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Interactions of Corporate Financing and Investment Decisions: A Dynamic Framework

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ABSTRACT

This article analyzes the interaction between a firm's dynamic investment, operating, and financing decisions in a model with operating adjustment and recapitalization costs. Using numerical analysis, we solve the model for cases that highlight interaction effects. We find that higher production flexibility (due to lower costs of shutting down and reopening a production facility) enhances the firm's debt capacity, thereby increasing the net tax shield value of debt financing. While higher financial flexibility (resulting from lower recapitalization costs) has a similar effect, production flexibility and financial flexibility are, to some extent, substitutes. We find that the impact of debt financing on the firm's investment and operating decisions is economically insignificant.

IN THEIR SEMINAL ARTICLE Modigliani and Miller (1958) prove that under perfect capital market assumptions corporate financing and investment decisions are completely separable. Since then, a central question in financial economics has been whether market imperfections establish a linkage between these decisions. On the one hand, empirical studies show that firm characteristics, such as profitability, volatility of firm value, collateral value of assets, growth opportunities, and technological uniqueness, which are a result of the firm's investment decisions, affect the firm's financing decisions (see, e.g., Bradley, Jarrell, and Kim (1984) and Titman and Wessels (1988)). On the other hand, the capital budgeting literature typically argues that because of the tax advantage of debt financing the mix of financing for a project may make the difference between project acceptance or rejection (see, e.g., Myers (1974)).

Theoretical models examining the investment-financing linkage have almost exclusively been static, i.e., investment and financing decisions are made at a single point in time and are irreversible. Furthermore, most of

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these models have only endogenized either the investment or the financing decision.[1] In this article, we analyze the interaction between investment and financing decisions in a multiperiod contingent claims model where the firm has the flexibility to dynamically manage both decisions over time. This framework allows us to address a number of interesting questions. For example, to what extent does the flexibility associated with a real investment affect the firm's financing policy—not only the initial capital structure, but also subsequent recapitalizations? How does one incorporate the value of interest tax shields into the valuation of a real investment if the firm's capital structure is changing over time as a result of the resolution of uncertainty and dynamic production decisions? How are investment and operating decisions affected by the firm's dynamic financing policy? To what extent are production and financial flexibility substitutes?

In our model, a firm produces a single commodity whose price is stochastic. The firm can shut down and reopen its production facility in response to price fluctuations by paying operating adjustment costs. The firm may issue both debt and equity and can alter its capital structure over time by paying recapitalization costs. In our setting, the optimal dynamic financing policy is characterized by a tradeoff between the tax advantage of debt financing and recapitalization and financial distress costs. We also examine the case of a firm that holds an option to invest in the production facility. By varying operating adjustment and recapitalization costs, we explore the impact of production and financial flexibility on the interactions between the firm's investment, operating, and financing decisions in a firm value maximization framework.

The complexity of the model precludes an analytic solution. We therefore use numerical solution techniques to solve the model and analyze interaction effects. Since our results are numerical rather than analytic, they depend on the choice of parameter inputs. As will become clear below, we examine cases that highlight possible interaction effects.

We find that production flexibility has a positive effect on the value of interest tax shields. The ability to shut down operations allows the firm to mitigate operating losses. Therefore, as operating adjustment costs decrease, firm value increases and firm value variance decreases, increasing the debt capacity of the firm and the associated net benefit of interest tax shields. However, production flexibility and financial flexibility are to some degree substitutes, since the effect of lower operating adjustment costs on net tax shield value is less pronounced the smaller are recapitalization costs. We also examine the effect of production flexibility on the firm's optimal dynamic recapitalization policy. As operating adjustment costs decrease, the average leverage ratio increases and the range over which the firm allows its optimal leverage ratio to vary without recapitalizing decreases.

In contrast, we find that debt financing has a negligible impact on the firm's investment and operating policies. For example, while a levered firm

[1]Examples of static models that allow for both investment and financing decisions are Hite (1977), Dotan and Ravid (1985), Dammon and Senbet (1988), and Aivazian and Berkowitz (1991).

has an incentive to invest earlier (i.e., at a lower commodity price) than an equivalent unlevered firm because it earns interest tax shields when it is operating, the benefit from doing so is largely offset by a loss in the value of waiting to invest. Therefore, the net benefit is not large enough to effect a significant change in investment policy. Similarly, our analysis indicates that any additional interest tax shields that a levered firm can earn by deviating from the optimal operating policy of an equivalent unlevered firm are counterbalanced by a loss in the value of its operating options. Thus a levered firm has little incentive to alter operating policy. From a practical standpoint, the implication is that the firm can determine exercise timing decisions on its real options, ignoring the effect of debt financing.

Our analysis significantly extends the literature on real options by endogenizing dynamic capital budgeting and capital structure decisions in a single unifying framework.[2] There are several articles that are related to our analysis, but they either lack the generality of our framework or focus on different issues. Brennan and Schwartz (1984) develop a model of firm value that allows for operating and financial flexibility. However, the focus of their article is on firm valuation in a setting where bond covenants restrict financial policy and influence investment policy. Fischer, Heinkel, and Zechner (1989a) analyze a firm's optimal dynamic financing policy in the presence of recapitalization costs. In contrast to the analysis in our article, investment decisions are exogenous in their model, which precludes interaction effects between investment and financing decisions. Finally, Mello and Parsons (1992) measure the agency costs of debt in a contingent claims framework where managers maximize equity value. Unlike our model, they examine a firm having no initial investment option and a fixed amount of debt in its capital structure, i.e., no financial flexibility.

The article is organized as follows. Section I presents an optimal control model in which a firm selects optimal dynamic operating, financing, and investment policies to maximize firm value. Section II investigates the interactions among these policies using numerical simulations. Section III discusses the robustness of our results to alternative model assumptions and parameter values, and Section IV concludes.

I. The Model

Consider a firm that produces a single commodity. The commodity is infinitely divisible and is produced continuously through time at the rate of one unit per year while the production facility is in operation. The cost of producing one unit of the commodity is C, which is constant through time. As

[2] The real options literature analyzes flexibility in real investments using a contingent-claims framework to simultaneously value and determine the optimal exercise policies of the firm's real options (see, e.g., Brennan and Schwartz (1985), McDonald and Siegel (1985, 1986), Majd and Pindyck (1987), Pindyck (1988), Paddock, Siegel, and Smith (1988), Dixit (1989), Sick (1989), Myers and Majd (1990), and Triantis and Hodder (1990)). However, this literature typically assumes all-equity financing and therefore ignores possible interactions between investment and financing decisions.

the commodity is produced, it is simultaneously sold in a perfectly competitive market at a price, P, per unit. The evolution of the commodity price through time follows geometric Brownian motion:

$$dP = \alpha P dt + \sigma P dz, \tag{1}$$

where the drift (α) and volatility (σ) parameters are constants and dz is the increment of a standard Wiener process. Once the commodity is produced, we allow for the possibility that it may be held in inventory by some firm for commercial purposes, and there is thus some net (of storage costs) benefit from an inventory of the commodity. This convenience yield is assumed to be a constant proportion, δ, of the commodity price.

We examine the firm during the time period $t \in [0, T]$, where t is measured in years. At any point in time during this period, the firm may decide to shut down operations by incurring a fixed "exit" cost of K_X. If it wishes to resume operating, the firm must pay an "entry" cost of K_N. Hereafter, we will refer to these costs as "operating adjustment costs," and to the ability to shut down and resume operations as "production flexibility." When the facility is operating, we assume that it is always at full capacity.[3] Finally, at time T the facility is sold for a salvage value of S, which is a fraction of the initial investment, I, that is required to build the facility. (We examine the initial investment decision later in this section).

The production facility can be financed with both debt and equity. The level of debt in the firm is denoted by B, which is the par value of outstanding bonds. All bonds mature at time T and pay interest at the instantaneous rate of ρ per year. The firm may adjust its debt-equity mix over time, but must pay recapitalization costs when it does so. These costs consist of a fixed recapitalization cost, κ_F, as well as a proportional cost on the debt level adjustment, either κ_{V1} when issuing equity to repurchase debt or κ_{V2} when issuing additional debt. Specifically, an upward adjustment of B by an amount ΔB costs $\kappa_F + \kappa_{V2}\Delta B$, and downward adjustment of ΔB (< 0) costs $\kappa_F + \kappa_{V1}|\Delta B|$. We assume that there are no informational asymmetries and that the bond indenture specifies that the firm selects operating and financing policies that maximize total firm value.

Operating profits net of interest payments to bondholders are taxed instantaneously at a constant rate, τ. For simplicity, we do not model economic depreciation of the production facility. We assume an asymmetric tax system in which there are no loss offset provisions.[4] The tax system motivates the firm to issue debt, since the firm may then shield part of its operating profits from taxation. After-tax profits net of interest payments are distributed to

[3]Adjustment costs associated with suspending and resuming production will cause the firm to produce in some states in which profits are negative. Thus, we impose the "full capacity" restriction to prevent the firm from avoiding paying the exit cost, K_X, simply by reducing production down to zero units.

[4]The U.S. tax system allows for at least a partial offset of operating losses through carryback and carryforward provisions. Although we ignore these provisions in our analysis, we discuss the effect of allowing for full loss offset provisions in Section III.

equityholders.[5] For convenience, the analysis assumes that interest and equity income are not taxed at the personal level.[6]

While taxes provide an incentive to issue debt in our model, we recognize that debt may be costly and that firms are therefore likely to choose interior debt-equity mixes. The costs attributed to debt are often associated with bankruptcy or financial distress in traditional capital structure models. However, our model also allows for the possibility that the costs associated with debt financing may be due to recapitalizations to prevent financial distress. We assume that there is a minimum net worth covenant in the bond indenture that requires that firm value must always be greater than the par value of debt. In the event that this covenant is violated, i.e., if the firm becomes insolvent, bondholders assume ownership of the firm, although deadweight costs are incurred in the process. These costs are equal to $G + gB$, where $G \geq 0$, $0 \leq g < 1$, and B is the face value of debt at the time of default. The bondholders may then choose a new capital structure, subject to recapitalization costs, that maximizes the value of the firm.

To value the firm and determine its optimal operating and financing policies, we assume that there are no investor transaction costs, investors may trade securities continuously, and existing securities span the price of the commodity. With respect to spanning, for convenience we assume that there is a traded futures contract on the commodity. Finally, we assume that there exists a riskless security that yields a constant instantaneous rate of return of r per year.

The firm chooses feasible operating and financing policies to maximize its value. Each policy consists of a sequence of decisions made over time contingent on the values of P and B. The set of operating policies is $\Phi = \{\phi(P, B, t) \in \{1,2\};\ P \geq 0,\ B \geq 0,\ t \in [0, T]\}$, where $\phi(P, B, t) = 1$ if the firm chooses to operate and $\phi(P, B, t) = 2$ otherwise. The set of financing policies is $\Psi = \{V - B \geq \psi(P, B, t) \geq -B;\ P \geq 0,\ B \geq 0,\ t \in [0, T]\}$, where $\psi(P, B, t)$ is the change in debt level. As will become clear below, the choice of optimal operating and financing policies depends on the costs of adjustment.[7]

We denote the value of the firm as $V^1(P, B, t)$ if the production facility is in operation and $V^2(P, B, t)$ if operations are idle. Using standard replication arguments, it is straightforward to derive the system of partial differential equations that, together with the appropriate boundary conditions, yield the optimal operating and financing policies and resulting firm values, V^1 and V^2. These equations, stated below in equations (2) and (3), result from

[5]We assume that if the firm's operating profit is at any time less than the coupon payment to be made to the bondholders (e.g., when the firm has suspended operations), equityholders transfer a portion of their ownership interest to the bondholders to cover the shortfall.

[6]The model can easily accommodate constant personal tax rates. The tax rate, τ, is then the *net* tax advantage of debt relative to equity after adjustment for personal taxes.

[7]The assumptions regarding the control variables, ϕ and ψ, and the associated costs of adjustment are such that our problem falls within the class of "impulse control" problems. This terminology denotes costly control applied at isolated points in time in a stochastic dynamic system (see, e.g., Harrison, Sellke, and Taylor (1983) and Hodder and Triantis (1993)).

forming a continuously rebalanced self-financing portfolio, consisting of the futures contract and the riskless security, that replicates the value of the firm. Thus, we have

$$\underset{\phi(P,B,t)\in\Phi,\ \psi(P,B,t)\in\Psi}{\mathrm{Max}} \Big\{\tfrac{1}{2}\sigma^2 P^2 V^1_{PP} + (r-\delta)PV^1_P + V^1_t$$
$$-rV^1 + (P-C) - (\tau)\max[0, P - C - \rho B]\Big\} = 0, \quad (2)$$

$$\underset{\phi(P,B,t)\in\Phi,\ \psi(P,B,t)\in\Psi}{\mathrm{Max}} \Big\{\tfrac{1}{2}\sigma^2 P^2 V^2_{PP} + (r-\delta)PV^2_P + V^2_t - rV^2\Big\} = 0, \quad (3)$$

where in equation (2), $(P - C)$ is the pretax operating profit and $(\tau)\max[0, P - C - \rho B]$ is the corporate tax bill, which assumes no loss offset.

The boundary conditions that V^1 and V^2 must satisfy are numerous, including free-boundary, smooth-pasting, and fixed-boundary conditions. There are two sets of free-boundary conditions, one for operating adjustments and the other for recapitalizations. We denote by $P_X(B,t)$ and $P_N(B,t)$ the output prices at which the firm will optimally suspend ("exit") and resume ("enter") operations, respectively. The functional dependence of these prices on B recognizes that the firm's financial policy may influence its operating policy, while the dependence on time is due to the finite horizon in our model. In addition, these prices may depend upon recapitalization costs and operating adjustment costs. The free-boundary conditions that firm value must satisfy when operations are suspended and resumed, respectively, are

$$V^1(P_X(B,t), B, t) = V^2(P_X(B,t), B, t) - K_X,\ B \ge 0,\ t \in [0,T], \quad (4)$$

$$V^2(P_N(B,t), B, t) = V^1(P_N(B,t), B, t) - K_N,\ B \ge 0,\ t \in [0,T]. \quad (5)$$

The set of free-boundary conditions that firm value must satisfy at times of recapitalization are

$$V^i(P, \underline{B}, t) = V^i(P, B^*, t) - \{\kappa_F + \kappa_{V2}(B^* - \underline{B})\},$$
$$P \ge 0,\ \underline{B} \ge 0,\ t \in [0,T],\ i = 1,2, \quad (6)$$

$$V^i(P, \overline{B}, t) = V^i(P, B^{**}, t) - \big\{\kappa_F + \kappa_{V1}(\overline{B} - B^{**})\big\},$$
$$P \ge 0,\ \overline{B} < V^i(P, \overline{B}, t),\ t \in [0,T],\ i = 1,2, \quad (7)$$

where $\underline{B}(P,t)$ and $\overline{B}(P,t)$ are "trigger" debt levels at which the firm will choose to increase or decrease leverage, respectively, and $B^*(P,t)$ and $B^{**}(P,t)$ are the "target" debt levels that the firm recapitalizes to in these two cases.[8] Note that trigger and target debt levels are functions of price and

[8] While it seems intuitive that $0 \le \underline{B}(P,t) \le B^*(P,t) \le B^{**}(P,t) \le \overline{B}(P,t)$, we have not been able to analytically prove the optimality of such a recapitalization policy. However, the numerical simulations that we present in Section II do not restrict the recapitalization policy to have this characterization, and yet we find that this particular policy is chosen as optimal. Furthermore, we find that the firm increases its leverage only when it is in the operating state ($i = 1$ in boundary condition (6)), which is intuitive given that with asymmetric taxes the firm does not earn interest tax shields when it is not operating.

time, although for notational simplicity this functional dependence is suppressed in equations (6) and (7).[9]

To solve for the optimal operating and financing policies, V^1 and V^2 must satisfy smooth-pasting conditions that are first-order conditions corresponding to firm value maximization along each of the free boundaries in equations (4) to (7). For brevity of exposition, we omit these conditions here since we use a numerical solution technique that ensures value maximization in a more direct manner. However, firm value must also satisfy two fixed-boundary conditions. First, we allow for the possibility that the firm may choose not to recapitalize despite the fact that it is approaching insolvency, and thereby risks violating the minimum net-worth covenant. Denoting the debt level at which default is triggered as B^d, upon default we have

$$V^i(P, B^d, t) = V^i(P, 0, t) - \{G + gB^d\},\ P \geq 0,\ B^d > 0,\ t \in [0, T],\ i = 1, 2. \tag{8}$$

Once bondholders assume control of the firm, they may choose to recapitalize from $B = 0$ to some positive debt level according to equation (6). Finally, we have the salvage value (of plant) condition at the horizon:

$$V^i(P, B, T) = S,\ P \geq 0,\ B \geq 0,\ i = 1, 2. \tag{9}$$

In addition to examining the interaction between operating and financing policies, we desire to analyze the effects of these policies on the firm's initial decision to invest in the production facility. This is accomplished by allowing the firm to make the initial investment decision during the time period $[0, T]$. Thus, the problem specified in equations (2) to (9) now includes an additional differential equation for the investment option and two additional boundary conditions:[10]

$$\operatorname*{Max}_{\xi(P,t) \in \Xi} \left\{ \tfrac{1}{2}\sigma^2 P^2 V^0_{PP} + (r - \delta) P V^0_P + V^0_t - r V^0 \right\} = 0, \tag{10}$$

subject to

$$V^0(P_I(t), t) = V^1(P_I(t), 0, t) - I,\ t \in [0, T], \tag{11}$$

$$V^0(P, T) = 0,\ P \geq 0, \tag{12}$$

where V^0 is the value of the option to invest, I is the exercise price, $P_I(t)$ is the time t commodity price at which the firm chooses to exercise the investment option, and $\Xi = \{\xi(P, t) \in \{0, 1\};\ P \geq 0,\ t \in [0, T]\}$ is the set of

[9] Our financing policy differs considerably from that in Fischer, Heinkel, and Zechner (1989a), where a constant leverage ratio range (i.e., region over which the firm allows its leverage ratio to vary without recapitalizing) and a single interior target leverage ratio uniquely characterize the firm's dynamic optimal capital structure. Their simpler structure results from an infinite horizon framework (including perpetual debt) and their assumption of a proportional recapitalization cost on the total debt level after a recapitalization, rather than on the debt level change.

[10] There is also a smooth-pasting condition that must be satisfied when the firm invests in the facility. We do not include it here since we employ a numerical solution technique that ensures that this condition is satisfied.

feasible investment policies, where $\xi(P,t) = 0$ if the firm delays investment and $\xi(P,t) = 1$ when the firm invests in the production facility. Once the firm exercises its investment option, it will likely take on debt immediately (subject to equation (6)), since it will have operating profits ($P_I(t) > C$) allowing it to earn interest tax shields. Further, since the choice of operating and financing policies affects the value of the firm once it invests (V^1), both policies may also affect the value of the option to invest (V^0) and the corresponding optimal investment decision ($P_I(t)$) through equation (11).

II. The Interaction Between Investment, Operating, and Financing Decisions

Due to the complexity introduced by the free-boundary conditions in our model (e.g., the functional dependence of the exit and entry prices on debt level and time), we are not able to obtain an analytic solution. Therefore, we employ a numerical technique to solve the model. The procedure is described in detail in the Appendix. We solve the model for a variety of different operating adjustment and recapitalization costs. For convenience, we assume that the cost of suspending operation (K_X) is equal to the cost of resuming operation (K_N), and that the proportional costs of increasing (κ_{V2}) or decreasing (κ_{V1}) the debt level are also equal. These and all other parameter values are shown in Table I.

In the table, note that the coupon rate of debt (ρ) is equal to the real risk-free rate of interest (r). This reflects the fact that the firm never defaults on its debt in the simulations but instead recapitalizes to avoid costly

Table I

Parameter Values for the Numerical Simulations

Parameter	Value
Annual rate of production	one unit per year
Production costs	$C = \$100$ per unit
Initial commodity price	$P(0) = \$100$
Commodity price volatility	$\sigma = 20\%$ annually
Time horizon	$T = 10$ years
Initial investment	$I = \$50$
Salvage value	$S = \$20$
Real riskless interest rate	$r = 3\%$ annually
Convenience yield	$\delta = 1\%$ annually
Coupon rate on debt	$\rho = 3\%$ annually
Financial distress costs	$G = \$1.0$, $g = 2\%$
Corporate tax rate	$\tau = 35\%$
Operating adjustment costs	$K_X = K_N = \$0, \$5, \$10, \$15, \$20$
Fixed recapitalization costs	$\kappa_F = \$0, \$0.1$
Proportional recapitalization costs	$\kappa_{V1} = \kappa_{V2} = 0, 0.5, 1.0, 1.5, 2.0\%$

bankruptcy.[11] In addition, observe that the initial commodity price equals the production cost of $100 per unit. Although somewhat arbitrary, this parameter specification ensures, for example, that the firm's operating option to shut down the facility is valuable and thereby allows for an examination of the effect of production flexibility on the firm's financial policy. Note that Section III contains a discussion of the robustness of the numerical results to alternative parameter values—including initial commodity prices.

We first examine the effect of production flexibility on the value of interest tax shields and on the firm's optimal dynamic financial policy. In the process, we also investigate whether financial flexibility has an effect on the firm's operating policy to shut down and reopen the production facility. We then introduce the option to invest and examine the effects of production and financial flexibility on the firm's optimal investment policy.

A. The Effect of Production Flexibility on the Value of Interest Tax Shields

Table II presents time-zero firm values (V^1) for different combinations of operating adjustment and recapitalization costs, assuming that the firm is operating at that time. Clearly, firm value increases as operating adjustment costs decrease.[12] This increase in value comes from two sources. First, the

Table II

Firm Values for Different Operating Adjustment and Recapitalization Costs

Firm values are computed at time zero assuming that the firm is operating. For all cases, the operating adjustment costs to shut down (K_X) and reopen (K_N) the production facility are equal, and the proportional recapitalization costs to increase the debt level (κ_{V2}) and decrease the debt level (κ_{V1}) are equal. For all nonzero proportional recapitalization cost cases, the fixed component of recapitalization costs (κ_F) is $0.1.

Operating Adjustment Cost ($)	Proportional Recapitalization Cost (%)					All Equity Financing
	0	0.5	1.0	1.5	2.0	
0	146.57	143.53	142.96	142.63	142.48	141.86
5	135.40	132.38	131.80	131.49	131.34	130.84
10	129.05	125.94	125.38	125.07	124.91	124.54
15	123.54	120.40	119.83	119.52	119.35	119.07
20	119.08	115.88	115.29	114.97	114.80	114.67

[11] In order for the firm to choose to risk default rather than recapitalize in our model, the cost of recapitalization would have to be significantly higher than the costs of financial distress, a rather unlikely situation that is not investigated here.

[12] As expected, the table also shows that firm value increases as recapitalization costs decrease. When such costs are low, the firm will take on higher debt levels since it is less costly to recapitalize to avoid insolvency. In addition, the lower are recapitalization costs, the sooner the firm will recapitalize to a higher debt level when the output price increases.

lower the cost of shutting down the facility, the sooner the firm will do so when operating cash flows become negative. This reduces the negative effect of operating losses on firm value. Similarly, the lower the cost of resuming operation, the sooner the firm will do so as the commodity price, and therefore operating profits, rebound.

Second, when the firm is levered, part of the increase in firm value as operating adjustment costs decrease is due to an increase in the value of interest tax shields. We isolate this effect in Table III, which shows net tax shield values (i.e., the present value of interest tax shields minus the present value of recapitalization costs) for the same combinations of operating adjustment and recapitalization costs as in Table II.[13] As can be seen in Table III, net tax shield values increase as operating adjustment costs decrease. This is attributable to the present value of the firm's operating profits increasing (as described above) and to the volatility of firm value decreasing (since the influence of low price outcomes on the firm's *realized* operating cash flows is attenuated) as operating adjustment costs decrease.[14] Everything else remaining the same, the combination of an increase in firm value and a decrease in firm value volatility reduces the threat of insolvency. Thus, the firm can maintain a higher level of debt through time, thereby increasing the value of interest tax shields.

Table III

Net Tax Shield Values for Different Operating Adjustment and Recapitalization Costs

The net tax shield value of debt financing is equal to the present value of interest tax shields minus the present value of recapitalization costs. For all cases, the operating adjustment costs to shut down (K_X) and reopen (K_N) the production facility are equal, and the proportional recapitalization costs to increase the debt level (κ_{V2}) and decrease the debt level (κ_{V1}) are equal. For all nonzero proportional recapitalization cost cases, the fixed component of recapitalization costs (κ_F) is \$0.1.

Operating Adjustment Cost	Proportional Recapitalization Cost (%)				
(\$)	0	0.5	1.0	1.5	2.0
0	4.71	1.67	1.10	0.77	0.61
5	4.56	1.54	0.96	0.66	0.50
10	4.53	1.41	0.85	0.54	0.38
15	4.47	1.33	0.77	0.46	0.29
20	4.41	1.21	0.62	0.30	0.14

[13] The Appendix describes the procedure that we use to compute net tax shield values in Table III.

[14] There is a significant decrease in firm value volatility as operating adjustment costs decrease. For example, the time-zero volatility of unlevered firm value is 0.82 when operating adjustment costs are \$20 and 0.58 when they are \$0. The Appendix describes the procedure that we use to compute firm value volatility.

Observe, however, that the effect of production flexibility on net tax shield value is less pronounced the lower are recapitalization costs (i.e., the higher is financial flexibility). For example, when proportional recapitalization costs are 2 percent, there is a \$0.47 increase in net tax shield value as operating adjustment costs decrease from \$20 to zero. The corresponding increase in net tax shield value when the firm has perfect financial flexibility (i.e., zero recapitalization costs) is only \$0.30. Intuitively, when the costs to dynamically manage capital structure are small, the hedging benefit of production flexibility has less of an impact on the net tax shield value of debt financing. Thus, production flexibility and financial flexibility are, at least to some degree, substitutes in terms of their effects on net tax shield value.

It is important to note that production flexibility would have no effect on net tax shield value if the firm never experiences operating losses, and therefore never uses its option to shut down the production facility. We have chosen the parameter values for the price process to ensure a positive probability of operating losses to illustrate interaction effects between production flexibility and financial policy. We discuss the effect of alternative parameter values on our results in Section III.

We can also investigate whether financial flexibility affects the firm's operating policy to shut down and reopen the production facility. If financial flexibility influences operating policy, we should observe that the present value of the firm's operating profits at any given level of operating adjustment costs depends on the level of recapitalization costs. The present value of operating profits (net of operating adjustment costs) is obtained by subtracting the net tax shield value in Table III from the corresponding levered firm value in Table II. We find that this value is identical (within rounding error) across all recapitalization cost cases for any given operating adjustment cost level. The implication is that financial flexibility has essentially no effect on the firm's operating policy.

This result may seem surprising since one might expect that a levered firm would enter operation somewhat sooner (i.e., at a lower commodity price) than an equivalent unlevered firm to start earning interest tax shields, especially when recapitalization costs are small. By entering sooner, however, the firm foregoes some of the benefit from waiting for further uncertainty resolution in the output price before committing the capital (K_N) to resume operation. Thus, although the firm could earn additional interest tax shields by entering operation sooner, it would also incur a loss in the present value of operating profits.[15] As such, a levered firm does not have much of an incentive to change operating policy; and therefore, financial policy has an insignificant effect on operating policy.

[15] In comparison with an unlevered firm, a levered firm might also wait for an even lower commodity price before shutting down operation in the hope that the price rebounds to a higher level that allows it to earn additional interest tax shields. However, there is an offsetting cost from sustaining larger operating losses that deters the firm from changing its operating policy.

B. The Effect of Production Flexibility on the Firm's Optimal Dynamic Financial Policy

We argue above that production flexibility enhances the net tax shield value of debt financing by enabling the firm to support a higher level of debt as operating adjustment costs decrease. To see this effect explicitly, we examine the firm's optimal dynamic financing policy. Since there are numerous commodity price paths through time, and since the firm may change its capital structure as firm value changes along these price paths, there are many possible leverage ratios for the firm at each price at a given point in time. Our numerical solution technique allows us to record at *each* price-time combination the maximum and minimum leverage ratios (which correspond to the trigger debt levels $\overline{B}$ and $\underline{B}$ in our model) and the average leverage ratio. The latter is computed using price path probabilities (see the Appendix for details).

Figure 1 displays the average leverage ratio at $t = 5$ years (midway through the time horizon) for commodity prices from \$100 to \$300 and for five different operating adjustment cost levels.[16] For the figure, recapitalization costs are held constant at \$0.10 plus 1 percent of the change in the face value of debt. Observe that the average leverage ratio increases monotonically as operating adjustment costs decrease and as the commodity price increases.[17] This increase in leverage is due to a decrease in firm value volatility under these circumstances. Note that for high commodity prices (above about \$250) the average leverage ratio approaches a level of around 45 percent and is insensitive to operating adjustment costs. At these price levels, the firm will likely continue to operate its facility without interruption until time T, and the volatility of firm value will asymptote to the level of the commodity price volatility.

The leverage ratio range, i.e., the difference between the maximum and minimum leverage ratios, is displayed in Figure 2 at $t = 5$ years for the same commodity prices and operating adjustment costs as in Figure 1. Notice that the range over which the firm allows its debt ratio to vary without recapitalizing decreases as operating adjustment costs decrease—especially at low price levels.[18] Since the volatility of firm value decreases as operating adjust-

[16] We do not report the average leverage ratio for prices below \$100, because for such price levels the firm typically has zero debt in its capital structure. The reason is that debt generates no tax shields for prices below \$100, given our assumption of asymmetric corporate taxes and a production cost of \$100. We do not report results for prices above \$300, because, given the parameters of the price process (see Table I), the probability of observing a price level greater than \$300 at $t = 5$ is significantly less than 1 percent.

[17] We also examine the effect of different recapitalization costs on the firm's average leverage ratio (not reported). As expected, the average leverage ratio is inversely related to the level of recapitalization costs.

[18] A careful inspection of Figure 2 reveals that the leverage ratio range is equal to zero when the exit cost is \$20 and the commodity price level is at or slightly above \$100. The reason is that the firm will have zero debt in its capital structure in these cases, regardless of the commodity price history. Notice that the average leverage ratio in Figure 1 is, of course, also equal to zero at the same exit cost and commodity price combinations.

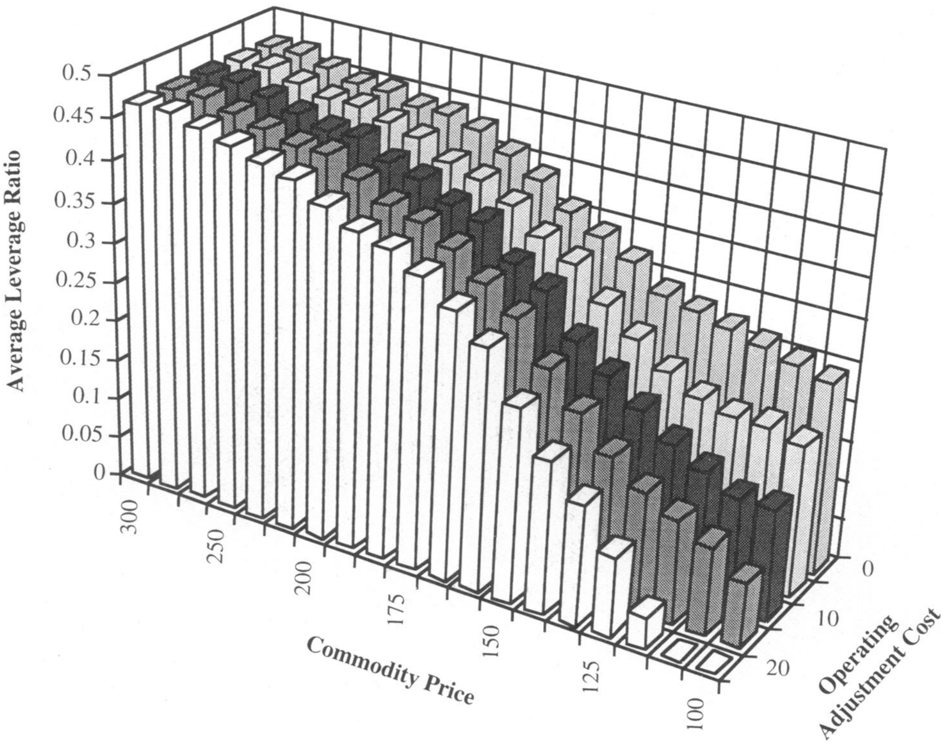

Figure 1. The average leverage ratio for different commodity prices and operating adjustment costs. Recapitalization costs are \$0.10 plus 1 percent of the change in the face value of debt, and the operating adjustment costs to shut down and reopen the production facility are assumed to be equal. The average leverage ratios are computed at $t = 5$ years (midway through the time horizon).

ment costs decrease, the firm can choose a narrower range for the no-recapitalization region without significantly increasing the frequency of costly recapitalizations.[19] Fischer, Heinkel, and Zechner (1989a) find a related result in their model, in that it predicts that the leverage ratio range is an increasing function of firm value volatility. While their prediction comes from

[19] Notice that the leverage ratio range in Figure 2 is a smooth, but nonmonotonic function of the commodity price. This pattern is explained by analyzing the behavior of the maximum and minimum leverage ratios across price levels. Regarding the maximum leverage ratio, since the volatility of firm value is a decreasing function of the commodity price level, the firm can maintain a higher maximum leverage ratio at high prices than at low prices while still avoiding insolvency. Thus, the maximum leverage ratio is an increasing function of commodity price. The minimum leverage ratio, however, is a decreasing function of commodity price for low price levels and an increasing function of commodity price for high price levels. At low prices, the firm is reluctant to increase its leverage since the expected increase in interest tax shields tends to be overshadowed by the fixed cost of recapitalization. Since firm value rises as the commodity price increases, and the minimum debt level remains unchanged, the minimum leverage *ratio*

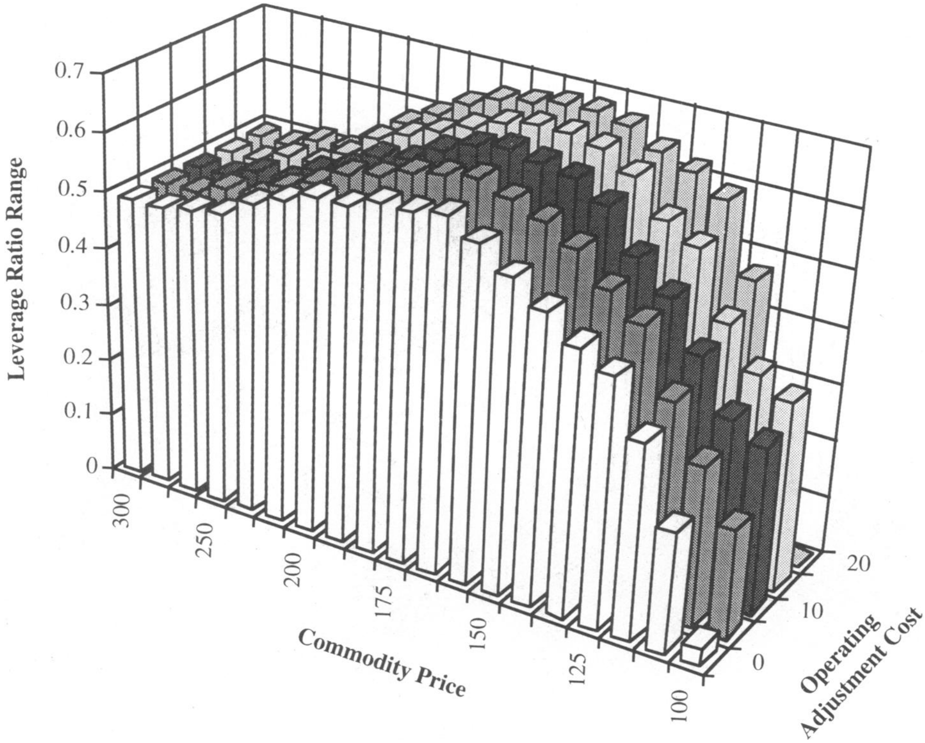

Figure 2. The leverage ratio range for different commodity prices and operating adjustment costs. The leverage ratio range is the difference between the maximum and the minimum leverage ratios. Recapitalization costs are \$0.10 plus 1 percent of the change in the face value of debt, and the operating adjustment costs to shut down and reopen the production facility are assumed to be equal. The leverage ratio ranges are computed at $t = 5$ years (midway through the time horizon).

a framework in which firm value volatility is exogenously specified, our model endogenizes firm value volatility as a function of production flexibility and the volatility of the underlying commodity price.

The positive relation between firm leverage and production flexibility has important empirical implications. Specifically, our model predicts that the firm's average leverage ratio is higher and the leverage ratio range (i.e., the

decreases. The combination of a decreasing minimum leverage ratio and an increasing maximum leverage ratio causes the leverage ratio range to quickly rise as the commodity price increases. Eventually, however, the opportunity cost of forgoing larger interest tax shields outweighs the fixed recapitalization cost, and the minimum leverage ratio will start to increase at a faster rate than the maximum leverage ratio. Thus, the leverage ratio range will start to decrease and, at very high price levels, will asymptote to about 50 percent for the parameter inputs used to construct Figure 2.

amount by which the firm allows its leverage ratio to vary without recapitalizing) is smaller, the higher the degree of production flexibility. To our knowledge, these are new predictions in the capital structure literature.

C. The Effect of Production and Financial Flexibility on the Firm's Investment Decision

We now introduce the option to invest, i.e., we assume that the initial investment in the facility has not been made prior to $t = 0$, to investigate the effects of production and financial flexibility on the firm's investment decision. For the analysis, we continue to use the parameter values in Table I. For that case, there is considerable value associated with waiting to invest, since the initial commodity price is equal to the per unit cost of production (i.e., \$100) and the cost of investing in the production facility (I) is \$50. As such, this case should highlight possible effects of production and financial flexibility on the value of the firm's option to invest in the production facility. Of course, we discuss the effect of alternative model assumptions and parameter values on our results in Section III.

Table IV presents the effect of production flexibility on both the exercise timing and value of the firm's option to invest in the production facility. In the table, operating adjustment costs are varied from \$0 to \$20, while recapitalization costs are held constant at \$0.1 plus 1 percent of the change in the face value of debt. Examining the second column, one can see that the commodity price at which the firm will optimally invest in the production

Table IV

The Effect of Operating Adjustment Costs on the Option to Invest

The variables are defined as follows: P_I is the time-zero commodity price at which the firm will optimally invest in the production facility; V^0 is the time-zero value of the option to invest assuming an optimal investment policy; and $V^1 - I$ is the time-zero value of the firm net of the initial investment expenditure if it invests immediately, i.e., at an initial commodity price ($P(0)$) of \$100. The last column shows the loss in the value of the option to invest if the firm invests immediately. For all cases, the operating adjustment costs to shut down (K_X) and reopen (K_N) the production facility are equal, the proportional recapitalization costs to increase the debt level (κ_{V2}) and decrease the debt level (κ_{V1}) are each equal to 1 percent, and the fixed component of recapitalization costs (κ_F) is \$0.1. The initial investment in the project (I) is \$50.

Operating Adjustment Cost (\$)	P_I	V^0	$V^1 - I$	$V^0 - [V^1 - I]$
0	119.06	101.06	92.96	8.10
5	124.35	97.65	81.80	15.85
10	126.96	95.85	75.38	20.47
15	128.93	94.75	69.83	24.92
20	130.51	93.93	65.29	28.64

facility, P_I, is an increasing function of operating adjustment costs.[20] Intuitively, the larger the cost to shut down and reopen the facility, the more costly it is for the firm to mitigate the risk of adverse movements in the commodity price. As a consequence, the firm will wait for an even higher commodity price before it is willing to commit \$50 to build the facility.

The last three columns in Table IV show, respectively, the time-zero value of the option to invest, the time-zero value of the firm if it does not delay investment in the facility (i.e., it invests immediately at a commodity price of \$100) net of the \$50 investment, and the difference between these two values. The last column measures the value of waiting to invest. Observe that the value of waiting to invest increases as operating adjustment costs increase. This indicates that the option to invest and the subsequent operating options to shut down and reopen the production facility are substitutes. Notice, however, that they are not perfect substitutes—the value of waiting to invest is still worth \$8.10 when operating adjustment costs are zero. The reason is that even when the firm has perfect production flexibility it will still find it valuable to wait to invest because of the fixed cost to build the production facility.

Debt financing may also affect the timing of exercise, and hence the value of the option to invest, because the firm earns interest tax shields only when it has operating profits. Since the level of debt financing that a firm can support increases as recapitalization costs decrease, a firm with low recapitalization costs may have an incentive to invest earlier (i.e., at a lower commodity price) than a firm with high recapitalization costs. Table V explores this conjecture by examining the relation between the price at which the firm will optimally invest in the production facility and the level of recapitalization costs.[21] In the table, the proportional component of recapitalization costs is varied from zero to 2 percent, while operating adjustment costs are held constant at $K_X = K_N = \$10$. Consistent with our intuition, the firm invests earlier when it has a high degree of financial flexibility. However, the effect on investment timing is very small. In fact, as recapitalization costs rise above 1 percent, there is effectively no change in P_I.[22]

This small difference in investment timing is likely to have an insignificant effect on the value of the option to invest. To measure this effect, we compute the time-zero value of the option to invest under two investment policies: first, we use the optimal investment policy for the levered firm, $P_I^L(t)$,

[20] Recall that the commodity price at which the firm exercises its option to invest, $P_I(t)$, is a function of time. We find that $P_I(t)$ is an increasing function of operating adjustment costs at every point in time, $t \in [0, T]$. Therefore, we report this commodity price only at time zero, and we refer to it as P_I.

[21] We find that the relationship between the commodity price at which the firm invests and the level of recapitalization costs is consistent through time, and therefore report P_I values at time zero.

[22] Indeed, if the firm were unlevered, it would optimally invest in the production facility when the commodity price reaches \$127 (not reported in the table), which is only slightly higher than the price at which it invests when recapitalization costs are 2 percent, i.e., \$126.98.

Table V

The Effect of Recapitalization Costs on the Option to Invest

The variables are defined as follows: P_I is the time-zero commodity price at which the firm will optimally invest in the production facility; $V^0(P_I^L(t))$ is the time-zero value of the option to invest assuming an optimal investment policy; and $V^0(P_I^U(t))$ is the time-zero value of the option to invest if the levered firm uses the optimal investment policy of an otherwise identical unlevered firm. The last column shows the loss in the value of the levered firm's option to invest from following the unlevered investment policy. For all cases, the operating adjustment costs to shut down (K_X) and reopen (K_N) the production facility are each equal to \$10; and for all nonzero proportional recapitalization cost cases, the fixed component of recapitalization costs (κ_F) is \$0.1.

Proportional Recapitalization Cost (%)	P_I	$V^0(P_I^L(t))$	$V^0(P_I^U(t))$	$V^0(P_I^L(t)) - V^0(P_I^U(t))$
0	125.47	99.16	99.07	0.09
0.5	126.77	96.46	96.44	0.02
1.0	126.96	95.85	95.84	0.01
1.5	126.98	95.51	95.51	0.00
2.0	126.98	95.33	95.33	0.00

$t \in [0, T]$, and second, we assume that the levered firm employs the unlevered firm's optimal investment policy, $P_I^U(t)$, $t \in [0, T]$. The latter policy is suboptimal for the levered firm since it delays investment too long. The last three columns in Table V report the values under these two scenarios ($V^0(P_I^L(t))$ and $V^0(P_I^U(t))$, respectively) and their difference. Observe that the loss in option value from following the "unlevered exercise policy" is extremely small and is essentially zero at anything less than perfect financial flexibility.

This result may seem surprising in light of the commonly held belief—drawn from static models in corporate finance—that tax-advantaged debt financing has a significant effect on investment decisions. However, in the "now or never" investment decision of a static model the present value of interest tax shields over the entire life of the investment project bear on the investment decision. By contrast, in a dynamic setting the firm foregoes earning interest tax shields only over the period of time that it chooses to delay the investment.

Furthermore, as we find, the value of interest tax shields is likely to be a rather modest fraction of total firm value. Here again, static models may be somewhat misleading in that they likely overestimate the value of interest tax shields. While our dynamic model allows the firm to recapitalize over time, it must also remain solvent at each point in time. By contrast, in a static model, solvency is only checked at a single future point in time to determine whether the firm is bankrupt. Everything else remaining the same, this allows the firm to choose a higher level of debt. Given the low, but empirically reasonable, levels of debt observed in our model, and, to a lesser extent, the absence of loss offset provisions, the value of interest tax shields is relatively small. Thus, it is not surprising that a levered firm does not invest significantly earlier than an equivalent unlevered firm to start earning interest tax shields when it is still valuable to wait for additional uncertainty resolution to reduce exposure to commodity price variation.

The lack of an economically significant effect of debt financing on the investment decision has an important practical implication for capital budgeting decisions. Specifically, it suggests that a levered firm may make its investment timing decisions ignoring the influence of debt financing, thus greatly simplifying the investment decision.

III. Robustness of Results

We now discuss the robustness of our results to alternative model assumptions and parameter values. For brevity, we do not provide additional tables and figures, but instead simply report whether any differences arise under alternative scenarios. We also briefly discuss the likely effects of altering our underlying assumption of firm value maximization.

An increase in the commodity price volatility (from 20 to 30 percent) decreases the firm's average leverage ratio, increases the leverage ratio range, and accentuates the impact of operating adjustment costs on these capital structure statistics and net tax shield values. For example, the increase in net tax shield value as operating adjustment costs decrease is more pronounced at a higher commodity price volatility. This is not surprising since the values of the firm's real options increase as volatility increases, which enhances their effect on the firm's financial policy. However, despite an increase in net tax shield value as commodity price volatility increases, there is still a negligible impact of financial policy on the firm's investment and operating policies. There are two reasons for this result. First, since operating firm value also increases as volatility increases, the higher net tax shield values are still a relatively small fraction of total firm value. Second, as volatility increases, the value of waiting for uncertainty resolution about the output price increases, which increases the opportunity cost to a levered firm from significantly deviating from the first-best investment and operating policies of an equivalent unlevered firm.

In the simulations, the initial commodity price is set equal to the marginal cost of producing one unit of the commodity (i.e., $100). An increase in the initial commodity price above that level results in higher firm values, higher leverage ratios and more valuable net tax shields. However, as the initial commodity price increases, we quickly get outside of cases where interaction effects matter, since the operating option to shut down the facility is further out-of-the-money and the value of waiting to invest decreases. Thus, as the initial commodity price increases, the positive effect of production flexibility on net tax shield value diminishes, and the effects of production and financial flexibility on the firm's investment timing decision are less pronounced than those that we report in Tables IV and V.

The convenience yield of the commodity affects the drift of the commodity price over time. For convenience yields smaller than the 1 percent that we assume in the simulations, the upward drift in the price is larger, resulting in higher average commodity price levels. As a consequence, the firm's operating options are less valuable, and the effect of production flexibility on financial policy and net tax shield value is less pronounced.

Our choice of an initial commodity price equal to the per unit production cost, coupled with the assumption that the firm has no assets other than its production facility, results in a fairly high initial volatility of firm value (see footnote 14). However, average firm value volatility decreases over time since the commodity price has a positive drift and firm value volatility is a decreasing function of the commodity price. Firm value volatility is also lower if the firm holds other assets that are less risky than the production facility. We investigate the effect of allowing the firm to hold riskless assets (with initial value $100) in addition to its production facility. While this results in significantly lower firm value volatility, it does not change the qualitative nature of our results.

While we assume an asymmetric tax system in which interest tax shields are lost if operating profits are negative, in practice firms may continue to earn at least part of these tax shields through carryback and carryforward provisions. Although including such provisions in our model would be interesting, solving the model would be much more difficult. We have, however, rerun all of our simulations assuming a symmetric tax system in which the firm always earns its interest tax shields. Not surprisingly, we find that net tax shield values are larger under a symmetric tax system, and that operating adjustment costs have an even more pronounced effect on net tax shield values. In addition, while the average leverage ratio is larger and the leverage ratio range is smaller under symmetric taxes, the effects of operating adjustment costs on these capital structure statistics are virtually identical to those displayed in Figures 1 and 2. Finally, the firm will invest in the production facility at a lower commodity price in the symmetric tax case than in the asymmetric tax case due to the enhanced value of interest tax shields. However, for any combination of operating adjustment and recapitalization costs, the difference in the optimal investment commodity prices for these two tax cases is always less than $1.00. More importantly, the effects of operating adjustment and recapitalization costs on the value of the option to invest are very similar to those reported in Tables IV and V.[23]

Although our results are robust to alternative assumptions and parameter inputs, we do not investigate the significance of our assumption of firm value maximization. Indeed, Brennan and Schwartz (1984), Fischer, Heinkel, and Zechner (1989b), and Mello and Parsons (1992) show that, under equity value

[23]Moving to a symmetric tax system does not change the result that financial policy has virtually no effect on the value of operating profits. The reason, of course, is that the firm continues to earn interest tax shields regardless of whether it is in or out of business. Therefore, interaction effects between financial policy and operating policy are least likely to be observed under symmetric taxes.

maximization, firms may make dynamic operating and financing decisions that are different from first-best policies under firm value maximization. For example, if we assume equity value maximization, debt financing may distort the firm's initial investment decision. However, it is not clear whether financial flexibility will have any more significant of an impact on this decision than in the first-best case we examine. In addition, while managers may make different recapitalization decisions than in the first-best case, we suspect that the presence of production flexibility in the firm will still increase its debt capacity. Nevertheless, allowing for agency problems under equity value maximization is an interesting avenue for future research.

IV. Conclusions

In this article we examine the interaction between dynamic capital budgeting decisions and dynamic capital structure decisions in a model where the firm has an initial investment option, operating options, and the ability to recapitalize over time. By varying operating adjustment and recapitalization costs, we isolate the effects of production flexibility and financial flexibility on the firm's investment, operating and financing policies. The analysis involves solving an optimal control problem with the objective of maximizing firm value. We calculate the levered firm's value using an adjusted present value approach, where the value of interest tax shields augments the net present value of the operating cash flow stream. However, unlike the standard static adjusted present value approach, in our model the debt level is changing over time and the discount rate for the operating cash flows is stochastic, due to the presence of operating options.

We find that production flexibility has a significant effect on financing decisions. Specifically, the firm's average leverage ratio is higher and the range over which the firm allows its leverage ratio to vary is narrower the smaller are operating adjustment costs. As a result, the present value of interest tax shields net of recapitalization costs increases as operating adjustment costs decrease. However, the increase in net tax shield value is smaller the lower are recapitalization costs. Intuitively, when the costs to dynamically manage capital structure are small, the hedging benefit of production flexibility has less of an impact on the net tax shield value of debt financing. This implies that production flexibility and financial flexibility are substitutes, albeit not perfect substitutes.

In contrast, we find that financial policy has a minimal effect on the firm's initial investment decision and subsequent operating decisions. Indeed, our analysis shows that if a levered firm uses the investment and operating policies of an equivalent unlevered firm, there is a negligible loss in firm value. These results are in sharp contrast to traditional static capital budgeting analyses that conclude that tax-advantaged debt financing can make the difference between project acceptance or rejection. In a static setting, however, the value of interest tax shields over the entire life of an investment project bear on the investment decision because the firm does not have the option to delay investment. By contrast, in a dynamic setting the firm foregoes earning interest tax shields only over the period of time that it chooses to delay the investment. Our analysis shows that the loss in tax shield value from waiting for additional uncertainty resolution is not large enough to encourage the firm to significantly deviate from the investment and operating policies of an equivalent unlevered firm.

Appendix

Numerical Solution Technique

We use a procedure based on the explicit finite difference method to approximate the solution of the problem in equations (2) to (12). First, we perform a logarithmic transformation of the differential equations (2), (3), and (10), using the logarithm of the output price, $p = \ln P$. As a result, these parabolic partial differential equations have constant coefficients, ensuring that the weights that we use in the explicit finite difference method are constant throughout the lattice. We use a three-dimensional lattice of nodes in the valuation procedure. On the first dimension, there are N_t equal time intervals of length $\Delta t = T/N_t$. On the second dimension, there are N_p logarithmic price levels spaced Δp apart. On the third dimension, there are $2N_b + 1$ nodes: for each time-price pair, there is one node for the case where the firm has yet to invest, N_b nodes for cases where the firm is operating and has one of N_b possible debt levels (including no debt), and N_b nodes representing debt level possibilities when the firm has suspended operation.

At time T, the firm values at the $(N_p)(2N_b + 1)$ nodes are given by the salvage value conditions (9) and (12). Working backward in time, i.e., starting at time $T - \Delta t$, for each of the N_p prices at time t we first calculate firm values at the $2N_b + 1$ debt level nodes assuming that the firm leaves its operating status and debt level unchanged over the next time step $(t, t + \Delta t)$. Following the explicit finite difference technique, firm values at time t and price P are computed by discounting the weighted average of firm values at time $t + \Delta t$, plus any cash flows realized at the end of the period, at three possible price outcomes, $Pe^{-\Delta p}$, P, and $Pe^{\Delta p}$. Thus, the discrete analogs of equations (10), (2), and (3), respectively, are

$$\begin{aligned} V^0(P,0,t) = e^{-(r-\delta)\Delta t}[&aV^0(Pe^{-\Delta p},0,t+\Delta t) \\ &+bV^0(P,0,t+\Delta t) + cV^0(Pe^{\Delta p},0,t+\Delta t)], \end{aligned} \tag{A1}$$

$$\begin{aligned} V^1(P,B,t) = e^{-(r-\delta)\Delta t}[&a\{V^1(Pe^{-\Delta p},B,t+\Delta t) \\ &+\{(Pe^{-\Delta p} - C) - (\tau)\max[0, Pe^{-\Delta p} - C - \rho B]\}\Delta t\} \\ &+b\{V^1(P,B,t+\Delta t) \\ &+\{(P - C) - (\tau)\max[0, P - C - \rho B]\}\Delta t\} \\ &+c\{V^1(Pe^{\Delta p},B,t+\Delta t) \\ &+\{(Pe^{\Delta p} - C) - (\tau)\max[0, Pe^{\Delta p} - C - \rho B]\}\Delta t\}], \end{aligned} \tag{A2}$$

$$V^2(P,B,t) = e^{-(r-\delta)\Delta t}[aV^2(Pe^{-\Delta p},B,t+\Delta t) \\ +bV^2(P,B,t+\Delta t) + cV^2(Pe^{\Delta p},B,t+\Delta t)], \quad \text{(A3)}$$

where the weights are

$$a = \frac{1}{2}\left[\left(\frac{\sigma}{\Delta p}\right)^2 - \frac{\left(r-\delta-\frac{1}{2}\sigma^2\right)}{\Delta p}\right]\Delta t;$$

$$b = 1 - \left(\frac{\sigma}{\Delta p}\right)^2 \Delta t; c = \frac{1}{2}\left[\left(\frac{\sigma}{\Delta p}\right)^2 + \frac{\left(r-\delta-\frac{1}{2}\sigma^2\right)}{\Delta p}\right]\Delta t.$$

To ensure stability and convergence of the solution technique, the time step, $\Delta t = 1/12$, and the logarithmic price step, $\Delta p = 0.06$, are chosen for the simulations so that the weights a, b, and c are positive given the values for r, δ, and σ (see Table I). Discretization of debt levels in our problem poses no dangers for stability. As the number of debt levels (N_b) increases, so does firm value, reflecting the larger number of debt level possibilities available to the firm. However, the marginal value of making an additional debt level available decreases very rapidly once there are already many levels in the lattice. We use $N_b = 250$ in the simulations.

Once the calculations in equations (A1) to (A3) are performed for each price at time t, boundary conditions (4) to (7) and (11)—and implicitly their corresponding smooth-pasting conditions—are then applied at each of the $2N_b + 1$ nodes at each price level, P, to determine whether an operating status and/or debt level change is warranted.[24] If at time t and price P the firm has operating status i and debt level B, we check to see whether it is optimal for the firm to switch to a different operating status, i', and/or debt level, B'. The firm will switch if:

$$\max_{i',B'}\left[V^{i'}(P,B',t) - \{K_N\text{Max}[0,i-i'] + K_X\text{Max}[0,i'-i]\} \\ -\{\kappa_F[1-J(B-B')] + \kappa_{V1}\text{Max}[0,B-B'] \\ +\kappa_{V2}\text{Max}[0,B'-B]\}\right] > V^i(P,B,t), i = 1,2, \quad \text{(A4)}$$

where the indicator function $J(B-B')$ is equal to one if $B = B'$ and zero otherwise, and all firm values, V^i, are as calculated in equations (A1) to (A3). If it is optimal to switch, then time t firm value is equal to the maximum value obtained on the left-hand side of equation (A4). A similar procedure is then followed for the initial investment decision, i.e., switching from $i = 0$ to $i = 1$. The firm will invest (and choose an optimal debt level $B \geq 0$) if:

$$\max_{B}\left[V^1(P,B,t) - I - \{\kappa_F[1-J(B)] + \kappa_{V2}B\}\right] > V^0(P,t), \quad \text{(A5)}$$

where the indicator function, $J(B)$, is equal to one if $B = 0$, and is zero otherwise. If the firm invests, then $V^0(P,t)$ is set equal to the maximum

[24]At each node with a nonzero debt level, we first check whether the firm is solvent, i.e., whether firm value exceeds the debt level. If it is not solvent, financial restructuring occurs according to equation (8).

value obtained on the left-hand side of equation (A5). Once all optimal recapitalizations and operating status changes are performed for each price at time t, the entire valuation procedure starting with equations (A1) to (A3) is repeated at times $t - q\,\Delta t,\ q = 1, \dots, t/\Delta t$.

We record the operating status and debt level changes in a "switch matrix." $SWITCH[i, B; P, t] = \{i', B'\}$ denotes that for price P at time t the firm optimally switches from operating state i to i' and from debt level B to B'. After the valuation procedure is complete, we use the *SWITCH* matrix to trace possible price paths forward in time (conditional on an initial price) to determine all possible path-dependent debt level and operating status configurations at a point in the horizon. We use the *SWITCH* matrix to compute leverage ratio ranges displayed in Figure 2, and using the weights a, b, and c to calculate price path probabilities, to compute average leverage ratios displayed in Figure 1. We also use the *SWITCH* matrix in Table V, where we force a levered firm to use the unlevered firm's optimal investment decision, i.e., operating status switch from $i = 0$ to $i = 1$.

We also separately compute the present value of interest tax shields net of recapitalization costs.[25] Analogous to the calculations for overall levered firm value in equations (A2) and (A3), the present values of net tax shields at time t and price P when the firm is operating and shut down, respectively, are

$$\begin{aligned} TS^1(P, B, t) = e^{-(r-\delta)\Delta t}[\, & a\{TS^1(Pe^{-\Delta p}, B, t + \Delta t) \\ & + \tau\,\mathrm{Max}[0, \mathrm{Min}(\rho B, Pe^{-\Delta p} - C)]\} \\ & + b\{TS^1(P, B, t + \Delta t) + \tau\,\mathrm{Max}[0, \mathrm{Min}(\rho B, P - C)]\} \\ & + c\{TS^1(Pe^{\Delta p}, B, t + \Delta t) + \tau\,\mathrm{Max}[0, \mathrm{Min}(\rho B, Pe^{\Delta p} - C)]\}] \end{aligned} \tag{A6}$$

$$\begin{aligned} TS^2(P, B, t) = e^{-(r-\delta)\Delta t}[\, & aTS^2(Pe^{-\Delta p}, B, t + \Delta t) \\ & + bTS^2(P, B, t + \Delta t) + cTS^2(Pe^{\Delta p}, B, t + \Delta t)] \end{aligned} \tag{A7}$$

where the weights a, b, and c are defined below equation (A3). If, according to equation (A4), the firm changes its operating status and/or debt level, net tax shield value is changed to

$$\begin{aligned} TS^i(P, B, t) = TS^{i'}(P, B', t) - \{ & \kappa_F[1 - J(B - B')] \\ & + \kappa_{V1}\mathrm{Max}[0, B - B'] + \kappa_{V2}\mathrm{Max}[0, B' - B]\},\ i = 1, 2. \end{aligned} \tag{A8}$$

Working backward in time using the operating status and debt level changes determined in equations (A4) and (A5), we are able to compute the time-zero present value of interest tax shields net of recapitalization costs. Table III reports time zero net tax shield values for the case where the firm is operating at that time, i.e. $TS^1(P, B, 0)$.

[25] In our simulations, the present value of financial distress costs is always equal to zero.

Finally, the volatility of firm value at a given commodity price, debt level and time is calculated as

$$\sigma_V(P, B, t) = \sigma \Omega_V(P, B, t), \tag{A9}$$

where σ is the volatility of the commodity price and $\Omega_V(P, B, t)$ is the elasticity of firm value with respect to price. The elasticity is computed as

$$\Omega_V(P, B, t) = \frac{[V^1(Pe^{\Delta p}, B, t) - V^1(Pe^{-\Delta p}, B, t)](P)}{[Pe^{\Delta p} - Pe^{-\Delta p}]V^1(P, B, t)}. \tag{A10}$$

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