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Robust Financial Contracting and the Role of Venture Capitalists

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ABSTRACT

We derive a role for inside investors, such as venture capitalists, in resolving various agency problems that arise in a multistage financial contracting problem. Absent an inside investor, the choice of securities is unlikely to reveal all private information, and overinvestment may occur. An inside investor, however, always makes optimal investment decisions if and only if he holds a fixed-fraction contract, where he always receives a fixed fraction of the project's payoff and finances that same fraction of future investments. This contract also eliminates any incentives of the venture capitalist to misprice securities issued in later financing rounds.

IN THIS ARTICLE WE develop a model of financial contracting with multiple investment decisions and show how conflicts of interest and informational asymmetries can be effectively resolved by an inside investor, i.e., an investor who not only provides capital but also works closely with the firm, monitors it frequently, and is generally very well informed about the firm's prospects and investment opportunities. Venture capitalists are obvious examples of inside investors, and their function has been an important motivation for this article. Banks, especially in Japan and Germany, are also involved in monitoring and can be viewed as playing the role of inside investors. In this article we examine the potential benefits of inside investors in a context where there are no restrictions beyond limited liability on the type of financial contracts that can be written.

We address a basic financial contracting problem involving an entrepreneur who has an idea for a project but does not have sufficient capital to fund it. An important assumption is that there are several stages in the project's development and that a decision must be made at each stage whether the project should be continued or abandoned. If the project is continued, the amount of additional capital to be invested must also be determined. We assume that the entrepreneur, as an insider, observes private information about the project's profitability at each stage. This information is not

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observed by outside investors and is not verifiable at any point in time so contracts cannot be written contingent on its realization.

If capital for the project is provided by outside investors, but the continuation decision is made by the better informed entrepreneur, then it is easy to see that the entrepreneur has an incentive to continue projects even when it is optimal to abandon them. This occurs because the entrepreneur is not putting up the money for the continuation but does stand to gain if the option to continue pays off. Since the entrepreneur has the incentive to overinvest, she cannot be expected to report her private information truthfully.[1]

It seems that the overinvestment problem described above might be solved if capital is obtained from an inside investor who establishes close ties with the firm, observes the private information about the project's profitability, and becomes involved in making the subsequent investment decisions. However, in general the involvement of an inside investor can create problems as well as solve them. First, assuming that he is not the sole owner of the firm, the inside investor may also make suboptimal investment decisions. For example, and analogous to Myers (1977), if the inside investor must finance in full any future investment but only receives a fraction of the payoffs, he will be inclined to underinvest. Second, the project-specific information that the inside investor acquires in his monitoring activities can give him significant bargaining power in subsequent rounds of financing.[2] If the inside investor is able to use this bargaining power to appropriate much of the firm's value, the entrepreneur may find it too expensive to use the inside investor as a source of capital. Third, if outside investors are also involved in providing capital to the firm, then they would be at an informational disadvantage relative to the entrepreneur and the inside investor, and this can again lead to suboptimal investment decisions.

Our objective in this article is to find the type of financial contracts that the entrepreneur and the inside investor can use to realize the advantages of inside financing and mitigate the disadvantages. Our main focus is on the problem of making optimal continuation and investment decisions, but we will also look at some other agency problems. In examining these issues we believe that it is important to consider the effects of the massive uncertainty that typically accompanies new ventures. This uncertainty makes it very difficult for agents to construct precise probabilistic models, especially if they are "boundedly rational," i.e., have limited processing powers. As a

[1]This tendency to overinvest is in many ways the mirror image of the Myers (1977) underinvestment problem. Myers argues that shareholders in a firm will tend to underinvest in situations where risky debt holders in the firm would benefit. In Myers the shareholders put up the money but do not realize all of the gains, while in our model outside investors put up the money, but some of the gains accrue to the entrepreneur.

[2]This bargaining power may come about, since outside investors observe the inside investor's behavior. If the latter refuses to finance new investments, the entrepreneur is likely to have difficulty raising funds from outside investors, since these investors are likely to conclude that the project is not worth financing. Sharpe (1990) and Rajan (1992) consider the effects of this bargaining problem in the context of banking contracts. (See below for a more detailed discussion of these papers.)

consequence, agents are unsure that any particular probabilistic model of the world is correct and complete. This leads us to seek contracts that are "robust." Roughly speaking, a contract is robust with respect to some property if it is not critical that some specific probabilistic model of the world be correct for the contract to have that property. We capture this notion of robustness by allowing the state space in our model to be extremely large, so that effectively "nothing can be ruled out," and by requiring that contracts have certain properties no matter what distributions are assessed.

Before considering the role played by an inside investor, we examine what the entrepreneur can achieve by dealing directly with outside investors at each stage of the investment process. We call this *entrepreneur-led financing*. It has been suggested, for example in Heinkel (1982), Brennan and Kraus (1987), Constantinides and Grundy (1989), and Thakor (1989), that private information held by entrepreneurs (or managers) can be communicated to outside investors through the set of securities they issue. We show, however, that fully revealing signaling equilibria are not robust in the sense described above, and, in particular, do not exist in those cases where firms' types are distinguished by the probability that the project will fail. Thus, we cannot generally expect signaling equilibria to resolve information asymmetries in the environment we seek to model. Unresolved information asymmetries are then shown to be costly to the entrepreneur, since they can lead to suboptimal investment decisions, lowering the initial value of the firm.

We proceed to introduce an inside investor who makes the continuation and investment decisions. *For concreteness we will refer to the inside investor as a venture capitalist for the remainder of the paper*. We assume that the venture capitalist observes the same information about the project's viability observed by the entrepreneur in the later stage, which means that no informational asymmetries exist between the entrepreneur and the venture capitalist. However, outside investors who might also be involved in financing the project do not observe this information. We show that there is a unique contract, which we call a *fixed-fraction contract*, that can be written between the entrepreneur and the venture capitalist that is robust with respect to optimal continuation. In other words, under this contract the venture capitalist will make the optimal investment decision in any contingency, and no other contract has this property. The fixed-fraction contract gives the venture capitalist an equity-like position in the firm, so that he is paid a fraction of the total payoffs and finances that same fraction of any future investment. In a sense, the venture capitalist becomes the sole owner of a fixed portion of the project and he provides all the capital needed for this portion. If the entrepreneur retains a financial stake in the firm, this contract requires that outside investors are involved in later financing stages. In the context of venture capital contracting, this explains why, even in a risk-neutral setting without capital constraints, later stages are not fully financed by the lead venture capitalist.

An important feature of the fixed-fraction contract is that although the venture capitalist purchases new securities in later financing rounds, his payoff is independent of the pricing of any newly issued securities. The pricing of new securities only affects the division of surplus between the entrepreneur and outside investors, where the entrepreneur gains if new securities are overpriced and outside investors gain if they are underpriced relative to their value given all available information. Intuitively, the venture capitalist's incentives as a "new shareholder" to underprice new securities (and thereby take value away from the entrepreneur) are exactly offset by his incentives as an "old shareholder" to overprice them (and thereby take away value from outside investors). Since he cannot gain by mispricing the new securities, the venture capitalist can be given the responsibility of pricing those securities and revealing the information he knows.[3]

Our model and the specific results we obtain are generally consistent with the stylized facts about the venture capital industry. First, many authors, including Sahlman (1990), Lerner (1992b), Macmillan, Kulow, and Khoylian (1988), and Gorman and Sahlman (1989), document the many ways in which venture capitalists are involved with their portfolio firms. For example, venture capitalists often serve on boards of directors, provide help in recruiting and compensating key individuals, work with suppliers and customers, get involved in establishing strategies, and play a major role in raising capital. In some situations, in fact, venture capitalists take complete control over the firm and dismiss the original entrepreneur.[4] This evidence supports our assumption that the venture capitalist is likely to be significantly better informed than other capital providers and that he effectively makes continuation decisions.

As is key in our model, the staging of commitment of capital is common in the venture capital industry. Sahlman (1988, 1990) argues that this is intended to give venture capitalists a role in making continuation decisions.[5] As our results suggest, venture capitalists typically hold equity-linked claims, and many initial agreements give them "preemptive rights and rights of first refusal" in future financing rounds.[6] Also consistent with our results, a number of papers suggest that venture capitalists are typically very involved in obtaining capital from outside investors, especially in later stages of the

[3] We also show that the fixed-fraction contract resolves the bargaining problem that potentially exists between the entrepreneur and the venture capitalist. Under the terms of the contract the venture capitalist has no ability to extract surplus from the entrepreneur. Surplus can only be extracted if the contract is violated or renegotiated. In Section IV we discuss how the contract can be written so that renegotiation does not occur.

[4] For models that address the issue of control in the context of venture capital contracting see Chan, Siegel, and Thakor (1990) and Berglöf (1992). For a model in which the venture capitalist is able to change the probability distribution of the outcome, see Amit, Glosten, and Müller (1991).

[5] Gompers (1992b) suggests that staging mitigates various agency problems that arise when there are intangible assets, and his empirical results generally confirm this proposition.

[6] Specifically, venture capitalists are often entitled to participate in later financing rounds "by buying newly issued shares from the company, often in proportion to their common-share equivalent holdings before the issuance of new equity-equivalent shares" (Sahlman (1990, p. 505)).

project (see Sahlman (1990), Macmillan, Kulow, and Khoylian (1988), Sapienza and Timmons (1989), and especially Gorman and Sahlman (1989)). Moreover, the literature on the role of venture capitalists in initial public offerings (IPOs) also supports our model's conclusions and especially the "certification" role of venture capitalists.[7]

We are aware of two papers in the literature on venture capital contracting that examine specifically the issue of optimal continuation decisions in a multistage investment problem. In Cooper and Carleton (1979), the venture capitalist is the sole provider of funds throughout the life of the project. Both the venture capitalist and the entrepreneur make decisions in each stage whether to continue investing (capital and effort respectively) in the project to get to the next stage. There is no asymmetric information, and the potential role of the venture capitalist as an intermediary between the entrepreneur and outside investors is not considered. Hansen (1991) analyzes a model of two-stage financing of start-up firms where the entrepreneur obtains private information after the first stage of the project and makes the continuation decision. Since there is asymmetric information between the entrepreneur and the venture capitalist in the second stage of the project, the venture capitalist in Hansen's model is completely analogous to the uninformed, outside investors in our model of entrepreneur-led financing (see Section II).

As mentioned earlier, banks may also serve as inside investors, and so our work is related to some articles in the banking literature.[8] The most closely related papers to our work in this literature are Sharpe (1990) and Rajan (1992). In these models banks learn information about their borrowers that other lenders do not observe. In Sharpe (1990) this allows the inside bank to capture some surplus in later period loans, but investment distortions arise because competition drives banks to write loans in the first period at lower than break-even rates. Rajan argues that borrowing from an inside bank reduces effort-incentives, since the bank is able to extract surplus from the firm in the second stage. On the other hand, he shows that borrowing from the bank leads to better liquidation decisions in the second stage.

There are a number of important differences between our model and those of Sharpe (1990) and Rajan (1992). First, the contracting environment in these models is different from the one we consider. Sharpe focuses mainly on situations in which banks cannot make multiperiod binding commitments. In

[7] Lim and Saunders (1990), Megginson and Weiss (1991), Barry, Muscarella, Peavey, and Vetsuypens (1991), Lerner (1992a), and Gompers (1992a) discuss the certification role, as well as other roles venture capitalists play in later financings, specifically in initial public offerings (IPOs). Note, however, that if capital constraints exist, an IPO may be the main way for the venture capitalist to liquidate at least some of his position in the firm. Also, because of the size of the company at this stage, risk aversion may become important. Thus, the venture capitalist may not keep his fixed fraction at the IPO stage. This may not be a problem since, in an IPO, investment bankers typically take the role of financial intermediaries and use their reputations to resolve informational asymmetries.

[8] For a comprehensive survey of this very large literature see Bhattacharya and Thakor (1994).

the second stage of his model all banks make unobservable loan contract offers both to the firms they lent to in the first period and to firms that received loans from other banks in the first period. Banks capture surplus from the high quality firms they lent to in the first period by making slightly more attractive offers than uninformed banks. Rajan assumes that an informed bank engages in negotiation with the entrepreneur at the second stage, with an exogenous specification of the bargaining outcome. By contrast, we allow initial financing agreements to be written in which a restricted menu of continuation options is available to the venture capitalist in the second stage. Also, both Sharpe and Rajan make specific parametric assumptions, while our main focus is on the choice of robust contracts.[9]

A crucial distinction between the banking literature, including the articles discussed above, and this article is that the banking literature has focused almost exclusively on debt contracts. This is consistent with banking regulations in the U.S. and can also be justified by assuming costly payoff observation. By contrast, we assume here that the securities held by the inside investor can have any form as long as they are measurable functions of final payoffs. Our results suggest that the fixed-fraction contract, which is essentially a linear, equity-like contract, is the best contract to give an inside investor when there are no regulatory restrictions and payoffs are verifiable. It should be noted that banks in Japan and Germany, which are not restricted to holding debt and are generally more involved in monitoring than banks in the U.S., do hold equity.[10] The debate on whether banks should be allowed to own equity involves a number of issues not addressed above and is beyond the scope of this paper. Nevertheless, our analysis suggests that a move towards "universal banking" might allow banks to be more effective in their role as inside investors. For further discussion and additional references on this issue see Berlin, John, and Saunders (1993).

Several authors have shown that even when one is restricted to debt-like contracts, there are ways to alleviate information asymmetries and at least partially solve the over- and underinvestment problems discussed earlier. For example, Bester (1985) and Chan and Thakor (1987) examine the role of collateral in signaling private information; Berkovitch and Greenbaum (1991) explore the use of loan commitments; Berkovitch and Kim (1990) study the effectiveness of seniority rules and dividend covenants in controlling investment incentives; and Stulz and Johnson (1985) suggest that secured debt can be used to reduce distortions.[11] The use of many of these mechanisms may be

[9]Also, in Section V we show that the additional information used by the venture capitalist for pricing securities under the fixed-fraction contract partly alleviates the agency problems that arise when entrepreneurial effort is not observed. In Rajan's model, by contrast, the effort problem is aggravated in the presence of the inside bank because of the extraction of surplus in the second stage.

[10]Hoshi, Kashyap, and Scharfstein (1990, 1991) document some of the effects of having close ties with banks on investment decisions and the possibility of financial distress for Japanese firms.

[11]For a survey of these issues in the context of firms' capital structure decisions, see Harris and Raviv (1991).

quite limited in the context of start-up companies and new ventures. For example, the use of secured debt and collateral is not likely to be very effective, since in most cases the assets of start-up companies are intangible, and the initial funding is used mostly to purchase supplies, meet a payroll, rent space, etc. Also, these approaches do not typically lead to robust contracts, since the properties of the contracts depend on specific distributional assumptions.

The robustness of the fixed-fraction contract is due to the fact that it is linearly related to both payoffs and capital input requirements. The special features of linear contracts have been recognized in several papers. For example, Dybvig and Zender (1991) consider the suboptimal investment problems identified in Myers and Majluf (1984). These occur when a manager makes investment decisions and the market is imperfectly informed about the value of the firm's existing assets and new investment opportunities. Dybvig and Zender argue that the source of the suboptimal decisions in the Myers-Majluf model is the manager's suboptimal contract. They suggest that a contract similar to our fixed-fraction contract will create the proper incentives for the manager to make the optimal investment decisions. They also observe that capital structure is irrelevant under the optimal contract, which is related to our observation that, with a fixed-fraction contract, the venture capitalist's payoff is independent of the payoff of outside investors.

Another related article that points to the special properties of linear contracts is Ravid and Spiegel (1992). This article considers a contracting problem with a one-period payoff and assumes that there is a limited supply of positive net present value (NPV) projects but a potentially infinite supply of nonpositive NPV projects. It shows that contracts that screen bad projects essentially must be linear, since deviations from linearity can be exploited by entrepreneurs creating bad projects. Although the setting is somewhat different, this result is complementary to ours, since it applies to the first period screening problem while we are considering conditions for optimal investment in later periods.

The article is organized as follows. The basic model is presented in Section I. In Section II we analyze entrepreneur-led financing. Section III introduces the notion of venture capital financing and shows that the fixed-fraction contract is the only contract that always induces optimal continuation. In Section IV we discuss issues related to enforcement and renegotiation of the contract. Section V illustrates by an example how the presence of a venture capitalist can partly alleviate the agency problem that arises when effort is unobservable. In Section VI we discuss briefly the effects of risk aversion on our results. Concluding remarks are found in Section VII.

I. The Model

There are three periods. Initially, the entrepreneur has access to a project that requires an immediate investment of K and, if the project is not subsequently abandoned, some additional investment in the second period.

The entrepreneur has no capital in either period. We assume that the entrepreneur will sell securities in the first period sufficient to raise capital K, but will not raise more than K.[12] If financing of the first stage is successful, the project proceeds to the next period, when a decision concerning the level of additional funding for the project must be made. (This includes the possibility that no further investment is made, i.e., the project is abandoned.) This decision is made in light of the realization of the state of the world s.[13] The state is unknown in the initial period, but it is observed by the entrepreneur and, if he is involved, by the venture capitalist before the continuation decision must be made.

If the project is abandoned, the value that can be recovered is a nonnegative random variable $\tilde{\psi}$, whose value is completely determined by the state—in state s it is given by ψ_s.[14] If the project is continued in state s and an investment of I is made in the second stage, then the total payoff realized in the final period is a nonnegative random variable $\tilde{\theta}$, whose distribution function is parameterized by I and s and is denoted by $F(\cdot|I, s)$. We assume that the level of investment I is a choice variable that can be determined in the second period after the state is observed. The only restriction we impose on the distributions of $\tilde{\psi}$ and $\tilde{\theta}$ is that both conditionally and unconditionally they have finite expected values.

We will consider two possibilities for the entrepreneur to obtain financing for the project. In an *entrepreneur-led financing* the entrepreneur approaches outside investors in each of the two stages. To obtain financing in the first stage the entrepreneur issues claims on the project's payoffs with the following general structure. If the project is abandoned in state s, then the initial investors receive $M(\psi_s)$ and the entrepreneur receives $\psi_s - M(\psi_s)$. If the project is financed at a level I in the second stage, then the initial investors receive $N_I(\tilde{\theta})$, and the providers of the second stage capital receive $R_I(\tilde{\theta})$ when the total final period payoff of the project is $\tilde{\theta}$. More generally, $R_I(\tilde{\theta})$ represents any refinancing of the outside investors' claims on the firm that is done in the second period, so that the total net claims held by security holders in the final period is $N_I(\tilde{\theta}) + R_I(\tilde{\theta})$ if an investment of I is made and the total payoff is $\tilde{\theta}$.

[12] This means that the entrepreneur does not sell out completely in the first stage. Obviously, if an inside investor such as a venture capitalist buys out the entrepreneur, then the problem of optimal continuation becomes trivial. In practice venture capitalists do not buy out entrepreneurs entirely in the initial stages of a new project. Outside investors may lack the knowledge and ability to run the project and, more generally, it is likely that for a variety of reasons (in particular, those related to effort incentives) the project cannot be carried out if the entrepreneur is bought out. On this issue see also our discussion in Section III following Proposition 2.

[13] The state can represent any information that becomes known in the second period about the viability and marketability of the product being developed and about the technical and managerial ability of the entrepreneur.

[14] This abandonment value could be the liquidation value of the project's assets or it could be the value of running the project with no further investment.

Another way for the entrepreneur to finance the project is to involve a venture capitalist. In this case the venture capitalist provides all of the capital needed for the first stage and possibly some of the capital needed in the second stage. In the first period, the entrepreneur and the venture capitalist enter into an *initial financing agreement*. This agreement is represented by a pair $\{A(\cdot), Z\}$. The function $A(\cdot)$ is similar to $M(\cdot)$ above and describes how the payoffs of the project are divided between the entrepreneur and the venture capitalist, if the project is abandoned in the second period. If the project is abandoned in state s, the venture capitalist receives $A(\psi_s)$, and the entrepreneur receives $\psi_s - A(\psi_s)$. The set Z specifies the financing arrangements that are permissible for financing the second stage. An element z of Z is a triple $\{I_z, \beta_z, B_z(\cdot)\}$ with the following interpretation. If the financing arrangement z is implemented, then a total of I_z will be raised, of which the venture capitalist will provide $\beta_z I_z$, and outside parties will provide $(1 - \beta_z)I_z$. The venture capitalist will then receive $B_z(\tilde{\theta})$, if the final value of the project is $\tilde{\theta}$. The initial financing agreement itself, including the venture capitalist's choice from the set Z, is assumed to be enforceable. (Section IV discusses this in more detail.) Table I summarizes the notation that will be used throughout the article.

We assume that in the case of entrepreneur-led financing, outside investors do not observe the state s. However, the venture capitalist does observe s when it becomes observable to the entrepreneur. This distinction arises from

Table I

Summary of Notation

K	Amount of capital needed in the first stage.
I	Amount of capital invested in the second stage, a choice variable.
s	The state of the world, realized after the first stage.
ψ_s	The total payoff of the project if it is abandoned after the first stage and the state is s.
$\tilde{\theta}$	The total payoff of the project if further investments are made in the second stage.
$F(\cdot\|I, s)$	The distribution of $\tilde{\theta}$ if a level of investment I is chosen and the state is s.
$M(\psi_s)$	The payoffs to providers of first-stage capital (in an entrepreneur-led financing) if the project is abandoned in state s.
$N_I(\tilde{\theta})$	The payoffs to providers of first-stage capital (in an entrepreneur-led financing) if the project is continued with an investment of I and its total payoff is $\tilde{\theta}$.
$R_I(\tilde{\theta})$	The payoffs to providers of second-stage capital (in an entrepreneur-led financing) if the project is continued with an investment of I and its total payoff is $\tilde{\theta}$.
$A(\psi_s)$	The payoff to the venture capitalist if the project is abandoned in state s.
Z	The set of financing arrangements for the second stage investment that the venture capitalist can choose from.
I_z	The level of second-stage investment if z is the chosen financing arrangement for the second stage.
β_z	The fraction of I_z which will be financed by the lead venture capitalist if z is the chosen financing arrangement for the second stage.
$B_z(\tilde{\theta})$	The payoff to the venture capitalist if z is the chosen financing arrangement for the second stage and the total payoff of the project is $\tilde{\theta}$.

our assumption that, unlike other investors, the venture capitalist works closely with the entrepreneur and hence is able to monitor the project and observe the state.[15] Based on his information, the venture capitalist makes the decision whether to continue the project or abandon it, and, if the project is continued, he chooses an element from the set Z to maximize his expected payoff given s.[16] As we will see, it is possible to construct the set Z so that the continuation decision made by the venture capitalist is the one that maximizes the expected payoffs of the project net of the investment given the state. This also implies that if the entrepreneur cannot commit to following the optimal continuation strategy, she would prefer to have the venture capitalist make the continuation decision, since this would maximize the ex ante surplus earned by the entrepreneur.

We make the following additional assumptions:

ASSUMPTION 1: (*Risk neutrality and no discounting*). *All capital providers as well as the entrepreneur are risk neutral. The rate at which future cash flows are discounted is zero.*

ASSUMPTION 2: (*Limited liability*). *The contracts under both financing regimes are consistent with limited liability. Specifically, each of $A(\tilde{\psi})$ and $M(\tilde{\psi})$ is nonnegative and not larger than $\tilde{\psi}$; each of $N_I(\tilde{\theta})$, $R_I(\tilde{\theta})$ and $B_z(\tilde{\theta})$ is nonnegative and not larger than $\tilde{\theta}$.*

ASSUMPTION 3: (*Competitive capital market*). *All capital providers obtain expected payoffs equal to their investment.*

ASSUMPTION 4: (*Asymmetric Information*). *If financing is entrepreneur led, the state of the world is observed only by the entrepreneur, but not by any provider of capital. If financing is done via a venture capitalist, then both the entrepreneur and the venture capitalist observe the state. Other providers of capital in the second stage do not observe the state.*

The risk neutrality assumption is relaxed in Section VI, where we discuss the effects of risk aversion. The assumption of no discounting is made purely for convenience and without loss of generality. The limited liability assumption is very important for our results. It simply says that no money other than that generated by the project can be paid out. If there were true unlimited liability, the entrepreneur could issue riskless debt and there would, of course, be no financing problems created by the asymmetric information. This, however, is inconsistent with the assumption that the

[15] We are not including in the analysis a cost for monitoring the project, i.e., a cost incurred by the venture capitalist in order to observe the state. See the concluding remarks for a discussion of this issue.

[16] Note that the formal statement of the initial financing agreement only addresses the role the venture capitalist plays in making continuation and investment decisions, but does not say anything about who makes other managerial decisions. In practice the entrepreneur and the venture capitalist are both actively involved in these other decisions, but we do not explicitly model how these other responsibilities are divided between them. See the concluding remarks for more discussion on the other roles the venture capitalist plays.

entrepreneur has no outside financial resources to draw upon. It also tends to distinguish the situation in a start-up company from the financing problem of an established firm that might have financial slack.[17] Assumption 3, that the capital market is competitive, however, is not important for much of our analysis. The basic results would not change much if, for example, the venture capitalist shares the ex ante surplus with the entrepreneur. As for Assumption 4 on the information structure, note that asymmetric information is assumed to exist in the second stage but not in the first stage. This allows us to focus on the second stage financing and investment decision. (See the concluding remarks for a discussion of asymmetric information at the first stage.) Note also that the fundamental distinction between entrepreneur-led financing and venture capital financing is that the venture capitalist is an inside investor who observes the state of the world, while investors in entrepreneur-led financing do not.

II. Entrepreneur-led Financing

We first consider the case where financing is led by the entrepreneur and involves only outside investors in each stage. In seeking capital for the second stage of the project, the entrepreneur has private information, since she observes the state s while investors do not. Heinkel (1982) and Brennan and Kraus (1987) have suggested that this type of asymmetric information can sometimes be overcome costlessly. This occurs in a signaling equilibrium in which the set of securities issued by the entrepreneur fully reveals her private information. Brennan and Kraus (1987) derive necessary and sufficient conditions on the type of private information held by the entrepreneur (or manager) and on the set of possible securities that can be issued for the existence of such a fully revealing signaling equilibrium.

We consider the following simple signaling game. First, nature chooses the state s, and the state is revealed to the entrepreneur. Then the entrepreneur chooses a set of securities to issue, where each security specifies the payoff to the claimholder as a function of the final payoff $\tilde{\theta}$. She also specifies the price for the securities. Investors then choose which claims to purchase given the price and their beliefs concerning the final payoff distribution. Investors face no capital constraints and are competitive, and the equilibrium concept we use is Bayesian Nash.

The next result shows that there is a sense in which fully revealing signaling equilibria are not robust. Specifically, whenever such an equilibrium exists, it is always possible to upset the equilibrium with the addition of one new state, and the equilibrium cannot be restored even if the entrepreneur is allowed to change the set of securities she issues. Moreover, the proposition and its proof imply that the conditions identified, in Brennan and

[17] Hansen (1991) analyzes a model in which the degree of liability for the entrepreneur is endogenous. In his model the entrepreneur may be liable for paying back wages earned during the life of the project.

Kraus (1987) and others for the existence of a fully revealing signaling equilibrium are quite severe; such equilibria do not exist for a large class of situations, including essentially all the realistic situations arising in the context of start-up firms. For example, new ventures tend to be very risky and typically have a positive probability of resulting in "total loss."[18] Some information about the likelihood of failure is often uncovered following the first few stages of the project. However, in the proof of the proposition below, we show that a fully revealing signaling equilibrium never exists if there are two states under which payoffs have the same distribution conditional on being positive but which imply different probabilities of zero payoffs. Two such states cannot be separated, because the worse type will always want to imitate the actions of the better type. Thus, in the context of entrepreneurial projects, we cannot expect asymmetric information to be completely resolved through costless signaling. The proof of this and subsequent results is in the appendix.

PROPOSITION 1: *Suppose financing is entrepreneur-led. Assume that there is a state $s \in S$ in which the optimal continuation level is I and, at this level of investment,* $\Pr(\tilde{\theta} - N_I(\tilde{\theta}) < I | I, s) > 0$. *Assume there exists a signaling equilibrium in which the set of contracts offered by the entrepreneur fully separates the states in S. Then it is possible to augment S by adding one other state s' so that with the augmented set of possible states there does not exist a fully revealing signaling equilibrium even if the set of possible contracts is allowed to change as well. In this sense the fully revealing signaling equilibrium is not robust to small changes in the model.*

The logic of the proof is straightforward: the new state s' is constructed so that it will be impossible to separate s from s' through financial contracts. State s' differs from state s, in that under s' some probability mass has been moved to an unfavorable outcome in which the firm does not cover the second stage investment in full given its initial commitments. It is easy to see that type s' will always want to imitate type s, breaking any potential separating signaling equilibrium. *This argument holds for any conjectured set of securities and refinancing possibilities*. There is no way that s and s' can be distinguished in equilibrium through the entrepreneur's choice of securities.[19]

We have established that if we rely on the communication of information through financial contracts, then asymmetric information between the

[18] Data on venture capital performance shows, for example, that of 383 investments made by 13 firms between 1969 and 1985, more than one-third resulted in an absolute loss (see Venture Economics (1988)).

[19] Note that the proposition requires that for some state there is a positive probability that the firm cannot cover the investment of I (after paying the initial security holders). In requiring this we eliminate cases in which the entrepreneur can finance the optimal second-stage investment with riskless debt in every state. If riskless debt can always be issued, then asymmetric information does not lead to any inefficiency. This does not seem to be a realistic situation—start-up firms are rarely in a position to finance their investment through riskless debt.

entrepreneur and capital providers is likely to persist in equilibrium. The main issue, however, is whether this information asymmetry can lead to suboptimal investment decisions by the entrepreneur. We now show by an example that in our setting the entrepreneur may indeed overinvest if her private information is not revealed to outside investors.

Assume that, in all possible states, $\tilde{\theta}$ can take on only two values, ten and zero. Also assume that the initial investment level is $K = 1$, that the second stage investment level is fixed at $I = 3$ and that the abandonment value ψ_s is 1 for all possible states. Finally, let the probability that $\tilde{\theta} = 10$ in state s be s. Assume that ex ante (in the initial period) s is distributed uniformly on [0,1]. In this example it is easy to see that the optimal continuation decision (in terms of the net expected payoff of the project) is to continue if and only if $s > (\psi + I)/10 = 0.4$. It is also straightforward to show that there is no signaling equilibrium that can fully reveal s through the entrepreneur's choice of contracts. We now show that in fact there is no equilibrium in which the entrepreneur makes the optimal continuation decision in all states of the world.

Let M be the payment made to the initial investors if the project is abandoned, let N be the payment made to initial investors if the project is continued and $\tilde{\theta} = 10$, and let R be the payment made to the providers of the second stage capital when $\tilde{\theta} = 10$. From our assumptions we must have $0 \leq M \leq \psi = 1$, both N and R must be nonnegative and $N + R \leq 10$. If the entrepreneur invests precisely in states s, where $s > 0.4$, then M, N, and R must solve the following equations

$$0.4M + 0.6(0.7)N = K = 1, \tag{1}$$

$$0.4(10 - N - R) = \Psi - M = 1 - M, \tag{2}$$

and

$$0.7R = I = 3. \tag{3}$$

Equation (1) states that the securities given to the initial investors must be worth K. With probability 0.4 the project will be abandoned, in which case the initial investors will receive M, and with probability 0.6 the project will be continued, in which case the probability of receiving N is 0.7 since, conditional on continuation, s is uniformly distributed on the interval [0.4, 1]. The next condition, equation (2), guarantees that when $s = 0.4$, the entrepreneur is indifferent between continuing and not continuing. The final equation guarantees that the providers of second-stage capital have an expected payoff equal to I. The unique solution to these three equations is

$$M = -0.2414, \qquad N = 2.6108, \qquad R = 4.2857. \tag{4}$$

Obviously, since M is negative, this solution violates limited liability.

In this example, the best payoff for the entrepreneur is defined when

$$M = 0.0, \qquad N = 2.2149, \qquad R = 4.5749, \tag{5}$$

which leads the entrepreneur to choose to continue whenever $s > 0.3115$. Thus, for $0.3115 \leq s < 0.4$, the entrepreneur continues the project, although in the first best world (where, for example, the entrepreneur has enough wealth to finance the project herself or s is directly observed by all parties) it is optimal to abandon it. This inefficiency is reflected in the ex ante value of the project—the project's ex ante value to the entrepreneur is reduced from 1.8 to 1.7608 as a consequence of the suboptimal continuation policy.

III. Financing with a Venture Capitalist

In this section we analyze the case where the financing of the first stage of the project, as well as possibly some of the second stage, is done by an inside investor such as a venture capitalist. Recall that in our model, after observing the state, the venture capitalist chooses a continuation course among the set of possible scenarios agreed upon initially. In order to maximize the net surplus from the project, the entrepreneur will wish to set up the initial financing agreement so that the venture capitalist has incentives to choose an investment level that maximizes the ex ante value of the project.

Below, we identify a particularly simple type of contract for the venture capitalist, in which, after the first investment of capital, the venture capitalist always obtains a fixed fraction of the *net* payoffs of the project. It is easy to see that this type of contract induces optimal continuation. We show, however, that this is the *only* robust contract with this property. Any other contract that might induce optimal continuation for a particular environment is not robust to small changes in the model such as adding another possible state of the world.

We first define precisely the notion that the initial financing agreement induces optimal continuation. Let S^N be the set of states for which it is optimal to abandon the project. That is, $s \in S^N$ iff

$$\sup_I(\mathrm{E}(\tilde{\theta}|I, s) - I) < \psi_s.$$

Definition: An initial financing agreement $\{A(\cdot), Z\}$ induces optimal continuation if and only if for any possible state of the world, i.e., for every ψ_s and any family of distributions $F(\cdot|I, s)$, the following holds:

(a) if $s \in S^N$, then

$$\sup_z\Big(\mathrm{E}\big(B_z(\tilde{\theta})|I_z, s\big) - \beta_z I_z\Big) \leq A(\psi_s).$$

and

(b) if $s \notin S^N$, then there exists $z \in Z$ such that

(i) $I_z \in \arg\max_I(\mathrm{E}(\tilde{\theta}|I, s) - I)$,

(ii) $\mathrm{E}(B_z(\tilde{\theta})|I_z, s) - \beta_z I_z \geq A(\psi_s)$,

(iii) for all $z' \in Z$

$$\mathrm{E}\big(B_z(\tilde{\theta})|I_z, s\big) - \beta_z I_z \geq \mathrm{E}\big(B_{z'}(\tilde{\theta})|I_{z'}, s\big) - \beta_{z'} I_{z'}$$

In words, an initial financing agreement induces optimal continuation if it always gives the venture capitalist incentives to make the optimal continuation decision in the second period. Such a contract will be robust with respect to optimal continuation. Condition (a) says that if $s \in S^N$, 'so that not continuing is the optimal strategy, then for every possible contract in the set Z, the venture capitalist (at least weakly) prefers that no continuation takes place. This ensures that no projects are continued that should be abandoned. Condition (b) says that if $s \notin S^N$, so that continuation is optimal at some level, then there exists a permitted financing arrangement $z = \{I_z, \beta_z, B_z(\cdot)\}$ that involves the optimal level of investment in the second stage and that leads to the highest payoffs to the venture capitalist among all possible financing arrangements.

The following result provides necessary and sufficient conditions for the set of initial financing agreements to induce optimal continuation.

PROPOSITION 2: *The initial financing agreement* $\{A(\cdot), Z\}$ *induces optimal continuation if and only if there exists* $\alpha \in (0,1]$ *such that*

(a) $A(\psi) = \alpha\psi$;

(b) *for all* $I > 0$, *there exists* $z \in Z$ *such that*

  (i) $I_Z = I$

  (ii) $\beta_z = \alpha$

  (iii) $B_z(\theta) = \alpha\theta$

(c) *for all* $z \in Z$ *and all* s, $E(B_z(\tilde{\theta})|I_z, s) - \beta_z I_z \leq \alpha(\mathrm{E}(\tilde{\theta}|I_z, s) - I_z)$.

This result identifies a special type of financing agreement involving an equity-like contract with the following feature: the venture capitalist obtains a fraction α of the total payoffs of the project no matter what the continuation strategy is. If the project is continued and an investment of I is needed in the second stage, then the venture capitalist puts up αI and the rest, $(1 - \alpha)I$, is financed by third parties. Thus, the fraction of the second stage investment that is put up by the original venture capitalist is equal to the fraction of the total payoffs that he would obtain if no continuation takes place, and this is the same fraction of the total payoffs that he obtains for any positive second stage investment level that is chosen. We will call the contracts identified in Proposition 2 *fixed-fraction contracts*. Note that these contracts are similar to equity in that they give the venture capitalist a fixed fraction of the payoff. However, *unlike common equity, the venture capitalist does not own a residual claim, since his shares have priority over all other claims issued by the firm.*[20]

It is easy to see why fixed-fraction contracts induce optimal continuation. After the initial investment, the venture capitalist simply owns under all circumstances a fixed fraction of the project, for which he is also the sole provider of capital. Dybvig and Zender (1991) have observed in another

[20] The fixed-fraction contract bears similarity to the notion of "strip financing," where certain agents receive the same fraction of all the securities issued by the firm.

context that linear contracts of this type given to managers induce optimal choice of projects.[21] Proposition 2 is stronger, however, since it asserts that not only does the fixed-fraction contract always induce optimal continuation, but this is the only type of contract which has this property. Part (c) says that if there are any other contracts in the set Z that are not fixed-fraction contracts, then these contracts are redundant. The entrepreneur is at best indifferent between choosing these alternative contracts and the fixed-fraction contract. Thus, the fixed-fraction contract must be in any set Z that induces optimal continuation and anything else is superfluous.

To obtain some intuition for the result, let us restrict the possible contracts for the venture capitalist to be linear in the total payoff. If, for example, the fraction of the new investment that the venture capitalist had to put up were higher than the fraction of the final payoffs he was promised, then it is easy to see that in some situations the venture capitalist would abandon projects that should be continued. On the other hand, if the venture capitalist obtained a higher fraction of the payoff than the fraction of the investment he puts up, then there would be projects that he would choose to continue, although it would be optimal to abandon them. Thus, with linear contracts optimal continuation necessarily entails the perfect alignment of the fraction of the investment put up by the venture capitalist and the fraction of the payoffs he obtains. This, of course, does not rule out the possibility that there are other nonlinear schemes that can achieve the same results. The demonstration that nonlinear contracts are not robust is more complicated, and the interested reader is referred to the proof.

It is important to note that *any* fixed-fraction contract, i.e., any positive level of α, will induce optimal continuation. How α is actually determined depends on the relative bargaining power of the entrepreneur and the venture capitalist in the initial period. Under the assumption of a competitive capital market (Assumption 3), the value of α is determined in the first period to give the venture capitalist an overall expected payoff equal to the initial investment K, taking into account that he will receive α of the net payoffs no matter which continuation decision is made. This also requires, as we have assumed, that the venture capitalist does not completely buy out the entrepreneur, which would effectively correspond to the entrepreneur "raising" more than K in capital (assuming that the project has a positive NPV overall). If we allow the venture capitalist to buy the entrepreneur out, then the resulting contract is also a fixed-fraction contract, with $\alpha = 1$, and where the entrepreneur obtains the expected value of her idea given initial information. In this case the venture capitalist is the sole owner and capital provider of the project. This of course is a trivial solution to the investment problem. Proposition 2 asserts that it is possible to obtain optimal continuation under all circumstances, even without having the venture capitalist become the sole

[21] The contract suggested in Dybvig and Zender (1991) involves a constant payment as well. This is due to the reservation utility assumed for the manager. In our model the reservation utility for the venture capitalist is zero. In fact, in our model if the investment might result in a total loss then it is not possible to pay a fixed positive payment because of limited liability.

owner and capital provider of the entire project. This is important, since for a variety of reasons, such as effort incentives, it may be preferable that the entrepreneur has a stake in the firm rather than being paid a fixed amount. (See footnote 2.)

If $\alpha < 1$, which, as argued above, is the typical case, then the venture capitalist only finances a fraction $\alpha < 1$ of the investment in the second stage. Outside investors must therefore be called for to put up some capital. This provides a rationale, even in our risk-neutral setting with no capital constraints, for the fact that new investors are typically recruited in later stages of financing for start-up companies.

When outside investors are involved in financing the second stage of the project, we have to confront the problem that arises because the entrepreneur and the venture capitalist have an informational advantage over outside investors at that stage. The asymmetric information problem here is similar to that in entrepreneur-led financing. However, the presence of a venture capitalist who holds a fixed-fraction contract makes it possible to resolve the problem. This follows because it turns out that with fixed-fraction contracts, the venture capitalist's payoff is independent of the pricing of any securities issued in the second stage. The pricing of these securities simply determines how the surplus from the project is divided between the entrepreneur and outside investors; the venture capitalist obtains a fixed fraction of the net surplus in all cases. The venture capitalist's incentives to overprice new securities, being an old shareholder like the entrepreneur, are exactly offset by his incentives to underprice new securities, being a capital provider in the second stage. This is another attractive feature of fixed-fraction contracts.

Given the above, it is reasonable to allow the venture capitalist to play a certification role in the issuance of new securities. In effect, the venture capitalist can be assumed to reveal the state information truthfully, since he has no incentives to misrepresent it. If any reputational assets are at stake either with outside investors or with the entrepreneur, the venture capitalist will clearly reveal the information truthfully, thereby resolving the informational asymmetry. Dybvig and Zender (1991) also observe that if a contract similar to the one we focus on is given to risk-neutral managers, they become indifferent to the pricing of new securities. As they show, this can lead to irrelevancy of capital structure even with asymmetric information. Similarly, our model so far does not predict the type of securities issued in the second stage. In Sections V and VI we will introduce considerations such as effort incentives and risk aversion that will bear on this issue, since the types of securities issued to capital providers determine the payoff structure to the entrepreneur who holds the residual claim on the firm.

IV. Enforcement and Renegotiation

A maintained assumption in our analysis has been that initial financing agreements are enforceable. Note that in our formulation, enforcement of the contract does not require that the state of the world be verifiable by the courts. That is, the contract does not call for specific actions to be taken in specific states of the world.[22] It only places state-independent restrictions on what the venture capitalist can do in financing the project. The actions taken by the venture capitalist (e.g., his choice for the second stage financing arrangement from the menu Z) are observable by third parties such as a court. Thus, it is reasonable to assume that the entrepreneur can use the courts to prevent a deviation from the contract that hurts her interests.

Despite the enforceability of the contract, a deviation from the initial agreement might still occur if *both* the entrepreneur and the venture capitalist find that they can gain by renegotiating the contract after the observation of the state. This leads us to examine the "renegotiation-proofness" of the contract, i.e., whether there exists a state of the world in which both parties can be made better off by renegotiating the contract's terms. In the context of our model, it is easy to see that the fixed-fraction contract is in fact renegotiation-proof as long as the outside investors can observe deviations from the financing arrangements allowed by the fixed-fraction contract.[23] First, note that the outcome of the fixed-fraction contract is Pareto optimal—since the optimal continuation decision is made in all states of the world, there is no way a deviation from the contract can increase the total expected net payoff from the project. The entrepreneur and the venture capitalist can only gain by splitting the surplus among themselves and the outside investors in a different way. However, any joint gain to the venture capitalist and the entrepreneur must come at the expense of the outside investors. Thus, upon observing a deviation, outsiders would realize that the securities offered to them are likely to be overpriced and would refuse to buy them.[24]

An additional issue concerning renegotiation is related to the bargaining problem raised in Rajan (1992). We have already observed that since he is informed, the venture capitalist might have some bargaining power in the second stage that would allow him to extract some surplus from the entrepreneur. Can this occur under the fixed-fraction contract? We have shown that under the fixed-fraction contract there is no way for the venture capitalist to extract value from the entrepreneur by mispricing securities. The only way the venture capitalist can gain is by bargaining with the entrepreneur to change the terms of the contract. For example, the venture capitalist might threaten to withhold new funding unless the entrepreneur agrees to

[22] We do, however, assume that the final payoff θ and the abandonment value ψ_s are verifiable so that payoff-contingent financial contracts such as $B_z(\theta)$ are enforceable.

[23] A deviation from the fixed-fraction contract occurs, for example, if the venture capitalist puts up a different fraction of the investment than is called for or acquires securities that do not have the proportional payoff. It is reasonable to assume, however, that outsiders can observe such deviation, since the fraction of new capital put up by the venture capitalist as well as the type of financial claim he ends up holding are easily observable.

[24] It is, of course, possible that the entrepreneur and venture capitalist might conspire to overprice new securities but agree to deviate from the contract after the securities have been issued, not before. This potential problem can be solved if the outsiders demand that a covenant be included in the securities they buy that prevents deviations from the financing agreement.

renegotiate and replace the original fixed-fraction contract with one that gives the venture capitalist a higher fraction of the firm. It is important to note that if the venture capitalist does not put up funds for continuation, he is not violating the initial contract. In other words, if the venture capitalist carries out his threat, the entrepreneur cannot take him to court.[25] The possibility therefore exists that the venture capitalist can use his bargaining power to force the entrepreneur to renegotiate. This potential problem can be at least partially resolved if the venture capitalist posts a bond up front, which would be forfeited to the entrepreneur if the contract were renegotiated. As long as the value of the bond is greater than the surplus the venture capitalist can extract, "forced" renegotiation will not occur.[26]

Finally we should note that, in a different but related context, Persons (1994) argues that the contract proposed by Dybvig and Zender (1991) as a solution to the suboptimal investment problem in Myers-Majluf (1984) does not survive renegotiation. Specifically, he shows that there are states of the world in which the old shareholders and the manager will all benefit by renegotiating the manager's compensation contract. He goes on to argue that no contract that induces the manager to invest optimally is renegotiation proof.

Given the similarities between the fixed-fraction contract and the one proposed by Dybvig and Zender, it is instructive to discuss the differences between our setting and Persons' that give rise to these seemingly contradictory results. The key difference lies in the observability of renegotiation. Persons assumes that outsiders do not observe renegotiation when it occurs. In the context of a managerial contract this may be a reasonable assumption, since the manager is an employee of the shareholders and it may be possible for the board of directors to alter the manager's total compensation in ways that are not directly observable by outsiders. By contrast, the venture capitalist and the entrepreneur are independent parties (neither is an employee of the other), and their financial relationship is defined by the securities they hold. Outsiders can observe the financial stakes each has at the time new securities are issued and changes in the structure of financial claims can readily be identified. As mentioned above, these considerations lead us to

[25] As in Rajan (1992), we have not explicitly modeled the bargaining game that might take place. Whether the venture capitalist's threats are credible depends on the exact form of the game.

[26] Note that if the entrepreneur has a crucial input into the project, she may have incentives to use her bargaining power in the second stage to extract more of the surplus from the venture capitalist or to pressure the venture capitalist into overpricing the new securities. The first problem is analogous to that alluded to in the text with respect to the venture capitalist, and a way to solve it is for the entrepreneur to sign a contract stating that if the financing agreement is renegotiated then she is penalized, e.g., she cannot start any business for a fixed period of time. This is analogous to the bond posted by the venture capitalist and does not require that the entrepreneur have capital of her own. As to the second problem, it may be resolved if the venture capitalist has sufficient reputational assets that would be affected if he consistently misprices securities. A full model of this is beyond the scope of this paper.

assume that deviations from the fixed-fraction contract will be observable by outside parties.

The nonobservability of renegotiation in Persons' setting gives rise to a second and related difference between our results and his. This concerns information asymmetries between outsiders and insiders. We have allowed the venture capitalist to price securities based on the information he knows, and we have shown that under the fixed-fraction contract he has no incentive to misprice. Persons assumes that the information asymmetries persist under the linear contract. This is because the market only observes the investment decision the manager makes, i.e., whether he invests or not. The manager does not reveal to the outside market what he knows about the value of the assets in place or the new investment project. The key question is whether reports made by the manager in Persons' setting would be credible. It is easy to see that they would not be if the manager and shareholders were in a position to renegotiate the linear contract in an unobservable way, since then the manager and the shareholders could collude to overprice the new shareholders' shares and the manager's compensation could be altered to allow him to share in the gain. With observable renegotiation this cannot occur.

V. Incentives for the Entrepreneur

So far we have focused on the agency problem of ensuring optimal continuation decisions. Clearly, other agency problems might be important in start-up firms. For example, the success of the project may depend on the amount of effort put into the project by the entrepreneur. If this is the case and if certain payoff-relevant actions by the entrepreneur are unobservable or cannot be contracted upon, then the contract between the entrepreneur and capital providers can have an affect beyond the continuation decision discussed in our analysis above. We will argue that an inside investor can partly alleviate this agency problem even if he cannot directly observe the effort.[27] Intuitively, if contracts in the second period can be made contingent on the state, the entrepreneur's payoff becomes more sensitive to the effort she puts into the project and this improves incentives.

We now present a simple example to illustrate this. Assume that there are two possible states of the world. The final payoff of the project in either state is either ten or zero. In state s_1 the payoff is ten with probability 0.9, while in state s_2 the payoff is ten with probability 0.6. An investment of $K = 1$ is required in the initial period and an investment of $I = 3$ is required in the second period. Finally, in both states the abandonment value of the project is zero. Thus, it is always optimal to continue the project.

Now assume that the entrepreneur can affect the likelihood of the two states by the amount of effort she puts into the project. The effort level is unobservable to all investors, including the venture capitalist. If the

[27] In Rajan (1992) the inside bank observes the entrepreneur's effort, but because the bank extracts surplus in the second stage, its presence actually leads to worse effort incentives.

entrepreneur does not expend effort, state s_1 occurs with probability 0.25, and state s_2 occurs with probability 0.75. If effort is expended, then states s_1 and s_2 are equally likely to occur. Thus, the investment of effort increases the probability that the final payoff is ten from $(0.25)(0.9) + (0.75)(0.6) = 0.6875$ to $(0.5)(0.9) + (0.5)(0.6) = 0.75$, and therefore increases the value of the project by $(10)(0.075) = 0.75$. Assume that the cost (in monetary terms) to the entrepreneur of expending effort is 0.5. Then the efficient solution is clearly that the entrepreneur expends effort.

Consider first entrepreneur-led financing. Using arguments similar to those used to prove Proposition 1, it can be shown that there is no equilibrium in which the choice of securities issued in the second period reveals the state. Note that, because of limited liability, the securities issued to capital providers will promise an amount $0 \leq g \leq 10$ if the project's payoff is ten and will promise zero otherwise. If the market assumes that the entrepreneur will expend effort, then $g = 4.0/0.75 = 5\frac{1}{3}$. If it is assumed that the entrepreneur does not put in the effort, then $g = 4.0/0.675 = 5.926$. Table II gives the payoff to the entrepreneur when she expends and does not expend effort under the two possible assumptions the market can make. Clearly, the only rational expectations equilibrium involves the entrepreneur not expending effort and receiving a payoff of 2.75.

Now suppose financing involves a venture capitalist. In this case the pricing of securities issued in the second period can be done by the venture capitalist and can, therefore, depend on the state. We assume that the venture capitalist obtains a fixed-fraction contract, which implies, as noted above, that he has no incentives to misrepresent the state. Prices of securities are then constructed so investors break even in each state. Note that we assume that the venture capitalist observes the state but not the effort. The state, however, is a noisy signal of whether or not effort was expended. Table III shows that the only rational expectations equilibrium now involves the entrepreneur expending effort. This follows from noticing in the last two rows

Table II

The Entrepreneur's Payoffs in the Example when Financing is Entrepreneur-led

Payoffs are calculated for the cases in which the entrepreneur expends effort or does not expend effort and for each of two possible market expectations regarding effort. The table shows that the only rational expectations equilibrium in this case is that no effort is expended.

	Market Assumes that Entrepreneur Expends Effort	Market Assumes that Entrepreneur Does Not Expend Effort
Entrepreneur expends effort	3.000	2.556
Entrepreneur does not expend effort	3.150	2.750

Table III

The Example with Venture Capital Financing

The first row gives the predicted value of the firm under the assumption that the initial investment of $K = 1$ is made. In calculating the next row, we assume that the total number of shares issued in the first stage is 1 so that this fraction represents the number of shares held by the venture capitalist. For example, if it is assumed that effort is expended, the initial value of the firm is 4.5 and since the venture capitalist is putting up 1, he retains $1/4.5 = 0.222$ shares. The next row gives the value of the firm in the second period if state s_1 is realized and the additional financing of $I = 3$ is raised. This is 9, which means that an additional 0.5 shares must be issued to obtain the needed investment, i.e., $(0.5/(1 + 0.5)) \times 9 = 3$. If the venture capitalist had an initial stake of 0.222 shares in the firm, then he will acquire $0.222 \times 0.5 = 0.1111$ of these new shares and the rest will be sold to outsiders. When the venture capitalist retains 0.222 in the initial financing, the expected payoff to the entrepreneur from the project if state s_1 occurs is $(0.778/1.5) \times 9 = 4.667$. The expected payoff to the entrepreneur if state s_2 occurs is calculated similarly. The last two rows give the ex ante payoff to the entrepreneur when she does and when she does not expend effort. For example, if she expends effort and it is assumed ex ante that she expends effort, then her ex ante payoff is $0.5(4.667) + 0.5(2.333) - 0.5 = 3.000$, since the states are equally likely, the expected payoff in state s_1 (resp. s_2) is 4.667 (resp. 2.333), and the cost of effort is 0.5.

	Initial Financing Based on Assumption That Effort Will Be Expended	Initial Financing Based on Assumption That Effort Will Not Be Expended
Period 1 value	4.500	3.750
Fraction of initial share retained by venture capitalist	0.222	0.267
Period 2 value if state s_1 is realized	9.000	9.000
Number of shares issued in period 2 in state s_1	0.500	0.500
Expected payoff to entrepreneur from project if state s_1 is realized	4.667	4.400
Period 2 value if state s_2 is realized	6.000	6.000
Number of shares issued in period 2 in state s_2	1.000	1.000
Expected payoff to entrepreneur from project if state s_2 is realized	2.333	2.200
Ex ante expected payoff to entrepreneur if effort is expended	3.000	2.800
Ex ante expected payoff to entreprenuer if effort is not expended	2.917	2.750

of the table that the entrepreneur's expected payoffs are higher if she expends effort both if the market believes that she expends effort or the market believes that she does not.

In this example the total unconditional expected payoff to the entrepreneur if financing is entrepreneur-led is 2.75, while her expected payoff if a venture capitalist is involved is 3.00. The gain is due to the fact that when the venture capitalist is involved, the prices of securities issued in period two depend directly on the state and therefore indirectly on the effort expended by the entrepreneur. The presence of the venture capitalist thus makes the entrepreneur's payoff more sensitive to her effort choice, and this sharpens incentives.

Although this example is very simple, it does capture an important aspect of the incentive problem in start-up firms. Much of the early effort expended by the entrepreneur probably serves to increase the chance of the ultimate success of the enterprise. At intermediate stages of investment the effects of entrepreneurial effort are probably not directly observable by outside parties who are not monitoring the firm, since these effects take the form of an increased probability of success that is hard to measure. The venture capitalist is more knowledgeable about these probabilities, and by making the prices of securities issued depend on the venture capitalist's knowledge, the incentive problem is at least partially solved.

Note that this example considers entrepreneurial effort only in the first stage. and it abstracts from the issue of optimal continuation, since it is never optimal to abandon the project. The optimal continuation problem generally becomes more complex when entrepreneurial effort is a significant input in the second stage. Let e be the level of effort (measured in monetary terms) contributed by the entrepreneur in the second stage, and let $\mathrm{E}(\tilde{\theta}|I, e, s)$ be the expectation of the final payoff if the state is s and I and e are, respectively, the levels of capital and entrepreneurial effort invested. Then in state s it is optimal to abandon if $\sup_{I,e}(\mathrm{E}(\tilde{\theta}|I, e, s) - I - e) < \Psi_s$. Otherwise I and e should be set at the level that maximizes $\mathrm{E}(\tilde{\theta}|I, e, s) - I - e$. The fixed-fraction contract leads the venture capitalist to trade off optimally the costs and benefits of investing *capital*, but it does not directly lead him to internalize the cost and benefits of entrepreneurial effort. This follows since the venture capitalist's payoff is $\alpha(\mathrm{E}(\tilde{\theta}|I, e, s) - I)$ and not $\alpha(\mathrm{E}(\tilde{\theta}|I, e, s) - I - e)$. If exactly I dollars of capital are raised, then in certain states it may be optimal to continue, but the entrepreneur's expected payoff net of her effort is negative. In these circumstances the venture capitalist might call for more than I to be raised and use the surplus to compensate the entrepreneur. It can be shown that in some cases the first best outcome can be achieved using this type of mechanism, but this cannot always be done. A full analysis is beyond the scope of this paper.

VI. Risk Aversion

Our results so far have been derived under the assumption that all parties are risk neutral. In this section we briefly explore how risk sharing concerns might affect our conclusions. First note that the fixed-fraction contract can still induce optimal continuation in a world where assets are priced based on risk characteristics. If the venture capitalist calculates values using the equilibrium state prices, then under the fixed-fraction contract he will still make the optimal continuation decision, where now optimal continuation involves maximizing the market value of the firm. (A similar observation is made for managerial contracts in Dybvig and Zender (1991).) Of course it is not clear whether the venture capitalist will value future cash flows according to the market's state pricing operator. However, the venture capitalists' financial backers (the limited partners), who tend to be wealthy investors, universities, and other institutional investors, are likely to hold well diversified portfolios and will possibly motivate this type of valuation.

Despite the notion that entrepreneurs tend to be risk-taking individuals, it seems likely that the venture capitalist and outside investors have a greater capacity to bear risk than the entrepreneur. In this case the contract design also needs to address risk-sharing considerations. The venture capitalist's fixed-fraction contract leaves little room for risk sharing to occur between the entrepreneur and the venture capitalist. However, there is no restriction on how payoffs are divided between the entrepreneur and outside investors who put up capital in the second stage. It would be possible, for example, to give the outside investors warrants, and this would tend to reduce the risk of the entrepreneur. In good states (high payoff realizations) the outside investors exercise their warrants and get a larger chunk of the firm, but in bad states (low payoff realizations) the outside investors get less than they would have gotten if they had simple equity. This makes the entrepreneur's payoff (the residual) less risky.

Risk sharing in the first stage of the project can be enhanced without affecting the continuation decisions if, instead of obtaining capital only from one venture capitalist, other investors participate in financing the first stage of the project as well as later stages. The "lead" venture capitalist, being an inside investor, would then be given the fixed-fraction contract and would make all of the continuation decisions. Other investors could be given securities that reduce the risk of the entrepreneur. Thus, the basic results of the previous sections may not be greatly affected when risk-sharing considerations are present. This is consistent with the notion that venture capitalists often form syndicates, with different types of contracts (and different responsibilities with respect to monitoring) for different capital providers.

VII. Concluding Remarks

This article has shown that with the appropriate financial contracts it is possible for inside investors such as venture capitalists to resolve agency problems related to investment decisions as well as informational asymmetries between outside investors and entrepreneurs. More specifically, our main results are the following:

(i) Fully revealing signaling equilibria in entrepreneur-led financing, where the entrepreneur interacts directly with less informed outside investors at each stage of the project, are not robust in the sense that the addition of one possible state of the world can upset the equilibrium, even if the entrepreneur is allowed to alter the set of securities she issues.

(ii) If informational asymmetries between the entrepreneur and outside investors persist in equilibrium, then suboptimal investment decisions may be made in later stages of a project.

(iii) The *only* contract for the venture capitalist that induces optimal continuation in all circumstances is a fixed-fraction contract, in which the venture capitalist owns the same fraction of the payoff independent of the continuation decision, and also finances that same fraction of any future investment. This implies that outside investors will necessarily be involved in later financing stages.

(iv) With the fixed-fraction contract the venture capitalist's payoff is independent of the pricing of any securities issued in later stages of the project. Thus, the venture capitalist has no incentives to misprice these securities given the information he has. This provides a rationale for the venture capitalist to be involved in arranging financing from outside investors in later stages of the project.

(v) The presence of an inside investor such as a venture capitalist can reduce some of the agency costs that arise if the entrepreneur's effort level is unobservable.

It is important to note that our results pertain to the potential *benefits* of financing by inside investors such as venture capitalists, while ignoring any potential *cost* that might be incurred in monitoring the company. Obviously, if monitoring is costly then the benefits we have identified must be balanced against these costs. Venture capitalists, however, are often not passive monitors—while they do not typically get involved in day-to-day operations, they do offer a variety of services to portfolio companies, including advice on strategic planning, management recruitment, and introduction to potential customers and suppliers. The information that the venture capitalist obtains can come as a by-product of other productive activities. This means that the effective cost of monitoring may be relatively low.

We have assumed that there is no asymmetric information between the entrepreneur and capital providers *in the first stage of the project*. It is quite possible, however, that the entrepreneur has superior information about her project or about her own ability in the first stage. The fixed-fraction contract cannot be used to solve the informational problems in the first stage, since at that stage the venture capitalist is not yet involved (and so he is not an insider). Moreover, the contract is not in force at that stage, because it is just negotiated at that point. A number of papers have examined how the entrepreneur might signal her private information through contract choice or other actions. In addition to costless signaling models such as Heinkel (1982) and Brennan and Kraus (1987), there are also costly signaling models such as Leland and Pyle (1977) and Amit, Glosten, and Müller (1990), which are based on the entrepreneur giving up some risk-sharing gains to signal her type. Note that the fixed-fraction contract we have identified can be modified to accommodate signaling in the first stage in ways similar to those discussed in Section VI in the context of risk aversion. That is, if additional investors are involved in financing in the first stage, then the inside investor can still obtain a fixed-fraction contract, while signaling or risk sharing can be accomplished through contracting between the entrepreneur and other investors. Signaling mechanisms, however, may not resolve all the asymmetry of information and, as shown in Section II, are not necessarily robust. If signaling is ineffective or too costly, then capital providers such as venture capitalists might engage in costly information acquisition, which would at least partly resolve the asymmetries. As compensation for his information-gathering activities, the venture capitalist might obtain a higher fraction of the firm in the fixed-fraction contract, but this would otherwise not affect our results.[28] Note also that it is possible that venture capitalists or other investors, especially those with extensive experience in the particular industry, might have better knowledge than the entrepreneur of the market conditions and the marketability of the product. How asymmetric information of this nature would affect the analysis remains open for future research.

Finally, the notion of robustness of contracts should be useful in a variety of contexts. It is often the case that contracts derived in specific models seem to depend crucially on (common) knowledge of probability distributions. Absent such knowledge, and especially if, as in our case, robust contracts have a particularly simple form and are not costly to write, it is reasonable to predict that robust contracts will be used whenever they are available.

Appendix

Proof of Proposition 1: Let $G(I, s) = \mathrm{E}(N_I(\tilde{\theta})|I, s)$ be the expected payoff to the original security holders in state s if I is invested in the second stage. In any refinancing that takes place in a fully revealing equilibrium the original security holders will receive securities that are worth $G(I, s)$. At the same time the assumption of competitive, risk-neutral capital markets implies that the securities issued to the new investors will be worth I.

Let $R_I^*(\cdot)$ be the optimal refinancing for the entrepreneur in state s. Since by definition $N_I(\theta) + R_I^*(\theta)$ is the total amount paid out to all security holders after the refinancing takes place and when the final value of the firm is θ, it follows that

$$\mathrm{E}\left(N_I(\tilde{\theta}) + R_I^*(\tilde{\theta})|I, s\right) = G(I, s) + I. \tag{A1}$$

From this we see that

$$\mathrm{E}\left(R_I^*(\tilde{\theta})|I, s\right) = I. \tag{A2}$$

[28] There is, of course, a bargaining problem that arises when the venture capitalist becomes more informed than other investors in the first stage. This problem is not addressed by our analysis and specifically by the fixed-fraction contract we have identified, since there is no contract in existence at this point.

Now let θ_0 be such that $\theta_0 - N_I(\theta_0) < I$. Since limited liability is imposed on $N_I(\cdot)$, any $\theta < I$ satisfies this restriction. Also, since the entrepreneur also has limited liability it follows that $R_I^*(\theta_0) < I$.

Consider now state s' in which $\tilde{\theta}$ has the following probability distribution: with probability $\pi > 0$, $\tilde{\theta} = \theta_0$ and with probability $(1 - \pi)$, $\tilde{\theta}$ has the distribution $F(\cdot|I, s)$. Note first that

$$G(I, s') = (1 - \pi)G(I, s) + \pi N_I(\theta_0). \tag{A3}$$

If the state is s' and the entrepreneur refinances with $R_I^*(\theta)$, then the aggregate value of the outstanding claims on the firm is

$$\begin{aligned}(1 - \pi)\mathrm{E}\big(N_I(\tilde{\theta}) + R_I^*(\tilde{\theta})|I, s\big) &+ \pi N_I(\theta_0) + \pi R_I^*(\theta_0)\\ &= G(I, s') + \pi I + \pi R_I^*(\theta_0)\\ &< G(I, s') + I\end{aligned} \tag{A4}$$

where the last inequality holds because $R_I^*(\theta_0) < I$. Thus, the entrepreneur can refinance the investment in state s' with overvalued securities if she uses those securities which would lead investors to believe the state is s. This means that there is no fully revealing signaling equilibrium. ■

Proof of Proposition 2: To prove this result we will need the following lemma:

LEMMA 1: *Assume that* $\{A(\cdot), Z\}$ *induces optimal continuation. Let* $\tilde{\theta}^*$ *be distributed so that* $\mathrm{E}(\tilde{\theta}^*) = x + I_o$ *for some positive* x *and* I_o. *Then*

(i) $\forall z \in Z$ *for which* $I_z = I_o$,

$$\mathrm{E}\big(B_z(\tilde{\theta}^*)\big) - \beta_z I_z \le A(x). \tag{A5}$$

(ii) $\exists z \in Z$ *such that* $I_z = I_o$ *and*

$$\mathrm{E}\big(B_z(\tilde{\theta}^*)\big) - \beta_z I_z = A(x). \tag{A6}$$

Proof: To prove part (i) of Lemma 1 assume that there exists $\hat{z} \in \mathrm{Z}$ such that $I_{\hat{z}} = I_o$ and

$$\mathrm{E}\big(B_{\hat{z}}(\tilde{\theta}^*)\big) = (1 + \delta)(A(x) + \beta_{\hat{z}} I_{\hat{z}}), \tag{A7}$$

where $\delta > 0$. Now consider a state s' in which an investment of I_o produces $\tilde{\theta}$, where $\tilde{\theta}$ has the same distribution as $\tilde{\theta}^*$ with probability $(1 + \frac{1}{2}\delta)/(1 + \delta)$ and $\tilde{\theta}$ is equal to zero with probability $\frac{1}{2}\delta/(1 + \delta)$. Also assume that $\psi_{s'} = x$ and $I_o \in \arg\max_{I > 0}(\mathrm{E}(\tilde{\theta}|I, s') - I)$. Then in state s' it is optimal to abandon since

$$\mathrm{E}\big(\tilde{\theta}|I_o, s'\big) = \left(\frac{1 + \frac{1}{2}\delta}{1 + \delta}\right)\mathrm{E}(\tilde{\theta}^*) = \left(\frac{1 + \frac{1}{2}\delta}{1 + \delta}\right)(x + I_o) < x + I_o. \tag{A8}$$

However, the venture capitalist will want to continue since

$$\mathrm{E}\big(B_{\hat{z}}(\tilde{\theta})|I_o, s'\big) = \left(\frac{1 + \frac{1}{2}\delta}{1 + \delta}\right)\mathrm{E}\big(B_{\hat{z}}(\tilde{\theta}^*)\big) = (1 + \tfrac{1}{2}\delta)(A(x) + \beta_{\hat{z}}I_{\hat{z}}), \quad (A9)$$

which implies that

$$\mathrm{E}\big(B_{\hat{z}}(\tilde{\theta})|I_o, s'\big) - \beta_{\hat{z}}I_{\hat{z}} = (1 + \tfrac{1}{2}\delta)A(x) + \tfrac{1}{2}\delta\beta_{\hat{z}}I_{\hat{z}} > A(x). \quad (A10)$$

Thus equation (A7) with $\delta > 0$ is inconsistent with optimal continuation.

To prove part (ii) assume that

$$\max_{\{z:\, I_z = I_o\}}\big(\mathrm{E}\big(B_z(\tilde{\theta}^*)\big) - \beta_z I_z\big) = A(x) - \delta, \quad (A11)$$

where $\delta > 0$. Let $\bar{\theta} > x + I_o$ and assume that in state s'' an investment of I_o produces $\tilde{\theta}$, where $\tilde{\theta}$ has the same distribution as $\tilde{\theta}^*$ with probability $1 - \epsilon$ and $\tilde{\theta} = \bar{\theta}$ with probability ϵ. Assume that $0 < \epsilon < \delta/(\bar{\theta} + \delta - A(x))$. (Note that by assumption $\bar{\theta} > x$ and by limited liability $x \geq A(x)$ so that $\bar{\theta} > A(x)$.) Finally, assume that $\psi_{s''} = x$ and that $\mathrm{E}(\tilde{\theta}|I_o, s'') - I_o > \mathrm{E}(\tilde{\theta}|I, s'') - I$ for all $I \neq I_o$. Since $\mathrm{E}(\tilde{\theta}|I_o, s'') = (1 - \epsilon)(x + I_o) + \epsilon\bar{\theta} > x + I_o$, under these assumptions the project should be continued and I_o is the optimal investment. However, the venture capitalist will not wish to continue at this optimal investment level since

$$\begin{aligned}
&\max_{\{z:\, I_z = I_o\}}\big(\mathrm{E}\big(B_z(\tilde{\theta})|I_z, s''\big) - \beta_z I_z\big) \\
&= \max_{\{z:\, I_z = I_o\}}\big((1 - \epsilon)\mathrm{E}\big(B_z(\tilde{\theta}^*)\big) + \epsilon B_z(\bar{\theta}) - \beta_z I_z\big) \\
&\leq \max_{\{z:\, I_z = I_o\}}\big((1 - \epsilon)\mathrm{E}\big(B_z(\tilde{\theta}^*)\big) + \epsilon\bar{\theta} - \beta_z I_z\big) \qquad (A12) \\
&< \max_{\{z:\, I_z = I_o\}}\big((1 - \epsilon)\big(\mathrm{E}\big(B_z(\tilde{\theta}^*)\big) - \beta_z I_z\big) + \epsilon\bar{\theta}\big) \\
&= (1 - \epsilon)(A(x) - \delta) + \epsilon\bar{\theta} \\
&< A(x).
\end{aligned}$$

The first (weak) inequality follows from the limited liability of $B(\cdot)$, which implies that $B_z(\bar{\theta}) \leq \bar{\theta}$. The last inequality follows from ϵ being sufficiently small, i.e., less than $\delta/(\bar{\theta} + \delta - A(x))$. We therefore conclude that equation (A11) is inconsistent with optimal continuation and this proves part (ii). ■

We now prove the proposition. Sufficiency is straightforward, and has been observed in another context by Dybvig and Zender (1991). To prove necessity we begin by showing that A(·) must be linear. Assume that $\mathrm{E}(\tilde{\theta}_1) = x + I_o$. By part (ii) of Lemma 1 there exists z_1 such that $I_{z_1} = I_o$ and

$$\mathrm{E}\big(B_{z_1}(\tilde{\theta}_1)\big) = A(x) + \beta_{z_1}I_{z_1}. \quad (A13)$$

Now assume that $\tilde{\theta}_2$ has the following distribution: for $\gamma \in (0, 1)$, $\tilde{\theta}_2$ has the same distribution as $\tilde{\theta}_1$ with probability $(\gamma x + I_o)/(x + I_o)$ and $\tilde{\theta}$ takes the value zero with probability $(1 - \gamma)x/(x + I_o)$. Then $\mathrm{E}(\tilde{\theta}_2) = \gamma x + I_o$, and by part (i) of Lemma 1 it follows that

$$\mathrm{E}\left(B_{z_1}(\tilde{\theta}_2)\right) = \left(\frac{\gamma x + I_o}{x + I_o}\right)\mathrm{E}\left(B_{z_1}(\tilde{\theta}_1)\right) \leq A(\gamma x) + \beta_{z_1} I_{z_1}, \tag{A14}$$

or, using equation (A13),

$$\left(\frac{\gamma x + I_o}{x + I_o}\right)\left(A(x) + \beta_{z_1} I_{z_1}\right) \leq A(\gamma x) + \beta_{z_1} I_{z_1}. \tag{A15}$$

Rearranging equation (A15) we have

$$\frac{A(x) + \beta_{z_1} I_{z_1}}{x + I_o} \leq \frac{A(\gamma x) + \beta_{z_1} I_{z_1}}{\gamma x + I_o}. \tag{A16}$$

Since the argument leading to equation (A16) holds for all I_o it follows that

$$\frac{A(x)}{x} \leq \frac{A(\gamma x)}{\gamma x}. \tag{A17}$$

Now by part (ii) of Lemma 1 there exists z_2 such that $I_{z_2} = I_o$ and

$$\mathrm{E}\left(B_{z_2}(\tilde{\theta}_2)\right) = \left(\frac{\gamma x + I_o}{x + I_o}\right)\mathrm{E}\left(B_{z_2}(\tilde{\theta}_1)\right) = A(\gamma x) + \beta_{z_2} I_{z_2}. \tag{A18}$$

By part (i) we know that

$$\mathrm{E}\left(B_{z_2}(\tilde{\theta}_1)\right) \leq A(x) + \beta_{z_2} I_{z_2}. \tag{A19}$$

Combining equations (A18) and (A19) we obtain

$$\frac{A(\gamma x) + \beta_{z_1} I_{z_1}}{\gamma x + I_o} \leq \frac{A(x) + \beta_{z_1} I_{z_1}}{x + I_o}. \tag{A20}$$

Again, this must hold for all I_o so that

$$\frac{A(\gamma x)}{\gamma x} \leq \frac{A(x)}{x}. \tag{A21}$$

Combining equations (A17) and (A21), we have

$$\frac{A(x)}{x} = \frac{A(\gamma x)}{\gamma x} \tag{A22}$$

Since this must hold for all x and for all $\gamma \in (0,1)$, it follows that $A(\cdot)$ is linear and this establishes part (a).

To establish part (b) assume that $\tilde{\theta}$ is continuously distributed with cumulative distribution function $F(\cdot)$ and assume that $0 < F(\tau) < 1$ all $\tau > 0$. (For example, $\tilde{\theta}$ could be distributed exponentially.) Let $\mathrm{E}(\tilde{\theta}) = \bar{\theta}$. By part (a) we know that for some $0 < \alpha < 1$, $A(\psi) = \alpha\psi$ and by part (ii) of Lemma 1 we know that there exists $\hat{z} \in Z$ such that $I_{\hat{z}} = I_o$ and

$$\mathrm{E}\left(B_{\hat{z}}(\tilde{\theta})\right) - \beta_{\hat{z}} I_{\hat{z}} = \alpha\left(\bar{\theta} - I_o\right). \tag{A23}$$

Now define $h_z(\theta) \equiv B_{\hat{z}}(\theta) - \alpha\theta + (\alpha - \beta_z)I_o$. Note that equation (A23) implies that $\mathrm{E}(h_z(\tilde{\theta})) = 0$. Now assume that over some interval $[0, \tau]$ where $\tau > 0$,

$$\int_0^\tau h_z(\theta)\mathrm{d}F(\theta) = \gamma \neq 0. \tag{A24}$$

It then follows that

$$\int_\tau^\infty h_z(\theta)\mathrm{d}F(\theta) = -\gamma. \tag{A25}$$

Now consider $\tilde{\theta}'$, where $\tilde{\theta}'$ has the following distribution: with probability $F(\tau) + \delta$ it is given by $\min(F(\theta)/F(\tau), 1)$, and with probability $1 - F(\tau) - \delta$ it is given by $\max((F(\theta) - F(\tau))/(1 - F(\tau)), 0)$. We assume that $|\delta|$ is sufficiently small so that both $F(\tau) + \delta$ and $1 - F(\tau) - \delta$ are probabilities.

Now let $\bar{\theta}_0^\tau \equiv \mathrm{E}(\tilde{\theta}'|\tilde{\theta}' \leq \tau)$ and let $\bar{\theta}_\tau^\infty \equiv \mathrm{E}(\tilde{\theta}'|\tilde{\theta}' > \tau)$. Then it is easily verified that $\mathrm{E}(\tilde{\theta}') = \bar{\theta} + \delta(\bar{\theta}_0^\tau - \bar{\theta}_\tau^\infty)$. By part (i) of Lemma 1 we must have

$$\mathrm{E}\left(B_{\hat{z}}(\tilde{\theta}')\right) - \beta_{\hat{z}} I_{\hat{z}} \leq \alpha\left(\bar{\theta} + \delta\left(\bar{\theta}_0^\tau - \bar{\theta}_\tau^\infty\right) - I_o\right). \tag{A26}$$

Using the definition of $h_z(\cdot)$ we have

$$\begin{aligned}
\mathrm{E}\left(B_{\hat{z}}(\tilde{\theta}')\right) - \beta_{\hat{z}} I_{\hat{z}} &= \mathrm{E}(\alpha\theta' - \alpha I_o + \beta_{\hat{z}} I_{\hat{z}} + h_z(\theta')) - \beta_{\hat{z}} I_{\hat{z}} \\
&= \alpha\left(\bar{\theta} + \delta\left(\bar{\theta}_0^\tau - \bar{\theta}_\tau^\infty\right) - I_o\right) + (F(\tau) + \delta)\mathrm{E}(h(\tilde{\theta}')|\tilde{\theta}' \leq \tau) \\
&\quad + (1 - F(\tau) - \delta)\mathrm{E}(h(\tilde{\theta}')|\tilde{\theta}' > \tau) \\
&= \alpha\left(\bar{\theta} + \delta\left(\bar{\theta}_0^\tau - \bar{\theta}_\tau^\infty\right) - I_o\right) + (F(\tau) + \delta)\left(\frac{\gamma}{F(\tau)}\right) \qquad (A27) \\
&\quad + (1 - F(\tau) - \delta)\left(\frac{-\gamma}{1 - F(\tau)}\right) \\
&= \alpha\left(\bar{\theta} + \delta\left(\bar{\theta}_0^\tau - \bar{\theta}_\tau^\infty\right) - I_o\right) + \delta\gamma\left(\frac{1}{F(\tau)} + \frac{1}{1 - F(\tau)}\right).
\end{aligned}$$

Together equations (A26) and (A27) imply that

$$\delta\gamma\left(\frac{1}{F(\tau)} + \frac{1}{1 - F(\tau)}\right) \leq 0. \tag{A28}$$

Since this must hold for both positive and negative δ, γ must be zero. Thus $\int_0^\tau h_z(\theta)\mathrm{d}F(\theta) = 0$ for all τ. This implies that $h_z(\theta) = 0$ for all θ and therefore $B_z(\cdot)$ is equal to $\alpha\theta + (\beta_z - \alpha)I_o$. By limited liability, $0 \leq B_z(\theta) \leq \theta$ for all $\theta > 0$. This can only be true if $\beta_z = \alpha$. This proves part (b). Part (c) follows directly from part (i) of Lemma 1. ■

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