ is higher in high-inflation circumstances and in low-frequency data.

To see the effect of the variability of relative money supply on δ , use the cash-in-advance constraint, $c_j(t) p_j(t) = m_j(t)$, $j = \{1, 2\}$ to note that:

$$\Delta \ln \frac{p_1(t)}{p_2(t)} = \Delta \ln \frac{m_1(t)}{m_2(t)} - \Delta \ln \frac{c_1(t)}{c_2(t)}.$$

This relation allows us to rewrite δ as:

$$\begin{aligned} \delta &\equiv \frac{\text{cov} \left[\Delta \ln \frac{m_1(t)}{m_2(t)}, \Delta \ln \frac{p_1(t)}{p_2(t)} \right]}{\text{var} \left[\Delta \ln \frac{p_1(t)}{p_2(t)} \right]} \\ &= \frac{\text{var} \left[\Delta \ln \frac{m_1(t)}{m_2(t)} \right] - \text{cov} \left[\Delta \ln \frac{m_1(t)}{m_2(t)}, \Delta \ln \frac{c_1(t)}{c_2(t)} \right]}{\text{var} \left[\Delta \ln \frac{m_1(t)}{m_2(t)} \right] - 2 \text{cov} \left[\Delta \ln \frac{m_1(t)}{m_2(t)}, \Delta \ln \frac{c_1(t)}{c_2(t)} \right] + \text{var} \left[\Delta \ln \frac{c_1(t)}{c_2(t)} \right]} \end{aligned} \quad (10)$$

When $\text{var}[\Delta \ln[m_1(t)/m_2(t)]] = 0$, the covariance term on the right-hand side is also zero, and δ is equal to zero. When $\text{var}[\Delta \ln[m_1(t)/m_2(t)]] = \text{var}[\Delta \ln[c_1(t)/c_2(t)]]$, the coefficient δ is equal to 0.5. Finally, when $\text{var}[\Delta \ln[m_1(t)/m_2(t)]]$ becomes very large

⁹ Given that most estimates of η exceed unity, it is possible for the slope coefficient to be negative. Apte *et al.* (1994) obtain eight negative coefficients (of 38) in their first-pass estimation of equation (7). They ascribe this to sampling error and a downward errors-in-variables bias, but it is also consistent with a negative value for $1 - \eta$.

relative to $\text{var}[\Delta \ln[c_1(t)/c_2(t)]]$, δ approaches unity. Given that $\gamma_1 = \eta \delta + (1 - \eta)$, it then follows that large variations in relative money growth rates lead to an expected value for γ_1 that approaches unity, even if $(1 - \eta)$ is substantially below unity. In short, under hyperinflation, the regressor $\Delta \ln[p_1(t)/p_2(t)]$ of the relative PPP test equation tracks almost perfectly the omitted variable, $\Delta \ln[(m_1(t), m_2(t))]$, and adds the effect η of this omitted variable to the effect $(1 - \eta)$ of the price change. Thus, the empirical slope coefficient between prices and the exchange rate is close to unity under hyperinflation circumstances as found, for example, by McNown and Wallace (1989).

A similar argument can be invoked to explain why tests that use low-frequency data generally produce estimates of γ_1 closer to unity than do tests using short observation intervals. Thus far we assumed that, holding constant the outputs, any change in money supply immediately feeds through into prices. In practice, however, in the short term the effect of changes in money supply on prices is rather weak and δ is close to zero. This implies that with high-frequency data γ_1 is heavily influenced by $(1 - \eta)$, which is below unity. Over longer intervals, however, the prices become increasingly more correlated with $\Delta \ln[(m_1(t)/m_2(t))]$; so δ becomes larger, and $\gamma_1 = \eta \delta + (1 - \eta)$ picks up more of the money-supply effect, η . Thus, for low-frequency data the estimated value of γ_1 is also closer to unity.

III. Conclusion

In complete financial markets, the real exchange rate equals the ratio of the marginal utilities of real consumption and can, therefore, be expressed as a function of the countries' current levels of real consumption. If commodity markets were frictionless, these marginal utilities would be equalized internationally and the real consumption quantities would, therefore, no longer show up in the real exchange rate equation. In reality, though, commodity trade is costly. As a result of these costs, international imbalances between marginal utilities will be left uncorrected as long as they are sufficiently small relative to the cost of shipping. Thus, in the real exchange rate equation, real consumptions—that is, nominal spending and price variables—do not cancel out across countries. The nominal exchange rate is, of course, obtained by multiplying this real exchange rate by the international price level ratio. Thus, the nominal exchange rate can be expressed as a function of each country's nominal spending and price level or—assuming constant velocities—each country's money stock and price level.

From this argument we see that price levels play a dual role in determining the nominal exchange rate. First, the price variables serve to translate the real exchange rate into nominal terms. Second, the price levels are needed, in conjunction with nominal spending variables, to track the real exchange rate. Because the traditional PPP model postulates a unit real exchange rate, it ignores this second role of the price variables and it likewise omits the money variables (and the velocities, if these are not constant) from the equation.

One implication of our analysis is that, if money supply effects are controlled for, the elasticity of the exchange rate with respect to the relative prices of tradable goods is below unity. We therefore argue that the low regression coefficients, commonly found when changes of exchange rates are regressed on inflation differentials, reflect to some extent a true relationship rather than just being the result of errors-in-variables. When the ratio of traded goods prices and relative money supplies are strongly correlated, as is the case during hyperinflation episodes, the inflation differential picks up the effect of the omitted factor, relative money supplies, and leads to an increase in the regression slope. A similar effect obtains when the observation interval for PPP tests is increased. Thus, our model provides a potential explanation for the low slope coefficients observed in tests of relative PPP, and for the higher values found in hyperinflation or low-frequency data.

The formal arguments developed here assume that there is only one good. However, our results also hold in a multi-good economy with frequent trade, because the crucial force that drives our model is the existence of output risk combined with a cost of shipping. To be able to define deviations from PPP in a multi-good economy, one needs homothetic utility functions—otherwise one cannot define a unique price index. With homothetic utility functions, one can identify a composite good (the composition of which is changing over time but is the same for all consumers in a given country on a single date) and a corresponding composite price. One can then show that the real exchange rate is the ratio of the marginal utilities with respect to each country's composite good, and that the composite price takes the place of the price of the single good in the model presented here. All our results then follow. First, in the presence of output uncertainty and shipping costs, it is unlikely that the ratio of the marginal utilities would be equal to unity at any given date t —that is, absolute PPP does not generally hold. Nor is it likely that this ratio would be a constant over time, because it is unlikely that, for each and every good, there is trade at all times and in the same direction. Thus, in general, relative PPP does not hold either. Lastly, a deviation between the marginal utilities will be related to a deviation between the composite prices in exactly the same way as it is in the one-good case.

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