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Source: *The Journal of Finance*, Vol. 49, No. 4 (Sep., 1994), pp. 1163-1211
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2329183
Accessed: 25/11/2009 10:39

THE JOURNAL OF FINANCE • VOL. XLIX, NO. 4 • SEPTEMBER 1994

A Theory of the Dynamics of Security Returns around Market Closures

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ABSTRACT

Numerous empirical studies document patterns in the means and variances of security returns measured over periods that are punctuated by market closures. This article develops a multiperiod model in which closures delay the resolution of uncertainty, thereby redistributing risk across time and agents. Since agents are risk averse in the model, this redistribution affects the equilibrium price, altering risk premia, liquidity costs, and the degree of informational asymmetry. As a consequence, closures alter both the means and variances of returns. The article demonstrates that closures can generate a variety of mean and variance effects, including those that mirror the empirical phenomena.

NUMEROUS EMPIRICAL STUDIES OF security returns have documented patterns in both the mean and the variance of returns. Interestingly, most of these patterns are punctuated by periods in which the market is regularly closed. For example, the variance of returns from the market close to the market opening is much less than the variance from open to close. Similarly, mean returns over weekends are abnormally low (in fact, significantly negative) relative to daily mean returns during the trading week.[1] Within a trading day, return variances are U-shaped with peaks occurring immediately after the market opens and just before it closes.[2] Further, French and Roll (1986) examine the variance of Tuesday-close to Thursday-close returns during a period in which the market was closed on Wednesdays and show that this variance is only slightly greater than half the Tuesday-close to Thursday-close variance when the market was open on Wednesdays.[3]

One explanation for the differences in the overnight and trading day variances is that market hours mostly coincide with business hours when

*University of Michigan. The author would like to thank the following individuals for their helpful comments: Alex Kane, Gautam Kaul, Naveen Khanna, John M. Marshall, Greg Niehaus, M. P. Narayanan, Michael Rothschild, Maxwell Stinchcombe, René Stulz (the editor), and an anonymous referee. I would also like to thank David Heike for research assistance. Obviously, any errors are the responsibility of the author.

[1]See, for example, French (1980), Gibbons and Hess (1981), Keim and Stambaugh (1984), Rogalski (1984), Wood, McInish, and Ord (1985), Terry (1986), Smirlock and Starks (1986), Harris (1986).

[2]See for example, Wood, McInish, and Ord (1985).

[3]The Wednesday closures occurred in the latter half of 1968 and were scheduled to allow the exchange to "catch up" on bookkeeping tasks. Except for the market closing, these Wednesdays were normal business days. The closures were fully anticipated and did not occur due to rare events such as extreme market stress, national holidays, or national crisis.

more events occur and/or information collection is less costly. The low Wednesday-closure variances, however, indicate that variance is more closely related to market hours than business hours, providing evidence that markets create fluctuations in prices that are independent of the arrival of information. One explanation for this phenomenon is that when markets are open, speculators' "animal spirits" are unleashed, creating either irrational price movements or speculative phenomena like bubbles. This explanation is unsatisfying, however, since it is hard to imagine "animal spirits" systematically varying in such a way as to create the U-shaped intraday variance pattern or the interday mean return patterns.

Recent variations of Kyle's (1985) monopolist insider trading model have generated some of these phenomena without relying on irrationality and/or bubbles by introducing strategic discretionary liquidity trading. For example, Admati and Pfleiderer (1988) provide one model that can generate U-shaped patterns in variance (and volume), while Admati and Pfleiderer (1989) present another model that implies mean return patterns. In both models, patterns occur when agents optimally concentrate trading in specific periods to minimize adverse selection costs. Although the predictions of these models are consistent with patterns in asset returns measured over periods that are punctuated by closures (since the periods in which agents choose to concentrate trade are arbitrary), these models do not provide any insights into why closures (as opposed to other periods) invariably punctuate the patterns. Foster and Viswanathan (1990), however, do examine a model in which weekend closures allow private information to accumulate. Although their model generates weekend variances similar to those in French and Roll (1986), it does not provide any mean return patterns.

In this article, we present an asymmetric information model that is devoid of irrationality, bubbles, and strategic liquidity trading but generates both mean and Variance phenomena that are consistent with the empirical regularities. Specifically, the article develops a multiperiod generalization of Grossman and Stiglitz (1980) in which informed agents receive private information about the future payoff of an asset in every period, regardless of whether the market is open or closed, and uninformed agents can only imperfectly infer the private information from market prices, which, due to random liquidity trades, are noisy. As in Foster and Viswanathan (1990), a closure allows private information to accumulate, altering the amount of adverse selection in the market and, thus, the equilibrium price process.

The critical difference between our model and the previously cited models is that we have risk-averse agents.[4] Risk aversion is important for three reasons. First, it allows for simultaneous variations in mean returns (risk

[4]The articles by Grundy and McNichols (1989) and Brown and Jennings (1989) consider a two-period model with a single risky asset and diffusely informed risk-averse agents. Although each trader receives a unique signal, all signals are drawn from the same distribution and, as such, no set of traders has superior (i.e., higher precision) information and no adverse selection problem exists. As is shown below, apart from risk aversion, the existence of adverse selection is needed to get the types of phenomena observed.

premia) and variances over time.[5] Second, since the risks associated with imperfect inference are priced, this model generates more complicated price dynamics than can exist in risk-neutral asymmetric information models. In fact, the model generates behavior that is consistent with the empirical observations, without relying on strategic liquidity trading (as in Foster and Viswanathan, 1990). Third, since risk aversion and uncertainty about the terminal payoff endogenously limit the positions agents take, we can obtain results for infinite agent economies in which all agents are price takers. Indeed, the article shows that the widely documented phenomena can occur in competitive environments devoid of non-price-taking behavior.

In the model, a closure alters investor uncertainty in the following ways. First, a closure changes the timing of the resolution of uncertainty, shifting risk across periods and across agents. We assume that uninformed agents can observe only the current market-clearing price and the private information possessed by informed agents the last time the market was open. Thus, after a closure, uninformed agents must infer two pieces of news from the single postclosure price. If the market had been open, uninformed agents would have learned the news received by informed agents during the closure, and, thus, they would only have to infer one piece of news. A closure therefore delays the release of closure news, subjecting uninformed agents to greater risk from adverse selection in the postclosure market.

Second, since any position taken in the preclosure market must be held over a period in which two pieces of news arrive, the risk faced by both informed and uninformed agents is greater prior to a closure. The risk faced by informed agents, however, increases proportionally more than the risk faced by uninformed agents. This is true since uninformed risk is generated from two sources—the arrival of future news and imperfect inference of current news—while informed risk generates from only the arrival of future news. Since a closure increases the variance of future news, but not the variance of current news, informed agents' risk increases proportionally more. As a consequence, some of the comparative advantage at risk bearing shifts from informed to uninformed agents, generating a third effect on risk. Given the shift in comparative advantage, additional trade based on risk-sharing motives occurs. Since these trades must take place in the presence of adverse selection and liquidity trading, which keep the price temporarily away from the fundamental value, a closure introduces an additional source of risk, referred to as insurance-trade risk.

The changes in risk induced by closures affect the variance of returns by altering the amount of private information that is reflected in price. A closure alters the informativeness of price in three ways. First, postclosure prices reflect a greater portion of the private information obtained on the day that the market reopens. This effect increases the return variance over a closure as the postclosure price moves more to reflect this information. Second, preclosure prices reflect less preclosure news. Since a smaller portion of the

[5]Also see LeRoy (1973).

preclosure news is reflected in the preclosure price, there is a larger impact on the postclosure price when this news becomes public information after the closure. Thus, this effect also increases the return variance over the closure. The final impact of the closure on the revelation of private information is that postclosure prices reflect less of the private information received during the closure. If the market had been open, the closure news would have become public and thus fully reflected in the postclosure price. But, because the closure news remains private following a closure, this news is only partially reflected in the postclosure price, and the component of the return variance that is due to the closure news falls. In total, whether the variance over the closure increases or decreases depends on the parameters of the model. When there are relatively few informed agents, the final effect dominates and a comparison of the closure and no-closure return variances is consistent with the phenomena documented in French and Roll (1986).

Closures also affect the variance of returns by altering both the pre- and postclosure liquidity costs. Specifically, postclosure liquidity costs are unambiguously higher because increased adverse selection causes uninformed agents to want to provide less liquidity; to satisfy liquidity demand, liquidity costs must increase. This effect increases the variance of returns. The effect of the closure on preclosure liquidity costs is ambiguous, however, because, depending upon the degree to which price reflects preclosure news, the risk associated with providing liquidity may increase or decrease. When preclosure prices reflect much less preclosure news than when the market does not close, more of the unexpected change in the preclosure price is due to liquidity shocks, and uninformed agents end up trading against informed agents less often. As a result, the cost of providing liquidity falls, thus reducing the return variance.

Both the mean level of prices and mean returns are also affected by closure. Specifically, the mean level of the price is lower both before and after the closure. This occurs because risk is greater on both sides of a closure; the risk associated with uninformed agents' imperfect inference increases following closures and insurance-trade risk occurs both before and after closures. Mean returns, however, depend upon the change in the level of risk, which, depending upon the percentage of informed agents in the market, can be either more or less than occurs in the no-closure case. When there are few informed agents, prices reflect very little current information and closures shift most of the risk associated with closure news to the postclosure market, thereby significantly increasing the uncertainty in the postclosure market and reducing the amount of uncertainty resolved over the closure. Consequently, mean returns over closures are smaller than they would be if the market had not closed, a result which is consistent with the "weekend effect."

Although this study examines the effect of closures, the model can be used to analyze the resulting price dynamics of a wide variety of underlying news and liquidity shock processes; a closure is only one example of an exogenous event that induces a change in the underlying processes. The closure results illustrate that changes in the underlying processes can lead to complicated price dynamics. These dynamics may or may not reflect expected relationships. For example, in a single risky asset economy, we would expect increases in variance to require greater mean returns. However, in our model, variance increases can be associated with decreases in mean return. These results complement the results in the static multiasset model of Admati (1985), where counterintuitive price behavior results from complicated cross-sectional interactions among assets. Furthermore, although the article only analyzes fully anticipated exogenously imposed closures, the results on the price behavior following closures also extend to endogenous closures due to circuit breakers and trading halts. The predictions of the model are consistent with recent empirical studies of price behavior around these types of events.[6]

The article is organized as follows. Section I describes the model. Section II defines and derives the equilibrium. A reader who is not interested in the technical details of the equilibrium can safely skip this section. Section III discusses the impact of closures by comparing the price processes that obtain in two cases: (1) the nonclosure case, where the market meets at regular intervals and the variance of news and liquidity shocks are constant on each trade date; and (2) the closure case, where the variance of news is larger following a closure. This section also discusses the implications of closure for volume. Section IV presents concluding comments.

I. The Model

The model consists of a market for two assets, one that is riskless and another that is risky. The riskless asset, M, is the numeraire which, without loss of generality, has a gross return of one. The risky asset, X, pays a single liquidation dividend of P_T per share in the terminal period, T. There are Q shares of the risky asset, each of which can be traded in a secondary market. The secondary market is open for an instant on $T - 1$ discrete dates (indexed $t = 1$ to $T - 1$), at which time a rational expectation price, P_t, clears the market.

In this model, there are $\hat{N}_I$ informed agents and $\hat{N}_U$ uninformed agents who trade in the secondary market but consume only in the terminal period T. Informed and uninformed agents are identical except for the information they receive. Both types of agents maximize expected negative exponential utility of terminal wealth. At every trading date, t, each agent of type $i (i = I$ for informed and U for uninformed) chooses a portfolio, (X_t^i, M_t^i), to solve:

$$\underset{\{X_t^i, M_t^i\}}{\text{Max}} E_t^i\left[U(W_T^i)\right] = E_t^i\left[-\exp(-\gamma W_T^i)\right]$$

$$\text{S.T.} \qquad W_t^i = X_{t-1}^i P_t + M_{t-1}^i = X_t^i P_t + M_t^i, \tag{1}$$

[6]For example, Lee, Ready, and Seguin (1994) shows that volume and volatility increase following New York Stock Exchange trading halts. Increased volatility is consistent with higher postclosure liquidity costs and increased volume is consistent with the extra risk-sharing-motivated trades induced by shifts in the comparitive advantage at bearing risk.

where γ is the coefficient of absolute risk aversion, $E_t^i[\cdot]$ is the expectation operator given agent i's information at time t, and W_T^i is agent i's terminal wealth, which equals the period-T value of the portfolio obtained at $T-1$ (i.e., $W_T^i = X_{T-1}^i P_T + M_{T-1}^i$). The constraint requires that the portfolio purchased at $t, (X_t^i, M_t^i)$, be affordable given the disposable wealth in period t, $X_{t-1}^i P_t + M_{t-1}^i$. Since we assume that there are no short sale restrictions or borrowing constraints, X_t^i and M_t^i may be negative.[7]

In addition to informed and uninformed agents, there are also $\hat{N}_L$ liquidity traders who trade in the secondary market. These traders are consumers/savers who demand (or supply) random amounts of liquidity each period. Specifically, at date t, the aggregate holdings of consumers/savers are $\hat{N}_L X_t^L = \hat{N}_L(\overline{X}_L + \omega_t)$, where $\overline{X}_L$ is a constant and ω_t is a mean zero normal random variable that is independent of information. This specification implies that the aggregate holdings of consumers/savers are stationary; as one subset of consumers/savers liquidates (or increases their holdings), another subset enters (leaves) the market and buys (sells) these shares. We assume that entry is not instantaneous, however, and that informed and uninformed agents will have to temporarily hold (or provide) the imbalance, ω_t.[8]

Given these three groups of agents, the market-clearing condition is $\hat{N}_I X_t^I + \hat{N}_U X_t^U + \hat{N}_L X_t^L = Q$. To examine price determination in a competitive setting, we consider an infinite agent economy (i.e., we let $\hat{N}_I$, $\hat{N}_U$, and $\hat{N}_L$ go to infinity) where all agents are price takers. In such economies, only the proportion of each type of agent affects the equilibrium price. Simple normalizations allow us to rewrite the market-clearing condition as the following function of these proportions:

$$N_I X_t^I + N_U X_t^U = \overline{X} + w_t, \tag{2}$$

where $N_I = \hat{N}_I/(\hat{N}_I + \hat{N}_U)$, $N_U = \hat{N}_U/(\hat{N}_I + \hat{N}_U)$, $\overline{X} = (Q - \hat{N}_L \overline{X}_L)/(\hat{N}_I + \hat{N}_U)$ and $w_t = -(\hat{N}_L/(\hat{N}_I + \hat{N}_U))\omega_t$. That is, $\overline{X}$ is the mean amount of the asset that must be held by informed and uninformed agents, and w_t is the temporary amount that informed and uninformed agents must hold to accommodate the temporary demand shocks of the liquidity traders. Note that when w_t is positive, liquidity traders are selling the asset and are demanding liquidity. w_t will be referred to as the liquidity shock at trade date t. We assume that $\overline{X} \geq 0$ and that the variance of w_t, denoted by $\sigma_{w_t}^2$, is finite and positive.

The demands of informed and uninformed agents depend upon the flow of information about the terminal value, P_T, which is modeled as follows:

$$P_T = \theta_0 + n_1 + n_2 + \cdots + n_T, \tag{3}$$

[7]The possibility of bankruptcy is ignored for simplicity.

[8]This specification differs from that in Brown and Jennings (1989), because in their model a liquidity shock permanently affects the future supply, while in our model the future mean supply is fixed at $\overline{X}$ regardless of the series of previous liquidity shocks.

where n_t, $t = 1, 2, \ldots, T$, is the new information received by informed agents before trading starts on the tth trade date and θ_0 is the signal of terminal value available to the informed agents prior to the beginning of the economy. The sequence of news, n_t $(t = 1, 2, 3 \cdots T)$, is drawn from independent normal distributions with mean zero and variance $\sigma_{n_t}^2$. In contrast to informed agents, uninformed agents only learn n_t after the tth trading period ends and must infer the current news, n_t, from the current price, P_t. Unobservable liquidity shocks that affect price make their inference imperfect, however.

The variance of news is subscripted t because private information flows at a steady rate independent of whether or not the market is open. Consider the time line of calendar dates, c, as depicted in Figure 1. At each calendar date, informed agents receive private information—a "piece" of news, η_c. The private information held by informed agents at a trade date t, n_t, is the sum of the pieces of news received on all calendar dates since the last time the market was open. For example, if the market had been closed on the previous calendar date only, then $n_t = \eta_c + \eta_{c-1}$, where trade date t and calendar date c are at the same point on the time line. When the variance of η_c is constant (say σ_η^2) for all c, the variance of the news known by informed agents after a closure is thus $\sigma_{n_t}^2 = 2\sigma_\eta^2$. When the market is open on each calendar date, however, $n_t = \eta_c$ and $\sigma_{n_t}^2 = \sigma_\eta^2$ for all t.

To get closed-form solutions for inferred values of news and equilibrium prices, we assume a linear price function of the following form:

$$P_t = \theta_{t-1} - \pi_0^t + \pi_1^t n_t - \pi_2^t w_t, \tag{4}$$

where the coefficients are indexed t since the price function may differ across periods. In the price function, π_0^t represents the risk discount; the greater the uncertainty, the larger the discount and the lower the mean level of price. The percentage of the current news that is reflected in the current price is given by π_1^t; when $\pi_1^t = 1$, P_t fully reflects n_t. Finally, π_2^t is the cost of liquidity in the market. When w_t is positive, liquidity traders sell shares, adding to the supply that must be held by informed and uninformed agents. Since prices must drop when supply is high, a sale by a liquidity trader depresses the price, imposing (on them) a liquidity cost equal to the amount by which their trades lower price. Integrating over all of the possible realizations of news, the expected liquidity cost per unit of w_t is π_2^t. Since π_1^t and π_2^t determine the response of price to news and liquidity shocks, they will sometimes be referred to as response rates.

Given this price function, the difference between prices in two adjacent trading dates, t and $t + 1$, can be broken into five components.

$$\Delta P_{t+1} \equiv P_{t+1} - P_t = \Delta^t + \pi_1^{t+1} n_{t+1} - (1 - \pi_1^t) n_t - \pi_2^{t+1} w_{t+1} + \pi_2^t w_t. \tag{5}$$

The first term on the right-hand side, $\Delta^t \equiv (\pi_0^t - \pi_0^{t+1})$, is the difference in the risk discount from $t + 1$ to t. As uncertainty is resolved, the risk discount gets smaller and the level of the price increases, generating positive mean returns (i.e., $\Delta^t > 0$). The second component of the price change, $\pi_1^{t+1} n_{t+1}$, is

No Closure

Trade dates	τ	$\tau+1$	$\tau+2$	$\tau+3$
	|—	—|—	—|—	—|—>time
Calendar dates	c	c+1	c+2	c+3
news "pieces"	η_c	η_{c+1}	η_{c+2}	η_{c+3}
news (n_t)	$n_\tau=\eta_c$	$n_{\tau+1}=\eta_{c+1}$	$n_{\tau+2}=\eta_{c+2}$	$n_{\tau+3}=\eta_{c+3}$
Var(n_t)	σ_η^2	σ_η^2	σ_η^2	σ_η^2

Closure at Calendar Date c+1

Trade dates	τ		$\tau+1$	$\tau+2$
	|—	—|—	—|—	—|—>time
Calendar dates	c	c+1	c+2	c+3
news "pieces"	η_c	η_{c+1}	η_{c+2}	η_{c+3}
news (n_t)	$n_\tau=\eta_c$		$n_{\tau+1}=\eta_{c+1}+\eta_{c+2}$	$n_{\tau+2}=\eta_{c+3}$
Var(n_t)	σ_η^2		$2\sigma_\eta^2$	σ_η^2

Figure 1. A Schematic for the arrival of news: Closure versus no closure. Regardless of whether the market is open or not, informed agents learn a "piece" of news, η_i, at the start of each calendar date i ($i = c, c+1, c+2, c+3, \ldots$). The variance of each "piece" of news is constant across all calendar dates and is denoted σ_η^2. The total news known by informed agents at each trade date $t(t = \tau, \tau+1, \tau+2, \tau+3, \ldots)$ is the accumulation (i.e., the sum) of all of the "pieces" of news received since the last trade date. This sum is denoted n_t, for $t = \tau$, $\tau+1$, $\tau+2$, $\tau+3, \ldots$. Thus, the news known by informed agents on trade date $\tau+1$ (calendar date $c+2$), which follows a closure on calendar date $c+1$, is $n_{\tau+1} = \eta_{c+1} + \eta_{c+2}$. Since each "piece" of news is independent, the variance of $n_{\tau+1}$ is $2\sigma_\eta^2$. If the market had been open on calendar date $c+1$, the news on trade date $t+1$ would consist solely of η_{c+1}, and the variance of $n_{\tau+1}$ would be only σ_η^2. Uninformed agents learn the accumulated news known by informed agents at trade date t just prior to the next market meeting. For example, if the market is closed on calendar date $c+1$, uninformed agents only learn the news known by informed agents at calendar date c, η_c. Thus, uninformed agents must infer $n_{\tau+1} = \eta_{c+1} + \eta_{c+2}$ when the market reopens on calendar date $c+2$.

due to the partial reflection of new private information, while the third component, $-(1-\pi_1^t)n_t$, represents the full reflection of the previous trading period's private information. The fourth component, $-\pi_2^{t+1}w_{t+1}$, is due to the temporary price pressure created by the liquidity shock at $t+1$, and the last component, $\pi_2^t w_t$, reflects the rebounding of price from the previous trading period's temporary liquidity price pressure. It follows that price changes have unconditional mean $E(\Delta P_{t+1}) = \Delta^t$ and variance given by

$$\mathrm{Var}(\Delta P_{t+1}) = (1-\pi_1^t)^2\sigma_{n_t}^2 + (\pi_1^{t+1})^2\sigma_{n_{t+1}}^2 + (\pi_2^{t+1})^2\sigma_{w_{t+1}}^2 + (\pi_2^t)^2\sigma_{w_t}^2. \quad (6)$$

Note that the variance of price changes is composed of two parts: the first depends upon the gradual reflection of private information, and the second is generated by liquidity-induced price pressures. In general, because liquidity shocks are temporary, prices will contain temporary components, resulting in nonzero covariance between adjacent price changes.[9] Prices follow a random walk (with drift) only when news is fully reflected at each time t and liquidity shocks do not affect price.

II. The Determination of the Equilibrium Price Functions

As is standard in rational expectations models, equilibrium is defined as a set of demand and price functions such that (i) demands solve the optimization problem given the information conveyed by price, and (ii) prices clear the market given the dependence of demands on price. Demands depend on price because price sets the rate of exchange between the risky and riskless assets, and it reveals private information. Given the normality of news and liquidity shocks and the linearity of the price function, we derive optimal demands as functions of the moments describing the joint distribution of future prices and demands. We determine closed-form expressions for these moments as functions of the price parameters π_0, π_1, and π_2 in equation (4), and we then determine demands as functions of the price parameters by substituting the closed-form expressions for the moments into the optimal demand functions. The equilibrium price functions are then obtained by imposing the market-clearing condition (equation (2)). The following subsections provide the details of each step in deriving the equilibrium.

A. Optimal Demands

The T-period horizon dynamic programming problem posed above reduces to a series of myopic one-period horizon optimization problems only when prices follow a random walk (with deterministic drift). Since prices, in general, do not follow a random walk, however, the solution for optimal demands is more complicated. The following theorem specifies optimal demands when serial correlation in price changes is allowed; the only restriction on the dynamic behavior of prices is that price changes be normally distributed.

THEOREM 1: *Optimal demands are given by*

$$X_t^i = \frac{E_t^i(P_t) - P_t}{\gamma G_{11}^{it}} + \frac{\Psi_t^i}{G_{11}^{it}} \tag{7}$$

where

$$\Psi_t^i = (G_{11}^{it} - G_{12}^{it}) E_t^i(X_{t+1}^i), \tag{8}$$

[9]The covariance of adjacent price changes is given by $\mathrm{Cov}(\Delta P_{t+1}, \Delta P_t) = (1 - \pi_1^t)\pi_1^t \sigma_{n_t}^2 - (\pi_2^t)^2 \sigma_{w_t}^2$. The temporary price movements caused by liquidity shocks (when π_2 is nonzero) induce a negative correlation, while the partial reflection of news (when $\pi_1 < 1$) induces positive correlation. The covariance is zero when $\pi_1 = 1$ and $\pi_2 = 0$.

and G_{11}^{it} *and* G_{12}^{it} *are elements of the conditional return covariance matrix,* G^{it}, G^{it} *is given by*

$$G^{it} \equiv \begin{pmatrix} G_{11}^{it} & G_{12}^{it} \\ G_{21}^{it} & G_{22}^{it} \end{pmatrix} \equiv \left(V_{t+1}^{i} + (\Omega^{it})^{-1}\right)^{-1}, \textit{ for } t \leq T-1, \tag{9}$$

where

$$V_{t+1}^{i} \equiv \frac{1}{G_{11}^{it+1}} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \tag{10}$$

and

$$\Omega^{it} \equiv \begin{pmatrix} \mathrm{Var}_t^i(P_{t+1}) & \mathrm{Cov}_t^i(P_{t+1}, E_{t+1}^i(P_{t+2})) \\ \mathrm{Cov}_t^i(P_{t+1}, E_{t+1}^i(P_{t+2})) & \mathrm{Var}_t^i(E_{t+1}^i(P_{t+2})) \end{pmatrix}, \tag{11}$$

for $t < T$, *and*

$$\Psi_{T-1}^{i} = 0 \textit{ and } V_T = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{12}$$

Proof: See Appendix A.

Although the form of the first part of the optimal demand in equation (7) is similar to myopic demand (see for example Grossman and Stiglitz (1980)), risk is measured by a complicated function of G_{11}^{it+1} and the elements of the conditional covariance matrix of the next period's price and future expected prices, Ω^{it}. In contrast, in the one-period horizon problem, risk is completely characterized by the variance of terminal price. Optimal dynamic demands differ from static demands by a second term, Ψ_t^i/G_{11}^{it}, as well. Both differences have important implications for the dynamic behavior of equilibrium prices and are discussed in detail after closed-form expressions for the demands are given.

Given the normality of the price function, we can calculate closed-form expressions for the expectations and variances that determine optimal demands. Substituting these expressions into the optimal demand expressions yields the following demand functions. Informed demand is given by:

$$X_t^I = \frac{\left[\Delta^t + (1-\pi_1^t)n_t + \pi_2^t w_t + \gamma \Psi_t^I\right]}{\gamma G_{11}^{It}}, \tag{13}$$

while uninformed demand is defined by:

$$X_t^U = \frac{\left[\Delta^t + (\lambda_1^t n_t + \lambda_2^t w_t) - \pi_1^t n_t + \pi_2^t w_t + \gamma \Psi_t^U\right]}{\gamma G_{11}^{Ut}}. \tag{14}$$

The closed-form expressions for Ψ_t^i and G_{11}^{it} ($i = I$ and U) are provided in Appendix B. The parameters λ_1^t and λ_2^t are defined by the optimal inference of n_t given P_t (i.e., uninformed agents solve $\lambda_1^t n_t + \lambda_2^t w_t = E(n_t|P_t)$).[10] Given the normality of the random variables, the form of the price function, and the flow of information, it follows that Ψ_t^i and G_{11}^{it} ($i = I$ and U) are all nonrandom constants that are functions of the future price coefficients.[11]

Each demand function can be divided into a part that depends upon the realized values of the news and liquidity shocks and a part that depends solely on parameters—not realizations. The former part is referred to as the conditional holding, while the latter part is referred to as the unconditional holding. Conditional holdings are purely temporary, since only the current values of the news and liquidity shocks affect them. Consequently, the risk associated with these holdings depends only on the amount of next period's variance that is not exploitable by the particular agent. For example, although informed agents do not know what the next period's news will be, only the portion of the news that gets reflected in the future price creates risk. Since their trades exploit the portion of news that is not revealed by price, this portion does not create risk. By incorporating the correlation between future prices and expected demand, G_{11}^{it} is the appropriate measure of risk. The variance of the next period's price alone, Ω_{11}^{it}, is not a sufficient measure of risk unless prices follow a random walk, where every price change is permanent.

In contrast to conditional holdings, unconditional holding are not temporary. Consequently, the risk associated with an unconditional position depends on only the fluctuations in future prices that are permanent. Since G_{11}^{it} includes the part of variance that is due to temporary fluctuations, it is not the appropriate measure of the risk associated with unconditional holdings. The term $(G_{11}^{it} - G_{12}^{it})$ in Ψ_t^i (see equation (9)), however, appropriately measures the risk by incorporating the correlation between next period's price and next period's expectation of future price. In other words, $(G_{11}^{it} - G_{12}^{it})$ measures the portion of the next period's price movement that is permanent.

Changes in the parameters of the underlying news and liquidity shock processes can induce changes in the unconditional holdings over time. Since these changes will be independent of the realized values of news and liquidity shocks, any transaction that results when an agent alters his/her unconditional holding subjects that agent to price risk. For example, if unconditional holding increase, then there is the risk that liquidity demand will be high and purchases will have to be made at temporarily high prices. Notice, however, that since the sequence of news and liquidity shock variances are known by agents, future unconditional holdings can be anticipated and should be approached in a way that minimizes the risk created by temporary shocks. If the approach is gradual, some purchases will be made when prices are

[10] Standard results on inference under normality imply that

$$\lambda_1^t = \frac{(\pi_1^t)^2 \sigma_{n_t}^2}{(\pi_1^t)^2 \sigma_{n_t}^2 + (\pi_2^t)^2 \sigma_{w_t}^2} \quad \text{and} \quad \lambda_2^t = \frac{-\pi_1^t \pi_2^t \sigma_{n_t}^2}{(\pi_1^t)^2 \sigma_{n_t}^2 + (\pi_2^t)^2 \sigma_{w_t}^2}.$$

[11] See Lemma 2 in Appendix B.

temporarily low and some will be made when prices are temporarily high, and the variance of the adjustment cost is lower than with a sudden adjustment. However, the longer it takes to adjust, the greater the risk created by permanent price changes caused by the arrival of news. These tradeoffs are optimally incorporated into Ψ_t^i, which depends upon the expected level of future unconditional demand.

Also note that, in general, risk can be broken down into two categories. The first is inference risk, which results when uninformed agents imperfectly infer current news. This risk is associated with unpredictable future price movements due to current news shocks. The second category is future risk, the risk associated with unpredictable price movements due to future news and liquidity shocks. Both informed and uninformed agents face future risk, but the magnitude differs across these two types of agents. The distinction between inference and future risk becomes important later when we consider the effect of closures on risk.

B. Equilibrium Price Functions

To obtain a closed-form expression for the equilibrium price function, we impose the market-clearing condition shown in equation (2). Substituting the closed-form demand functions into the market-clearing condition and rearranging the terms yields:

$$\Pi_0^t + \Pi_1^t n_t + \Pi_2^t w_t = 0, \tag{15}$$

where

$$\Pi_0^t = \frac{N_I(\Delta\pi_0^{t+1} + \gamma\Psi_t^I)}{\gamma G_{11}^{It}} + \frac{N_U(\Delta\pi_0^{t+1} + \gamma\Psi_t^U)}{\gamma G_{11}^{Ut}} - \bar{X}, \tag{16}$$

$$\Pi_1^t = \frac{N_I(1 - \pi_1^t)}{\gamma G_{11}^{It}} + \frac{N_U(\lambda_1^t - \pi_1^t)}{\gamma G_{11}^{Ut}}, \tag{17}$$

and

$$\Pi_2^t = \frac{N_I(\pi_2^t)}{\gamma G_{11}^{It}} + \frac{N_U(\lambda_2^t + \pi_2^t)}{\gamma G_{11}^{Ut}} - 1. \tag{18}$$

Because the equilibrium price function must be such that the market clears for any realization of news and liquidity shocks, equation (15) must also hold for any realization of n_t and w_t. Therefore, equations (16), (17), and (18) must all equal zero at each trade date t. That is, $\Pi_0^t = \Pi_1^t = \Pi_2^t = 0$ for all t. This requirement generates a system of $3T$ equations in the $3T$ unknowns, π_0^t, π_1^t, and π_2^t for $t = 1, 2 \cdots T$. The following theorem provides the solution to this system of equations.

THEOREM 2: *The system of equations is recursive in that the price function at t depends upon the price function at t + 1. This dependence exists through G_{11}^{it} and Ψ_t^i ($i = I, U$), which, in equilibrium, are solely functions of the coefficients of the price function at t + 1. Given values for G_{11}^{it} and Ψ_t^i ($i = I, U$),* *the equilibrium price function at t is defined by the following coefficients:*

$$\pi_1^t = \left[\delta_I^t + (1 - \delta_I^t)\lambda_1^t\right], \tag{19}$$

$$\pi_2^t = \left(\frac{\gamma G_{11}^{It}}{N_I}\right)\pi_1^t, \tag{20}$$

and

$$\Delta^t = \gamma\left[\overline{XG}_{11}^{t} - \overline{\Psi}_t\right], \tag{21}$$

where $\overline{G}_{11}^t$ is the harmonic mean of G_{11}^{It} and G_{11}^{Ut} (i.e., $\overline{G}_{11}^t \equiv 1/[N_I(1/G_{11}^{It}) + N_U(1/G_{11}^{Ut})]$), and $\overline{\Psi}_t$ is the weighted average of Ψ_t^I and Ψ_t^U with weights $\delta_I^t \equiv (N_I/G_{11}^{It})/[(N_I/G_{11}^{It}) + (N_U/G_{11}^{Ut})]$ and $(1 - \delta_1^t)$, respectively.

Proof: See Appendix B.

Given a specific terminal price function and a sequence of values for $\sigma_{n_t}^2$ and $\sigma_{w_t}^2$, the equilibrium price coefficients for any t can be obtained by solving equations (19) through (21) recursively. Since we want to rule out bubbles, we impose the transversality condition that the terminal price equal the firm's "intrinsic" value. That is, we set $P_T = \theta_T = \theta_{T-1} - n_T$, which implies that the terminal price function is given by $\pi_1^T = 1$, $\pi_2^T = 0$, and $\pi_0^T = 0$. Using these values as seeds, the price coefficients at $T - 1$ are derived given values for $\sigma_{n_T}^2$, $\sigma_{w_T}^2$, $\sigma_{n_{T-1}}^2$, and $\sigma_{w_{T-1}}^2$, and so on.

III. Price Dynamics

Given the procedure for obtaining the equilibrium price coefficients outlined in the previous section, we next develop equilibrium price functions for two specific sequences of the news and liquidity shock variances. The two sequences allow us to gauge the impact of closures on price dynamics. The first sequence is the nonclosure sequence in which the news and liquidity shock variances are constant, i.e., information and liquidity shocks flow at a steady rate and the market opens at every calendar date. The second sequence is the closure sequence. Let dates τ and $\tau + 1$ be the pre- and postclosure trade dates. Also let c be the calendar date of the preclosure trade date and $c + 2$ be the calendar date of the postclosure trade date. Under the assumption that private news is an elapsed-time process, $\sigma_{n_{\tau+1}}^2 = 2\,\sigma_n^2$, where σ_n^2 is the variance per calendar date.[12] For the closure case, we

[12] The variances of liquidity demands on either side of the closure, $\sigma_{w_\tau}^2$ and $\sigma_{w_{\tau+1}}^2$, may not follow an elapsed-time process in general. Liquidity demands from the day of closure must be satisfied either in some other manner or on some other trading day. Let a portion α of the liquidity demand from the day of closure be anticipated and arrive at the market before the closure (i.e., on τ) and let β percent of the liquidity demand from the day of closure spill over to the next trading day (i.e., on $\tau + 1$) where $\alpha + \beta \leq 1$. Because the variance of the liquidity demand shock is constant for all units of time, $\sigma_{w_\tau}^2 = (1 + \alpha^2)\sigma_w^2$ and $\sigma_{w_{\tau+1}}^2 = (1 + \beta^2)\sigma_w^2$, where σ_w^2 is the variance of the liquidity demand per unit of time. Note that, if liquidity demand must be satisfied immediately, $\beta = 0$; if liquidity demand cannot be anticipated, then $\alpha = 0$. This analysis does not allow discretionary liquidity trades (e.g., as in Admati and Pfleiderer (1988)). For the purposes of this article, we assume $\alpha = \beta = 0$.

also assume that, for calendar dates $c + 3$ and beyond, the market is open and the news and liquidity shock variances are constant per trade date.

The impact of the closure on price dynamics is evaluated by comparing the means and the variances of the returns over the closure with those that obtain in the constant element sequences over the same unit of time. That is, the means and the variances of returns from c to $c + 2$ are calculated and compared for periods when the market is open and closed at $c + 1$. This comparison is similar to those conducted by French and Roll (1986), with calendar date $c + 1$ corresponding to the Wednesdays when the stock market was closed.

A. Constant Element Sequence: Steady Response Equilibrium

This section uses the model developed above to examine the behavior of return moments when the variances of news and liquidity shocks are constant for each trade date. Here we only examine equilibria in which the response rates and mean price changes are self-perpetuating. That is, we examine only equilibria in which the next period's response rates equal the current response rates and the change in the risk discount is constant. An equilibrium that satisfies this condition will be referred to as a steady response equilibrium (SRE). Technically, a SRE only obtains in the limit as T goes to infinity.[13] Approximate SRE values are obtained by iterating back from T until the response rates of the last iteration do not differ from the response rates generated by the current iteration by more than 0.000001. Typically, numerical solutions obtain after very few steps (i.e., 20 to 30), and we easily obtain values that are arbitrarily close to the SRE values.[14]

Two interesting results obtain in SREs. First, for a fixed asset supply and risk aversion, SRE mean returns depend solely upon the variance of news, σ_n^2; they are not functions of the percentage informed (N_I) and/or the variance of the liquidity shocks (σ_w^2). (See Figure 2.) This is striking since,

[13]Although each agent's utility function is defined on his/her terminal wealth at T, it is not necessary that agents live forever for a SRE to obtain. The specification used here is equivalent to a situation in which agents maximize utility of wealth at intermediate points in their life, since maximizing the value function at an intermediate point in an agent's life is equivalent to maximizing expected utility of intermediate wealth. All that is required for a SRE is that the *economy* have an infinite life and the price at the end of each generation's life be a SRE price. The latter condition requires that the percentage of informed agents be the same in each generation.

[14]One could solve equations (19) and (20) for SRE values π_1 and π_2 such that $\pi_1 = \pi_1^t = \pi_1^{t+1}$ and $\pi_2 = \pi_2^t = \pi_2^{t+1}$. This, however, may admit more equilibria than will obtain given conditions on the terminal price. Further, no closed-form solutions are available because these equations imply polynomials of a degree higher than 4. Therefore, we use numerical methods to obtain solutions. Interestingly, there are sets of parameters for which no SRE exists. For some parameters, π_2 will go to negative infinity, while π_1 oscilates between two values. This usually occurs for high values of γ; the risk created by future liquidity costs is large enough to make uninformed agents require larger current liquidity payments. In some cases, both π_1 and π_2 oscilate. To ensure that the comparison between the closure and no-closure cases are not affected by nonstable price functions, we do not examine these equilibria.

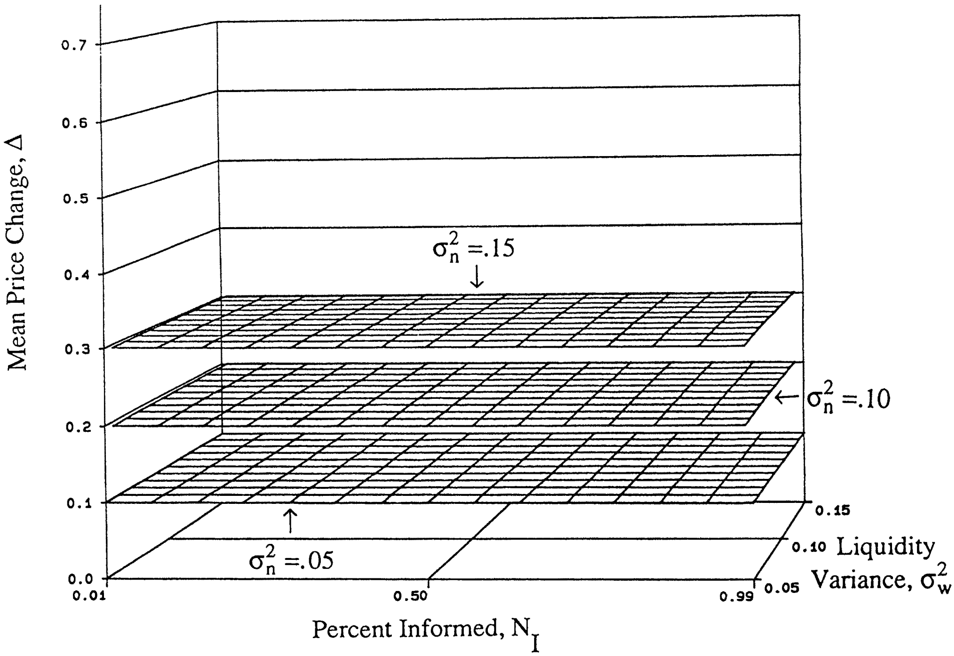


Figure 2. Steady response equilibrium mean price changes, Δ. Each surface corresponds to a different value for the variance of news. A point on each surface corresponds to a different combination of the percent informed (N_I) and the variance of liquidity shocks (σ_w^2). For all points, the coefficient of risk aversion is $\gamma = 2$, and the mean supply of the asset is $\bar{X} = 1$.

counter to what is expected, mean returns do not depend on either the informativeness of price or the degree of adverse selection. The second result is that return variances do depend on N_I and σ_w^2 (see Figures 3*A* and 3*B*). Consequently, it is easy to construct examples in which the mean return of one asset is less than another even though its variance is higher. (See Table I and compare case 6 with cases 12 and 13; and compare case 9 with case 11.) Thus, the usual positive relationship between variance and return need not hold, in spite of the fact that our economy contains a single risky asset and there is no consumption.

Although we cannot demonstrate the mean return result using closed-form expressions, the following intuitive argument illustrates why it holds.[15] By Theorem 2, the mean price change is equal to an average of the informed and uninformed risks associated with permanent price changes (i.e., the risks measured by G_{11}^i *and* Ψ^i, for $i = I$ and U) scaled by the coefficiént of risk aversion. In an SRE, however, all agents' unconditional holdings are constant.[16] Since no agent enters the market to alter these unconditional hold-

[15] Closed-form expressions are not available because SRE values for G_{11}^I and G_{11}^U are solutions to very high degree polynomials for which closed-form solutions do not exist.

[16] See Lemma 3 in Appendix B.

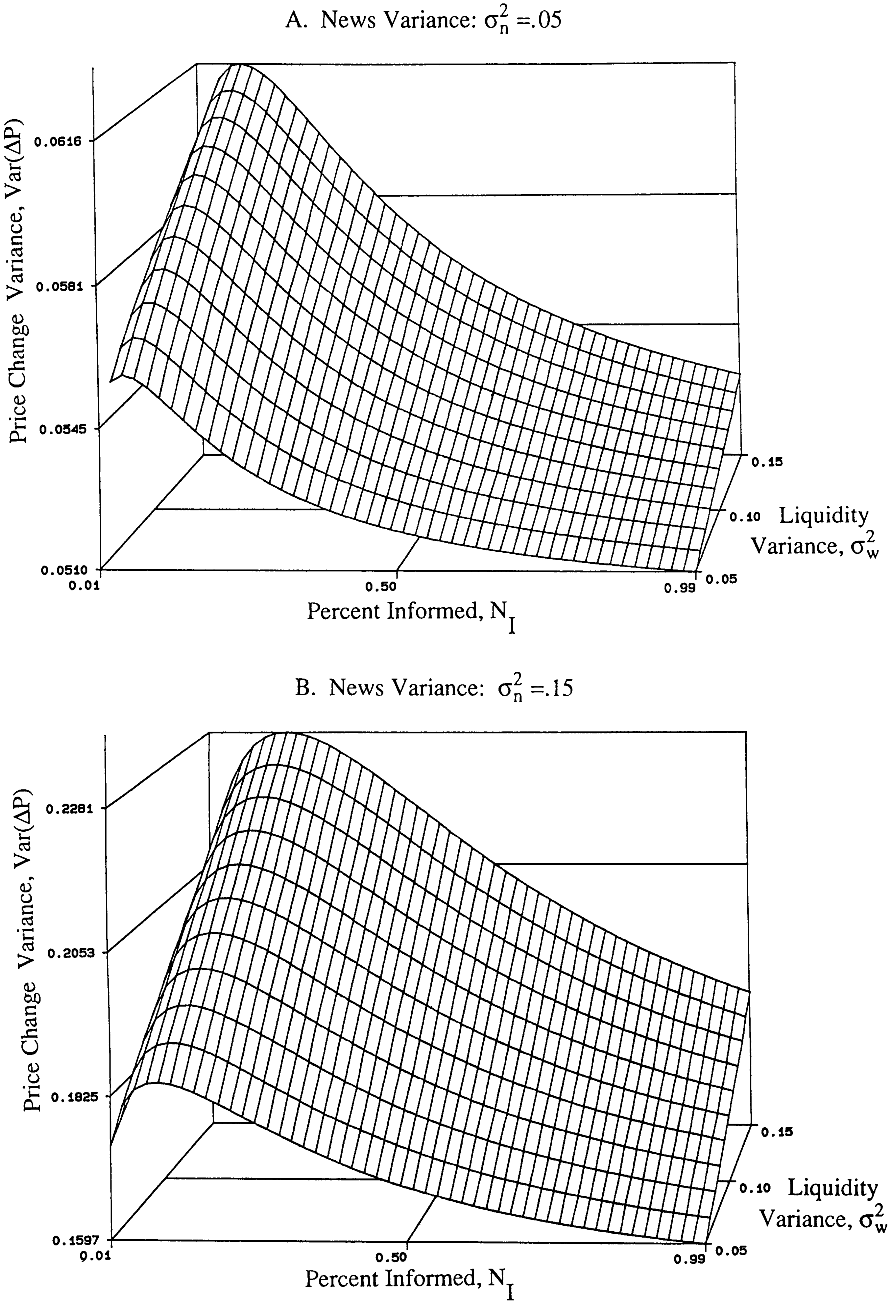


Figure 3. Steady response equilibrium (SRE) variances, Var(ΔP). Each figure plots the variance of price changes between adjacent trading dates in a SRE as a function of the percentage informed (N_I) and the variance of liquidity shocks (σ_w^2). In Panel A, the variance of news is 0.05 and in Panel B, the variance of news is 0.15. For both panels, the coefficient of risk aversion is $\gamma = 2$, and the mean asset supply is $\overline{X} = 1$.

Table I

Values for Selected Steady Response Equilibria

This table presents the steady response equilibrium (SRE) values of the following items for various combinations of the percentage informed (N_I), the variance of news (σ_n^2), and the variance of liquidity shocks (σ_w^2): (1) the SRE response rates—i.e., the coefficients on the news and liquidity shocks in the SRE price function, π_1 and π_2, respectively. (2) The aggregate measures of risk that, by Theorem 2, affect the mean price change. The first measure of aggregate risk is the harmonic average of the SRE values of each agent's conditional variance of next period's price—i.e., $\overline{G}_{11} = 1/[N_I(I/G_{11}^I) + N_U(1/G_{11}^U)]$, where G_{11}^I and G_{11}^U are the SRE values of the conditional variances of the informed and uninformed, respectively. The second aggregate risk measure is the common SRE value for Ψ^I and Ψ^U, the term in each agent's demand that takes into account the amount of the future price variation that is permanent. This common value is denoted Ψ. (3) The mean (Δ) and the variance (Var(ΔP)) of the distribution of price changes between adjacent trade dates. For comparability across tables, each case number corresponds to a unique combination of the exogeneous parameters, N_I, σ_n^2, and σ_w^2. For all cases, the coefficient of risk aversion is $\gamma = 2$, and the mean asset supply is $\overline{X} = 1$.

	Parameters			Response Rates		Risk		Moments	
Case No.	N_I	σ_n^2	σ_w^2	π_1	π_2	$\overline{G}_{11}$	Ψ	Var(ΔP)	Δ
1	0.001	0.07	0.07	0.028	0.254	0.07249	0.00249	0.0752	0.14
2		0.07	0.10	0.023	0.219	0.07306	0.00306	0.0764	0.14
3		0.15	0.10	0.014	0.378	0.16133	0.01133	0.1744	0.30
4	0.02	0.07	0.05	0.265	0.601	0.07421	0.00421	0.0789	0.14
5		0.07	0.06	0.249	0.547	0.07460	0.00460	0.0798	0.14
6	0.05	0.07	0.15	0.325	0.408	0.07857	0.00857	0.0892	0.14
7		0.10	0.11	0.320	0.571	0.11257	0.01257	0.1283	0.20
8		0.10	0.14	0.294	0.516	0.11445	0.01445	0.1331	0.20
9	0.10	0.06	0.15	0.515	0.390	0.06702	0.00702	0.0756	0.12
10		0.07	0.15	0.493	0.430	0.07902	0.00902	0.0904	0.14
11	0.90	0.07	0.05	0.998	0.157	0.07110	0.00110	0.0722	0.14
12		0.08	0.05	0.998	0.180	0.08144	0.00144	0.0829	0.16
13		0.08	0.14	0.995	0.185	0.08414	0.00414	0.0887	0.16

ings, such holdings are not subject to adverse selection. Therefore, the risk discount that makes all agents want to hold the fixed mean supply is independent of the proportion of the market that is informed. In particular, SRE mean returns equal the mean returns that occur when $N_I = 1$. Since the amount of uncertainty that is resolved between dates $t - 1$ and t is simply $\sigma_{n_t}^2$, the risk discount falls by the certainty equivalent of $\sigma_{n_t}^2$. As a result, mean price changes are $\gamma \bar{X} \sigma_{n_t}^2$; that is, mean returns do not depend upon either N_I or σ_w^2.

The variance of returns, however, does depend on both N_1 and σ_w^2 through their effects on the SRE response rates, π_1 and π_2. Specifically, in an SRE, the variance simplifies to

$$\mathrm{Var}(\Delta P_{t+1}) = \left[(1 - \pi_1)^2 + (\pi_1)^2\right]\sigma_n^2 + 2(\pi_2)^2 \sigma_w^2. \tag{22}$$

This expression implies that, as π_1 increases, the portion of the daily variance due to current news increases while the portion due to old news falls. The information component of variance is minimized when $\pi_1 = 0.5$ and maximized when returns either fully reflect current news (and thus reflect no old news) or when they reflect no current news (and thus only reflect old news). The component of the variance that is due to liquidity shocks, however, is everywhere increasing in liquidity costs. As the liquidity costs increase, a given liquidity shock moves price by more, thus creating a larger temporary fluctuation and increasing the total variance. To understand how the variances depend upon the parameters of the model, Figures 4 and 5 provide plots of π_1 and π_2 as a functions of N_I and σ_w^2 for various values of σ_n^2.[17]

Figures 4*A* and 4*B* show that the percentage of the news that gets reflected in price is increasing in N_I and decreasing in σ_n^2 and σ_w^2. This is true because, by Theorem 2, π_1 is simply the weighted average of the sensitivities to current news of informed and uninformed agents' expectation of news. That is, π_1 is a weighted average of 1 (the sensitivity of $E_t^I(n_t)$ to n_t) and λ_1^t (the sensitivity of $E_t^U(n_t)$ to n_t). The weights depend on the relative intensity with which each group trades on its expectation. For example, the weight on the informed sensitivity depends upon the magnitude of N_I/G_{11}^I relative to N_U/G_{11}^U. Thus, π_1 is increasing in N_I, because as N_I increases more weight is placed on 1 and prices reflect more news.[18] Further,

[17] The effect of changes in the coefficient of risk aversion, γ, on the response rates is straightforward. In the interest of brevity, we do not provide any discussion or any plots that describe the impact of γ.

[18] Note that there is a moderating impact due to increases in G_{11}^I resulting from increases in π_1. G_{11}^I increases because less current and future news is exploitable the higher the π_1. Thus, as the increase in N_I pushes π_1 up, the resulting increase in G_{11}^I pushes it back. It is never the case, however, that the effect of G_{11}^I will negate the effect of N_I. This is true since, if it did, then G_{11}^I would fall, increasing π_1 and causing a contradiction.

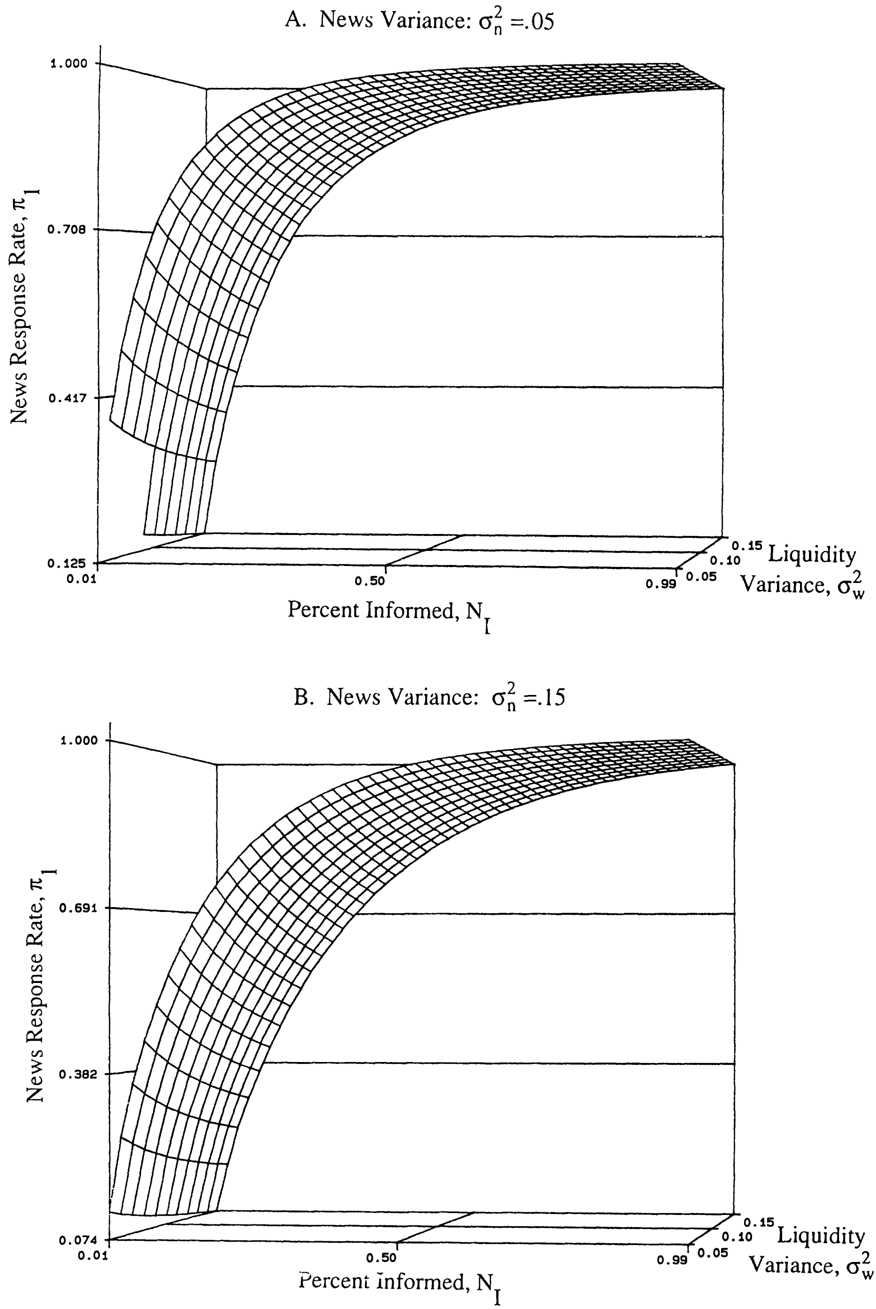
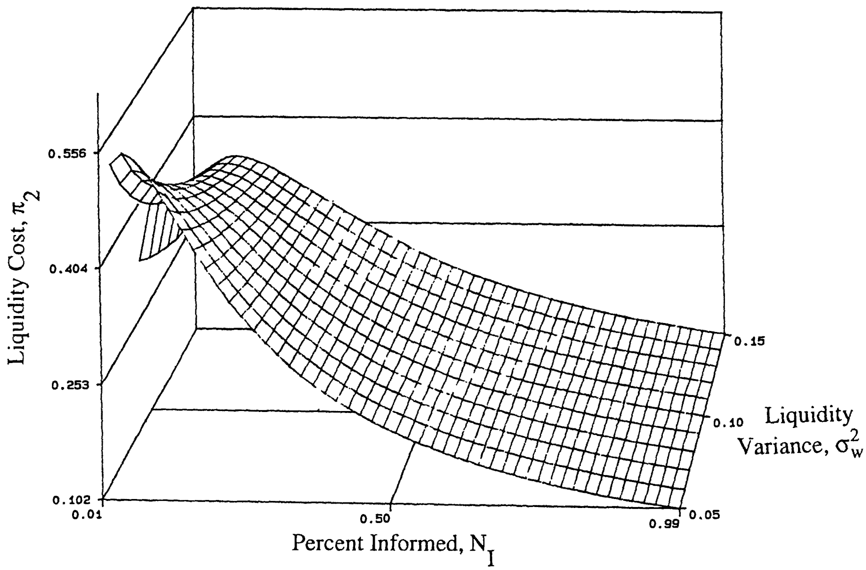


Figure 4. Steady response equilibrium (SRE) news response rates, π_1. Each figure plots the SRE response rate to news (π_1) as a function of the percentage informed (N_I) and the variance of liquidity shocks (σ_w^2). In Panel A, the variance of news is 0.05, and in Panel B, the variance of news is 0.15. For both panels, the coefficient of risk aversion is $\gamma = 2$, and the mean asset supply is $\overline{X} = 1$.

A. News Variance: $\sigma_n^2 = .05$



B. News Variance: $\sigma_n^2 = .15$

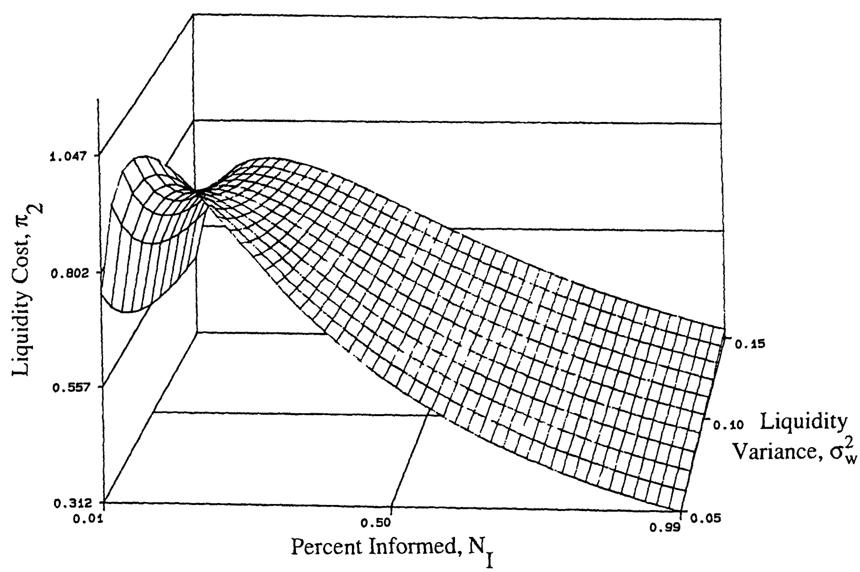


Figure 5. Steady response equilibrium (SRE) liquidity costs, π_2. Each figure plots the SRE liquidity cost (π_2) as a function of the percentage informed (N_I) and the variance of liquidity shocks (σ_w^2). The liquidity cost is the amount that a liquidity-motivated trade affects price. That is, the liquidity cost is the coefficient in the price function on the liquidity shock. In Panel A, the variance of news is 0.05 and in Panel B, the variance of news is 0.15. For both panels, the coefficient of risk aversion is $\gamma = 2$, and the mean asset supply is $\bar{X} = 1$.

as either σ_n^2 or σ_w^2 increase, uninformed agents' expectation of news becomes less accurate (i.e., λ_1^t falls), and inference risk increases.[19] As a result, prices reflect less news.

Figures 5*A* and 5*B* show that liquidity costs first increase and then decrease with increases in N_I. The market-clearing condition (equation (17)) provides intuition for this result. It implies that the sensitivity of aggregate uninformed demand to news is equal but opposite to the sensitivity of aggregate informed demand to news, i.e., $(N_I(1 - \pi_1))/(\gamma G_{11}^I) = -(N_I(\lambda_I - \pi_1))/(\gamma G_{11}^U)$. Since current news is only partially reflected in current prices, informed traders buy (sell) on good (bad) news and profit from subsequent price increases (decreases) when the news is fully reflected in prices. Uninformed traders, however, take the opposite position, buying (selling) whenever the price is lower (higher) than expected. Prices are lower than expected when news is bad or when liquidity traders are demanding liquidity and selling (i.e., when the liquidity trader's demand shock is positive—$w_t > 0$). When news is bad, uninformed agents buy, thereby subjecting themselves to future losses when the news becomes fully reflected in the future price. When the liquidity traders are selling, however, uninformed agents buy from these liquidity traders at temporarily deflated prices, thereby receiving profits by providing liquidity. If, on average, the losses exceed the gains, uninformed agents will not actively trade, and thus will not provide any liquidity. As a result, all temporary liquidity shocks would be absorbed by informed agents, making them bear additional risk. Since there are gains from sharing the risk associated with holding a temporary liquidity position, the equilibrium price must be such that uninformed agents are induced to provide liquidity. That is, on average, contrarian strategies will be profitable in equilibrium.[20]

The profitability of contrarian strategies, in turn, implies a relationship between the responsiveness of price to news (π_1) and the liquidity costs (π_2). By adopting a contrarian strategy, uninformed agents trade against informed agents more frequently the greater the portion of the unexpected price change that is due to news. As N_I increases, π_1 increases, and news changes price by more. Without an increase in π_2, uninformed traders are trading against informed traders more, and the profitability of the contrarian strategy falls. Consequently, π_2 must also increase with N_I so that there are additional gains to providing liquidity to compensate for the extra losses of the uninformed agents. However, higher π_1 leads to a more informative price and better inferences by uninformed traders. As informativeness increases, the precision of the uninformed agents' inference increases, and the risk associated with taking a temporary position when providing liquidity falls. Consequently, the cost of liquidity, π_2, falls too. Figures 5*A* and 5*B* show

[19] In equilibrium, $G_{11}^U = G_{11}^I + (1 - \lambda_1)\sigma_n^2$ where $(1 - \lambda_1)\sigma_n^2$ is the inference risk (see the proof of Lemma 3). The reader can easily verify that $(1 - \lambda_1)\sigma_n^2$ is increasing in both σ_n^2 and σ_w^2.

[20] Campbell, Grossman, and Wang (1993) presents a model without asymmetric information in which contrarian strategies are optimal in equilibrium.

that the effect of increased price informativeness dominates when N_I (and π_1) are high.

The variance plots (i.e., Figures $3A$ and $3B$) indicate that SRE variances first increase and then decrease with N_I, as do SRE liquidity costs. Thus, the dominant component of variance is generated by temporary price pressures caused by liquidity shocks. Although these fluctuations create risk for short-term positions (e.g., a positive short-term position may have to be created when liquidity shocks temporarily inflate prices and/or reversed when liquidity shocks temporarily deflate prices), these fluctuations do not affect long-run unconditional holdings, and, thus, do not affect SRE mean returns. Further, since most of these fluctuations are due to agents getting compensated for providing liquidity, these fluctuations do not create risk and are not priced in mean returns. Thus, the positive relationship between return variances and means implied by symmetric information models (e.g., the capital asset pricing model) need not hold in this model. In this model, the variance is not a sufficient statistic for risk since risk differs (1) across agents and (2) by reason for trade; risk is measured by G_{11}^{it} when the trade is a temporary conditional position or $(G_{11}^{it} - G_{12}^{it})/G_{11}^{it}$ when the trade results from change in the optimal unconditional holding.

B. Closure Equilibrium

The preceding discussion of steady-response equilibria characterizes situations under which stock return means and variances are constant over time. In this section, we show that relatively simple perturbations to the underlying news process (such as result with closures) lead to dramatic and complex dynamic effects on both the mean and the variance of returns by affecting risk. Specifically, a closure delays the resolution of uncertainty by postponing the public release of closure news. As a result, risk is redistributed across time and agents. Since agents are risk averse in our model, this redistribution affects the equilibrium price, altering risk premia, liquidity costs, and the degree of informational asymmetry. As a consequence, a closure alters both the means and variances of returns. In fact, when there are relatively few informed agents, a closure generates phenomena that are consistent with documented empirical regularities. For example, consistent with the results on weekend mean returns (see French (1980)) and Wednesday closure variances (see French and Roll (1986)), both the mean and the variance of returns over closures are abnormally low relative to the mean and variance of nonclosure returns when N_I is small (see, for example, cases 1 through 5 in Table II). In addition, closures can also cause counter-intuitive dynamic behavior. For example, a closure can cause mean returns to fall while variances increase (see, for example, cases 6, 7, and 8 in Table II).

To get intuition for certain dynamics that result, it is important to understand how closures affect the risks of both types of agents. First, by delaying the release of private information, a closure causes postclosure uninformed risk to increase. Postclosure informed risk is unaffected by the closure,

Table II

Values For Selected Closure Equilibria

This table presents the equilibrium values that obtain prior to and after a closure for the following items for various combinations of the percentage informed (N_I), the variance of news (σ_n^2), and the variance of liquidity shocks (σ_w^2): (1) the coefficients on the news and liquidity shocks in the price function, π_1 and π_2, respectively, and (2) the aggregate measures of risk, $\overline{G}_{11}$ and Ψ. $\overline{G}_{11}$ is the harmonic average of the values of each agent's conditional variance of next period's price—i.e., $\overline{G}_{11} = 1/[N_I(1/G_{11}^I) + N_U(1/G_{11}^U)]$, where G_{11}^I and G_{11}^U are the values of the conditional variances of the informed and uninformed, respectively. The second aggregate risk measure is the common value for Ψ^I and Ψ^U, the term in each agent's demand that takes into account the amount of the future price variation that is permanent. This common value is denoted Ψ. The values that obtain prior to a closure are listed under "Preclosure," and the values that obtain after a closure are listed under "Postclosure." This table also presents the ratio of the mean and variance of the returns over closures to the mean and variance over a two-day period in a steady response equilibrium (SRE). $\text{Var}(\Delta P_{\tau+1})$ is the variance of the difference in the pre- and postclosure price, while $\text{Var}(P_{c+2} - P_c)$ is the variance of the change in price over two days in a SRE. Δ^τ represents the mean price change over a closure, while 2Δ is the mean price change over two trading dates in a SRE. For compatibility with Table I, the cases (i.e., the different combinations of N_I, σ_n^2, and σ_w^2) are the same as those considered in Table I. For all cases, the coefficient of risk aversion is $\gamma = 2$, and the mean asset supply is $\overline{X} = 1$.

	Parameters			Preclosure				Postclosure				Moment Comparisons	
												Variance	Mean
Case No.	N_I	σ_η^2	σ_w^2	π_1	π_2	$\overline{G}_{11}$	Ψ	π_1	π_2	$\overline{G}_{11}$	Ψ	$\frac{\text{Var}(\Delta P_{\tau+1})}{\text{Var}(P_{c+2} - P_c)}$	$\frac{\Delta^\tau}{2\Delta}$
1	0.001	0.07	0.07	0.006	0.202	0.08587	0.00871	0.054	0.485	0.13687	0.00249	0.612	0.551
2		0.07	0.10	0.006	0.194	0.08656	0.01071	0.045	0.419	0.13845	0.00306	0.620	0.542
3		0.15	0.10	0.004	0.407	0.19579	0.03842	0.027	0.719	0.30541	0.01133	0.671	0.525
4	0.02	0.07	0.05	0.069	0.459	0.13177	0.00899	0.411	0.935	0.10513	0.00421	0.931	0.877
5		0.07	0.06	0.066	0.428	0.13013	0.01004	0.391	0.860	0.10728	0.00460	0.920	0.858
6	0.05	0.07	0.15	0.124	0.412	0.14399	0.01591	0.477	0.598	0.10456	0.00857	1.034	0.915
7		0.10	0.11	0.123	0.580	0.20531	0.02347	0.471	0.840	0.15043	0.01257	1.033	0.909
8		0.10	0.14	0.118	0.544	0.20360	0.02768	0.438	0.769	0.15617	0.01445	1.032	0.880
9	0.10	0.06	0.15	0.243	0.439	0.13581	0.01025	0.668	0.506	0.77777	0.00702	1.144	1.046
10		0.07	0.15	0.234	0.487	0.15769	0.01344	0.647	0.564	0.09292	0.00902	1.141	1.030
11	0.90	0.07	0.05	0.994	0.311	0.14143	0.00110	0.999	0.158	0.07110	0.00110	1.025	1.002
12		0.08	0.05	0.993	0.356	0.16186	0.00144	0.999	0.180	0.08144	0.00144	1.029	1.003
13		0.08	0.14	0.983	0.357	0.16510	0.00415	0.997	0.186	0.08416	0.00414	1.078	1.006

however, since informed risk depends only on the future price function and future variances, and these values are SRE values. Second, by reducing opportunities to trade, a closure increases *both* informed and uninformed preclosure risk, albeit differentially. In particular, informed risk increases proportionally more than uninformed risk. This is true because informed risk contains only future risk, while uninformed risk consists of both future risk and inference risk, and the closure doubles the variance of future news while maintaining the variance of current news and liquidity shocks. When the future news variance doubles, the future risk for both informed and uninformed approximately doubles. However, because inference risk depends on only the variance of the current news and liquidity shocks, the inference risk of uninformed agents is mostly unaffected by the closure. Thus, since informed risk contains only future risk, the closure approximately doubles informed risk, while, because uninformed risk consists of both inference risk and future risk, uninformed risk does not increase proportionally as much.[21] Given this result, some of the informed agents' comparative advantage at risk bearing shifts to uninformed agents. This shift in comparative advantage leads to insurance-motivated trades, introducing insurance-trade risk; since insurance-motivated trades take place in the presence of adverse selection and liquidity trading, agents risk losing wealth by trading at temporarily inflated or deflated prices. Without the closure, there is no shift in comparative advantage, and thus no insurance-motivated trades or insurance-trade risk. The next two sections examine how these changes in risk affect the equilibrium price functions and the resulting means and variances of returns over closures.

B.1. Variance Effects

The effect of the closure on return variances can be broken into two components. Specifically, the difference in the variances when the market is open and when it is closed on $c + 1$ is

$$\operatorname{Var}(\Delta P|O) - \operatorname{Var}(\Delta P|C) = \Delta^N + \Delta^L, \tag{23}$$

where

$$\Delta^N \equiv \left[1 - (\pi_1^{\tau+1})^2\right]\sigma_n^2 + \left[(1 - \pi_1)^2 - (1 - \pi_1^{\tau})^2\right]\sigma_n^2 + \left[(\pi_1)^2 - (\pi_1^{\tau+1})^2\right]\sigma_n^2 \tag{24}$$

and

$$\Delta^L \equiv \left[(\pi_2)^2 - (\pi_2^{\tau})^2\right]\sigma_w^2 + \left[(\pi_2)^2 - (\pi_2^{\tau+1})^2\right]\sigma_w^2, \tag{25}$$

[21] This is a consequence of the following simple relationship: for $b > 0$, $(2a + b)/(a + b) < 2c/c = 2$. Here a and c represent the risk due to the arrival of future news faced by uninformed and informed agents, respectively, and b represents the inference risk faced by uninformed agents.

where π_1 and π_2 denote SRE values and π_k^τ and $\pi_k^{\tau+1}$ $(k = 1, 2)$ denote pre- and postclosure response rates. The first component, Δ^N, reflects the difference in variances due to differences in the news response rates. The second component, Δ^L, specifies the effect due to changes in pre- and postclosure liquidity costs. In equilibrium, the sign of both components can be negative or positive; as a result, a closure may increase or decrease the variance of returns. When the percentage of informed agents is small, however, variances typically fall.

To examine why variances fall when there are few informed agents, we examine how the closure affects the information response rate and the liquidity costs. By doing so, we consider how N_I affects the magnitude of the closure effects. We first examine Δ^N, which is comprised of three parts: (1) the effect caused by the interruption of the release of closure news to uninformed traders; (2) the effect due to differences in preclosure response rates; and (3) the effect of changes in postclosure news responsiveness. The first effect (the first term in equation (24)) reduces the variance. Since the news received by informed traders during the closure does not become public until after the close of the next trade date, closure news is only partially reflected (by $\pi_1^{\tau+1}$) in the postclosure price. However, if the market had been open, this news would have been fully reflected in the price on calendar date $c + 2$. The difference between full and partial reflection of the closure news reduces the variance of returns. The last two effects, however, increase variance since, as is shown in Propositions 1 and 2 and Figures 6*A* through 6*D*, the news response rate decreases prior to a closure but increases after a closure.

PROPOSITION 1: *After a closure, prices are more responsive to news. That is,* $\pi_1^{\tau+1} > \pi_1$.

Proof: See Appendix B.

Because informed demand after a closure is the same as in a SRE (but for changes in the price coefficients), informed agents trade with the same aggressiveness as in a SRE. However, because uninformed risk is greater after a closure than during a SRE, uninformed demands are less sensitive to news and liquidity shocks, i.e., they sell less on good news and buy less on bad news. Consequently, if news is good, there is excess demand and the price is bid up. If news is bad, there is excess supply and prices fall. As a result, postclosure prices reflect a greater proportion of postclosure news, increasing the variance of returns over closures (i.e., the third term in equation (24) is positive).

PROPOSITION 2: *The preclosure price function is less responsive to news. That is,* $\pi_1^\tau < \pi_1$.

Proof: See Appendix B.

Because informed risk increases proportionally more than uninformed risk prior to a closure, the sensitivity of informed demand to news drops more than the sensitivity of uninformed demand. This decrease in the informed

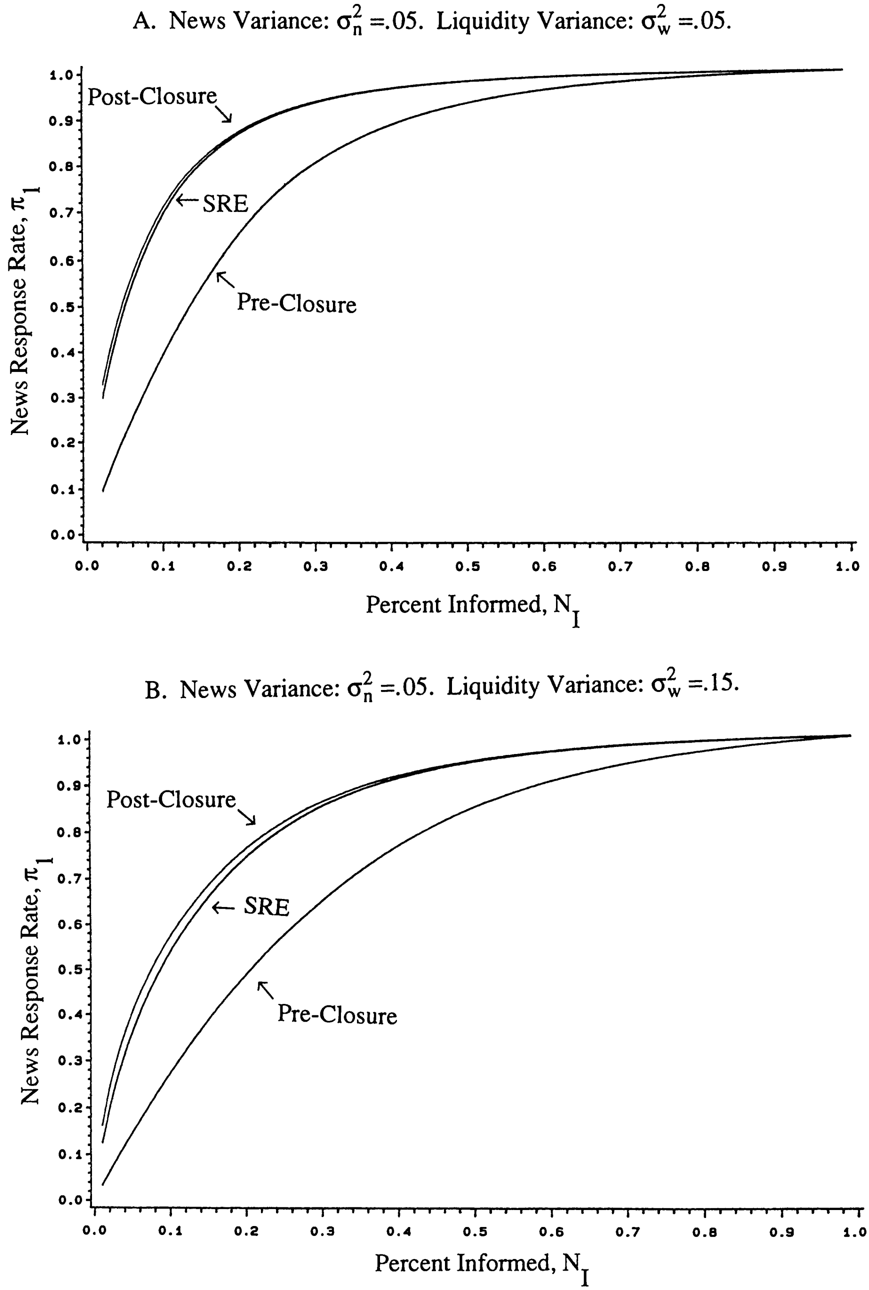
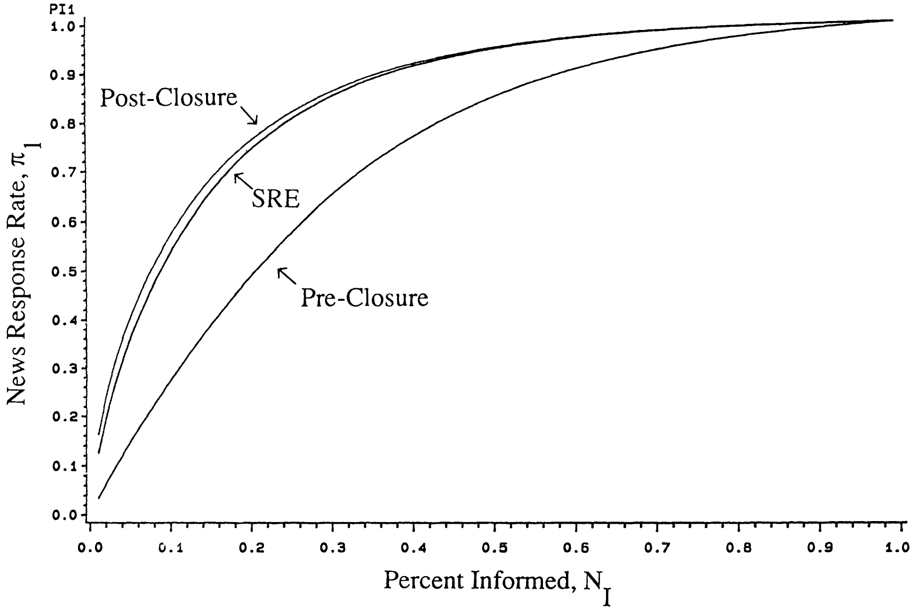


Figure 6. Pre- and postclosure news response rates, π_1^τ and $\pi_1^{\tau+1}$, as a function of the percentage informed (N_I). Each panel corresponds to a different combination of the variance of news and liquidity shocks. In all panels, the coefficient of risk aversion is $\gamma = 2$, and the mean asset supply is $\overline{X} = 1$. For comparison, the steady response equilibrium (SRE) news response rate function is also provided.

C. News Variance: $\sigma_n^2 = .15$. Liquidity Variance: $\sigma_w^2 = .05$.



D. News Variance: $\sigma_n^2 = .15$. Liquidity Variance: $\sigma_w^2 = .15$.

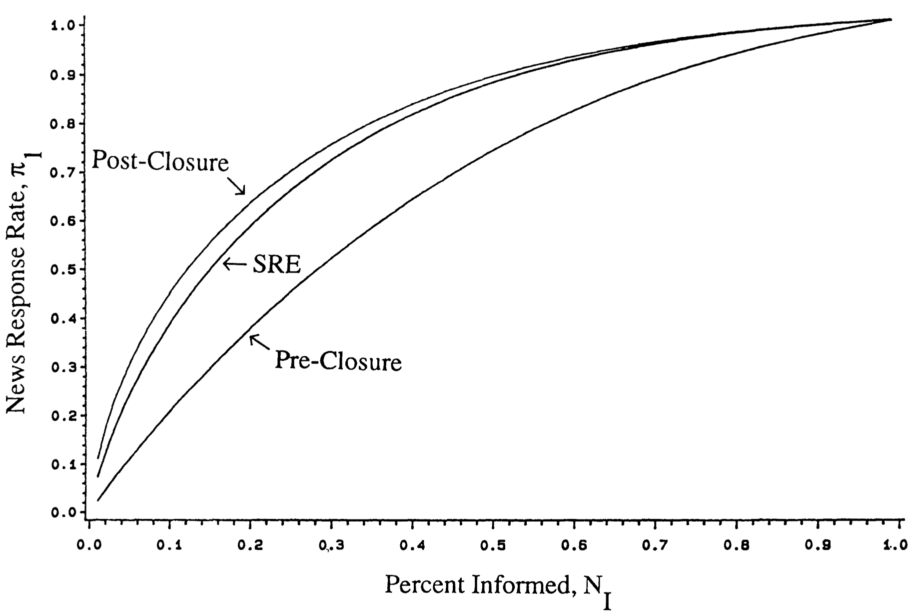


Figure 6—*Continued*

demand sensitivity causes excess supply (demand) when news is good (bad). Consequently, when news is good (bad), prices are lower (higher), reflecting less of the good (bad) preclosure news. In this case, since the preclosure price starts further away from the fully revealing price, the postclosure price moves more when the preclosure news becomes fully reflected, thus increasing the variance over closures (i.e., the second term in equation (24) is also positive).

The net effect of the closure is to reduce return variances when there are very few informed agents. In these cases, prices reflect very little current news. Consequently, the difference between the full and partial reflection of closure news (i.e., the first effect) is large, while the changes in pre- and postclosure responsiveness (the second and third effects) are minor. When the percentage of the market that is informed is large, however, prices reflect most of the current news and the difference between full and partial reflection of closure news is minor compared to the increase in the closure variance caused by changes in pre- and postclosure news responsiveness. In these cases, the component of variance due to news increases over closure. The larger the percentage of informed agents, the smaller the difference becomes, however. In fact, when $N_I = 1$, $\pi_1^t = 1$ (for all t regardless of closures) and $\Delta^N = 0$—the component of variance due to information is unaffected by closures.

The following propositions show how closures affect liquidity costs. Specifically, Proposition 3 implies that postclosure liquidity costs unambiguously increase the variance of postclosure prices, while Proposition 4 shows that the effect on preclosure liquidity costs are ambiguous. Hence, the net effect on that component of the difference in variance that is due to liquidity shocks is also ambiguous.

PROPOSITION 3: *Liquidity costs are larger after a closure than during a SRE. That is,* $\pi_2^{\tau+1} > \pi_2$.

Proof: See Appendix B.

Since the risk faced by uninformed agents is greater following a closure than during a SRE, uninformed demand becomes less sensitive to unexpected price changes after a closure. Consequently, uninformed traders are less contrarian than they are during a SRE—they sell less on unexpected price increases and buy less on unexpected price drops. This implies that they provide less liquidity. When price is unexpectedly low due to liquidity traders selling, uninformed traders are buying fewer of the shares and are thus providing less liquidity. Further, because informed risk is unaffected by the closure, the amount of liquidity that informed traders provide is the same after a closure as in a SRE. The reduction in the amount of liquidity satisfied by uninformed traders creates excess liquidity demand. To induce agents to provide more liquidity, liquidity costs must increase and price must become more sensitive to liquidity shocks. Thus, ceteris paribus, the postclosure price is more variable than in a SRE.

Proposition 4, however, implies that the preclosure price can be less variable.

PROPOSITION 4: *The effect of a closure on liquidity costs prior to a closure is ambiguous.*

Proof: This proposition is proved by demonstration. See Figures 7*A* through 7*D* and the following discussion. Q.E.D.

The effect on preclosure liquidity costs is examined in Figures 7*A* through 7*D*, which show SRE and preclosure liquidity costs as functions of N_I for a variety of combinations of σ_w^2 and σ_n^2.[22] These figures show that preclosure liquidity costs are smaller than SRE liquidity costs when N_I is small. In these cases, the postclosure news response rates are much bigger than those that obtain in a SRE. This implies that a larger portion of the news received during the closure and just prior to the market reopening is not exploitable by informed traders, increasing the risk associated with an informed position taken prior to the closure. As a result, informed agents trade less aggressively on their private information in the preclosure market, and the price reflects less preclosure news. With this drop in the preclosure news response rate, the portion of the unexpected price change that is caused by news falls while the portion created by temporary liquidity shocks increases. Holding risk constant, uninformed traders become more aggressive contrarians, supplying more liquidity and reducing liquidity costs. If the risk faced by uninformed agents is much larger prior to the closure, then uninformed agents will supply less liquidity, and liquidity costs will rise. When N_I is small, however, uninformed risk does not increase much prior to a closure, and liquidity costs fall.[23]

Figures 8*A* through 8*D* show how the net effect of both the information and liquidity shock components depend upon the percentage informed in the market. When N_I is small, both the information and liquidity shock components imply lower variance. As N_I increase, variances increase reflecting increases in preclosure liquidity costs and the reduction of the impact of the closure news. In fact, when $N_I = 1$ (and $\pi_1^t = 1$ for all t regardless of closures), preclosure liquidity costs are higher (i.e., $\pi_2^\tau < \pi_2$) but postclosure liquidity costs are unchanged (i.e., $\pi_2^{\tau+1} = \pi_2$). In this case, $\Delta^N = 0$ and $\Delta^L < 0$. Therefore, the variance from c to $c + 2$ is larger with a closure than in a SRE.

[22] For comparison, the postclosure liquidity costs are also provided on these figures. As indicated by Proposition 3, postclosure liquidity costs always exceed the liquidity costs in a SRE.

[23] When N_I is small, prices reflect very little current information and most of the information-based volatility of future price is due to current news. Thus, the inference risk component of uninformed risk is large relative to the future risk component. As a result, the increase in future risk prior to the closure does not affect an uninformed agent's total risk very much. Since the variance of the preclosure news is the same as in a SRE, uninformed risk is very close to SRE risk. See, for example, cases 1, 2, and 3 in Table II. In case 1, $G_{11}^{Ut} = 0.0862$ while $G_{11}^{Ut+1} = 0.1410$ and $G_{11}^{U} = 0.0736$; in case 2, $G_{11}^{Ut} = 0.0869$ while $G_{11}^{Ut+1} = 0.1425$ and $G_{11}^{U} = 0.0741$; in case 3, $G_{11}^{Ut} = 0.1964$ while $G_{11}^{Ut+1} = 0.3122$ and $G_{11}^{U} = 0.1631$.

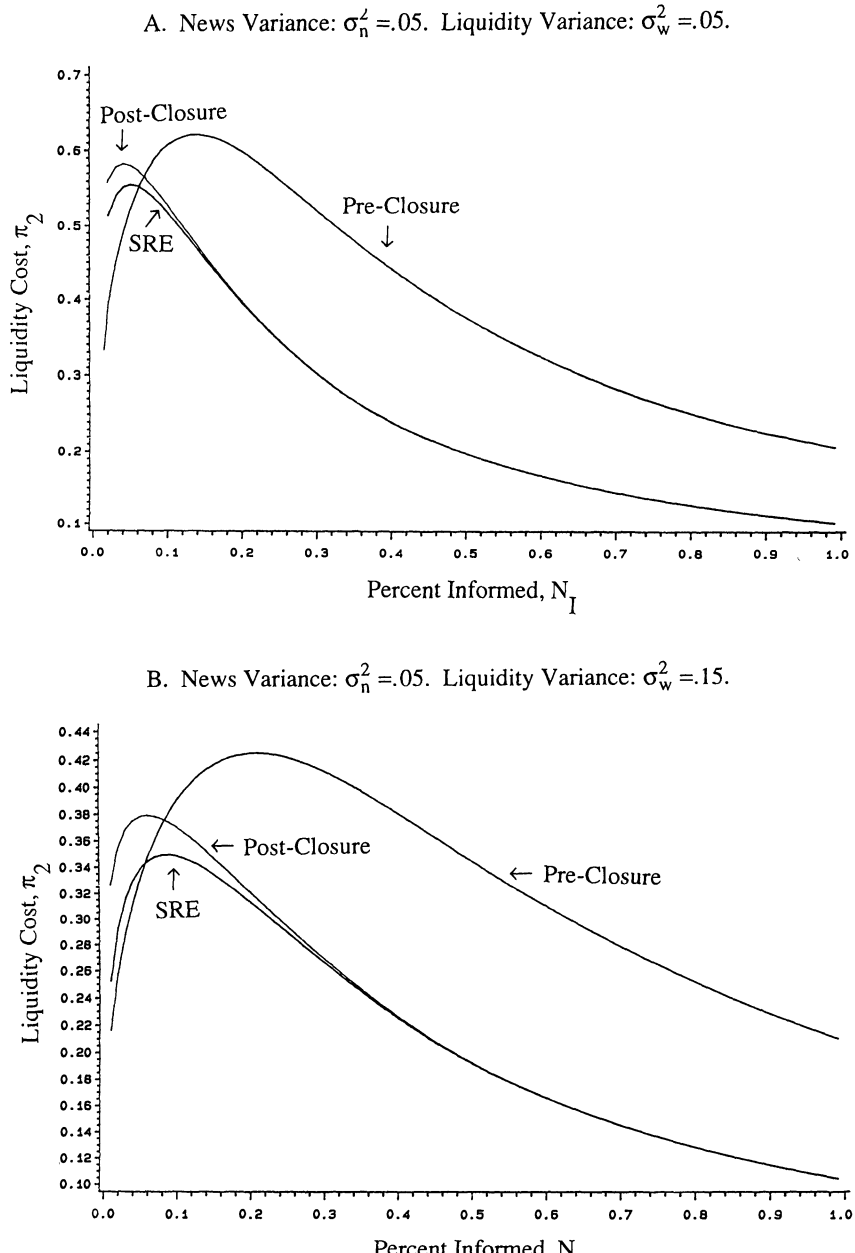
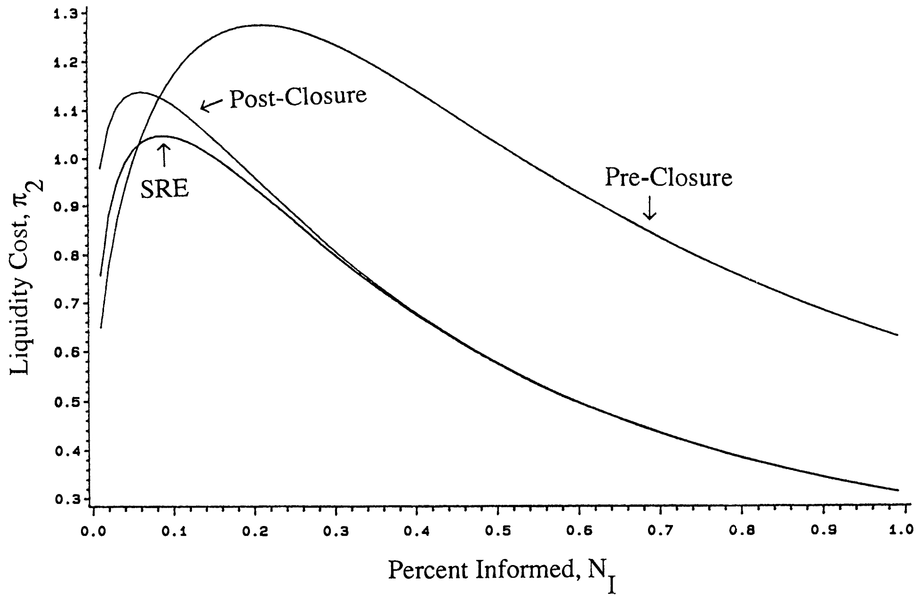


Figure 7. Pre- and postclosure liquidity costs, π_2^τ and $\pi_2^{\tau+1}$, as a function of the percentage informed (N_I). The liquidity cost is the amount that a liquidity-motivated trade affects price. That is, the liquidity cost is the coefficient in the price function on the liquidity shock. Each panel corresponds to a different combination of the variance of news and liquidity shocks. In all panels, the coefficient of risk aversion is $\gamma = 2$, and the mean asset supply is $\overline{X} = 1$. For comparison, the steady response equilibrium (SRE) liquidity cost function is also provided.

C. News Variance: σ_n^2 =.15. Liquidity Variance: σ_w^2 =.05.



D. News Variance: σ_n^2 =.15. Liquidity Variance: σ_w^2 =.15.

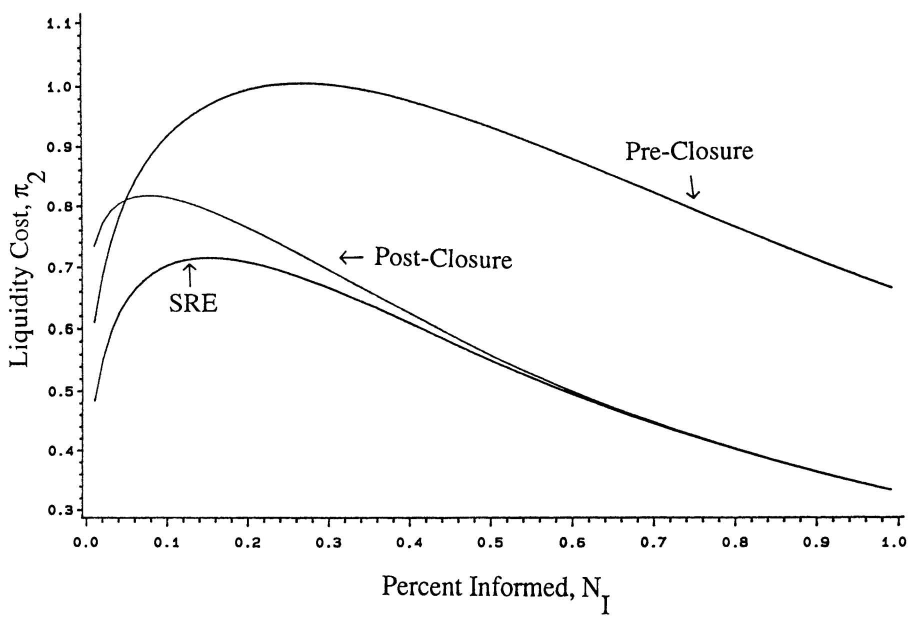


Figure 7—*Continued*

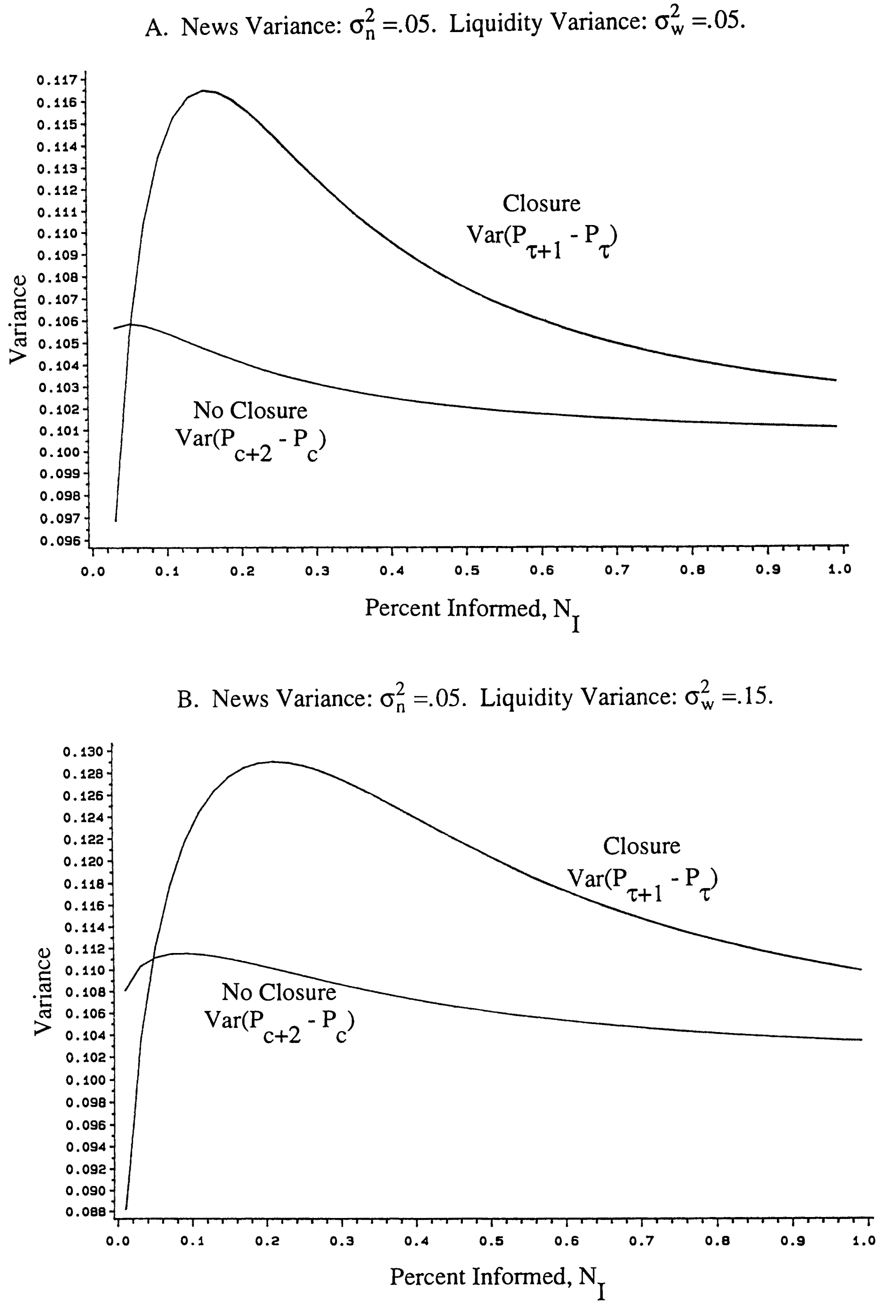
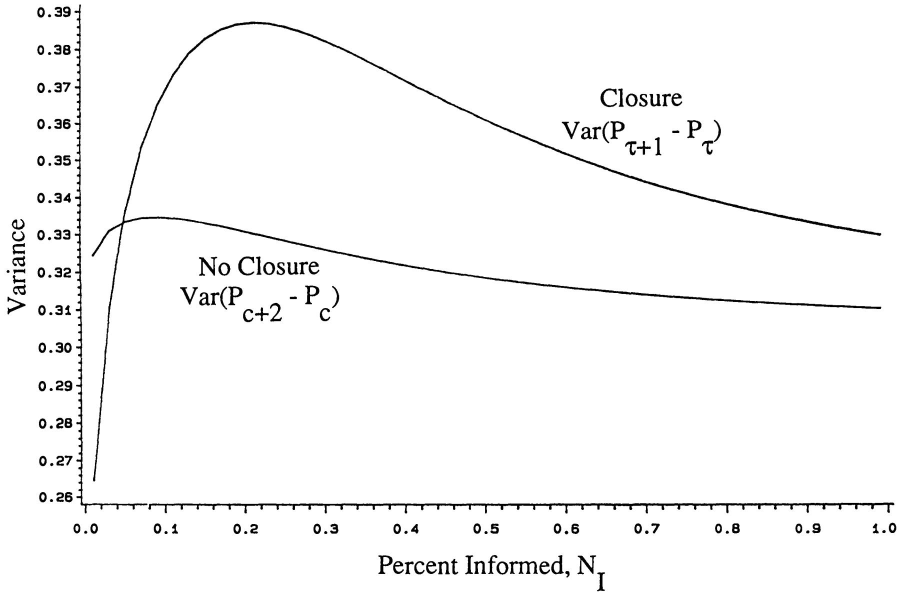


Figure 8. Variance comparisons: Closure versus no closure, as a function of the percentage informed (N_I). Each panel corresponds to a different combination of the variance of news and liquidity shocks. In all panels, the coefficient of risk aversion is $\gamma = 2$, and the mean asset supply is $\overline{X} = 1$. For comparison, the steady response equilibrium (SRE) variance function is also provided.

C. News Variance: $\sigma_n^2 = .15$. Liquidity Variance: $\sigma_w^2 = .05$.



D. News Variance: $\sigma_n^2 = .15$. Liquidity Variance: $\sigma_w^2 = .15$.

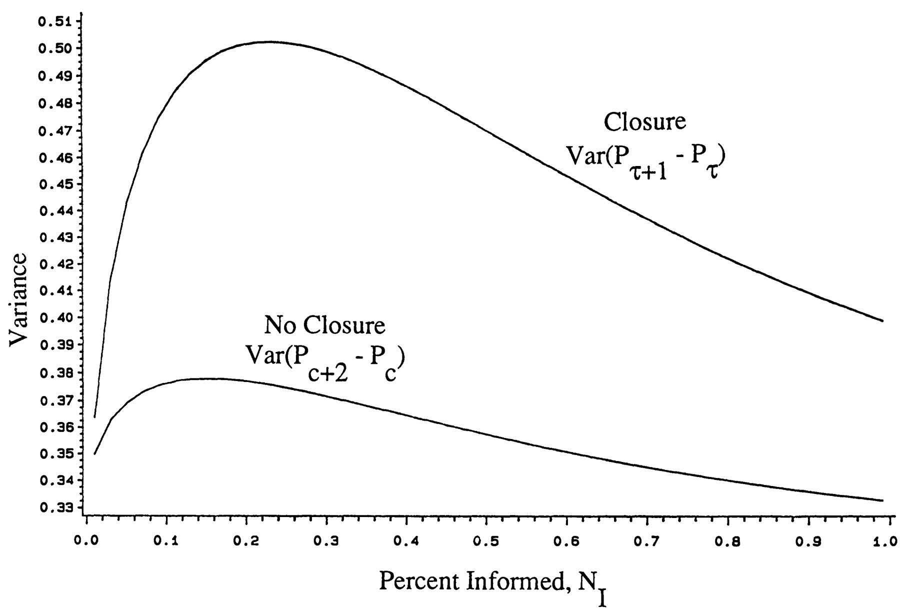


Figure 8—*Continued*

B.2. Mean Return Effects

Figures 9 and 10 illustrate how closures affect risk discounts and the mean price change by showing plots of the mean level of price and the mean price change for the closure and nonclosure (SRE) cases as a function of N_I. These figures illustrate the following results. First, Figures 9*A* and 9*B* show that

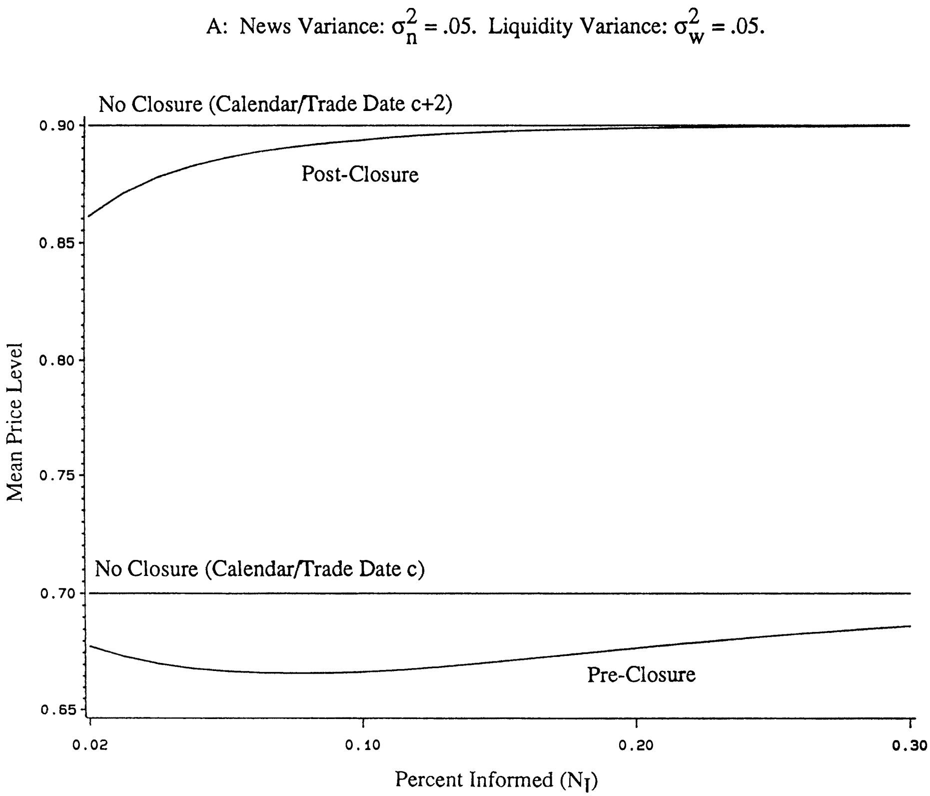


Figure 9. Mean price level; Closure versus no closure. The "Preclosure" *curve* plots the mean level of price as a function of the percentage informed (N_I) on the trade date prior the closure (trade date τ) while the "Postclosure" *curve* plots the mean level of price on the trade date after the closure (trade date $\tau + 1$). The "No Closure" *curves* plot the mean level of price as a function of the percentage informed on calendar and trade dates c and $c + 2$. Calendar dates c and $c + 2$ correspond to the pre- and postclosure trade dates τ and $\tau + 1$, with the market being closed on calendar date $c + 1$. Thus, to compare the mean level of price with a closure to the mean level that obtains in a steady response equilibrium (SRE) without a closure, compare the "Preclosure" *curve* to the "No Closure (Calendar/Trade Date c)" *curve*, and compare the "Postclosure" *curve* to the "No Closure (Calendar/Trade Date $c + 2$)" *curve*. In each figure, the price level on calendar date $c + 3$ is normalized to 1. Each panel corresponds to a different combination of variances for news (σ_n^2) and liquidity shocks (σ_w^2). In all panels, the coefficient of risk aversion is $\gamma = 2$, and the mean asset supply is $\bar{X} = 1$.

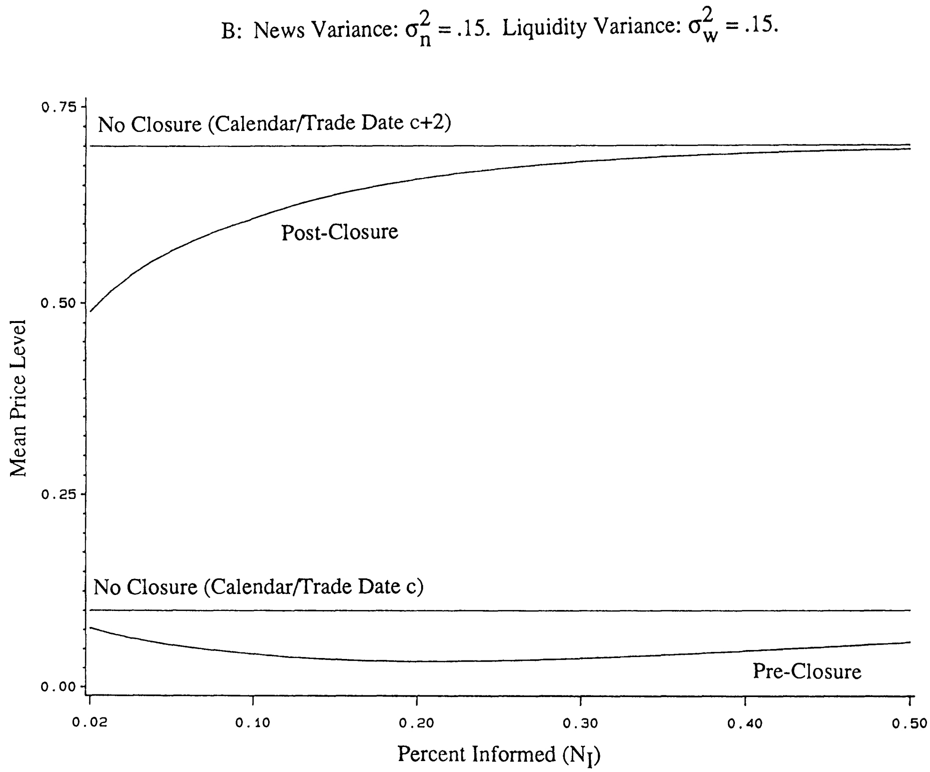


Figure 9—*Continued*

the risk discounts on either side of the closure are greater (i.e., the mean level of price is lower) than they are in a SRE. This reflects the fact that there is more risk on both sides of a closure; inference risk increases following closures, and insurance-trade risk exists both before and after closures. Second, the magnitude of the increase in pre- and postclosure risk discounts depends upon N_I. The postclosure risk discount increases the most when N_I is small, falling monotonically to the SRE level as N_I increases. The preclosure risk discount, however, increases the least when N_I is small, rising and then falling as N_I increases. Taken together, the mean price change, as plotted in Figure 10, is smaller over a closure than during SREs when N_I is small because the postclosure risk discount increases by more than the preclosure risk discount. As N_I increases, the closure mean returns increase and exceed the SRE mean returns as the increase in the preclosure risk discount exceeds the increase in the post-closure discount. As N_I increases further, the closure mean returns still exceed the SRE mean returns, but

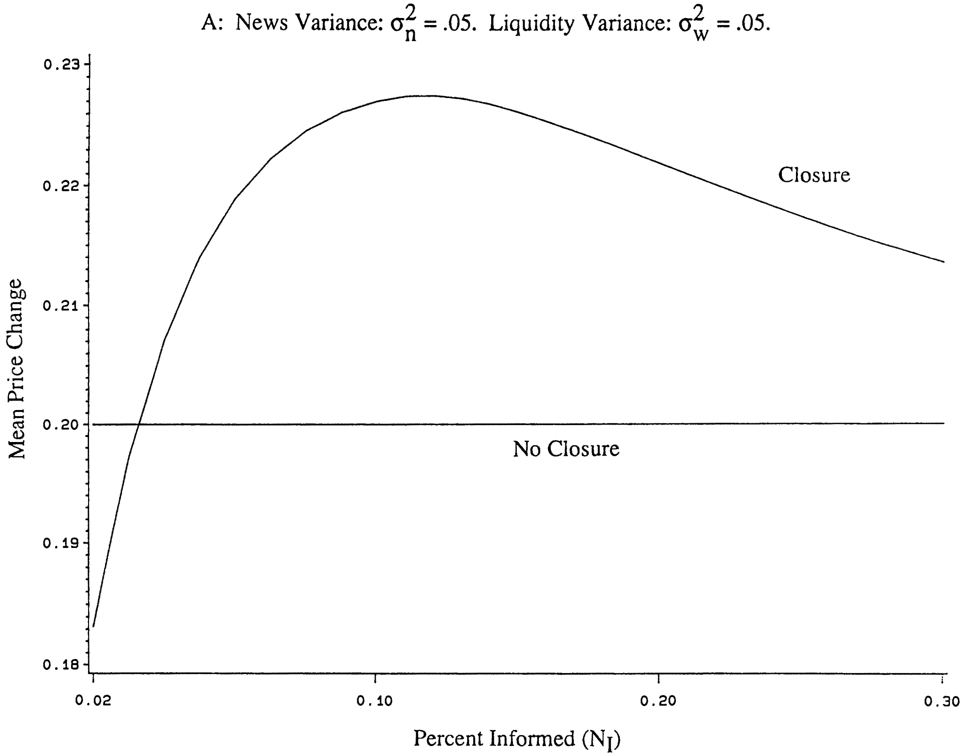


Figure 10. Mean price changes; Closure versus no closure. The "No Closure" *curve* plots the mean price change between calendar/trade dates c and $c + 2$ when the market does not close on $c + 1$ as a function of the percentage informed (N_I). The "Closure" *curve* plots the mean price change between calendar dates c and $c + 2$ when the market is closed on calendar date $c + 1$ (i.e., the mean price change between trade dates τ and $\tau + 1$). Each panel corresponds to a different combination of variances for news (σ_n^2) and liquidity shocks (σ_w^2). In all panels, the coefficient of risk aversion is $\gamma = 2$, and the mean asset supply is $\overline{X} = 1$.

begin to fall, converging to the SRE mean returns at $N_I = 1$, as both the pre- and postclosure risk discounts converge to the SRE values.

To understand how the mean returns depend upon N_I, we first consider its effect on the postclosure risk discount. Since the economy returns to a SRE the day after the market reopens, the amount of future variance due to temporary components is the same as in a SRE. Consequently, informed risk is unaffected by the closure and $\Psi^I_{\tau+1}$ and $\Psi^U_{\tau+1}$ equal their SRE values (see Lemma 4 in Appendix B). This implies that informed unconditional demand, as a function of the price coefficients, is the same as in a SRE. The increase in inference risk faced by uninformed agents, however, reduces uninformed unconditional demand. As a result, there is excess supply and the postclosure risk discount must increase for the market to clear. Since the increase in

B: News Variance: $\sigma_n^2 = .15$. Liquidity Variance: $\sigma_w^2 = .15$.

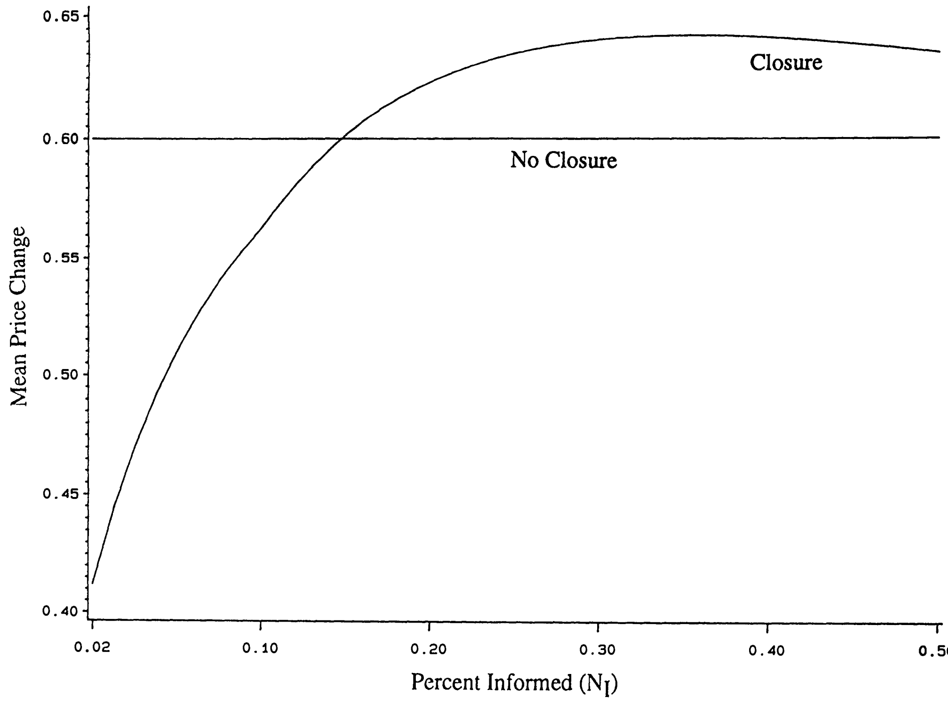


Figure 10—*Continued*

inference risk is greatest when N_I is small, it follows that the risk discount must increase the most in these cases.[24]

We next consider the preclosure risk discount. Lemma 4 in Appendix B implies that the preclosure risk discount is given by

$$\pi_0^{\tau} = \pi_0^{\tau+1} + \gamma\left(\overline{XG}_{11}^{\tau} - \Psi_{\tau}\right), \tag{26}$$

where $\gamma(\overline{XG}_{11}^{\tau} - \Psi_{\tau})$ is the average risk faced by informed and uninformed agents scaled by the coefficient of risk aversion and the asset supply.[25] For a

[24] Mathematically, the results of Lemma 4 (see Appendix B) and equation (21) imply that the risk discount following a closure is given by $\pi_0^{t+1} = \pi_0^{t+2} + \gamma[\overline{XG}_1^{t+1} - \Psi]$, where Ψ is the common SRE value of Ψ^I and Ψ^U. If the market had not been closed at $\tau + 1$, the risk discount at $\tau + 2$ (also trade date $t + 2$) would be $\pi_0^{t+2} = \pi_0^{t+3} - \gamma[\overline{XG}_{11} - \Psi]$, where $\overline{G}_{11}$ is the harmonic mean of G_{11}^I and G_{11}^U. Since informed risk is the same as in a SRE (i.e., $G_{11}^{It+1} = G_{11}^I$), but uninformed risk increases (i.e., $G_{11}^{Ut+1} > G_{11}^U$), it follows that $\overline{G}_{11}^{t+1}$ is bigger than $\overline{G}_{11}$, and, consequently, that the postclosure risk discount is larger than it would be in a SRE. Since uninformed inference risk increases the most when N_I is small (i.e., G_{11}^{Ut+1} is much larger than G_{11}^U), $\overline{G}_{11}^{t+1}$ is much bigger than $\overline{G}_{11}$ and closures induce the largest drop in price in these cases.

[25] Specifically, $\overline{G}_{11}^{\tau}$ is the harmonic mean of the preclosure values of $G_{11}^{I\tau}$ and $G_{11}^{U\tau}$ and Ψ_{τ} is the common value of the preclosure value of Ψ_{τ}^I and Ψ_{τ}^U.

given amount of average risk, the risk discount prior to the closure is larger than the SRE discount simply because the postclosure discount, as is argued above, is larger. The difference in the risk discounts (i.e., the mean price change), however, depends only on the average risk. Compared to the mean price change over the same length of time in a SRE (i.e., two times the daily SRE mean price change which, given the results of Lemma 3, is simply two times the average daily risk—i.e., $2\Delta = 2\gamma(\overline{XG}_{11} - \Psi)$, where Ψ is the common SRE value of Ψ^I and Ψ^U), the closure mean price change is smaller than the nonclosure mean price change when the average long-term risk over the closure (i.e., over two days) is less than twice the average daily long-term risk in a SRE.

The amount of risk faced over a closure relative to the amount of risk faced over two SRE days depends on N_I. When N_I is small, inference risk is high and the closure shifts most of the risk associated with the closure news into the postclosure market. That is, the risk associated with the part of the closure news that does not get reflected in the postclosure price is borne after the closure. If the market had been open, this risk would have been borne on the closure date instead. As a result, the amount of uncertainty resolved over a closure is much smaller than during a SRE, resulting in small mean price changes. Furthermore, when N_I is small the aggregate number of shares that informed agents want to sell to shed risk is small simply because there are few informed agents. Consequently, insurance-trade risk is small and does not significantly affect the mean price change.

As N_I increases, the portion of news that gets reflected in price increases, and thus less of the risk associated with closure news gets shifted to the postclosure market, resulting in larger mean price changes. In addition, since there are more informed agents who want to shed risk by trading with uninformed traders for insurance purposes, insurance-trade risk increases, increasing the preclosure risk discount further, accentuating the increase in the mean price change. As N_I approaches 1, however, the preclosure risk discount falls back to the SRE level as the risks faced by informed and uninformed agents become very similar. In these cases, there are fewer insurance-motivated trades (and, thus, little insurance-trade risk), and prices almost fully reflect news. Consequently the closure has a very small impact on risk and mean price changes.[26]

[26] In fact, when $N_I = 1$, it can be shown that $\Delta^t = \gamma\bar{X}\sigma^2_{n_{t+1}}$ even when $\sigma^2_{n_{t+1}}$ is not constant for all t. Thus, the mean return over a closure is $\Delta^t = \gamma\bar{X}\sigma^2_{n_{\tau+1}} = \gamma\bar{X}2\sigma_\eta^2$. The mean return from c to $c + 2$ in a SRE is two times the daily SRE mean return: $\pi_0^c - \pi_0^{c+2} = [(\pi_0^c - \pi_0^{c+1}) + (\pi_0^{c+1} - \pi_0^{c+2})] = 2\gamma\bar{X}\sigma_\eta^2$. Thus, a closure has no effect on mean returns when $N_I = 1$. Intuitively, this result holds because, when $N_I = 1$, there are no opportunities to exploit information even when the market is open. Furthermore, insurance-motivated trades do not occur since all agents are identical. Thus, the closure does not affect the returns and does not introduce insurance-motivated trade risk. Consequently, the risk at calendar date c is the same whether or not the market is closed at $c + 1$.

C. Additional Implications

An interesting implication of our model is that, in general, a closure affects the mean and variance of returns differently. That is, variances and mean returns over closures do not necessarily move in the same direction. For example, compare case 2 and case 6 in Table I. In case 6, both the mean and variance over the closure are smaller. Specifically, the ratio of closure-to-open variances is $0.621 < 1$, while the closure-to-open mean ratio is 0.542. In case 2, however, the mean return is lower (87.1 percent of the open mean) while the variance is greater (1.031 times larger than the open variances). This discrepancy occurs because of the difference between short-term and long-term risks. That is, short-term risk (i.e., G_{11}^{i}) affects the news response rate and the liquidity costs, which determine variance, while long-term risk (which depends on G_{11}^{i} *and* Ψ^{i}) affects mean returns.

In addition to mean and variance, a closure also affects trading volume. Since a closure alters the relative size of informed and uninformed risk, it creates insurance-motivated volume. Specifically, since informed postclosure risk is unaffected by a closure, while uninformed risk increases, uninformed traders reduce their unconditional holdings by selling to informed agents. This behavior creates volume that is independent of liquidity-induced volume. Furthermore, a closure also affects volume in the preclosure market. Relative to a SRE, the risk and return tradeoff becomes less favorable for both informed and uninformed agents prior to a closure. This is true because a closure (a) forecloses on one opportunity to trade (thereby reducing the ability of informed agents to exploit private information), (b) increases future inference risk, and (c) creates insurance-trade risk. Given this, it follows that both types of agents will attempt to reduce their preclosure unconditional holdings. Consequently, the risk discount must increase to restore equilibrium. Since the closure affects uninformed agents less than informed agents, the uninformed agents want to reduce their unconditional holding by less than informed agents do. Thus, in equilibrium, the risk discount increases to the point where uninformed agents actually increase their unconditional holdings by buying some of the informed agent's unconditional holdings. As this happens, preclosure volume increases.

IV. Conclusion

In this article, we develop a multiperiod model in which closures delay the resolution of uncertainty, redistributing risk across time and between different agents. In particular, after a closure, informed agents have a greater informational advantage, imposing more risk onto uninformed agents. Also, prior to a closure, risk increases for both informed and uninformed agents because there are fewer opportunities to trade. Informed agents' risk increases proportionally more, resulting in insurance-motivated trades that shift risk from informed agents to uninformed agents. Since these insurance-motivated trades must take place in the presence of adverse selection and liquidity trading, the closure introduces an additional source of risk by creating a need for insurance trades. This type of risk is also present in the postclosure market since uninformed agents want to shed risk due to the increase in postclosure inference risk.

Since agents are risk averse in the model, the redistribution of risk affects the equilibrium price, altering risk premia, liquidity costs, and the degree of informational asymmetry. As a result, both the mean and the variance of returns are affected by a closure. We demonstrate a number of mean and variance effects that are possible around closures, some of which are consistent with documented empirical regularities. Overall, the results indicate that empirical observations are most consistent with the presence of relatively few informed agents in the market. Furthermore, although we have explicitly examined closures, the results of the article show that any changes in the underlying processes of the arrival of news and liquidity shocks can lead to interesting and complicated price dynamics.

Appendix A

Proof of Theorem 1. Each trader maximizes his expected utility by picking asset holdings in each trading period. Traders solve for optimal demands by backward recursion. That is, the optimization problem at $T - 1$ is solved first. Then, the optimal demands at $T - 1$ are substituted into the objective function, thus defining the value function at $T - 1$. Then, given the information at $T - 2$, optimal demands at $T - 2$ are chosen to maximize the expected value of the $T - 1$ value function. This process continues back until period t. The maximization problem at $T - 1$ is simply a one-period optimization problem. It is well-known (see Grossman and Stiglitz (1980)) that the solution is

$$X_{T-1}^{*} = \frac{E_{T-1}(P_T) - P_{T-1}}{\gamma \mathrm{Var}_{T-1}(P_T)}, \tag{A1}$$

and the value function at $T - 1$ (defined as $J_{T-1} = \mathrm{Max}_{X_{T-1}} E_{T-1}[U(W_T)]$) is

$$J_{T-1} = -\exp\left\{ -\gamma \left[W_{T-1} + \frac{1}{2} \frac{(E_{T-1}(P_T) - P_{T-1})^2}{\gamma \mathrm{Var}_{T-1}(P_T)} \right] \right\}. \tag{A2}$$

At $T - 2$, the problem is to maximize $E_{T-2}[\tilde{J}_{T-1}]$ by choosing X_{T-2}. This problem is solved in Brown and Jennings (1989) and Grundy and McNichols (1989). They show that optimal demands are

$$X_{T-2}^{*} = \frac{E_{T-2}(P_{T-1}) - P_{T-2}}{\gamma G_{11}^{T-2}} + \frac{G_{11}^{T-2} - G_{12}^{T-2}}{G_{11}^{T-2}} E_{T-2}(X_{T-1}), \tag{A3}$$

where G_{11}^{T-2} and G_{12}^{T-2} are as defined in the text.

To solve the problem at T-3, substitute X^*_{T-2} into $E_{T-2}(\tilde{J}_{T-1})$. This yields

$$\begin{aligned}
J_{T-2} &\equiv \text{Max } E_{T-2}\left(\tilde{J}_{T-1}\right) \\
&= -K'_{T-2}\exp\Bigg\{-\gamma W_{T-2} + \gamma\left(\frac{E_{T-2}(P_{T-1}) - P_{T-2}}{\gamma G_{11}^{T-2}} + R_{T-2}\right)P_{T-2} \\
&\quad - \gamma\left(\frac{E_{T-2}(P_{T-1}) - P_{T-2}}{\gamma G_{11}^{T-2}} + R_{T-2}\right)E_{T-2}(P_{T-1}) \\
&\quad - \frac{1}{2\,\text{Var}_{T-1}(P_T)}\{E_{T-2}(E_{T-1}(P_T) - P_{T-1})\}^2 \\
&\quad + \frac{1}{2}\Bigg\{\gamma^2 G_{11}^{T-2}\left[\frac{E_{T-2}(P_{T-1}) - P_{T-2}}{\gamma G_{11}^{T-2}} + R_{T-2} - E_{T-2}(X_{T-2})\right]^2 \\
&\quad + 2\gamma^2 G_{12}^{T-2}\left[\frac{E_{T-2}(P_{T-1}) - P_{T-2}}{G_{11}^{T-2}} + R_{T-2}\right] \\
&\quad \cdot[E_{T-2}(X_{T-1})] + \gamma^2 G_{22}^{T-2}[E_{T-2}(X_{T-1})]^2\Bigg\} \\
&= -K'_{T-2}\exp\Bigg\{-\gamma(W_{T-3} + X_{T-3}(P_{T-2} - P_{T-3})) \\
&\quad - \frac{1}{2}\frac{(E_{T-2}(P_{T-1}) - P_{T-2})^2}{G_{11}^{T-2}} \\
&\quad + M_{T-2} - \gamma R_{T-2}[E_{T-2}(P_{T-1}) - P_{T-2}]\Bigg\} \qquad \text{(A4)}
\end{aligned}$$

where

$$R_{T-2} = \left(\frac{G_{11}^{T-2} - G_{12}^{T-2}}{G_{11}^{T-2}}\right)E_{T-2}(X_{T-1}) \quad \text{and}$$

$$\begin{aligned}
M_{T-2} \equiv \frac{\gamma^2}{2}[E_{T-2}(X_{T-1})]^2\Bigg[&-\text{Var}_{T-1}(P_T) + (G_{11}^{T-2} + G_{22}^{T-2}) \\
&+ \frac{(G_{11}^{T-2} - G_{12}^{T-2})^2}{G_{11}^{T-2}} \\
&+ \left(\frac{G_{11}^{T-2} - G_{12}^{T-2}}{G_{11}^{T-2}}\right) \\
&(2G_{12}^{T-2} - 2G_{11}^{T-2})\Bigg].
\end{aligned}$$

M_{T-2} is a constant since $E_{T-2}(X_{T-1}) = -\pi_0^{T-1}/\gamma\sigma_{P_T}^2$ is a constant with respect to information at $T-2$. Thus J_{T-2} can be written as

$$\begin{aligned} J_{T-2} &= -K'_{T-2}\exp\{-\gamma W_{T-3} + \gamma X_{T-3}P_{T-3} + M_{T-2}\} \\ &\quad \cdot \exp\left\{-\gamma X_{T-3}P_{T-2} - \frac{1}{2}\frac{(E_{T-2}(P_{T-1}) - P_{T-2})^2}{G_{11}^{T-2}} \right. \\ &\quad \left. -(E_{T-2}(P_{T-1}) - P_{T-2})\gamma R_{T-2}\right\} \\ &= -K'_{T-2}\exp\{-\gamma W_{T-3} + \gamma X_{T-3}P_{T-3} + M_{T-2}\} \\ &\quad \cdot \exp\{D'_{T-3}Z_{T-3} - Z'_{T-3}V_{T-2}Z_{T-3} - Z'_{T-3}V_{T-2}Y_{T-2}\}, \qquad \text{(A5)} \end{aligned}$$

where

$$\begin{aligned} D'_{T-3} &\equiv (-\gamma X_{T-3}, 0), Z'_{T-3} \equiv (P_{T-2}, E_{T-2}(P_{T-1})), \\ V_{T-2} &= \frac{1}{2G_{11}^{T-2}}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \text{ and} \\ Y'_{T-3} &\equiv \left(-2\gamma(G_{11}^{T-2} - G_{12}^{T-2})E_{T-2}(X_{T-1}), 0\right). \end{aligned}$$

The expectation of $\tilde{J}_{T-2}$ at $T-3$ is

$$\begin{aligned} E_{T-3}\left[\tilde{J}_{T-2}\right] &= -K_{T-3}\exp\{-\gamma W_{T-3} + \gamma X_{T-3}P_{T-3} + M_{T-2}\} \\ &\quad \cdot \int \exp\left\{D'_{T-3}Z_{T-3} - Z'_{T-3}V_{T-2}Z_{T-3} - Z'_{T-3}V_{T-2}Y_{T-3} \right. \\ &\quad \left. -\frac{1}{2}\left(Z_{t-3} - \bar{Z}_{t-3}\right)'\Omega_{T-3}^{-1}\left(Z_{T-3} - \bar{Z}_{T-3}\right)\right\} dF(Z_{T-3}), \qquad \text{(A6)} \end{aligned}$$

where K_{T-3} is such that the p.d.f. integrates to 1, and

$$\bar{Z}'_{T-3} = E_{T-3}(Z'_{T-3}) = (E_{T-3}(P_{T-2}), E_{T-3}(E_{T-2}(P_{T-1}))).$$

Completing the square yields

$$\begin{aligned} E_{T-3}\left[\tilde{J}_{T-2}\right] &= -K_{T-3}\exp\left\{-\gamma W_{T-3} + \gamma X_{T-3}P_{T-3} + M_{T-2} \right. \\ &\quad \left. + D'_{T-3}\bar{Z}_{T-3} - C'_{T-3}\bar{Z}_{T-3} + \bar{Z}'_{T-3}V_{T-2}\bar{Z}_{T-3}\right\} \\ &\quad + \int \exp\left\{(D_{T-3} - C_{T-3})'\left(Z_{T-3} - \bar{Z}_{T-3}\right) \right. \\ &\quad -\left(Z_{T-3} - \bar{Z}_{T-3}\right)'V_{T-2}\left(Z_{T-3} - \bar{Z}_{T-3}\right) \\ &\quad \left. -\frac{1}{2}\left(Z_{T-3} - \bar{Z}_{T-3}\right)'\Omega_{T-3}^{-1}\left(Z_{T-3} - \bar{Z}_{T-3}\right)\right\} dF(Z_{T-3}) \end{aligned}$$

$$= -K_{T-3}\exp\Big\{-\gamma W_{T-3} + \gamma X_{T-3}P_{T-3} + M_{T-2} + (D_{T-3} - C_{T-3})'\bar{Z}_{T-3} + \bar{Z}'_{T-3}V_{T-2}\bar{Z}_{T-3} + \frac{1}{2}(D_{T-3} - C_{T-3})'G^{T-3}(D_{T-3} - C_{T-3})\Big\}, \tag{A7}$$

where $G^{T-3} = (2V_{T-2} + \Omega_{T-3}^{-1})^{-1}$, and $C_{T-3} = V_{T-2}(2\bar{Z}_{T-3} + Y_{T-3}) = \begin{pmatrix} -\gamma E_{T-3}(X_{T-2}) \\ \gamma E_{T-3}(X_{T-2}) \end{pmatrix}$, and Ω_{T-3} is as defined in the text. Rewriting all of the terms yields

$$\begin{aligned} E_{T-3}\big[\tilde{J}_{T-2}\big] &= J_{T-3} \\ &= -K_{T-3}\exp\Big\{-\gamma W_{T-3} + \gamma X_{T-3}P_{T-3} + \big(M_{T-2} + C'_{T-3}\bar{Z}_{T-3}\big) \\ &\quad -\gamma X_{T-3}E_{T-3}(P_{T-2}) + \frac{1}{2G_{11}^{T-2}}[E_{T-3}(E_{T-2}(P_{T-1}) - P_{T-2})]^2 \\ &\quad + \frac{1}{2}\Big[\gamma^2 G_{11}^{T-3}X_{T-3}^2 - 2X_{T-3}\gamma^2 E_{T-3}(X_{T-2})\big[G_{11}^{T-3} - G_{12}^{T-3}\big] \\ &\quad + \gamma^2[E_{T-3}(X_{T-2})]^2\big[G_{22}^{T-3} - 2G_{12}^{T-3} + G_{11}^{T-3}\big]\Big]\Big\} \\ &\propto X_{T-3}(E_{T-3}(P_{T-2}) - P_{T-3}) - \frac{1}{2}\big[\gamma G_{11}^{T-3}X_{T-3}^2 \\ &\quad -2X_{T-3}\gamma E_{T-3}(X_{T-2})(G_{11}^{T-3} - G_{12}^{T-3})\big]. \end{aligned} \tag{A8}$$

The first-order condition is

$$(E_{T-3}(P_{T-2}) - P_{T-3}) - \gamma G_{11}^{T-3}X_{T-3} + \gamma(G_{11}^{T-3} - G_{12}^{T-3})E_{T-3}(X_{T-2}) = 0,$$

which implies

$$X_{T-3} = \frac{E_{T-3}(P_{T-2} - P_{T-3})}{\gamma G_{11}^{T-3}} + \left(\frac{G_{11}^{T-3} - G_{12}^{T-3}}{G_{11}^{T-3}}\right)E_{T-3}(X_{T-2}). \tag{A9}$$

The problem at $t, t < T-2$, can be solved in a similar manner by defining, for every t, $D'_t = (-\gamma X_t, 0)$, $Z'_t = (P_{t+1}, E_{t+1}(P_{t+2}))$, $\bar{Z}'_t = E_t(Z'_t) = (E_t(P_{t+1}), E_t(E_{t+1}(P_{t+2})))$, $Y'_t = (-2\gamma(G_{11}^{t+1} - G_{12}^{t+1})E_{t+1}(X_{t+2}), 0)$, $C'_t = (-\gamma E_t(X_{t+1}), \gamma E_t(X_{t+1}))$, and V_t, G^t, Ω_t are as defined in the text. Q.E.D.

Appendix B

I. Closed-form Demands

Given the form of the price function, closed-form expressions of the elements of Ω^{it} that define G_{11}^{it} can easily be obtained. Closed-form expressions for the elements of Ω^{It} are

$$\Omega_{11}^{It} = (\pi_1^{t+1})^2 \sigma_{n_{t+1}}^2 + (\pi_2^{t+1})^2 \sigma_{w_{t+1}}^2, \tag{B1}$$

$$\Omega_{12}^{It} = \Omega_{21}^{It} - \pi_1^{t+1} \sigma_{n_{t+1}}^2, \tag{B2}$$

and

$$\Omega_{22}^{It} = \sigma_{n_{t+1}}^2, \tag{B3}$$

while closed-form expressions for the elements of Ω^{Ut} are

$$\Omega_{11}^{Ut} = \Omega_{11}^{It} + (1 - \lambda_1^t)^2 \sigma_{n_t}^2 + (\lambda_2^t)^2 \sigma_{w_t}^2, \tag{B4}$$

$$\Omega_{12}^{Ut} = \Omega_{21}^{Ut} = (1 - \lambda_1^t)^2 \sigma_{n_t}^2 + (\lambda_2^t)^2 \sigma_{w_t}^2 + \pi_1^{t+1} \lambda_1^{t+1} \sigma_{n_{t+1}}^2 - \pi_2^{t+1} \lambda_2^{t+1} \sigma_{w_{t+1}}^2, \tag{B5}$$

and

$$\Omega_{22}^{Ut} = (1 - \lambda_1^t)^2 \sigma_{n_t}^2 + (\lambda_2^t)^2 \sigma_{w_t}^2 + (\lambda_1^{t+1})^2 \sigma_{n_{t+1}}^2 + (\lambda_2^{t+1})^2 \sigma_{w_{t+1}}^2. \tag{B6}$$

Further, Ψ_t^I also depends upon the elements of Ω^{It} through G_{11}^{It} and G_{12}^{It}, as well as future values of Ψ^I and G_{11}^I. Ψ_t^i, $i = I$ and U, is given by

$$\Psi_t^i = (G_{11}^{it} - G_{12}^{it}) \left(\frac{\Delta^{t+1} + \gamma \Psi_{t+1}^i}{\gamma G_{11}^{it+1}} \right), \tag{B7}$$

where the elements of the matrix G^{it} depend upon the elements of Ω^{it} and G_{11}^{it+1} according to

$$G^{it} = \begin{pmatrix} \dfrac{h_t^i + \Omega_{11}^{it} G_{11}^{it+1}}{g_t^i} & \dfrac{h_t^i + \Omega_{12}^{it} G_{11}^{it+1}}{g_t^i} \\[2ex] \dfrac{h_t^i + \Omega_{12}^{it} G_{11}^{it+1}}{g_t^i} & \dfrac{h_t^i + \Omega_{22}^{it} G_{11}^{it+1}}{g_t^i} \end{pmatrix} \tag{B8}$$

with $h_t^i \equiv \Omega_{11}^{it} \Omega_{22}^{it} - (\Omega_{12}^{it})^2$ and $g_t^i \equiv \Omega_{11}^{it} + \Omega_{22}^{it} - 2\Omega_{12}^{it}$.

II. Proof of Theorem 2

The proof of theorem 2 requires the following two lemmas.

LEMMA 1: $\pi_2^t = \pi_1^t (\gamma G_{11}^{It} / N_I)$.

Proof: Define Q as

$$Q = \frac{N_I}{\gamma G_{11}^{It}} + \frac{N_U}{\gamma G_{11}^{Ut}}\left[1 - \frac{\pi_1^t \sigma_{n_t}^2}{(\pi_1^t)^2 \sigma_{n_t}^2 + (\pi_2^t)^2 \sigma_{w_t}^2}\right]. \tag{B9}$$

Equation (17) set to zero implies $\pi_1 \gamma G_{11}^{It}/N_I = 1/Q$ while equation (18) set to zero implies $\pi_2 = 1/Q$. Equating left-hand sides and rearranging the terms yields the relationship between π_1 and π_2 specified in the lemma. Q.E.D.

LEMMA 2: *λ_1^t, λ_2^t, and G_{11}^{Ut} are independent of π_1^t and π_2^t.*

Proof: We first show that λ_1^t and λ_2^t are independent of π_1^t and π_2^t. Given Lemma 1, it can be shown that $\lambda_2^t = \lambda_1^t(\pi_2^t/\pi_1^t)$. Thus, if λ_1^t is independent of π_1^t and π_2^t, then λ_2^t is also independent of π_1^t and π_2^t since, by Lemma 1, (π_2^t/π_1^t) is independent of π_1^t and π_2^t. Replacing $\pi_2^t \sigma_{w_t}^2$ by $(\pi_1^2)^2(\gamma G_{11}^{It}/N_I)\sigma_{w_t}^2$ in the definition of λ_1^t yields

$$\lambda_1^t = \frac{(\pi_1^t)^2 \sigma_{n_t}^2}{(\pi_1^t)^2\left[\sigma_{n_t}^2 + \left(\dfrac{\gamma G_{11}^{It}}{N_I}\right)\sigma_{w_t}^2\right]} = \frac{\sigma_{n_t}^2}{\sigma_{n_t}^2 + \left(\dfrac{\gamma G_{11}^{It}}{N_I}\right)^2 \sigma_{w_t}^2}, \tag{B10}$$

which, because G_{11}^{It} depends only on future values of π_1 and π_2, does not contain either π_1^t or π_2^t, thus yielding the result. The only argument of G_{11}^{Ut} that is possibly a function of π_1^t and π_2^t is Ω_{11}^{Ut} (see equations (B4) through (B6)). The only part of Ω_{11}^{Ut} that is possibly a function of π_1^t and π_2^t is $(1 - \lambda_1^t)^2\sigma_{n_t}^2 + (\lambda_2^t)^2\sigma_{w_t}^2$, which, by the first part of this lemma, is not a function of π_1^t or π_2^t. Q.E.D.

Proof of Theorem 2. Part (a) is obvious given equations (B1), (B2), (B3), and (B7). Part (b) obtains after algebraically manipulating equations (16), (17), and (18) with $\Pi_0^t = \Pi_1^t = \Pi_2^t = 0$, using the results of Lemmas 1 and 2. Q.E.D.

III. Proofs for Propositions 1, 2, and 3

The effect of a closure on the price function can be examined using the total differential of equations (17) and (18) when $\Pi_1^t = \Pi_2^t = 0$. The total differential of π_1 from equation (17), evaluated at SRE values, is

$$d\pi_1 = \left(\frac{1}{\dfrac{N_I}{G_{11}^I} + \dfrac{N_U}{G_{11}^U}}\right)\left\{\left[\frac{N_I}{(G_{11}^I)^2}(\pi_1 - 1) - \frac{N_U}{G_{11}^U}\frac{\partial \lambda_1}{\partial G_{11}^I}\right]dG_{11}^I + \left[\frac{N_U}{(G_{11}^U)^2}(\pi_1 - \lambda_1)\right]dG_{11}^U\right\}. \tag{B11}$$

The total differential of π_2 from equation (18) is:

$$d\pi_2 = \left(\frac{1}{\frac{N_I}{G_{11}^I} + \frac{N_U}{G_{11}^U}}\right)\left\{\left[\frac{N_1}{G_{11}^I}\frac{\pi_2}{G_{11}^I} - \frac{N_U}{G_{11}^I}\frac{\partial\lambda_2}{\partial G_{11}^I}\right]dG_{11}^I + \left[\frac{N_U}{G_{11}^U}\frac{\lambda_2 + \pi_2}{G_{11}^U}\right]dG_{11}^U\right\}. \tag{B12}$$

Proof of Proposition 1. Since informed risk after a closure is the same as in a SRE, $dG_{11}^I = 0$. However, uninformed risk is bigger, and $dG_{11}^U > 0$. Since the coefficient on dG_{11}^U in equation (B11) is positive, $d\pi_1$ is positive, and price reflects more news. Q.E.D.

Proof of Proposition 2. The coefficient on dG_{11}^U in equation (B12) is also positive. Thus, since $dG_{11}^I = 0$ and $dG_{11}^U > 0$ after a closure, $d\pi_2$ is positive, and postclosure liquidity costs are greater than SRE liquidity costs. Q.E.D.

Proof of Proposition 3. By equation (19) in theorem 2, π_1 is the weighted average of 1 and λ_1, with weights that depend upon the relative size of each type of agent's per-agent risk (N_I/G_{11}^I). The effect of the closure on π_1 depends upon how it affects the weights and λ_1. It can easily be shown that λ_1 is a decreasing function of G_{11}^I. Consequently, if the weight on λ_1 increases, π_1 unambiguously falls. It can also be shown that the weight on λ_1 is larger when G_{11}^U increases proportionally less than G_{11}^I. Since this last condition holds prior to a closure, preclosure π_1 is less than the SRE value of π_1. Q.E.D.

IV. Mean Return Results

The following two Lemmas are required to derive mean return implications. The proof of Proposition 5, which describes the postclosure risk discount, follows these Lemmas.

LEMMA 3: *In a SRE, the values of Ψ_t^I and Ψ_t^U are constant and equal for all t. That is,*

$$\Psi_t^I = \Psi_t^U = \Psi \equiv \frac{(G_{11}^I - G_{12}^I)}{G_{12}^I}\frac{\Delta}{\gamma} \tag{B13}$$

for all t, where Δ, G_{11}^I, and G_{12}^I are the SRE values of the mean return, G_{11}^{It} and G_{12}^{It}, respectively.

Proof of Lemma 3. The SRE value of Ψ_t^i ($i = I$ and U) is obtained by finding the value Ψ^i that satisfies equation (B7) and the additional conditions that $\Psi_t^i = \Psi_{t+1}^i = \Psi^i$, and G^{it}, G^{it+1}, and Δ^{t+1} are their SRE values (i.e., when $G^{it} = G^{it+1} = G^i$ and $\Delta^{t+1} = \Delta$). The solution is

$$\Psi^i = \left(\frac{G_{11}^i - G_{12}^i}{G_{12}^i} \right) \frac{\Delta}{\gamma}, \tag{B14}$$

for $i + I$ and U. Although Δ^t is a function of Ψ_t^I and Ψ_t^U according to equation (21), it is shown below that SRE Δ is independent of Ψ^I or Ψ^U. As such, equation (B14) represents a reduced-form solution for Ψ^i.

SRE values for $(G_{11}^{It} - G_{12}^{It})/G_{12}^{It}$ and $(G_{11}^{Ut} - G_{12}^{Ut})/G_{12}^{Ut}$ are obtained by first finding SRE values of the elements of the matrix Ω_t^i during a SRE. Let Ω^i denote this matrix. Replacing σ_n^2 for $\sigma_{n_t}^2$ and $\sigma_{n_{t+1}}^2$ and σ_w^2 for $\sigma_{w_t}^2$ and $\sigma_{w_{t+1}}^2$ in equations (B1) through (B3) and (B4) through (B6) yields the elements of the matrix Ω^i. The matrix G^{it} that occurs during a SRE is the matrix G^i that satisfies equation (B8) and $G_{11}^{it} = G_{11}^i$, where G_{11}^i is the 1, 1 element of G^i. Solving for each element of G^i as a function of G_{11}^I and rearranging yields

$$\frac{G_{11}^I - G_{12}^I}{G_{12}^I} = \frac{\Omega_{11}^I - \Omega_{12}^I}{\Omega_{22}^I - \Omega_{12}^I + G_{11}^I} \tag{B15}$$

and

$$\frac{G_{11}^U - G_{12}^U}{G_{12}^U} = \frac{\Omega_{11}^I - \Omega_{12}^I}{\Omega_{22}^I - \Omega_{12}^I + G_{11}^U - (1 - \lambda_1)\sigma_n^2}. \tag{B16}$$

Comparing G_{11}^I and G_{11}^U as defined by equation (B8) also yields $G_{11}^I = G_{11}^U - (1 - \lambda_1)\sigma_n^2$.

Substituting $G_{11}^I = G_{11}^U - (1 - \lambda_1)\sigma_n^2$ into equation (B16) implies that the right-hand sides of equations (B15) and (B16) are equal. Thus, $(G_{11}^{It} - G_{12}^{It})/G_{12}^{It} = (G_{11}^{Ut} - G_{12}^{Ut})/G_{12}^{Ut}$, and $\Psi^I = \Psi^U = \Psi$. Q.E.D.

LEMMA 4:

(a) *After a closure*, $\Psi_{\tau+1}^I = \Psi_{\tau+1}^U = \Psi$.
(b) *Prior to a closure* $\Psi_\tau^I = \Psi_\tau^U$.

Proof of Lemma 4(a). By equation (B7),

$$\Psi_{\tau+1}^i = (G_{11}^{i\tau+1} - G_{12}^{i\tau+1}) \left[\frac{\Delta^{\tau+2} + \gamma \Psi_{\tau+2}^i}{\gamma G_{11}^{i\tau+2}} \right]. \tag{B17}$$

Since we assume that the economy returns to a SRE the second period following the closure, $G_{11}^{I\tau+1}$, $G_{12}^{I\tau+1}$, $G_{11}^{I\tau+2}$, $\Delta^{\tau+2}$, and $\Psi_{\tau+2}^I$ are SRE values. Thus, the right-hand side of equation (B17) is the definition of a SRE Ψ^I. As a result, $\Psi_{\tau+1}^I = \Psi^I$. For uninformed agents, the closed-form expression of $G_{11}^{U\tau+1} - G_{12}^{U\tau+1}$ can be calculated in the manner outlined in the proof of Lemma 3. The result shows that $G_{11}^{U\tau+1} - G_{12}^{U\tau+1}$ is independent of $\sigma_{n_{\tau+1}}^2$ and $\sigma_{w_{\tau+1}}^2$. Thus, it depends only on the future news and liquidity shock variances. Since the future shock variances are the SRE constants, $G_{11}^{U\tau+1} - G_{12}^{U\tau+1}$ is the same as the SRE value. Furthermore, since $G_{11}^{U\tau+2}$, $\Delta^{\tau+2}$ and $\Psi_{\tau+2}^{U}$ are SRE values, the right-hand side of equation (B17) is the definition of a SRE Ψ^{U}. Consequently, $\Psi_{\tau+1}^{U} = \Psi^{U}$. Because $\Psi^{U} = \Psi^{I} = \Psi$ by Lemma 3, $\Psi_{\tau+1}^{I} = \Psi_{\tau+1}^{U} = \Psi$. Q.E.D.

Proof of Lemma 4(b). The definition of Ψ_t^I and Ψ_t^U from equation (B7) and the result that $\Psi_{\tau+1}^{U} = \Psi_{\tau+1}^{I} = \Psi$ implies that

$$\Psi_\tau^i = \left(\frac{G_{11}^{i\tau} - G_{12}^{i\tau}}{G_{11}^{i\tau+1}}\right)\left[\frac{\Delta^{\tau+1}}{\gamma} + \Psi\right]. \tag{B18}$$

Since the term in square brackets is the same for $i = I$ and U, the second part of the lemma holds if $(G_{11}^{I\tau} - G_{12}^{I\tau})/G_{11}^{I\tau+1} = (G_{11}^{U\tau} - G_{12}^{U\tau})/G_{11}^{U\tau+1}$. Using equation (B8), it can be verified that

$$\frac{G_{11}^{I\tau} - G_{12}^{I\tau}}{G_{11}^{I\tau+1}} = \frac{G_{11}^{I\tau} - G_{12}^{I\tau}}{G_{11}^{I}} = \frac{\Omega_{11}^{I\tau} - \Omega_{12}^{I\tau}}{\Omega_{22}^{I\tau} - \Omega_{12}^{I\tau} + G_{11}^{I}}, \tag{B19}$$

$$\frac{G_{11}^{U\tau} - G_{12}^{U\tau}}{G_{11}^{U\tau+1}} = \frac{\Omega_{11}^{I\tau} - \Omega_{12}^{I\tau}}{\Omega_{22}^{I\tau} - \Omega_{12}^{I\tau} + G_{11}^{U\tau+1} - (1 - \lambda_1^{\tau+1})2\sigma_n^2}, \tag{B20}$$

and

$$G_{11}^{U\tau+1} = G_{11}^{I} + (1 - \lambda_1^{\tau+1})2\sigma_n^2. \tag{B21}$$

Substituting equation (B21) into equation (B20) shows that the right-hand sides of equations (B19) and (B20) are equal and, thus, $\Psi_\tau^I = \Psi_\tau^U$. However, since $G_{11}^{I\tau} \neq G_{11}^{I}$ and $G_{11}^{U\tau} \neq G_{11}^{U}$, $\Psi_\tau^I = \Psi_\tau^U$ do not equal Ψ. Q.E.D.

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