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Asset-pricing Puzzles and Incomplete Markets

CHRIS I. TELMER*

ABSTRACT

The representative agent theory of asset pricing is modified to incorporate heterogeneous agents and incomplete markets. The model features two types of agents who differ up to a nontradable, idiosyncratic component in their endowment processes. Numerical solutions indicate that individuals are able to diversify a substantial portion of their idiosyncratic income risk through riskless borrowing and lending alone. Restrictions on the variability of intertemporal marginal rates of substitution (Hansen and Jagannathan (1991)) are used to argue that incomplete markets, as modeled here, cannot account for the properties of asset returns that are anomalous from the perspective of representative agent theory.

A LARGE LITERATURE in financial economics has investigated the extent to which variants of the Lucas (1978) general equilibrium theory of asset pricing can account for the joint behavior of aggregate U.S. consumption and asset returns. By and large, this literature has concluded that a theory based upon frictionless Arrow-Debreu markets and/or a representative agent makes strongly counterfactual predictions regarding even the simple (unconditional) properties of this relationship. Mehra and Prescott (1985), for instance, show that the theory drastically underpredicts the average excess rate of return on U.S. stocks—the so-called “equity premium puzzle.” Weil (1989) points out, as did Mehra and Prescott, that the magnitude of the average risk-free rate is far below that predicted by the theory: this has been dubbed the “risk-free rate puzzle.” Numerous authors, including Grossman, Melino, and Shiller (1987) and Backus, Gregory, and Zin (1989), show that the theory cannot account for term premia inherent in the yield curve. Hodrick (1987) surveys a large literature which documents the theory’s shortcomings in accounting for excess expected returns in foreign exchange markets.

The notion that the representative agent model cannot account for excess returns across a wide array of asset markets can be made more precise by using tools developed by Hansen and Jagannathan (1991) and Shiller (1982).

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These authors show that the absolute value of the Sharpe ratio (the ratio of expected return to standard deviation of return) associated with any balanced portfolio puts a lower bound on the ratio of the standard deviation of an investor's intertemporal marginal rate of substitution (IMRS) to its expected value. Many properties of asset returns that are anomalous within the representative agent framework can be interpreted in this light. For instance, since large excess returns on equity tend to imply large Sharpe ratios, an alternative way to view the equity premium puzzle is that the IMRS of the representative agent exhibits insufficient variability (under standard restrictions on tastes and technology). Papers by Breen, Glosten, and Jagannathan (1989), Hansen and Jagannathan (1991), and Heaton (1992) use U.S. stock and bond market data to document evidence in this regard. Backus, Gregory, and Telmer (1993) and Bekaert and Hodrick (1992) find that this interpretation extends to the theory's inability to account for other excess returns, those from forward foreign currency markets in particular. This line of research shows that the extent to which an asset-pricing model can generate substantial excess returns depends on its ability to generate substantial variation in the theoretical IMRS. The representative agent model is typically found to be deficient in these terms.

One branch of subsequent research has explored the possibility that relaxing auxiliary assumptions, within the representative agent framework, can help bridge the gap between theory and data. Papers by Cecchetti, Lam, and Mark (1990), Kandel and Stambaugh (1990), and Rietz (1988) examine alternative assumptions about the driving processes for aggregate consumption. Labadie (1989) investigates the effects of stochastic inflation on asset prices. Rouwenhorst (1989) takes into account production decisions and examines asset prices in a version of the neoclassical growth model. Benninga and Protopapadakis (1990) and Kocherlakota (1990) allow for firm leverage and unconventional values for the rate of time preference. Finally, the effects of alternative preference structures are examined by a number of authors, including Abel (1990), Constantinides (1990), Dunn and Singleton (1986), Epstein and Zin (1989), Heaton (1992), Nason (1988), and Weil (1989). While this line of research has offered valuable insights, such as the role played by habit formation in generating variability in the IMRS, many issues remain unresolved. From the point of view of a theory based on a representative agent, the stochastic properties of aggregate U.S. consumption appear to be inconsistent with those of asset returns.

This paper steps outside the representative agent/complete markets paradigm and investigates the extent to which certain deviations from frictionless Arrow-Debreu markets can help to account for variability in the IMRS. It is assumed that an agent's endowment process contains an idiosyncratic component that cannot be perfectly diversified due to incomplete asset markets. Aggregation (in the sense of Constantinides (1982)) is invalid and the valuation model depends on idiosyncratic factors that cannot be detected by observing aggregate consumption alone. The focal point of the analysis is the extent to which these individual specific endowment shocks are diversifi-

able, conditional upon a specific incomplete asset market structure. Should a particular market structure not allow for a substantial amount of diversification, one would expect the IMRS to display increased variability relative to the complete markets outcome, thereby providing a valuation model with the potential to reconcile low variability in per capita consumption with large Sharpe ratios on zero net investment portfolios. This approach borrows from Mankiw (1986) who examines a two-period model in which idiosyncratic income shocks are completely undiversifiable and, by virtue of the two-period assumption, permanent. Mankiw finds that both the variability of individual IMRSs and the excess return on equity can be substantially greater than they would be otherwise, should financial markets allow for risk sharing. This paper asks whether or not such results will obtain in a multiperiod model in which the (assumed) transitory nature of idiosyncratic income shocks, in addition to the potential for future trade, permit financial markets to play an important role in the allocation of risk.¹

There are a number of reasons for which undiversifiable income risk may play an important role in asset valuation. Foremost is the issue of moral hazard. Should endowment shocks be unobservable, complete risk sharing, and therefore aggregation, may not be feasible due to incentive compatibility issues.² In addition, there are a number of empirical regularities which suggest that incomplete markets may be useful in reconciling theory and data. In the macroeconomic literature on consumption-savings dynamics a number of papers, including Flavin (1981) and Zeldes (1989a, 1989b), document evidence suggesting that consumers face binding constraints on their portfolio choice problems. Papers by Cochrane (1991) and Mace (1991) suggest that individuals may not be insured against certain types of consumption risks. Mankiw and Zeldes (1991) find evidence suggestive of substantial cross-sectional heterogeneity in both portfolio composition and consumption behavior. This body of evidence casts doubt on the appropriateness of valuation models based on complete markets in which risk is priced in terms of covariation with measures of aggregate consumption.

The results of the paper are that a very limited set of securities have the potential to go a long way in terms of providing a vehicle for the sharing of endowment risks. Specifically, it is found that almost all of an agent's idiosyncratic income risk can be diversified through trade in a single riskless discount bond market with constraints on borrowing. The incremental amount

¹Related literature on idiosyncratic risk and asset pricing includes the following papers. Aiyagari and Gertler (1990) and Huggett (1990) use dynamic models with a continuum of agents and no aggregate uncertainty. Kahn (1990) and Weil (1990) study overlapping generations economies and two-period economies, respectively. Finally, Lucas (1990) and Marcet and Singleton (1990), in independent work, use a two-agent economy similar to this paper to examine the equity premium puzzle and several other issues.

²See Townsend (1982). Several recent papers that examine dynamic information constrained economies include Green (1987), Atkeson and Lucas (1992), and Phelan and Townsend (1991). While this paper does not explicitly model such informational frictions, they provide a useful framework in which to think about the underlying rationale for market incompleteness.

of variability in the IMRS achieved as a result of market incompleteness is very small relative to the amount of variability in the idiosyncratic component of individual income. For an economy calibrated to U.S. time series data, the population standard deviation of the theoretical IMRS remains substantially below any reasonable estimate of a Hansen-Jagannathan bound. The implication is that market incompleteness of the form modeled here is not likely to provide a resolution of either the equity premium puzzle or any other excess return anomaly associated with the complete markets model. This conclusion relies on the conjecture that should the model allow for trade in additional assets, such as equity, risk-sharing opportunities would be enhanced, thereby reducing variability in the IMRS relative to the single asset economy. In this sense the single asset assumption provides an upper bound on the extent to which incomplete markets can account for Hansen-Jagannathan bounds and therefore for asset-pricing puzzles in general. While this conjecture seems reasonable, the reader should be aware that no proof is available.

The remainder of the paper is organized as follows. Section I derives a simple version of a Hansen-Jagannathan bound. Empirical evidence on the implied statistical properties of the IMRS is reviewed in order to provide a metric with which to evaluate the model. The incomplete markets asset-pricing model is outlined in Section II and a computational solution algorithm is developed in Section III and in an Appendix. Section IV conducts a series of quantitative experiments by calibrating the model to monthly U.S. per capita consumption data and varying, among other things, the amount of heterogeneity inherent in individual income processes. Section V conducts a brief sensitivity analysis and Section VI concludes and discusses future work.

I. Hansen-Jagannathan Bounds

This section makes use of tools developed by Hansen and Jagannathan (1991) and Shiller (1982) in order to develop a simple diagnostic that can be used to evaluate a broad class of asset-pricing models. The procedure has the added advantage of being able to summarize the implications of data on returns from many different asset markets in a parsimonious manner. Empirical evidence from a number of different sources is reviewed in order to provide a metric with which the model of Section II can be evaluated.

A fundamental building block of finance is the assumption that securities markets do not admit any riskless arbitrage opportunities. The consequence (Harrison and Kreps (1979), Ross (1976)) is that there exists a positive random variable, say m_t , which satisfies the following conditional moment restriction:

$$p_t = E_t(m_{t+1}X_{t+1}). \quad (1)$$

In equation (1) p_t represents a vector of asset prices, p_t^i , and X_t represents a vector of payoffs, x_t^i , on a set of securities indexed by i . The conditional

expectations operator, E_t , is defined in terms of an information set, $\mathcal{I}_t$, that represents the intersection of all individual information sets. Random variables subscripted with a t are assumed to be adapted to the information structure, $F \equiv \{\mathcal{I}_t; t = 0, 1, 2, \dots\}$. The variable m_t will be referred to as the pricing kernel. Knowledge of the exact form of m_t is sufficient to value any asset or portfolio of assets.

The linear pricing condition (1) can be used to derive a mean variance frontier for the pricing kernel which can be completely characterized using data from securities markets alone. Writing (1) in terms of returns, it is easily shown that, for any two assets, the following relation must hold:

$$E_t(m_{t+1}[R_{t+1}^i - R_{t+1}^j]) = 0 \quad (2)$$

where $R_{t+1}^i \equiv x_{t+1}^i/p_t^i$ is the gross rate of return on asset i between time t and $t + 1$. Define r_{t+1} as the return differential, $r_{t+1} \equiv R_{t+1}^i - R_{t+1}^j$. Since (2) holds in conditional mean, it holds unconditionally as well. The definition of covariance implies the following:

$$E(m)E(r) + \text{Cov}(r, m) = 0, \quad (3)$$

where the absence of subscripts denotes an unconditional moment. Since m_t is a positive random variable, and correlations are less than unity in absolute value, (3) implies the inequality

$$\sigma_m/E(m) \geq |E(r)|/\sigma_r, \quad (4)$$

where σ denotes standard deviation. The expression (4) implies that the absolute value of the "Sharpe ratio," $|E(r)|/\sigma_r$, provides a lower bound on the ratio of σ_m to its mean.

The inequality restriction (4) applies to any random variable, or pricing kernel, that satisfies the pricing relation (1). The essential link between (1) and the consumption-based asset-pricing model is that, in general, the latter identifies the pricing kernel, m_t , with the ratio of an investor's marginal utilities of consumption at two points in time: her intertemporal marginal rate of substitution (IMRS). One important exception to this statement involves constraints on asset holdings, or corner solutions. In this case the pricing kernel is a quantity that involves more than one individual. This is made precise in Section II. The important point for now is that in a wide class of theoretical economies, including those with heterogeneous agents and/or incomplete markets, there always exists a random variable (which is not in general unique) that satisfies the definition of a pricing kernel. Should the pricing kernel from a particular economy fail to satisfy the statistical restriction (4), given the Sharpe ratio from some portfolio of assets, then this economy can be deemed inconsistent with the data in these dimensions. For expositional clarity it will be useful to refer to the pricing kernel as an IMRS

whenever the definitions of the two coincide. In all other cases the more general terminology will be used.

This framework suggests that many “asset-pricing puzzles” can be reinterpreted as asking, from the point of view of a particular model, why we observe such high implied variability in the pricing kernel. For instance, one implication of equation (4) is that large excess returns on equity tend to imply substantial pricing kernel variability. This suggests that an alternative interpretation of Mehra and Prescott’s (1985) “equity premium puzzle” is that the representative agent model with additively time-separable preferences cannot generate realistic variability in the IMRS (Hansen and Jagannathan (1991) provide evidence in this regard). Similar reasoning allows one to interpret the model’s inability to account for predictable variation in many other excess returns in the same light (Gallant, Hansen, and Tauchen (1990)). This work shows that the extent to which an asset-pricing theory will be able to account for interesting properties of the cross-sectional distribution of returns depends first on its ability to account for variation in the pricing kernel, and next on its ability to account for covariation between the pricing kernel and excess returns. In this sense a Hansen-Jagannathan bound can be viewed as a necessary, but not sufficient, condition that a valuation model should satisfy.

A number of different approaches have been used to obtain estimates of Hansen-Jagannathan bounds. Several authors’ results, based on monthly data, are now reviewed for use in the subsequent empirical analysis. Hansen and Jagannathan (1991) and Heaton (1992) use a generalization of the above procedure to derive bounds based on U.S. stock and bond market data. They infer a lower bound of approximately 0.15, conditional on a mean of roughly unity. Breen, Glosten, and Jagannathan (1989) make use of the predictive power of Treasury bill returns over stock market returns to construct balanced portfolios based on a dynamic trading strategy. The Sharpe ratios for these portfolios constitute valid Hansen-Jagannathan bounds. They report estimates in the 0.13 to 0.14 range. Backus, Gregory, and Telmer (1993) use similar methods and foreign currency market data to obtain estimated Sharpe ratios ranging from 0.09 to 0.36. Finally, Bekaert and Hodrick (1992) make extensive use of conditioning information as well as a wide array of domestic and foreign securities returns to infer lower bounds on the pricing kernel’s variability as high as 0.776.

To summarize, using data on monthly asset returns, the Hansen-Jagannathan procedure allows one to infer that a reasonable model of consumption and asset returns should feature a pricing kernel with a standard deviation of at least 0.150 and a mean between 0.995 and unity. This value for the standard deviation is chosen in part to account for sampling variability inherent in the above estimates. The value of 0.15 is well below the 95 percent confidence interval associated with some of the larger point estimates of the Hansen-Jagannathan bound (see Bekaert and Hodrick (1992)). The range for the mean implies an average risk-free return between 0 and 6 percent, at annual rates. This characterization of the admissible

region of the mean-standard deviation space for the pricing kernel will prove useful in the quantitative analysis which follows the development of the model.

II. Asset-pricing Model

Consider an endowment economy in which many rational investors trade in competitive securities markets and solve optimal portfolio allocation problems in order to determine their asset holdings and consumption profiles. It is assumed that individuals, indexed by k , derive utility from the consumption of a single good, in terms of which asset prices and payoffs are denominated. Denote $\tilde{m}_{k,t}$ as investor k 's IMRS: the ratio of her time t marginal utility of consumption to that obtained at time $t - 1$. If individuals are at interior solutions to their portfolio allocation problems $\tilde{m}_{k,t}$ satisfies the definition of a pricing kernel.

$$p_t = E_t(\tilde{m}_{k,t+1}X_{t+1}). \quad (5)$$

Equilibria featuring corner solutions, in which the usual Kuhn-Tucker conditions replace (5), are discussed below.

The asset-pricing relation (5) is consistent with a wide class of models, the two of immediate interest being the Mehra-Prescott representative agent model and the incomplete markets model derived below. In order to see the relationship between the two, consider the following additional structure to the model economy. Suppose there are two types of agents, $k = 1, 2$, and that agents of each type are identical in all respects. It is valid to construct two different representative agents, again indexed by $k = 1, 2$, and deal strictly in terms of this two-agent economy. Next, assume that these two agents are each endowed with a claim to an exogenously determined income stream, hereafter called labor income, and that incomes are not perfectly correlated across individuals. For reasons outlined further below, claims to individual labor income are assumed to be nontradable. Finally, assume that preference orderings over the single good are the same across agents 1 and 2: the determinant of agent type is solely related to endowments. At any point in time the following resource constraint must hold:

$$y_{1,t} + y_{2,t} = c_{1,t} + c_{2,t}, \quad (6)$$

where $c_{k,t}$ and $y_{k,t}$ are, respectively, individual k 's period t consumption and labor income. Assuming, for now, that agents are not at corner solutions, equations (5) and (6), along with the requirements that all assets are held and that both agents obey their intertemporal budget constraints, define a competitive equilibrium.

If we further assume that agents are able to trade in a complete set of Arrow-Debreu markets for contingent claims, then the Mehra-Prescott model results. As noted, the only characteristic that distinguishes agents 1 and 2 is their nontradable labor income. However, since markets are complete, individual IMRSs will be equated across states of the world, and no idiosyncratic

risk will be priced in equilibrium. A unique equilibrium consumption sequence is easily identified in which each agent gets a deterministic fraction of the aggregate endowment in every state. Supporting asset prices are then obtainable using (5). The Mehra-Prescott exercise consists of placing further structure on preferences and endowment processes such that the pricing functional implicitly defined by (5) can be solved and exact numerical solutions can be obtained.

The remainder of this paper will be concerned with an environment in which a complete set of contingent claims are not marketed. The resulting problem becomes somewhat more complex. Since agent 1 and agent 2's IMRS need not be equated in equilibrium, standard welfare theorems from microeconomics cannot be relied upon to provide a tractable two-stage solution strategy. That is, equilibrium consumption sequences and asset prices must be solved for simultaneously: the invalidity of the welfare theorems renders centralized methods for finding equilibrium consumption allocations inapplicable.³

Additional structure on preferences, technology, and asset markets is now required in order to make (5) operational. For both expository and tractability reasons, an attempt is made to keep the model economy similar to that of Mehra and Prescott. The process that is observable at the aggregate level is outlined first, followed by a description of how the idiosyncratic component of individual labor income evolves.

A. Labor Income and Asset Markets

The specification of the endowment processes for agents 1 and 2 borrows from Mankiw (1986). Aggregate labor income evolves according to some exogenous stochastic process while individual income is generated by a stochastic sharing rule. One attractive feature of this approach is that it forces the model to account for both individual and aggregate data. The links to the more easily measured aggregate data will be helpful in choosing values for the model's technological parameters. At the same time, since theoretical aggregate data will be inappropriate for asset pricing, the model has the potential to generate asset prices that are substantially different than those of the representative agent framework.

Aggregate income, y_t , is assumed to grow at (gross) rate λ_t , where the stochastic process $\{\lambda_t\}_{t=0}^{\infty}$ is governed by a first-order Markov chain with transition probabilities π_{ij} . Therefore, $y_{t+1} = \lambda_{t+1}y_t$. The random variable λ_t is restricted to take on one of two values, λ_1 or λ_2 , which define two states of the world for aggregate labor income, high and low growth, respectively.

The stochastic sharing rule is specified to capture Mankiw's notion that bad aggregate shocks are unevenly distributed across the population. Conditional on the growth rate of aggregate income being λ_2 , agent 1 either

³Many other properties of the Arrow-Debreu model, such as efficiency, also fail to hold in the incomplete markets environment. See Duffie (1990) and Geanakoplos (1990) for surveys of these and other important aspects of general equilibrium with incomplete markets.

receives a fraction γ , $1/2 \leq \gamma < 1$, of y_t with probability $1/2$, or a fraction $(1 - \gamma)$ of y_t with probability $1/2$. Agent 2 receives the remainder in both of these idiosyncratic states of the world. When aggregate income growth is high ($\lambda_t = \lambda_1$) both agents receive $y_t/2$. It will be convenient to define a random variable, $Q_{1,t}$, as follows:

$$\begin{aligned} Q_{1,t} &= 1/2 && \text{with probability } \pi_{i1} \\ &= \gamma && \text{with probability } \pi_{i2}/2 \\ &= (1 - \gamma) && \text{with probability } \pi_{i2}/2. \end{aligned}$$

Defining $Q_{2,t}$ as $(1 - Q_{1,t})$, it is valid to write $y_{k,t} = Q_{k,t} y_t$. $Q_{k,t}$, the stochastic fraction of aggregate income that individual k receives, is referred to as the idiosyncratic shock process.

The parameter γ plays an important role in the subsequent analysis in that it determines the degree of heterogeneity in the model. Should $\gamma = 1/2$ the model collapses to the representative agent model of Mehra and Prescott (1985). As the value of γ increases, from $1/2$ towards unity, individuals become heterogeneous in that they are endowed with different random labor incomes. Larger values of γ imply more variability in individual income processes, and an increased amount of (potentially diversifiable) idiosyncratic labor income risk.

Given these exogenous sources of idiosyncratic risk, risk-averse agents will seek to trade intertemporally with one another. A crucial assumption is that agents cannot directly trade in claims to their individual specific labor income. An obvious interpretation of such a restriction is that moral hazard problems result in an inability to write incentive-compatible contracts based upon idiosyncratic outcomes. Agents can however self-insure to a certain extent by trading in perfectly competitive asset markets. It is assumed that the only asset available is a riskless discount bond that pays one unit of the consumption good with certainty one period after being issued. This assumption is sufficient to ensure that asset markets are dynamically incomplete (see Huang and Litzenberger (1988)). Restricting trade to a single riskless asset is useful in that it is likely to provide an upper bound on the extent to which incomplete markets can account for variability in the pricing kernel. Additional assets, such as claims to a dividend stream, would provide for enhanced intertemporal trading opportunities, resulting in less idiosyncratic risk being priced in equilibrium.

It is worth noting the conditions under which additional assets would "complete the market" and result in an equilibrium isomorphic to that of the representative agent model. A heuristic word on the underlying mathematics will be useful at this point. The economy can be thought of as being endowed with two information structures, an increasing set of σ -algebras, $F \equiv \{\mathcal{F}_t: \mathcal{F}_t \subseteq \mathcal{F}_{t+1}\}$, that corresponds to the evolution of both the idiosyncratic and aggregate shocks, and another set, $G \equiv \{\mathcal{G}_t: \mathcal{G}_t \subseteq \mathcal{G}_{t+1}\}$, that corresponds only to the evolution of the aggregate process. Note that $\mathcal{G}_t \subset \mathcal{F}_t$. Note also that $Q_{k,t}$ is $\mathcal{F}_t$ measurable but is not $\mathcal{G}_t$ measurable. It is understood that the

probability measure associated with the operator $E_t(\cdot)$ is associated with the set $\mathcal{F}_t$. The assumption that drives the results is that while asset prices are $\mathcal{F}_t$ measurable, asset payouts are restricted to be $\mathcal{G}_t$ measurable. The requirement that contracts contingent upon the idiosyncratic information structure cannot be written seems natural, given the moral hazard motive discussed above. As long as this restriction is maintained, asset markets will be dynamically incomplete, regardless of the number of securities traded.

B. Competitive Equilibrium

Denote $b_{k,t}$ as the number of bonds held by individual k between time t and $(t + 1)$, and p_t as the time t price of one discount bond. Individual k 's intertemporal budget constraint can be written as follows:

$$y_{k,t} + b_{k,t-1} \geq c_{k,t} + p_t b_{k,t}. \quad (7)$$

In order to formulate the model in terms of stationary processes, (7) is normalized by aggregate income, y_t .

$$Q_{k,t} + b_{k,t-1}^* / \lambda_t \geq c_{k,t}^* + p_t b_{k,t}^*. \quad (8)$$

Variables with asterisks are expressed relative to aggregate income. For example, $b_{k,t}^*$ is individual k 's period t purchases of discount bonds, expressed as a fraction of y_t . For notational simplicity, the asterisks will be omitted hereafter.

Individual optimization problems are standard. Taking prices as given, agent k chooses a consumption sequence, $\{c_{k,t}\}_{t=0}^\infty$, supported by an admissible trading strategy, $\{b_{k,t}\}_{t=0}^\infty$, to maximize the following additively time-separable expected utility function:

$$U(c_k) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_{k,t}). \quad (9)$$

The momentary utility function, $u(\cdot)$, is assumed to take the form $u(c) = c^{1-\alpha}/(1-\alpha)$ for both agents. The IMRS inherent in (5) is therefore $\tilde{m}_{k,t+1} \equiv \beta(c_{k,t+1}/c_{k,t})^{-\alpha}$. Finally, it is assumed that $b_{k,-1} = 0$, $k = 1, 2$, or that the initial distribution of wealth is uniform.⁴ Prior to receiving their period 0 endowments, agents 1 and 2 are therefore ex ante identical in that they both hold no financial wealth, have identical preferences, and are endowed with labor income streams that have identical present values. Once their period 0 endowments are realized, individuals make their consumption-savings decisions. Should the realization of $Q_{1,0}$ (the sharing rule that determines individual income) be $1/2$, no trade in bonds occurs and individuals remain identical. Otherwise, trade occurs, the distribution of wealth becomes skewed, and period 1 decisions reflect this additional aspect of heterogeneity.

A competitive equilibrium with incomplete markets is defined as a sequence of consumption plans, $\{c_{k,t}\}_{t=0}^\infty$ ($k = 1, 2$), financed by trading rules

⁴None of the results reported below are affected by different assumptions regarding the initial distribution of wealth.

$\{b_{k,t}\}_{t=0}^{\infty}$ ($k = 1, 2$) and supported by a price process, $\{p_t\}_{t=0}^{\infty}$, that satisfy the following conditions at each point in time:

$$p_t \geq E_t(\tilde{m}_{k,t+1}), \quad k = 1, 2, \quad (10)$$

$$c_{k,t} = Q_{k,t} + b_{k,t-1}/\lambda_t - p_t b_{k,t}, \quad k = 1, 2, \quad (11)$$

$$b_{1,t} + b_{2,t} = 0, \quad (12)$$

$$b_{k,t} \geq -\hat{b}. \quad (13)$$

In addition, it is assumed that both agents agree on all aspects of the stochastic process describing prices. Equations (10) and (11) are, respectively, first-order conditions (Euler equations) and intertemporal budget constraints from each agent's intertemporal choice problem. Equation (12) requires that markets for bonds and goods (by Walras Law) clear at all times. Finally, some sort of terminal condition must be satisfied in order to rule out equilibria which admit unbounded borrowing, or Ponzi schemes. This is accomplished by equation (13) which places an upper bound, $\hat{b}$, on the number of one-period bonds that an agent can issue. This upper bound, which will be referred to as a borrowing constraint, plays an important role in the subsequent analysis: the solution method outlined below cannot be employed without putting (implicit) constraints on asset holdings. However, in practice, the value of $\hat{b}$ can be chosen to be large enough so that the constraint is almost never binding. Consequently, this framework is capable of examining economies with and without constraints on asset holdings, the former of which are of interest in and of themselves.

In states of the world when (13) is binding for one agent, it cannot be binding for the other agent: there is no restriction on lending, only borrowing. Consequently, bond prices in these states are uniquely defined by the unconstrained agent's first-order condition which will hold with equality. For the constrained agent, (10) will take the form of a strict inequality and (13) will hold with equality. Finally, the pricing kernel can now be defined explicitly. Define the indicator variables, $I_{k,t}$, $k = 1, 2$, as being equal to unity whenever agent k is unconstrained in the bond market, and zero otherwise. Two pricing kernels can be defined, for $k = 1, 2$, as

$$m_{k,t} = I_{k,t} \tilde{m}_{k,t} + (1 - I_{k,t}) \tilde{m}_{\bar{k},t}, \quad (14)$$

where $\bar{k} = 2$ when $k = 1$ and $\bar{k} = 1$ when $k = 2$. By definition, $m_{k,t}$ will satisfy (1) with equality at each point in time. Its standard deviation is therefore bounded by the Sharpe ratio as was shown in (4). It will be useful for subsequent sections to note that a measure of the net effect of the borrowing constraint on the economy is provided by the difference between the pricing kernel, $m_{k,t}$, and the IMRS, $\tilde{m}_{k,t}$.

III. Solving the Model

Solving the model involves characterizing the trading rule, $b_{1,t}$, and the equilibrium pricing function, p_t , as stationary functions of the current state

of the world. The trading rule for individual 2, $b_{2,t}$, is then given by the market-clearing condition. The trading rule and the pricing function will be referred to as the policy functionals. The following random vector constitutes a set of state variables sufficient to describe the system at time t and predict its subsequent evolution:

$$S_t \equiv [Q_{1,t} \quad b_{1,t-1}].$$

The policy functionals can be expressed as $b_{1,t}(Q_{1,t}, b_{1,t-1})$ and $p_t(Q_{1,t}, b_{1,t-1})$. The tractability obtained by using a two-agent model should now be apparent. Unlike standard asset-pricing theory, asset prices in the current framework depend on the distribution of wealth. The two-agent assumption allows the cross-sectional distribution of wealth to be characterized by the single variable, $b_{1,t-1}$.

The system (10) to (13) can be written as a bivariate system of functional equations to which the policy functionals represent a solution. Obtaining the solution involves computing the conditional mean of a nonlinear function of an endogenous state variable (the wealth variable). Closed form solutions to such expressions do not exist in general. This paper follows much of the recent literature on dynamic stochastic general equilibrium theory by using a numerical solution technique to obtain approximate solutions for the "true" policy functionals. Taylor and Uhlig (1990) present a summary of related methods that have been used to solve various specifications of the neoclassical growth model.

The solution algorithm developed here involves iterating on a mapping, defined on the system of first-order conditions (10) to (13), to which the policy functionals represent a fixed point. An intuitive way of thinking about the algorithm is that the equilibrium policy functionals for a sequence of finite period problems are characterized. The functionals describing equilibrium behavior for the infinite period problem are then computed as the limit of this sequence. The reason that the solution is an approximation is that, for tractability reasons, the state space is discretized to a certain extent. Let $\mathcal{B}$ denote the set of η points defining a uniform partition of the interval $[-\hat{b}, \hat{b}]$. The state variable $b_{1,t-1}$ is restricted to take on values in this set. The other state variable, $Q_{1,t}$, has already been restricted to take on values in the set $\mathcal{Q} \equiv \{1/2 \gamma (1 - \gamma)\}$. The domain for the policy functionals is therefore the set $\mathcal{S} \equiv \mathcal{B} \times \mathcal{Q}$. The functionals can be represented as $b_1: \mathcal{S} \rightarrow \mathcal{B}$ and $p: \mathcal{S} \rightarrow \mathbb{R}_{++}$. The nature of the approximation is that the trading rule is restricted to map from $\mathcal{S}$ into the predetermined set of points, $\mathcal{B}$. The algorithm does not, however, restrict the range of the pricing functional in this manner. Although the bond price will map to values in a countable set (by virtue of its countable domain), these values are determined endogenously by the algorithm. Finally, note that the approximation error can be made arbitrarily small as the number of points, η , defining the partition becomes large. This is of course conditional on the finite set $\mathcal{Q}$ being treated as a fundamental of the economy, which it is throughout the paper. A more detailed description of the algorithm is provided in the Appendix, where it is also argued that the value

chosen for η is sufficiently large so as to make the magnitude of the approximation error insignificant.

IV. Numerical Results

This section asks whether or not incomplete markets can provide an explanation for the variability of the pricing kernel implied by the Sharpe ratios reported in Section I. It was argued there that a standard deviation of at least 0.150 is required in order to claim even modest success. Since agents in the theoretical economy can have incomes that are highly variable (in growth rates) relative to the per capita aggregate, one might imagine that the model would do quite well in terms of generating variability in individual IMRSs and, as a result, in the pricing kernel. However, as will become clear, this conclusion depends on the extent to which individuals are able to pool idiosyncratic risk by trading in the bond market.

The numerical exercise can be summarized as follows. First, suitable values for the discount factor and the parameters describing the aggregate endowment process are selected. The robustness of the results to perturbations in these values is discussed in Section V. Next, the effects of incomplete markets are characterized by varying the heterogeneity parameter from 0.50 (representative agent) to 0.65 (extreme heterogeneity). The effects of increased risk aversion are also explored by varying the curvature parameter, α , from 2.0 to 4.0. These experiments are conducted in a sequence of economies that are indexed by differing assumptions about the opportunities to trade intertemporally. The case of complete markets is examined first, followed by the opposite extreme, complete absence of trading opportunities, or autarky. Several intermediate cases are then examined: unrestricted trade in bond markets (no borrowing constraints), followed by two examples in which borrowing constraints are imposed. Theoretical population moments are reported for each of these economies and are compared to the sample moments reported in Section I.

As in Mehra and Prescott (1985) the process for aggregate income (which equals aggregate consumption in this environment) is chosen to match the mean, standard deviation, and coefficient of first-order autocorrelation from U.S. data. The difference is that in the current model the consumption data are monthly rather than annual. A complete description of all data sources is provided in the notes to Table I. The mean, standard deviation, and autocorrelation of the consumption growth rate, for the period 1959 to 1986, are 1.00142, 0.00465, and -0.2257 , respectively. The transition matrix, Π , is restricted to be symmetric, which implies that it is completely described by one parameter defined by $\phi = \pi_{11} = \pi_{22}$. Trivially, $\pi_{12} = \pi_{21} = (1 - \phi)$. The parameter values that satisfy these criteria are $\lambda_1 = 1.0061$, $\lambda_2 = 0.9968$, $\phi = 0.39$. The discount factor, β , is set to 0.9983. This value was chosen to represent an annualized discount rate of approximately 2 percent.

Consider now a sequence of heterogeneous agent economies indexed by the parameter γ . Recall that the value of $(\gamma - 0.50)$ can be interpreted as a

Table I
Complete Markets (Population Moments)

The values reported for the U.S. economy are robust GMM estimates. Standard errors are in parentheses. All data are monthly over the sample 1959 to 1986. The consumption series is per capita nondurables and services exclusive of clothing, shoes, and medical services and was obtained from CITIBASE. The corresponding implicit price deflator was used to convert all nominal quantities into real quantities. The risk-free rate corresponds to the 30-day Treasury bill rate which was obtained from the FAMA CRSP tape. The mean IMRS was computed as the sample average of the implied Treasury bill price series. The standard deviation of the IMRS is a lower bound and is intended to be representative of the findings of the various studies cited in Section I.

The remainder of the table reports population moments from theoretical economies in which markets are assumed to be complete. The definitions of the pricing kernel and the IMRS of the representative agent coincide in these cases. The moments are computed from the Markov chain governing the evolution of the process for per capita consumption growth. In Panel B, parameter values (given in the text) are chosen to match the mean, standard deviation, and coefficient of first-order autocorrelation of the theoretical consumption process to the corresponding sample moments from the monthly CITIBASE consumption series. The case in Panel C uses the same parameter values as the case in Panel B, except that the standard deviation of consumption growth has been doubled. The parameter α denotes the representative agent's coefficient of relative risk aversion. The discount factor is set to 0.9983 in all cases.

		Pricing Kernel		Consumption Growth		Risk-free Rate	
		Mean	Standard Deviation	Mean	Standard Deviation	Mean	Standard Deviation
Panel A. U.S. Economy							
		0.9993 (0.0003)	0.150	0.142 (0.021)	0.465 (0.021)	0.991 (0.342)	3.748 (0.259)
Panel B. Theoretical Economies (Base Case)							
α							
(i)	2	0.9955	0.009	0.142	0.465	5.506	2.479
(ii)	4	0.9929	0.018	0.142	0.465	8.952	5.015
(iii)	10	0.9854	0.046	0.142	0.465	19.220	12.964
(iv)	25	0.9703	0.112	0.142	0.465	43.421	35.188
Panel C. Theoretical Economies (High-Consumption Variability)							
α							
(i)	2	0.9957	0.018	0.142	0.930	5.270	5.015
(ii)	4	0.9935	0.037	0.142	0.930	8.147	10.256
(iii)	10	0.9889	0.092	0.142	0.930	14.519	27.389
(iv)	25	0.9907	0.226	0.142	0.930	13.548	80.079

measure of the degree of heterogeneity, or incompleteness of asset markets, associated with a particular economy. The magnitude of γ also determines the variability of individual income growth rates. Figure 1 reports the population standard deviation of individual income growth for the range $\gamma = 0.50$ to $\gamma = 0.65$, in increments of 0.01. Mean income growth is 0.142 percent per

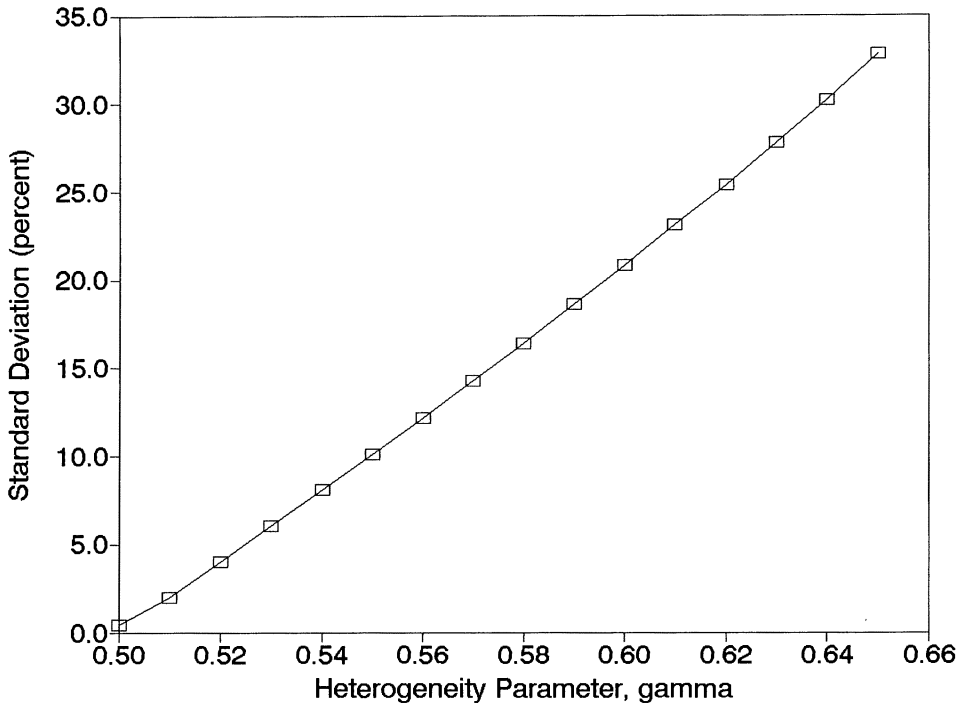


Figure 1. Standard deviation of individual income growth with different values of the heterogeneity parameter.

month for all values. Due to the symmetric nature of the economy, all moments associated with agent 1 and 2 are identical. Only one set of numbers are therefore reported for the remainder of the paper. One way in which to choose an appropriate value for γ is to match the variability of the theoretical income processes (Figure 1) to those observed in some panel data set. The strategy here, however, is to take an agnostic position and report results for the wide range of γ given in the graph.

Taking individual endowment processes as given, we now can examine the implications of alternative asset market structures. Tables I and II report results from two extreme cases: complete markets and autarky.⁵ As was discussed in Section II, the complete markets equilibrium for this model is isomorphic to an equilibrium with a representative agent who has the same utility function as agents 1 and 2 and who consumes the aggregate endow-

⁵Complete markets would result if agents were allowed to trade in at least three assets with linearly independent payoffs. Alternatively, an economy with trade in a single asset with payoffs that are perfectly correlated with $Q_{1,t}$ would be isomorphic to a complete markets economy. Trade in this single asset would be equivalent to allowing individuals to directly trade in claims to their endowments. Note that both of these market structures violate the measurability restriction discussed in Section II.

Table II
Incomplete Markets: No Intertemporal Trade
(Population Moments)

These data correspond to autarkic economies with idiosyncratic labor income shocks. All values are based on monthly data. "Mean" and "Std. Dev." denote population mean and standard deviation, respectively. Computation is straightforward using the Markov chain described in the text. The moments for the pricing kernel and the risk-free rate are computed using equations (10) and (14). That is, in a given state of the world the IMRS of the agent with the highest shadow price defines both the pricing kernel and the bond price. The moments correspond to both individual 1 and individual 2: due to the symmetric nature of the economy, unconditional moments are equalized across agents. The mean of consumption growth is 0.142 percent for all cases. The discount factor, β , is set to 0.9983 for all cases. The parameters α and γ describe risk aversion and heterogeneity, respectively.

		Income Growth Std. Dev. (percent)	Consumption Growth Std. Dev. (percent)	Pricing Kernel		Risk-free Rate	
				Mean	Std. Dev.	Mean (percent)	Std. Dev. (annualized)
Panel A. $\alpha = 2$							
	γ						
(i)	0.50	0.465	0.465	0.9955	0.009	5.506	2.479
(ii)	0.55	10.105	10.105	1.0159	0.206	-73.970	181.150
(iii)	0.60	20.779	20.779	1.0810	0.444	-96.319	538.089
(iv)	0.65	32.774	32.774	1.2047	0.762	-99.805	1102.769
Panel B. $\alpha = 4$							
	γ						
(i)	0.50	0.465	0.465	0.9929	0.018	8.952	5.015
(ii)	0.55	10.105	10.105	1.0762	0.439	-95.967	597.621
(iii)	0.60	20.779	20.779	1.3678	1.143	-99.996	2683.355
(iv)	0.65	32.774	32.774	2.0356	2.667	-99.999	7596.221

ment each period. As a result, the parameter γ plays no role. Bond prices are equal to the conditional mean of the representative agent's IMRS: $E_t(\beta \lambda_{t+1}^{-\alpha})$. In Table I, Panel B reports the mean and standard deviation of the pricing kernel (the IMRS), consumption growth and the risk-free interest rate for several different values of the risk aversion parameter, including an extreme case ($\alpha = 25$). In addition, sample moments from the U.S. economy are reported in Panel A. The results indicate that the model with complete markets can only generate realistic variability in the IMRS when the agent exhibits extreme risk aversion and the average real rate of interest is unrealistically large.⁶ This is the essence of Weil's (1989) "risk-free rate"

⁶This statement does not strictly apply for values of α greater than 71, at which point the mean risk-free rate of interest begins to decrease in α . A value of α equal to 147 will match the U.S. sample moment for the risk-free rate (0.991 percent) and generate a standard deviation for the IMRS of 0.590. This effect is also apparent in Panel C of Table I.

puzzle. The appropriateness of such large values for α has been extensively discussed in the literature and will not be pursued further here. For instance, Mankiw and Zeldes (1991) report anecdotal evidence in terms of the certainty equivalent value of a lottery, suggesting that values of α greater than 10 are highly unrealistic. The remainder of the paper will focus on low values for α .

Within the time and state-separable power utility framework, the implications of Table I are quite robust. The results on the variability of the IMRS are not affected by perturbations to the assumed mean and autocorrelation of consumption growth. This is not the case with respect to the standard deviation. Table I, Panel C reports moments from four economies that are identical to those in Panel B, except that the standard deviation of consumption growth has been doubled. We see that the standard deviation of the IMRS rises in roughly a proportional manner. In the most extreme case, $\alpha = 25$, the standard deviation increases to a value in excess of the Sharpe ratio of 0.15. Finally, it is straightforward to show that the mean risk-free rate is decreasing in the mean of consumption growth. These observations suggest that measurement error resulting in an underestimation of the mean and variance of consumption growth could contribute to the anomalous behavior of the IMRS and the risk-free rate in the complete markets model. The magnitude of this measurement error would have to be quite large however.

Turning now from complete markets to the other extreme, Table II reports the results of not allowing individuals to trade at all. When no intertemporal trade is allowed, equilibrium is autarkic: $c_{k,t} = y_{k,t}$. Consequently, the standard deviation of consumption growth is equal to the standard deviation of income growth. Bond prices in these economies are uniquely defined by the agent who has the highest shadow price. In other words, this is just the limiting case of the economy with borrowing constraints. The agent who is on the long side of market (or desires to be) is by definition unconstrained: her first-order conditions therefore determine the market-clearing price. The pricing kernel is then defined exactly as in (14).

Table II reports moments for $\gamma = 0.50, 0.55, 0.60$, and 0.65 , with the parameter α being set at either 2 or 4. The moments reported in Table II are not of interest in and of themselves. It is quite obvious that in the limit, as intertemporal trade tends to zero, high income variability leads to large standard deviations for the pricing kernel and highly unrealistic interest rate behavior. Table II confirms this. What is of interest is the contrast between the results in Table II and those in which only limited opportunities for trade are available via bond markets with and without constraints on borrowing.

Table III reports the results of allowing for unconstrained trade in the riskless bond market. In other words, the borrowing constraint, $\hat{b}$, is chosen so that it is almost never binding (see the Appendix for details). Moments are reported for the same range of values for α and γ as were used in Table II. The central message of Table III is that the incomplete markets equilibrium is one in which individuals are able to share a substantial portion of their idiosyncratic income risk through borrowing and lending alone. Consumption

Table III
Incomplete Markets: Trade in Bonds—
Unconstrained Borrowing (Population Moments)

These data correspond to economies with trade in a single riskless bond market in which borrowing constraints are effectively nonbinding. All values are based on monthly data. "Mean" and "Std. Dev." denote mean and standard deviation, respectively. Population moments are obtained as sample averages from an extremely long (100,000 periods) simulation of the artificial data-generating process. The borrowing constraint is chosen so that it is binding in less than 0.01 percent of the time periods in the simulation. A value for b of 4.5 satisfied this criterion. The moments correspond to both individual 1 and individual 2: due to the symmetric nature of the economy, unconditional moments are equalized across agents. The mean of consumption growth is 0.142 percent for all cases. The discount factor, β , is set to 0.9983 for all cases. The parameters α and γ describe risk aversion and heterogeneity, respectively.

		Income Growth Std. Dev. (percent)	Consumption Growth Std. Dev. (percent)	Pricing Kernel		Risk-free Rate	
				Mean	Std. Dev.	Mean (percent)	Std. Dev. (annualized)
Panel A. $\alpha = 2$							
γ							
(i)	0.50	0.465	0.465	0.9955	0.009	5.506	2.479
(ii)	0.55	10.105	0.474	0.9955	0.010	5.331	2.580
(iii)	0.60	20.779	0.549	0.9955	0.011	5.328	2.648
(iv)	0.65	32.774	0.880	0.9956	0.018	5.132	3.098
Panel B. $\alpha = 4$							
γ							
(i)	0.50	0.465	0.465	0.9929	0.018	8.952	5.015
(ii)	0.55	10.105	0.474	0.9928	0.019	8.666	5.157
(iii)	0.60	20.779	0.494	0.9928	0.020	7.928	5.320
(iv)	0.65	32.774	0.528	0.9929	0.021	7.601	5.348

variability is quite small, despite a large amount of variability in incomes. For low risk aversion ($\alpha = 2$) the most extreme case is $\gamma = 0.65$, where the standard deviation of individual income growth is 32.77 percent. The standard deviation of consumption growth in this economy is only 0.88 percent. Although income variability is more than 60 times greater than in the no-heterogeneity case ($\gamma = 0.50$), consumption variability has not even doubled. Furthermore, this effect becomes more pronounced as risk aversion is increased. With $\alpha = 4$, consumption variability rises proportionally less as γ is increased than in the $\alpha = 2$ case. This result reflects the fact that as risk aversion increases the benefits attributable to risk sharing also increase.

A more precise measure of the extent to which idiosyncratic risk is priced in equilibrium is the variability of the pricing kernel. Note that since the borrowing constraint (13) is nonbinding for these economies, the pricing

kernel (which is not unique) is equal to an agent's IMRS. Table III reports that, for the $\alpha = 2$ case, the standard deviation of the kernel at most doubles relative to the complete markets case, but is still almost an order of magnitude lower than the Sharpe ratio of 0.15. Again, as risk aversion increases, the proportional increase in the variability of the pricing kernel (IMRS) diminishes. Table III also shows that the mean of the pricing kernel increases slightly as the departure from complete markets becomes larger. This reflects the fact that the mean risk-free rate diminishes slightly as γ increases (column 5 of Table III). In contrast to the representative agent model, the incomplete markets model tends to generate increased pricing kernel variability (albeit not much) while decreasing the risk-free rate. This effect will become more pronounced as borrowing constraints are introduced and markets become "more incomplete."

The final asset market structure to be examined allows for trading in bond markets with binding borrowing constraints. Table IV reports moments from economies with two different constraints. The first, referred to as static solvency, specifies the borrowing constraint to be equivalent to the terminal condition which applies in a finite lived problem $\hat{b} = \min(Q_{k,t})$. This condition ensures that an individual can liquidate her bond market position at any point in time. The second borrowing constraint is simply $\hat{b} = 0.25$, which implies that the most an individual can borrow is 1/2 of her average monthly income. The results in Table IV show that the effect of a borrowing constraint is to increase the variability of the pricing kernel and equilibrium consumption growth, and to decrease, in some cases quite drastically, the mean risk-free rate. For instance, in the low risk aversion case with extreme heterogeneity ($\gamma = 0.65$) the standard deviation of the pricing kernel is approximately seven times larger than that in the model with a representative agent. As was the case with no borrowing constraints, this effect is less pronounced when risk aversion is increased. Figure 2 illustrates the combined effect of incomplete markets and borrowing constraints on the mean and standard deviation of the pricing kernel. This graph corresponds to the economy with the static solvency borrowing constraint and low risk aversion ($\alpha = 2$).

The fact that the locus of points in Figure 2 is upward sloping indicates that, for a given level of risk aversion, the incomplete markets economy can achieve higher variability in the pricing kernel while at the same time reducing the mean risk-free rate. In this sense the model provides a potential explanation for the "risk-free rate puzzle." This effect is driven primarily by the borrowing constraint. When one agent is constrained in equilibrium, the agent on the long side of the market must be persuaded not to accumulate larger credit balances. The reduction in the rate of return, relative to an equilibrium with no constraint, accomplishes this. Another measure of the economic significance of the borrowing constraint is provided by the difference between the pricing kernel, m , and the IMRS of an agent, $\tilde{m}$. For the case $\alpha = 2$, $\gamma = 0.65$, the mean of $\tilde{m}$ is 0.9980 whereas the mean of m is 1.0040. As a loose indicator of the magnitude of this difference, suppose that

Table IV
Incomplete Markets: Trade in Bonds—
Borrowing Constraints (Population Moments)

These data correspond to economies with trade in a single riskless bond market with two different specifications for the borrowing constraint. In Panels A and B, the constraint matches that which would apply in the last date of a finite-period economy: $\hat{b} = \min(Q_{k,t})$. In Panels C and D, the borrowing constraint is set to 0.25, implying that agents cannot borrow in excess of half of their average monthly income. All values are based on monthly data. "Mean" and "Std. Dev." denote mean and standard deviation, respectively. Population moments are obtained as sample averages from an extremely long (100,000 periods) simulation of the artificial data generating process. The moments correspond to both individual 1 and individual 2: due to the symmetric nature of the economy, unconditional moments are equalized across agents. The mean of consumption growth is 0.142 percent for all cases. The discount factor, β , is set to 0.9983 for all cases. The parameters α and γ describe risk aversion and heterogeneity, respectively.

		Income Growth Std. Dev. (percent)	Consumption Growth Std. Dev. (percent)	Pricing Kernel		Risk-free Rate	
				Mean	Std. Dev.	Mean (percent)	Std. Dev. (annualized)
Panel A. $\alpha = 2$, Static Solvency							
	γ						
(i)	0.50	0.465	0.465	0.9955	0.009	5.506	2.480
(ii)	0.55	10.105	0.550	0.9957	0.011	5.344	3.688
(iii)	0.60	20.779	1.339	0.9973	0.026	3.329	13.183
(iv)	0.65	32.774	3.234	1.0040	0.062	− 4.041	35.354
Panel B. $\alpha = 4$, Static Solvency							
	γ						
(i)	0.50	0.465	0.465	0.9929	0.018	8.952	5.015
(ii)	0.55	10.105	0.521	0.9931	0.021	8.624	6.168
(iii)	0.60	20.779	1.135	0.9962	0.045	4.706	20.921
(iv)	0.65	32.774	2.557	1.0081	0.103	− 8.448	58.321
Panel C. $\alpha = 2$, 50% of Average Monthly Income							
	γ						
(i)	0.50	0.465	0.465	0.9955	0.009	5.506	2.480
(ii)	0.55	10.105	0.717	0.9961	0.014	4.836	6.161
(iii)	0.60	20.779	2.155	1.0001	0.042	0.039	23.322
(iv)	0.65	32.774	4.373	1.0100	0.085	− 10.661	51.524
Panel D. $\alpha = 4$, 50% of Average Monthly Income							
	γ						
(i)	0.50	0.465	0.465	0.9929	0.018	8.952	5.015
(ii)	0.55	10.105	0.678	0.9940	0.027	7.591	10.875
(iii)	0.60	20.779	1.776	1.0014	0.071	− 1.133	38.599
(iv)	0.65	32.774	3.499	1.0205	0.142	− 20.258	90.631

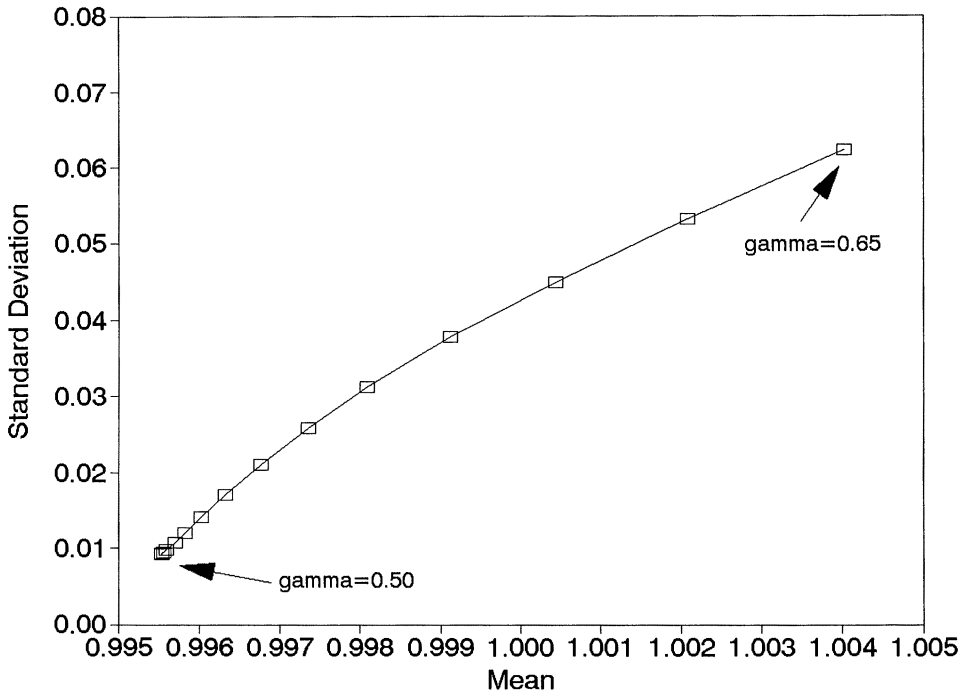


Figure 2. Mean and standard deviation of the pricing kernel with different values of the heterogeneity parameter. Each point denotes a different value for the heterogeneity parameter, gamma (γ), ranging from 0.50 (no heterogeneity) to 0.65 (extreme heterogeneity).

one mistakenly computed the mean risk-free rate based on $\tilde{m}$. The implied interest rate is roughly 2.43 percent: 6.47 percent higher than the mean of the true market-clearing rate (-4.041 percent). In economies with less heterogeneity this effect is not as pronounced. For $\gamma = 0.60$ the above difference is roughly 2 percent and for values of γ between 0.55 and 0.50 the effect is virtually zero.

Despite a substantial increase in the variability of the pricing kernel relative to the complete markets model, the main implication of these results remains that very little idiosyncratic risk is priced in equilibrium. The largest standard deviation for the pricing kernel (for $\alpha = 2$) is still roughly half the (liberally chosen) Sharpe ratio reported in Section I. This is true in spite of highly variable individual income processes and fairly restrictive constraints on borrowing. Equilibrium consumption variability also reflects the extent to which idiosyncratic risk is not priced in equilibrium. Figure 3 graphs the ratio of consumption variability to income variability for the economy with $\alpha = 2$ and the most restrictive borrowing constraint. This ratio is also plotted for the representative agent model. The small difference between the two loci indicates that a substantial portion of each agent's idiosyncratic income risk is diversifiable through trading in the bond market.

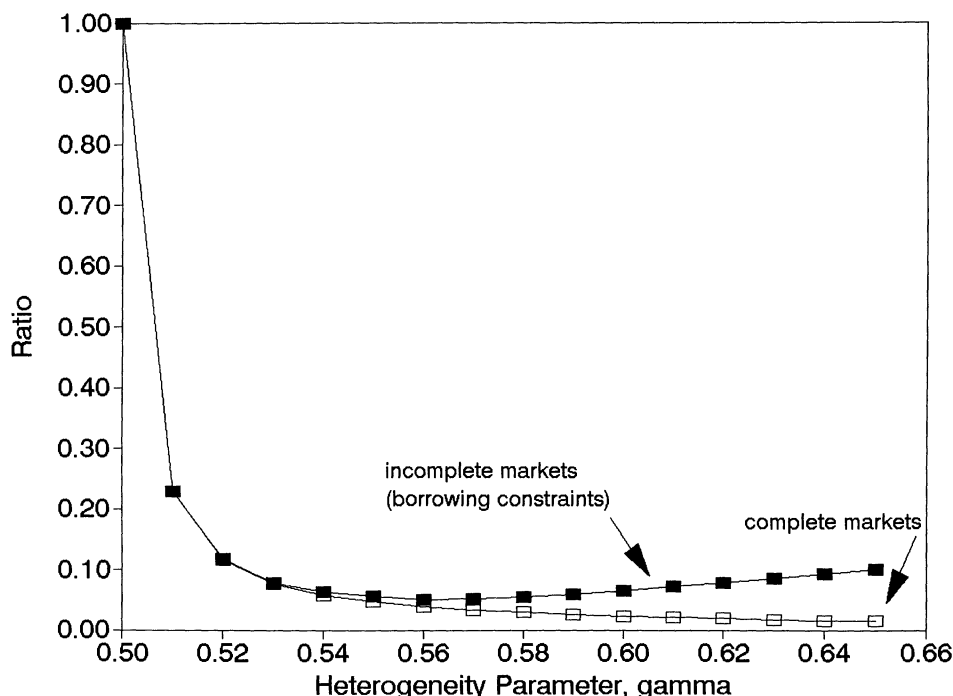


Figure 3. Ratio of the standard deviation of consumption growth to income growth with different values of the heterogeneity parameter.

V. Sensitivity to Parameter Value Assumptions

This section briefly discusses the sensitivity of the results to perturbations to the assumed values for the model's parameters. Turning first to the preference parameters, the effects of varying the risk aversion parameter, α , have already been described. Increasingly large values for α sharpen the results in that less idiosyncratic risk gets priced in equilibrium. The effects of changing β also are straightforward. The ratio of the standard deviation to the mean of the pricing kernel (equation (4)) is independent of β . The risk-free rate however, is monotonically decreasing in β . This property of the model underlies the success enjoyed by several studies in terms of accounting for the equity premium puzzle using values for β in excess of unity. The important difference here is that, in economies with a substantial amount of heterogeneity, the mean risk-free rate is actually too low instead of too high. Consequently, if one is provided the flexibility to choose values for β less than that used here ($\beta = 0.9983$), it is possible to match the U.S. sample moment for the risk-free rate, and generate substantial variation in the pricing kernel. For instance, with $\alpha = 4$, $\gamma = 0.65$, and $\hat{b} = 0.25$, a value for β of 0.9775 will generate an average risk-free rate of 0.773 percent and a

standard deviation for the pricing kernel of 0.141. Note however, that the standard deviation of the risk-free rate is highly implausible in this case (see Table IV). Furthermore, the results on the mean risk-free rate are probably quite sensitive to the assumed asset market structure. Should individuals have the opportunity to trade in alternative assets, the negative effect of the borrowing constraint on the risk-free rate is likely to be dampened substantially.

Turning to the parameters describing the endowment processes, the range of values reported for γ were comprehensive. Economies for which $\gamma > 0.65$ imply a standard deviation of income growth greater than 33 percent per month, which seems excessive. The results are not sensitive to the assumption that agents receive idiosyncratic endowment shocks with probability 1/2 (see Section II). Alternative values for this probability change relative wealth levels but not consumption growth rates, which drive the model. Variations in the parameters describing the stochastic process for aggregate consumption are equally innocuous. All second moments in the model are unaffected by perturbations to the mean and autocorrelation of aggregate consumption growth. As is the case for the representative agent model, the location of the risk-free rate is decreasing in the mean of the consumption growth process. Perturbations to the assumed standard deviation of aggregate consumption growth have exactly the same effects as those outlined in Table I for the representative agent model. Increased variability enhances the model's ability to account for variation in the pricing kernel. Several of the economies with borrowing constraints could achieve a standard deviation in excess of 0.150 if the assumed standard deviation of consumption growth were doubled. This is not the case for economies without borrowing constraints (Table III), where extreme risk aversion would still be required, even if consumption growth variability were substantially underestimated in the data set.

Finally, the effects of increasingly restrictive borrowing constraints are to increase the variability of the pricing kernel in all cases, but also to generate a mean and standard deviation for the risk-free rate that are highly implausible. This is obvious given the moments reported in Tables III and IV.

VI. Conclusions

This paper was motivated by the thought that departures from frictionless Arrow-Debreu markets may be helpful in accounting for the properties of consumption and asset returns that are anomalous within the representative agent framework. A generalization of the representative agent model was formulated which featured two types of frictions: incomplete asset markets and constraints on asset holdings. It was found that in almost every economy considered the standard deviation of the theoretical pricing kernel was insufficient in terms of satisfying a Hansen-Jagannathan bound, or Sharpe ratio, obtained from actual securities markets data. This is important since, as was shown in Section I, many anomalies that arise in the context of

dynamic asset-pricing theory may be a result of inadequate variability in pricing kernel. The properties of asset returns that are anomalous within the complete markets/representative agent model are therefore likely to remain anomalous within the incomplete markets environment.

The overall conclusion of the paper is that when markets are incomplete the equilibrium allocation of risk can be very close to first-best. While this result is limited to the particular type of market incompleteness modeled here, it is robust in a number of different ways. First, one can imagine many more sophisticated asset market structures than were used here, each of which can conceivably give rise to different equilibrium processes for prices and quantities. By only allowing for trade in bonds, the above results are likely (recall that a proof was not available) to put an upper bound on what any of these environments can achieve in terms of accounting for variability in the pricing kernel. Additional assets, such as a claim on a dividend stream (equity), will in all likelihood provide for enhanced risk-sharing opportunities, thereby reducing pricing kernel variability. The implication is that incomplete markets as modeled here cannot offer an explanation for the "equity premium puzzle."

The results are also robust to the existence of an upper bound on the number of bonds that an agent can issue. Even in the presence of fairly restrictive borrowing constraints, agents are still able to pool a great deal of idiosyncratic risk. This result, along with the findings on the behavior of interest rates, suggests possible limitations of models based on short sales, or borrowing constraints. For example, Deaton (1991) examines (in a partial equilibrium context) the consumption-savings decision of an investor facing a constant interest rate and a lower bound on the net value of her assets. He finds that if individual endowment processes are nonstationary in levels, which they are here, very little consumption smoothing occurs. The ratio of consumption variability to income variability in Deaton's model is much higher than in any case documented above, including those with restrictive borrowing constraints. This study points out that when interest rates are allowed to vary in response to changing IMRSs, the conclusions for the dynamic properties of consumption can be drastically different. Furthermore, should constraints on asset holdings have a substantial impact on consumption allocations, the model suggests that the stochastic properties of market-clearing prices may be strongly counterfactual. The general equilibrium approach emphasizes that merely endowing individuals with highly variable incomes and not allowing them to trade intertemporally may be insufficient for explaining the behavior of consumption and asset returns simultaneously.

One important caveat concerns the transitory nature of the idiosyncratic component of individual income used here. Constantinides and Duffie (1992) study a similar environment in which idiosyncratic income shocks are permanent and find that competitive equilibrium is autarkic. As a result they can generate any amount of variability in the IMRS by judiciously selecting values for the parameters that describe their income processes. Rietz (1988)

has also shown, in a representative agent context, that large permanent shocks to aggregate consumption that occur with a small probability can have a substantial effect on the conditional distribution of returns.⁷ The risk-sharing role performed by incomplete markets may therefore be quite sensitive to assumptions regarding the persistence of idiosyncratic income processes. An important question for future work involves identifying the empirical contribution to idiosyncratic income variation due to transitory and permanent components.

This paper also suggests that future research may find it beneficial, perhaps even necessary, to investigate additional sources of friction, such as transactions costs, liquidity constraints, or other market microstructure-related phenomena. Progress has already been made along these lines by authors such as Aiyagari and Gertler (1991), Grossman and Laroque (1990), Heaton and Lucas (1992), and Marshall (1990). It should prove interesting to see which of these approaches, if any, can provide a realistic explanation for the relationship between asset returns and fundamental sources of risk.

Appendix: Computational Algorithm

The algorithm developed to solve the model is a modified version of that suggested by Bertsekas (1976) and used recently by Baxter, Crucini, and Rouwenhorst (1990) to solve a version of the neoclassical growth model. An intuitive way to understand the solution procedure is that the equilibrium policy functionals for the infinite period problem are computed as the limit of a sequence of finite period functionals that are increasing in time horizon. Each element of this sequence is characterized by iterating on the system of first-order conditions that arise from individual optimization problems. This stands in contrast to the commonly used approach of defining a centralized dynamic programming problem and using the resulting Bellman equation to derive the optimal policy functionals. The invalidity of the second welfare theorem renders centralized solution methods inapplicable.

As was noted in the text, the random vector, $S_t \equiv [Q_{1,t}, b_{1,t-1}]$ (hereafter referred to as the state vector), is sufficient to describe the state of the system at any point in time. Knowledge of $Q_{1,t}$ is sufficient to infer the realization of both $Q_{2,t}$ and λ_t whereas knowledge of $b_{1,t}$ is sufficient to determine the exact value of $b_{2,t}$, due to the market-clearing condition. The task at hand is to characterize the time-invariant equilibrium policy functionals, $b_1(S_t)$ and $p(S_t)$. The bivariate system of functional equations describing an equilibrium

⁷As Mehra and Prescott (1988) note, a drawback of Rietz's representative agent analysis is that large shocks to aggregate consumption are not observed in U.S. time series data. Note however, that in a heterogeneous agent framework, it is possible for extreme events to occur at the individual level while remaining unobservable in aggregate data.

can be written as follows:

$$p(S_t) = \frac{\beta E_t[Q_{1,t+1}\lambda_{t+1} + b_1(S_t) - \lambda_{t+1}p(S_{t+1})b_1(S_{t+1})]^{-\alpha}}{[Q_{1,t} + b_{1,t-1}/\lambda_t - p(S_t)b_1(S_t)]^{-\alpha}} \quad (A1)$$

$$p(S_t) = \frac{\beta E_t[Q_{2,t+1}\lambda_{t+1} - b_1(S_t) + \lambda_{t+1}p(S_{t+1})b_1(S_{t+1})]^{-\alpha}}{[Q_{2,t} - b_{1,t-1}/\lambda_t + p(S_t)b_1(S_t)]^{-\alpha}} \quad (A2)$$

Equations (A1) and (A2) were derived by substituting the two budget constraints, (11), and the market-clearing condition, $b_{1,t} + b_{2,t} = 0$, into the Euler equations, or first-order conditions, of each agent. For the time being it is assumed that interior solutions are obtained so that equations (A1) and (A2) will hold with equality.

The first step in the algorithm is to choose numerical values for all parameters associated with tastes and technology. Any "solution" to the model is always contingent on a specific set of parameter values. Next, it is assumed that the state variables for the economy take on values in a countable, finite set. Let $\mathcal{B}$ denote the set of η points defining a uniform partition of the interval $[-\hat{b}, \hat{b}]$. The state variable $b_{1,t-1}$ is restricted to take on values in this set. The other state variable, $Q_{1,t}$, is already restricted to take on values in the set $\mathcal{S} \equiv \{1/2 \gamma (1 - \gamma)\}$. The state vector is thus assumed to lie in the space $\mathcal{S} \equiv \mathcal{B} \times \mathcal{S}$, of which a typical element will be denoted $\{s\}$. Consequently, the policy functionals, $b_1(s)$ and $p(s)$ will take the form of discrete-valued mappings: $b_1: \mathcal{S} \rightarrow \mathcal{B}$ and $p: \mathcal{S} \rightarrow \mathbb{R}_{++}$.

Next, define $b_{1,T-j}(s_{T-j})$ and $p_{T-j}(s_{T-j})$, ($j = 0, 1, 2, \dots$), as a sequence of policy functionals that correspond to a j -period economy that begins at period $T-j$ and terminates at period T . At period T debt has no value and $b_{1,T}(s_T) = p_T(s_T) = 0$. Now consider solving the problem as if it began at period $T-1$ (so that there are two model periods). The third term in the numerator of (A1) and (A2) is zero implying that every term in the system has a well defined conditional mean. As a result, the only difficult aspect of the problem becomes solving the nonlinear stochastic differential equations. This is accomplished by individually selecting each point in the state space, $\mathcal{S}$, and solving the system conditional on that point. This is obviously feasible only insofar as $\mathcal{S}$ is countable.

Given a point $s \in \mathcal{S}$, the task is to find a point in $\mathcal{B}$ and a price, $p \in \mathbb{R}_{++}$, that satisfy both (A1) and (A2). The procedure first picks an arbitrarily chosen price, say p' . Then each equation is evaluated at various points in $\mathcal{B}$, and the two points, $\mathcal{E}_1(p')$ and $\mathcal{E}_2(p')$, which minimize the absolute deviation between p' and the right-hand side of (A1) and (A2), respectively, are selected as candidate equilibrium points. This minimization can actually be done in a rather efficient manner since the function defined by the absolute deviation is monotonic in the argument $b_1(s)$. Consequently, if (A1) and (A2) are evaluated sequentially at points on the partition, any point that increases the value of the objective function (after more than two points have been examined) identifies a minimum.

The next step is to check if $\mathcal{E}_1(p') = \mathcal{E}_2(p')$. If so, p' and $\mathcal{E}_1(p')$ represent a two-period equilibrium for $s \in \mathcal{S}$. If not, another monotonicity property of the economy can be exploited. The difference, $\mathcal{E}_1(p') - \mathcal{E}_2(p')$, can loosely be interpreted as an excess demand function. This function is monotonically decreasing in the bond price. Consequently, if $\mathcal{E}_1(p') > \mathcal{E}_2(p')$ ($\mathcal{E}_1(p') < \mathcal{E}_2(p')$), then there exists another price, say p'' , that is greater than (less than) p' , which results in $\mathcal{E}_1(p'') < \mathcal{E}_2(p'')$ ($\mathcal{E}_1(p'') > \mathcal{E}_2(p'')$). The equilibrium price, p^* , will lie between p'' and p' . This is where the "linear interpolation" aspect of the algorithm arises. The price, $\bar{p}$, is computed as that which would result in $\mathcal{E}_1 = \mathcal{E}_2$ should the "excess demand" function be linear. Having done so, $\mathcal{E}_1(\bar{p})$ and $\mathcal{E}_2(\bar{p})$ are computed and the algorithm checks for an equilibrium: $\mathcal{E}_1(\bar{p}) = \mathcal{E}_2(\bar{p})$. If $\mathcal{E}_1(\bar{p}) \neq \mathcal{E}_2(\bar{p})$ (the excess demand function is typically not linear) the procedure sets $p' = \bar{p}$ and continues until p^* is found. The pair, $(p^*, \mathcal{E}_1(p^*))$, represents a two-period equilibrium for $s \in \mathcal{S}$.

The policy functionals $p_{T-1}(s_{T-1})$ and $b_{1,T-1}(s_{T-1})$ are well defined once the above procedure has been carried out for each point in $\mathcal{S}$. Consider now the problem as if it began at period $T - 2$ (so that there are three model periods). The crucial thing to note is that the conditional mean of the third term in the numerator of (A1) and (A2) can now be computed. These terms are functions of $p_{T-1}(s_{T-1})$ and $b_{1,T-1}(s_{T-1})$, where $s_{T-1} = [Q_{1,T-1}b_{1,T-2}]$. The first element in this vector has a well-defined conditional distribution, given by the joint Markovian transition matrix for the idiosyncratic and aggregate income processes, and the second term is in the period $T - 2$ information set. Calculating the conditional mean of the third terms in (A1) and (A2) simply involves adding up a finite number of terms, each weighted by a conditional probability measure.

The period $T - 2$ component of the algorithm therefore consists of substituting the policy functionals from the $T - 1$ iteration into (A1) and (A2), and proceeding as was outlined above: for each point in $\mathcal{S}$ the functionals $p_{T-2}(s_{T-2})$ and $b_{1,T-2}(s_{T-2})$ are computed using the linear interpolation method. Upon completion, the algorithm checks the following convergence criteria: $b_{1,T-2}(s_{T-2}) = b_{1,T-1}(s_{T-1})$ and $p_{T-2}(s_{T-2}) = p_{T-1}(s_{T-1})$. Should these relations be satisfied, the finite-period policy functionals will have converged to the time-invariant functionals corresponding to the infinite-period problem. The algorithm then terminates. The exactness of the criteria is a result of the discrete-valued nature of the functionals. Should the convergence criterion not be satisfied, the procedure continues by solving the $T - j$, ($j = 3$) period problem, and so on. The value of j required to achieve convergence is typically greater than 100.

Until now we have assumed that (A1) and (A2) hold with equality. The following modification incorporates states in which the borrowing constraint is binding. At the end of each iteration (or the end of each j -period problem), the algorithm will associate with each point in the state space one of two alternatives: either an interior solution or a *potential* corner solution. The former does not present a problem: an interior point has been identified in

which both (A1) and (A2) are satisfied with equality. The latter defines a candidate point for which the borrowing constraint may be binding in equilibrium. The algorithm temporarily assumes that this is the case. A shadow price is then computed (exactly) as that at which the unconstrained agent is content to hold the maximum number of bonds, $\hat{b}$. Next, the algorithm doubles back and checks whether or not, at this new price, equilibrium is actually associated with an interior outcome. If so, the interior outcome is characterized, the period- j policy functionals are updated, and the procedure continues on to the $j + 1$ th iteration. If not, the value of the pricing functional for the state in question is set equal to the shadow price, the trading rule maintains that the constraint is binding, and the algorithm subsequently proceeds.

Several additional technical issues are as follows. The borrowing constraint for the economies in Table III is chosen so that the unconditional probability of it binding is less than 0.01 percent. In addition, these economies are subject to a solvency issue: in the sequence of finite-period economies that are used to obtain approximate solutions, there exist states in which an investor is insolvent in the final period of life. This problem is addressed by using the equilibrium policy functionals from the static-solvency solutions as starting values. The number of grid points, η , used in the partition for the trading rule is chosen to satisfy an accuracy criterion. This number varies according to how restrictive the borrowing constraint is specified. For the static-solvency economies, 150 grid points per interval of length 0.10 turned out to be sufficient. The accuracy criterion involves computing the state-contingent conditional mean of the pricing kernel (which is feasible since the algorithm yields the discrete state Markovian transition matrix) and comparing it to the state-contingent bond price. Because of the number of points in the state space (over 3000 in all cases) the accuracy criterion uses summary statistics from these state-contingent approximation errors. The criterion is that the number of grid points must be large enough so that a two standard deviation interval around the mean (which should be very close to zero) does not admit an interest rate error of greater than 0.0001 percent. Additional details are available in Telmer (1991). Finally, the computer program written to implement the algorithm is written in FORTRAN and is available from the author. Solutions for most economies were obtained on a 486-25 personal computer in less than 30 minutes.

REFERENCES

- Abel, Andrew, 1990, Asset prices under habit formation and catching up with the Joneses, *American Economic Review* 80, 38-42.
- Aiyagari, S. Rao, and Mark Gertler, 1990, Asset returns with transactions costs and uninsured individual risk, *Journal of Monetary Economics* 27, 311-331.
- Atkeson, Andrew, and Robert E. Lucas, Jr., 1992, On efficient distribution with private information, *Review of Economic Studies* 59, 427-453.
- Backus, David K., Allan W. Gregory, and Chris I. Telmer, 1993, Accounting for forward rates in markets for foreign currency, *Journal of Finance* 48, 1887-1908.

- Backus, David K., Allan W. Gregory, and Stanley E. Zin, 1989, Risk premiums in the term structure—evidence from artificial economies, *Journal of Monetary Economics* 24, 371–399.
- Baxter, Marianne, Mario J. Crucini, and K. Geert Rouwenhorst, 1990, Solving the stochastic growth model by a discrete state space Euler equation approach, *Journal of Business and Economic Statistics* 8, 19–22.
- Bekaert, Geert, and Robert J. Hodrick, 1992, Characterizing predictable components in excess returns on equity and foreign exchange markets, *Journal of Finance* 47, 467–509.
- Benninga, Simon, and Aris Protopapadakis, 1990, Leverage, time preference and the 'equity premium puzzle', *Journal of Monetary Economics* 25, 49–58.
- Bertsekas, D. P., 1976, *Dynamic Programming and Stochastic Control* (Academic Press, New York).
- Breen, William, Lawrence R. Glosten, and Ravi Jagannathan, 1989, Economic significance of predictable variations in stock index returns, *Journal of Finance* 44, 1177–1190.
- Cecchetti, Stephen G., Pok-sang Lam, and Nelson C. Mark, 1990, The equity premium and the riskfree rate: Matching the moments, Mimeo, Ohio State University.
- Cochrane, John H., 1991, A simple test of consumption insurance, *Journal of Political Economy* 99, 957–976.
- Constantinides, George M., 1982, Intertemporal asset pricing with heterogeneous consumers and without demand aggregation, *Journal of Business* 55, 253–267.
- , 1990, Habit formation: A resolution of the equity premium puzzle, *Journal of Political Economy* 98, 519–543.
- and Darrell Duffie, 1992, Asset pricing with heterogeneous consumers, Mimeo, University of Chicago.
- Deaton, Angus, 1991, Saving and liquidity constraints, *Econometrica* 59, 1221–1248.
- Duffie, Darrell, 1990, The nature of incomplete security markets, Mimeo, Stanford University.
- Dunn, Kenneth, and Kenneth J. Singleton, 1986, Modelling the term structure of interest rates under nonseparable utility and durability of goods, *Financial Economics* 17, 27–55.
- Epstein, Larry G., and Stanley E. Zin, 1989, Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.
- Flavin, M., 1981, The adjustment of consumption to changing expectations about future income, *Journal of Political Economy* 89, 974–1009.
- Gallant, A. R., Lars P. Hansen, and George Tauchen, 1990, Using conditional moments of asset payoffs to infer the volatility of intertemporal marginal rates of substitution, *Journal of Econometrics* 45, 141–179.
- Geanakoplos, John, 1990, An introduction to general equilibrium with incomplete asset markets, *Journal of Mathematical Economics* 19, 1–38.
- Green, Edward J., 1987, Lending and the smoothing of uninsurable income, in Edward C. Prescott and Neil Wallace, eds.: *Contractual Arrangements for Intertemporal Trade* (University of Minnesota Press, Minneapolis), pp. 3–25.
- Grossman, Sanford J., and Guy Laroque, 1990, Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption goods, *Econometrica* 58, 25–51.
- Grossman, Sanford J., Angelo Melino, and Robert J. Shiller, 1987, Estimating the continuous time consumption based asset pricing model, *Journal of Business and Economic Statistics* 5, 315–327.
- Hansen, Lars P., and Ravi Jagannathan, 1991, Implications of security market data for models of dynamic economies, *Journal of Political Economy* 99, 225–262.
- Harrison, J. Michael, and David M. Kreps, 1979, Martingales and arbitrage in multiperiod securities markets, *Journal of Economic Theory* 20, 381–408.
- Heaton, John, 1992, The interaction between time-nonseparable preferences and time aggregation, *Econometrica*, Forthcoming.
- and Deborah J. Lucas, 1992, Evaluating the effects of incomplete markets on risk sharing and asset prices, Mimeo, MIT.
- Hodrick, Robert J., 1987, *The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets* (Harwood Academic Publishers, New York).

- Huang, Chi-fu, and Robert H. Litzenberger, 1988, *Foundations for Financial Economics* (North Holland, Amsterdam).
- Huggett, Mark, 1989, The risk-free rate in heterogeneous agent economies, Mimeo, University of Minnesota.
- Kahn, James A., 1990, Moral hazard, imperfect risk sharing, and the behavior of asset returns, *Journal of Monetary Economics* 26, 27–44.
- Kandel, Shmuel, and Robert F. Stambaugh, 1990, Expectations and volatility of consumption and asset returns, *Review of Financial Studies* 3, 207–232.
- Kocherlakota, Narayana R., 1990, On the 'discount factor' in growth economies, *Journal of Monetary Economics* 25, 43–47.
- Labadie, Pamela, 1989, Stochastic inflation and the equity premium, *Journal of Monetary Economics* 24, 277–298.
- Lucas, Deborah J., 1990, Estimating the equity premium with undiversifiable income risk and short sales constraints, Mimeo, Northwestern University.
- Lucas, Robert E., Jr., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–1445.
- Mace, Barbara J., 1991, Full insurance in the presence of aggregate uncertainty, *Journal of Political Economy* 99, 928–956.
- Mankiw, N. Gregory, 1986, The equity premium and the concentration of aggregate shocks, *Journal of Financial Economics* 17, 211–219.
- and Stephen P. Zeldes, 1991, The consumption of stockholders and non-stockholders, *Journal of Financial Economics* 29, 97–112.
- Marcet, Albert, and Kenneth J. Singleton, 1990, Optimal consumption savings decisions and equilibrium asset prices in a model with heterogeneous agents subject to short-sales constraints, Mimeo, Stanford University.
- Marshall, David A., 1990, Asset pricing anomalies with very small costs of consumption adjustment, Mimeo, Northwestern University.
- Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–161.
- , 1988, The equity premium puzzle: A solution?, *Journal of Monetary Economics* 22, 133–136.
- Nason, James, 1988, The equity premium and time-varying risk behavior, Mimeo, Board of Governors of the Federal Reserve System.
- Phelan, Christopher, and Robert M. Townsend, 1991, Computing multi-period, information constrained optima, *Review of Economic Studies* 58, 853–881.
- Rietz, Thomas A., 1988, The equity premium puzzle: A solution, *Journal of Monetary Economics* 22, 117–131.
- Ross, Stephen A., 1976, The arbitrage theory of capital asset pricing, *Journal of Economic Theory* 13, 341–360.
- Rouwenhorst, K. Geert, 1989, Asset returns and business cycles: A general equilibrium approach, Mimeo, University of Rochester.
- Shiller, Robert J., 1982, Consumption, asset markets, and macroeconomic fluctuations, *Carnegie-Rochester Conference Series on Public Policy* 17, 203–238.
- Taylor, John B. and Harald Uhlig, 1990, Solving nonlinear stochastic growth models: A comparison of alternative methods, *Journal of Business and Economic Statistics* 8, 1–18.
- Telmer, Chris I., 1991, Asset pricing puzzles and incomplete markets, Working paper no. 206, Queen's University.
- Townsend, Robert M., 1982, Optimal multiperiod contracts and the gain from enduring relationships under private information, *Journal of Political Economy* 90, 1166–1186.
- Weil, Philippe, 1989, The equity premium puzzle and the riskfree rate puzzle, *Journal of Monetary Economics* 24, 401–421.
- , 1990, Equilibrium asset prices with undiversifiable labor income risk, Discussion paper no. 1507, Harvard University.
- Zeldes, S., 1989a, Consumption and liquidity constraints: An empirical investigation, *Journal of Political Economy* 97, 305–346.
- , 1989b, Optimal consumption with stochastic income: Deviations from certainty equivalence, *Quarterly Journal of Economics* 104, 275–298.