



American Finance Association

Arbitrage Chains

Author(s): James Dow and Gary Gorton

Source: *The Journal of Finance*, Vol. 49, No. 3, Papers and Proceedings Fifty-Fourth Annual Meeting of the American Finance Association, Boston, Massachusetts, January 3-5, 1994 (Jul., 1994), pp. 819-849

Published by: Blackwell Publishing for the American Finance Association

Stable URL: <http://www.jstor.org/stable/2329208>

Accessed: 25/11/2009 10:38

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/action/showPublisher?publisherCode=black>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.



Blackwell Publishing and American Finance Association are collaborating with JSTOR to digitize, preserve and extend access to *The Journal of Finance*.

<http://www.jstor.org>

Arbitrage Chains

JAMES DOW and GARY GORTON*

ABSTRACT

A privately informed trader will engage in costly arbitrage, that is, trade on his knowledge that the price of an asset is different from the fundamental value if: (1) his order does not move the price immediately to reflect the information; and (2) he can hold the asset until the date when the information is reflected in the price. We study a general equilibrium model in which all agents optimize. In each period, there may be a trader with a limited horizon who has private information about a distant event. Whether he acts on his information, and whether subsequent informed traders act, is shown to depend on the possibility of a sequence or *chain* of future informed traders spanning the event date. An arbitrageur who receives good news will buy only if it is likely that, at the end of his trading horizon, a subsequent arbitrageur's buying will have pushed up the expected price. We show that limited trading horizons result in inefficient prices, because informed traders do not act on their information until the event date is sufficiently close. We also show that limited horizons can arise because of the cost-of-carry associated with holding an arbitrage portfolio over an extended period of time.

THE EFFICIENCY OF SECURITY prices depends upon arbitrage, that is, trading based upon knowledge that the price of an asset is different from its fundamental value. (Although the term "arbitrage," strictly speaking, refers to an entirely riskless speculation, we use the term in the broader sense common among practitioners.) For example, suppose an agent has private information about a high future dividend to be paid by a firm. If the current stock price does not reflect this information, then the agent can profit by buying the stock, if his purchase does not instantly raise the price, and holding it until the dividend is paid.

The argument depends on two assumptions. First, the arbitrageur's information cannot be instantly reflected in the price he pays upon submission of the order. Second, the arbitrageur must be able to hold the security until the dividend is paid (in order to realize the profit).

The first assumption has been widely studied (Ausubel (1990), Diamond and Verrechia (1981), Grossman and Stiglitz (1976), Kyle (1985)). The second assumption, that the arbitrageur's horizon span the event date (i.e., the date at which the dividend arrives in the example) has received less attention (the relevant literature is discussed below). But this second assumption is also crucial for the argument. There are several reasons why the informed trader's trading horizon may not span the event date. A portfolio manager may have

*Dow is from London Business School, and Gorton is from the Wharton School of the University of Pennsylvania. We thank Bruno Biais, Sam Orez, Jean-Charles Rochet, and José Scheinkman for helpful discussions.

to liquidate his portfolio to make large distributions to pensioners. Alternatively, he may need to show good performance over a short horizon at the end of which his performance is assessed. A third explanation, which we explore in detail below, is that the cost-of-carry of an arbitrage portfolio may make long-term arbitrage prohibitively costly.

Although the second assumption is often not satisfied, arbitrage may still be profitable if the price adjusts to reflect the information by the time the arbitrageur must close out his position. On the one hand, an informed trader with a limited horizon will not trade on his information (if arbitrage is costly) if he believes that tomorrow's price, when he must sell the stock, will not reflect the information. Since he does not buy the stock to start with, the price cannot reflect his information. On the other hand, he may believe that tomorrow another informed trader will arrive and buy the stock, pushing the price up so as to make the arbitrage profitable. In this case, he trades on his information, and this may have the effect of making the price more informative immediately. Of course, the complication is that if the next informed trader does not believe that by the end of his horizon yet another informed trader will push up the price, then *he* will not buy and the chain of arbitrages will unravel.

With limited trading horizons arbitrageurs' decisions to trade are influenced by their beliefs about the distribution of the private information across other (future) arbitrageurs with different, perhaps overlapping, trading horizons. If there is the possibility of a sequence of arbitrageurs with overlapping trading horizons that span the event date, then the decision of the first arbitrageur in the sequence depends on his beliefs about the subsequent chain of arbitrageurs and their decisions. The question we address is whether the actions of arbitrageurs in the chain can replicate the behavior of a single long-lived arbitrageur, or whether it is possible that there could be private information that is not acted upon.

The conventional wisdom is that short horizons, in themselves, will not cause assets to be mispriced. The reasoning is as follows. Suppose that a cash flow in twenty periods is not correctly priced currently. It will certainly be priced correctly in period twenty. Then in the nineteenth period an agent with a one-period horizon will insure that the asset is priced correctly. Since the asset will be priced in the nineteenth period, an agent with a one period horizon in the eighteenth period will insure that it is also correctly priced then. By backward induction, the asset must be correctly priced today. Our article questions this conventional wisdom by examining this argument in more detail. In particular, while this argument certainly applies to cases in which information is public, we argue that it does not extend to the case of private information.

Examples of situations where private information is too long-term relative to traders' horizons include the following. Foreign exchange traders may be unwilling to speculate on anticipated exchange-rate movements in two years' time. Government-bond arbitrageurs may be unwilling to speculate on perceived mispricings that are not corrected within days. Equity portfolio man-

agers may pay little attention to information concerning the prospects of companies twenty years from now unless they believe it will be reflected in the price within a year or two.

We study a general equilibrium model where informed traders (arbitrageurs) have limited horizons and in which there are no exogenous “noise” traders. The model is one of overlapping generations where prices are formed by a marketmaker. Arbitrage is costly (either because of a brokerage fee or because of a borrowing cost). In the model risk-averse uninformed traders choose an amount to trade based on hedging motives. Each period there is a probability of an informed trader (arbitrageur) arriving who may choose to trade. Informed traders have private information about a future dividend that, for the first such trader, will arrive beyond his lifetime. Since arbitrageurs profit at the expense of the hedgers, the amount the hedgers trade can change depending on whether they believe there are informed traders operating or not.

The main result of the article is that informed agents will *not* engage in arbitrage a long time in advance of the event. In particular, only if the probability of another arbitrageur arriving next period is high enough will an informed agent trade this period. Consequently, information can arrive privately that has no chance of being impounded in prices because the arbitrageur finds it too costly to trade.

We then develop a variant of the basic model with long horizons and use this to compare equilibria with long and short horizons. The fact that arbitrage can only be accomplished via a chain of traders dramatically reduces arbitrage profitability, compared to the case where arbitrage does not require the asset to be resold because the arbitrageur’s lifetime spans the event date. One reason is that for arbitrage to be profitable, the market must be relatively “deep” so that the arbitrageur can buy without simultaneously pushing up the price. With a chain, however, profitable arbitrage also requires selling the asset when a subsequent arbitrageur pushes up the price in the next period. This requires that the market not be too deep.

Another reason for the reduced profitability of short-term arbitrage is that profits also depend on the likelihood that another trader will soon receive the same information. That is, the chain must be unbroken.

We have assumed that there is a transaction cost for trading the risky security. The transaction cost may be interpreted as a brokerage fee, as a bid-ask spread, or as a borrowing cost (cost-of-carry). In any model transaction costs are likely to cause price inefficiencies. In our model, however, we show that the effect of the transaction cost is multiplied by the two factors discussed above: the need to carry out arbitrage via a chain, and the possibility that the chain may be broken. This causes a given transaction cost to have a larger impact on the efficiency of asset prices. This conclusion is based on a comparison of arbitrage profits for an investor with a short horizon and an investor with a long horizon.

While we have given several reasons why traders’ horizons may be limited, the model allows us to explore one explanation in more detail. Interpreting

the transaction cost as a cost-of-carry, we show that a trader with a long horizon may optimally choose to conduct only short-term arbitrages because the cost-of-carry makes long-term arbitrages unprofitable.

The article proceeds as follows. In Section I we set out the model. Section II describes equilibrium order flows, while Section III describes price formation. Section IV describes the trading strategies of the arbitrageurs and proves our main result: informed agents will not engage in arbitrage a long time in advance of the event. Section V derives the quantities traded by the hedgers; Section VI discusses out-of-equilibrium beliefs. Section VII discusses the results and the related literature. In Section VIII we modify the model so that arbitrageurs have long-term horizons. We first show how short horizons reduce arbitrage profitability compared to long horizons. We then show how the cost-of-carry can make long-term arbitrage unprofitable. Section IX concludes.

I. The Model

A. Definitions and Notation

We consider an economy with an infinite sequence of periods $t = -\infty, \dots, \infty$. There is a stock that pays a dividend of 1 or 0 every period, with probabilities π and $1 - \pi$, respectively. Dividend realizations are serially independent. The other available asset is a riskless asset which earns r . We will focus on the periods preceding the period distinguished as $t = T$. The dividend realization at T may become known in advance to some agents, as described below.

Agents live for only two periods. Thus there are overlapping generations of young and old agents. Consumption occurs in old age. In each generation there are 0, 1, or 2 people. Agents may or may not receive a private piece of information. The probability that there is one uninformed hedger is $\frac{1}{2}$; the probability that there is no uninformed hedger is $\frac{1}{2}$. The probability that there is one informed arbitrageur in generation t is δ_t ; with probability $1 - \delta_t$ there is no arbitrageur. These realizations are independent of each other, and serially. Thus, a generation may contain nobody (probability $\frac{1}{2}(1 - \delta_t)$), one informed arbitrageur and no uninformed hedger (probability $\frac{1}{2}\delta_t$), one uninformed hedger and no informed arbitrageur (probability $\frac{1}{2}(1 - \delta_t)$), or one arbitrageur and one hedger (probability $\frac{1}{2}\delta_t$).

Uninformed hedgers have initial wealth W . They also receive a wage income in old age of either 0 or 1 that is perfectly negatively correlated with that period's dividend. Thus the uninformed start life by buying a portfolio that they will liquidate in old age. However, we emphasize that they are not forced to participate in the stock market at all. If they want, they can simply invest at the riskless return r .

Uninformed agents are risk averse and so will have an incentive to buy stock to hedge their income. The reader may wish to read the first part of the article assuming that they are *infinitely* risk averse to simplify the analysis:

in this case, they hedge perfectly by buying 1 unit of stock. In Section V, we explain how the quantity traded is determined for the case of finite risk aversion; this is quite straightforward and separate from the rest of the model. In particular, we stress that the equilibrium prices and the period at which information starts to be revealed in equilibrium are not affected.

For technical reasons, an uninformed agent should be viewed as representing a mass of infinitely small, identical, uninformed agents. This assumption will be needed in the derivation of the equilibrium quantities traded by the uninformed in Sections V and VI, and plays no role in the rest of the article. To avoid lengthy circumlocutions, elsewhere in the article we will simply refer to “an uninformed agent” or “a hedger” rather than “a mass of uninformed agents,” etc.

Informed arbitrageurs are risk neutral. Like the uninformed, they receive an endowment W when young (since they are risk neutral, we assume for simplicity that they receive no wage income as this will not affect their decisions). An arbitrageur, if there is one, receives private information about the dividend to be received at a fixed date T . “Good news” means that he learns it will be high (equal to 1); “bad news” means it will be zero. As time approaches date T , the probability δ_t that there is an informed agent is increasing. The reader may find it convenient to think of δ_t as following the relation:

$$\delta_t = \epsilon \delta_{t+1} = \epsilon^{(T-t)},$$

which converges smoothly to 1 at date T , though the analysis does not require us to use any specific process.

Apart from these agents, there is an institution for pricing and trading stocks. In the stock market, prices are set by a risk-neutral marketmaker who faces Bertrand competition and who has an inventory of stocks and cash (see Kyle (1985)). He observes all the (market) orders for the stock, and then posts a price and meets the net order out of his inventory. Because of competition, the price in each period is equal to his expected value of the asset. (His inventory is discussed further below.) We assume that the marketmaker observes all buy and sell orders separately (as will be seen below, in this model it would be equivalent to assume the marketmaker could only observe the net aggregate order).

Notice that when we refer to “agents,” we do not include the marketmaker in this terminology.

There is a per-share transaction cost, c , to trading the stock. For notational simplicity c is the cost for a round-trip transaction (since all stocks bought when young will be sold the next period in old age).

The timing of events within each period may be summarized thus:

Start of period t

1. Dividends, old wage income, and young endowment arrive.
2. Information arrives (if there is an informed trader).
3. Orders are submitted.

4. Marketmaker sets the price.
 5. Trades are executed.
 6. Consumption occurs.
- End of period t

B. Comments on Assumptions

Here we comment briefly on several aspects of the assumptions that may be questioned by the reader:

1. Why are informed arbitrageurs risk neutral and the uninformed, risk averse?

The risk aversion of uninformed agents makes them want to trade so as to hedge their wage income shock. If they were risk neutral they would simply invest at the risk-free rate and there would be no “liquidity” trades for the arbitrageurs to hide behind. Arbitrageurs are risk neutral for simplicity (it will be clear that the results do not depend on this).

2. Why not have a lump-sum, rather than proportional, transactions cost? Why is the proportional transactions cost per share, not per dollar?

A lump-sum transaction cost seems less realistic and would complicate matters. The per-share assumption is made for analytical tractability. We have solved the model with proportional costs per dollar, but the closed-form solutions of the model are much more complex. The results are unchanged except that the formula in Proposition 3 is modified appropriately.

Although a proportional cost per dollar might seem more plausible a priori, there is empirical evidence to the contrary. Dimson and Marsh's (1989) data indicate that the bid-ask spread on the London Stock Exchange is approximately six pence regardless of the share price. Brennan and Hughes (1991) argue that brokerage commissions in the United States, expressed as a percentage of the value of the transaction, fall for shares with higher prices (“big” shares).

In Section VII we show that the cost may be interpreted as a borrowing cost, since the per-dollar version of the transaction cost is exactly equivalent to an interest charge on a loan.

3. Why is δ_t increasing over time?

Our model does not require that δ_t increase over time, but it is more plausible and simplifies the exposition to assume that it does. If it is not increasing, it is extremely easy to derive equilibria where long-term information is ignored. For example, if there is always an informed arbitrageur at $T - 2$ but never in any other period, there will be such a nonrevealing equilibrium. The harder question is whether information may be ignored even when it is increasingly likely to arrive as we approach the event. Not only is this question harder, but it is more important since increasing δ_t seems more plausible.

4. Does the model depend on all agents having short horizons?

The answer is no. While both the uninformed hedgers and the informed arbitrageurs have limited horizons, the marketmaker is infinitely lived. Thus, as we discuss in Section VII, the results cannot be due to all agents having short horizons. What matters is that the privately informed arbitrageurs have short horizons and that there may be periods in which the arbitrage chain will be broken.

5. Why construct the hedging demand rather than simply adding a “noise” term to demand (as in Kyle (1985))?

Trading profitably on the basis of private information requires that other agents trade and lose money on average. The standard device in the literature has been to model these “noise traders” as a random exogenous component of asset demand that prevents asset prices from being fully revealing. The exact motivation of these noise traders, or liquidity traders, is not fully specified and they are sometimes interpreted as irrational (e.g., De Long *et al.* (1990)). This article is about short-term biases in asset pricing, but it is not about irrationality in asset pricing. On the contrary, we investigate the possibility of short-term biases in a fully rational setting. We therefore consider it important to explicitly model the origin of the “liquidity” trade.

The model endogenizes the “liquidity” trade in an attractive way. In our multiperiod framework, agents start without assets and do not have to participate in the stock market if they do not want to. If they do participate, however, in order to hedge, then in the second period of their lives they must unwind their positions, i.e., sell their holdings. This distinguishes our model from Biais and Hillion (1992). They interpret the liquidity trade as coming from rational agents who hedge shocks to nontradeable assets, similar to our wage-income shocks. However, theirs is a one-period model where agents start with an endowment of assets, trade once, then the assets pay liquidating dividends and the agents consume.

Models based on multiperiod versions of Kyle (1985) contain a serially independent demand shock representing the liquidity trade. One benefit of our approach is that we impose the restriction that every security bought must be sold later. Thus, we derive the appropriate negatively serially correlated liquidity demand.

Another important aspect of the liquidity trade is the responsiveness of these traders to the amount lost (on average) to the informed traders. In our model the uninformed traders will trade a decreasing amount over time in response to the progressive increase in the profitability of arbitrage activity.

II. Order Flow in Equilibrium

We will now describe the equilibrium order flow and stock prices. The equilibrium has the property that arbitrageurs who receive good news act on the information starting a fixed number of periods from the end (date T), but

not before. We call this date at which arbitrageurs start to act “date K .” Subsequently we will show that this is indeed an equilibrium, determine K , and show that this is the unique equilibrium of the model.

It is possible that, after K , the arbitrageur’s actions will completely reveal the news to the marketmaker and the price will immediately jump to the full information price from then on. In that case, subsequent arbitrageurs will not act. But unless this happens, in our model arbitrageurs with good news will always act after K for all other price histories.

Let x_t be the amount of stock bought in equilibrium by an uninformed (risk-averse) agent arriving at date t . Because of the transaction cost, the agent will only choose to hedge fully ($x_t = 1$) if he is infinitely risk averse. The determination of x_t is discussed in Section V below. However, the prices and other properties of equilibrium, including the determination of date K , will not be affected by x_t .

What are the possible orders after period K ? If no agents arrive in period t , no orders will be submitted. If there is an uninformed hedger, he will buy x_t . We will show in Section VI below that, in order to disguise himself, an arbitrageur who receives good news will also buy x_t . Thus the possible buy orders are 0, x_t , or $2x_t$. In other words, at all dates orders will be in multiples 0, 1, or 2 of x_t . (Notice that this implies that the total stockholding of the traders in equilibrium is therefore either 0, x_t , or $2x_t$ shares.)

Arbitrageurs who receive bad news do not act because they would immediately be identified by the marketmaker. They do not sell the asset short. This is because in equilibrium the sell orders at date t will be the same as the buy orders from date $t - 1$. If an arbitrageur with bad news did sell short, sell orders at date t would exceed buys in $t - 1$, so the information would be completely revealed, and he would have incurred the transaction cost. (The model could easily be extended slightly to allow for short sales by arbitrageurs, but we do not pursue that.)

How could these multiples 0, 1, or 2 occur, and what are their probabilities?

Buy = 0

This can occur in two ways: either no agents arrive (probability $\frac{1}{2}(1 - \delta_t)$), or an arbitrageur arrives and receives bad news ($\frac{1}{2}\delta_t(1 - \pi)$):

$$\frac{1}{2}(1 - \delta_t) + \frac{1}{2}\delta_t(1 - \pi) = \frac{1}{2}(1 - \delta_t\pi).$$

Buy = 1

This can occur in three ways: only an uninformed hedger arrives ($\frac{1}{2}(1 - \delta_t)$); only an arbitrageur arrives and he receives good news ($\frac{1}{2}\delta_t\pi$); both a hedger and an arbitrageur arrive, and the arbitrageur receives bad news ($\frac{1}{2}\delta_t(1 - \pi)$)

$$\frac{1}{2}(1 - \delta_t) + \frac{1}{2}\delta_t\pi + \frac{1}{2}\delta_t(1 - \pi) = \frac{1}{2}.$$

Buy = 2

This occurs if both an arbitrageur and a hedger arrive, and the arbitrageur receives good news

$$\frac{1}{2}\delta_t\pi.$$

This description of the order flows now allows us to derive the marketmaker's beliefs and hence prices.

III. Stock Prices

If we are more than $T - K$ periods from T , by hypothesis, arbitrageurs do *not* act on their information. In that case there is no information in a buy order (there will be either 0 or 1 orders) and the price is given by

$$p = \pi/r,$$

which is the marketmaker's expected valuation.

We now consider prices less than $T - K$ periods from T . Suppose first that the marketmaker knows for sure that the dividend at time T will be 1. Then the price is

$$p_t = \pi/r + (1/(1+r))^{T-t}(1-\pi).$$

If the dividend is known to be zero, the price is

$$p_t = \pi/r - (1/(1+r))^{T-t}\pi.$$

The former case will happen in equilibrium if the marketmaker observes two buy orders—which can only arise when both an uninformed hedger and an informed arbitrageur with good news arrive. The latter case will not arise in equilibrium, since arbitrageurs do not act on bad news.

If the marketmaker observes 0 or 1 buy orders, he does not know the date T dividend for sure. However, the order flow is informative and will be used to update the marketmaker's belief. For example, an order flow of 0 could arise if there is an arbitrageur with bad news or no arbitrageur, but not if there is an arbitrageur with good news. So 0 orders will cause the belief to be revised downwards.

Let β_t be the marketmaker's belief at date t that the date T dividend will be 1. This belief is formed at date t , having observed the order flow at date t and is used to set price p_t . The stock price will be a weighted average of the above prices,

$$\begin{aligned} p_t &= \beta_t \left[\pi/r + (1/(1+r))^{T-t}(1-\pi) \right] + (1-\beta_t) \left[\pi/r - (1/(1+r))^{T-t}\pi \right] \\ &= \pi/r + (1/(1+r))^{T-t}(\beta_t - \pi) \end{aligned}$$

for $t \geq T - K$. Note that $\beta_{T-K-1} = \pi$. We now derive the updating rule for subsequent beliefs.

Note that many of the formulas for the probabilities derived below are similar to those in Section II above, but those were marginal probabilities

(i.e., not conditional on the realized history of orders in previous periods), while these are the marketmaker's beliefs. So these formulas have β_t where the previous ones had π .

If There Are 0 Buy Orders

As explained in Section II above, this can occur either if no agents arrive, or if only an arbitrageur arrives and he receives bad news. The probability that no agents arrive is $\frac{1}{2}(1 - \delta_t)$. The probability that only an arbitrageur arrives and he receives bad news is $\frac{1}{2}\delta_t(1 - \beta_{t-1})$. So the probability that the dividend is high and that there is a buy order of 0 is $\frac{1}{2}(1 - \delta_t)\beta_{t-1}$. The probability of 0 buy orders is

$$\frac{1}{2}(1 - \delta_t) + \frac{1}{2}\delta_t(1 - \beta_{t-1}) = \frac{1}{2}(1 - \delta_t\beta_{t-1}).$$

So

$$\beta_t = [\frac{1}{2}(1 - \delta_t)\beta_{t-1}] / [\frac{1}{2}(1 - \delta_t\beta_{t-1})] = (1 - \delta_t)\beta_{t-1} / (1 - \delta_t\beta_{t-1}).$$

If There Is 1 Buy Order

This can occur in three ways: if only a hedger arrives (probability $\frac{1}{2}(1 - \delta_t)$, if only an arbitrageur arrives and he receives good news (probability $\frac{1}{2}\delta_t\beta_{t-1}$), or both an arbitrageur and a hedger arrive, but the arbitrageur receives bad news (probability $\frac{1}{2}\delta_t(1 - \beta_{t-1})$). The probability that there is 1 buy order and the dividend is high is

$$\frac{1}{2}(1 - \delta_t)\beta_{t-1} + \frac{1}{2}\delta_t\beta_{t-1} = \frac{1}{2}\beta_{t-1}.$$

The probability of 1 buy order is

$$\frac{1}{2}(1 - \delta_t) + \frac{1}{2}\delta_t\beta_{t-1} + \frac{1}{2}\delta_t(1 - \beta_{t-1}) = \frac{1}{2}.$$

So

$$\beta_t = (\frac{1}{2}\beta_{t-1}) / \frac{1}{2} = \beta_{t-1}.$$

If There Are 2 Buy Orders

As discussed above, this can occur only if there are both a hedger and an arbitrageur with good news present. This event, therefore, reveals that the date T dividend is high and so $\beta_t = 1$.

This completes the description of beliefs and, hence, describes the evolution of stock prices as a function of the history of information arrival. It remains to determine K .

IV. Arbitrageurs' Trading Strategies

We now consider the decision problem of an arbitrageur. First, if in the next period an arbitrageur (if there is one) would not act, then an arbitrageur

this period will not act. Because the next period's price cannot reflect more information than today's, he would simply incur the transaction cost without any benefit. This is Proposition 1.

On the other hand, if in the next period an arbitrageur *would* act, then an arbitrageur this period may or may not act, depending on the likelihood of next period's price reflecting more information than today's and on the size of the capital gain in that event, balanced against the transaction cost.

It may be that the price already completely reflects the information. Clearly in this case the arbitrageur will not trade. But if the price does not already completely reflect the information, then we show in Proposition 2 that if an arbitrageur in the *last* period would have decided to act (because the expected capital gain outweighed the transaction cost), then an arbitrageur this period will also decide to act. In other words, the expected capital gain increasingly outweighs the transaction cost.

Combining Propositions 1 and 2, the equilibrium must have the property that there exists a critical date K before which arbitrageurs never act and after which they always act, except in the event that the price is fully revealing. It remains to show in Proposition 3 how date K is determined.

PROPOSITION 1: *Suppose that the probability of an arbitrageur arriving and acting on his good news next period ($t + 1$) is zero. Then an arbitrageur will not act on his good news this period (t).*

Proof: Since, by hypothesis, an arbitrageur at date $t + 1$ will not act, $E\beta_{t+1} = \beta_t$. Note that β_t may or may not be updated from date $t - 1$. So

$$p_t = \pi/r + (1/(1+r))^{T-t}(\beta_t - \pi),$$

and p_{t+1} is nonrandom:

$$p_{t+1} = \pi/r + (1/(1+r))^{T-(t+1)}(\beta_t - \pi).$$

It follows that

$$p_t(1+r) = (1+r)\pi/r + (1/(1+r))^{T-(t+1)}(\beta_t - \pi) = \pi + p_{t+1},$$

so if the agent acts, his expected wealth at $t + 1$ is

$$\begin{aligned} & \pi[(W - p_t x_t)(1+r) + x_t(p_{t+1} - c + 1)] \\ & + (1 - \pi)[(W - p_t x_t)(1+r) + x_t(p_{t+1} - c)] = W(1+r) - x_t c. \end{aligned}$$

This is less than his wealth if he simply invests in the riskless asset, $W(1+r)$. In other words, the expected return on the share (including the expected dividend) is the same as on the riskless asset (r), but ignoring the transactions cost. When the transaction cost is included, the return on the share is less. Q.E.D.

We next consider the arbitrageur's wealth in case he acts or does not act at time t , assuming that an arbitrageur next period would act. If he does not act, his wealth is simply $W(1+r)$. If he acts and buys x_t shares, his wealth

depends on whether there is also a hedger present at time t . With probability $\frac{1}{2}$, there is a hedger present at time t . In this case, the arbitrageur's order will reveal the information and the marketmaker's belief will jump to 1 and remain there, so the arbitrageur will earn the safe rate of return r , but will also incur a transaction cost $x_t c$. His wealth will be

$$W(1 + r) - x_t c.$$

With probability $\frac{1}{2}$ there is no hedger present. The price at time t will not reveal the information. We will use the notation, $*$, to denote conditioning on the event that there is no hedger present at time t . The marketmaker's current belief, and the corresponding price, are not random, conditional on this event. We denote them β_t^* and p_t^* (as shown in Section III, the marketmaker's belief in this event is the same as last period's belief β_{t-1}). The marketmaker's belief next period, and the corresponding price, conditional on this event, are random and we denote their expectations as $E^* \beta_{t+1}$ and $E^* p_{t+1}$.

With this notation we can now write the arbitrageur's expected wealth (in the event there is no hedger present at t) as:

$$\begin{aligned} (W - p_t^* x_t)(1 + r) + x_t [E^* p_{t+1} - c + \pi] \\ = W(1 + r) + x_t [E^* p_{t+1} - p_t^*(1 + r) - c + \pi]. \end{aligned}$$

Averaging over the two events, the arbitrageur's expected wealth if he acts is

$$\frac{1}{2} [W(1 + r) - x_t c] + \frac{1}{2} [W(1 + r) + x_t [E^* p_{t+1} - p_t^*(1 + r) - c + \pi]].$$

If he does not act on his information he receives $W(1 + r)$. Comparing these, the arbitrageur will act if

$$E^* p_{t+1} + \pi - p_t^*(1 + r) > 2c.$$

This decision rule will enable us to characterize the equilibrium.

PROPOSITION 2: *Suppose that there is an initial date K at which an arbitrageur will act on good news. Then an arbitrageur will act on good news at all subsequent dates, if the price is not already fully revealing.*

Proof: An arbitrageur with good news at date $t \geq K$ will act if

$$E^* p_{t+1} + \pi - p_t^*(1 + r) > 2c,$$

i.e.,

$$\begin{aligned} (\pi/r)(1 + r) + (1/(1 + r))^{T-(t-1)}(E^* \beta_{t+1} - \pi) \\ - (1 + r) \left[(\pi/r) + (1/(1 + r))^{T-t}(\beta_t^* - \pi) \right] > 2c, \end{aligned}$$

or

$$\begin{aligned} (1/(1 + r))^{T-(t+1)}(E^* \beta_{t+1} - \pi - \beta_t^* + \pi) > 2c \\ E^* \beta_{t+1} - \beta_t^* > 2c(1 + r)^{T-(t+1)}. \end{aligned}$$

By definition of K this expression holds for $t = K$. Note that the right-hand side of this inequality falls with time. We will show that the left-hand side increases over time regardless of how β_t evolves (so long as it does not reach 1).

Suppose first that an arbitrageur acting at date K is followed by an uninterrupted sequence of single buy orders until date $t > K$. We now consider the decision problem of an arbitrageur at date t . The updating rule for β_t implies that $\beta_t^* = \beta_K = \pi$. Note that from the expression given above for the value of $E^*\beta_{t+1}$,

$$E^*\beta_{t+1} - \beta_t^* = \delta_{t+1}(1 - \beta_t^*)^2 / [2(1 - \delta_{t+1}\beta_t^*)].$$

Thus,

$$\partial[E^*\beta_{t+1} - \beta_t^*] / \partial\delta = (1 - \beta_t^*)^2 / [2(1 - \delta_{t+1}\beta_t^*)^2] > 0.$$

It follows immediately that

$$E^*\beta_{t+1} - \beta_t^* > E^*\beta_{K+1} - \beta_K > 2c(1+r)^{T-(K+1)} > 2c(1+r)^{T-(t+1)},$$

and therefore an arbitrageur at date t will act.

If the sequence of buy orders between date K and date t include some dates at which there were no buys, then $\beta_t^* < \beta_K$. The reason is that a buy of 0 causes the marketmaker to revise his beliefs downward, while a buy of 1 causes beliefs to remain unchanged. (Note that a buy of 2 reveals the information, so beliefs reach 1, but we are not describing this case here.) However, when beliefs are revised downward this simply increases $E^*\beta_{t+1} - \beta_t^*$, and the result remains true:

$$\begin{aligned} \partial[E^*\beta_{t+1} - \beta_t^*] / \partial\beta_t^* \\ = \delta_{t+1}(1 - \beta_t^*)(\delta_{t+1} + \delta_{t+1}\beta_t^* - 2) / [2(1 - \delta_{t+1}\beta_t^*)^2] < 0, \end{aligned}$$

since

$$\delta_{t+1} + \delta_{t+1}\beta_t^* < 2.$$

Q.E.D.

By Proposition 1 arbitrageurs will not act on good news before date K , and by Proposition 2 they will act afterward, except if the price is fully revealing. Note that Proposition 2 implies there are no mixed strategy equilibria (except possibly in the negligible case that the agent is indifferent at date K).

We will now find the largest δ_t (and hence the last date t) at which an arbitrageur who receives good news is unwilling to act, assuming that at date $t + 1$ an arbitrageur will act. This determines date K .

PROPOSITION 3: *K is the first date t for which:*

$$\frac{1}{4}\delta_{t+1}(1 - \pi)^2 / (1 - \delta_{t+1}\pi) > c(1+r)^{T-(t+1)}.$$

Proof: Since by hypothesis the arbitrageur is not supposed to act on his information in $t - 1$, the marketmaker will not update his beliefs when he

sees a buy order of 1, unless there is also a hedger at $t - 1$. So long as there is no hedger at $t - 1$, the hedger will therefore purchase the share at price:

$$p_{t-1} = \pi/r.$$

(Of course, if there is a hedger at $t - 1$ then there will be two buy orders and the good news will be revealed.)

An arbitrageur will choose *not* to act if:

$$E^*p_{t+1} + \pi - p_t^*(1 + r) < 2c,$$

or, substituting for E^*p_{t+1} using the formula for p_{t+1} ,

$$(1/(1 + r))^{T-(t+1)}[E^*\beta_{t+1} - \pi] < 2c$$

since prices are linear in beliefs. We now compute $E^*\beta_{t+1}$.

Period $t + 1$ is, by hypothesis, the first date at which an arbitrageur with good news will act. Therefore, depending on whether trading volume at $t + 1$ is 0, 1, or 2 (times x_{t+1}), the marketmaker's belief will be:

$$\begin{aligned}\beta_{t+1} &= (1 - \delta_{t+1})\pi/(1 - \delta_{t+1}\pi), \\ \beta_{t+1} &= \pi,\end{aligned}$$

or

$$\beta_{t+1} = 1,$$

respectively. Since the arbitrageur is informed (he knows the dividend at date T will be 1) the probabilities he attaches to each of these three beliefs (and corresponding prices) occurring are different from the probabilities attached to these events by the uninformed marketmaker. From the point of view of the arbitrageur their chances of occurrence are as follows:

Buy = 0: No agents arrive, $\frac{1}{2}(1 - \delta_{t+1})$.

Buy = 1: Only one agent arrives and he is informed, $\frac{1}{2}\delta_{t+1}$, or only one agent arrives and he is uninformed, $\frac{1}{2}(1 - \delta_{t+1})$. Total probability = $\frac{1}{2}$.

Buy = 2: Two agents, one uninformed and one informed, $\frac{1}{2}\delta_{t+1}$.

Thus

$$E^*\beta_{t+1} = \frac{1}{2}(1 - \delta_{t+1})[(1 - \delta_{t+1})\pi/(1 - \delta_{t+1}\pi)] + \frac{1}{2}\pi + \frac{1}{2}\delta_{t+1}$$

So

$$\begin{aligned}E^*\beta_{t+1} - \pi &= \frac{1}{2}(1 - \delta_{t+1})[(1 - \delta_{t+1})\pi/(1 - \delta_{t+1}\pi)] - \frac{1}{2}\pi + \frac{1}{2}\delta_{t+1} \\ &= \frac{1}{2}\delta_{t+1}(1 - \pi)^2/(1 - \delta_{t+1}\pi).\end{aligned}$$

K is therefore defined as the first time index t for which:

$$\left[\frac{1}{2}\delta_{t+1}(1 - \pi)^2/(1 - \delta_{t+1}\pi)\right]/(1 + r)^{T-(t+1)} > 2c.$$

Q.E.D.

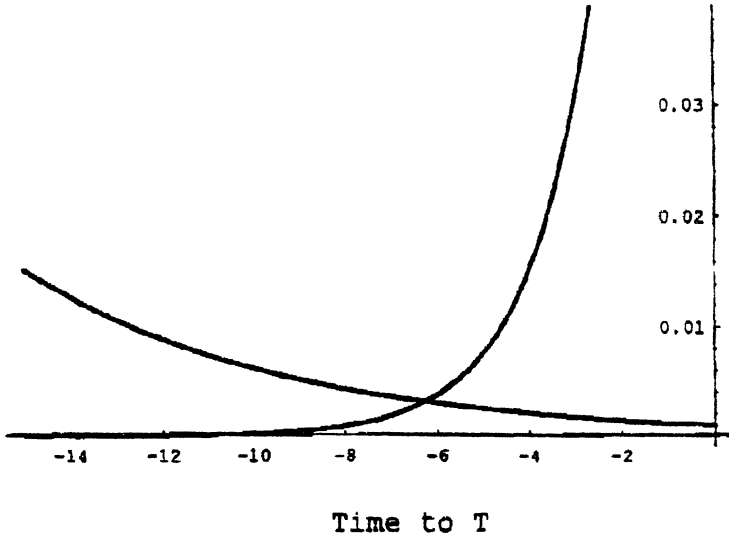


Figure 1. Inequality relationship to determine K .

Figure 1 illustrates the determination of K . Viewing time as a continuous variable, it graphs the left-hand side and right-hand side of the inequality defining K :

$$\frac{1}{4}\delta_{t+1}(1 - \pi)^2 / (1 - \delta_{t+1}\pi) > c(1 + r)^{T-(t+1)},$$

for $c = 0.001$, $\pi = 0.01$, $r = 0.01$, and $\delta_t = 0.5^{T-t}$. Note that the (uninformative) price for the asset is $\pi/r = 1$, so c is approximately 0.1 percent of the asset price. The information cannot be revealed more than 6 periods from the event date T .

Date K is essentially determined by the tradeoff between the chance of another arbitrageur arriving next period (the left-hand side) and the transaction cost (the right-hand side). To see this, note that the left-hand side is almost equal to a constant times δ_{t+1} (because the denominator rapidly approaches 1). On the other hand, the solution is not very sensitive to the interest growth term on the right-hand side. By ignoring this term we get a lower bound for K (the true date K happens *later*).

V. Uninformed Hedgers' Trading Strategies: Determination of x_t

To this point, the amount traded by the hedgers, x_t , has been taken as given. In this section the amount these risk-averse agents will trade each period will be determined. As was seen above, x_t does not affect the equilibrium prices and strategies so long as it is positive. But if the uninformed agents are only slightly risk averse, they might choose not to hedge at all.

Therefore, we also provide conditions on their utility function that guarantee that they are sufficiently risk averse to hedge.

In periods before $K - 1$, hedgers buy and sell at the same price, π/r . Their expected return on the risky asset (ignoring the transaction cost) is r , the same as on the bond. Thus, a hedger who is infinitely risk averse will choose a portfolio that completely insures him against risk, which in this case means buying one unit of the asset (since the dividend exactly offsets the wage income). If the hedger is not infinitely risk averse, he will choose to hedge only partly because of the transaction cost.

From date $K - 1$ to K , a hedger will get an expected return to r , but this time the return is uncertain. However, the price variability is independent of the wage-income shock.

From date K onwards arbitrageurs will act on their information so that the price uninformed hedgers buy at will not be fair. Unless they are infinitely risk averse, they will not fully hedge but will buy some number of shares $x_t < 1$ even if the transaction cost is zero.

In summary, there are three cases:

1. Before date $K - 1$.
2. At date $K - 1$.
3. From date K onwards.

The details of the derivations of x_t in the three cases, and the conditions for x_t to be strictly positive, are given in the Appendix.

VI. Out-of-Equilibrium Beliefs

To this point we have only considered the possibility that all agents trade a multiple 0, 1, or 2 of x_t . To complete the construction of the equilibrium, it remains to verify that no agent has an incentive to deviate by trading other quantities. Recall that the belief maps β_{t-1} to β_t as a function of the total quantity traded at time t . We proceed, as usual, by specifying beliefs for the marketmaker at other quantities, as follows:

1. For trading volume less than or equal to 0, the marketmaker has the same beliefs as he does at 0: $\beta_t = (1 - \delta_t)\beta_{t-1}/(1 - \delta_t\beta_{t-1})$.
2. For trading volume greater than 0, but less than or equal to x_t , the marketmaker has the same beliefs as he does at x_t , so $\beta_t = \beta_{t-1}$.
3. For trading volume greater than x_t , the marketmaker believes the asset is of high value, $\beta_t = 1$.

There are two possible deviations an arbitrageur can make. He can trade more than x_t , in which case the price will immediately become fully revealing, and he will earn the riskless return r less the transaction cost. This is clearly not a profitable deviation.

He can trade less than x_t , in which case the price will be unchanged and he will earn the same percentage return on a smaller quantity. Again, this is clearly suboptimal.

Finally, the hedgers have no incentive to deviate by definition of x_t . Since the hedgers are a continuum of infinitesimal agents (see Section I.A), an individual cannot affect the aggregate trading volume and so cannot change the marketmaker's beliefs. The quantity x_t was derived under precisely this assumption.

VII. Discussion of the Results

A. Pricing of Public Information and Horizon Length

Our analysis has investigated the formation of asset prices in a model where agents with private information have short horizons. However, our analysis does not assume, or require, that all agents have short horizons. It should be stressed that the risk-neutral marketmaker does not have a limited horizon; he is infinitely lived. The price is determined by the information set of the marketmaker; public information is "properly" priced. The marketmaker may be viewed as a "sea" of uninformed, risk-neutral traders (this interpretation is given by Kyle (1985)). Our analysis only requires that the privately informed agents have limited horizons.

B. The Marketmaker's Inventory

The total stockholding of the traders in equilibrium is either 0, 1, or 2 shares. Hence the number of shares in existence could be as few as 2, in which case the marketmaker would hold 2, 1, or 0 shares (respectively) in inventory. In particular, it is not necessary for the marketmaker to hold an infinite inventory, as in Kyle (1985) and Glosten and Milgrom (1985). A more important difference is that in those models the marketmaker's inventory has no mean reversion because it follows a random walk. The reason for this difference is that our uninformed hedgers "unwind" their positions, whereas in the above models the "liquidity" trade each period is an independent random variable.

C. Interpretation of the Transaction Cost

In the model we interpreted the transaction cost essentially as a brokerage fee for trading, but this is not essential. The cost could also be interpreted as a borrowing cost, i.e., c is the interest rate premium on a loan (above r). In other words, an arbitrageur who borrowed $x_t p_t$ would repay $x_t p_t (1 + r + c)$. This could arise because of transaction costs or, possibly, because of default risk. In any case, this would be equivalent to making the transaction cost proportional to the value of the shares rather than the number of shares. As discussed above in Section I.B.2, we have solved the model for this case and the results are qualitatively unchanged, although the algebra is considerably more complicated.

D. Path Dependence of the Hedgers' Trades

The arbitrageurs' decisions on whether to act is path dependent only in the following, limited, sense. If a previous arbitrageur has already pushed up the price, then subsequent arbitrageurs will not act. In general, the decision on whether to act would have more complicated path dependencies, but the structure of our model has been chosen to avoid these complications.

However, the quantity, x_t , traded by the hedgers will, in general, be path dependent in our model after date K , depending on whether the sequence of past orders was 0 in every period, 1 in every period, or any of the other possible combinations. Note, however, that the magnitude of x_t does not affect the arbitrageurs' decisions on whether to act; nor does it affect the price.

E. Speculation on Bad News

In our model arbitrageurs do not sell short upon receiving bad news. This is because the hedgers never sell short, so that the marketmaker could always infer that a short sale originated with an arbitrageur. This is a pure modelling feature that would be different if hedgers' demand did not have the simplified form we have assumed here.

F. Interpretation of the Information Event

We took the information to be knowledge of the dividend at date T . The results would not be significantly changed if we assumed any form of uncertainty in which "news" became public at the event date T . For example, it could be that the probability of the dividend being 1 could increase at every date starting with date T . Alternatively, the dividend growth rate could change at date T .

In our model, the uncertainty relates to the dividend at a single fixed date, T . One could imagine a variant of the model in which information could arrive about dividends in different periods. This scenario is more realistic, but agents' inferences in such a model would be considerably more complex. However, there is no reason to believe that the properties of the model would be fundamentally altered.

G. Related Literature

A number of articles have, explicitly or implicitly, analyzed asset pricing with short horizons. These include Allen and Gorton (1993), Hart and Kreps (1986), and Froot *et al.* (1992). Allen and Gorton (1993) show that asset price bubbles can occur when there are short horizons and contracting problems with professional money managers. Hart and Kreps (1986) show that introducing long-horizon arbitrageurs with a commodity storage technology can actually increase price volatility. Froot *et al.* (1992) show that if traders have short horizons then they will coordinate on producing the same piece of information so that it will be impounded in the price when they unwind their

position. In other words, they ask whether some types of information are more valuable, and hence more worthwhile to produce, than others. The same question could be asked in the different setting of our model: is short-term information more valuable than long-term information? We have not addressed this question since it would be quite easy to argue that agents should prefer to produce short-term rather than long-term information. For example, it is plausible that time spent producing short-term information is more productive: short-term information may be easier to produce than long-term information. Instead, we sought to obtain the stronger result that information may actually be worthless to an agent if it is too long term.

Other articles emphasize the effects of the presence of irrational traders interacting with rational traders who have short horizons. In these articles the question is whether informed traders can profit from the mispricing of assets in the presence of irrational traders. In the model of De Long *et al.* (1990) prices can deviate from fundamentals because irrational traders trade on incorrect beliefs. In one equilibrium of the model two identical assets trade at different prices. Rational traders, who are risk averse, do not engage in arbitrage because the irrational traders may drive the price even further from fundamentals within their horizons. This would be impossible in our setting where prices are set by a risk-neutral, long-lived marketmaker.

Shleifer and Vishny (1990) consider a model in which noise traders drive initial prices away from fundamental values because they are either “optimistic” or “pessimistic.” Arbitrageurs, who borrow in order to trade, prefer short-term assets because the loan can be paid off sooner, both because the interest cost of carrying the arbitrage position is lower and because they face rationing in their borrowing of funds for arbitrage (we model a similar argument about the cost-of-carry in Section VIII, below). As in De Long *et al.* (1990), the mispricing of the assets depends on the noise traders’ beliefs being exogenously wrong.

Our intention is somewhat different. We wish to investigate the possibility of short-term mispricing in a model where all agents are rational. We show that irrationality is not necessary for assets to be mispriced in the short term. Short-term mispricing of private information can plausibly occur when all agents are fully rational.

VIII. Limited Horizons

In this section we discuss two modifications to the above model to address the following two questions. The first question concerns the effects of introducing transaction costs into an environment with short-term horizons. At first sight, it might be argued that transaction costs alone explain the inefficiency that we have analyzed. We show that the effect of transaction costs on short-term speculators is greatly multiplied compared to the effect on speculators with long horizons. Thus, our result is not simply a description of

the effect of a transaction cost. It describes the effect of limited horizons, rather than long horizons, in an environment with transaction costs.

Second, we have assumed that horizons are limited. In the introduction we mentioned that short horizons may emanate from monitoring portfolio managers in the presence of agency problems. In this section we provide another explanation for why horizons are limited: the cost-of-carry of an arbitrage portfolio will rapidly accumulate, making longer-term arbitrages inherently less profitable. Arbitrageurs prefer short-term arbitrages to long-term ones. This provides a justification for our assumption of short horizons.

To address the first question, we compare the short-lived agent of our model to a long-lived agent who also faces a fixed round-trip transaction cost. To address the second question, we compare our short-lived agent to a long-lived agent who incurs a per-period transaction cost. Both of these comparisons are based on a version of the model modified to include long-lived agents.

A. Introducing Long Horizon Arbitrageurs

To derive the long-term model, suppose that arbitrageurs live forever rather than for two periods. Each period there is a probability, δ_t , of an arbitrageur arriving (as before—so the marketmaker's beliefs are the same). If the arbitrageur arrives he will mimic a liquidity trader by buying x_t shares. However, since he has no incentive to sell the shares next period, δ_t has no effect on his decision.

The details for modifying the model so that the equilibrium prices, and the inferences of the marketmaker, will be exactly the same as in the short-term model are as follows. Hedgers are also long lived. They consume at date T . When a hedger arrives, at date t , he will buy some shares to hedge the wage-income shock that will occur at date T . Hedgers are assumed to have preferences displaying constant absolute risk aversion. Since CARA preferences have no income effects, hedgers will not adjust their positions even though their wealth evolves over the interval from date t to date T .

The modification to the hedgers' utility functions is necessary because otherwise a marketmaker observing the mismatch between buy orders one period and sell orders the next period could infer the presence of an arbitrageur. The equilibrium of the model would then be completely different and not directly comparable to our basic model. Note that in the long-term model the hedgers will choose different x_t , but, as will be seen below, this does not affect either the prices or the arbitrageurs' decision concerning whether to act on information.

B. Transaction Costs and Speculative Horizons

The point of this subsection is to show that our above results are not simply due to the presence of transaction costs. In any model transaction costs may prevent arbitrageurs from acting on their information a long time in advance (because the present value of a mispriced dividend in the distant future is

To clarify the comparison between the short-term and long-term models, consider the decision rule of an arbitrageur in the short-term model when $\delta_t = 1$, for all t . In other words, assume that an arbitrageur *will* always arrive next period, but the arbitrageurs have short horizons, and their profits depend on reselling the asset. K is determined by setting $\delta_{t+1} = 1$ in the expression in Proposition 3; it is the first period t for which

$$\frac{1}{4}(1 - \pi)/(1 + r)^{T-(t+1)} > c.$$

Comparing the two inequalities, we can see the effect of the arbitrageurs' short horizons that make profit depend on the chain of future arbitrageurs. Since we are examining the case where $\delta_t = 1$ at all t , the chain is certain to be unbroken. However, the profitability of the arbitrage depends not only on buying the asset cheaply at time t (which happens half the time) but also on selling it at a high price at time $t + 1$ (which happens only half the time, conditional on having bought cheaply at t). This will only happen if the arbitrageur next period coincides with the arrival of an uninformed hedger who buys (in which case the information is revealed). Thus the dependence on the chain reduces the profitability of arbitrage by 50 percent, relative to the long-term model, even when the chain is certain to be unbroken. This is why $\frac{1}{4}$ appears in the determination of K , rather than $\frac{1}{2}$ in the determination of K^* .

When $\delta_t < 1$ the date K at which information may first be revealed is delayed even further. The reason is clear: the short-term arbitrage cannot succeed unless the next arbitrageur is present to push up the price. When $\delta_t = 1$, the problem for the arbitrageur is that next period's arbitrageur may not push up the price because there may be no hedging demand. When $\delta_t < 1$, there is the additional problem that there may be no arbitrageur next period. Thus, we have shown:

PROPOSITION 4: *When the transaction cost is charged per stock purchase, and $\delta_t = 1$, then the profitability of short-term arbitrage is one-half the profitability of long-term arbitrage. When $\delta_t < 1$, the profitability of short-term arbitrage is further reduced. Thus, the effect of a given transaction cost implies a later date for the beginning of information revelation with short-term arbitrage than with long-term arbitrage.*

We can illustrate this effect using the numerical example given in Section IV. Recall that date K was six periods before date T in that example, i.e., with short-term horizons information cannot be revealed more than six periods before the event. Using the same parameter values, K^* , the date at which the long-term arbitrageur is willing to act on his information is determined by

$$\frac{1}{2}(1 - 0.99) = 0.001(1.01)^{T-(K+1)}.$$

Or, in other words, $K = 624$. The information may be revealed as early as 624 periods before the event, rather than six periods. This illustrates how short horizons may affect the efficient pricing of information. Of course, this example is only an illustration; it is sensitive to the parameters.

small). However, we will show that the effect of transaction costs in our model is magnified because of short horizons.

The short horizons of the arbitrageurs makes the chain of future arbitrageurs important for arbitrage. An additional factor concerns the likelihood of future arbitrageurs arriving. Each of these factors is important for the result that prices are inefficient prior to K . Moreover, each factor is important in delaying trading by arbitrageurs with short horizons, relative to the decision that would be made by an arbitrageur facing the same transaction cost, but without a short horizon.

We analyze these issues by comparing the long-term model, defined above, to the short-term model. In this subsection long-term arbitrageurs will face the same round-trip transaction cost, cx_t , as in the short-term model. We will denote by K^* the date at which the arbitrageur starts to act in the long-term model.

There are two differences between the two models. First, in the short-term model arbitrage must be carried out via a chain. Second, there is a positive probability, $(1 - \delta_t)$, that the next link in the chain may be missing. We can separate these two effects by analyzing the short-term model in the case where the probability of an arbitrageur arriving next period is 1 (i.e., $\delta t = 1$, for all t).

Consider the arbitrageur's decision problem. If he buys at time t , then with probability $\frac{1}{2}$ he submits an order at the same time that an uninformed hedger submits a buy order. In this case, he loses cx_t (because the market-maker will price the asset to give a return, r , conditional on the information having been revealed); the value of his wealth at $t + 1$ is $W(1 + r) - cx_t$. With probability $\frac{1}{2}$ he is the only trader, and he buys at price p_t . Then he holds the shares until the date T dividend; the value of his wealth at $t + 1$ is:

$$W(1 + r) + x_t \left[\pi/r + 1/(1 + r)^{(T-(t+1))} (1 - \pi) - p_t(1 + r) - c + \pi \right].$$

Therefore, he buys if

$$\frac{1}{2} [W(1 + r) - cx_t] + \frac{1}{2} \left[W(1 + r) + x_t \left(\pi/r + 1/(1 + r)^{(T-(t+1))} (1 - \pi) - p_t(1 + r) - c + \pi \right) \right] > W(1 + r),$$

or

$$\pi/r + 1/(1 + r)^{(T-(t+1))} (1 - \pi) - p_t(1 + r) + \pi > 2c.$$

K^* is determined by the previous expression with $p_t = \pi/r$; it is the first period t for which

$$\frac{1}{2} (1 - \pi)/(1 + r)^{T-(t+1)} > c.$$

Note that the left-hand side, representing the benefit of the arbitrage, has the term " $\frac{1}{2}$ " in it because half the time the arbitrageur reveals the information when he buys the shares at the same time as a hedger.

A different specification of the model might have a greater chance of revealing the information at $t + 1$. We have taken the probability of a hedger arriving to be $\frac{1}{2}$. If this probability were higher, increasing the likelihood of next period's price being fully revealing, then the chance of the arbitrageur being able to buy the asset cheaply at time t would be reduced. A short-term arbitrageur, like a long-term arbitrageur, wants to buy in a deep market so that his purchase does not push up the price too much. But, unlike a long-term arbitrageur, he cannot profit if the market is too deep, because there is a smaller likelihood that a subsequent arbitrageur's purchase will push up the price by the time he needs to sell. In other words, the presence of short horizons has the effect of reducing the profitability of arbitrage and thereby multiplying the effect of the transaction cost on the revelation of information.

C. Cost-of-Carry and the Origin of Limited Horizons

One reason for short horizons concerns the cost of financing an arbitrage position for a long period of time. In this subsection we argue that this cost will rapidly mount up, making longer-term arbitrage unprofitable. This motivates our assumption of exogenous fixed horizons. In order to compare the borrowing costs of long-term and short-term horizons we need to interpret our transaction cost, c , as a borrowing cost. The difference when c is interpreted as a borrowing cost is that the costs should accumulate every period, rather than once per transaction. As discussed above in Section VII.A, it is possible to perform the analysis when the transaction cost is proportional to the cost of the shares rather than the quantity alone, but the closed-form expressions are more complicated without fundamentally changing any insights. For this reason we shall maintain the assumption that the cost is proportional to the number of shares, but not to the price paid for them.

We now develop a "cost-of-carry" model in which arbitrageurs can choose their horizon, but face a cost of financing their portfolios. Strictly speaking, a portfolio costing $p_t x_t$ at time t , held until t' , financed at a premium, c , above the interest rate, r , should accumulate debt of $(1 + r + c)^{t'-t} p_t x_t$. However, to maintain comparability with the rest of the article, we will instead assume that an arbitrageur who holds x_t shares for n periods incurs transaction costs of $c x_t$ in each of the n successive periods. The main simplification is that the interest is not compounding every period. However, this simplified form of transaction cost captures the principal feature of cost-of-carry, namely that the cost accumulates every period.

We will calculate the arbitrage profits accruing to a long-term trader who borrows to finance his trade. A long-term trader is a trader whose horizon spans the event date, T . To make the cost-of-carry interpretation more natural, we will assume that he has zero initial wealth, but unlimited liability. (Consideration of arbitrage with the option to default is beyond the scope of this article.)

We compare the arbitrage profits of the long-term trader to the profit earned by a short-term trader (as given in Proposition 3). (Although we did not assume, in Proposition 3, that the arbitrageur had zero initial wealth, it can easily be seen that the level of initial wealth was simply a constant in the risk-neutral arbitrageur's decision problem.)

Suppose that the long-term trader with cost-of-carry buys the asset at date t when the marketmaker's belief is π . One-half the time he will buy the asset without revealing his information; otherwise he will buy the shares when there is a buying hedger present, fully revealing his information and pushing up the price. (These features are exactly the same as in the short-term model worked out above.)

If he is able to buy the asset at the uninformative price, $p_t = \pi/r$, he will gain/lose the following amounts of money per share: at time t he pays π/r ; from $t + 1$ onwards he gains the dividend stream, π , in perpetuity, except at date T when the dividend will be 1; finally, from date $t + 1$ to T he loses a transaction cost, c , each period. The present value of these cash flows sums to

$$-(\pi/r)(1+r) + \pi + \pi/r + (1-\pi)/(1+r)^{T-(t+1)} - c(1+A(t+1, T))$$

where $A(t+1, T)$ is the present value at date $t+1$ of an annuity of \$1 starting the following period and continuing until date T

$$A(t+1, T) = (1/r) \left[1 - (1/(1+r))^{T-(t+2)} \right].$$

The other half of the time the agent will buy the asset at the fully revealing price. Clearly, it is better in this eventuality to liquidate the position the following period incurring only a single period's borrowing cost, c . Averaging across both cases, the agent's expected arbitrage profits (as of time $t+1$) are

$$\begin{aligned} & \frac{1}{2} \left\{ -(\pi/r)(1+r) + \pi + \pi/r + (1-\pi)/(1+r)^{T-(t+1)} \right. \\ & \quad \left. - c(1+A(t+1, T)) \right\} + \frac{1}{2} \{-c\} \\ & = \frac{1}{2}(1-\pi)/(1+r)^{T-(t+1)} - c - \frac{1}{2}cA(t+1, T). \end{aligned}$$

Compare this to the arbitrage profits per share of the short-term trader (given in Proposition 3)

$$= \frac{1}{4}(1-\pi)/(1+r)^{T-(t+1)} - c.$$

There are two differences. The first, as discussed above, is that the arbitrage is successful twice as often in the case of a long-term trader. Second, the borrowing costs born by the long-term trader have been repeated every period between $t+1$ and T . Clearly, if T is distant from t , then the long-term arbitrage is inherently unprofitable compared to one period arbitrage, relying on the chain. Thus, we have shown:

PROPOSITION 5: *If T is sufficiently distant that*

$$A(t+1, T) > \frac{1}{2}(1-\pi)/(1+r)^{T-(t+1)}c,$$

then the arbitrageur will unwind his position after one period rather than waiting till T .

In other words, with cost-of-carry that accumulate every period, only short-term arbitrages will be profitable. This provides a justification for the exogenous short-term horizons assumed in the main model above.

For expositional purposes, Proposition 5, and the above analysis, only allow the arbitrageur to choose between holding the position until date T and unwinding the position after one period. In fact, the arbitrageur will sometimes choose to unwind the position at an intervening date, i.e., if and when a subsequent arbitrageur has pushed the price up. If that happens, the asset will be priced to give the fair return, r , conditional on the information, so there is no point in continuing to incur the cost of carry. We now give the details of the actual expected cost-of-carry taking this into account.

In the analysis above the probability that the transaction cost, cx_i , was paid at date $t+n$ was given as

$$\begin{aligned} 1 & \text{ for } n = 1; \\ \frac{1}{2} & \text{ for } 1 < n \leq T - t. \end{aligned}$$

In fact, the probability that an uninformed hedger and an arbitrageur simultaneously buy in one of the intervening periods, i , is $\frac{1}{2}(1-\delta_i)$. When this first happens the price will be fully revealing, and in the following period our original arbitrageur will close out his position. Thus, the probability of incurring the transaction cost at period $t+n$ is

$$\frac{1}{2} \left[\left(\frac{1}{2} \right)^{n-1} \prod_{i=t+1}^{n-1} (1-\delta_i) \right].$$

Clearly, the expected transaction cost will be larger than in the short-term model but it will not be as large as in the discussion preceding Proposition 5.

IX. Final Remarks

The conventional wisdom is that short-term horizons do not cause short-term biases in the kind of information that is incorporated in asset prices. This reasoning is based on the principle that a chain of arbitrageurs with short horizons should, by backward induction, replicate the actions of long-term arbitrageurs.

This conventional wisdom implies that short-term biases in asset pricing are incompatible with rationality. Thus, some analyses have attempted to explain short-term biases by describing the effects of irrational traders, while others have asserted that the sophistication of financial market participants is such as to make short-term biases unlikely.

By examining the backward induction argument in greater detail, we have shown that private information, in the presence of transaction costs, will

prevent short-term arbitrageurs from acting like long-term traders. Prices may not be informative about events that are too far distant. Our article does not explicitly endogenize the trading horizons of arbitrageurs, but we show how the cost-of-carry of an arbitrage portfolio can make long-term arbitrage unprofitable. Contracting problems with portfolio managers may also lead them to focus on short-term profit opportunities.

Appendix

This appendix gives the details of the derivation of x_t and the condition for x_t to be strictly positive, as described in Section V. Note that since the hedgers are a continuum of small agents they take prices as given.

Case 1: The agent buys and sells a quantity, x , at $p = \pi/r$. There is no uncertainty in the price. If the dividend is high, his wealth is

$$\begin{aligned}(W - px)(1 + r) + x + px - cx &= W(1 + r) + x - pxr - cx \\ &= W(1 + r) + x - x\pi - cx.\end{aligned}$$

If the dividend is low, his wealth is

$$\begin{aligned}(W - px)(1 + r) + 1 + px - cx &= W(1 + r) + 1 - pxr - cx \\ &= W(1 + r) + 1 - x\pi - cx.\end{aligned}$$

Thus, his expected utility is

$$\pi U[W(1 + r) + x - x(\pi + c)] + (1 - \pi)U[W(1 + r) + 1 - x(\pi + c)].$$

So for t prior to $K - 1$, x_t is defined to be the maximizer of this function. Taking the derivative with respect to x

$$\begin{aligned}\pi U'[W(1 + r) + x - x(\pi + c)](1 - \pi - c) \\ + (1 - \pi)U'[W(1 + r) + 1 - x(\pi + c)](-\pi - c).\end{aligned}$$

Note that the transaction cost will prevent the agent from hedging completely. If $c = 0$, it is easy to verify that the derivative is zero when wealth is equal in the two states, hence $x = 1$. This is the usual local risk neutrality argument.

On the other hand, if $c > 0$, the derivative at $x = 1$ is

$$\begin{aligned}\pi(1 - \pi - c)U'[W(1 + r) + 1 - (\pi + c)] \\ + (1 - \pi)(-\pi - c)U'[W(1 + r) + 1 - (\pi + c)] \\ = [\pi(1 - \pi - c) - (1 - \pi)(\pi + c)]U'[W(1 + r) + 1 - (\pi + c)] \\ = -cU'[W(1 + r) + 1 - (\pi + c)] < 0\end{aligned}$$

showing that the agent does not completely hedge.

Next we derive the condition for the agent to hedge when $c > 0$. The derivative at $x = 0$ will be positive and the agent will hedge if:

$$\pi(1 - \pi - c)U'[W(1 + r)] > (1 - \pi)(\pi + c)U'[W(1 + r) + 1]$$

i.e.,

$$U'[W(1 + r)]/U'[W(1 + r) + 1] > [(1 - \pi)(\pi + c)]/[\pi(1 - \pi - c)].$$

Our analysis therefore requires that agents be sufficiently risk-averse for this condition to hold. Otherwise there will be no trading.

Case 2: The agent buys at price $p_{K-1} = \pi/r$ and sells at a price that will depend on the number of traders in period K .

In every period, the return earned by agents in aggregate is r . Since the marketmaker earns an expected return of r , so do the other agents combined. Since there are no arbitrageurs active from $K-1$ to K , the uninformed hedgers earn an expected return of r over this period. Recall also that the variation in the expected price at period K is independent of the dividend and the uninformed agent's wage-income shock.

In period K , the trading volume may be a multiple 0, 1, or 2 of x_K . In each case there will be a different price, which we denote p_K^i , for $i = 0, 1, 2$. The corresponding probabilities are denoted Pr_K^i . (The formulas for these prices and probabilities were computed in Sections II and III.)

The expected utility of a hedger is therefore

$$\pi \sum_i Pr_K^i U[W(1+r) + x_{K-1}(p_K^i - \pi/r - \pi - c + 1)] \\ + (1 - \pi) \sum_i Pr_K^i U[W(1+r) + x_{K-1}(p_K^i - \pi/r - \pi - c) + 1].$$

x_{K-1} is therefore defined to be the maximizer of this function. The derivative is

$$\pi \sum_i Pr_K^i U'[W(1+r) + x_{K-1}(p_K^i - \pi/r - \pi - c + 1)](p_K^i - \pi/r - \pi - c + 1) \\ + (1 - \pi) \sum_i Pr_K^i U'[W(1+r) + x_{K-1}(p_K^i - \pi/r - \pi - c) + 1](p_K^i - \pi/r - \pi - c).$$

By evaluating at $x_{K-1} = 0$, we obtain that the agent will hedge a positive amount if

$$\pi U'[W(1+r)](Ep_K^i - \pi/r - \pi - c + 1) \\ > (1 - \pi)U'[W(1+r) + 1](c + \pi + \pi/r - Ep_K^i),$$

because of the independence of dividends and trader arrivals. Furthermore, since (as explained above) $Ep_K^i = \pi/r$, this condition becomes

$$\pi(1 - \pi - c)U'[W(1+r)] > (1 - \pi)(\pi + c)U'[W(1+r) + 1]$$

as in Case 1.

Case 3: There are two subcases. First, when the hedger buys at time t , an arbitrageur may also submit an order. In this case, the purchase is made at the fully revealing price, and in the next period the shares are sold at the fully revealing price. The second subcase occurs when only the hedger submits an order at time t . Then when he sells the share at time $t+1$ there are three possible prices.

We start by computing expected utility conditional on each of these subcases. If good news is revealed at time t , the asset earns an expected return of

r thereafter. The expected utility conditional on this subcase is, as in Case 1 above,

$$\pi U[W(1+r) + x_t - x_t(\pi + c)] + (1 - \pi)U[W(1+r) + 1 - x_t(\pi + c)],$$

where we have used the fact that, since the return on the stock is r , $p_t(1+r) = p_{t+1} + \pi$. In this subcase, x_t will therefore be the same as in periods before $K - 1$.

If there is no other trade at time t (the other subcase), then the hedger buys at price p_t and sells at one of three prices at time $t + 1$. As before we use p_{t+1}^i , $i = 0, 1, 2$, to denote the three prices corresponding to the trading volume multiples of 0, 1, or 2 of x_t . The corresponding probabilities are denoted Pr_{t+1}^i . The expected utility conditional on this subcase is

$$\begin{aligned} & \pi \sum_i Pr_{t+1}^i U[W(1+r) + x_t(p_{t+1}^i - p_t(1+r) - c + 1)] \\ & + (1 - \pi) \sum_i Pr_{t+1}^i U[W(1+r) + x_t(p_{t+1}^i - p_t(1+r) - c) + 1]. \end{aligned}$$

The first subcase, where there is an arbitrageur submitting an order in addition to the uninformed, occurs with probability $\delta_t \beta_t$. The second subcase, where there is no other agent submitting an order, therefore has probability $1 - \delta_t \beta_t$. The overall expected utility is therefore the expectation, using these two probabilities, of the above conditional expected utilities. x_t is defined to be the maximizer of this expected utility; note that the actual quantity x_t will be sample-path dependent because Pr_{t+1}^i and p_{t+1}^i depend on the marketmaker's beliefs (we give the formulas for these below).

The derivative with respect to x_t is

$$\begin{aligned} & \delta_t \beta_t \{ \pi U'[W(1+r) + x_t - x_t(\pi + c)](1 - \pi - c) \\ & + (1 - \pi) U'[W(1+r) + 1 - x_t(\pi + c)](-\pi - c) \} \\ & + (1 - \delta_t \beta_t) \{ \pi \sum_i Pr_{t+1}^i U'[W(1+r) + x_t(p_{t+1}^i - p_t(1+r) - c + 1)] x \\ & \times (p_{t+1}^i - p_t(1+r) - c + 1) + (1 - \pi) \sum_i Pr_{t+1}^i U'[W(1+r) \\ & + x_t(p_{t+1}^i - p_t(1+r) - c) + 1] (p_{t+1}^i - p_t(1+r) - c) \}. \end{aligned}$$

We now derive the condition for x_t to be positive. Evaluated at $x_t = 0$, the derivative becomes

$$\begin{aligned} & \delta_t \beta_t \{ \pi U'[W(1+r)](1 - \pi - c) + (1 - \pi) U'[W(1+r) + 1](-\pi - c) \} \\ & + (1 - \delta_t \beta_t) \{ \pi U'[W(1+r)](E^\# p_{t+1} - p_t(1+r) - c + 1) \\ & + (1 - \pi) U'[W(1+r) + 1](E^\# p_{t+1} - p_t(1+r) - c) \\ & = [(E^\# p_{t+1} - p_t(1+r))(1 - \delta_t \beta_t) - c - \pi \delta_t \beta_t] [\pi U'[W(1+r)] \\ & + (1 - \pi) U'[W(1+r) + 1]] + \pi U'[W(1+r)], \end{aligned}$$

where

$$E^\# p_{t+1} = \sum_i Pr_{t+1}^i p_{t+1}^i$$

denotes the expectation of p_{t+1} , conditional on the event that there was no arbitrageur acting at t . This expectation is analogous to E^*p_{t+1} discussed above, but here expectations are from the point of view of the hedger.

Therefore, the uninformed trader will choose to hedge a positive amount if

$$\pi\delta_t\beta_t - (1 - \delta_t\beta_t)(E^\#p_{t+1} - p_t(1 + r)) \\ < \pi U'[W(1 + r)]/[(1 - \pi)U'[W(1 + r) + 1] + \pi U'[W(1 + r)]] - c.$$

We can show that this is stronger than the corresponding condition for dates prior to K (case 1):

$$\pi < \pi U'[W(1 + r)]/[(1 - \pi)U'[W(1 + r) + 1] + \pi U'[W(1 + r)]] - c,$$

since the expected return to the hedger after date K is less than the market return, $1 + r$

$$(E^\#p_{t+1} - p_t + \pi)/p_t < r,$$

or, equivalently

$$E^\#p_{t+1} - p_t(1 + r) < -\pi.$$

Thus,

$$\pi\delta_t\beta_t - (1 - \delta_t\beta_t)(E^\#p_{t+1} - p_t(1 + r)) > \pi.$$

We can derive an explicit expression for the derivative by computing an expression for $E^\#p_{t+1} - p_t(1 + r)$ in terms of $E^\#\beta_{t+1}$

$$p_{t+1} = \pi/r + (1/(1 + r))^{T-(t+1)}(\beta_{t+1} - \pi) \\ p_t(1 + r) = \pi/r + (1/(1 + r))^{T-(t+1)}(\beta_t - \pi).$$

So

$$E^\#p_{t+1} - p_t(1 + r) = (1/(1 + r))^{T-(t+1)}(E^\#\beta_{t+1} - \beta_t).$$

We now compute $E^\#\beta_{t+1}$. At time $t + 1$, there may be 0, 1, or 2 buy orders and corresponding values of β_{t+1} . The conditional probabilities of these events are

Buy = 0:

$$Pr_{t+1}^0 = \frac{1}{2}\{(1 - \delta_{t+1}) + \delta_{t+1}[1 - \beta_t(1 - \delta_t)/[(1 - \delta_t) + \delta_t(1 - \beta_t)]]\} \\ = \frac{1}{2}[1 - \delta_{t+1}\beta_t(1 - \delta_t)/(1 - \delta_t\beta_t)].$$

Buy = 1:

$$Pr_{t+1}^1 = \frac{1}{2}(1 - \delta_{t+1}) + \frac{1}{2}\{\delta_{t+1}[1 - \beta_t(1 - \delta_t)/[(1 - \delta_t) + \delta_t(1 - \beta_t)]]\} \\ + \frac{1}{2}\delta_{t+1}\beta_t(1 - \delta_t)/[1 - \delta_t + \delta_t(1 - \beta_t)] \\ = \frac{1}{2}.$$

Buy = 2:

$$\begin{aligned} Pr_{t+1}^2 &= \frac{1}{2} \delta_{t+1} \beta_t (1 - \delta_t) / [(1 - \delta_t) + \delta_t (1 - \beta_t)] \\ &= \frac{1}{2} \delta_{t+1} \beta_t (1 - \delta_t) / (1 - \delta_t \beta_t). \end{aligned}$$

The marketmaker's beliefs in these three events are:

$$\text{Buy} = 0: \quad \beta_{t+1} = (1 - \delta_{t+1}) \beta_t / (1 - \delta_{t+1} \beta_t).$$

$$\text{Buy} = 1: \quad \beta_{t+1} = \beta_t.$$

$$\text{Buy} = 2: \quad \beta_{t+1} = 1.$$

So the expected belief from the hedger's viewpoint is

$$\begin{aligned} E^{\#} \beta_{t+1} &= [(1 - \delta_{t+1}) \beta_t / (1 - \delta_{t+1} \beta_t)] \times \frac{1}{2} [1 - \delta_{t+1} \beta_t (1 - \delta_t) / (1 - \delta_t \beta_t) \\ &\quad + \beta_t \times \frac{1}{2} + 1 \times \frac{1}{2} [1 - \delta_{t+1} \beta_t (1 - \delta_t) / (1 - \delta_t \beta_t)] \\ &= \frac{1}{2} \beta_t [1 + (1 - \delta_{t+1}) \beta_t / (1 - \delta_{t+1} \beta_t)] [1 + \delta_{t+1} (1 - \delta_t) / (1 - \delta_t \beta_t)]. \end{aligned}$$

Using this expression, $E^{\#} p_{t+1} - p_t(1 + r)$ may be evaluated to verify the condition that uninformed agents choose to hedge a positive amount. We therefore require that the hedgers are sufficiently risk averse that this condition holds for all $t > K$ for all possible nonrevealing price paths. Of course, if the information has already been revealed prior to t , the prices are set to ensure the riskless return r and the hedging conditions for the uninformed is ensured by the analysis in Case 1.

REFERENCES

- Allen, Franklin, and Gary Gorton, 1993, Churning Bubbles, *Review of Economic Studies* 60, 813-836.
- Ausubel, Lawrence, 1990, Partially-revealing Rational Expectations Equilibrium in a Competitive Economy, *Journal of Economic Theory* 50, 1022-1041.
- Biais, Bruno, and Pierre Hillion, 1992, Insider Trading and Stock Options Markets, Working paper, Groupe HEC.
- Brennan, Michael, and Patricia Hughes, 1991, Stock Prices and the Supply of Information, *Journal of Finance* 46, 1665-1691.
- De Long, Bradford, Andrei Shleifer, Lawrence Summers, and Robert Waldman, 1990, Noise Trader Risk in Financial Markets, *Journal of Political Economy* 98, 703-738.
- Diamond, Douglas, and Robert Verrecchia, 1981, Information Aggregation in a Noisy Rational Expectations Economy, *Journal of Financial Economics* 9, 221-235.
- Dimson, Elroy, and Paul Marsh, 1989, The Marketability of Smaller Companies in the UK, Working paper, London Business School.
- Dow, James, and Gary Gorton, 1993, Noise Trading, Professional Portfolio Management and Economic Welfare, Working paper, London Business School.
- Froot, Kenneth, David Scharfstein, and Jeremy Stein, 1992, Herd on the Street: Informational Inefficiencies in a Market with Short-Term Speculation, *Journal of Finance* 47, 1461-1484.
- Glosten, Lawrence, and Paul Milgrom, 1985, Bid, Ask, and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders, *Journal of Financial Economics* 14, 71-100.

- Grossman, Sanford, and Joseph Stiglitz, 1976, Information and Competitive Price Systems, *American Economic Review* 66, 246–253.
- Hart, Oliver, and David Kreps, 1986, Price Destabilizing Speculation, *Journal of Political Economy* 94, 393–408.
- Kyle, Albert, (1985), Continuous Auctions and Insider Trading, *Econometrica* 53, 1315–1335.
- Shleifer, Andrei, and Robert Vishny, 1990, Equilibrium Short Horizons of Investors and Firms, *American Economic Review* 80, 148–153.
- Vives, Xavier, 1992, Short-term Investment and the Informational Efficiency of the Market, Working paper, Institut d'Anàlisi Econòmica, Universitat Autònoma de Barcelona.