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Moral Hazard and the Portfolio Management Problem

NEAL M. STOUGHTON*

ABSTRACT

This paper investigates the significance of nonlinear contracts on the incentive for portfolio managers to collect information. In addition, the manager must be motivated to disclose this information truthfully. We analyze three contracting regimes: (1) first-best where effort is observable, (2) linear with unobservable effort, and (3) the optimal contract within the Bhattacharya-Pfleiderer quadratic class. We find that the linear contract leads to a serious lack of effort expenditure by the manager. This underinvestment problem can be successfully overcome through the use of quadratic contracts. These contracts are shown to be asymptotically optimal for very risk-tolerant principals.

AN IMPORTANT JUSTIFICATION FOR the employment of portfolio managers is their skill at acquiring and interpreting information related to the movements in security prices. Investors designing appropriate fee contracts would want to take into account the impact of the contract design on the information acquisition process, yet, they face a complication since neither the effort spent nor the information itself is typically observable to anyone but the portfolio manager. The contract must therefore serve two functions: (1) to transfer the benefits of better information from manager to investor, and (2) to provide incentives to the manager to work at acquiring such information. In the presence of moral hazard and asymmetric information, it is known that there are two mechanisms that have the incentive compatibility property that the manager has an incentive to disclose her private information truthfully. However, the ability of these contracts to motivate the manager to work at acquiring information is not known. In this paper, we compare these regimes to the first-best case in which the manager's effort is observable.

To benefit from the manager's private information, the investor can delegate the portfolio decision to the manager and utilize a linear sharing rule on the portfolio returns. With this scheme, the manager's information is revealed truthfully, but indirectly, through her portfolio choice. Alternatively, the investor can offer the manager a quadratic contract of the type identified in Bhattacharya and Pfleiderer (1985) in exchange for her information. The manager reveals her information directly to the investor who uses the

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information to make his portfolio choice. Our results indicate that although these schemes induce truthful revelation of the manager's information, they have very different implications for the manager's motivation to acquire such information. For example, use of the nonlinear contract can overcome an underinvestment problem in information collection. We demonstrate that although the quadratic contract leads to suboptimal risk sharing, this negative aspect is economically less significant when the investor is a large (risk-tolerant) principal.

Much of the literature on portfolio management has focused on measurement questions rather than the form of the contract. By contrast, we utilize the agency framework of *delegated expertise* developed by Demski and Sappington (1987) in which the agent's information collection is costly, while the action is costless. Other papers applying agency theory to portfolio management include Allen (1990), Amershi (1984), Kihlstrom (1986), and Zender (1987).

The agency problem is set up in Section I where the first-best optimal level of effort and the optimal managerial fee is determined. Section II considers the delegation problem where the agent's effort is unobservable and an optimal linear sharing rule is employed. In Section III, we take up the issue of the quadratic contracts employed by Bhattacharya and Pfleiderer. Then, in Section IV, we compare the performance of these second-best contracts to the first-best contracts. Section V concludes the paper. Technical proofs are relegated to the Appendix.

I. The Agency Problem

The portfolio management problem we consider has two individuals: principal and agent. A more general problem with many principals can be handled similarly provided that the preferences satisfy the syndicate requirements of Wilson (1968). Both principal and agent are strictly risk averse and have exponential utility functions defined over monetary wealth. The problem to be solved involves the selection of the proportions of the principal's wealth to be invested in a risky asset and a riskless asset. The riskless rate of return is denoted by R.

The feature that distinguishes principal and agent is that the agent has access to some information technology that the principal does not. Upon input of an amount of effort, ρ, the agent learns the value of a signal, $\tilde{S}$, that is correlated with the true net rate of return of the risky asset, $\tilde{P}$,

$$\tilde{S} = \tilde{P} + \tilde{\varepsilon},$$

where $\tilde{\varepsilon}$ represents a random error. All variables are assumed to be jointly normal. For simplicity, we assume that $\tilde{\varepsilon}$ has zero mean and is uncorrelated with $\tilde{P}$. We define $E(\tilde{P}) = \bar{P}$, $E[(\tilde{P} - \bar{P})^2] = \sigma_P^2$ and $E(\tilde{\varepsilon}^2) = \sigma_\varepsilon{}^2$. The notation is the same as Bhattacharya and Pfleiderer (1985), except that we use net rather than gross rates of return. As they show, the Bayesian posteriors are given by

$$H = \frac{\sigma_P^2 + \sigma_\varepsilon^2}{\sigma_P^2 \sigma_\varepsilon^2} \tag{1}$$

$$M(S) = KS + (1 - K)\bar{P} \tag{2}$$

$$K \equiv \frac{\sigma_P^2}{\sigma_P^2 + \sigma_\varepsilon^2},$$

where H denotes the posterior *precision* and $M(S)$ denotes the conditional posterior mean.

Effort, ρ, is modeled as a linear increase in the posterior precision. Or, equivalently, the variance of the noise term is a hyperbolic function of effort exhibiting diminishing marginal returns. Specifically, we assume

$$\rho = \sigma_P^2 / \sigma_\varepsilon^2 \tag{3}$$

which has the implication that

$$H(\rho) = \frac{\rho + 1}{\sigma_P^2} \tag{4}$$

$$K = \frac{\rho}{\rho + 1}. \tag{5}$$

Moral hazard is present in the model by virtue of the agent's experiencing disutility of effort. The respective utility functions of the agent and principal are assumed to be exponential:

$$U_A(\tilde{W}_A) = -\exp\{-a\tilde{W}_A + V(a, \rho)\}$$
$$U_B(\tilde{W}_B) = -\exp\{-b\tilde{W}_B\}$$

where $\tilde{W}_A$ is the end-of-period wealth of the agent, $\tilde{W}_B$ is the end-of period wealth of the principal, a and b are the respective risk aversion coefficients, and $V(a, \rho)$ denotes the disutility of effort. We assume that $V(a, 0) = 0$, $V_\rho(a, 0) = 0$, $V_\rho(a, \rho) > 0$, $V_{\rho\rho}(a, \rho) \geq 0$ and $V_a(a, \rho) \geq 0$. In the first-best problem, ρ is observable to both principal and portfolio manager.

Disutility for expending effort ρ may be interpreted as follows. With constant absolute risk aversion coefficient a, $(1/a)V(a, \rho)$ is the amount of monetary compensation that must be paid to the agent to compensate for the disutility of expending effort ρ. In addition to the usual interpretation $V(a, \rho)$ may also be thought of as the agent's opportunity cost of becoming informed. Thus, this could represent the cost of purchasing information from a third party such as a research firm.

The principal moves first in this model by choosing the fee schedule. Subsequently, the agent expends effort, observes the signal, and then either makes a portfolio choice or reports the value of the signal to the principal. At the end of the period, the portfolio is liquidated and the manager is compensated.

A. The First-Best Problem

In the first-best problem, effort is observable, and the agency problem winds up being a simple problem of risk sharing between principal and agent. The optimal contract in this case consists of a fixed payment, $\Phi(\rho)$, depending on the level of effort and a sharing rule, $\phi(\tilde{W})$, depending on the ending portfolio value, $\tilde{W}$. Notice that the contract does not depend on the signal observed by the agent. This is due to the fact that with exponential utility, linear sharing rules are Pareto optimal and, moreover, the principal and agent have homogeneous likelihood functions. As Milgrom and Stokey (1982) and Hakansson, Kunkel and Ohlson (1982) show, under these conditions, side bets about the signal are not optimal.

We are now ready to state the first-best problem. Given some initial wealth level, W_0, the principal maximizes his expected utility subject to a constraint on the optimal portfolio choice and the reservation utility of the agent:

$$\max_{\rho, \Phi, \phi} E\left[U_B\left(\tilde{W}_B\right)\right],$$

subject to

$$\tilde{W}_A = \Phi(\rho) + \phi(\tilde{W}) \tag{6}$$

$$\tilde{W}_B = W_0(1 + R) + \tilde{W} - \phi(\tilde{W}) - \Phi(\rho) \tag{7}$$

$$\tilde{W} = X(S)(\tilde{P} - R) \tag{8}$$

$$X(S) \in \text{argmax } E\left[U_B\left(\tilde{W}_B\right) \middle| \tilde{S} = S\right] \tag{9}$$

$$E\left[U_A\left(\tilde{W}_A\right)\right] \geq -1. \tag{10}$$

Equations (6) and (7) are the definitions of end-of-period wealths. Equation (8) is the definition of the ending portfolio value.[1] Equation (9) is the constraint representing optimal portfolio choice.[2] The reservation utility constraint of the agent, (10) is set so as to yield the same utility as in a situation where the agent was not hired and received no payment. Obviously the value on the right-hand side of (10) will not change the solution.

B. The First-Best Solution

As was demonstrated by Wilson (1968), the optimal sharing rule calls for each individual to receive the fraction of the risky asset equal to his risk tolerance divided by the aggregate social risk tolerance. This implies that the

[1] For convenience, we segregate the principal's holding of the riskless asset from the portfolio. Because of the freedom in selecting the fixed component of the contract, this is without loss of generality.

[2] Note that in the first-best problem it does not matter who makes the portfolio choice.

variable part of the agent's fee is

$$\phi(\tilde{W}) = \frac{(1/a)}{(1/a) + (1/b)} \tilde{W}.$$

Since the maximization of conditional expected exponential utility is equivalent to maximizing an objective function with a constant marginal rate of substitution between mean and variance of wealth, we get the following solution to the problem embodied in (9):

$$X(S) = \left[\frac{1}{a} + \frac{1}{b}\right] H(M(S) - R). \tag{11}$$

Substituting into the agent's utility function and evaluating the expectation (using the appendix to Bhattacharya and Pfleiderer) yields the following expression for the unconditional expected utility of the agent:

$$E\left[U_A\left(\tilde{W}_A\right)\right] = \frac{-1}{\sigma_P \sqrt{H}} \exp\left\{-a\Phi(\rho) - \frac{(\bar{P} - R)^2}{2\sigma_P^2} + V(a, \rho)\right\}. \tag{12}$$

Similarly, unconditional expected utility of the principal is

$$E\left[U_B\left(\tilde{W}_B\right)\right] = \frac{-1}{\sigma_P \sqrt{H}} \exp\left\{-bW_0(1 + R) - b\Phi(\rho) - \frac{(\bar{P} - R)^2}{2\sigma_P^2}\right\}. \tag{13}$$

Using the agent's reservation utility, solving for $\Phi(\rho)$ from (12) and substituting into (13) yields

$$\begin{aligned} E\left[U_B\left(\tilde{W}_B\right)\right] = &-\left[\sigma_P \sqrt{H}\right]^{(-1-b/a)} \\ &\times \exp\left\{-bW_0(1 + R) - \left[1 + \frac{b}{a}\right] \frac{(\bar{P} - R)^2}{2\sigma_P^2} + \frac{b}{a} V(a, \rho)\right\}. \end{aligned} \tag{14}$$

After dropping the constant multiplicative factors and using a monotone transformation, the optimal effort problem can be written as

$$\max_{\rho} - \left[\sqrt{H(\rho)}\right]^{-a/b-1} e^{V(a, \rho)}.$$

The first-order necessary condition is

$$-\left[-\frac{a}{2b} - \frac{1}{2}\right] \frac{1}{H} \frac{dH}{d\rho} - V_\rho(a, \rho) = 0,$$

which can be simplified using the definition of effort (equation (4)). We then obtain the following implicit equation for the optimal effort:

$$\frac{1}{2}\left[\frac{a}{b} + 1\right]\frac{1}{1+\rho} = V_\rho(a, \rho). \tag{15}$$

It is immediately evident that there is a unique (strictly positive) solution to (15) since the left-hand side is monotonically decreasing in ρ with range $(0, \frac{1}{2}(a/b + 1)]$ while the right-hand side is monotonically increasing starting from 0.

The left-hand side of equation (15) represents the marginal gain from expending effort and contracting between the principal and agent. The right-hand side is the marginal disutility. It can be seen that the gain due to effort expenditure is only a function of the ratio of risk aversion coefficients. Because portfolio choice is optimized after the signal is received, all prior information about the payoff of the risky asset is irrelevant. It is straightforward to see that as the principal gets relatively less risk averse, the optimal effort expenditure increases. This is because the value to the principal of having additional information becomes more important as he increases the proportion invested in the risky asset.

The comparative static with respect to the agent's risk aversion coefficient depends on the functional form of $V(a, \rho)$. If V_a is small, then the gain will outweigh the disutility and a more risk-averse agent will expend more effort. On the other hand, if V_a is large, the disutility effect dominates and more risk-averse agents expend lower effort.

II. Linear Contracts

The previous section considered the case where the agent's effort is observable and hence it is feasible to pay the portfolio manager a fee contingent on effort. Now we turn to the case where the effort and signal are unobservable and the principal *delegates* the portfolio selection decision to the manager. Under these circumstances, we investigate the use of the linear sharing rule that is optimal in the absence of moral hazard (Haugen and Taylor (1987)). Bhattacharya and Pfleiderer (1985) document that this contract would essentially give all the surplus to the agent in a screening model with asymmetric information. We show that this contract also has a serious limitation with moral hazard. The results of this section are certainly more general than the preference and distributional assumptions made here.[3]

[3] Dybvig and Spatt (1986) have set forth the condition (preference similarity) that is necessary and sufficient for first-best sharing rules to be linear. From the results of Ross (1973) and Wilson (1979), it is known that this condition is equivalent to there not being welfare losses in a delegation problem. Because preference similarity depends on endowments, which are customarily assumed to be observable in an agency framework, it is more general than the usual HARA utility restriction.

The mathematical description of the second-best delegated portfolio problem with linear contracts is

$$\max_{\Phi,\, \phi} E\left[U_B\left(\tilde{W}_B\right)\right]$$

subject to equations (6), (7), (8), and (10) with the fixed amount Φ replacing $\Phi(\rho)$ and

$$X(S) \in \operatorname{argmax}\ E\left[U_A\left(\tilde{W}_A\right)\middle|\tilde{S} = S\right] \tag{16}$$

$$\rho \in \operatorname{argmax}\ E\left[U_A\left(\tilde{W}_A\right)\right]. \tag{17}$$

This problem differs from the first-best problem in three ways. The fixed portion of the compensation contract is independent of ρ and portfolio choice (16) and effort (17) decisions are explicitly made by the agent.

Given that linear contracts are used, the function ϕ is specified as

$$\phi(\tilde{W}) = \phi'\tilde{W},$$

where ϕ' is the slope of the sharing rule. Upon observation of the signal, S, the portfolio manager selects the dollar amount in the risky asset, $X(S)$, to maximize (16). The solution to this subproblem is

$$X(S) = H\left[\frac{M(S) - R}{a\phi'}\right].$$

The distribution of portfolio value then satisfies

$$\tilde{W} = H\left[\frac{M(S) - R}{a\phi'}\right](\tilde{P} - R),$$

and the agent's wealth is

$$\tilde{W}_A = \Phi + H\left[\frac{M(S) - R}{a}\right](\tilde{P} - R).$$

An important aspect of the use of linear contracts is that the stochastic part of the agent's wealth distribution is not a function of the share, ϕ'. Using procedures similar to that of the previous section, we can now evaluate the ex ante expected utility of the agent:

$$E\left[U_A\left(\phi(\tilde{W})\right)\right] = \frac{-1}{\sigma_P\sqrt{H}}\exp\left\{-a\Phi - \frac{(\bar{P} - R)^2}{2\sigma_P^2} + V(a, \rho)\right\}.$$

An equivalent objective function for the effort choice of the agent is thus

$$\max_{\rho} \frac{-1}{\sqrt{H(\rho)}} e^{V(a,\, \rho)},$$

which yields the following first-order necessary condition for optimal effort in the linear case:

$$\frac{1}{2}\left[\frac{1}{1+\rho}\right] = V_\rho(a, \rho). \tag{18}$$

Equations (15) and (18) can be compared to determine the extent of the welfare loss due to the moral hazard problem. As long as the agent is strictly risk averse, there is underinvestment in effort relative to the first-best case. This results from the inability for the agent to internalize the benefits accruing to the principal from the expenditure of effort. The magnitude of this "shirking" problem can be seen to be more severe the greater is the ratio a/b.

In the standard agency paradigm—where the agent's action and disutility occur simultaneously—the solution to the incentive problem is to increase the share of output faced by the agent. It is easy to see that this will not work in the portfolio management problem. No matter what the slope of the linear sharing rule is, the agent will invest the same amount of effort. The difficulty is that because the agent's portfolio choice is undertaken after the effort is expended, the agent has the incentive to "undo" the incentive effect of the linear contract. Because trading allows linear modifications to the contract, there is no way for linear contracts to control the effort disincentive problem of the portfolio manager. The inability to transfer benefits from the agent to the principal significantly lowers the principal's expected utility, with the effect being greatest when the principal's risk aversion coefficient is small. It is clear that no matter what the potential for valuable information acquisition, effort is expended as if the agent only works for herself.

III. Nonlinear Contracts

In this section, we undertake an examination of the quadratic contracts proposed by Bhattacharya and Pfleiderer (1985) and compare their performance under moral hazard to that of the first-best and linear contracts considered in the previous sections. We explore the possibility that these contracts allow for internalizing of the externality created by the agent's effort, thus succeeding where the linear contracts fail. While these quadratic contracts do not optimally solve the moral hazard problem in general, they are of interest for two reasons. First, these are the only nonlinear contracts known to have the incentive compatibility property of truthful elicitation of the signal observed by the agent. Second, as we will demonstrate in the next section, they are asymptotically optimal in the limit for a very large principal. In fact, these quadratic contracts approach the first-best optimum.

It is helpful in this section to represent the portfolio manager as a security analyst who reports the value of her information to the principal, rather than making a portfolio decision. The security analyst contract can be viewed as a direct revelation mechanism whereby the principal precommits to a fee schedule depending upon the observable (payoff-relevant) state of nature, $\tilde{P}$, and an announced signal, S. The agent must then be given the incentive to report the true value of the signal, $\tilde{S}$.

The Bhattacharya-Pfleiderer contract is denoted by

$$f(M, \tilde{P}) = \gamma_1 - \gamma_2(\tilde{P} - M)^2, \tag{19}$$

where M is the reported conditional mean of the risky asset. Clearly, by equation (2), this contract can also be written as a (measurable) function of $\tilde{P}$ and S since there is a one-to-one relation between conditional means and the observed signal. The agent's ex ante expected utility is

$$\underset{M}{\operatorname{argmax}} E\left[U_A\left(f(M, \tilde{P})\right) \middle| \tilde{M}\right],$$

where $\tilde{M}$ is the actual conditional mean. The main result (Proposition 2) of Bhattacharya and Pfleiderer is that the maximizer is $M = \tilde{M}$ for all increasing and strictly concave utility functions. In the sequel, we shall use M and $\tilde{M}$ synonymously.

A. *Optimal Effort Inducement*

We will first obtain a necessary condition for the agent's optimal effort conditional on the contract parameter γ_2. Clearly γ_1 will not have any direct effect on effort. Therefore, the agent solves the following problem in choosing effort:

$$\max_{\rho} E\left[U_A\left(\tilde{W}_A\right)\right] = -E\left[\exp\left\{-a\gamma_1 + a\gamma_2(\tilde{P} - \tilde{M})^2 + V(a, \rho)\right\}\right].$$

From results in the appendix to Bhattacharya and Pfleiderer (1985), this expression is computed as

$$E\left[U_A\left(\tilde{W}_A\right)\right] = -\left(1 - \frac{2a\gamma_2}{H}\right)^{-1/2} e^{-a\gamma_1 + V(a, \rho)}. \tag{20}$$

Therefore, the following first-order necessary condition obtains:

$$\frac{1}{2}\left(1 - \frac{2a\gamma_2}{H}\right)^{-3/2}\left[\frac{2a\gamma_2}{H^2}\frac{dH}{d\rho}\right] - \left(1 - \frac{2a\gamma_2}{H}\right)^{-1/2} V_\rho(a, \rho) = 0.$$

This yields

$$\frac{1}{2}\left(1 - \frac{2a\gamma_2}{H}\right)^{-1} \frac{2a\gamma_2}{H}\frac{1}{1+\rho} = V_\rho(a, \rho). \tag{21}$$

It is clear from inspection of equation (21) that the way to get the agent to work harder and achieve a more precise estimate of the value of the risky asset is to increase the coefficient, γ_2. On the other hand, this will increase the nondiversifiable risk of the agent's portfolio, since she is unable to hedge through security trading. Notice also that the manager's effort choice is not a function of the risk/return characteristics of the risky asset. This is because her contract influences her behavior only through the error of her prediction; she does not actually hold the risky asset. We shall return to equation (21) when we present our asymptotic results.

B. The Principal's Portfolio Choice

Unlike the agent, the principal trades for himself in this direct mechanism and selects the dollar investment in the risky asset conditional on the report of the agent. In order to solve for the principal's optimal investment, $X(S)$, we need to find an expression for the conditional expected utility. The principal's wealth distribution is

$$\tilde{W}_B = W_0(1 + R) - \gamma_1 + \gamma_2(\tilde{P} - M)^2 + X(S)(\tilde{P} - R). \tag{22}$$

It is obvious that the principal is exposed to nonlinear risk in the return of the risky asset. He will attempt to hedge this through his portfolio choice. Thus, the holdings of the risky asset are distorted from the first-best levels. The following proposition contains our results concerning the principal's portfolio choice and optimized expected utility.

PROPOSITION 1: *The optimal portfolio choice, $X(S)$, of the principal satisfies*

$$X(S) = \frac{(M - R)}{b}(2b\gamma_2 + H). \tag{23}$$

Optimal conditional expected utility is

$$E\left[U_B\left(\tilde{W}_B\right)\middle| M\right] = -\frac{\sqrt{H}}{\sqrt{2b\gamma_2 + H}} \times \exp\left\{-\frac{1}{2}(2b\gamma_2 + H)(M - R)^2 - bW_0(1 + R) + b\gamma_1\right\}. \tag{24}$$

Proof: See the Appendix.

We see that the effect of the nonlinearity still yields a portfolio selection rule that is linear in the deviation of the posterior mean from the riskless rate of return. The modification from first-best is that *more* is invested in the risky asset as long as γ_2 is strictly positive. Notice that the principal's wealth distribution is a positive function of this coefficient times the prediction error. The reason that this does not lead to a tendency to sell short the risky asset for hedging purposes is that the principal's wealth distribution is decreasing in the return to the risky asset when the return is lower than predicted. Concavity of the utility function causes this decreasing portion of the contract to affect the hedging decision more than the increasing portion which occurs when the return is greater than expected. As a result, the principal counters the decreasing section by investing more rather than less. Of course, the principal, like the portfolio manager, is unable to reduce risk back to the level that is achievable under the first-best optimal contract.

To compute the principal's expected utility, we evaluate the conditional expectation of (24), and substitute the value for γ_1 obtained by equating the agent's expected utility (20) to the reservation level.

The result of this computation is stated as Proposition 2.

PROPOSITION 2: *The principal's ex ante expected utility is equal to*

$$E\left[U_B(\tilde{W}_B)\right] = -\left(1 - \frac{2a\gamma_2}{H}\right)^{-b/(2a)}\left(\frac{2b\gamma_2}{H} + 1\right)^{-1/2}\left(1 + \left(\frac{2b\gamma_2}{H} + 1\right)\rho\right)^{-1/2}$$

$$\times \exp\left\{-\frac{1}{2}(\bar{P} - R)^2\left[H\frac{\frac{2b\gamma_2}{H} + 1}{1 + \left(\frac{2b\gamma_2}{H} + 1\right)\rho}\right]\right.$$

$$\left. -bW_0(1 - R) + \frac{b}{a}V(a, \rho)\right\}. \tag{25}$$

Proof: See the Appendix.

It is this expression that the principal maximizes with respect to γ_2 and ρ while adopting the constraint given by equation (21). The principal's expected utility differs from the first-best level given by equation (14) in essentially two ways. First, as evidenced by the term in the exponential, there is incomplete risk sharing. In this case, the difference between the posterior and prior precision matter, as compared to the first-best situation. Notice, though, that as effort expenditure gets very large, the deviation goes to zero. Intuitively, in this case, the risky asset becomes riskless in the limit and incomplete risk sharing becomes less of an issue. Second, the multiplicative terms represent the benefits associated with utilizing prior information in the portfolio decision. It is clear from these results that the risk aversion of both principal and agent matter to both expected utility and effort expenditure, as in the first-best case. This stands in contrast to the linear contract case and forms an important basis for comparing the respective contracts.

IV. Comparison

In the first-best contract, we saw that optimal effort would be greater the greater is the risk tolerance of the principal as compared to the agent. Moreover optimal effort does not depend on the risk/return tradeoff of the risky vs. riskless asset since both principal and agent can achieve Pareto-optimal risk sharing. With use of the linear contract, once again Pareto-optimal risk sharing will obtain, but with an inferior level of effort investment by the portfolio manager. The optimal quadratic contract will depend on the prior statistical relationship between risky and riskless asset, because of the lack of perfect risk sharing. This is traded off against the benefits associated with an increased effort expenditure.

While it is generally difficult to compute the agency loss from quadratic contracting with the first-best, we have been able to obtain asymptotic results for the case in which the principal is much less risk averse than the agent. This is the relevant asymptotic case either for a single very risk-tolerant principal or for a large number of risk-averse principals. In the latter case, abstracting from informational and agency problems between the principals, it is known that provided their preferences are drawn from the HARA class, they form a Wilson (1968) syndicate. As such, the collection of principals is equivalent to a single aggregate principal whose risk tolerance is equal to the sum of the individual risk tolerances. Hence, holding individual risk tolerances constant, increasing the number of individuals causes the aggregate risk tolerance to increase. In the limit as the number of individuals gets large, the syndicate risk aversion goes to zero. Since most pension and mutual funds are composed of many investors, it is therefore relevant to consider the case where the principal's risk aversion is small.

In order to compare the principal's expected utility under the quadratic contract with that under the first-best contract, suppose (although this would not be strictly optimal) that γ_2 were chosen so that the effort expenditures by the agent are identical in the two situations. Using equations (15) and (21) it is clear that γ_2 must satisfy

$$\frac{2a\gamma_2}{H}\left(1 - \frac{2a\gamma_2}{H}\right)^{-1} = \frac{a}{b} + 1. \tag{26}$$

Since this value of γ_2 is suboptimal for the quadratic contract, the use of it leads to a lower bound for the expected utility of the principal. However, as is demonstrated in the following proposition, expected utility approaches that of the first-best contract as the principal's risk aversion tends to zero.

PROPOSITION 3: *For fixed risk aversion of the portfolio manager, the use of the quadratic contract achieves the first-best outcome in the limit as the risk aversion of the principal tends to zero.*

Proof: See the Appendix.

By using a quadratic contract, the principal does not achieve optimal risk sharing; however, the cost of this lack of diversification is not as significant for a principal who is nearly risk neutral. Furthermore, the information signals have greater value for a principal who tends toward risk neutrality since his desired portfolio becomes relatively more sensitive. These two effects reinforce each other in the quadratic contract. Performance of the optimal second-best contract improves as the size of the syndicate representing the principal increases. Our demonstration that the Bhattacharya-Pfleiderer contract achieves the first-best asymptotically is significant, as it illustrates the importance of nonlinear contracting and shows that restricting the contract space to the class of quadratic contracts may not be an important limitation for a large principal.

We now provide a numerical illustration of solutions for finite levels of the principal's risk aversion. The quadratic contract performance is compared to that of the first-best case on two grounds: (1) with respect to effort (and hence precision), and (2) with respect to expected utility of the principal. A comparative statics analysis with respect to the principal's risk aversion coefficient is performed. For the purpose of illustration, we chose the following values: $a = 1.0$, $\sigma_P^2 = 0.04$, $\bar{P} = 0.15$, $R = 0.05$, and $V(a, \rho) = (1/2)a\rho^2$, i.e., marginal cost is increasing in effort expenditure.[4] Although we assume explicit values for $\bar{P}$ and R, the optimal contract and effort expenditure depend only on the difference (risk premium).

Figure 1 depicts the effort expenditure for a range of principal risk tolerances from $1/b = 0$ to $1/b = 100$. Recall that posterior precision, H, is linear in effort (equation (4)) so the graph of precision is just a linear rescaling of this figure. Note that optimal effort in the linear case is equal to 0.366, and is independent of the agent's risk tolerance. It is evident that effort in the quadratic Bhattacharya-Pfleiderer is strictly greater than the linear contract for risk aversion ratios greater than two. Thus, only when principal and

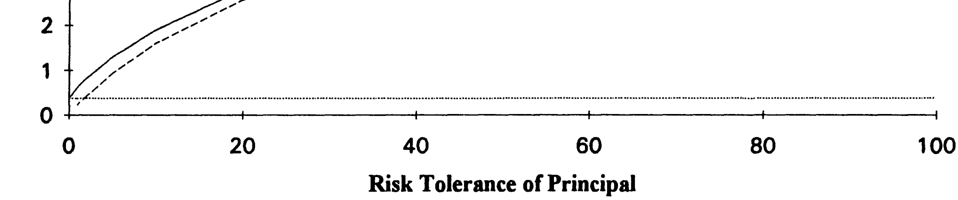

Figure 1. Optimal effort as a function of the principal's risk tolerance. The optimal effort in the three contracting regimes is graphed as a function of the principal's risk tolerance, for the parameter values in the numerical example. The *solid line* is the first-best case where effort is observable. The *dotted line* indicates effort in the linear contracting situation and the *dashed line* shows effort when the optimal quadratic contracts are used.

[4] In an earlier version of the paper, we also consider the case where $V(a, \rho) = (1/2)a\rho$, which has the interpretation as a situation where the manager has access to an information market where units of precision trade at fixed prices (Admati and Pfleiderer (1987)). The results are qualitatively similar.

agent are approximately of equal risk aversions, would the linear contract yield a greater amount of effort than the quadratic contract.

Other facts can be deduced from Figure 1. For example, for a principal whose risk tolerance is $1/b = 3$, the first-best contract leads to a posterior precision that is roughly twice as precise ($\rho = 1$) as the prior. To achieve this same doubling of precision in the quadratic case, the principal is required to be almost five times as risk tolerant as the agent. Another implication of the analysis appears to be that the absolute difference in precision between the first-best and quadratic contracts is roughly constant and independent of the principal's risk tolerance. This implies that the underinvestment in effort due to the moral hazard problem is relatively constant and independent of the potential for risk sharing. This difference is decreasing in a relative sense as the principal approaches risk neutrality. For very risk-tolerant principals the underinvestment problem appears to be rather slight indeed.

Next, we consider the cost of the agency problem in terms of the respective objective functions. The metric chosen for comparison purposes is the percentage difference in certain equivalent wealth. This measure is computed by finding the amount of (period one) wealth that would compensate the principal for the lower utility in the quadratic and linear situations as compared to first-best and dividing by the first-best certain equivalent wealth. Figure 2 graphs this measure against the risk tolerance of the principal starting from the point where both principal and agent have equal risk aversion coefficients ($a = b = 1$). It is evident that the loss increases with the principal's risk tolerance for the linear case, while it decreases for the quadratic case. In this case, whenever the risk tolerance of the principal is at least five times that of

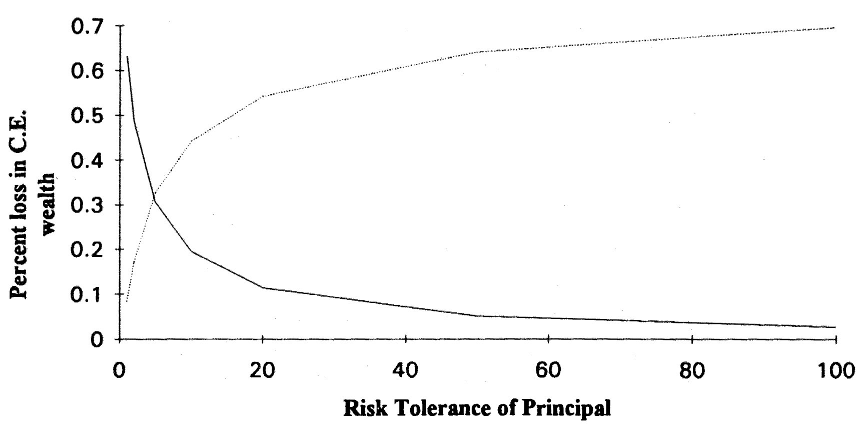

Figure 2. Percentage loss in certain equivalent wealth due to the agency problem. This graph shows the certain equivalent (C.E.) wealth loss as a function of the principal's risk tolerance for the numerical illustration. The percent loss in certain equivalent wealth is the amount of second-period wealth measured as a percentage of the first-best wealth required to equate the principal's utility under the quadratic and linear contracts with the first-best contract. The linear contract C.E. loss is indicated by the *dotted line*, while the quadratic contract C.E. loss is indicated by the *solid line*.

the manager, use of the nonlinear contract is preferred. The agency cost under the quadratic contract drops off rapidly as a function of the principal's risk tolerance. In fact, the loss in certain equivalent wealth is less than 3 percent for a principal whose risk tolerance is 100. Figure 2 thus illustrates our asymptotic results on the near optimality of the quadratic contract.

V. Conclusion

This paper has been devoted to studying the moral hazard problem in a portfolio management context. We formulate the problem in a nonstandard framework in which the agent invests effort to become informed, rather than to undertake some action. Due to information asymmetry, the moral hazard problem has two components. First, effort undertaken to acquire information is unobservable. Second, once the information is learned, the agent has to be motivated to reveal it to the principal. These two components create a nontrivial agency problem. We consider the use of the two contracts known to solve the information revelation aspect of the problem. First, a linear contract is evaluated in which the portfolio choice is delegated to the agent. Unfortunately, the linearity of the contract and the investment opportunity available to the agent leads to a serious underinvestment problem. Not surprisingly, the agent behaves as if she were operating by herself.

To ameliorate such concerns, we next investigate the quadratic contracts of Bhattacharya and Pfleiderer. It is appropriate to interpret these contracts in a security analyst framework. These second-best contracts work by imposing a risk of prediction error so that the manager has the incentive to report her private information accurately. The benefit from the use of these contracts stems from their ability to motivate the agent to undertake effort. In comparison to the first-best fixed fee contract, we discover that the underinvestment problem becomes less severe the more risk tolerant is the principal. In fact, the major result of this paper is that the quadratic contracts achieve nearly the first-best utility allocations as the principal approaches risk neutrality.

There has been some recent interest, both in academic and institutional circles, concerning the impact of the new regulations permitting managers of mutual and pension funds to be compensated in the form of contracts contingent on some benchmark index.[5] The analysis of this paper shows that without selectivity motives, contracts that are linearly related to the benchmark index will not improve performance at all as long as the benchmark is one of the components of the portfolio choice. Therefore moral hazard would seem to lead to an important motivation for the use of nonlinear contracts. Nevertheless, for those to work, it is also necessary to prevent managers from duplicating the nonlinearities through their ex post trading practices. For

[5] The Investment Advisors Act of 1940 generally prohibited incentive-based performance fees, including any compensation based on performance relative to a benchmark index. This act was amended in 1985 exempting those advisers whose clients have at least $500,000 under management.

example, continuous trading could negate the beneficial aspects of the Bhattacharya-Pfleiderer contract. Since option positions can be effectively created through continuous trading, it seems also advisable to institute some restrictions such as a maximum number of trades or a minimum average holding period. The current trend toward allowing managers to hold option and futures positions requires careful consideration.

Finally, although the quadratic contracts seems to perform well in a security analyst context, they are not directly implementable via a strict portfolio management framework. This is because the ending wealth distribution of the agent is not decomposable into a part measurable in the signal and another part that is linear in the risky asset's return. Finding an equivalent contract in the delegation environment would seem to be an interesting avenue for future research.

Appendix

In this appendix, we provide proofs of Propositions 1, 2, and 3.

Proof of Proposition 1: We need the following technical lemma:

LEMMA 1: *Expected utility of the principal conditional on H and M is*

$$E\left(U_B\left(\tilde{W}_B\right)\middle|S\right) = -\left(1 + \frac{2b\gamma_2}{H}\right)^{-1/2} \times \exp\left\{W_0(1+R) - \gamma_1 + \frac{b^2X(S)^2}{4(b\gamma_2 + H/2)} - bX(S)(M-R)\right\}. \tag{27}$$

Proof: First consider the following sequence of identities:

$$\begin{aligned}
&-b\gamma_2(\tilde{P}-M)^2 - bX(S)(\tilde{P}-R) - \frac{H}{2}(\tilde{P}-M)^2 \\
&\quad = -\left(b\gamma_2 + \frac{H}{2}\right)(\tilde{P}-M)^2 - bX(S)(\tilde{P}-M) - bX(S)(M-R) \\
&\quad = -\left(b\gamma_2 + \frac{H}{2}\right)\left[(\tilde{P}-M) + \frac{bX(S)}{2(b\gamma_2 + H/2)}\right]^2 \\
&\qquad + \frac{b^2X(S)^2}{4(b\gamma_2 + H/2)} - bX(S)(M-R).
\end{aligned}$$

Therefore,

$$E\left[\exp\left\{-b\gamma_2(\tilde{P}-M)^2 - bX(S)(\tilde{P}-R)\right\}\middle|S\right]$$

$$= \exp\left\{\frac{b^2X(S)^2}{4(b\gamma_2+H/2)} - bX(S)(M-R)\right\}$$

$$\times\frac{\sqrt{H}}{2\pi}\int_{-\infty}^{+\infty}\exp\left\{-\frac{1}{2}\left(b\gamma_2+\frac{H}{2}\right)\left[(\tilde{P}-M)+\frac{bX(S)}{2(b\gamma_2+H/2)}\right]^2\right\}d\tilde{P}$$

$$= \frac{\sqrt{H}}{\sqrt{2b\gamma_2+H}}\exp\left\{\frac{b^2X(S)^2}{4(b\gamma_2+H/2)} - bX(S)(M-R)\right\}$$

$$\times\frac{\sqrt{2b\gamma_2+H}}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\exp\left\{-\frac{1}{2}\left(b\gamma_2+\frac{H}{2}\right)\right.$$

$$\left.\times\left[(\tilde{P}-M)+\frac{bX(S)}{2(b\gamma_2+H/2)}\right]^2\right\}d\tilde{P}$$

$$= \frac{\sqrt{H}}{\sqrt{2b\gamma_2+H}}\exp\left\{\frac{b^2X(S)^2}{4(b\gamma_2+H/2)} - bX(S)(M-R)\right\},$$

because the integral times the fraction in front of it is equal to one. We get the result of the lemma by multiplying through by minus one and the nonstochastic factors. □

Using the lemma, we can show that the first-order condition for optimal portfolio selection is

$$\frac{2b^2}{4(b\gamma_2+H/2)}X(S) - b(M-R) = 0.$$

Thus, we get equation (23). To find the expression for optimized conditional expected utility, note that

$$\frac{b^2X^2}{4(b\gamma_2+H/2)} - bX(M-R) = -\frac{1}{2}(2b\gamma_2+H)(M-R)^2.$$

Substituting back into equation (27) yields equation (24) in the text. □

Proof of Proposition 2: In order to prove the result, we need another technical lemma.

LEMMA 2: *Let z be a random variable that is normally distributed with mean μ and variance σ^2. Then for $-\infty < s < 1/(2\sigma^2)$,*

$$E[e^{sz^2}] = \frac{\exp\left\{\dfrac{\mu^2 s}{1 - 2s\sigma^2}\right\}}{\sqrt{1 - 2s\sigma^2}}.$$

Proof: Available from the author upon request. □

Lemma 2 is now used to compute the unconditional expected utility of the principal. Consider the following expectation:

$$E\left[\exp\left\{-\frac{1}{2}(2b\gamma_2 + H)(\tilde{M} - R)^2\right\}\right].$$

Notice that the random variable $\tilde{M} - R$ is normal with mean $\bar{P} - R$ and variance $K^2(\sigma_P^2 + \sigma_\varepsilon^2)$. Put $s = -(1/2)(2b\gamma_2 + H)$, and we get the following:

$$E\left[\exp\left\{-\frac{1}{2}(2b\gamma_2 + H)(\tilde{M} - R)^2\right\}\right]$$
$$= \frac{\exp\left\{\dfrac{-(1/2)(\bar{P} - R)^2(2b\gamma_2 + H)}{1 + (2b\gamma_2 + H)(K^2)(\sigma_P^2 + \sigma_\varepsilon^2)}\right\}}{\sqrt{1 + (2b\gamma_2 + H)K^2(\sigma_P^2 + \sigma_\varepsilon^2)}}. \tag{28}$$

It will be helpful at this point to record a couple of algebraic simplifications:

$$1 + (2b\gamma_2 + H)K^2(\sigma_P^2 + \sigma_\varepsilon^2) = 1 + (2b\gamma_2 + H)\frac{\sigma_P^4}{\sigma_P^2 + \sigma_\varepsilon^2}$$
$$= 1 + \left(\frac{2b\gamma_2}{H} + 1\right)\rho.$$

Also,

$$\frac{2b\gamma_2 + H}{1 + (2b\gamma_2 + H)K^2(\sigma_P^2 + \sigma_\varepsilon^2)} = \frac{\dfrac{2b\gamma_2}{H} + 1}{\dfrac{1}{H} + \left(\dfrac{2b\gamma_2}{H} + 1\right)\dfrac{\sigma_P^2}{H\sigma_\varepsilon^2}}$$
$$= H\frac{\dfrac{2b\gamma_2}{H} + 1}{1 + \left(\dfrac{2b\gamma_2}{H} + 1\right)\rho}.$$

Equating the expected utility of the agent, equation (20) with this contract to her reservation utility, we obtain

$$e^{b\gamma_1} = \left[1 - \frac{2a\gamma_2}{H}\right]^{-b/(2a)} e^{(b/a)V(a,\rho)}.$$

Now, substituting this expression for γ_1 in equation (24) and using (28), we arrive at equation (25) in the text. □

Proof of Proposition 3: Taking the limit of expected utility in the first-best case, equation (14), as b goes to zero yields

$$E\left[U_B\left(\tilde{W}_B\right)\right] = -\left[\sigma_P\sqrt{H}\right]^{-1}\exp\left\{-bW_0(1+R) - \frac{(\bar{P}-R)^2}{2\sigma_P^2} + \frac{b}{a}V(a,\rho)\right\}. \tag{29}$$

For the quadratic contract, taking the limit as $b \to 0$ in equation (25) yields the principal's expected utility for this setting:

$$E\left[U_B\left(\tilde{W}_B\right)\right] = -(1+\rho)^{-1/2}\left(1 - \frac{2a\gamma_2}{H}\right)^{-b/(2a)} \times \exp\left\{-bW_0(1+R) - \frac{(\bar{P}-R)^2}{2\sigma_P^2} + \frac{b}{a}V(a,\rho)\right\}. \tag{30}$$

In taking this limit, we have used equation (26) to determine γ_2; this implies that the previous expression is actually a lower bound to the *optimal* quadratic contract. Notice that (26) implies that as $a/b \to \infty$, $(2a\gamma_2)/H \to 1$. Hence the following term from (30) approaches

$$\left[1 - \frac{2a\gamma_2}{H}\right]^{-b/(2a)} = \left[\frac{\frac{2a\gamma_2}{H}}{\frac{a}{b}+1}\right]^{b/(2a)} \to \left[\frac{1}{\frac{a}{b}+1}\right]^{b/(2a)}$$

It can be demonstrated that this term goes to one as $b \to 0$. Using the definition of H, we therefore have shown equations (29) and (30) converge as $b \to 0$, which implies that the principal's utility under the first-best and quadratic contracts are asymptotically identical. □

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