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Expected Returns, Time-varying Risk, and Risk Premia

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ABSTRACT

A new empirical model for intertemporal capital asset pricing is presented that allows both time-varying risk premia and betas where the latter are identified from the dynamics of the conditional covariance of returns. The model is more successful in explaining the predictable variations in excess returns when the returns on the stock market and corporate bonds are included as risk factors than when the stock market is the single factor. Although changes in the covariance of returns induce variations in the betas, most of the predictable movements in returns are attributed to changes in the risk premia.

THIS PAPER PRESENTS A new empirical model based upon the multiple beta, or intertemporal capital asset pricing model (ICAPM), to study the predictability of stock, bill, and bond returns. The model allows for both time-varying betas and risk premia. The risk premia are modeled as linear functions of a set of instrumental variables, estimated by rolling regressions to allow for possible structural changes. The betas are identified from estimates of the conditional covariance matrix of returns. The model is estimated with an asymptotically efficient two-step procedure.

Recent research in financial economics has been directed towards explaining the predictability of asset returns. Bollerslev, Engle, and Wooldridge (1988), Harvey (1989), Ng (1991), Bodurtha and Mark (1991), and Chan, Karolyi, and Stulz (1992) explore the role of changing betas within the context of the conditional CAPM where the market is the sole risk factor. This article's model allows for multiple risk factors that can account for possible variations in the investment opportunity set that are ignored by the conditional CAPM.[1] It also allows examination of the contribution that conditional heteroskedasticity makes to the predictability of returns through variations in the betas.

I examine the behavior of monthly returns for five portfolios of common stocks, two Treasury bills, and a portfolio of long-term Treasury bonds from January 1964 through December 1990. I find that more of the predictable variations in excess returns can be accounted for when the returns on the

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[1] Models by Engle, Ng, and Rothschild (1990, 1991), Buse and Turtle (1991), and Nanisetty (1991) also identify the betas from the covariance matrix of returns and allow for multiple risk factors.

stock market and corporate bonds are included as risk factors than when the stock market is the single risk factor. In fact, the two-factor model can explain the predictable variations in excess returns on all the stocks, long-term bonds, and six-month bills. On *average*, the risk premia associated with interest rate risk have the strongest effect on expected bill and bond returns, while the expected returns on the common stocks are most strongly influenced by the risk premium associated with the stock market. There is some evidence that the betas vary systematically over the business cycle. But, changes in risk contribute relatively little to the predictability of returns through the betas. Instead, like Ferson and Harvey (1991), the primary source of predictability can be traced to variations in the risk premium.

The next section presents and discusses the empirical specification. Section II describes how the model is estimated. The empirical results are presented and discussed in Sections III and IV. Section V concludes with a summary of the main results.

I. The Model

The empirical model is motivated by the ICAPM restrictions that originate from the investor's first-order conditions. For any asset i these first-order conditions can be expressed as $E_t[M_{t+1}(1 + R_{i,t+1})] = 1$, where M_{t+1} is the discounted marginal rate of substitution between consumption in periods $t + 1$ and t, $R_{i,t+1}$ is the real return from t to $t + 1$ on asset i, and E_t denotes the expectation conditioned on information available at t. This condition holds for all returns, including the return on a benchmark asset $R_{m,t+1}$, which is perfectly conditionally correlated with M_{t+1} and the conditionally risk free return $R_{0,t+1}$ (known at t).

The following restrictions on excess returns ($r_{i,t+1} \equiv R_{i,t+1} - R_{0,t+1}$ and $r_{m,t+1} \equiv R_{m,t+1} - R_{0,t+1}$) are implied:

$$E_t r_{i,t+1} = \beta_{i,m,t}\ E_t r_{m,t+1}, \tag{1a}$$

$$\beta_{i,m,t} = \mathrm{cov}_t(r_{i,t+1}, r_{m,t+1})/\mathrm{var}_t(r_{m,t+1}) \tag{1b}$$

where $\mathrm{cov}_t(.)$ and $\mathrm{var}_t(.)$ denote the conditional covariance and variance given information available at t (see, for example, Hansen and Hodrick (1983) and Campbell (1987)). Thus, variations in the expected excess return on an asset may be due to changes in the expected excess return on the benchmark or in the asset's beta, $\beta_{i,m,t}$.

Three auxiliary assumptions are needed to empirically examine equation (1). These concern (i) the identity of the benchmark portfolio, (ii) the behavior of the $\beta_{i,m,t}$, and (iii) the link between realized and expected returns.

(i) I follow Gibbons and Ferson (1985), Campbell (1987), and Shanken (1991) by assuming that the benchmark portfolio is a time-varying weighted combination of the risk-free asset, and k fixed observable hedge portfolios with (possibly correlated) returns $H_{1,t}, \ldots H_{k,t}$. Thus, $R_{m,t+1} = w_{0,t}R_{0,t+1} + w_{1,t}H_{1,t+1} + \cdots + w_{k,t}H_{k,t+1} + \varepsilon_{m,t+1}$ where $w_{j,t}$ are scalars and $\Sigma_{j=0}^{k} w_{j,t} =$

1. $\epsilon_{m,t}$ is a benchmark residual that is assumed to be uncorrelated with $H_{k,t}$ and $R_{i,t}$. The excess return on the benchmark is

$$r_{m,t+1} = \mathbf{w}_t \mathbf{h}_{t+1} + \varepsilon_{m,t+1} \tag{2}$$

where $\mathbf{w}_t \equiv [w_{1,t}, \ldots w_{k,t}]$ and $\mathbf{h}_t \equiv [h_{1,t}, \ldots h_{k,t}]'$ are k-dimensional vectors of weights and excess returns on the individual hedge portfolios. (Hereafter, all vectors and matrices are identified with bold type.)

Let $\mathbf{r}_t \equiv [r_{1,t}, \ldots r_{q,t}]'$ be a $q \times 1$ vector of individual excess returns to be studied. If we substitute equation (2) into equation (1), and note that equation (1) holds for all excess returns including $\mathbf{h}_t$, the ICAPM restrictions can be rewritten as

$$E_t \mathbf{r}_{t+1} = \boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t} E_t \mathbf{h}_{t+1} \tag{3a}$$

where the $q \times k$ conditional beta matrix is

$$\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t} \equiv \mathrm{cov}_t(\mathbf{r}_{t+1}, \mathbf{h}'_{t+1}) \mathrm{var}_t(\mathbf{h}_{t+1})^{-1}. \tag{3b}$$

Below, different sets of returns will be employed to identify the benchmark portfolio and diagnostic tests used to evaluate the suitability of each choice.

(ii) The second auxiliary assumption concerns the identification of $\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t}$. Earlier empirical tests of the ICAPM either assumed that the betas were constant (Gibbons and Ferson (1985)) or that they could be expressed as linear functions of observable variables (Shanken (1990)). Here I identify the beta matrix as part of the time-varying conditional covariance matrix of returns. For this purpose, assume that the k-dimensional vector of expected portfolio returns, $E_t \mathbf{h}_{t+1}$, is available. Under rational expectations, the theoretical restrictions imposed on the set of q individual returns by equation (3) can be written as

$$\mathbf{Y}_{t+1} = \mathbf{A}_t E_t \mathbf{h}_{t+1} + \mathbf{e}_{t+1}, \tag{4}$$

where $\mathbf{Y}_{t+1} \equiv [\mathbf{r}'_{t+1} \; \mathbf{h}'_{t+1}]'$, $\mathbf{e}_{t+1} \equiv [\mathbf{er}'_{t+1} \; \mathbf{eh}_{t+1}{}']'$ is the $(q+k)$-dimensional vector of forecasts errors with $E_t \mathbf{e}_{t+1} = \mathbf{0}$, and $\mathbf{A}_t \equiv [\boldsymbol{\beta}'_{\mathbf{r},\mathbf{h},t} \mathbf{I}_k]'$ is a $(q+k) \times k$ matrix.

The beta matrix depends on elements of the conditional covariance of $\mathbf{Y}_{t+1}$, which, under rational expectations, is $E_t[\mathbf{e}_{t+1} \mathbf{e}'_{t+1}] = \boldsymbol{\Omega}_t \equiv [\Omega_{i,j,t}]$. Hence, partitioning $\boldsymbol{\Omega}_t$ conformably with $\mathbf{r}_{t+1}$ and $\mathbf{h}_{t+1}$, equation (3b) implies that $\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t} \equiv \boldsymbol{\Omega}_{\mathbf{r},\mathbf{h},t}(\boldsymbol{\Omega}_{\mathbf{h},\mathbf{h},t})^{-1}$. Any variations in $\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t}$ must therefore be reflected in the dynamics of $\boldsymbol{\Omega}_t$. To model these dynamics I use least squares theory to write the errors in forecasting the individual assets returns as $\mathbf{er}_{t+1} = \boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t} \mathbf{eh}_{t+1} + \mathbf{u}_{t+1}$, where $\mathbf{u}_{t+1}$ is a vector of asset-specific shocks that are uncorrelated with the errors in forecasting the excess returns on the hedge portfolios, i.e., $\mathrm{cov}_t(\mathbf{u}_{t+1}, \mathbf{eh}'_{t+1}) = \mathbf{0}$. The covariance matrix for the $q+k$ vector of forecast errors, $\mathbf{e}_{t+1}$, can now be written as

$$\boldsymbol{\Omega}_t \equiv \begin{bmatrix} \boldsymbol{\Omega}_{\mathbf{u},\mathbf{u},t} + \boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t} \boldsymbol{\Omega}_{\mathbf{h},\mathbf{h},t} \boldsymbol{\beta}'_{\mathbf{r},\mathbf{h},t} & \boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t} \boldsymbol{\Omega}_{\mathbf{h},\mathbf{h},t} \\ \boldsymbol{\Omega}_{\mathbf{h},\mathbf{h},t} \boldsymbol{\beta}'_{\mathbf{r},\mathbf{h},t} & \boldsymbol{\Omega}_{\mathbf{h},\mathbf{h},t} \end{bmatrix} \tag{5}$$

where $\boldsymbol{\Omega}_{\mathbf{u},\mathbf{u},t}$ is the conditional covariance matrix of $\mathbf{u}_{t+1}$. This specification insures that $\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t}\boldsymbol{\Omega}_{\mathbf{h},\mathbf{h},t} = \boldsymbol{\Omega}_{\mathbf{r},\mathbf{h},t}$ so that the restrictions in equation (3b) are satisfied for any beta and covariance matrices, $\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t}$ and $\boldsymbol{\Omega}_{\mathbf{h},\mathbf{h},t}$.

In the empirical specification I assume that the covariance matrices $\boldsymbol{\Omega}_{\mathbf{u},\mathbf{u},t}$ and $\boldsymbol{\Omega}_{\mathbf{h},\mathbf{h},t}$ follow asymmetric GARCH processes:

$$\boldsymbol{\Omega}_{\mathbf{u},\mathbf{u},t} = \boldsymbol{\Omega}_{\mathbf{u},\mathbf{u}} + \mathbf{G}_0 \bullet (\mathbf{u}_t + \mathbf{G}_2)(\mathbf{u}_t + \mathbf{G}_2)' + \mathbf{G}_1 \bullet \boldsymbol{\Omega}_{\mathbf{u},\mathbf{u},t-1} \tag{6a}$$

and

$$\boldsymbol{\Omega}_{\mathbf{h},\mathbf{h},t} = \boldsymbol{\Omega}_{\mathbf{h},\mathbf{h}} + \mathbf{F}_0 \bullet (\mathbf{eh}_t + \mathbf{F}_2)(\mathbf{eh}_t + \mathbf{F}_2)' + \mathbf{F}_1 \bullet \boldsymbol{\Omega}_{\mathbf{h},\mathbf{h},t-1}. \tag{6b}$$

Here $\boldsymbol{\Omega}_{\mathbf{u},\mathbf{u}}$, $\mathbf{G}_0$, and $\mathbf{G}_1$ are symmetric $q \times q$ matrices, $\boldsymbol{\Omega}_{\mathbf{h},\mathbf{h}}$, $\mathbf{F}_0$, and $\mathbf{F}_1$ are symmetric $k \times k$ matrices, $\mathbf{G}_2$ is a $q \times 1$ vector, and $\mathbf{F}_2$ is a $k \times 1$ vector. "$\bullet$" denotes element-by-element multiplication. This specification allows the conditional covariance matrix to respond asymmetrically to positive and negative shocks, $\mathbf{eh}_t$ and $\mathbf{u}_t$. In particular, when the elements of the vectors $\mathbf{G}_2$ and $\mathbf{F}_2$ are negative, the conditional variances increase more in response to a negative error than to a positive error of the same size (see Engle, and Ng (1991)). The asymmetric effects of shocks on the conditional variance of asset returns has been noted by French, Schwert, and Stambaugh (1987) and Schwert (1989) and is often referred to as the "leverage effect."

The final element driving $\boldsymbol{\Omega}_t$ is the beta matrix. I assume that both asset-specific and market shocks affect the betas. For asset i's beta on the kth, hedge portfolio (i.e., the i,kth element of $\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t}$):

$$\beta_{i,k,t} = \beta_{i,k} + \lambda_{i,k,0} eh_{k,t} + \lambda_{i,k,1} u_{i,t} + \lambda_{i,k,2}\, \beta_{i,k,t-1} \tag{6c}$$

where $\mathbf{eh}_t \equiv [eh_{k,t}]$ and $\mathbf{u}_t \equiv [u_{i,t}]$. When the scalars $\lambda_{i,k,0}$ and $\lambda_{i,k,1}$ are negative, the conditional beta will rise in response to unanticipated negative returns on both the individual assets and the hedge portfolios and fall in response to positive returns. Chan (1988), Ball and Kothari (1989), and Braun, Nelson, and Sunier (1990) have argued that such movements are consistent with a leverage effect.

(iii) The third auxiliary assumption concerns identification of the expected excess hedge portfolio returns or risk premia, $E_t\mathbf{h}_{t+1}$. I assume that conditional expectations are linear functions of a set of instrumental variables. The estimates of $E_t\mathbf{h}_{t+1}$ are obtained as the out-of-sample forecasts from the linear regression of $\mathbf{h}_\tau$ for $\tau \le t$ on instrumental variables known at t and before. This procedure insures that the estimates $E_t\mathbf{h}_{t+1}$ are valid conditional forecasts.[2]

[2] The linearity of conditional expectations is a restrictive assumption that may be inaccurate in practice (see Harvey (1990)). To assess its importance, Evans (1992) examines models that include nonparametric estimates of $E_t\mathbf{h}_{t+1}$. The results from these models are very similar to those presented below.

II. Econometric Issues

It is useful to think about estimating the model in two stages: (i) estimate the risk premia and (ii) find the maximum likelihood estimates for the parameters in the covariance and beta processes in equation (6) using the estimates from stage (i). The difficulty associated with a two-stage procedure is that, in general, inferences drawn from the second stage estimates must account for the sampling properties of the first stage estimates.

Fortunately, the inference problem posed by the two-step procedure can be dealt with by exploiting the structure of the model. Let $\boldsymbol{\mu}_t(\boldsymbol{\theta})$ represent the regression estimate of $E_t\mathbf{h}_{t+1}$ that depends upon the parameter vector $\boldsymbol{\theta}$, and let $\boldsymbol{\phi}$ represent the parameters of the processes in equation (6) that govern the dynamics of $\boldsymbol{\Omega}_t$. Assuming that the vector of forecast errors $\mathbf{e}_{t+1} \equiv [\mathbf{er}'_{t+1}\ \mathbf{eh}'_{t+1}]'$ is drawn from a multivariate normal distribution $N(0, \boldsymbol{\Omega}_t)$, the sample log likelihood is (ignoring the constant)

$$\begin{aligned} L_T(\boldsymbol{\phi}, \boldsymbol{\theta}) = &-\frac{k+q}{2} \textstyle\sum_{t=1}^{T} \log|\boldsymbol{\Omega}_t(\boldsymbol{\phi})| \\ &-\frac{1}{2} \textstyle\sum_{t=1}^{T} [\mathbf{Y}_{t+1} - \mathbf{A}_t(\boldsymbol{\phi})\boldsymbol{\mu}_t(\boldsymbol{\theta})]'\boldsymbol{\Omega}_t(\boldsymbol{\phi})^{-1} \\ &\times [\mathbf{Y}_{t+1} - \mathbf{A}_t(\boldsymbol{\phi})\boldsymbol{\mu}_t(\boldsymbol{\theta})]. \end{aligned} \tag{7}$$

where $\mathbf{Y}_{t+1} \equiv [\mathbf{r}'_{t+1}\ \mathbf{h}'_{t+1}]'$ and $\mathbf{A}_t(\boldsymbol{\phi}) \equiv [\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t}(\boldsymbol{\phi})'\ \mathbf{I}_k]'$. This sample log likelihood can be written as the sum of the marginal log likelihood for the k hedge portfolio returns and the conditional log likelihood for the q assets:

$$\begin{aligned} L_T(\boldsymbol{\phi}, \boldsymbol{\theta}) \\ = &-\frac{k}{2} \textstyle\sum_{t=1}^{T} \log|\boldsymbol{\Omega}_{\mathbf{h},\mathbf{h},t}(\boldsymbol{\phi})| \\ &-\frac{1}{2} \textstyle\sum_{t=1}^{T} [\mathbf{h}_{t+1} - \boldsymbol{\mu}_t(\boldsymbol{\theta})]'\boldsymbol{\Omega}_{\mathbf{h},\mathbf{h},t}(\boldsymbol{\phi})^{-1}[\mathbf{h}_{t+1} - \boldsymbol{\mu}_t(\boldsymbol{\theta})] \\ &-\frac{q}{2} \textstyle\sum_{t=1}^{T} \log|\boldsymbol{\Omega}_{\mathbf{u},\mathbf{u},t}(\boldsymbol{\phi})| \\ &-\frac{1}{2} \textstyle\sum_{t=1}^{T} [\mathbf{r}_{t+1} - \boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t}(\boldsymbol{\phi})\mathbf{h}_{t+1}]'\boldsymbol{\Omega}_{\mathbf{u},\mathbf{u},t}(\boldsymbol{\phi})^{-1}[\mathbf{r}_{t+1} - \boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t}(\boldsymbol{\phi})\mathbf{h}_{t+1}] \end{aligned} \tag{8}$$

Here we see that the conditional log likelihood for the asset returns only depends on the parameters in $\boldsymbol{\phi}$. Notice also that in the marginal log likelihood of the portfolio returns, the conditional mean depends on $\boldsymbol{\theta}$ and the conditional variance on $\boldsymbol{\phi}$. The information matrix for $L_T(\boldsymbol{\phi}, \boldsymbol{\theta})$ defined over $(\boldsymbol{\phi}, \boldsymbol{\theta})$ will therefore be block diagonal. Block diagonality is sufficient to establish the asymptotic efficiency of the two-step procedure (see Evans (1992)).

Although this result means that the sampling properties of the estimated risk premia can be ignored when drawing asymptotic inferences, the two-step procedure may affect the parameter estimates in finite samples. Any sample correlation between the estimation error in the betas and the expected portfolio returns could induce a finite sample bias in the estimated forecast errors $\mathbf{er}_{t+1}$. To allow for this possibility, I include a constant in the asset return equations. The estimable form of equation (4) becomes:

$$\mathbf{Y}_{t+1} = \mathbf{A}_0 + \mathbf{A}_t E_t \mathbf{h}_{t+1} + \mathbf{e}_{t+1}, \tag{4'}$$

where $\mathbf{A}_0 \equiv [\boldsymbol{\alpha}' \ \ \mathbf{0}']'$ with $\boldsymbol{\alpha}$ a $q \times 1$ vector of constants.[3]

III. Data and Empirical Results

A. The Data

I examine the behavior of monthly stock, Treasury bills, and bond returns from January 1964 to December 1990. The stock returns are value-weighted portfolio returns of common stocks on the New York Stock Exchange (NYSE), formed from size deciles. The deciles are based on share prices at the beginning of the year. I include the returns from the 1st, (smallest value), 3rd, 5th, 8th, and 10th deciles. I also study the monthly returns on two and six-month Treasury bills and a portfolio of five-to-ten year Treasury bonds. All these series were obtained from the Center for Research in Security Prices (CRSP) tapes. Excess returns are calculated by subtracting the one-month Treasury bill rate from each of the monthly returns.

Two sets of returns are used to identify the return on the benchmark portfolio. These are the excess returns on the value-weighted index of common stocks on the NYSE from CRSP and the excess returns on a corporate bond portfolio from Ibbotson Associates. Panel A of Table I presents statistics for the portfolio and individual excess returns.

A wide selection of information variables are used to estimate the risk premia. In addition to lagged excess returns, they are: the lagged realized real return on a one-month Treasury bill RR_t, the spread between the yield on one- and six-month Treasury bills ($R_{1-6,t}$), the spread between the yields on Moody's AAA- and BAA-rated bonds as reported by the Federal Reserve Bulletin ($R_{A-B,t}$), the spread between the dividend-price ratio on the Standard and Poor's 500 and the one-month Treasury bill rate ($R_{DIV,t}$),[4] and a January dummy (Jan_t).[5] The bottom of Panel A shows that none of the sample correlations between these instruments are very large. This suggests that the instruments are not redundant and that the regression estimates of the risk premia will not be subject to multicollinearity problems.

[3] The block diagonality of the information matrix is not affected by using equation (4′) rather than equation (4). I am grateful to an anonymous referee for pointing out the potential finite sample problem that led me to modify the model in this way. The other possible effects of the two-step procedure are examined in Evans (1992) with the aid of some Monte Carlo experiments.

[4] The dividend price ratio comes from table 1.35 of the *Federal Reserve Bulletin*. The measure is defined as dividends over the past year divided by the market price. The *Federal Reserve Bulletin* reports the average of the daily ratios each month.

[5] Previous research showed that these variables have high correlations with the excess returns studied here. The studies include: Fama and French (1988), Keim and Stambaugh (1986) and Campbell (1987).

Table I

Summary Statistics for Excess Returns 1964:1 to 1990:12

Excess returns are calculated with respect to the one-month Treasury bill rate. The instrument variables are: the real return on a one-month Treasury bill, RR; the spread between the yields on six- and one-month Treasury bills, R_{1-6}; the spread between the yields on Moody's AAA- and BAA-rated bonds, R_{A-B}; and the difference between the dividend yield on Standard and Poor's 500 stock index and the one-month Treasury bill's rate, R_{DIV}. Returns are expressed in units of percent per year by multiplying by 1,200. Panel B presents statistics on the out-of-sample projection estimate of $E_t r_{i,t+1}$ using the instruments reported in the right-hand column. The autocorrelations of the forecast errors $r_{i,t+1} - E_t r_{i,t+1}$ at lag i are reported under ρ_i. The column header χ^2 reports Lagrange Multiplier statistics for the predictability of the estimated forecast errors. Marginal significance levels are shown in parentheses.

Panel A: Excess Returns

	Symbol $r_{i,t}$	Mean	Std.	Autocorrelations ρ_1	ρ_2	ρ_3	ρ_6
Individual excess returns							
Stock decile 1	r_{ST1}	10.136	25.428	0.124	−0.002	−0.037	0.008
Stock decile 3	r_{ST3}	8.086	21.321	0.152	−0.019	−0.020	0.008
Stock decile 5	r_{ST5}	7.230	19.449	0.159	−0.039	−0.009	−0.010
Stock decile 8	r_{ST8}	6.383	18.359	0.108	−0.055	−0.027	−0.012
Stock decile 10	r_{ST10}	3.673	14.815	0.015	−0.036	0.007	−0.029
2-month T-bill	r_{TB2}	0.580	0.281	0.546	0.344	0.367	0.328
6-month T-bill	r_{TB6}	0.554	1.181	0.040	0.100	0.269	0.329
Long-term T-bonds	r_{TBL}	0.809	5.796	0.129	−0.085	−0.079	0.041
Portfolio excess returns							
Stock market	h_{SM}	4.511	15.530	0.062	−0.047	−0.003	−0.033
Corporate bonds	h_{CB}	0.640	9.796	0.141	−0.018	−0.077	0.069
Instrumental variables							
Excess dividend yield	R_{DIV}	−2.529	0.602	0.963	0.931	0.905	0.860
AAA-BAA spread	R_{A-B}	1.104	0.141	0.989	0.980	0.973	0.855
Term structure spread	R_{1-6}	−0.612	0.952	0.124	−0.065	0.158	0.357
1-month real rate	RR	1.491	0.901	0.626	0.532	0.450	0.443
Correlation matrix		R_{DIV}	R_{A-B}	R_{1-6}	RR	r_{SM}	r_{TB2}
Excess dividend yield	R_{DIV}	1.000					
AAA-BAA spread	R_{A-B}	−0.385	1.000				
Term structure slope	R_{1-6}	0.275	−0.176	1.000			
1-month real rate	RR	−0.301	0.223	−0.045	1.000		
Stock market	h_{SM}	0.164	0.150	0.161	0.118	1.000	
2-month T-bill	r_{TB2}	−0.234	0.282	−0.125	0.119	−0.080	1.000

Panel B: Expected Excess Returns

	Symbol	Residual Autocorrelations ρ_1	ρ_2	ρ_3	ρ_6	χ^2 (sig.)	Instruments
Stock market	h_{SM}	−0.031	−0.094	−0.030	−0.039	5.698 (0.458)	$R_{DIV}, R_{A-B}, r_{TB2}, RR$
Corporate bonds	h_{CB}	−0.054	−0.031	−0.065	0.045	5.897 (0.435)	$R_{DIV}, R_{A-B}, r_{TB2}, R_{1-6}$

Panel B of Table I presents summary statistics for the estimates of expected excess returns used to identify the benchmark portfolio. The instruments used to estimate each expected excess return are listed in the right hand column. They were chosen as the only statistically significant regressors (allowing for heteroskedasticity) in regressions of the excess return on a constant and all the instruments over the whole sample period. There is little evidence of serial correlation in the estimated forecast errors as measured by the autocorrelations ρ_i. I also conducted Lagrange Multiplier (LM) tests to check whether the forecast errors could be predicted using R_{DIV}, R_{A-B}, R_{1-6}, and one lagged value of RR, r_{SM}, and r_{TB2} as instruments. As the table shows, none of the χ^2 statistics are significant at the 5 percent level.

B. Estimates of the ICAPM with a Single Hedge Portfolio

The results from estimating models with 8 assets and the value-weighted index of common stocks on the NYSE as the single hedge portfolio are reported in Table II. Panel A reports the maximum likelihood estimates of the intercepts α_i and the parameters in the beta and variance processes:

$$\beta_{i,t} = \beta_i + \lambda_{i,0} eh_t + \lambda_{i,1} u_{i,t} + \lambda_{i,2}\,\beta_{i,t-1} \tag{9a}$$

$$\Omega_{i,t} = \Omega_i + G_{i,0}\Omega_{i,t-1} + G_{i,1}(u_{i,t} + G_{i,2})^2 \tag{9b}$$

where $\beta_{i,t}$ are the row elements of $\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t}$, and $\Omega_{i,t}$ are the diagonal elements of $\boldsymbol{\Omega}_{\mathbf{u},\mathbf{u},t}$. In this model asset-specific shocks are assumed to be uncorrelated with one another so that $\mathrm{cov}_t(u_{i,t+1}, u_{j,t+1}) = 0$, for $i \neq j$, making $\boldsymbol{\Omega}_{\mathbf{u},\mathbf{u},t}$ a diagonal matrix. This restriction is introduced for parsimony and is tested below. Below the parameter estimates, the table reports the robust standard errors developed by Boolerslev and Wooldridge (1989) that allow for the possibility that the forecast errors are not conditionally normal.[6]

Column 1 shows that the estimates of the intercepts α_i are only significantly different from zero in the equations for bills. This may indicate that there is a sizable finite sample correlation between the estimation errors in the betas and risk premia or the presence of an average "pricing error" due to model misspecification. The estimates of $\lambda_{i,0}$ and $\lambda_{i,1}$ reported in columns 3 and 4 indicate that asset-specific and aggregate shocks generally have statistically significant effects on the conditional betas of the stock equations; aggregate shocks, eh_t, raise the betas, while asset-specific shocks, $u_{i,t}$, lower the betas.[7] There is therefore some evidence to support the presence of leverage effects in the betas. By contrast, neither shock appears to signifi-

[6] The Appendix describes how the robust standard errors and robust test statistics (presented below) are calculated.

[7] When the market return is included as a risk factor, the multiple-beta model implies that $\boldsymbol{\omega}_t\boldsymbol{\beta}_{sm,t} = 1$ for all t where $\boldsymbol{\omega}_t$ is a vector of time-varying weights such that $r_{sm,t+1} = \boldsymbol{\omega}_t\mathbf{r}_{t+1}$. This means that the covariation between the betas and any other variable x, must obey the restriction $\boldsymbol{\omega}\,\mathrm{cov}(\boldsymbol{\beta}_{sm,t}, x) \approx -\mathrm{cov}(\boldsymbol{\omega}_t, x)\boldsymbol{\beta}_{sm}$, where $\boldsymbol{\omega}$ and $\boldsymbol{\beta}_{sm}$ are the sample averages of $\boldsymbol{\omega}_t$ and $\boldsymbol{\beta}_{sm,t}$. This implicit restriction on the dynamics of the betas applies to all the models I estimate.

cantly affect the betas for bills. In the case of long-term bonds, asset-specific shocks do have a significantly positive effect on the betas. Column 5 shows that the conditional betas for all but the bill returns are significantly autocorrelated. Interestingly, the estimates imply that the betas on the smaller decile returns are more positively autocorrelated than the betas on the larger deciles.

There is some evidence of asymmetric GARCH effects in the conditional variance of asset-specific shocks. The estimates of $G_{i,2}$ in column 9 are large, negative, and significantly different from zero in four of the stock equations. These estimates imply that negative shocks $u_{i,t}$, raise $\mathbf{\Omega}_{i,t}$ more than positive shocks of the same magnitude. The estimated processes for $\mathbf{\Omega}_{i,t}$ are highly persistent, as measured by the sum of the $G_{i,0}$ and $G_{i,1}$ coefficients.[8]

Panel B of Table II reports the results of a number of specification tests. The estimated model assumes that expected excess returns, $E_t\mathbf{r}_{t+1}$, move one for one with variations in $\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t}E_t\mathbf{h}_{t+1}$. To test this hypothesis, I first conducted LM tests on the restriction $\psi_i = 1$ in

$$r_{i,t+1} = \alpha_i + \psi_i\,\beta_{i,t}E_t h_{t+1} + er_{i,t+1}. \tag{10}$$

Column 1 of the table reports robust LM statistics developed by Wooldridge (1990) that allow for model misspecification. Few of the statistics are significant at the 5 percent level. To examine the joint hypothesis that $\alpha_i = 0$ and $\psi_i = 1$, I tested the conditional moment condition $E[r_{i,t+1} - \beta_{i,t}E_t h_{t+1}\,|\,\mathbf{x}_t] = 0$ where the vector $\mathbf{x}_t$ includes a constant and $\beta_{i,t}E_t h_{t+1}$. The robust conditional moment (CM) test statistics shown in column 2 are significant at the 5 percent level in the bill equations.

The statistics in columns 3 and 4 allow us to examine whether changes in the betas and risk premia account for all of the predictability in $r_{i,t+1}$. If the model is correctly specified, the estimated residuals, $er_{i,t+1}$, should not be predictable using any variable in the time t information set. I use a vector of variables known at t, $\mathbf{x}_t$, to test the CM restriction $E[er_{i,t+1}\,|\,\mathbf{x}_t] = 0$. The CM tests in column 3 look for serial correlation in $er_{i,t+1}$, so the $\mathbf{x}_t$ vector includes $er_{i,t-j}$, for $j = 0$ to 6. For the CM tests in column 4, $\mathbf{x}_t$ includes: the lagged real rate, the dividend yield spread, AAA–BAA yield spread, the slope of the term structure, and the January dummy (when testing the stock equations). Overall, the results indicate that the single portfolio model does rather poorly in explaining the predictability of returns. The test statistics are highly significant in the bill and bond equations indicating that shocks to these equations are serially correlated and predictable with other variables.

To examine whether the estimated model adequately captures the dynamics in the conditional covariance matrix of returns, I also calculated CM tests for $E[(er_{i,t+1})^2 - (\Omega_{i,t} + \beta_{i,t}\Omega_{h,h,t}\beta_{i,t})\,|\,\mathbf{x}_t] = 0$, $E[er_{i,t+1}eh_{t+1} -$

[8] This measure of persistence is discussed in Bollerslev, Chou, and Kroner (1992). The estimated value of $F_{i,0} + F_{i,1}$ is 0.824 (with standard error, 0.053) indicating that the process for $\Omega_{h,h,t}$, the conditional variance of the market return in equation (6b), is also highly persistent.

Table II

Estimates and Specification Tests of the ICAPM with One Hedge Portfolio

Panel A reports the maximum likelihood estimates of the model:

$$\mathbf{r}_{t+1} = \alpha + \beta_{r,h,t} E_t r_{SM,t+1} + \beta_{r,h,t} eh_{t+1} + \mathbf{u}_{t+1} \quad \mathrm{var}_t(\mathbf{u}_{t+1}) \equiv \mathrm{diag}[\Omega_{i,t}], \Omega_{i,t} = \Omega_i + G_{i,0}\Omega_{i,t-1} + G_{i,1}(u_{i,t} + G_{i,2})^2,$$

$$h_{SM,t+1} = E_t h_{SM,t+1} + eh_{t+1}, \quad \mathrm{var}_t(eh_{t+1}) \equiv \Omega_{h,h,t} = \Omega_{h,h} + F_0(eh_t + F_2)^2 + F_1\Omega_{h,h,t-1},$$

$$\beta_{r,h,t} \equiv [\,\beta_{i,t}\,], \qquad \beta_{i,t} = \beta_i + \lambda_{i,0} eh_t + \lambda_{i,1} u_{i,t} + \lambda_{i,2}\beta_{i,t-1}.$$

The vector $\mathbf{r}_t$ contains the excess returns on the 1st, 3rd, 5th, 8th, and 10th decile portfolios of NYSE firms, the excess returns on two- and six-month Treasury bills and a portfolio of long-term Treasury bonds. $E_t h_{SM,t+1}$ is regression estimate of the expected excess return on the NYSE, $h_{SM,t+1}$. The estimates of α_i are multiplied by 1,200. Robust standard errors are reported below the parameter estimates.

Columns 1 and 2 of Panel B report χ^2 statistics for the hypothesis that expected excess returns on asset i are proportional to its beta multiplied by the risk premium, $\beta_{i,t}u_t$. χ^2 tests for serial correlation and predictability in the equation residuals are shown in columns 3 and 4. Misspecification tests for the conditional variance, covariance, and beta processes are reported in columns 5, 6, and 7. All tests are robust to nonnormality.

Panel A: Maximum Likelihood Estimates									
i =	α_i (1)	β_i (2)	$\lambda_{i,0}$ (3)	$\lambda_{i,1}$ (4)	$\lambda_{i,2}$ (5)	Ω_i (6)	$G_{i,0}$ (7)	$G_{i,1}$ (8)	$G_{i,2}$ (9)
$ST1$	0.812	0.457	0.196	−0.171	0.605	25.045	0.318	0.399	−37.014
	(2.646)	(0.148)	(0.074)	(0.048)	(0.129)	(6.838)	(0.230)	(0.200)	(5.950)
$ST3$	1.019	0.666	0.217	−0.137	0.435	4.169	0.012	0.935	−19.621
	(1.831)	(0.074)	(0.058)	(0.055)	(0.067)	(1.373)	(0.015)	0.048)	(11.303)
$ST5$	1.856	0.462	0.123	−0.087	0.597	4.380	0.019	0.978	−20.748
	(1.437)	(0.092)	(0.048)	(0.039)	(0.082)	(2.130)	(0.019)	(0.052)	(11.996)

Table II–*Continued*

Panel A: Maximum Likelihood Estimates

$i =$	α_i (1)	β_i (2)	$\lambda_{i,0}$ (3)	$\lambda_{i,1}$ (4)	$\lambda_{i,2}$ (5)	Ω_i (6)	$G_{i,0}$ (7)	$G_{i,1}$ (8)	$G_{i,2}$ (9)
*ST*8	0.793	0.688	0.098	−0.082	0.386	3.443	0.136	0.801	−6.088
	(0.833)	(0.054)	(0.037)	(0.031)	(0.050)	(1.067)	(0.042)	(0.051)	(3.072)
*ST*10	−0.443	1.200	0.076	−0.085	−0.212	2.409	0.062	0.869	0.022
	(0.460)	(0.075)	(0.037)	(0.042)	(0.049)	(0.608)	(0.031)	(0.061)	(2.339)
TB2	0.245	0.000	−0.000	0.067	−0.190	0.070	0.140	0.825	0.049
	(0.028)	(0.001)	(0.001)	(0.063)	(6.388)	(0.019)	(0.076)	(0.050)	(0.091)
TB6	0.235	0.001	−0.001	0.061	−0.414	0.144	0.119	0.805	0.158
	(0.081)	(0.003)	(0.001)	(0.056)	(1.552)	(0.035)	(0.037)	(0.034)	(0.303)
TBL	−1.300	0.021	0.016	0.133	0.638	1.248	0.131	0.866	−0.735
	(0.921)	(0.020)	(0.013)	(0.062)	(0.416)	(0.330)	(0.077)	(0.070)	(3.598)

Panel B: Specification Tests

					Misspecification in the Process for		
$i =$	Proportionality (1)	(2)	Serial Correlation (3)	Predictability (4)	$\text{var}(er_i)$ (5)	$\text{cov}(er_i, eh_{SM})$ (6)	$\beta_{i,SM}$ (7)
*ST*1	1.723	2.843	12.061	7.345	4.526	2.819	4.915
*ST*3	4.810	0.433	4.800	5.709	3.390	1.793	2.747
*ST*5	5.010	0.433	5.979	6.648	4.499	3.228	3.923
*ST*8	7.768	0.758	9.621	10.681	6.636	4.132	7.410
*ST*10	3.164	0.112	11.537	4.384	3.195	3.373	7.362
*TB*2	2.189	22.985	23.671	43.698	6.435	1.014	4.521
*TB*6	0.077	2.985	22.343	11.880	5.001	0.283	9.931
TBL	1.424	3.519	12.057	17.487	9.294	5.771	10.500
Degrees of Freedom	1	2	6	7	6	6	6
5%	3.84	5.99	12.59	14.07	12.59	12.59	12.59
1%	6.63	9.21	16.81	18.48	16.81	16.81	16.81

$\beta_{i,t}\Omega_{h,h,t}\,|\mathbf{x}_t] = 0$, and $E[er_{i,t+1}eh_{t+1} - \beta_{i,t}(eh_{t+1})^2\,|\mathbf{x}_t] = 0$.[9] These tests are designed to look for evidence of misspecification in the process for the variance of $er_{i,t+1}$, the covariance between $er_{i,t+1}$ and eh_{t+1} and the betas. Columns 5–7 show that the test statistics are not significant at the 5 percent level. Thus, there is little evidence of serious misspecification in the processes for the betas and covariance matrices.

As a final specification test, I examined whether the estimated asset-specific shocks are uncorrelated with one another by testing whether $E[u_{i,t+1}, u_{j,t+1}\,|\mathbf{x}_t] = 0$ for all $i \neq j$ with different choices for $\mathbf{x}_t$. Since there are 8 assets, there are 28 moment restrictions of this type to jointly test. None of the test statistics were significant at the 5 percent level.

Overall, these results suggest that, if the ICAPM is correct, the stock market is an inadequate proxy for the benchmark portfolio. When combined with the dynamics of the estimated stock market premium, the movements in betas appear unable to explain the predictability of stock, bond, and bill returns. It seems likely, therefore, that the predictable variations in excess returns are attributable to additional risk factors. To investigate this, I next consider a model with more than one hedge portfolio.

C. Estimates of the ICAPM with Two Hedge Portfolios

The new model adds corporate bonds as the second hedge portfolio. Two reasons suggest this choice. First, the true market portfolio should contain a fixed income component. Second, the returns on corporate bonds may be good indicators of variable interest rate risk. Ferson and Harvey (1991), for example, find that much of the predictability in bond returns can be attributed to this source of risk.

In this model, each row of the beta matrix $\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t}$ comprises the stock beta $\beta_{i,SM,t}$ and corporate bond beta $\beta_{i,CB,t}$, which are assumed to follow

$$\beta_{i,SM,t} = \beta_{i,SM} + \lambda_{i,SM,0}eh_{SM,t} + \lambda_{i,SM,1}u_{i,t} + \lambda_{i,SM,2}\,\beta_{i,SM,t-1} + \lambda_{i,SM,3}R_{A-B,t} \tag{11a}$$

$$\beta_{i,CB,t} = \beta_{i,CB} + \lambda_{i,CB,0}eh_{CB,t} + \lambda_{i,CB,1}u_{i,t} + \lambda_{i,CB,2}\,\beta_{i,CB,t-1} + \lambda_{i,CB,3}R_{A-B,t}, \tag{11b}$$

where $eh_{SM,t}$ and $eh_{CB,t}$ are the shocks to the excess return on the stock market and corporate bonds, and $R_{A-B,t}$ is the AAA–BAA spread. The spread is only included in the equations for the three smallest stock deciles

[9] The $\mathbf{x}_t$ vector includes ω_{t-j} for $j = 1$, to 6, where $\omega_{t+1} = (er_{i,t+1})^2 - (\Omega_{i,t} + \beta_{i,t}\Omega_{h,h,t}\beta_{i,t})$ for the tests in column 5, $\omega_{t+1} = er_{i,t+1}eh_{t+1} - \beta_{i,t}\Omega_{h,h,t}$ for the tests in column 6, and $\omega_{t+1} = er_{i,t+1}eh_{t+1} - \beta_{i,t}(eh_{t+1})^2$ for the tests in column 7. I also conducted tests where $\mathbf{x}_t$ included a constant and lagged values of RR, R_{1-6}, R_{DIV} and R_{A-B}. As none of the test statistics were significant, the results are not reported to conserve space. Similarly, a test of $E[(eh_{t+1})^2 - \Omega_{h,h,t}\,|\mathbf{x}_t] = 0$ indicated that the process for $\Omega_{h,h,t}$ was correctly specified.

($i = ST1$, $ST3$ and $ST5$).[10] The covariance process for the hedge portfolios follow

$$\mathbf{\Omega}_{\mathbf{h},\mathbf{h},t} = \mathbf{\Omega}_{\mathbf{h},\mathbf{h}} + \mathbf{F}_0 \bullet (\mathbf{eh}_t + \mathbf{F}_2)(\mathbf{eh}_t + \mathbf{F}_2)' + \mathbf{F}_1 \bullet \mathbf{\Omega}_{\mathbf{h},\mathbf{h},t-1}, \tag{11c}$$

here $\mathbf{\Omega}_{\mathbf{h},\mathbf{h}}$, $\mathbf{F}_0$ and $\mathbf{F}_1$ are 2×2 symmetric matrices and $\mathbf{F}_2$ is a 2×1 vector. I continue to assume that asset-specific shocks are uncorrelated [i.e., $\text{cov}_t(u_{i,t+1}, u_{j,t+1}) = 0 \ i \neq j$] so that $\mathbf{\Omega}_{\mathbf{u},\mathbf{u},t}$ is diagonal. The variances follow

$$\Omega_{i,t} = \Omega_i + G_{i,0}(\Omega_{i,t-1} - G_{i,2} Jan_{t-1}) + G_{i,1}(u_{i,t})^2 + G_{i,2} Jan_t. \tag{11d}$$

The January dummy, Jan_t, is included because initial estimates of the model showed that asset-specific shocks have a larger variance during this month in the returns to the three smallest stock deciles. The dummy is excluded from the specification in the other equations. The estimates also revealed that it is unnecessary to allow for asymmetric effects of shocks on the conditional variance of $u_{i,t}$.

The maximum likelihood estimates of the constants, α_i, and the parameters for the beta processes in equations (11a) and (11b) are reported in Panel A of Table III.[11] Except for the two-month bill equation, column 1 shows that the estimates of α_i are insignificantly different from zero. The estimates in columns 2–6 indicate that the $\beta_{i,SM,t}$'s are significantly raised by $eh_{SM,t}$ shocks and lowered by $u_{i,t}$ shocks in the equations for the small stock deciles. A rise in the AAA–BAA spread also significantly lowers the betas. These results suggest that the leverage effects found in Table II are robust to the inclusion of other variables driving the betas and to the addition of a second hedge portfolio. Neither shock appears to have a significant influence on the $\beta_{i,SM,t}$'s in the bill equations, and only the asset-specific shock significantly affects the stock beta for long-term bonds.

The estimates for the $\beta_{i,CB,t}$ process reported in columns 7–11 show a different pattern. In the equations for the small stock deciles, the estimates of $\lambda_{i,CB,1}$ and $\lambda_{i,CB,3}$ indicate that the betas significantly increase in response to negative $u_{i,t}$ shocks and rises with the AAA–BAA spread. Thus negative asset-specific shocks appear to increase the susceptibly of equity returns to the interest rate risk associated with the risk premium on corporate bonds. This finding seems consistent with the effects of financial leverage. Interestingly, the estimates of $\lambda_{i,CB,1}$ are insignificantly different from zero in the equations for the other stock deciles. In the bill and bond equations, the asset-specific shocks exert the strongest influence on the corporate bond betas.

Panel B presents the results of specification tests. The robust CM statistics reported in columns 5–9 indicate that the model successfully captures the

[10] Initially, I estimated models where the betas were only affected by the $u_{i,t}$, $eh_{SM,t}$ and $eh_{CB,t}$ shocks. I then conducted a series of robust LM tests to see whether any of the instrumental variables should be added to the beta specification. The tests revealed that $R_{A-B,t}$ was the most significant omitted variable.

[11] The estimates of the other parameters in the models are not reported to conserve space. The interested reader is referred to Evans (1992) for the complete set of estimation results.

Table III

Estimates and Specification Tests of the ICAPM with Two Hedge Portfolios

Panel A reports the maximum likelihood estimates of the model:

$$\mathbf{r}_{t+1} = \alpha + \beta_{\mathbf{r},SM,t} E_t h_{SM,t+1} + \beta_{r,CB,t} E_t h_{CB,t+1} + \mathbf{er}_{t+1} \qquad \mathrm{var}_t(\mathbf{u}_{t+1}) \equiv \mathrm{diag}[\,\Omega_{i,t}], \qquad \Omega_{i,t} = \Omega_i + G_{i,0}(\Omega_{i,t-1} - G_{i,2} Jan_{t-1}) + G_{i,1}(u_{i,t})^2 + G_{i,2} Jan_t$$

$$\mathbf{er}_{t+1} = [\,\beta_{\mathbf{r},SM,t}, \beta_{\mathbf{r},CB,t}]\mathbf{eh}_{t+1} + \mathbf{u}_{t+1} \qquad \mathrm{var}_t(\mathbf{eh}_{t+1}) \equiv \Omega_{\mathbf{h},\mathbf{h},t} = \Omega_{\mathbf{h},\mathbf{h}} + \mathbf{F}_0 \cdot (\mathbf{eh}_t + \mathbf{F}_2)(\mathbf{eh}_t + \mathbf{F}_2)' + \mathbf{F}_1 \cdot \Omega_{\mathbf{h},\mathbf{h},t-1}.$$

$$\mathbf{h}_{t+1} = [E_t h_{SM,t+1} \;\; E_t h_{CB,t+1}]' + \mathbf{eh}_{t+1}. \qquad \beta_{\mathbf{r},SM,t} \equiv [\,\beta_{i,SM,t}], \qquad \beta_{i,SM,t} = \beta_{i,SM} + \lambda_{i,SM,0} eh_{SM,t} + \lambda_{i,SM,1} u_{i,t} + \lambda_{i,SM,2}\beta_{i,SM,t-1} + \lambda_{i,SM,3} R_{A-B,t},$$

$$\beta_{\mathbf{r},CB,t} \equiv [\,\beta_{i,CB,t}], \qquad \beta_{i,CB,t} = \beta_{i,CB} + \lambda_{i,CB,0} eh_{CB,t} + \lambda_{i,CB,1} u_{i,t} + \lambda_{i,CB,2}\beta_{i,CB,t-1} + \lambda_{i,CB,3} R_{A-B,t},$$

The vector $\mathbf{r}_t$ contains the excess returns on the 1st, 3rd, 5th, 8th, and 10th decile portfolios of NYSE firms, the excess returns on two- and six-month Treasury bills and a portfolio of long-term Treasury bonds. The vector $\mathbf{h}_{t+1}$ contains the excess return on the NYSE and corporate bonds. $[E_t h_{SM,t+1} \;\; E_t h_{CB,t+1}]$ is the corresponding vector of regression estimates of expected excess returns. R_{A-B} is the spread between the yields on Moody's AAA- and BAA-rated bonds. The estimates of α_i are multiplied by 1,200. Robust standard errors are reported below the parameter estimates. Columns 1 and 2 of Panel B report χ^2 statistics for the hypothesis that expected excess returns on asset i are proportional to $\beta_{i,SM,t}E_t h_{SM,t+1}$ and $\beta_{i,CB,t}E_t h_{CB,t+1}$. χ^2 tests for serial correlation and predictability in the equation residuals are shown in columns 3 and 4. Misspecification tests for the conditional variance, covariance, and beta processes are reported in columns 5–9. All tests are robust to nonnormality.

Panel A: Maximum Likelihood Estimates

	Stock Market Beta						Corporate Bond Beta				
$i =$	α_i (1)	$\beta_{i,SM}$ (2)	$\lambda_{i,SM,0}$ (3)	$\lambda_{i,SM,1}$ (4)	$\lambda_{i,SM,2}$ (5)	$\lambda_{i,SM,3}$ (6)	$\beta_{i,CB}$ (7)	$\lambda_{i,CB,0}$ (8)	$\lambda_{i,CB,1}$ (9)	$\lambda_{i,CB,2}$ (10)	$\lambda_{i,CB,3}$ (11)
$ST1$	−1.474	0.755	0.275	−0.163	0.528	−0.137	−0.556	−0.060	−0.101	0.003	0.259
	(2.353)	(0.218)	(0.077)	(0.061)	(0.131)	(0.067)	(0.262)	(0.056)	(0.050)	(0.243)	(0.150)
$ST3$	−0.462	0.792	0.225	−0.133	0.453	−0.106	−0.528	−0.021	−0.096	0.120	0.294
	(1.798)	(0.199)	(0.062)	(0.060)	(0.135)	(0.054)	(0.423)	(0.050)	(0.048)	(0.262)	(0.120)
$ST5$	0.685	0.642	0.146	−0.087	0.527	−0.073	−0.383	−0.016	−0.054	0.295	0.241
	(1.470)	(0.194)	(0.054)	(0.056)	(0.145)	(0.042)	(0.358)	(0.054)	(0.045)	(0.279)	(0.125)

Table III–*Continued*

Panel A: Maximum Likelihood Estimates

	Stock Market Beta						Corporate Bond Beta				
$i =$	α_i (1)	$\beta_{i,SM}$ (2)	$\lambda_{i,SM,0}$ (3)	$\lambda_{i,SM,1}$ (4)	$\lambda_{i,SM,2}$ (5)	$\lambda_{i,SM,3}$ (6)	$\beta_{i,CB}$ (7)	$\lambda_{i,CB,0}$ (8)	$\lambda_{i,CB,1}$ (9)	$\lambda_{i,CB,2}$ (10)	$\lambda_{i,CB,3}$ (11)
*ST*8	1.536	0.655	0.052	−0.046	0.402	0.000	0.005	−0.008	−0.012	0.424	0.000
	(0.788)	(0.046)	(0.027)	(0.038)	(0.046)		(0.009)	(0.040)	(0.037)	(0.280)	
*ST*10	−0.155	1.138	−0.027	0.017	−0.208	0.000	0.002	0.006	0.032	0.621	0.000
	(0.420)	(0.021)	(0.014)	(0.022)	(0.028)		(0.005)	(0.022)	(0.027)	(0.030)	
*TB*2	0.221	−0.003	−0.001	−0.083	−0.666	0.000	0.005	0.001	0.221	−0.388	0.000
	(0.031)	(0.001)	(0.001)	(0.054)	(0.207)		(0.005)	(0.002)	(0.085)	(2.423)	
*TB*6	0.237	0.000	−0.003	0.096	−0.805	0.000	0.011	−0.001	−0.055	0.325	0.000
	(0.183)	(0.003)	(0.002)	(0.046)	(0.315)		(0.007)	(0.006)	(0.100)	(2.068)	
TBL	0.183	−0.053	−0.003	0.057	−0.944	0.000	0.394	−0.034	0.134	0.040	0.000
	(0.408)	(0.027)	(0.004)	(0.027)	(0.091)		(0.334)	(0.024)	(0.103)	(5.578)	

Panel B: Specification Tests

	Proportionality		Serial Correlation	Predictability	Tests for Misspecification in the Processes for				
$i =$	(1)	(2)	(3)	(4)	$\text{var}(er_i)$ (5)	$\text{cov}(er_i, eh_{SM})$ (6)	$\text{cov}(er_i, eh_{CB})$ (7)	$\beta_{i,SM}$ (8)	$\beta_{i,CB}$ (9)
*ST*1	3.586	2.182	8.545	8.264	6.942	2.504	1.974	4.872	1.746
*ST*3	3.200	3.701	7.109	7.366	6.600	2.571	0.655	5.055	4.400
*ST*5	2.501	2.780	8.556	11.317	5.610	4.290	2.624	7.991	1.591
*ST*8	3.277	1.435	5.041	5.541	3.794	3.614	0.271	9.401	8.062
*ST*10	3.358	1.369	6.380	4.659	3.642	3.001	0.011	9.787	9.252
*TB*2	1.735	25.394	14.704	36.132	9.133	3.795	2.670	3.198	8.111
*TB*6	0.026	5.550	5.958	9.892	0.869	1.940	0.097	6.926	5.353
TBL	3.319	4.981	6.015	11.246	5.652	4.074	8.539	7.475	6.161
Degrees of Freedom	2	3	6	7	6	6	6	6	6
5%	5.99	7.82	12.59	14.07	12.59	12.59	12.59	12.59	12.59
1%	9.21	11.34	16.81	18.48	16.81	16.81	16.81	16.81	16.81

dynamics of the betas and the covariance matrices.[12] Column 1 presents LM tests of $\psi_{SM,i} = \psi_{CB,i} = 1$, in

$$r_{i,t+1} = \alpha_i + \psi_{SM,i}\,\beta_{i,SM,t}E_t h_{SM,t+1} + \psi_{CB,i}\,\beta_{i,CB,t}E_t h_{CB,t+1} + er_{i,t+1}.$$

None of the test statistics are significant at the 5 percent rate. Column 2 presents CM statistics of $\alpha_i = 0$, $\psi_{SM,i} = \psi_{CB,i} = 1$. The test statistics strongly reject the restrictions in the two-month bill equation. Columns 3 and 4 report the results of conditional moment tests on the residuals, $er_{i,t+1}$. In contrast to the results in Table II, there is no statistically significant evidence that the errors in the equations for the stock deciles can be predicted with the instrumental variables. In this sense, the addition of the second hedge portfolio improves the ICAPM's ability to explain the predictable variations in stock returns. The same cannot be said for bill returns. The residuals from the two-month bill equation are both serially correlated and predictable.[13]

IV. Changing Risk as a Source of Predictable Excess Returns

In this section, I will use the model estimates to examine how changes in the structure of risk contribute to the predictability of excess returns through variations in the betas. Table III showed that negative asset-specific shocks increase the susceptibility of equity returns to stock market risk, a result consistent with the "leverage effects" reported by Chan (1988) and Ball and Kothari (1989).[14] If variations in the betas significantly contribute to the predictability of excess returns, changing "leverage" may affect expected returns, as Chan, Ball, and Kothari argue.

A. Beta Dynamics

Table IV reports summary statistics for the conditional betas implied by the maximum likelihood estimates in Table III. For the returns on the smaller stock deciles (i = ST1, ST3, and ST5), the stock market betas are above one on average, and the corporate bond betas are negative. Both sets of betas are quite variable, as measured by their sample standard deviations.

[12] As in Table II, the CM tests in the table use six lags of the residuals ω_t as instruments. I also conducted CM tests where $\mathbf{x}_t$ included a constant and lagged values of RR, R_{1-6}, R_{DIV}, and R_{A-B}. To check the specification for the $\mathbf{\Omega}_{\mathbf{h},\mathbf{h},t}$ I conducted CM tests on $E[(eh_{SM,t+1})^2 - \Omega_{SM,SM,t}\,|\,\mathbf{x}_t] = 0$, $E[(eh_{CB,t+1})^2 - \Omega_{CB,CB,t}\,|\,\mathbf{x}_t] = 0$, and $E[eh_{CB,t+1}eh_{SM,t+1} - \Omega_{CB,SM,t}\,|\,\mathbf{x}_t] = 0$. I also tested whether $E[u_{i,t+1}u_{j,t+1}\,|\,\mathbf{x}_t] = 0$ for $i \neq j$. None of the robust test statistics were significant at the 5 percent level.

[13] These statistics reflect the fact that R_{DIV} is a very good instrument for predicting excess bill returns. This is hard for the model to explain, because the betas and R_{DIV} do not appear closely related. The CM tests support the beta specifications in equation (11) that omit R_{DIV}.

[14] Interestingly, Braun, Nelson, and Sunier (1990) were unable to find any evidence of leverage effects in the returns to industry portfolios. They conjecture that by aggregating to the industry level they miss the asymmetric response of conditional betas of individual firms to good and bad news. Here the use of size deciles appears to have avoided the problem, at least for the smaller deciles. Kothari and Shanken (1992) find that betas of return-ranked portfolios vary with unexpected market returns. These findings appear broadly consistent with the effects of $eh_{SM,t}$ on the betas estimated here.

Table IV

Summary Statistics from the Maximum Likelihood Estimates of the ICAPM with Two Hedge Portfolios

The conditional betas and variances are calculated from the maximum likelihood estimates of the model in Table III. $E_t h_{SM,t+1}$ and $E_t h_{CB,t+1}$ are the risk premia on the stock market and corporate bonds. $\Omega_{SM,SM,t}$ and $\Omega_{CB,CB,t}$ are associated conditional variances. $\Omega_{i,t}$ is the conditional variance of asset-specific stocks. The interaction effect is calculated as cov($\beta_{i,SM,t}$, $E_t h_{SM,t+1}$) + cov($\beta_{i,CB,t}$, $E_t h_{CB,t+1}$), where the risk premia are expressed in annual percent.

	Stock Betas					Corporate Bond Betas					
			Correlations with					Correlations with			
$i =$	Mean (1)	Standard Deviation (2)	$E_t h_{SM,t+1}$ (3)	$\Omega_{SM,SM,t}$ (4)	$\Omega_{i,t}$ (5)	Mean (6)	Standard Deviation (7)	$E_t h_{CB,t+1}$ (8)	$\Omega_{CB,CB,t}$ (9)	$\Omega_{i,t}$ (10)	Interaction Effects (11)
*ST*1	1.250	0.280	−0.329	−0.383	−0.102	−0.278	0.216	0.380	0.126	−0.059	0.170
*ST*3	1.221	0.187	−0.348	−0.385	−0.004	−0.236	0.189	0.485	0.165	0.037	0.238
*ST*5	1.182	0.130	−0.398	−0.415	−0.037	−0.167	0.131	0.550	0.140	0.038	0.131
*ST*8	1.096	0.027	−0.146	−0.195	−0.113	0.010	0.031	0.013	−0.110	−0.026	0.017
*ST*10	0.942	0.013	−0.018	−0.046	−0.043	0.005	0.040	−0.021	0.010	−0.041	−0.058
*TB*2	−0.001	0.002	−0.137	−0.328	−0.443	0.001	0.004	0.234	0.470	0.542	0.020
*TB*6	−0.001	0.009	0.010	0.137	−0.255	0.018	0.015	0.140	−0.354	0.276	−0.020
TBL	−0.004	0.019	−0.035	−0.042	−0.039	0.521	0.039	−0.145	−0.007	−0.041	−0.042

For the returns on the larger stock deciles (i = ST8, and ST10), the stock market betas are closer to one on average, and the corporate bond betas are closer to zero. These betas show much less sample variability.

In the bill and bond equations the average stock market betas are smaller than the average corporate bond betas and less variable. Both betas in the two-month bill equations have sample means and standard deviations close to zero. As the estimated model was unable to capture all the predictable variation in the returns to two-month bills, these statistics suggest that there may be some other factor missing from the choice of benchmark, or, as Evans and Lewis (1994, 1992) argue, that some of the predictable movements in excess two-month bill returns are unrelated to risk.

Columns 3 to 5 and 8 to 10 of Table IV examine how variations in the conditional betas are related to the estimated risk premia and measures of aggregate and asset-specific risk. Column 3 shows that for most equations the stock market betas are negatively correlated with the stock market premia, $E_t h_{SM,t+1}$. By contrast, column 8 shows that, generally, the corporate bond betas are positively correlated with the risk premia, $E_t h_{CB,t+1}$. Columns 4, 5, 9, and 10 show how variations in the betas are associated with movements in asset-specific risk measured by $\Omega_{i,t}$ and aggregate risk measured by $\Omega_{SM,SM,t}$ and $\Omega_{CB,CB,t}$. None of the correlations are particularly large except those between $\Omega_{SM,SM,t}$ and $\beta_{i,SM,t}$ in the stock equations. Column 11 reports the sum of the covariances between the betas and risk premia, $\mathrm{cov}(\beta_{i,SM,t}, E_t h_{SM,t+1}) + \mathrm{cov}(\beta_{i,CB,t}, E_t h_{CB,t+1})$, where the risk premia are expressed in annual percent. This term provides us with a measure of the total interaction effect between changes in the betas and changes in the risk premia. The table shows that the interaction effects are largest in the small firm equations, a result consistent with Ferson and Harvey (1991).

There is some evidence that the betas in the equations for smaller deciles (i = ST1, ST3, and ST5) vary over the business cycle. The stock market betas appear mildly pro-cyclical. For the 1st decile stocks, the mean change in the beta during booms is estimated to be 0.048 per year with a standard error of 0.012, and -0.143 with a standard error of 0.087 during recessions. By contrast, the corporate bond betas tend to move counter-cyclically. During booms, the mean change in the 1st decile beta is estimated to be -0.031 per year with a standard error of 0.009. During recessions, the estimated mean change is 0.049 with a standard error of 0.051.[15]

[15] I obtain similar results for the betas in equations i = ST3 and ST5. The estimates are obtained from a_0 and a_1 in the regression

$$\beta_{i,t} = a_0 t d_t + a_1 t(1 - d_t) + \sum_{j=1}^{10} b_j d_j + \xi_t$$

where $d_t = 1$ from trough to peak (as determined by the National Bureau of Economic Research) and zero otherwise. d_j are also dummy variables that take the value of zero except in regime j, where the regimes are defined by the 10 booms and recessions in the sample. I allow for serial correlation (an MA(3) process) and heteroskedasticity in the residuals, ξ_t, when calculating the standard errors.

B. Sources of Predictability

In principle, variations in expected asset returns may be due to changing risk premia, betas, or both. To investigate how much of the predictable changes in excess returns can be attributed to these different sources, Table V reports the following variance ratios:

$$R_1 = \frac{\mathrm{Var}(\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t} E_t \mathbf{h}_{t+1})}{\mathrm{Var}(\Pr(r_{i,t+1} \mid \mathbf{x}_t))}, \qquad R_2 = \frac{\mathrm{Var}(\Pr(r_{i,t+1} - \boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t} E_t \mathbf{h}_{t+1} \mid \mathbf{x}_t))}{\mathrm{Var}(\Pr(r_{i,t+1} \mid \mathbf{x}_t))}, \tag{12a}$$

$$R_3 = \frac{\mathrm{Var}(\boldsymbol{\beta}_{r,SM,t} E_t h_{SM,t+1})}{\mathrm{Var}(\Pr(r_{i,t+1} \mid \mathbf{x}_t))}, \qquad R_4 = \frac{\mathrm{Var}(\boldsymbol{\beta}_{r,CB,t} E_t h_{CB,t+1})}{\mathrm{Var}(\Pr(r_{i,t+1} \mid \mathbf{x}_t))}, \tag{12b}$$

$$R_5 = \frac{\mathrm{Var}(\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t} \mathbf{h})}{\mathrm{Var}(\Pr(r_{i,t+1} \mid \mathbf{x}_t))}, \qquad R_6 = \frac{\mathrm{Var}(\boldsymbol{\beta} E_t \mathbf{h}_{t+1})}{\mathrm{Var}(\Pr(r_{i,t+1} \mid \mathbf{x}_t))}, \tag{12c}$$

where; $\Pr(r_{i,t+1} \mid \mathbf{x}_t)$ is the projection estimate of expected excess returns found by regressing $r_{i,t+1}$ on instruments $\mathbf{x}_t$ over the whole sample, $\Pr(r_{i,t+1} - \boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t} E_t \mathbf{h}_{t+1} \mid \mathbf{x}_t)$ is a similar projection estimate of the predictable components in excess returns not captured by the ICAPM, $\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t} E_t \mathbf{h}_{t+1}$ is the expected excess return implied by the estimates of the ICAPM, $\boldsymbol{\beta}_{r,\mathbf{h},t}\mathbf{h}$ is the predicted value from a regression of the excess return $r_{i,t+1}$ on a constant and the two betas, and $\boldsymbol{\beta} E_t \mathbf{h}_{t+1}$ is predicted value from a regression of $r_{i,t+1}$ on a constant and the two risk premia.[16] Notice that $\boldsymbol{\beta}_{\mathbf{r},\mathbf{h},t}\mathbf{h}$ and $\boldsymbol{\beta} E_t \mathbf{h}_{t+1}$ are effectively the predicted returns from constant premia and constant beta models.

The ratios calculated from the maximum likelihood estimates are shown in the top row of each cell in Table V. The second row in each cell reports the value of the ratio we should expect to see if the ICAPM held true. These statistics are calculated from Monte Carlo experiments that assume excess returns obey the restrictions imposed by the estimated specifications of the ICAPM.[17] The bottom row of each cell reports the relative frequency with

[16] The instruments $\mathbf{x}_t$ were chosen as the only statistically significant regressors (allowing for heteroskedasticity) in the regressions of excess returns $r_{i,t+1}$ on a constant and all the instruments. For all the stock returns I include $R_{DIV,t}$, $R_{A-B,t}$, $R_{1-6,t}$, $r_{tb2,t}$. In addition, for i = ST1 ST3 and ST5, I include $r_{SM,t}$ and Jan_{t+1}. For the bond and bill returns I used $R_{DIV,t}$, $r_{tb2,t}$ and $R_{1-6,t}$.

[17] For each experiment, I randomly drew a new parameter vector from the robust asymptotic distribution of the maximum likelihood estimates. With these new parameter values, I use the estimated risk premia to recursively generate a sample of realized and expected excess returns equal to the size of the actual data sample. Then, I ran regressions using actual data for the instruments $\mathbf{x}_t$ and generated data for returns and the betas to calculate the six ratios. The mean values of each ratio calculated from the 500 experiments are reported in the second row of each cell. These experiments assume that the estimated risk premia are equal to the true risk premia. Other experiments allowing for sampling variation in the estimated risk premia give similar results and are reported in Evans (1992).

Table V

Variance Ratios from the Maximum Likelihood Estimates of the ICAPM with Two Hedge Portfolios

The first row of each cell reports the ratio calculated using the model estimates from Table III. The second row reports the mean value of the ratio from 500 replications of a Monte Carlo experiment that assumes excess returns are generated by the ICAPM specification in Table III. The last row reports the relative frequency with which the Monte Carlo experiments produced ratios less than those found in the actual data (shown in the first row). $\Pr(r_{i,t+1} \mid x_t)$ is the projection estimate of expected excess returns found by regressing $r_{i,t+1}$ on instruments $\mathbf{x}_t$ over the whole sample, $\Pr(r_{i,t+1} - \beta_{r,\mathbf{h},t} E_t \mathbf{h}_{t+1} \mid \mathbf{x}_t)$ is a similar projection estimate of the predictable components in excess returns not captured by the ICAPM, $\beta_{r,\mathbf{h},t} E_t \mathbf{h}_{t+1}$ is the expected excess return implied by the estimates of the ICAPM, $\beta_{r,\mathbf{h},t}\mathbf{h}$ is the predicted value from the a regression of the excess return $r_{i,t+1}$ on a constant and the two betas, and $\boldsymbol{\beta} E_t \mathbf{h}_{t+1}$ is the predicted value from a regression of $r_{i,t+1}$ on a constant and the two risk premia.

$i =$	$R_1 = \frac{\mathrm{Var}(\beta_{r\mathbf{h},t} E_t \mathbf{h}_{t+1})}{\mathrm{Var}(\Pr(r_{i,t+1} \mid x_t))}$ (1)	$R_2 = \frac{\mathrm{Var}(\Pr(r_{i,t+1} - \beta_{r,\mathbf{h},t} E_t \mathbf{h}_{t+1} \mid x_t))}{\mathrm{Var}(\Pr(r_{i,t+1} \mid x_t))}$ (2)	$R_3 = \frac{\mathrm{Var}(\beta_{r,SM,t} E_t h_{SM,t+1})}{\mathrm{Var}(\Pr(r_{i,t+1} \mid x_t))}$ (3)	$R_4 = \frac{\mathrm{Var}(\beta_{r,CB,t} E_t h_{CB,t+1})}{\mathrm{Var}(\Pr(r_{i,t+1} \mid x_t))}$ (4)	$R_5 = \frac{\mathrm{Var}(\beta_{r,h,t} h)}{\mathrm{Var}(\Pr(r_{i,t+1} \mid x_t))}$ (5)	$R_6 = \frac{\mathrm{Var}(\beta E_t h_{t+1})}{\mathrm{Var}(\Pr(r_{i,t+1} \mid x_t))}$ (6)
*ST*1	0.419	0.328	0.336	0.019	0.086	0.467
	1.020	0.155	0.185	0.378	0.205	0.887
	0.302	0.796	0.838	0.112	0.274	0.104
*ST*3	0.650	0.275	0.685	0.029	0.227	0.966
	1.110	0.102	0.313	0.491	0.223	0.927
	0.112	0.864	0.854	0.134	0.520	0.508
*ST*5	0.728	0.177	0.757	0.016	0.316	0.964
	1.105	0.528	0.511	0.178	0.370	1.201
	0.394	0.306	0.740	0.214	0.454	0.378
*ST*8	0.941	0.108	0.931	0.001	0.235	1.088
	1.115	0.116	1.102	0.003	0.206	1.100
	0.098	0.480	0.110	0.118	0.726	0.422
*ST*10	1.298	0.092	1.270	0.003	0.039	1.159
	1.112	0.111	1.085	0.011	0.058	1.111
	0.942	0.366	0.910	0.148	0.488	0.848
*TB*2	0.085	0.932	0.014	0.091	0.482	0.003
	0.855	0.915	0.144	0.919	0.505	0.470
	0.054	0.620	0.058	0.056	0.484	0.004
*TB*6	0.456	0.576	0.040	0.137	0.127	0.319
	0.838	0.121	0.369	0.715	0.307	0.823
	0.328	0.668	0.128	0.142	0.128	0.186
TBL	0.542	0.093	0.001	0.541	0.038	0.940
	1.205	0.118	0.006	1.201	0.103	1.296
	0.204	0.422	0.054	0.094	0.104	0.104

which Monte Carlo experiments produced a ratio that was less than the value found in the actual data.

The ratios R_1 and R_2 reported in Table V show that both ICAPM specifications explain sizable proportions of the predictable variations in the common stock and long-term bond returns. In the equations for the larger stock deciles (i = ST5, ST8, and ST10), the R_1 ratios are close to one, and their complements, the R_2 ratios, are close to zero. In the other stock equations, the R_1 ratios are lower and fall into the left-hand tail of the distributions. Thus, the model appears more successful in "explaining" the predictable variations in returns on the larger stock deciles than the smaller stock deciles. This can be partially attributable to the "January effect," which the model finds hard to explain. When the January dummy is excluded from the instruments used to calculate the projections, the R_1 ratios are higher and the R_2 ratios are lower.

While the estimates of R_1 indicate that the ICAPM accounts for between 50 percent and 80 percent of the predictable variations in bond returns, the models are much less successful in explaining the predictable variation in the bill returns. The R_1 ratio implies that only 5 to 8 percent of predictable variations in two-month bill returns are captured by the ICAPM specification. It is very unlikely that these low ratios could be observed by chance, since they fall well into the left-hand tail of the Monte Carlo distributions. The R_1 ratios also show that the ICAPM specifications capture more than 40 percent of the predictable variations in six-month returns. The Monte Carlo experiments indicate that this ratio could be quite easily observed by chance.

The ratios R_3 and R_4 show that risks associated with the stock market contribute most to the predictable variation in stock decile returns and the risk associated with corporate bonds contribute most to the predictability of bill and bond returns. In the stock decile equations, the estimates of R_3 are a good deal larger than the estimates of R_4, while the reverse is true in the bill and bond equations.

The estimated R_5 and R_6 ratios show that most of the predictable variation in excess returns can be attributed to the risk premia rather than the betas. With the exception of the two-month bill equations, the estimates of R_6 are a good deal larger than the estimates of R_5. They indicate that movements in the risk premia account for over 90 percent of the predictable variations in five of the excess returns. Thus, while the model estimates suggest that leverage contributes to the variations in the conditional betas, there is no evidence that leverage effects materially contribute to the predictability of the excess returns included in this study. There is also no evidence that heteroskedasticity contributes very much to the predictability of returns through variations in the betas. This suggests, in accordance with the findings of Ferson and Harvey (1991), that models with constant betas may approximate the behavior of expected returns reasonably well. So if conditional heteroskedasticity plays a major role in the determination of expected returns, it must be through the risk premia.

V. Summary

This article presents a new econometric specification for the ICAPM to examine the behavior of expected returns on stocks, bills, and bonds. Unlike earlier multiple beta models, the conditional betas are identified from the time-varying conditional covariance matrix of returns. Overall, the model estimates appear to support a specification for the ICAPM that includes two hedge portfolios.

The model estimates show that asset-specific shocks significantly affect both betas in the equations for smaller stock deciles. These results are consistent with the "leverage effects" that Chan (1988) and Ball and Kothari (1989) find when studying individual stock betas in the context of the CAPM. The estimates also show that the covariance structure of returns determines the influence of different risk premia on individual excess returns. On average, the risk premia associated with interest rate risk appear to have the strongest effect on expected bill and bond returns, while the expected returns on the stock deciles are most strongly influenced by the risk premium associated with the stock market. Although there are variations in the betas, changes in risk contribute relatively little to the predictable variation in returns through the betas. Instead, the primary source of predictability can be traced to variations in the risk premia. Future research should therefore concentrate on the forces driving the risk premia if we wish to gain a fuller understanding of the co-movements in expected returns.

Appendix

Robust Standard Errors

Stage (ii) of the estimation procedure assumes that the forecast errors are drawn from a multivariate normal distribution. If this assumption is incorrect, Bollerslev and Wooldridge (1989) show that the parameter estimates continue to be consistent and asymptotically normal provided that the conditional means and covariances are correctly specified. Under these circumstances, they show that an estimate of the covariance matrix for $\hat{\boldsymbol{\phi}}_T$, $\mathbf{VCV}_T$, can be obtained as $T^{-1}(\hat{\mathbf{h}}_T)^{-1}\hat{\mathbf{S}}_T(\hat{\mathbf{h}}_T)^{-1}$ where $\hat{S}_T$ is the sample score matrix, $\hat{\mathbf{h}}_T = T^{-1}\sum_{t=1}^{T} \mathbf{h}_t(\hat{\boldsymbol{\phi}}_T)$ for sample size T, and

$$\mathbf{h}_t(\hat{\boldsymbol{\phi}}_T) \equiv \nabla_\phi \mathbf{m}(t;\hat{\boldsymbol{\phi}}_T)'\boldsymbol{\Omega}(t;\hat{\boldsymbol{\phi}}_T)^{-1}\nabla_\phi \mathbf{m}(t;\hat{\boldsymbol{\phi}}_T) + \frac{1}{2}\nabla_\phi \boldsymbol{\Omega}(t;\hat{\boldsymbol{\phi}}_T)'\left[\boldsymbol{\Omega}(t;\hat{\boldsymbol{\phi}}_T)^{-1} \otimes \boldsymbol{\Omega}(t;\hat{\boldsymbol{\phi}}_T)^{-1}\right]\nabla_\phi \boldsymbol{\Omega}(t;\hat{\boldsymbol{\phi}}_T).$$

$\mathbf{m}(t;\hat{\boldsymbol{\phi}}_T)$ is the estimated conditional mean of $\mathbf{Y}_{t+1}$ (the $k+q$-vector of returns in the model). $\boldsymbol{\Omega}(t;\hat{\boldsymbol{\phi}}_T)$ is the estimated conditional covariance matrix of returns at t. $\nabla_\phi \mathbf{m}(t;\hat{\boldsymbol{\phi}}_T)$ is the gradient of $\mathbf{m}(t;\hat{\boldsymbol{\phi}}_T)$ with respect to $\boldsymbol{\phi}$, and $\nabla_\phi \boldsymbol{\Omega}(t;\hat{\boldsymbol{\phi}}_T)$ is the gradient of vec$(\boldsymbol{\Omega}(t;\hat{\boldsymbol{\phi}}_T))$ with respect to $\boldsymbol{\phi}$. The robust standard errors reported in Tables III and V are derived from $\mathbf{VCV}_T$.

Robust Conditional Moment Tests

Wooldridge (1990) shows how to calculate conditional moment test statistics that are robust to departures from the distributional assumptions that are not being tested (i.e., normality in the present context).

The general method is as follows: Let $\mathbf{\Pi}(t) = [\pi_1(t), .. \pi_s(t) ... \pi_S(t)]'$ be a S-dimensional vector of moments with $E[\pi_s(t)|\mathbf{x}_t] = 0 \ \forall \ s$ under the null hypothesis of a correct specification where $\mathbf{x}_t$ is a g-dimensional vector of conditioning variables. To implement Wooldridge's procedure we need to construct the sequence of gradient matrices $\mathbf{\Psi}(t) = \nabla_\phi \mathbf{\Pi}(t)$ (evaluated at the parameter estimates $\hat{\boldsymbol{\phi}}_T$), $S \times S$ weighting matrices $\mathbf{C}(t)$, and $S \times g$ indicator matrices $\mathbf{\Lambda}(t)$, with each row equal to $\mathbf{x}_t$. Next, the matrix regression of $\mathbf{C}(t)^{1/2}\mathbf{\Lambda}(t)$ on $\mathbf{C}(t)^{1/2}\mathbf{\Psi}(t)$ is run for $t = 1$ to T and the residuals saved as $\mathbf{u}(t)$. Then, a $T \times 1$ vector of ones is regressed on $\mathbf{\Pi}(t)'\mathbf{u}(t)$ for $t = 1$ to T. Wooldridge shows that T minus the sum of the squared residuals in this regression is asymptotically χ^2 with g degrees of freedom.

The weighting matrices used to test each conditional moment are tabulated below:

Moment being tested $\Pi(t)$	Weighting matrix $C(t)$
$er_{i,t+1}$	$(\Omega_{i,i,t})^{-1}$
$(er_{i,t+1})^2 - (\Omega_{u,u,t} + \beta_{i,t}\Omega_{h,h,t}\beta_{i,t})$	$(\Omega_{i,i,t})^{-2}$
$er_{i,t+1}eh_{SM,t+1} - \beta_{i,SM,t}\Omega_{SM,SM,t}$	$(\Omega_{i,i,t} \ \Omega_{SM,SM,t})^{-1}$
$er_{i,t+1}eh_{CB,t+1} - \beta_{i,CB,t}\Omega_{CB,CB,t}$	$(\Omega_{i,i,t} \ \Omega_{CB,CB,t})^{-1}$
$er_{i,t+1}eh_{SM,t+1} - \beta_{i,SM,t}(eh_{SM,t+1})^2$	$(\Omega_{i,i,t} \ \Omega_{SM,SM,t})^{-1}$
$er_{i,t+1}eh_{CB,t+1} - \beta_{i,SM,t}(eh_{CB,t+1})^2$	$(\Omega_{i,i,t} \ \Omega_{CB,CB,t})^{-1}$

Robust LM Tests

Robust LM tests are calculated along very much the same lines as robust conditional moment tests. Let $\mathbf{\Pi}(t) = [\mathbf{m}(t; \hat{\boldsymbol{\phi}}_T)' \ \text{vec}(\mathbf{\Omega}(t; \hat{\boldsymbol{\phi}}_T))']$ be the vector of conditional mean and variance functions. The LM tests requires the construction $\mathbf{\Psi}_t = \nabla_\phi \mathbf{\Pi}(t)$ the matrix of gradients when the restrictions on the parameter vector $\boldsymbol{\phi}$ are not imposed, and $\mathbf{\Lambda}_t = \nabla_\phi \mathbf{\Pi}(t)$, the matrix of gradients where they are. Both matrices are evaluated at the restricted parameter estimates. We also need to form the vector of "errors" $\boldsymbol{\eta}(t) \equiv [\mathbf{e}_{t+1}, \text{vec}(\mathbf{e}_{t+1}\mathbf{e}'_{t+1} - \mathbf{\Omega}(t; \hat{\boldsymbol{\phi}}_T))]$. Next, the matrix regression of $\mathbf{C}(t)^{-1/2}\mathbf{\Lambda}(t)$ on $\mathbf{C}(t)^{-1/2}\mathbf{\Psi}(t)$ is run for $t = 1$ to T and the residuals saved as $\mathbf{u}(t)$. Then, a $T \times 1$ vector of ones is regressed on $(\mathbf{C}(t)^{-1/2}\boldsymbol{\eta}(t))'\mathbf{u}(t)$ for $t = 1$ to T. Wooldridge shows that T minus the sum of the squared residuals in this regression is asymptotically χ^2 with g degrees of freedom where g is the number of restrictions on $\boldsymbol{\phi}$ being tested. He suggests that the weighting matrix $\mathbf{C}(t)^{-1}$ is set equal to a block diagonal matrix with $\mathbf{\Omega}(t; \hat{\boldsymbol{\phi}}_T)^{-1}$, and $^1/_2\mathbf{\Omega}(t; \hat{\boldsymbol{\phi}}_T)^{-1} \otimes \mathbf{\Omega}(t; \hat{\boldsymbol{\phi}}_T)^{-1}$ in the diagonal blocks.

REFERENCES

Ball, R., and S. P. Kothari, 1989, Nonstationary expected returns: Implications for tests of market efficiency and serial correlation of returns, *Journal of Financial Economics* 25, 51–74.

Bodurtha, J. N., Jr., and N. C. Mark, 1991, Testing the CAPM with time-varying risks and returns, *Journal of Finance* 46, 1485–1505.

Braun, P. A., D. B. Nelson, and A. M. Sunier, 1990, Good news, bad news, volatility, and betas, Working paper, University of Chicago.

Bollerslev, T., R. Y. Chou, and K. F. Kroner, 1992, Arch modeling in finance: A review of the theory and empirical evidence, *Journal of Econometrics* 52, 5–59.

Bollerslev, T., R. F. Engle, and J. M. Wooldridge, 1988, A capital asset pricing model with time-varying covariances, *Journal of Political Economy* 96, 116–131.

Bollerslev, T., and J. M. Wooldridge, 1989, Quasi-maximum likelihood estimation of dynamic models with time-varying covariances, Working paper, Department of Economics, M.I.T.

Buse, K., and H. Turtle, 1991, An intertemporal asset pricing model with time-varying moments, Working paper, University of Manitoba.

Campbell, J. Y., 1987, Stock returns and the term structure, *Journal of Financial Economics* 18, 373–399.

Chan, K. C., 1988, On the contrarian investment strategy, *Journal Business* 461, 147–163.

———, G. A. Karolyi, and R. M. Stulz, 1992. Global financial markets and the risk premium on U.S. equity, *Journal of Financial Economics* 32, 137–167.

Engle, R. F., and V. Ng, 1993, Measuring and testing the impact of news on volatility, *Journal of Finance*, 48, 1749–1788.

———, and M. Rothschild, 1990, Asset pricing with a factor ARCH covariance structure: Empirical evidence for treasury bills, *Journal of Econometrics* 45, 213–237.

———, 1991, A multidynamic factor model for stock returns, Working paper, Economics Department, University of California, San Diego.

Evans, M. D. D., 1992, Expected returns, time-varying risk and risk premia, Working paper, Stern School of Business, New York University.

———, and K. K. Lewis, 1994, Do risk premia explain it all? Evidence from the term structure, *Journal of Monetary Economics*, Forthcoming.

———, 1992, Peso problems and heterogeneous trading: Evidence from excess returns in foreign exchange and Euromarkets, National Bureau of Economic Research Working Paper No. 4003.

Fama, E. F., and K., R. French, 1988, Dividend yields and expected stock returns, *Journal of Financial Economics* 22, 3–25.

Ferson, W. E., and C. Harvey, 1991, The variation of economic risk premiums, *Journal of Political Economy* 99, 385–415.

French, K. R., G. W. Schwert, and R. F. Stambaugh, 1987, Expected stock returns and volatility, *Journal of Financial Economics* 14, 217–236.

Gibbons, M. R., and W. Ferson, 1985, Testing asset pricing models with changing expectations and an unobservable market portfolio, *Journal of Financial Economics* 14, 217–236.

Hansen, L. P., and R. J. Hodrick, 1983, Risk averse speculation in the forward foreign exchange market: An econometric analysis of linear models, in J. A. Frenkel, ed.: *Exchange Rates and International Macroeconomics* (University of Chicago Press, Chicago).

Harvey, C., 1989, Time-varying conditional covariances in tests of asset pricing models, *Journal of Financial Economics* 24, 289–317.

———, 1990, On the linearity of conditionally expected market returns, Working paper, Fuqua School of Business, Duke University.

Keim, D. B., and R. F. Stambaugh, 1986, Predicting returns in the bond and stock markets, *Journal of Financial Economics* 17, 357–390.

Kothari, S. P., and J. Shanken, 1992, Stock return variation and expected dividends, *Journal of Financial Economics* 31, 177–210.

Nanisetty, P., 1991, The effect of time-varying covariances on asset risk premia: A test of an intertemporal CAPM, Working paper, Indiana University.

Ng, L., 1991, Test of the CAPM with time-varying covariances: A multivariate GARCH approach, *Journal of Finance* 46, 1507–1521.

Schwert, G. W., 1989, Why does stock market volatility change over time?, *Journal of Finance* 44, 1115–1153.

Shanken, J., 1990, The intertemporal capital asset pricing model: An empirical investigation, *Journal of Econometrics* 45, 99–120.

Shanken, J., 1991, On the estimation of beta-pricing models, *Review of Financial Studies* 5, 1–33.

Wooldridge, J. M., 1990, A unified approach to robust, regression-based specification tests, *Econometric Theory* 6, 17–43.