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Tax-Induced Intertemporal Restrictions on Security Returns

PETER BOSSAERTS and ROBERT M. DAMMON*

ABSTRACT

This article derives testable restrictions on equilibrium asset prices when investors have the option to time the realization of their capital gains and losses for tax purposes. The tax-timing option alters both the magnitude and timing of equity returns relative to those in a tax-free model. The tax-induced restrictions are empirically examined, and the tax rates and preference parameters are estimated. While the tax-free model can be rejected in favor of the tax-based model as the specified alternative, the tax-based model is still unable to adequately explain cross-sectional differences in asset returns.

THE EFFECT OF TAXES on the equilibrium pricing of financial assets is an important, yet unresolved, issue. In the late 1970s, the discussion focused primarily on the effect of dividend yields on the equilibrium before-tax expected rates of return on common stocks (e.g., Brennan (1973), Litzenberger and Ramaswamy (1979, 1982), Black and Scholes (1974), and Miller and Scholes (1978, 1982)). More recently, the effect of the capital gains tax on asset prices has also attracted considerable attention. Capital gains taxation, unlike dividend taxation, provides investors with a potentially valuable tax-timing option. While dividends are taxed when received, capital gains and losses are not taxed until the investor sells the asset, which gives the investor the opportunity to time his asset sales so as to minimize the present value of his net tax payments. To the extent that these tax-timing options are valuable to investors, they should be reflected in equilibrium asset prices.

Constantinides (1983, 1984), Constantinides and Scholes (1980), and Dammon and Spatt (1994) discuss the optimal trading of stocks in the

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presence of personal taxes and explore the effect of optimal realization decisions on equilibrium stock prices. Constantinides and Ingersoll (1984) and Litzenberger and Rolfo (1984) extend the analysis to riskless government bonds. The results indicate that tax-timing options can represent a large fraction of the total benefits associated with holding capital assets. The implication is that taxes should not be ignored when estimating and testing asset pricing models. The purpose of this article is to empirically explore the tax-induced restrictions on asset prices in the presence of tax timing.

Despite the potential importance of taxes in determining asset prices, the recent empirical work in the asset pricing literature has largely ignored taxes. For example, over the past decade there have been a number of empirical studies of the traditional consumption-based asset pricing model without taxes. While the traditional tax-free model survives tests involving a single asset, it generally fails in tests with multiple assets (e.g., Hansen and Singleton (1982) and Dunn and Singleton (1986)). For example, in tests involving an equity index and U.S. Treasury bills, the difference in the conditional mean returns on the two assets is inconsistent with the difference in the conditional covariances of the returns with the marginal rate of substitution.

Attempts to improve the fit of the model by acknowledging measurement errors in the consumption data (e.g., Grossman, Melino, and Shiller (1987)), seasonality in consumption (e.g., Miron (1986), English, Miron, and Wilcox (1989), and Ferson and Harvey (1992)), time nonseparabilities in preferences (e.g., Dunn and Singleton (1986), Ferson and Constantinides (1991), and Heaton (1993)), or state nonseparabilities in preferences (e.g., Epstein and Zin (1991)) have produced mixed results. Notice, however, that these studies focus on problems with the way in which consumption or utility are measured. While we do not question the validity and insights of these approaches, we alter the focus by considering whether *returns* are properly measured. In particular, we acknowledge the fact that taxes alter both the magnitude and timing of the cash flows that assets provide investors. By modifying the timing of the cash flows, taxes alter the covariance of returns with the marginal rate of substitution. Our empirical analysis examines whether measuring returns on an after-tax basis improves the fit of the consumption-based asset pricing model. We also provide estimates of the effective dividend and capital gains tax rates facing the marginal investor.[1]

Under symmetric taxation (i.e., when the long-term and short-term tax rates are equal), Constantinides (1983) shows that the optimal tax-trading policy is to realize losses immediately and to defer gains. We derive the restrictions implied by this optimal tax-trading policy for equilibrium asset

[1]While previous studies examine consumption-based models using after-tax measures of return (e.g., Grossman and Shiller (1981), Mankiw, Rotemberg, and Summers (1985), and Rotemberg (1984)), they specify the relevant tax rates exogenously and ignore the tax-timing option associated with capital gains taxation by assuming that all capital gains and losses are taxed each period. Not surprisingly, the results of these studies are virtually identical to those from studies that completely ignore taxes.

prices. These restrictions involve the covariability between the marginal rate of substitution and (1) after-tax dividends and (2) the payoffs on a sequence of one-period put options, each with a stochastic exercise price equal to the minimum price attained by the security subsequent to its date of purchase. This sequence of one-period put options reflects the value of the investor's tax-timing option. It is primarily this tax-timing option that alters the covariability of returns with the marginal rate of substitution.

Using monthly consumption and return data from January 1959 to December 1990, we employ Hansen's (1982) generalized method of moments (GMM) procedure to estimate and test the after-tax moment conditions involving (1) an equally weighted index of all New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) stocks and (2) a one-month Treasury bill. Consistent with the findings of Litzenberger and Ramaswamy (1979, 1982), the estimate of the dividend tax rate is economically and statistically significant. However, the estimate of the capital gains tax rate is imprecise and economically implausible. The estimate of the coefficient of relative risk aversion is not significantly different from zero (i.e., risk neutrality), and the pricing restrictions implied by the model are rejected by the data.

One possible reason for the failure of our tax-based model to produce reliable estimates of the capital gains tax rate is that it assumes symmetric taxation of long-term and short-term capital gains and losses, whereas over much of the sample period the long-term and short-term tax rates differ. While a theoretical model that allows for asymmetric long-term and short-term tax rates is clearly warranted, it is difficult to derive the optimal tax-trading strategy for this case. The difficulty arises in deriving the cutoff levels, in both the long-term and short-term regions, below which it is optimal to realize all capital gains and losses and above which it is optimal to defer all capital gains and losses.[2]

In light of this difficulty, we investigate a simple, alternative tax-based model in which all capital gains are assumed to be realized after one year, while all capital losses are assumed to be realized as they occur. We assume that any realized gains are long term and taxed at 40 percent of the short-term capital gains tax rate. While the estimates of the coefficient of relative risk aversion and the dividend tax rate are qualitatively indistinguishable from those that are obtained under symmetric taxation, the estimate of the capital gains tax rate is dramatically improved. The estimate of the short-term capital gains tax rate, while low, is positive and statistically significant. Despite these improvements, the model is still rejected. Further investigation reveals that these rejections are due in part to the predictability of the after-tax real return on the one-month Treasury bill. However, in a

[2]Constantindides (1984) shows that it can be optimal for investors to realize some long-term gains in order to increase one's tax basis and restart the option to realize subsequent losses at the short-term tax rate. Dammon and Spatt (1994) show that it can be optimal to defer some short-term losses to avoid restarting the short-term holding period prematurely, even in the absence of transaction costs.

direct test of the null hypothesis that the tax-free model is correct against the asymmetric tax-based model as the specific alternative, we are able to reject the tax-free model in favor of the asymmetric tax-based model at the 0.1 percent level.

The remainder of the article is organized as follows. In Section I, we derive restrictions on equilibrium asset prices given the optimal tax realization policy described by Constantinides (1983) under symmetric taxation. In Section II, we develop the empirical tests of the pricing restrictions. In Section III, we describe our data set and instrument selection procedure. The empirical results are presented in Section IV. Section V examines the robustness of the results, and Section VI offers some concluding remarks.

I. The Model

In the standard consumption-based asset pricing model, the first-order conditions for the representative agent's expected utility maximization problem are:

$$E_t\Big[m(t+1,t)R_j(t+1,t) - 1\Big] = 0 \quad \text{for all } j, \tag{1}$$

where $m(t+1,t)$ is the marginal rate of substitution of consumption at date t for consumption at date $t+1$, $R_j(t+1,t)$ is the gross real rate of return on asset j from date t to date $t+1$, and $E_t[\cdot]$ denotes the conditional expectation operator. With taxes, $R_j(t+1,t)$ is the gross *after-tax* real rate of return on asset j. To determine this after-tax return it is necessary to specify a particular tax environment and tax-trading strategy for investors. The purpose of this section is to derive the after-tax version of the above Euler equation for an environment with symmetric taxation.

We assume that all investors are identical and face the same tax rates.[3] The tax environment is a simplification of the actual U.S. tax code. Dividend income is fully taxed at the constant rate of τ_d, where $\tau_d \in (0,1)$, and *realized* capital gains and losses are taxed at the constant rate of τ_c, where $\tau_c \in (0,1)$. To be as general as possible, we leave the relation between τ_d and τ_c unspecified. We assume that all *unrealized* capital gains and losses remain untaxed, which gives investors the option to time the realization of their capital gains and losses for tax purposes. Initially, we assume that there is no distinction between the long-term and short-term tax rates (i.e., symmetric taxation). Throughout the analysis, we ignore the capital loss limit (currently \$3,000 per year) imposed by the actual U.S. tax code.

[3] While the assumption that all investors face the same tax rate is clearly unrealistic, it allows us to avoid the well-known difficulties arising from tax arbitrage when investors face different tax rates (e.g., Schaefer (1980) and Dammon and Green (1987)). Alternatively, if investors face different tax rates, then the pricing restrictions we derive can be thought of as the restrictions that must be satisfied in equilibrium by the marginal investor. The necessary and sufficient conditions for the existence of an equilibrium in the presence of differential taxation are derived in Dammon and Green (1987).

Constantinides (1983) shows that under these conditions the optimal policy is to realize losses immediately and to defer capital gains. Consequently, the payoffs on a security include the future after-tax dividends and tax rebates on the realization of capital losses. Because investors are taxed on nominal quantities, we calculate tax payments and rebates on nominal dividends, interest, and capital gains and losses. Investors then deflate these nominal after-tax payoffs to determine their optimal consumption plans. This implies that the price of security j must satisfy the following Euler equation in order for the security market to be in equilibrium at date t:

$$P_j(t) = E_t\left\{ \sum_{s=t+1}^{\infty} m(s,t)[\pi_t/\pi_s]\Big[(1-\tau_d)d_j(s) + \tau_c \max\big[0, \min\big[P_j(t),\ldots,P_j(s-1)\big] - P_j(s)\big]\Big]\right\} \tag{2}$$

where π_s denotes the consumption price index at date s, $m(s,t)$ denotes the marginal rate of substitution between consumption at date s and consumption at date t, $d_j(s)$ denotes the pretax dividend on security j at date s, and $P_j(s)$ denotes the market price of security j at date s. Future tax rebates on capital losses are represented in equation (2) as a sequence of one-period put options, where the exercise price at date s is equal to the minimum price reached by the security over the period from date t (the current date) to date $s-1$. Recall that under the optimal realization policy, the investor sells and repurchases the asset only when he has a loss. Consequently, $\min[P_j(t),\ldots,P_j(s-1)]$ is the investor's tax basis at date s under the optimal realization policy.

In equilibrium, the price of the security at date t reflects the future cash flows to the marginal investor, including the future tax rebates on the optimal realization of capital losses. This formulation of the equilibrium price is valid only if investors can consume and rebalance their portfolios without the need to liquidate positions with embedded capital gains. Constantinides (1983) derives a set of conditions under which this will be true in his model. In our model, if investors can sell short against their current portfolio holdings, with full use of the short-sale proceeds, then the consumption, investment, and liquidation decisions will be independent.[4] This independence result is empirically important because it allows the model to be tested without knowledge of the portfolio holdings or basis values of investors.

[4]To prevent tax arbitrage, investors who establish simultaneous long and short positions in the same asset must not be allowed to subsequently liquidate only one side of this transaction. If this behavior is not restricted, then investors can exploit their tax-timing options against the government while maintaining a zero net position in the asset. One way to restrict this behavior is to require that any liquidations of long (short) positions must first be offset by corresponding liquidations of any short (long) positions that the investor may have in the same asset. This treatment is consistent with current U.S. tax laws.

Our goal is to transform the equilibrium pricing relation in equation (2) into a form involving after-tax returns, as in equation (1). To do this, we rewrite equation (2) in the following convenient form:

$$P_j(t) = E_t\left\{ \sum_{s=t+1}^{\infty} m(s,t)[\pi_t/\pi_s]x_j(s,t) \right\}, \tag{3}$$

where

$$x_j(s,t) = d_j(s)(1-\tau_d) + \tau_c \max\left[0, \min\left[P_j(t),\ldots,P_j(s-1)\right] - P_j(s)\right] \tag{4}$$

is the total after-tax cash flow on security j at date s from an investment made at date $t < s$. Adding and subtracting $E_t\{m(t+1,t)[\pi_t/\pi_{t+1}]P_j(t+1)\}$ from the right-hand side of equation (3) and rearranging terms then yields:

$$P_j(t) = E_t\left\{m(t+1,t)[\pi_t/\pi_{t+1}]\left[x_j(t+1,t) + P_j(t+1) - z_j(t+1,t)\right]\right\} \tag{5}$$

where both $P_j(t)$ and $P_j(t+1)$ conform to equation (3) and $z_j(t+1,t)$ is a nonnegative random variable given by the conditional expectation:

$$z_j(t+1,t) \equiv E_{t+1}\left\{ \sum_{s=t+2}^{\infty} m(s,t+1)[\pi_{t+1}/\pi_s]\left[x_j(s,t+1) - x_j(s,t)\right] \right\}. \tag{6}$$

The interpretation of $z_j(t+1,t)$ is of particular interest. From equation (4), the cash flow difference $[x_j(s,t+1) - x_j(s,t)]$ appearing in equation (6) is the difference in the payoffs on two put options, both written on security j and maturing at date s. The first put option is the option to realize losses at date s for an investor who purchases the stock at date $t+1$. Its random exercise price is $\min[P_j(t+1),\ldots,P_j(s-1)]$. The second put option is the option to realize losses at date s for an investor who purchases the stock at date t. Its random exercise price is $\min[P_j(t),\ldots,P_j(s-1)]$. Since the exercise price on the first put option is at least as high as that on the second, $[x_j(s,t+1) - x_j(s,t)] \geq 0$ for all $s > t+1$. From equation (6), $z_j(t+1,t)$ is the present value of the difference in these future tax rebates. Therefore, $z_j(t+1,t)$ can be interpreted as the decline in the value of the tax-timing option between dates t and $t+1$ resulting from the embedded capital gain of $[P_j(t+1) - P_j(t)]$ on shares purchased at date t.

Clearly, $z_j(t+1,t) \geq 0$ and is equal to zero only if $P_j(t+1) \leq P_j(t)$. In this case, the investor optimally realizes the capital loss at date $t+1$ and reestablishes a new position in the security, thereby restarting the tax-timing option and resetting the basis at $P_j(t+1)$. For $P_j(t+1) > P_j(t)$, $z_j(t+1,t)$ is strictly positive and increasing in the difference between $P_j(t+1)$ and $P_j(t)$. However, because the investor has the option to defer capital gains, the limiting value of $z_j(t+1,t)$ can be no higher than the tax liability that would be owed if the investor suboptimally realized gains. Therefore, $z_j(t + 1, t) \leq \tau_c[P_j(t + 1) - P_j(t)]$ for all $P_j(t + 1) > P_j(t)$.

The interpretation of the pricing relation in equation (5) is now intuitively clear. The investor values the stock at date t based upon the expected after-tax cash flow at date $t + 1$, $x_j(t + 1, t)$, the expected price of the stock at date $t + 1$, $P_j(t + 1)$, and the expected loss in the value of the tax-timing option at date $t + 1$, $z_j(t + 1, t)$. The difference $[P_j(t + 1) - z_j(t + 1, t)]$ is the present value (at date $t + 1$) of the future dividends and tax rebates on capital losses that an investor can expect to receive from purchasing the stock at date t. This difference can be interpreted as the investor's *personal valuation* at date $t + 1$ of the shares purchased at date t.

Equation (5) can now be rewritten in terms of the after-tax rate of return on security j. Dividing both sides of equation (5) by $P_j(t)$, subtracting one from both sides, and rearranging terms produces equation (1), where

$$R_j(t + 1, t) \equiv [\pi_t/\pi_{t+1}]\Big(1 + \mu_j^d(t + 1, t)(1 - \tau_d) + \mu_j^c(t + 1, t) - \tau_c\,\mu_j^z(t + 1, t) - \tau_c \min\big[0, \mu_j^c(t + 1, t)\big]\Big). \tag{7}$$

Equation (7) provides the appropriate measure of the gross *after-tax* real rate of return on security j from date t to date $t + 1$. In equation (7), $\mu_j^c(t + 1, t)$ is the pretax nominal capital gain return on security j from date t to date $t + 1$, $\mu_j^d(t + 1, t)$ is the pretax nominal dividend return on security j from date t to date $t + 1$, and $\tau_c\,\mu_j^z(t + 1, t) \equiv z_j(t + 1, t)/P_j(t)$ is given by

$$\tau_c\,\mu_j^z(t + 1, t) = \tau_c E_{t+1}\Bigg\{\sum_{s=t+2}^{\infty} m(s, t + 1)[\pi_{t+1}/\pi_s] \times\Big(\max\big[0, \min\big[\mu_j^c(t + 1, t), \ldots, \mu_j^c(s - 1, t)\big] - \mu_j^c(s, t)\big] - \max\big[0, \min\big[0, \mu_j^c(t + 1, t), \ldots, \mu_j^c(s - 1, t)\big] - \mu_j^c(s, t)\big]\Big)\Bigg\}. \tag{8}$$

As with $z_j(t + 1, t)$, the value of $\mu_j^z(t + 1, t)$ is always nonnegative and is equal to zero only if $\mu_j^c(t + 1, t) \leq 0$. Moreover, because of the ability to defer gains, $\mu_j^z(t + 1, t) < \mu_j^c(t + 1, t)$ for all $\mu_j^c(t + 1, t) > 0$.

II. Testable Restrictions of the Model

In this section, we derive the testable restrictions implied by the equilibrium pricing relations of the previous section and describe our empirical methodology. According to equation (1), equilibrium requires $[m(t + 1, t) R_j(t + 1, t) - 1]$ to be orthogonal to all elements of investors' information sets at date t, y_t. That is,

$$E\Big(\big[m(t + 1, t)R_j(t + 1, t) - 1\big]\,y_t\Big) = 0 \quad \text{for all } j, \tag{9}$$

where $E[\cdot]$ denotes the unconditional expectation. The orthogonality condition in equation (9) restricts the comovements between aggregate consumption and the after-tax real returns on the financial assets. In particular, equation (9) restricts the mean of $m(t+1,t)R_j(t+1,t)y_t$ to equal the mean of y_t for all j.

To exploit the restrictions in equation (9) empirically, we specify the functional form of investors' utility as in Hansen and Singleton (1982, 1984). In particular, we assume that investors possess power utility,

$$U(c_t) = \frac{[c_t]^\gamma}{\gamma}, \tag{10}$$

where $1-\gamma > 0$ is the coefficient of relative risk aversion. Under power utility, the marginal rate of substitution $m(s,t)$ is given by

$$m(s,t) = \beta^{s-t}[c_s/c_t]^{\gamma-1} \tag{11}$$

and, therefore, equation (9) becomes

$$E\left(\left[\beta[c_{t+1}/c_t]^{\gamma-1}R_j(t+1,t) - 1\right]y_t\right) = 0 \quad \text{for all } j. \tag{12}$$

The tests of the equilibrium relation, equation (12), conducted in this article rely on the GMM procedure proposed by Hansen (1982) and Hansen and Singleton (1982). A brief description of this procedure is provided below.[5]

Let Λ be an n-vector of unknown parameters estimated from the model. For our model, $n = 4$ and $\Lambda = \{\beta, \gamma, \tau_c, \tau_d\}$. Define

$$u_{jt+1} \equiv \beta[c_{t+1}/c_t]^{\gamma-1}R_j(t+1,t) - 1, \tag{13}$$

to be the disturbances for our econometric analysis. Equation (12) implies that $E[u_{jt+1}y_t] = 0$ for all instruments, y_t, and for all assets, j. After selecting m ($m > n$) instruments to be used in estimating Λ, the parameter estimates are chosen by GMM to minimize a quadratic form in the sample means of $u_{jt+1}y_{it}$, $i = 1,\ldots,m$. However, with more orthogonality conditions than parameters to be estimated, the model is overidentified, and not all m orthogonality conditions will be set equal to zero in the estimation. Nevertheless, the $(m-n)$ overidentifying restrictions should be close to zero if the model is "correct." Hansen (1982) and Hansen and Singleton (1982) show how a chi-square goodness-of-fit statistic can be used to test these overidentifying restrictions.

[5]In estimating the model, GQOPT's Davidon-Fletcher-Powell algorithm (see Powell (1971)) is used to search for the optimum. The criterion to decide whether an optimum is reached is solely based on the length of the gradient. The time-preference parameter, β, is transformed to $\beta' = \exp(10\beta)$ in the optimization. This transformation improves convergence speed markedly. Analytical derivatives are used in the calculations of the gradients and the standard errors. In addition to improving the convergence over numerical derivatives, analytical derivatives often lead to smaller standard errors.

An advantage of the GMM procedure is that it allows for the conditional variance of the disturbances u_{jt+1} to be an arbitrary function of the elements of investors' information sets at date t. This allows for the possibility that conditional variances and covariances of returns and consumption vary over time and change sign. Thus, the GMM procedure is capable of testing asset pricing models that allow for time-varying risk premia. Moreover, it is not necessary to specify how these variances and covariances change over time as a function of the investors' information sets, nor is it necessary to make any particular distributional assumptions about returns and consumption, unlike the log-linear models of asset prices (e.g., Hansen and Singleton (1983) and Ferson (1983)).

III. The Data and Instrument Selection

Various versions of the model are estimated using monthly data on consumption, stock returns, and Treasury-bill returns, covering the period January 1959 to December 1990. The consumption data are taken from the CITIBASE tape. Consumption of nondurables and services is divided by total civilian population to obtain estimates of per capita consumption. The stock return series are taken from the CRSP monthly returns tape. Returns with and without dividends are collected for all NYSE and (from 1962 on) all AMEX firms that traded during this period. In total, there are 3,482 securities in our sample. An equally weighted portfolio of common stocks is constructed by adding newly listed firms as they are introduced, assigning a zero return if a security did not trade during a particular month, and deleting securities as they are delisted. This procedure minimizes survival bias.[6] Fama and Bliss's risk-free returns file on the CRSP bond tape is the source for the yields on a one-month and three-month Treasury bill. Returns are deflated using the Consumption Price Index implicit in the data on consumption of nondurables and services from CITIBASE. Table I provides descriptive statistics on the monthly growth rate in real per capita consumption, the monthly before-tax real return on the equally weighted portfolio of NYSE and AMEX stocks, the before-tax real return on a one-month Treasury bill, and the monthly inflation rate.

GMM directly estimates and tests moment conditions, such as equation (12), for different instrumental variables, y_t. While straightforward, instrumental variables estimation poses a major problem: from a possibly innumerable set of instruments, we have to pick a limited set. Recent Monte Carlo evidence suggests that the choice of instruments affects the small sample properties of the estimates and the χ^2 statistic (see Tauchen (1986), Kocherlakota (1990), and Ferson and Foerster (1992)). Hence, it is important to choose instruments carefully.

[6] Earlier versions of this article focus on portfolios of continuously traded NYSE stocks (only 388 securities). Our earlier results are virtually identical to those reported here, which seems to indicate that survival bias is not a serious issue.

Table I

Descriptive Statistics of Selected Variables

The variables are: one plus the growth rate in real per capita consumption, c_{t+1}/c_t; the before-tax real rate of return on an equally-weighted index of all NYSE and AMEX common stocks, $r_p(t+1,t)$; the before-tax real rate of return on the one-month Treasury bill, $r_f(t+1,t)$; and one plus the inflation rate, π_{t+1}/π_t; for the period from January 1959 through December 1990. All variables are measured on a monthly basis.

	Mean	Median	St. Dev.	Minimum	Maximum	Autocorrelation
c_{t+1}/c_t	1.00189	1.00180	0.00394	0.98993	1.01891	−0.258
$r_p(t+1,t)$	0.00759	0.00877	0.05329	−0.26015	0.28573	0.139
$r_f(t+1,t)$	0.00107	0.00119	0.00265	−0.00533	0.01067	0.369
π_{t+1}/π_t	1.00385	1.00358	0.00283	0.99632	1.01152	0.482

To illustrate our selection process, it is informative to linearize the marginal rate of substitution as a function of consumption growth as in Singleton (1989).[7] This yields:

$$m(t+1,t) \cong \beta + \beta(\gamma-1)(c_{t+1}/c_t) - \beta(\gamma-1). \tag{14}$$

Substituting equation (14) into equation (9) gives:

$$(2-\gamma)E\left[R_j(t+1,t)y_t\right] + (\gamma-1)E\left[(c_{t+1}/c_t)R_j(t+1,t)y_t\right] = \beta^{-1}E[y_t], \tag{15}$$

for all j. Our first instrument choice is the unit vector. Substituting $y_t = 1$ into the above expression yields:

$$(2-\gamma)E\left[R_j(t+1,t)\right] + (\gamma-1)E\left[(c_{t+1}/c_t)R_j(t+1,t)\right] = \beta^{-1}, \tag{16}$$

for all j. Comparing equations (15) and (16), it is clear that an instrument does not add any restriction beyond the one provided by the unit vector if it is uncorrelated with both $R_j(t+1,t)$ and $(c_{t+1}/c_t)R_j(t+1,t)$ for all assets. If it is possible to satisfy equation (16) for some choice of parameter values and one adds a moment condition using any instrument that is uncorrelated with both $R_j(t+1,t)$ and $(c_{t+1}/c_t)R_j(t+1,t)$ for all j, the newly introduced moment condition will hold for the same parameter values. No additional restriction is provided. Consequently, our selection procedure chooses instruments that are correlated with $R_j(t+1,t)$ and/or $(c_{t+1}/c_t)R_j(t+1,t)$ for at least one asset.

While this procedure of choosing instruments based upon their correlation with $R_j(t+1,t)$ and $(c_{t+1}/c_t)R_j(t+1,t)$ utilizes data that is subsequently used to test the model, it does not bias the results in favor of rejection. If the

[7]Singleton (1989) uses the linearization in equation (14) to investigate the reasons for the rejections of the consumption-based term structure model. He justifies this linearization of the marginal rate of substitution based upon the observation that aggregate consumption growth exhibits little variability.

model is correct, the cross-sectional restrictions provided by equations (15) and (16) should hold for some choice of γ and β (ignoring sampling variation), regardless of the instruments selected. The advantage of our instrument selection procedure is that it provides power. If the model is incorrect so that there are no values of γ and β that satisfy equations (15) and (16) simultaneously, then the test is likely to uncover this. On the other hand, if instruments are chosen that are uncorrelated with both $R_j(t+1, t)$ and $(c_{t+1}/c_t)R_j(t+1, t)$ for all j, then equations (15) and (16) will hold for some choice of γ and β, leading to the incorrect inference that the model is correct.

To test the traditional consumption-based asset pricing model without taxes, instruments should be selected based upon their correlations with before-tax real returns and the product of before-tax real returns and consumption growth. We reject the traditional model using instruments selected in this way. Our interest is in determining whether the rejections of the traditional model using these instruments can be attributed to the fact that the model ignores taxes.[8]

We consider the following instruments as candidates for our empirical analysis:

1) consumption growth lagged once, c_t/c_{t-1};
2) consumption growth lagged twice, c_{t-1}/c_{t-2};
3) one plus the lagged before-tax real return on the equally weighted portfolio of common stocks, $1 + r_p(t, t-1)$;
4) one plus the lagged before-tax real return on a one-month Treasury bill, $1 + r_f(t, t-1)$;
5) one plus the contemporaneous before-tax nominal return on a one-month Treasury bill, $1 + \mu_f(t+1, t)$;
6) one plus the contemporaneous before-tax yield on a three-month Treasury bill in excess of that on a one-month Treasury bill, $1 + q_t^3 - q_t^1$; and
7) one plus the lagged before-tax yield on a three-month Treasury bill in excess of that on a one-month bill, $1 + q_{t-1}^3 - q_{t-1}^1$.

The latter two variables relate term structure information, which recent evidence (e.g., Campbell (1987) and Ferson and Harvey (1991)) shows to be important in predicting future stock returns. Table II reports the sample correlations of these variables with (1) before-tax real returns and (2) the product of consumption growth and before-tax real returns.

[8] One may wonder whether the instruments that are correlated with before-tax returns are also correlated with after-tax returns. This is difficult to determine a priori since after-tax returns, unlike before-tax returns, depend upon the unknown tax rates and preference parameters that are to be estimated (see equations (7) and (8)). If we generate an after-tax return series using initial guesses for the tax rates and preference parameters, then additional interpretive problems arise. If the initial guesses for the tax rates and preference parameters are close to the final GMM estimates, then the correlations between the instruments and the after-tax returns will be close to zero if the after-tax model is correct. Roughly speaking, this is because the tax rates and preference parameters are chosen by GMM to minimize these correlations.

Table II

Descriptive Statistics of the Power of Various Instruments

Panel A gives the correlation between various instrumental variables and (1) one plus the before-tax real return on the equally weighted index of all NYSE and AMEX common stocks, $1 + r_p(t+1, t)$, and (2) one plus the before-tax real return on the one-month Treasury bill, $1 + r_f(t+1, t)$, for the period January 1959 through December 1990. Panel B gives the correlation between various instrumental variables and (1) one plus the before-tax real return on the equally weighted index of all NYSE and AMEX common stocks times one plus consumption growth, $[1 + r_p(t+1, t)][c_{t+1}/c_t]$, and (2) one plus the before-tax real return on the one-month Treasury bill times one plus consumption growth, $[1 + r_f(t+1, t)][c_{t+1}/c_t]$, for the period January 1959 through December 1990. The instrumental variables, y_t, are: one plus consumption growth lagged once, c_t/c_{t-1}; one plus consumption growth lagged twice, c_{t-1}/c_{t-2}; one plus the lagged before-tax real return on the equally weighted index of all NYSE and AMEX common stocks, $1 + r_p(t, t-1)$; one plus the lagged before-tax real return on the one-month Treasury bill, $1 + r_f(t, t-1)$; one plus the contemporaneous before-tax nominal return on the one-month Treasury bill, $1 + \mu_f(t+1, t)$; one plus the contemporaneous before-tax nominal yield differential between a three-month and a one-month Treasury bill, $1 + q_t^3 - q_t^1$; and one plus the lagged before-tax nominal yield differential between a three-month and a one-month Treasury bill, $1 + q_{t-1}^3 - q_{t-1}^1$. All variables are measured on a monthly basis.

	$R_j(t+1, t)$	
y_t	$1 + r_p(t+1, t)$	$1 + r_f(t+1, t)$
	Panel A: corr$[R_j(t+1, t), y_t]$	
c_t/c_{t-1}	0.061	0.003
c_{t-1}/c_{t-2}	−0.031	0.014
$1 + r_p(t, t-1)$	0.139	−0.006
$1 + r_f(t, t-1)$	0.084	0.370
$1 + \mu_f(t+1, t)$	−0.077	0.372
$1 + q_t^3 - q_t^1$	0.114	0.025
$1 + q_{t-1}^3 - q_{t-1}^1$	0.099	0.169
	Panel B: corr$[(c_{t+1}/c_t)R_j(t+1, t), y_t]$	
c_t/c_{t-1}	0.041	−0.208
c_{t-1}/c_{t-2}	−0.030	0.020
$1 + r_p(t, t-1)$	0.139	0.013
$1 + r_f(t, t-1)$	0.080	0.177
$1 + \mu_f(t+1, t)$	−0.085	0.102
$1 + q_t^3 - q_t^1$	0.108	−0.030
$1 + q_{t-1}^3 - q_{t-1}^1$	0.096	0.079

From Table II it is clear that the following instruments are powerful in testing the moment conditions involving the equally weighted portfolio of common stocks: $1 + r_p(t, t-1)$ and $1 + q_t^3 - q_t^1$. In contrast, several instruments can be used for moment conditions involving the one-month Treasury bill: c_t/c_{t-1}, $1 + r_f(t, t-1)$, $1 + \mu_f(t+1, t)$, and $1 + q_{t-1}^3 - q_{t-1}^1$.[9] These instruments and the unit vector are used to construct a total of eight moment conditions: three involving the equally weighted portfolio of common stocks and five involving the one-month Treasury bill (the unit vector is used as an instrument for both the portfolio of common stocks and the one-month Treasury bill).

IV. Parameter Estimates and Test Results

Our empirical analysis uses monthly returns on the equally weighted portfolio of NYSE and AMEX stocks and the one-month Treasury bill over the sample period January 1959 to December 1990. The traditional tax-free model and various versions of the after-tax model are tested, and the results are reported below.

A. Tests of the Traditional Tax-Free Model

As a benchmark, we test the first-order conditions for a representative consumer in a world without taxes. In other words, we jointly test the following stochastic Euler equations:

$$E_t\Big(m(t+1,t)\big[1+r_p(t+1,t)\big]-1\Big)=0 \tag{17}$$

and

$$E_t\Big(m(t+1,t)\big[1+r_f(t+1,t)\big]-1\Big)=0 \tag{18}$$

where $r_p(t+1,t)$ is the before-tax real return from date t to date $t+1$ on the equally weighted portfolio of stocks, $r_f(t+1,t)$ is the before-tax real return on a one-month Treasury bill from date t to date $t+1$, and $m(t+1,t)$ is the marginal rate of substitution given by equation (11). This particular version of the model is the subject of the Hansen and Singleton (1982, 1984) studies.

There are eight orthogonality conditions but only two parameters (β and γ) to be estimated. This leaves us with six overidentifying restrictions (degrees of freedom). The first column (Model 1) of Table III reports the estimation results.[10] The estimate of the risk-aversion parameter, $\hat{\gamma}=1.176$, is insignificantly greater than 1.0. A value of γ above 1.0 indicates that the representative consumer is risk loving, while a value of γ below 1.0 indicates

[9] There is some argument about whether consumption growth lagged once is an admissible instrument. In particular, consumption data may be aggregated and, hence, lead to spurious predictability one period ahead (e.g., Grossman, Melino, and Shiller (1987)). We investigate this issue in more detail later.

[10] While qualitatively the same, the point estimates in the first column of Table III do differ from the ones found in Hansen and Singleton (1982, 1984) for the following reasons:

1) Instruments used to estimate and test the model are different.
2) A longer time series is used in the present article (extending beyond 1978).
3) The CITIBASE and CRSP pre-1978 data have been updated substantially.
4) We use a different population measure. Instead of the total population, we look only at total *civilian* population, which excludes the military. A great deal of the consumption of military personnel comes out of the defense budget, which is excluded from the consumption series reported by CITIBASE.
5) We match consumption over a given month by the end-of-month population, not by the population at the beginning of the month. The matching of consumption and returns, on the other hand, does not differ from that in Hansen and Singleton (1982, 1984). Consumption over the month of January, for example, is matched with the January stock return.

Table III

Generalized Method of Moments (GMM) Estimation and Test Results for the Moment Conditions Representing Alternative Asset Pricing Models

Moment conditions for both an equally weighted index of all NYSE and AMEX common stocks and the one-month Treasury bill are estimated using monthly data for the period January 1959 through December 1990. Model 1 is the tax-free consumption-based asset pricing model of Hansen and Singleton (1982). Models 2 to 4 are after-tax versions of this model. In Model 2, capital gains and losses are assumed to be taxed every period, without the benefit of tax timing. In Model 3, capital losses are realized each period and all capital gains are deferred indefinitely. In Model 4, capital losses are realized each period, and all capital gains are realized after one-year and taxed at 40 percent of the short-term capital gains tax rate. The parameters to be estimated are: the personal rate of time preference, β; the risk-aversion coefficient, γ; the short-term capital gains tax rate, τ_c; and the dividend tax rate, τ_d. Standard errors are in parentheses under the parameter estimates and the probability value is in parentheses under the chi-square, χ^2, statistic. DOF refers to the number of degrees of freedom.

	Asset Pricing Model			
	1	2	3	4
Parameter estimates				
$\hat{\beta}$	0.998	1.004	1.000	1.000
	(0.001)	(0.001)	(0.001)	(0.001)
$\hat{\gamma}$	1.176	0.821	0.916	0.982
	(0.371)	(0.306)	(0.126)	(0.114)
$\hat{\tau}_c$		1.003	−0.423	0.099
		(0.004)	(0.623)	(0.022)
$\hat{\tau}_d$		1.010	0.335	0.320
		(0.020)	(0.104)	(0.113)
Correlations				
$\hat{\beta}, \hat{\gamma}$	−0.848	−0.839	−0.732	−0.743
$\hat{\beta}, \hat{\tau}_d$		0.023	0.763	0.813
$\hat{\tau}_c, \hat{\tau}_d$		0.925	−0.001	−0.200
$\hat{\tau}_c, \hat{\gamma}$		0.074	0.063	0.013
Goodness-of-fit				
χ^2	33.424	13.601	12.212	11.154
	(< 0.001)	(0.009)	(0.016)	(0.025)
DOF	6	4	4	4

that the representative consumer is risk averse. The estimate of the time-preference parameter, $\hat{\beta} = 0.998$, is highly correlated with the estimate of γ. The tax-free model is strongly rejected. These parameter estimates and test results are broadly consistent with the findings of earlier studies (e.g., Hansen and Singleton (1982, 1984)). The model is unable to relate differences in conditional mean returns to differences in the conditional covariances of returns with the marginal rate of substitution.[11]

[11] In the estimation of the Euler equation for a single asset, as opposed to the joint estimation of multiple assets, we are unable to reject the null hypothesis that the tax-free model is correct. The tax-free model is rejected only when testing cross-sectional restrictions. These results are consistent with the findings of earlier studies (e.g., Hansen and Singleton (1982, 1984)).

B. Tests of the Tax-Adjusted Model without Tax Timing

We next estimate the Euler equations for the equally weighted stock index and the one-month Treasury bill for a representative consumer in a world where capital gains and losses are taxed each period (i.e., monthly), without the option to defer capital gains. Although this model ignores the tax-timing option available to investors, it is of independent interest because it incorporates taxes in a simple, straightforward manner. Dividends and interest are assumed to be taxed at a rate of τ_d, and capital gains and losses are assumed to be taxed at a rate of τ_c. We jointly test the following stochastic Euler equations:

$$E_t\left\{m(t+1,t)[\pi_t/\pi_{t+1}]\frac{1}{J_t}\sum_{j=1}^{J_t}\Big(1+\mu_j^d(t+1,t)(1-\tau_d) + \mu_j^c(t+1,t)(1-\tau_c)\Big) - 1\right\} = 0 \tag{19}$$

and

$$E_t\Big(m(t+1,t)[\pi_t/\pi_{t+1}]\big[1+(1-\tau_d)\mu_f(t+1,t)\big] - 1\Big) = 0. \tag{20}$$

where J_t in equation (19) is the number of stocks in the equally weighted equity portfolio at date t, and $\mu_f(t+1,t)$ in equation (20) is the before-tax nominal return on the one-month Treasury bill from date t to date $t+1$.

The results are reported in the second column (Model 2) of Table III.[12] The estimate of the risk-aversion parameter, $\hat{\gamma} = 0.821$, is not significantly different from 1.0. The estimates of the dividend and capital gain tax rates, $\hat{\tau}_d = 101.0$ percent and $\hat{\tau}_c = 100.3$ percent, are not significantly different from 100 percent and are obviously economically implausible. The overall model is rejected at the 1 percent level.

It is not surprising that this model fails to produce reasonable results. To understand the intuition underlying the parameter estimates for this model, consider once again equations (19) and (20). Inspection of these two equations reveals that they collapse to the same restriction if the after-tax nominal

[12] The estimates reported in Table III for Models 2 to 4 are sensitive to the starting value for β, the time preference parameter. Two optima emerge: one with β below 1.0 and the other with β above 1.0. The criterion function is generally lower for the latter. Consequently, only the optima with β above 1.0 are reported in Table III. The results for the optima with β below 1.0 are different in one important respect: the dividend tax rate, τ_{d}, is estimated with high standard error. The point estimate often turns out to be (insignificantly) negative. The difference in point estimates from those reported in Table III is to be expected, however, since the estimates of β and τ_{d} are highly positively correlated in most of the models.

return on all assets can be driven to zero. This, in fact, is exactly what GMM does by setting both the dividend and capital gain tax rates approximately equal to 100 percent. In this case, equations (19) and (20) reduce to $E_t[m(t+1,t)[\pi_t/\pi_{t+1}] - 1] = 0$, and GMM becomes a test of whether $[m(t+1,t)[\pi_t/\pi_{t+1}] - 1]$ is orthogonal to past information. Since asset returns are highly correlated with past information, eliminating them from the orthogonality conditions makes it more difficult to reject the model. Thus GMM mechanically chooses parameter estimates that best fit the model, even though the parameter choices may not be economically plausible. The fact that the model is still rejected indicates that consumption growth (and, hence, the marginal rate of substitution) is correlated with past information (e.g., lagged consumption growth).

C. Tests of the Tax-Adjusted Model with Tax Timing

We next estimate and test the symmetric tax model of Section I, where investors are assumed to have the option to time the realization of their capital gains and losses. When the long-term and short-term tax rates are equal (i.e., symmetric taxation), the optimal trading strategy is to realize losses immediately and defer gains indefinitely. The Euler equation for the one-month Treasury bill is given by equation (20). The Euler equation for the equally weighted stock index is given by:

$$E_t\left\{m(t+1,t)[\pi_t/\pi_{t+1}]\frac{1}{J_t}\sum_{j=1}^{J_t}\Big(1+\mu_j^d(t+1,t)(1-\tau_d)+\mu_j^c(t+1,t)\right.$$
$$\left. -\tau_c\mu_j^z(t+1,t)-\tau_c\min\big[0,\mu_j^c(t+1,t)\big]\Big)-1\right\}=0. \tag{21}$$

As equation (21) indicates, the Euler equation for the stock index assumes that capital gains and losses are computed separately for each individual stock in the index, as opposed to computing capital gains and losses for the index as a whole. This maximizes the value of the investor's tax options for essentially the same reason that a portfolio of options is more valuable than an option on a portfolio. Consequently, the after-tax rate of return on the index may include tax rebates on capital losses even though the before-tax rate of return on the index itself is positive.

Before the above model can be tested, however, there is one more complication that must be resolved. Notice that equation (21) involves the nominal quantity $\tau_c\mu_j^z(t+1,t)$, which measures the decline in the value of the investor's tax-timing option between dates t and $t+1$. Formally, the value of $\tau_c\mu_j^z(t+1,t)$ is given by equation (8). According to equation (8), if security j suffers a capital loss between dates t and $t+1$ (i.e., $\mu_j^c(t+1,t) \le 0$), the value of $\mu_j^z(t+1,t) = 0$. However, if security j experiences a capital gain between dates t and $t+1$ (i.e., $\mu_j^c(t+1,t) > 0$), the value of $\mu_j^z(t+1,t)$ is positive, and it is then necessary to estimate an infinite sum in equation (8).

To operationalize equation (8), we approximate $\mu_j^z(t+1,t)$ by terminating the infinite sum after $T = t + 13$ periods.[13] Substituting this approximation for $\mu_j^z(t+1,t)$ into equation (21) and applying the law of iterated expectations produces the following stochastic Euler equation for the equity portfolio:

$$\begin{aligned} E_t\Bigg\{ & m(t+1,t)[\pi_t/\pi_{t+1}]\frac{1}{J_t}\sum_{j=1}^{J_t}\Bigg(1 + \mu_j^d(t+1,t)(1-\tau_d) + \mu_j^c(t+1,t) \\ & -\tau_c \sum_{s=t+2}^{T} m(s,t+1)[\pi_{t+1}/\pi_s] \\ & \times\Big(\max\big[0,\min\big[\mu_j^c(t+1,t),\ldots,\mu_j^c(s-1,t)\big] - \mu_j^c(s,t)\big] \\ & -\max\big[0,\min\big[0,\mu_j^c(t+1,t),\ldots,\mu_j^c(s-1,t)\big] - \mu_j^c(s,t)\big]\Big) \\ & -\tau_c\min\big[0,\mu_j^c(t+1,t)\big]\Bigg) - 1\Bigg\} = 0. \end{aligned} \tag{22}$$

where $T = t + 13$. This moment condition, together with the moment condition involving the Treasury bill (see Equation (20)), are estimated using the same set of instrumental variables as before.[14]

The results of jointly estimating equations (20) and (22) are reported in the third column (Model 3) of Table III. The results are encouraging on some dimensions and disappointing on others. The estimate of the risk-aversion parameter, $\hat{\gamma} = 0.916$, is precise but insignificantly different from 1.0. The estimate of the dividend tax rate, $\hat{\tau}_d = 33.5$ percent, is significantly greater than zero and economically plausible. However, the estimate of the capital gains tax rate, $\hat{\tau}_c = -42.3$ percent, is economically implausible, but insignificantly different from zero. The model is rejected at the 2 percent level.

One possible explanation for the disappointing results for Model 3 is that our tax environment may be too simplistic to capture the true effect of taxes on investors' trading behavior and asset prices. We investigate this possibility below.

D. Tests of the Tax-Adjusted Model with Differential Tax Rates

In contrast to the actual tax code in existence over the sample period, our model does not distinguish between long-term and short-term capital gains

[13] To investigate whether this approximation has any material effect on the results, we also approximate $\mu_j^z(t+1,t)$ by setting $T = t + 19$ and $T = t + 31$. The parameter estimates and test results are qualitatively unchanged. Since $\mu_j^z(t+1,t)$ is a concave function of $\mu_j^c(t+1,t)$, and $\mu_j^z(t+1,t) = 0$ whenever $\mu_j^c(t+1,t) = 0$, we also reestimate Model 3 with the following analytic approximation for $\mu_j^z(t+1,t)$:

$$\mu_j^z(t+1,t) = \{\max[0,\mu_j^c(t+1,t)]\}^{1/2}.$$

Again the estimation results do not change qualitatively.

[14] Since the random variables in equation (22) now overlap in time, we adjust the GMM weighting matrix for the moving average this creates.

and losses. Prior to the Tax Reform Act of 1986, long-term capital gains and losses were taxed at lower rates than short-term capital gains and losses. Once the distinction between long-term and short-term capital gains and losses is made, it may be optimal for investors to realize some of their smaller long-term capital gains to reset their tax bases and restart the option to realize potential future losses short term (see Constantinides (1984)). It may also be optimal for investors to defer the realization of some of their smaller short-term losses to avoid restarting the short-term holding period prematurely, even in the absence of transaction costs (see Dammon and Spatt (1994)). In our model, however, it is suboptimal for investors to realize any capital gains or to defer the realization of any losses. Perhaps they are realizing too many losses and too few gains relative to investors in the real world.

Unfortunately, our model becomes analytically intractable once a distinction is made between the long-term and short-term tax rates. The difficulty arises in solving simultaneously for the optimal realization policies and the equilibrium stock price process. Constantinides (1984) solves for the optimal realization policy for a world in which annual stock price changes follow an exogenous binomial process and investors are allowed to trade only once per year. With the long-term tax rate equal to 40 percent of the short-term tax rate, Constantinides (1984) shows that for high- and medium-variance stocks it is optimal to realize all long-term capital gains each year. Using a model in which trading and stock price changes occur more frequently than once per year (e.g., monthly), Dammon and Spatt (1994) analytically solve for the optimal realization policy in the long-term region in the presence of differential long-term and short-term tax rates. They find an optimal cutoff level, which depends upon the parameters of the stock price process and the level of transaction costs, above which all long-term capital gains are deferred and below which all long-term capital gains are realized. In the short-term region, however, they are forced to use dynamic programming and numerical techniques to solve for the optimal cutoff level.

To determine whether the absence of capital gains realizations is the source of the disappointing results reported earlier, we reestimate equations (20) and (22) assuming that the representative agent is forced to realize all embedded capital gains at the end of one year (i.e., at the horizon date $T = t + 13$). We assume that these embedded capital gains are taxed at 40 percent of the capital gains tax rate, τ_c, and discount the resulting taxes using the marginal rate of substitution between dates t and T, $m(t, T)$.[15] We assume that all short-term losses are realized as they are incurred.

The results are reported in the fourth column (Model 4) of Table III. The estimate of the risk-aversion parameter, $\hat{\gamma} = 0.982$, is not significantly different from 1.0. The estimate of the dividend tax rate, $\hat{\tau}_d = 32.0$ percent, is

[15]The assumption that the long-term capital gains tax rate is 40 percent of the short-term capital gains tax rate is consistent with the U.S. tax code between January 1979 and December 1986. We later investigate the sensitivity of the results to this assumption.

economically plausible and significantly greater than zero. The estimate of the short-term capital gains tax rate, $\hat{\tau}_c = 9.9$ percent, is relatively low, but significantly greater than zero. Despite these improvements, the model is still rejected (at the 2.5 percent level).

E. Additional Statistical Tests

To better understand the rejection of Model 4, we ran four separate ordinary least squares (OLS) regressions with (1) the before-tax real returns on the equally weighted portfolio of stocks and the one-month Treasury bill and (2) the marginal rate of substitution times the after-tax real returns on these same assets as dependent variables and the instruments as explanatory variables. We calculate the marginal rate of substitution and after-tax returns using the parameter estimates from Model 4 in Table III and report the results of these regressions in Table IV. The former regressions involving the before-tax real returns are used as a benchmark. The parameter estimates of the latter regressions should all be insignificantly different from zero if Model 4 is correct, but we already know from the GMM tests of Model 4 in Table III that this is not the case.

While very little of the variation in the equity returns, both before and after the adjustment for taxes and risk, can be explained by the independent variables, the real return on the one-month Treasury bill still exhibits significant autocorrelation, even after these adjustments. Although the adjustments for taxes and risk reduce the R^2 of the regression, an R^2 of 10.4 percent is still quite large. This indicates that the rejections of Model 4 are due, in part, to the fact that the model cannot accommodate the autocorrelation of the real return on a one-month Treasury bill.

While the regression results and the significance levels of the χ^2 tests indicate that the after-tax model (Model 4) outperforms the tax-free model (Model 1), this is not a formal statistical test for comparing the fit of the two models. To make this comparison rigorous, we compute the χ^2 statistic proposed in Newey and West (1987), which tests the null hypothesis that the tax-free model is correct against the after-tax model as the specific alternative. The χ^2 statistic (with two degrees of freedom) is 34.611. We, therefore, reject the tax-free model in favor of the after-tax model at the 0.1 percent level.[16]

V. Robustness of the Results

A. Time-Varying Tax Rates and Year-End Tax Payments

In our tests of the models with taxes, we restrict the tax parameters so they are constant over the entire sample period. Moreover, in Model 4 we fix the long-term capital gains tax rate at $0.4\tau_c$. These restrictions obviously are

[16] We also ran a test of the tax-free model (Model 1) against the tax-based model (Model 4) using only equity returns. In this case, the χ^2 statistic (with two degrees of freedom) is 2.353, and we are unable to reject the tax-free model in favor of the tax-based model.

Table IV
Regression Analysis of the Model with Asymmetric Long-Term and Short-Term Tax Rates

This table reports the results of regressing (1) before-tax real returns and (2) risk-adjusted after-tax real returns on to the instruments used in the GMM estimation of the model with asymmetric long-term and short-term tax rates (Model 4) in Table III. The risk-adjusted after-tax real returns, $\beta[c_{t+1}/c_t]^{\gamma-1}R_j(t+1,t)-1$, are calculated using the parameter estimates from Model 4 of Table III: $\hat{\beta}=1.000$, $\hat{\gamma}=0.982$, $\hat{\tau}_c=0.099$, and $\hat{\tau}_d=0.320$. The instruments used as the independent variables in the regressions are: the unit vector, 1; one plus lagged consumption growth, c_t/c_{t-1}; one plus the lagged before-tax real return on the equally weighted index of all NYSE and AMEX common stocks, $1+r_p(t,t-1)$; one plus the lagged before-tax real return on the one-month Treasury bill, $1+r_f(t,t-1)$; one plus the contemporaneous before-tax nominal return on the one-month Treasury bill, $1+\mu_f(t+1,t)$; one plus the contemporaneous before-tax nominal yield differential between a three-month and a one-month Treasury bill, $1+q_t^3-q_t^1$; and one plus the lagged before-tax nominal yield differential between a three-month and a one-month Treasury bill, $1+q_{t-1}^3-q_{t-1}^1$. The regressions are conducted for both the equally weighted index of common stocks and the one-month Treasury bill using monthly data for the period January 1959 through December 1990. Standard errors are in parentheses under the parameter estimates.

	Dependent Variables			
	Before-Tax Returns		Risk-Adjusted After-Tax Returns	
Independent Variables	Equity	T-Bill	Equity	T-Bill
1	−1.436	−0.656	−1.159	−0.347
	(0.565)	(0.072)	(0.518)	(0.072)
c_t/c_{t-1}		0.020		0.025
		(0.032)		(0.031)
$1+r_p(t,t-1)$	0.143		0.136	
	(0.051)		(0.047)	
$1+r_f(t,t-1)$		0.285		0.283
		(0.049)		(0.049)
$1+\mu_f(t+1,t)$		0.301		−0.011
		(0.057)		(0.057)
$1+q_t^3-q_t^1$	1.293		1.014	
	(0.558)		(0.511)	
$1+q_{t-1}^3-q_{t-1}^1$		0.049		0.049
		(0.026)		(0.026)
R^2	0.034	0.221	0.032	0.104

at odds with the substantial variation that actually occurs in the tax rates and in the treatment of long-term capital gains over the sample period (see Table V). To investigate whether the time variation in tax rates has any material affect on our empirical results, we reestimate Model 4 assuming that the dividend and short-term capital gains tax rates for the marginal investor are a constant proportion of the maximum statutory tax rates that were in effect over the sample period. However, we allow the constant of proportionality to differ across dividends and short-term capital gains. We denote these constants of proportionality as α_d and α_c, respectively. The ratio of the long-term tax rate to the short-term tax rate is set equal to the

Table V

Maximum Statutory Tax Rates and Long-Term Capital Gains Tax Rates, 1959 to 1990

Time Period	Maximum Short-Term Tax Rate (%)	Long-Term Tax Rate as a Percentage of the Short-Term Tax Rate (%)
1959–1963	91.0	50.0[a]
1964	77.0	50.0[a]
1965–1978	70.0[b]	50.0[a]
1979–1981	70.0[c]	40.0
1982–1986	50.0	40.0
1987	38.5	100.0[a]
1988–1990	28.0[d]	100.0

[a] The maximum long-term tax rate is capped at 25 percent prior to 1969 and at 28 percent after 1986.

[b] Excludes a surcharge of 7.5 percent in 1968, 10 percent in 1969, and 2.5 percent in 1970.

[c] Excludes a tax credit of 1.25 percent in 1981.

[d] Excludes the effects of the phaseout of the 15 percent tax bracket and the personal exemptions.

statutory ratio given by the prevailing tax code. The estimation results are as follows (standard errors and p-values in parentheses):

$$\hat{\beta} = 1.001\ (0.001) \qquad \hat{\alpha}_c = 0.116\ (0.030)$$

$$\hat{\gamma} = 1.089\ (0.114) \qquad \hat{\alpha}_d = 0.830\ (0.307)$$

$$\chi^2 = 11.546\ (0.021)$$

A comparison of the above results to those of Model 4 (see Table III) indicates that they are not qualitatively different. The short-term capital gains tax rate is estimated to be 11.6 percent of the maximum statutory tax rate, and the dividend tax rate is estimated to be 83.0 percent of the maximum statutory tax rate. Thus, the estimate of the dividend tax rate for the marginal investor varies between 75.5 percent (in the 1959 to 1963 period) and 23.2 percent (in the post-1988 period), while the estimate of the short-term tax rate varies between 10.6 percent (in the 1959 to 1963 period) and 3.2 percent (in the post-1988 period).

As with the other tax-based models we consider, Model 4 assumes that taxes are paid as they are incurred, rather than at the end of each tax year. To investigate whether this timing difference has any material effect on our results, we reestimate Model 4 assuming that taxes are paid at the end of each December (which may be more appropriate for individuals who do not pay withholding tax on their investment income during the year, but are still required to pay at least 90 percent of their yearly tax liability by the end of December). The timing adjustment is obtained by discounting the tax payments and rebates in the months prior to December by the rate of return on one-month Treasury bills rolled over until the end of the year.[17] Our results are virtually identical to those reported in Table III, which suggests that by simplifying the timing of the tax payments and rebates we do not seriously bias the results.

B. Alternative Instrumental Variables

Previous researchers document that before-tax common stock returns exhibit seasonalities. In particular, there is a strong January seasonal, which some have attributed to taxes. Because of this possibility, we investigate the sensitivity of our results to the introduction of a January dummy as an additional instrument for the portfolio of common stocks.[18] Reestimating Model 4 with the January dummy does not alter the results qualitatively: the model is rejected (at the 2 percent level) and the estimates of the tax parameters are $\hat{\tau}_d = 41.6$ percent and $\hat{\tau}_c = 9.0$ percent.

Similarly, there is evidence to suggest that lagged dividend yield is also a good predictor of common stock returns on a before-tax basis. This predictability may be related to the fact that dividends are taxed more heavily than capital gains. On an after-tax basis, however, we would expect this predictability to dissipate. To investigate this, we reestimate Model 4 with the lagged dividend yield as an additional instrument for the portfolio of common stocks. Again, the results do not change qualitatively: the model is rejected (at the 5 percent level), and the tax parameters are estimated to be $\hat{\tau}_d = 32.0$ percent and $\hat{\tau}_c = 9.9$ percent.

Some researchers would object to the fact that we consider consumption growth lagged one period to be in the investor's information set. In particular, consumption data may be aggregated and, hence, lead to spurious predictability one period ahead (e.g., Grossman, Melino, and Shiller (1987)). To investigate the sensitivity of our results to the use of consumption growth lagged once, we replace it with consumption growth lagged twice and then reestimate Model 4. From Table II, however, we know that consumption growth lagged twice is not a good instrument for either our portfolio of common stocks or the one-month Treasury bill.

Although the χ^2 statistic has a similar p-level and the parameter estimates are virtually identical, the standard errors and correlations of the parameter estimates increase dramatically. Consequently, the use of consumption growth lagged twice as an instrument, instead of consumption growth lagged once, mainly leads to a drop in the precision of the estimates but does not otherwise affect our conclusions.

[17] Although the rate of return on one-month Treasury bills, rolled over until the end of the year, is not known in advance, this discounting method should be a reasonable approximation with which to study the sensitivity of our results to the timing issue.

[18] Strictly speaking, a January dummy should not be used as an instrument because it is a nonstationary random variable and, therefore, violates the assumptions underlying the asymptotic properties of GMM.

VI. Summary

This article investigates whether the fit of the standard consumption-based asset pricing model can be improved by incorporating dividend and capital gains taxation. The model acknowledges that dividends are taxed as they are paid, whereas capital gains and losses are taxed only when realized, which gives investors the option to optimally time their asset sales so as to minimize the present value of the net tax payments. We argue that this tax-timing option alters the covariance between the marginal rate of substitution and asset returns. When the long-term and short-term tax rates are equal, investors optimally realize capital losses as they occur and defer capital gains indefinitely. While this version of our model produces an estimate of the dividend tax rate that is significantly greater than zero and economically plausible, the estimate of the capital gains tax rate is imprecise and economically implausible. The overall model is rejected.

To incorporate differential long-term and short-term tax rates into the model, we construct an alternative model in which investors realize all capital gains after one year, while all capital losses are assumed to be realized as they occur. We also assume that any realized gains are long term and taxed at preferential rates. This version of our model produces an estimate of the dividend tax rate that is virtually identical to that in the symmetric tax model. The estimate of the capital gains tax rate, while low, is dramatically improved; however, the model is still rejected at standard confidence levels. A closer look at these rejections indicates that they are due, in part, to the autocorrelation in the after-tax real return on the one-month Treasury bill. In a direct test of the tax-free model against the asymmetric tax-based model as the specific alternative, the tax-free model is rejected in favor of the tax-based model at the 0.1 percent level.

While our results indicate that taxes and the options they create are potentially important in testing asset pricing models, adding taxes alone does not completely explain the cross-sectional differences in asset returns. However, our model assumes that investors have time-separable utility, which may not be realistic. Perhaps it is time to go back to the specification of preferences and merge the tax-based model with a more general form of intertemporal utility. The recent success of time-nonseparable preferences in explaining patterns of consumption measured at different frequencies (e.g., Heaton (1993)) indicates a potentially fruitful enrichment of the tax-based model. We leave this for future research.

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