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Author(s): Gautam Goswami, Thomas Noe, Michael Rebello

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Debt Financing under Asymmetric Information

GAUTAM GOSWAMI, THOMAS NOE, and
MICHAEL REBELLO*

ABSTRACT

We analyze the optimal design of debt maturity, coupon payments, and dividend payout restrictions under asymmetric information. We show that, if the asymmetry of information is concentrated around long-term cash flows, firms finance with coupon-bearing long-term debt that partially restricts dividend payments. If the asymmetry of information is concentrated around near-term cash flows and there exists considerable refinancing risk, firms finance with coupon-bearing long-term debt that does not restrict dividend payments. Finally, if the asymmetry of information is uniformly distributed across dates, firms finance with short-term debt.

THE RELIANCE OF CORPORATIONS on debt financing has more than doubled over the last half-century. The reliance on long-term debt has more than kept pace, with the long-term debt to long-term capital ratio more than tripling over the same period.[1] The resulting heavy dependence on long-term debt financing is documented by Barclay and Smith (1995), who find that 37 percent of the total debt outstanding for the average firm has a maturity of greater than 5 years. A majority of long-term debt instruments impose dividend restrictions and mandate coupon payments. Malitz (1986) provides evidence that 55 percent of nonconvertible corporate bonds issued between 1960 and 1980 included dividend restrictions among their covenants. De and Kale (1993) point to the fact that less than 10 percent of recently issued bonds failed to include coupon provisions.

In this article, we examine the impact of informational asymmetries on the design of debt contracts with a view to explaining these features of debt financing. Our explanation is based not on the *existence* of informational asymmetry per se, but on the *temporal distribution* of asymmetric information, i.e., the relative degree of asymmetric information regarding short and long-term cash flows. We show that the popularity of long-term debt, coupon

* Goswami is on the faculty of Fordham University. Noe is on the faculty of Georgia State University and is a visiting scholar at the Federal Reserve Bank of Atlanta. Rebello is on the faculty of Georgia State University. He was at Wasserstein Perella when some of the work on this article was completed. The first author is grateful to the Georgia State University College of Business Research Program Council for support. We are grateful to the editor, René Stulz, two anonymous referees, and participants of the Atlanta Finance Workshop for their helpful suggestions. The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of Atlanta or the Federal Reserve System. The usual disclaimer applies.

[1] See Tables 1–3, pages 8–9 in Masulis (1988).

payments, and dividend restrictions can be parsimoniously explained by the temporal distribution of asymmetric information without recourse to any capital market frictions such as transactions costs, taxes, and agency conflicts.[2] In our setting, the standard debt securities, short-term debt, uncovenanted coupon bearing long-term debt that incorporates *no* dividend restrictions, and covenanted coupon bearing long-term debt that does restrict dividend payments, are optimal designs.

To model the effect of the temporal distribution of information asymmetry, we consider a firm that receives cash flows at two dates, an intermediate date and a terminal date. The firm has private information regarding the distribution of its intermediate and terminal-date cash flows. The degree of information asymmetry regarding the cash flows at the two dates may vary. For example, asymmetry of information may be greater for terminal cash flows. This is likely when short-term cash flows are determined by past decisions, regarding which there is little informational asymmetry. Moreover, long-run cash flows may depend on the entrepreneurial skills of its incumbent and future management. Because there could exist considerable asymmetry of information regarding management skills, there could exist substantial asymmetry of information regarding long-run cash flows. Greater asymmetry of information regarding long-run cash flows is not the only temporal pattern of informational asymmetry. In fact, the firm could be characterized by the opposite pattern of informational asymmetry—greater asymmetry of information regarding short-run cash flows. For example, there could be asymmetric information regarding the date at which production comes on line. In this case, there may be considerable information asymmetry regarding short-run cash flows, but little or no informational asymmetry regarding long-run cash flows. We demonstrate that the debt financing choices in these cases would be distinct and driven by differences in the temporal structures of informational asymmetry.

Our results indicate that, when the asymmetry of information primarily concerns long-run prospects, the firm prefers financing with covenanted long-term debt that restricts dividend payments. On the other hand, when there is greater informational asymmetry regarding the short run, the firm prefers either short-term debt or uncovenanted long-term debt that does not restrict dividends. The basic intuition underlying these results is that short-term debt has two unique features: first, there is the "updating" feature. Unlike long-term debt, short-term debt allows for the financing terms to be reset at the intermediate date. This allows for more information to be utilized in setting the terms of the debt issue. Second, there is a "long-term-cash flow dependence feature." Because it can be rolled over, short-term debt may require both intermediate and terminal date payments. Thus, terminal cash flows secure both intermediate date payments, by permitting the firm to roll

[2] For a model of debt financing that includes transactions costs see Flannery (1986), for models that include taxes see Brick and Ravid (1985, 1991) and Wiggins (1990), and for models that include agency conflicts see Barnea, Haugen, and Senbet (1980) and Diamond (1991, 1993).

over its debt, and terminal date payments. Intermediate date cash flows, however, secure only intermediate date payments. Thus, the default premia on short-term debt tends to be more dependent on long-term rather than short-term cash flows. In contrast, in the case of covenanted long-term debt, a large portion of intermediate cash flows is retained to secure the terminal repayment of face value, allowing intermediate and long-term cash flows to contribute roughly equally to securing the debt. Long-term debt also lacks the updating feature of short-term debt.

Asymmetric information enters into the pricing of debt issues only through its effect on default premia. If informational asymmetries are primarily concentrated around long-term cash flows, little information is revealed by observing short-run cash flows, minimizing the updating advantage of short-term debt. Further, the long-term cash flow dependence feature of short-term debt implies that asymmetry of information about long-term cash flows has a disproportionately large effect on the degree of mispricing of short-term debt issues. Consequently, when the rate at which informational asymmetry increases over time is rapid enough, the combined effect of these two considerations may make long-term covenant protected debt the financing instrument of choice.

In contrast, when informational asymmetry is concentrated on intermediate cash flows, in order to lower its mispricing, the firm has an incentive to issue claims that depend only on long-term cash flows. If there is no risk of the firm being unable to rollover short-term debt at the intermediate date, the default risk on the short-term debt is dependent only on long-term cash flows. Moreover, the updating feature favors short-term debt over uncovenanted long-term debt, whose default risk is also tied solely to terminal cash flows. Thus, in the absence of rollover risk, short-term debt is the preferred instrument. However, if rollover risk becomes sufficiently great, the rollover feature loses much of its value and the default premium on short-term debt exhibits reduced dependence on long-term cash flows. When there is a great deal of asymmetric information regarding short-term cash flows, this becomes a major drawback and can lead to a preference for long-term uncovenanted debt.

Our results give rise to a number of interesting empirical implications. Researchers who have investigated the patterns of corporate debt financing have for the most part focused on the effect of the existence of informational asymmetry on the choice of debt maturity structure (see, for example, Barclay and Smith (1995), Crabbe and Helwege (1993), and Mitchell (1991, 1993)). Our analysis, however, points to the importance of the temporal distribution of informational asymmetry as the major factor determining the design of debt contracts. The key to incorporating this factor into the empirical analysis of debt maturity is to find proxies for the distribution of informational asymmetry. One important determinant of the temporal distribution of informational asymmetry may be the degree to which a firm is established. Crude proxies for this would be firm size and age. The older and larger the firm, the less asymmetry one would expect regarding short-term cash flows.

Another proxy for the temporal distribution of informational asymmetry is industry membership. Firms in industries which exhibit considerable cyclicality in short-run demand, such as consumer durables and housing, should exhibit relatively higher levels of informational asymmetry regarding short-run prospects. Further, firms that are subject to stricter reporting requirements, because accounting data has relatively more predictive power for short-run rather than long-run performance, should exhibit relatively less asymmetry regarding the short term (see, for example, Brown and Rozeff (1979) and Coates (1972)).

Using the proxies for the intertemporal distribution of cash flows described above, a number of empirical predictions can be drawn from our analysis. If informational asymmetry is greater for periods further into the future, covenanted long-term debt is more likely to be the financing vehicle of choice. For firms facing relatively more informational asymmetry regarding the near term, short-term debt is more likely to be the mode of financing. Uncovenanted long-term debt is most likely to be observed when the degree of informational asymmetry is fairly large regarding the short-run and fairly small regarding distant cash flows. These hypotheses can be empirically investigated using polychotomous dependent variable techniques such as multinomial logit/probit models (see, for example, Kennedy (1992), Chapter 15). Further, the tightening of accounting standards would tend, through the imposition of increased reporting, to decrease informational asymmetry regarding short-term cash flows, leading us to predict that such changes would tend to increase corporate dependence on covenanted long-term debt.

The analysis also makes fairly sharp empirical predictions regarding dividend restrictions in bond contracts. If the uncertainty regarding the firm's cash flows around a bond's maturity date are large, firms will prefer to restrict dividend payments. Thus, payout restrictions will be more likely when the uncertainty of cash flows around the maturity date is large. On the other hand, the range of parameters over which covenanted long-term debt is optimal is decreasing in refinancing risk. Because refinancing risk is captured by the probability of intermediate-date default on debt obligations, and is reflected in a firm's credit rating, we hypothesize that firms will be less likely to incorporate dividend restrictions in their bond covenants when they have lower credit ratings and there exists considerable informational asymmetry regarding short-term cash flows.

In terms of the announcement effects of debt issues, our results indicate that covenanted long-term debt issues should signal unfavorable information while long-term uncovanented debt will tend to have positive announcement effects. On the other hand, because short-term debt can be issued in pooling equilibria or as the contract of choice for either high- or low-quality firms, the announcement effects from short-term debt issues should be mixed or zero. Because long-term debt is a superior vehicle for signaling private information, and because announcement effects should only be observed when the firm signals private information, our results predict a larger variation of announcement returns accompanying long-term debt as opposed to short-term debt. These signaling effects could alternatively be tested by analyzing innovations in the time series of earnings for firms after the announcement of a debt maturity decision.[3]

The remainder of the article is organized as follows: in Section I, we develop the sequential signaling framework used to examine the debt financing decision. In Section II, we characterize the pooling equilibria generated by our model. Section III contains a characterization of the separating equilibria. Some concluding remarks are presented in Section IV. Proofs of all claims are presented in Appendix 1. Appendix 2 contains some technical results.

I. A Model of Debt Maturity Choice

A two-period, three-date setting is considered with dates indexed by $d = 0, 1, 2$. All agents in the economy are risk neutral and the risk-free interest rate is 0. Financial markets are competitive in that, whenever possible, projects are financed at zero expected profits for investors. At date 0, a firm has access to a positive net present value (NPV) project that requires an initial investment of I. The firm's type, denoted by $t \in T = \{G, B\}$, can be either good (G) or bad (B) and is private information. The market assigns a probability distribution over firm types, and we identify the firm's initial credit rating with the market's date 0 assessment that it is type G. This probability is represented by π.

If financed, the project generates cash flows X^d at $d = 1$ and 2. We assume that the firm is unable to liquidate its assets either at date 0 or at date 1. We assume that, at each date, the cash flow distribution has the two point support $\{L, H\}$, where $0 < L < H$. If the firm is type t, the probability of realizing cash flow H at date 1 is represented by P_t and the probability of realizing cash flow L is represented by $1 - P_t$.[4] At date 1, after the cash flow realization, the firm can experience a shock. This shock is publicly observable. The probability of the shock is independent of date 1 cash flows, and the probability of receiving the shock, θ, does not vary with firm type.[5] If the shock occurs, the firm's date 2 cash flow equals L with probability 1. Thus, the shock provides no information regarding the firm's type. Further, once the shock is observed, it is common knowledge that the firm will realize cash

[3] See Ofer and Natarajan (1987) for an example of this approach to testing signaling models.

[4] This amounts to assuming the firm's cash flows are independent across dates. Cash flows may, however, display some intertemporal dependence, as would be the case if cash flows followed a random walk. Weakening this independence assumption would leave our basic results unchanged as long as the degree of intertemporal cash flow correlation is similar across types. This requirement will be satisfied in the random walk case if the cash flow increments have similar variances. The robustness of our result follows because, as will become more apparent from our analysis, only differences in characteristics of cash flow distributions across types, not common characteristics of the distributions, affect the optimal design.

[5] This assumption is made so that we can separate out the effect of informational asymmetry, measured by the relative probabilities of upticks across the two types, and refinancing risk, measured by θ. In this formulation, the terms of the refinancing vary with the firm's information, allowing the probability of refinancing to vary across the two types. Thus, modifying this assumption to permit type-dependent refinancing risk would not produce any new qualitative results. However, it would greatly complicate the analysis.

flow L at date 2 and thus the firm's private information is irrelevant. Because neither the realization of the shock nor the effect of the shock discriminates between firm types, its observation does not add to the information provided by the date 1 cash flow realization. In the absence of a shock, the probability of the firm realizing cash flow H at date 2 is denoted by P_t^2 if the firm is type t. To formalize the type ordering of cash flows, we assume that type G's cash flows uniformly dominate those of type B, i.e., $P_G^2 > P_B^2$ and $P_G > P_B$. This restriction on the cash flow distributions implies that type G's total cash flow distribution dominates that of type B in the sense of first-order stochastic dominance (FSD).

In order to finance its project, the firm issues debt claims against its date 1 and date 2 cash flows and sets dividend covenants.[6] Its objective is to maximize the intrinsic value of its equity as of date 0. A date d debt payment, which we represent by F^d, is a binding obligation to pay F^d at date d. If the firm's date 1 cash flow is too low to meet its date 1 obligation, F^1, the firm can attempt to "roll over" the debt, that is, provide the short-term claim holders with the available cash flow and a new claim with a market value equal to the shortfall between the promised payment and the realized cash flow.[7] This requires the issuance of new debt with a promised date 2 payment of F^r. This attempt at financing the shortfall occurs after the market learns whether the firm has suffered a shock. If it is successful in rolling over its debt, the newly issued debt has a lower priority than outstanding debt claims against the firm's date 2 cash flow.[8] If the firm is unable to roll over its date 1 debt, the shareholders lose their stake in the firm, with the residual ownership rights being transferred to the owners of the date 1 claims. The dividend covenants restrict the firm to retaining all or part of its date 1 cash flows after debt claims have been satisfied. The dividend restriction imposed by these covenants is represented by τ, where τ represents the cash flows that must be retained before any dividend payments can be made.[9] Funds re-

[6] Introducing equity financing into our model would have little effect on our results because, from the perspective of minimizing mispricing, equity financing is a suboptimal choice. To see this, note that equity finance is equivalent to writing a contingent claim on total cash flows whose terms are not updated at the intermediate date. Long-term covenanted debt on the other hand is a debt claim on total cash flows. More precisely, under our distributional assumptions, the sums of the date 1 and date 2 cash flows are ordered by conditional stochastic dominance. This implies that, over all increasing claims written on total cash flows, debt minimizes mispricing (see Nachman and Noe (1994)).

[7] This transaction can also be interpreted as paying off the balance of the obligation to the date 1 claimants ($F^1 - L$) with the proceeds from a new issue secured by the date 2 cash flows.

[8] In our setting, the firm's objective is to minimize the total value of the claims issued to outsiders and there are no dissipative costs from the inability to roll over debt. Thus, unlike, Diamond (1993), seniority has no impact on the results.

[9] Ex ante restrictions on payout policies are, in fact, required to prevent payouts because, ex post, after the firm has obtained financing the residual dividend policy is always optimal. If the firm does not follow the residual dividend policy, in the date two bankruptcy event, it would have to hand over a portion of the retained cash flows to the bondholders. Thus, retention reduces the total expected cash flow to equityholders. Further, after date 0 there are no further incentives to signal using dividends as the firm is only concerned with maximizing the intrinsic value of its equity and its financing terms have been fixed at date 0.

tained as a result of the dividend restriction are invested at the risk-free rate of return.

Let the firm's financing policy be represented by a vector $p \in \mathscr{P} \equiv [0, H]^3$. The first component of this vector represents the contracted date 1 obligation (F^1), the second component represents the date 2 debt obligation (F^2), and the third represents the dividend restriction (τ). The mix of debt financing policies allowed in this framework is very rich. By setting $F^2 = 0$ and $F^1 > 0$, the firm can finance with pure short-term debt. On the other hand, by setting $F^2 > 0$ and $F^1 = 0$ the firm can finance with zero-coupon long-term debt. By setting $0 < F^1 < F^2$ the firm can finance using long-term debt with a coupon payment of F^1. By varying τ, the firm can restrict dividend payments either partially of completely so as to increase the security of the long-term debtholders' claim.

Because the securities market is competitive, security prices are set so as to ensure that investors break even. Thus, in the absence of asymmetric information regarding the firm's cash flows, the firm's financing decision is irrelevant. However, in the presence of asymmetric information, claims may be mispriced. In any Nash equilibrium, claims are priced correctly on average, i.e., the expected value of claims issued, conditioned on the equilibrium financing strategy of the firm, equals the funds raised. This implies that, if the firm's financing choice varies with type, its claims will be correctly priced. However, because type G's cash flows dominate those of type B, when it is type B the firm will always obtain a mispricing gain from mimicking the security choice made when it is type G. Thus, *only pooling equilibria obtain* (see, for example, Nachman and Noe (1994)). Because its claims are priced at "pooled terms," the firm suffers a mispricing loss if it is type G. In such a setting, the intrinsic value of the outstanding equity claim is equal to the NPV of the project less the mispricing of the firm's debt, which is the difference between the intrinsic value of claims issued and the amount of funds raised by the firm. Thus, when considering pooling equilibria, it is natural to restrict attention to equilibria in which contracts that minimize the mispricing losses are issued.[10]

The mispricing of the firm's claims is crucially dependent upon the market's assessment of the firm's type at the time of its financing decision. The firm always raises funds at date 0, and may need financing at date 1 if it receives a low cash flow and needs to rollover its debt. In a pooling equilibrium, the financing policy does not convey any information regarding the firm's type. Thus, at date 0, the market's prior belief regarding the firm's type remains unchanged. On the other hand, in a separating equilibrium the firm's private information is fully revealed by its financing decision and the market's prior belief is revised accordingly. At date 1, the market's belief is updated on the basis of the realized cash flow. After the observation of date 1 cash flow L.

[10] Mispricing minimization has commonly been used as a normative criteria for security design, see, e.g., Myers and Majluf (1984) and Flannery (1986). Nachman and Noe (1994) provide a formal justification of the mispricing minimization criteria in a setting similar to ours.

Bayes' rule implies that ϕ, the market's date 1 assessment that the firm is type G, is

$$\phi(\delta) \equiv \frac{\delta(1 - P_G)}{\delta(1 - P_G) + (1 - \delta)(1 - P_B)}, \tag{1}$$

where δ represents the market's date 0 assessment of the probability that the firm is type G.[11]

The mispricing loss suffered by the firm if it is type G results from the incorrect assessment of its default risk. To rule out the uninteresting case in which the firm can finance with riskless debt, and hence avoid incurring mispricing losses, we assume that $I > 2L$. We also place some additional restrictions on the parameter values to ensure that date 1 debt can always be rolled over if no shock is experienced at date 1, and the feasible set of financing policies available to the firm is as extensive as possible, including pure short-term debt at one extreme and zero-coupon long-term debt at the other. This also permits us to interpret the shock as a measure of the refinancing risk faced by the firm if it issues short-term debt. More formally, we assume that the project is sufficiently profitable to ensure that

$$\begin{gathered} I < (1 - \theta)P_t(1 - P_t^2)H, I < (1 - \theta)P_t P_t^2 H, \\ \text{and } I < (1 - \theta)(1 - P_t)P_t^2 H \text{ for } t = B, G.^{12} \end{gathered} \tag{2}$$

Let $v_t^d(p, \delta)$ represent the intrinsic value of the payoffs received by bondholders as a result of their claim against the firm's date d cash flow. Because of their claim against date 1 cash flows, bondholders receive $\text{Min}\{x^1, F^1\}$ if the cash flow x^1 is realized at date 1. Because $F^1 \in [0, H]$, if the cash flow H is realized at date 1, the bondholders receive no more payments to satisfy their claims against the date 1 cash flow, i.e., $F^r = 0$. Similarly, if $F^1 \leq L$, the bondholder's claims can be satisfied out of the date 1 cash flow regardless of its realization. On the other hand, if the cash flow L is realized at date 1 and if $F^1 > L$, then in addition to receiving the cash flow L, bondholders also receive a claim against the firm's date 2 residual cash flow. Let $[x - F]^+ \equiv \text{Max}\{x - F, 0\}$ and let the actual cash flow retained at date 1 be represented by $R(F^1, \tau, x^1)$ where $R(F^1, \tau, x^1) \equiv \text{Min}\{[x^1 - F^1]^+, \tau\}$. Contingent on the realization of cash flows L at date 1 and x^2 at date 2, the date 2 cash flow available to service first period claims is given by $[R(F^1, \tau, L) + x^2 - F^2]^+$. It follows that the bondholders receive a date 2 payment of $\text{Min}\{[R(F^1, \tau, L) + x^2 - F^2]^+, F^r\}$ if cash flow L is realized at date 1 and x^2 is realized at date 2. On the other hand, if $F^1 > L$ and cash flow L and

[11] This assessment is not modified by the observation of the liqudity shock as the conditional probability of the liquidity shock is the same for both types.

[12] While these assumptions simplify the analysis, they are not required to establish the subsequent results.

shock are realized at date 1, bondholders become residual claimants against the firm's date 2 cash flow. As a result they receive a date 2 payoff of $\text{Min}\{[R(F^1, \tau, L) + L - F^2]^+, F^r\} = [R(F^1, \tau, L) + L - F^2]^+$.[13] Thus,

$$v_t^1(p,\delta) \equiv E\{\text{Min}[X_t^1, F^1]\} + E\Big\{\text{Min}\Big(\big[R(F^1,\tau,X_t^1) + X_t^2 - F^2\big]^+, F^r(X_t^1)\Big)\Big\}, \tag{3}$$

where $F^r(x)$, $x = H, L$ is defined implicitly by

$$\phi(\delta)E\Big\{\text{Min}\Big(\big[R(F^1,\tau,X_G^1) + X_G^2 - F^2\big]^+, F^r(x)\Big)\Big\} + (1-\phi(\delta)) \times E\Big\{\text{Min}\Big(\big[R(F^1,\tau,X_B^1) + X_B^2 - F^2\big]^+, F^r(x)\Big)\Big\} = [F^1 - x]^+.\text{[14]} \tag{4}$$

Let the intrinsic value of the bondholders' claim against the firm's date 2 cash flow be represented by $v_t^2(p, \delta)$. Because, the bondholders' date 2 claim entitles them to first priority on the firm's date 2 cash flow, contingent on the realization of cash flow x^d at date d, they hold a claim against a cash flow of $R(F^1, \tau, x^1) + x^2$. Thus, their claim entitles them to a date 2 payoff of $\text{Min}\{R(F^1, \tau, x^1) + x^2, F^2\}$. The expected value of this claim is presented by equation (5).

$$v_t^2(p,\delta) \equiv E\{\text{Min}\{R(F^1,\tau,X_t^1) + X_t^2, F^2\}\} \tag{5}$$

The intrinsic value of the bondholders' claims against the firm is the sum of the value of their claims against date 1 and date 2 cash flows. Let this value be represented by $v_t(p, \delta)$, where $v_t(p, \delta) \equiv v_t^1(p, \delta) + v_t^2(p, \delta)$. The date 0 market value of bondholders' claims is based on their beliefs regarding the firms type. Given that they assess probability δ to the firm being type G at date 0, let the market value of their claims be represented by $V(p, \delta)$, where $V(p, \delta) \equiv \delta v_G(p, \delta) + (1 - \delta)v_B(p, \delta)$.

II. Optimal Debt-Contract Designs

In this section we characterize the firm's debt-financing decision. Regardless of the financing contract it chooses, the firm has lower default risk if it is type G. This implies that there exist only pooling equilibria, and in these equilibria, the firm realizes a mispricing loss if it is type G. Given that its bonds are mispriced, if the firm is type G, its objective is to choose a contract that minimizes mispricing. Given that the market is competitive, the market value of the bonds issued must equal the funds raised, i.e. $V(p, \pi) = I$. Given

[13] Note that because the firm's debt is risky, $F^1 + F^2 > 2L$, or equivalently $F^1 - L > L - F^2$. This implies that if $F^1 > L$, then $[R(F^1, \tau, L) + L - F^2]^+ = L - F^2 < F^1 - L$. Because $F^r \geq F^1 - L$, it follows that $F^r > \text{Min}[R(F^1, \tau, L) + L - F^2]^+$.

[14] Note that because $F^1 \in [0, H]$, it follows that $\text{Max}\{F^1 - H, 0\} = 0$ and consequently $F^r(H) = 0$. This implies that if the cash flow H is realized at date 1, the firm does not need to roll over its debt.

that the market has a posterior belief that the firm is type G with probability δ, let $\mathscr{FP}(\delta)$ be the set of feasible policies available to the firm, where $\mathscr{FP}(\delta) \equiv \{p \in \mathscr{P} \mid V(p, \delta) = I\}$. Type G's problem, (MPM), reduces to the following

$$\begin{gathered} \operatorname{Min}_p v_G(p, \pi) \\ p \in \mathscr{FP}(\pi). \end{gathered} \tag{6}$$

Using basic principles of nonlinear programming, it can be shown that the firm's mispricing can be minimized by one of three contract designs. These mispricing minimizing designs are (i) pure short-term debt ($F^2 = 0$), (ii) uncovenanted long-term debt with a date 1 coupon payment of L($F^1 = L$, $F^2 > L$, and $\tau = 0$), and (iii) covenanted long-term debt with a date 1 coupon payment of L that commits the firm to retaining at least F^2 whenever it has sufficient cash flow to do so ($F^1 = L$, $F^2 > L$, and $\tau = F^2 - L$).

LEMMA 1: *There exists a policy p^* that solves (MPM) under which the firm issues either (i) short-term debt ($F^{2*} = 0$), (ii) uncovenanted coupon bearing long-term debt ($F^{1*} = L$, $F^{2*} > L$, and $\tau^* = 0$), or (iii) covenanted coupon bearing long-term debt ($F^{1*} = L$, $F^{2*} > L$, and $\tau^* = F^{2*} - L$).*

The proof of Lemma 1 is somewhat tedious because of the large number of policy options that must be considered. However, the intuition behind the proof is fairly straightforward. First, note that the default risk from claims on riskless cash flows is 0. Thus, using riskless cash flows to finance the investment, as opposed to paying it out as a dividend, lowers the mispricing losses accruing to type G. It follows that mispricing can be minimized by setting $F^1 \geq L$. For this reason, attention can be restricted to contracts in which $F^1 \geq L$. A parallel incentive does not exist in the case of the riskless portion of the date 2 cash flow. In any case, regardless of financing policy, this cash flow is implicitly pledged to bondholders whenever it is required to stave off default. In fact, when $F^2 < L$, raising F^2 to equal L merely redistributes the value of the riskless component of the date 2 cash flow between date 1 and date 2 claims.[15]

This result, combined with the capital raising constraint, $V(p, \pi) = I$, and our parameter restrictions, imply that the value of the bonds is concave in the financing policy. Thus, it is possible to transform (MPM) into a concave programming problem that can be solved using standard optimization techniques. Because the value of the bonds is concave in the financing policy, minima obtain at extreme points of the feasible contract set. All the possible bond values that can be attained at extreme points of the feasible set can be attained by one of the three contract designs represented in Lemma 1. Given this result, we can restrict our attention to these three contract designs. Therefore, we proceed to characterize the conditions under which each of these contracts minimizes mispricing.

[15] See Lemma A3 in Appendix 2 for a formal proof of this result.

We first consider contract (i) in Lemma 1, short-term debt. Diamond (1991) and Flannery (1986) show that, in a frictionless adverse selection context similar to ours, short-term debt financing is optimal. The following proposition shows the extent to which this result remains valid in our setting, which allows for both intermediate date cash flows and nonstationary cash flow distributions. When the asymmetry of information is the same for both dates, i.e., P_G/P_B and P_G^2/P_B^2 are "similar" in magnitude, there is no advantage to the firm from tying the payments on its bonds to its cash flows at a particular date. Thus, the mispricing of its bonds depends solely on the quality of information used in pricing them. Because short-term debt may be refinanced at the intermediate date, when more information is made available to investors, it allows for more information to be incorporated in the investors' pricing decisions. Thus, it minimizes mispricing.

LEMMA 2: *Whenever*

$$\frac{(1 - P_G)}{(1 - P_B)} \frac{P_G^2}{P_B^2} < \frac{P_G}{P_B} \leq \frac{P_G^2}{P_B^2}. \tag{7}$$

short-term debt pooling is an equilibrium outcome. Moreover, pooling with coupon bearing long-term debt, either covenanted or uncovenanted, is not an equilibrium outcome.

As Lemma 2 indicates, the optimality of short-term debt financing is crucially dependent on the uniformity of the degree of information asymmetry across dates. This is measured by the magnitude of $P_G^2/P_B^2 - P_G/P_B$. When this difference is sufficiently large, the information asymmetry is concentrated around the firm's date 2 cash flow. Thus, if the firm is type G, it can reduce its mispricing losses by issuing a bond whose payoffs are closely linked to the distribution of the date 1 cash flow, about which there is less informational asymmetry. This rules out financing with uncovenanted long-term debt, whose default risk is based solely on date 2 cash flows. Thus, the firm's decision is one of choosing between short-term debt and covenanted long-term debt.

Consider the case where there is no refinancing risk and no informational asymmetry regarding date 1 cash flows. Because there is no informational asymmetry regarding the intermediate cash flow, it provides no information to investors, and thus, the updating feature that favors short-term debt is absent. Because there is no refinancing risk, the initial short-term debt issue commands no risk premium and thus has a face value of I. When cash flow L is realized at date 1, the firm pays out the cash flow and repays the balance owed by issuing a claim against date 2 cash flows. When cash flow H is realized, the firm pays off its short-term debt obligation of I. Similarly, when it issues long-term covenanted debt, at date 1 the firm always pays out L if cash flow L is realized. It pays a coupon of L and retains $F^2 - L$, enough cash to cover the face value of the long-term debt, if cash flow H is realized. This debt is risky, and thus, commands a risk premium, i.e., $F^2 > I$. Consequently, the payment out of date 1 cash flows is uniformly higher with long-term covenanted debt than with short-term debt. Because payments out of date 1 cash flows minimize mispricing, long-term covenanted debt is optimal.

Increases in the risk of the firm being unable to refinance its short-term debt lead to an increase in the risk-premium on short-term debt repayable at the intermediate date. However, this increase never results in a risk premium greater than the risk-premium on covenanted long-term debt. To see this, note that if $\theta = 1$, the firm is unable to refinance its short-term debt after the realization of cash flow L at the intermediate date. Consequently, ownership of terminal cash flows is transferred to the bondholders. This is also the case when the firm issues covenanted long-term debt. Because both short-term and long-term covenanted debt provide identical payoffs at the terminal date and can only be satisfied if cash flow H is realized at date 1, they command the same risk premium. This demonstrates that long-term covenanted debt minimizes mispricing regardless of the level of refinancing risk.[16]

PROPOSITION 1: *Whenever*

$$\frac{(1 - P_G)}{(1 - P_B)} \frac{P_G^2}{P_B^2} > \frac{P_G}{P_B} \tag{8}$$

covenanted long-term debt is an equilibrium pooling outcome. Moreover, short-term and uncovenanted long-term debt are never equilibrium outcomes.

Proposition 1 provides a rationale for the issuance of long-term debt. This rationale does not rely on the existence of any dissipative costs or frictions in the capital markets (see, for example, Diamond (1991), and Flannery (1986)). Instead, it relies on a fairly natural specification of informational asymmetry which postulates that informational asymmetry is larger for more distant cash flows. As shown in Proposition 1, the range of cash flow distributions under which pooling with covenanted long-term debt is optimal is independent of the refinancing risk associated with short-term debt. Thus, so long as the asymmetry of information increases at a sufficiently high rate over time, it is optimal to finance with covenanted long-term debt regardless of the level of refinancing risk.

[16] This result appears to contradict the conventional wisdom that start-up firms are characterized both by high degrees of informational asymmetry regarding long-term cash flows and by an extensive reliance on short-term debt financing. However, there are two important caveats to this observation. The first is the stylized fact that short-term financing for small firms tends to be collateralized (see Hayes (1977, p. 156) and McConnell and Petit (1984, p. 118)). This fact is important because, while informational asymmetry regarding the firm's operating cash flows may be growing over time, there is no reason to believe that the informational asymmetry regarding highly liquid collateralized assets is also increasing over time. In this case, our results suggest that it may be optimal to rely on short-term debt financing collateralized by liquid assets. Second, while they may not tend to place heavy reliance on long-term debt financing, its use by small business is by no means insignificant (see Andrews and Eisemann (1984)).

In contrast to the case of covenanted long-term debt, in which refinancing risk plays essentially no role, the viability of pooling equilibria in which uncovenanted long-term debt is issued is crucially dependent on the existence of refinancing risk. Because the elimination of dividend restrictions reduces the dependence of bondholders' payoffs on date 1 cash flows, uncovenanted long-term debt is a candidate for minimizing mispricing losses when there is greater informational asymmetry regarding near-term cash flows. Thus, uncovenanted long-term debt is optimal whenever it is mispriced to a lesser extent than short-term debt.

However, when there is little or no refinancing risk, the default risk on short-term debt is also primarily dependent on date 2 cash flows. Further, because it allows for superior information to be used in repricing debt at the intermediate date, short-term debt minimizes mispricing losses. On the other hand, when refinancing risk is sufficiently large, the informational advantage of short-term debt is eroded because of the reduced likelihood of the firm being able to refinance its debt. In this case, because it places greater weight on date 2 cash flows, uncovenanted long-term debt is optimal.

PROPOSITION 2: *Pooling with uncovenanted long-term debt is an equilibrium outcome if $P_G/P_B > P_G^2/P_B^2$ and*

$$\theta > \left[\frac{P_G^2 - P_B^2}{P_G^2}\right]\left[\frac{P_G - P_B}{P_G}\right]^{-1}. \tag{9}$$

For θ that satisfy these inequalities, short-term and covenanted long-term debt are not equilibrium outcomes.

The above expressions demonstrate that pooling with uncovenanted long-term debt exists only if the degree of informational asymmetry is concentrated on near-term cash flows. Further, they demonstrate that these equilibria exist if there exists sufficiently high refinancing risk.

The following numerical example (Figure 1) illustrates the above results. In the simulations, we examine the impact of both changes in refinancing risk and the distribution of informational asymmetry across dates. The level of asymmetric information at dates 1 and 2 are represented by the horizontal and vertical axes of each of the graphs, respectively. Refinancing risk varies across the graphs. As is apparent from Proposition 1, the range of parameters over which covenanted long-term debt is optimal is independent of the level of refinancing risk. However, the optimality of uncovenanted long-term debt is crucially dependent upon the level of refinancing risk. In fact, the region over which uncovenanted long-term debt is optimal is empty when there is no refinancing risk and approaches the entire region below the diagonal as the level of refinancing risk converges to 1.

III. Signaling with Debt Contracts

In the absence of the assumption that type G has a higher expected cash flow at both dates, it is possible that type G may have lower cash flows at

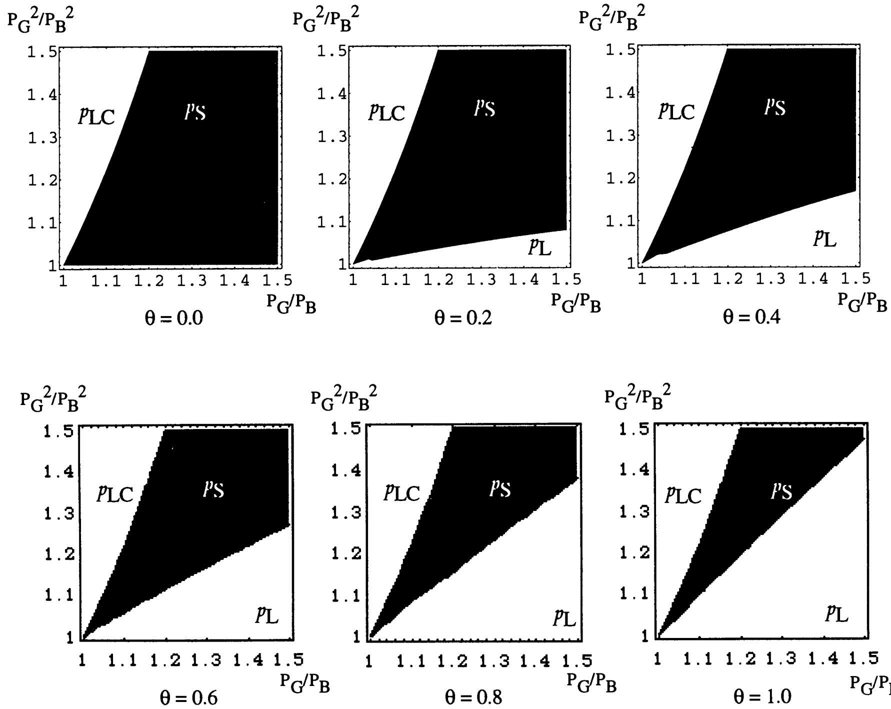

Figure 1. The effect of refinancing risk and information asymmetry on debt maturity. The series of graphs presented in Figure 1 are the results of a simulation capturing the effects of variations in the intertemporal distribution of information asymmetry and refinancing risk on the design of debt securities. Throughout the simulations, the probability of the high cash flow if the firm is type B is fixed at 0.5 for both dates ($P_B = P_B^2 = 0.5$), and the prior assessment that the firm is type G is also fixed at 0.5 ($\pi = 0.5$). The horizontal axis in each graph represents the degree of informational asymmetry at date 1 (P_G/P_B). The vertical axis on each graph represents the degree of informational asymmetry at date 2 (P_G^2/P_B^2). These ratios vary between 1 and 1.5. The six panels in the graph are obtained by varying the level of refinancing risk (θ). The three regions of the graphs represent parameter values for which pooling by issuing short-term debt (p_S), covenanted long-term debt (p_{LC}), and uncovenanted long-term debt (p_L) is the unique pooling outcome.

either date 1 or date 2. When this is the case, the relative value of a contract issued by the firm may be lower if it is type G. This leads to the emergence of separating equilibria. These equilibria are characterized by the issuance of contracts that place maximal weight on cash flows the firm is least likely to realize contingent on the realizations of its private information (see, for example, Brennan and Kraus (1987)). For simplicity, we restrict attention only to the optimal contract designs identified in Lemma 1.

To allow for separating equilibria, we drop the uniform dominance assumption ($P_G > P_B$ and $P_G^2 > P_B^2$). We now assume that $P_G \neq P_B$ and $P_G^2 \neq P_B^2$. To maintain the assumption that the cash flow distributions across types are distinct and type G's distribution is dominant, we assume that the cash flow distributions of the two types are ordered in terms of *strict* FSD, i.e., we impose the following dominance conditions

$$P_G P_G^2 > P_B P_B^2, \tag{10}$$

and

$$P_G + (1-\theta)(1-P_G)P_G^2 > P_B + (1-\theta)(1-P_B)P_B^2. \tag{11}$$

Condition (10) captures the notion that the probability of type G realizing a total cash flow of $2H$ is greater than the corresponding probability for type B. Condition (11) captures the restriction that the probability of the firm realizing a cash flow of at least $H + L$ is greater if it is type G.

In separating equilibria, the firm's type is fully revealed at date 0 through its contract choice. Thus, in these equilibria there is no room for the revision of beliefs at date 1. This eliminates the primary advantage associated with short-term debt financing. Because uncovenanted long-term debt places maximal weight on date 2 cash flows, it is issued by the type that has lower date 2 cash flows. On the other hand, covenanted long-term debt, because it places maximal weight on date 1 cash flows, is issued by the type that has lower date 1 cash flows.

PROPOSITION 3: *(i) Dividend covenants never signal favorable information. (ii) Parameter values that support a separating outcome in which the short-term debt is issued, also support separating outcomes in which only long-term debt is issued.*

COROLLARY 1: *In separating equilibria, the absence of dividend covenants signals favorable information.*

Proposition 3 demonstrates that long-term debt dominates short-term debt as a vehicle for signaling information. In the separating equilibria characterized in the proposition, information is signaled by the presence or absence of dividend covenants. When a cash flow stream with a higher expected value has a longer duration, neither does the presence of dividend covenants signal unfavorable information, nor does their absence signal favorable information. However, when a cash flow stream with a higher expected value has a shorter duration, the presence of dividend covenants signals unfavorable information and their absence signals favorable information. Thus, whether or not the incorporation of dividend covenants in the debt contract is a favorable signal depends on the characteristics of the firm. It follows that the announcement effect is firm specific, like the signals in the models of Bagnoli and Khanna (1992) and Cooney and Kalay (1993).[17] Consequently, the results of empirical analyses of our predictions would be sample-specific, and the failure to correctly segment the sample could lead to the observance of insignificant average announcement effects. If each analyst's forecast of expected earnings corresponds to the expected earnings of the firm under a particular realization of private information, the magnitude of the serial

[17] See Bagnoli and Khanna (1992) for a discussion of the distinction between firm-specific and action-specific signaling models.

correlation of analysts' forecasts would be related to the variance of announcement effects from debt issues. In any case, our results predict that the variance of announcement effects should be higher for long-term debt than it is for short-term debt.

IV. Conclusion

In this article, we examine the role of debt maturity, coupon payments, and payout restrictions in signaling a firm's private information. We show that the design of the firm's optimal debt structure is crucially linked to the temporal distribution of informational asymmetry. Coupon-bearing long-term debt with dividend covenants is optimal when there is greater informational asymmetry regarding long-term cash flows, and coupon-bearing uncovenanted long-term debt is optimal when there is greater informational asymmetry regarding near-term cash flows. Short-term debt financing is optimal only when the informational asymmetry is uniformly distributed over time. Further, the use of covenant restrictions on long-term debt dominates the use of short-term debt as a means of signaling information.

These results are in stark contrast to those of Diamond (1991) and Flannery (1986), who argue that the ex ante determination of the terms of debt contracts is a distinct disadvantage in the presence of informational asymmetries. In a setting with only a terminal cash flow, they show that by issuing short-term debt against long-term cash flows, firms are able to minimize mispricing by delaying the determination of payment terms. In order to justify the use of long-term debt financing, Flannery (1986) and Diamond (1991) associate dissipative costs (in the form of loss of control rights in Diamond (1991) and transactions costs in Flannery (1986)) with short-term debt financing. In our analysis, long-term debt can minimize mispricing and thus be preferred even in the absence of dissipative costs. The consideration of the impact of intermediate-date cash flows also sets our model apart from those of Diamond (1991) and Flannery (1986). Because it reveals information to investors, the intermediate-date cash flow acts very much like the noisy signal in their models. However, the introduction of these cash flows also introduces into the analysis important issues such as the role of corporate payout policies and dividend restrictions.

Our analysis abstracts from firms' operating discretion. We assume that the firm is unable to prematurely liquidate assets. Liquidation may be infeasible either because the costs from premature liquidation are relatively large, or all available debt instruments contain covenants precluding dissipative asset transformations. If covenants restricting asset transformation were not enforceable, and assets had sufficient liquidation value, uncovenanted long-term debt might be infeasible as a financing alternative. Another restriction imposed on the firm's discretion is the assumption that retained funds are invested in riskless assets. This assumption is analogous to imposing sinking fund provisions on the firm. Sinking funds often require firms to invest funds earmarked for bondholders in low-risk assets. When such sinking fund restrictions are infeasible, the firm would have an incentive to invest retained funds in risky ventures. Recognizing this incentive, investors would appropriately discount the firm's *covenanted* long-term debt. This could reduce its attractiveness as a financing alternative. This is an interesting avenue for future theoretical investigation.

An interesting avenue for future empirical research would be to generalize the notion of the temporal distribution of informational asymmetry with a view to constructing better proxies for the temporal distribution. Under one scenario, the evolution of the variance of firm cash flows can act as a proxy for the distribution of asymmetric information. This would be the case when the average quality of inside information is the same for all firm types across all dates.[18] In this case, increased cash flow volatility over time, which could be estimated using GARCH techniques or the term structure of implied volatility estimated using option prices, would imply greater information asymmetry regarding long-term cash flows. Conversely, decreasing cash flow variance over time would imply lower informational asymmetry regarding long-term cash flows. In this fashion one could relate the term structure of volatility to the maturity structure of corporate debt.

Appendix 1

Definition A1: Let the policies $p_S(\delta)$, $p_{S''}(\delta)$, $p_L(\delta)$, and $p_{LC}(\delta)$ be defined as follows: $p_S(\delta) \equiv (F_S^1(\delta), 0, 0)$, where $F_S^1(\delta)$ is the unique F^1 which solves $V(F^1, 0, 0, \delta) = I$; $p_{S''}(\delta) \equiv (F_{S''}^1(\delta), L, 0)$, where $F_{S''}^1(\delta)$ is the unique F^1 which solves $V(F^1, L, 0, \delta) = I$; $p_L(\delta) \equiv (L, F_L^2(\delta), 0)$, where $F_L^2(\delta)$ is the unique solution to $V(L, F^2, 0, \delta) = I$; and $p_{LC}(\delta) \equiv (L, F_{LC}^2(\delta), F_{LC}^2(\delta) - L)$, where $F_{LC}^2(\delta)$ is the unique solution to $V(L, F^2, F^2 - L, \delta) = I$.[19]

LEMMA A1: *Suppose that p' is a policy such that $V(p', \delta) = I$. Then it must be the case that (i) $R(F^{1\prime}, F^{2\prime} - L, H) + L = F^{2\prime}$, (ii) $H - F^{2\prime} > F^{r\prime}(L)$, (iii) $H > F^{1\prime} + F^{2\prime}$, and (iv) $F^{2\prime} < H$.*

Proof of Lemma A1: (i) First note that $R(F^{1\prime}, F^{2\prime} - L, H) + L = \text{Min}\{H - F^{1\prime}, F^{2\prime} - L\} + L \leq F^{2\prime}$. Now suppose that $R(F^{1\prime}, F^{2\prime} - L, H) + L = \text{Min}\{H - F^{1\prime}, F^{2\prime} - L\} + L < F^{2\prime}$. This implies that $H - F^{1\prime} \leq F^{2\prime} - L$, or equivalently, $R(F^{1\prime}, F^{2\prime} - L, H) = H - F^{1\prime}$. In this case $v_t(p', \delta) \geq P_t F^{1\prime} + (1 - \theta)P_t(1 - P_t^2)\text{Min}\{R(F^{1\prime}, F^{2\prime} - L, H) + L, F^{2\prime}\} = P_t F^{1\prime} + (1 - \theta)P_t(1 - P_t^2)(H - F^{1\prime} + L) > (1 - \theta)P_t(1 - P_t^2)(F^{1\prime} + H - F^{1\prime} + L) > (1 - \theta)P_t(1 - P_t^2)H$. By condition (2), $(1 - \theta)P_t(1 - P_t^2)H > I$. This contradicts $V(p', \delta) = I$.

[18] That is $\text{Var}(X^d \mid t)$ is constant across all d and t.

[19] When analyzing pooling equilibria, the terms of the policies $p_S(\cdot)$, $p_{S''}(\cdot)$, $p_L(\cdot)$, and $p_{LC}(\cdot)$ are always dependent on the market's prior beliefs, π. Hence, to simplify the presentation of our results on pooling equilibria, henceforth, the dependence of the terms of these policies on the market's beliefs will be suppressed. However, to avoid confusion, this dependence is made explicit when discussing separating equilibria.

(ii) We establish the desired result by means of a contradiction. Suppose that $H - F^{2\prime} \leq F^{r\prime}(L)$. This implies that $\text{Min}\{H - F^{2\prime}, F^{r\prime}(L)\} = H - F^{2\prime}$. Then it must be the case that $v_t(p', \delta) \geq (1-\theta)(1-P_t)P_t^2 \text{ Min}\{H - F^{2\prime}, F^{r\prime}(L)\} + v_t^2(p', \delta) \geq (1-\theta)(1-P_t)P_t^2(H - F^{2\prime}) + (1-\theta)(1-P_t)P_t^2 F^{2\prime} = (1-\theta)(1-P_t)P_t^2 H$. By condition (2), $(1-\theta)(1-P_t)P_t^2 H > I$. This contradicts $V(p', \delta) = I$.

(iii) We prove this result by means of a contradiction. Suppose that $H \leq F^{1\prime} + F^{2\prime}$. Then $v_t(p', \delta) \geq P_t F^{1\prime} + (1-\theta)P_t^2 F^{2\prime} \geq (1-\theta)P_t P_t^2[F^{1\prime} + F^{2\prime}] \geq (1-\theta)P_t P_t^2 H$. By condition (2), $(1-\theta)P_t P_t^2 H > I$. This contradicts $V(p', \delta) = I$.

(iv) Suppose that $H \leq F^{2\prime}$. Then $v_t(p', \delta) \geq v_t^2(p', \delta) \geq (1-\theta)(1-P_t)P_t^2 \text{ Min}\{F^{2\prime}, H\} = (1-\theta)(1-P_t)P_t^2 H$. By condition (2), $(1-\theta)(1-P_t)P_t^2 H > I$. This contradicts $V(p', \delta) = I$.

Sketch of Proof of Lemma 1: A formal proof follows directly from Lemmas A6 and A9 in Appendix 2. The proof is fairly straightforward. First, note that the default risk of claims on riskless cash flows is 0. Thus, using riskless cash flows to finance the investment lowers the mispricing losses accruing to type G. The objective of paying out these cash flows can be attained by setting $F^1 \geq L$. For this reason attention can be restricted to contracts in which $F^1 \geq L$. A parallel incentive does not exist in the case of the riskless portion of the date 2 cash flow, which, regardless of financing policy, is implicitly pledged to bondholders in any case.

This result, combined with the capital raising constraint, $V(p, \pi) = I$, and our parameter restrictions, implies that the value of the bonds is concave in the financing policy. In fact, it takes the functional form $U(\cdot)$ in Definition A2 in Appendix 2. Thus, it is possible to transform (MPM) into a concave programming problem which can be solved using standard optimization techniques. Because the value of the bonds is concave in the financing policy, from Rockafellar (1970) Theorem 32.2 it follows that minima obtain at extreme points of the feasible contract set. Because the parameter restrictions ensure $F^d < H$ for $d = 1$ or 2, it follows that it must be the case that either $F^1 > L$ and $F^2 = 0$, or $F^1 = L$ and $F^2 > L$. All the possible bond values that can be attained at extreme points of the feasible set can be attained by one of the three contract designs represented in Lemma 1. Given that we can restrict our attention to these three contract designs, we proceed to characterize the conditions under which each of these contracts minimizes mispricing.

Result 1: Let $P(\delta) \equiv \delta P_G + (1-\delta)P_B$, $P^2(\delta) = \delta P_G^2 + (1-\delta)P_B^2$, and $P_H(\delta) \equiv \delta(1-P_G)P_G^2 + (1-\delta)(1-P_B)P_B^2$, then

$$v_G(p_S(\delta), \delta) = 2L + \frac{P_G(I - 2L)}{1 - \theta(1 - P(\delta))} + \frac{(1-\delta)(I - 2L)(1 - P_G)P_G^2(1 - P(\delta))}{[1 - \theta(1 - P(\delta))]P_H(\delta)}, \tag{A1}$$

$$v_G(p_{LC}(\delta), \delta) = 2L + \frac{\left[P_G + (1-\theta)(1-P_G)P_G^2\right](I-2L)}{P(\delta) + (1-\theta)P_H(\delta)}, \tag{A2}$$

$$v_G(p_L(\delta), \delta) = 2L + \frac{P_G^2(I-2L)}{P^2(\delta)}, \tag{A3}$$

$$v_B(p_S(\delta), \delta) = 2L + \frac{P_B(I-2L)}{1-\theta(1-P(\delta))} + \frac{(1-\theta)(I-2L)(1-P_B)P_B^2(1-P(\delta))}{[1-\theta(1-P(\delta))]P_H(\delta)}, \tag{A4}$$

$$v_B(p_{LC}(\delta), \delta) = 2L + \frac{\left[P_B + (1-\theta)(1-P_B)P_B^2\right](I-2L)}{P(\delta) + (1-\theta)P_H(\delta)}, \tag{A5}$$

$$v_B(p_L(\delta), \delta) = 2L + \frac{P_B^2(I-2L)}{P^2(\delta)}. \tag{A6}$$

Proof of Result 2: The proof follows by using definition A1 to obtain values for $F_S^1(\delta)$, $F_{LC}^2(\delta)$, and $F_L^2(\delta)$ and substituting these values into v_G and v_B.

Proof of Lemma 2: To establish the existence of short-term pooling as the mispricing-minimizing outcome and to establish that neither covenanted nor uncovenanted long-term pooling is a mispricing minimizing outcome, we only need to establish that $v_G(p_S, \pi) < v_G(p'', \pi)$ where $p'' = p_L, p_{LC}$. From definitions (A1) and (A2) it follows that $v_G(p_{LC}, \pi) - v_G(p_S, \pi) > 0$ if $(1-P_B)P_B^2P_G - (1-P_G)P_G^2P_B > 0$, or equivalently if

$$\frac{P_G}{P_B} > \frac{(1-P_G)P_G^2}{(1-P_B)P_B^2}. \tag{A7}$$

Similarly, from equations (A1) and (A3) it follows that $v_G(p_L, \pi) - v_G(p_S, \pi) > 0$ if $P_BP_G^2 - P_GP_B^2 > 0$, or equivalently if $P_G^2/P_B^2 > P_G/P_B$. Because these conditions are the ones imposed in the Lemma, our proof is complete.

Proof of Proposition 1: To establish the existence of covenanted long-term pooling as the mispricing-minimizing outcome and to establish that neither uncovenanted long-term pooling nor short-term pooling is a mispricing minimizing outcome, we only need to establish that $v_G(p_{LC}, \pi) < v_G(p'', \pi)$ where $p'' = p_L, p_S$. From (A1) and (A2), it follows that $v_G(p_{LC}, \pi) - v_G(p_S, \pi) < 0$ if $P_G/P_B < ((1-P_G)P_G^2)/((1-P_B)P_B^2)$, or equivalently if

$$\frac{P_G^2}{P_B^2} > \frac{P_G(1-P_B)}{P_B(1-P_G)}. \tag{A8}$$

From equations (A2) and (A3) it follows that $v_G(p_{LC}, \pi) - v_G(p_L, \pi) < 0$ if $P_G P_B^2 - P_B P_G^2 - (1 - \theta)P_B^2 P_G^2(P_G - P_B) < 0$, or equivalently if

$$P_B P_B^2\left(\frac{P_G}{P_B} - \frac{P_G^2}{P_B^2}\right) - (1 - \theta)P_B^2 P_G^2(P_G - P_B) < 0. \tag{A9}$$

Because equation (A9) is satisfied by the assumptions of Proposition 1, and $(1 - P_B) > (1 - P_G)$, it follows that $P_G/P_B - \mathrm{P}_G^2 P_B^2 < 0$, implying that the above inequality is satisfied so long as the assumptions of Proposition 3 are satisfied.

Proof of Proposition 2: To establish this result, note that from equations (A2) and (A3) it follows that $v_G(p_{LC}, \pi) - v_G(p_L, \pi) > 0$ if $\theta > 1 - [P_B/(P_G^2(P_G - P_B))][P_G/P_B - P_G^2/P_B^2]$. Similarly, from equations (A1) and (A3) it follows that $v_G(p_L, \pi) - v_G(p_S, \pi) < 0$ if

$$\theta > \frac{P_G^2 - P_B^2}{(P_G - P_B)P_G^2 P_B^2} Q(\pi), \tag{A10}$$

where

$$Q(\pi) \equiv \left[\frac{\pi(1 - P_G)P_G^2 P_B + (1 - \pi)(1 - P_B)P_B^2 P_G}{\pi(1 - P_G) + (1 - \pi)(1 - P_B)}\right]. \tag{A11}$$

Next note that because the numerator and denominator of $Q(\pi)$ are affine in π and the denominator is positive for all permissible parameter values, it follows that $Q(\pi)$ is monotone. Note that $Q(1) = P_G^2 P_B$ and $Q(0) = P_B^2 P_G$. It follows that $Q(0) - Q(1) = P_B P_B^2[P_G/P_B - P_G^2/P_B^2] > 0$ if $P_G/P_B > P_G^2/P_B^2$. Next note that if $P_G/P_B > P_G^2/P_B^2$, then $1 - [P_B/(P_G^2(P_G - P_B))][P_G/P_B - P_G^2/P_B^2] < 1$ and further

$$\begin{aligned}\frac{P_G^2 - P_B^2}{(P_G - P_B)P_G^2 P_B^2} Q(1) &= \left[\frac{P_G^2 - P_B^2}{P_G^2}\right]\left[\frac{P_G - P_B}{P_G}\right]^{-1} \\ &> 1 - \left[P_B/(P_G^2(P_G - P_B))\right]\left[P_G/P_B - P_G^2/P_B^2\right].\end{aligned} \tag{A12}$$

Thus, if $P_G/P_B > P_G^2/P_B^2$ and $\theta > \left[\frac{P_G^2 - P_B^2}{P_G^2}\right]\left[\frac{P_G - P_B}{P_G}\right]^{-1}$, then long-term debt minimizes mispricing.

Proof of Proposition 3: First, we establish (i). Then we establish (ii).

(i) To see that long-term covenanted debt cannot signal favorable information note that in any separating equilibrium in which the firm issues long-term covenanted debt if it is type G, it must be the case that $v_B(p_{LC}(1), 1) \geq v_B(p''(0), 0) = I$. From equation (A5) it follows that this condition can only be satisfied if $P_G + (1 - \theta)(1 - P_G)P_G^2 - P_B + (1 - \theta)(1 - P_B)P_B^2 < 0$. However, from the dominance conditions (10) and (11) it follows that this condition can never be satisfied.

(ii) Consider the case where $P_G < P_B$, and $P_G^2 > P_B^2$. We show that in this case there exist no separating equilibria in which short-term debt is issued. Note that a separating outcome in which the firm issues short-term debt if it is type G and either covenanted long-term debt or uncovenanted long-term debt if it is type B can only be sustained if (1) $v_B(p_S(1), 1) \geq v_B(p''(0), 0)$ where $p''(0) = p_{LC}(0)$, $p_L(0)$ and (2) $v_G(p_S(1), 1) \leq v_G(p''(0), 0)$ where $p''(0) = p_{LC}(0)$, $p_L(0)$. From (A4) it follows that (1) can only be satisfied if

$$0 \geq (1 - P_B)(P_G^2 - P_B^2) - \theta\left[(1 - P_G)P_G^2 - (1 - P_B)P_B^2\right]. \qquad \text{(A13)}$$

However, $(1 - P_B)(P_G^2 - P_B^2) - \theta[(1 - P_G)P_G^2 - (1 - P_B)P_B^2] - [P_G - P_B + (1 - \theta)[(1 - P_G)P_G^2 - (1 - P_B)P_B^2]] = (1 - P_G^2)(P_B - P_G) \geq 0$. Thus, from the dominance conditions (10) and (11) it follows that equation (A13) cannot be satisfied.

Next consider a separating equilibrium in which the firm issues short-term debt if it is type B and uncovenanted long-term debt if it is type G (the separating outcome in which the firm issues covenanted long-term debt if it is type G being ruled out by (i)). For this outcome to be supported by an equilibrium, it must be the case that (1) $v_B(p_S(0), 0) \leq v_B(p_L(1), 1)$ and (2) $v_G(p_L(1), 1) \leq v_G(p_S(0), 0)$. From equation (A6) it follows that (1) can only be satisfied if $P_G^2 < P_B^2$, contradicting our hypothesis. Thus, there exist no separating outcomes in which short-term debt is issued.

Now consider the case where $P_G^2 < P_B^2$ and $P_G > P_B$. Suppose that there exists a separating equilibrium in which short-term debt is being issued by type G. Consider an alternate equilibrium in which the firm issues uncovenanted long-term debt if it is type G and covenanted long-term debt if it is type B. Further, let short-term debt be priced as if it were being issued by type G. Note that the necessary condition for short-term debt to be issued by type G in the original equilibrium will ensure that the firm will not defect to short-term debt if it is type B. Further, because both short-term and uncovenanted long-term debt are correctly priced if the firm is type G, it too will not have an incentive to defect to short-term debt. Further note that, from equation (A2) it follows that $v_G(p_L(1), 1) - v_G(p_{LC}(0), 0) \leq 0$ if and only if

$$P_G + (1 - \theta)(1 - P_G)P_G^2 - P_B - (1 - \theta)(1 - P_B)P_B^2 \geq 0. \qquad \text{(A14)}$$

From the dominance conditions (10) and (11) it follows that equation (A14) is always satisfied. Also note that from equation (A5) it follows that $v_B(p_L(1), 1) - v_B(p_{LC}(0), 0) \geq 0$ if and only if $P_G^2 \leq P_B^2$. This establishes that if there exists a separating equilibrium in which short-term debt is being issued by the firm if it is type G, then there also exists a separating equilibrium in which only long-term debt is issued. A virtually identical proof establishes the desired result for the case where there exists a separating equilibrium in which short-term debt is issued by type B. This establishes (ii).

Appendix 2

LEMMA A2: *Let p' be a policy such that $R(F^{1\prime}, \tau', L) > 0$. Then there exists a policy p'' such that, for $t = B, G$, $v_t(p'', \delta) = v_t(p', \delta)$ and $R(F^{1\prime\prime}, \tau'', L) = 0$.*

Proof of Lemma A2: Let $F^{1\prime\prime} = F^{1\prime} + R(F^{1\prime}, \tau', L)$, $F^{2\prime\prime} = F^{2\prime} - R(F^{1\prime}, \tau', L)$, and $\tau'' = \tau' - R(F^{1\prime}, \tau', L)$. Note that if $R(F^{1\prime}, \tau', L) > 0$, then $L > F^{1\prime}$ and $F^{1\prime\prime} = F^{1\prime} + R(F^{1\prime}, \tau', L) = F^{1\prime} + \text{Min}\{L - F^{1\prime}, \tau'\} \leq L$. Because $L > F^{1\prime}$, it follows that $F^{r\prime}(H) = F^{r\prime}(L) = 0$, and because $L \geq F^{1\prime\prime}$, it follows that $F^{r\prime\prime}(H) = F^{r\prime\prime}(L) = 0$. Further, note that $R(F^{1\prime\prime}, \tau'', X^1) = \text{Min}\{X^1 - F^{1\prime\prime}, \tau''\} = \text{Min}\{X^1 - F^{1\prime}, \tau'\} - R(F^{1\prime}, \tau', L) = R(F^{1\prime}, \tau', X^1) - R(F^{1\prime}, \tau', L)$. Consequently, for $t = B, G$,

$$\begin{aligned} v_t(p'', \delta) &\equiv E\left\{\text{Min}\left[X_t, F^{1\prime\prime}\right]\right\} + E\left\{\text{Min}\left(\left[X_t^2 - F^{2\prime\prime}\right]^+, 0\right)\right\} \\ &\quad + E\{\text{Min}\{R(F^{1\prime\prime}, \tau'', X^1) + X^2, F^{2\prime\prime}\}\} \\ &= F^{1\prime\prime} + E\{\text{Min}\{R(F^{1\prime}, r', X^1) - R(F^{1\prime}, \tau', L) + X^2, \\ &\qquad F^{2\prime} - R(F^{1\prime}, \tau', L)\}\} \\ &= F^{1\prime} + R(F^{1\prime}, \tau', L) + E\{\text{Min}\{R(F^{1\prime}, \tau', X^1) + X^2, F^{2\prime}\}\} \\ &\quad - R(F^{1\prime}, \tau', L) \\ &= F^{1\prime} + E\{\text{Min}\{R(F^{1\prime}, \tau', X^1) + X^2, F^{2\prime}\}\} = v_t(p', \delta). \end{aligned} \tag{A15}$$

LEMMA A3: *Given any p' such that $F^{2\prime} < L$, there exists a policy p'' such that $F^{2\prime\prime} = L$ and $v_t(p'', \delta) = v_t(p', \delta)$.*

Proof of Lemma A3: To see this, first note that because $F^{2\prime} < L$ it must be the case that $F^{1\prime} > L$. Now let $F^{1\prime\prime} = F^{1\prime} - (L - F^{2\prime})$, $F^{2\prime\prime} = L$, and $\tau'' = \tau'$. Note also that because $F^{2\prime\prime} = L$ and $F^{2\prime\prime} + F^{1\prime\prime} > 2L$, it follows that $F^{1\prime\prime} > L$. Because, p' is feasible, from equation (4) it follows that $F^{r\prime}(L)$ satisfies

$$\begin{aligned} F^{1\prime} - L &= \phi(\delta)\left[P_G^2 F^{r\prime}(L) + (1 - P_G^2)(L - F^{2\prime})\right] \\ &\quad + (1 - \phi(\delta))\left[P_B^2 F^{r\prime}(L) + (1 - P_B^2)(L - F^{2\prime})\right] \\ &= L - F^{2\prime} + P^{u2}(\delta)[F^{r\prime}(L) + F^{2\prime} - L], \end{aligned} \tag{A16}$$

where $P^{u2}(\delta) \equiv \phi(\delta)P_G^2 + (1 - \phi(\delta))P_B^2$. Further, by (4) it follows that $F^{1\prime\prime} - L = P^{u2}(\delta)F^{r\prime\prime}(L)$. Thus, because $F^{1\prime} - L - (L - F^{2\prime}) = F^{1\prime\prime} - L$, it follows that $P^{u2}(\delta)[F^{r\prime}(L) + F^{2\prime} - L] = P^{u2}(\delta)F^{r\prime\prime}(L)$, or equivalently, $F^{r\prime\prime}(L) = F^{r\prime}(L) + F^{2\prime} - L$. Thus, for $t = B, G$,

$$\begin{aligned} v_t^1(p'', \delta) &= P_t[F^{1\prime} - (L - F^{2\prime})] \\ &\quad + (1 - \theta)(1 - P_t)P_t^2[L + F^{r\prime}(L) - (L - F^{2\prime})] \\ &\quad + \left[(1 - \theta)(1 - P_t)(1 - P_t^2) + \theta(1 - P_t)\right] \end{aligned}$$

$$\begin{aligned} &\times[L + (L - F^{2\prime}) - (L - F^{2\prime})] \\ &= v_t^1(p', \delta) - (L - F^{2\prime}). \end{aligned} \tag{A17}$$

Similarly, $v_t^2(p'', \delta) = L = F^{2\prime} + (L - F^{2\prime}) = v_t^2(p', \delta) + (L - F^{2\prime})$. Thus, $v_t(p'', \delta) = v_t^1(p', \delta) - (L - F^{2\prime}) + v_t^2(p', \delta) + (L - F^{2\prime}) = v_t(p', \delta)$ for $t = B, G$.

Definition A2: Define the function U_t: $[0, H]^3 \to \mathbb{R}$, $t = G, B$, as follows:

$$\begin{aligned} U_t(p, \delta) \equiv\ & P_t F^1 + (1 - P_t)\text{Min}\{F^1, L\} + (1 - \theta)(1 - P_t)P_t^2 F^r(L) \\ & + (1 - \theta)P_t^2 F^2 + (1 - \theta)(1 - P_t^2)\big[(1 - P_t)L + P_t \text{Min}\{F^2, \tau\}\big] \\ & + \theta\big[(1 - P_t)L + P_t \text{Min}\{F^2, \tau\}\big], \end{aligned} \tag{A18}$$

where

$$F^r(L) \equiv \frac{[F^1 - L]^+}{P^2(\delta)}.$$

Further let,

$$U(p, \delta) = \delta U_G(p, \delta) + (1 - \delta)U_B(p, \delta).$$

LEMMA A4: (*a*) *If* $p \in \mathscr{FP}(\delta)$, $R(F^1, r, L) = 0$, $F^2 \geq L$, *then there exists* $\varepsilon > 0$ *such that if* p' *is a policy such that* $\tau' = \tau$, $F^{1\prime} \in [\text{Max}\{F^1 - \varepsilon, 0\}, F^1 + \varepsilon)$ *and* $F^{2\prime} \in [\text{Max}\{F^2 - \varepsilon, L\}, F^2 + \varepsilon)$, *then* $v_t(p', \delta) = U_t(p', \delta)$ *for* $t = B, G$. (*b*) *Similarly, if* $U(p, \delta) = I$, *and* $R(F^1, \tau, L) = 0$, $F^2 \geq L$, *then there exists* $\varepsilon > 0$ *such that if* p' *is a policy such that* $\tau' = \tau$, $F^{1\prime} \in [\text{Max}\{F^1 - \varepsilon, 0\}, F^1 + \varepsilon)$ *and* $F^{2\prime} \in [\text{Max}\{F^2 - \varepsilon, L\}, F^2 + \varepsilon)$, *then* $v_t(p', \delta) = U_t(p', \delta)$ *for* $t = B, G$.

Proof of Lemma A4: From Lemma A1 it follows that (i) $R(F^{1\prime}, F^{2\prime} - L, H) + L = F^{2\prime}$, (ii) $H - F^{2\prime} > F^{r\prime}(L)$, (iii) $H > F^{1\prime} + F^{2\prime}$, and (iv) $F^{2\prime} < H$. This implies that over the restricted parameter set given by the hypothesis of the lemma, $v_t(\cdot)$ has the functional form $U_t(\cdot)$ for $F^{1\prime} \in [\text{Max}\{F^1 - \varepsilon, 0\}, F^1 + \varepsilon)$ and $F^{2\prime} \in [\text{Max}\{F^2 - \varepsilon, L\}, F^2 + \varepsilon)$. (b) Follows by an identical argument noting that the hypothesis of Lemma A1 that $V(p, \delta) = I$ can be replaced with the hypothesis that $U(p, \delta) = I$, without affecting the validity of the inference.

LEMMA A5: *There exists a solution to* (*MPM*), p^*, *such that* $F^{1*} \geq L$.

Proof of Lemma A5: First note that the continuity of v_t and the compactness of the feasible $\mathscr{FP}(\delta)$ implies that a solution to (MPM) exists. Let p^* be any such solution. By Lemmas A1 and A3 there exists p' such that $F^{2\prime} \geq L$, $R(F^{1\prime}, \tau, L) = 0$, and $v_t(p', \delta) = v_t(p^*, \delta)$ for $t = G$ and B. Clearly, because p^* solves (MPM) so does p', and thus $V(p', \delta) = I$. We now show that the Lagrange necessary conditions imply that $F^{1\prime} \geq L$. To see this suppose that $F^{1\prime} < L$, then Lemma A4 implies the derivatives of $v_t(\cdot)$ with respect to F^1

and F^2 are identical to those of $U_t(\cdot)$. Thus the Lagrange associated with (MPM) evaluated at p' is given by

$$L(\lambda, F^{1\prime}, F^{2\prime}, \tau') = U_G(p', \delta) - \lambda[U(p', \delta) - I]. \qquad \text{(A19)}$$

Whenever $F^{1\prime} < L$, feasibility requires that $F^{2\prime} > L$. For this reason, the following conditions are necessary for the policy p' with $F^{1\prime} < L$ to minimize the Lagrange:

$$D_{F^1} L(\lambda, F^{1\prime}, F^{2\prime}, \tau') = D_{F^1} U_G(p', \delta) - \lambda D_{F^1} U(p', \delta) = 0, \qquad \text{(A20)}$$

$$D^-_{F^2} L(\lambda, F^{1\prime}, F^{2\prime}, \tau') = D^-_{F^2} U_G(p', \delta) - \lambda D^-_{F^2} U(p', \delta) \le 0 \qquad \text{(A21)}$$

where $D^-_{F^2}$ represents the left-hand partial derivative with respect to F^2 and D_{F^1} represents the derivative with respect to F^1. Next note that, because $F^{1\prime} < L$, $D_{F^1} U_G(p', \delta) = D_{F^1} U(p', \delta) = 1$, and thus, $\lambda = 1$. It follows that, $D^-_{F^2} U_G(p', \delta) - D^-_{F^2} U(p', \delta) \le 0$ is a necessary condition for optimality. Note that this expression has the same sign as $D^-_{F^2} U_G(p', \delta) - D^-_{F^2} U_B(p', \delta)$. Simple algebra shows that this expression is always strictly positive, contradicting the necessary condition for the optimality of p'.

LEMMA A6: $\mathrm{Min}\{U_G(p, \delta) \mid p \in \mathcal{FP}^m(\delta)\} = \mathrm{Min}\{v_G(p, \delta) \mid p \in \mathcal{FP}(\delta)\}$, *where*

$$\mathcal{FP}^m(\delta) \equiv \{p \in \mathcal{P} | p \in [L, H]^2 \times [0, H] \text{ and } U(p, \delta) \ge I\}, \qquad \text{(A22)}$$

Proof of Lemma A6: Let $\mathcal{FP}^{m=}(\delta) \equiv \{p \in \mathcal{P} \mid p \in [L, H]^2 \times [0, H]$ and $U(p, \delta) = I\}$. By Lemma A1 and A3 it follows that $\mathrm{Min}\{v_G(p, \delta) \mid p \in \mathcal{FP}(\delta)\} = \mathrm{Min}\{v_G(p, \delta) \mid p \in \mathcal{FP}(\delta), F^1 \ge L, F^2 \ge L\}$. It follows from Lemma A4 that $\{p \in \mathcal{FP}(\delta) \mid F^1 \ge L,\ F^2 \ge L\} = \mathcal{FP}^{m=}(\delta)$. Further, by Lemma A4, $v_t \equiv U_t$ for $t = B, G$ on $\{p \in \mathcal{FP}(\delta) \mid F^1 \ge L,\ F^2 \ge L\}$. Thus, $\mathrm{Min}\{U_G(p, \delta) \mid p \in \mathcal{FP}^m = (\delta)\} = \mathrm{Min}\{v_G(p, \delta) \mid p \in \mathcal{FP}(\delta)\}$. The result follows by noting that the minimum of $\{U_G(p, \delta) \mid p \in \mathcal{FP}^m(\delta)\}$ occurs when $p \in \mathcal{FP}^{m=}(\delta)$.

LEMMA A7: $\mathrm{Min}\{U_G(p, \delta) \mid p \in \mathcal{FP}^m(\delta)\} = \mathrm{Min}\{U_G(p, \delta) \mid p \in \mathrm{ext}\{\mathcal{FP}^m(\delta)\} \cap \mathcal{FP}^{m=}\}$, *where* $\mathrm{ext}\{\mathcal{FP}^m(\delta)\}$ *represents extreme points of the set* $\mathcal{FP}^m(\delta)$.

Proof of Lemma A7: First note that $\mathcal{FP}^m(\delta)$ is convex. To see this, note first that U_t is concave in p for $t = G, B$. Thus the upper contour sets associated with U_t are convex. Because $\mathcal{FP}^m(\delta)$ is the intersection of an upper contour set with a cube in $\mathbb{R}^3$, which is also a convex, it is convex. Next note that because U_G is concave, the problem $\mathrm{Min}\{U_G(p, \delta) \mid p \in \mathcal{FP}^m(\delta)\}$ is a problem of minimizing a concave function over a convex set. Thus, the minima are always attained at extreme points in the feasible set.

LEMMA A8: *If* $p \in \mathrm{ext}\{\mathcal{FP}^m(\delta)\} \cap \mathcal{FP}^{m=}(\delta)$, *either* $F^1 = L$ *and* $\tau \notin (0, F^2 - L)$, *or* $F^2 = L$ *and* $\tau \notin (0, F^2 - L)$.

Proof of Lemma A8: We will prove the contrapositive statement: if $F^1 > L$ and $F^2 > L$ then $p \notin \mathrm{ext}\{\mathcal{FP}^m(\delta)\} \cap \mathcal{FP}^{m=}(\delta)$. This proof is completed in two parts. First we consider the case where $\tau \neq F^2 - L$, then we consider the case where $\tau = F^2 - L$.

Note that from Lemma A1 it follows that for $p \in \mathscr{FP}^{m=}(\delta)$ it must be the case that $H > F^2 + F^1$. Now suppose that $\tau \neq F^2 - L$. Define the map K_τ: $\mathbb{R}^2 \to \mathbb{R}$ by $K_\tau(x, y) \equiv U(x, y, \tau, \delta)$. The function U is affine in a sufficiently small neighborhood N, of (F^1, F^2, τ), where N is a subset of an open interval in $\mathscr{FP}^{m=}(\delta)$. This implies there exists an open set N' such that N' is a subset of a neighborhood which lies in the interior of $[L, H]^2$, and the map K_τ is affine when restricted to N'. By assumption $K_\tau(F^1, F^2) = I$. Let $D_n K_\tau$ represent the partial derivative of K_τ with respect to its nth argument. The fact that $D_1 K_\tau(F^1, F^2)$ and $D_2 K_\tau(F^1, F^2) \neq 0$ implies by the implicit function theorem that there exists an affine function g: $N^{1\prime} \to \mathbb{R}$ such that $K_\tau(x, g(x)) = I$ for $x \in N^{1\prime}$, where $N^{1\prime} \equiv \{x \in \mathbb{R} \mid (x, y) \in N'\}$. We can thus pick $x_- < F^1 < x_+$ and $y_- < F^2 < y_+$ such that $U(x_-, y_-, \tau, \delta) = I = U(x_+, y_+, \tau, \delta)$. Finally, pick $\lambda \in (0, 1)$ such that $\lambda x_- + (1 - \lambda)x_+ = F^1$. Because $\{(x, y) \in N' \mid U(x, u, \tau, \delta) = I\}$ is a one-dimensional affine set, it must be the case that $\lambda y_- + (1 - \lambda)y_+ = F^2$. This shows that (F^1, F^2, τ) is not extremal.

Next suppose that $\tau = F^2 - L$. Define the map K_0: $\mathbb{R}^2 \to \mathbb{R}$ by $K_0(x, y) \equiv U(x, y, y - L, \delta)$. Noting first that $F^2 > L$ by hypothesis and $F^2 < H$ by Lemma A1, the desired result follows by a proof virtually identical to the proof given above.

Finally, suppose that $\tau \in (0, F^2 - L)$. First note that it cannot be the case that both $F^1 = F^2 = L$. Suppose without loss of generality that $F^1 > L$. Because $F^1 < H$, it is possible to define a map K_F by $K_F(x, y) = U(x, F^2, y, \delta)$ and again use the local linearity of K_F to show that p can be expressed as a strict convex combination of policies p' and p'' such that $U(p', \delta) = U(p'', \delta) = I$.

LEMMA A9: $\text{Min}\{U_G(p, \delta) \mid p \in \text{ext}\{\mathscr{FP}^m(\delta)\} \cap \mathscr{FP}^{m=}(\delta)\} = \text{Min}\{v_G(p_S(\delta), \delta), v_G(p_L(\delta), \delta), v_G(p_{LC}(\delta), \delta)\}$.

Proof of Lemma A9: This follows from the stronger result that for all $p \in \text{ext}\{\mathscr{FP}^m(\delta)\} \cap \mathscr{FP}^{m=}(\delta)$, $U_G(p, \delta) \in \{U_G(p_{S''}(\delta), \delta), U_G(p_L(\delta), \delta), U_G(p_{LC}(\delta), \delta)\}$. To see this note by Lemma A8, if $p \in \text{ext}\{\mathscr{FP}^m(\delta)\} \cap \mathscr{FP}^{m=}(\delta)$, either $F^1 = L$ or $F^2 = L$. If $F^1 = L$, and $\tau \in (0, F^2 - L)$ then p is not extremal. On the other hand, if $F^1 = L$ and $\tau \in \{0, F^2 - L\}$ then $p \in \{p_L(\delta), p_{LC}(\delta)\}$. Finally, if $\tau > F^2 - L$, then from the definition of U_G and U it follows that $U(p, \delta) = U(p_{LC}(\delta), \delta)$ and $U_G(p, \delta) = U_G(p_{LC}(\delta), \delta)$. Next suppose that $F^2 = L$. The fact that $p \in \mathscr{FP}^{m=}(\delta)$ implies that $U(F^1, L, \tau, \delta) = I$. Because U_G and U are independent of τ in this case, $U(p, \delta) = U(F^1, L, 0, \delta) = I$ and $U_G(p, \delta) = U_G(F^1, L, 0, \delta)$. But by the definition of $p_{S''}(\delta)$ it follows that $p_{S''} = (F^1, L, 0)$. This establishes that $\text{Min}\{U_G(p, \delta) \mid p \in \text{ext}\{\mathscr{FP}^m(\delta)\} \cap \mathscr{FP}^{m=}(\delta)\} = \text{Min}\{U_G(p_{S''}(\delta), \delta), U_G(p_L(\delta), \delta), U_G(p_{LC}(\delta), \delta)\}$. By Lemma A4 it follows that $U_G(p_{S''}(\delta), \delta) = v_G(p_{S''}(\delta), \delta)$, $U_G(p_L(\delta), \delta) = v_G(p_L(\delta), \delta)$, and $U_G(p_{LC}(\delta), \delta) = v_G(p_{LC}(\delta), \delta)$. Further, by Lemma A3 it follows that $v_G(p_{S''}(\delta), \delta) = v_G(p_S(\delta), \delta)$, implying that $U_G(p_{S''}(\delta), \delta) = v_G(p_{S''}(\delta), \delta) = v_G(p_S(\delta), \delta)$. The desired result follows.

Proof of Lemma 1: From Lemmas A6, A7, and A9 it follows that $\text{Min}\{v_G(p, \delta) \mid p \in \mathscr{FP}(\delta)\} = \text{Min}\{v_G(p_S(\delta), \delta), v_G(p_L(\delta), \delta), v_G(p_{LC}(\delta), \delta)\}$. By construction $p_S(\delta)$, $p_L(\delta)$, and $p_{LC}(\delta)$ are feasible. It follows that one of these policies is optimal.

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