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Short Selling and Efficient Sets

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ABSTRACT

The effect of short selling on the composition and location of the efficient set has been analyzed in a variety of ways. However, the situation typically facing investors where the initial margin requirement is less than 100 percent and the riskfree interest rate that is paid on the short proceeds is less than the rate paid on initial margin has not previously been considered. The Elton-Gruber-Padberg algorithm (1976, 1978), subject to certain modifications, is shown here to be capable of identifying the efficient set under such conditions.

WHEN HARRY MARKOWITZ (1952) developed the notion of efficient sets as a method of identifying an investor's optimal portfolio, he did so in a setting where short selling was prohibited. Since then the effect of short selling on the composition and location of the efficient set has been considered in a variety of ways. For example, Lintner (1965) suggested that short sales be treated from the perspective of a broker-dealer where an amount of money equal to the short proceeds must be put up as initial margin with the short seller receiving riskfree interest on both the short proceeds and the margin. Alternatively, Black (1972) and Merton (1972) assumed that the short seller did not have to put up any initial margin and that the short seller actually got the use of the short proceeds. More recently, Dyl (1975) has assumed that the short seller must put up initial margin that is less than the short proceeds, but receives no interest on either the margin or short proceeds.

This paper is concerned with the generation of efficient sets where short selling is allowed but with certain restrictions attached to it, referred to hereafter as restricted short selling. In particular, initial margin is required but can be met by depositing interest-bearing securities with the brokerage firm where the short seller's account is kept. Furthermore, the model allows the short seller to earn interest on the short proceeds. Accordingly, the model is more realistic than those previously examined.[1]

The approach taken builds on the market model-based algorithm of Elton, Gruber, and Padberg (1976, 1978, 1991). This algorithm has been shown to be capable of quickly identifying the efficient set when there is either (1) a given

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[1] For a description of short selling procedures, see Markowitz (1983, 1987), Price (1989), Weiss (1991), and Sharpe (1991, p. 500).

riskfree borrowing and lending rate, or (2) a riskfree borrowing rate that is higher than the riskfree lending rate, or (3) only a riskfree lending rate. It will be shown here that this algorithm, subject to certain modifications, can also be used in all three of these situations when restricted short selling is permitted.

The rest of the paper is organized as follows. Section I discusses short selling in terms of mean-variance analysis, while Section II discusses the Elton-Gruber-Padberg algorithm and how it can be used to identify the efficient set associated with a riskfree borrowing and lending rate when restricted short selling is allowed. Section III expands the analysis to consider cases where riskfree borrowing is either not allowed or is allowed but at a rate that is greater than the riskfree lending rate. Lastly, Section IV presents the conclusion.

I. Short Selling

When an investor short sells a share of stock, the investor must deposit cash or securities earning the riskfree rate r_{f1} in his or her account at a brokerage firm in order to meet an initial margin requirement.[2] Here the amount deposited must be at least equal to the proportion of m (where $m \leq 1$) of the sales proceeds. Furthermore, the sales proceeds are deposited into the investor's account and thereafter provide the investor with interest at the riskfree rate r_{f2} where $0 \leq r_{f2} \leq r_{f1}$.[3] Lastly, if the firm whose shares were sold short subsequently declares a cash dividend, then the short seller must pay the brokerage firm an amount equal to the per share cash dividend times the number of shares sold short. Hence the return to the short seller of firm i, r_{iS}, is equal to:

$$r_{iS} = (P_{i0} - P_{i1} - D_i)/mP_{i0} + r_{f1}mP_{i0}/mP_{i0} + r_{f2}P_{i0}/mP_{i0} \qquad (1a)$$

$$= -r_{iL}/m + r_{f1} + r_{f2}/m \qquad (1b)$$

where P_{i0} is the beginning-of-period selling price, P_{i1} is the end-of-period buying price, D_i is the cash dividend paid during the period, and r_{iL} is the return that the investor would earn from a long position in the stock of firm i. It is this model that is the focus of this paper, as it accurately describes the situation faced by many investors.[4]

[2] An investor who already has a brokerage account that is invested in a portfolio can short sell stock (up to a limit) without having to deposit anything in the account. By letting r_{f1} also be the riskfree borrowing rate, the model to be presented is capable of accommodating such a situation.

[3] Weiss (1991) reports that typically the interest earned on the short proceeds is less than the interest earned on the initial margin deposit.

[4] The model of Dyl (1975) is captured by equations (1a) and (1b) by letting $r_{f1} = r_{f2} = 0$. Similarly, the case where riskfree interest is earned on the initial margin deposit but not on the short proceeds (a situation that is faced by most small investors, as reported by Weiss (1991)) can be considered by letting $r_{f2} = 0$.

From equation (1b) it follows that the expected return from short selling a share of stock is equal to:

$$E(r_{iS}) = -E(r_{iL})/m + r_{f1} + r_{f2}/m. \tag{2}$$

Note how a stock that has a positive expected return if held long can also have an expected return that is positive if it is sold short due to the interest earned on the initial margin and the leveraged short proceeds.

Again using equation (1b), the variance of returns associated with short selling a share of stock i is equal to:

$$\sigma_{iS}^2 = \sigma_{iL}^2/m^2. \tag{3}$$

Since m is typically less than one, the variance associated with short selling a stock is higher than that associated with a long position in the stock.

When thinking about the covariances associated with short selling, there are three cases that need to be examined. First, consider the covariance of returns between a short position in one stock and a long position in a different stock:

$$\sigma_{iS,jL} = -\sigma_{iL,jL}/m \text{ where } i \neq j. \tag{4}$$

Since most stocks have positive covariances with each other, short selling creates a set of negative covariances that have potential risk reduction benefits when constructing a portfolio. This suggests that in the context of a portfolio's risk, short selling may not be as risky as one might think when looking at just the variance of a short position as in equation (3).

Second, consider the covariance of returns between a short and long position in the same stock. In this case, i can be set equal to j in equation (4), resulting in:

$$\sigma_{iS,iL} = -\sigma_{iL}^2/m. \tag{5}$$

Intuitively, this covariance is negative because the correlation between a long and short position in the same stock is -1.

Third, consider the covariance between a short position in one stock and a short position in a different stock:

$$\sigma_{iS,jS} = \sigma_{iL,jL}/m^2 \text{ where } i \neq j. \tag{6}$$

In this case, the covariance of the short positions in the two stocks is larger than the covariance of the long positions as long as $m < 1$. Hence, a portfolio of just short positions in several stocks will have a substantially larger variance than a similar portfolio involving just long unmargined positions in the same stocks. The reason for this is the leverage associated with the short positions as shown in equations (3) and (6), since if $m = 1$, the two portfolios would have the same variance.

II. The Elton-Gruber-Padberg Algorithm

In 1976 Elton, Gruber, and Padberg developed an algorithm for determining the composition of the tangency portfolio p associated with a given riskfree borrowing and lending rate. Based on the assumption that security returns can be accurately described by the market model and that a riskfree borrowing and lending rate r_{f1} exists, it seeks to maximize the following objective function:

$$\underset{\{X_i\}}{\text{MAX}}\,\Theta = \frac{E(r_p) - r_{f1}}{\sigma_p}. \tag{7}$$

Hence, the algorithm seeks out the portfolio with the highest expected excess return-to-standard deviation ratio. In doing so it utilizes the following relationships arising from the market model:

$$\sigma_i^2 = \beta_i^2\sigma_M^2 + \sigma_{\epsilon i}^2 \tag{8a}$$

$$\sigma_{i,j} = \beta_i\,\beta_j\sigma_M^2 \qquad i \neq j \tag{8b}$$

where β_i and $\sigma_{\epsilon i}^2$ denote the systematic and unsystematic risk of stock i, respectively, and σ_M^2 denotes the variance of the market portfolio. This allows the objective function to be restated as:

$$\underset{\{X_i\}}{\text{MAX}}\,\Theta = \frac{\sum_{i=1}^{N} X_i\left[E(r_i) - r_{f1}\right]}{\left[\sum_{i=1}^{N} X_i^2\beta_i^2\sigma_M^2 + \sum_{i=1}^{N}\sum_{\substack{j=1\\ i\neq j}}^{N} X_iX_j\beta_i\,\beta_j\sigma_M^2 + \sum_{i=1}^{N} X_i^2\sigma_{\epsilon i}^2\right]^{1/2}}. \tag{9}$$

As can be seen from this objective function, the parameters $E(r_i)$, β_i, and $\sigma_{\epsilon i}^2$ must be estimated for each one of the N risky securities. In addition, the market parameters σ_M^2 and r_{f1} must be estimated.

A. Lintnerian Short Sales

When Lintnerian short selling is allowed, Elton, Gruber, and Padberg show that the objective function can be solved by determining the values of Z_i in the following equation:

$$Z_i = \frac{\beta_i}{\sigma_{\epsilon i}^2}\left[\frac{E(r_i) - r_{f1}}{\beta_i} - \phi_N\right] \tag{10}$$

where

$$\phi_N = \sigma_M^2\frac{\sum_{i=1}^{N}\frac{E(r_j) - r_{f1}}{\sigma_{\epsilon j}^2}\beta_j}{1 + \sigma_M^2\sum_{i=1}^{N}\frac{\beta_i^2}{\sigma_{\epsilon i}^2}}. \tag{11}$$

Once the values of Z_i have been determined for $i = 1,\ldots,N$, the weights X_i

(negative weights denote short selling) are calculated as follows:

$$X_i = Z_i / \sum_{i=1}^{N} |Z_i|. \tag{12}$$

B. No Short Selling

Elton, Gruber, and Padberg also indicate how their algorithm needs to be modified when short selling is prohibited. Since the objective function given in (9) is now supplemented by nonnegativity constraints, Kuhn-Tucker conditions must be employed. Assuming the stocks have been ordered so that the first k stocks have nonzero weights in the optimal solution, equation (10) becomes:

$$Z_i = \frac{\beta_i}{\sigma_{\epsilon i}^2}\left[\frac{E(r_i) - r_{f1}}{\beta_i} - \phi_k\right] + \mu_i \tag{13}$$

where

$$\phi_k = \sigma_M^2 \frac{\sum_{i=1}^{k} \frac{E(r_i) - r_{f1}}{\sigma_{\epsilon i}^2}\beta_i}{1 + \sigma_M^2 \sum_{i=1}^{k} \frac{\beta_i^2}{\sigma_{\epsilon i}^2}}. \tag{14}$$

In equations (13) and (14), k is set so that $[E(r_i) - r_{f1}]/\beta_i > \phi_k$ for $i = 1, \ldots, k$, and $[E(r_i) - r_{f1}]/\beta_i < \phi_k$ for $i = k+1, \ldots, N$. Hence securities $i = 1, \ldots, k$ have positive weights while securities $i = k+1, \ldots, N$ have weights of zero in the solution. The algorithm identifies securities 1 through k by ranking the securities in descending order by the magnitude of $[E(r_i) - r_{f1}]/\beta_i$, setting k equal to one, and then increasing k one by one as securities are added from this ranking. The final set k is determined as soon as it is noted that the addition of the next security in the ranking would result in a situation where $[E(r_i) - r_{f1}]/\beta_i$ is less than ϕ_k.[5]

C. Restricted Short Selling

When restricted short selling is allowed, it is not possible to simply let short sales be denoted by negative weights as was done for Lintnerian short sales. This is because the associated margin constraint cannot be treated at an aggregate portfolio level. Consider the expected excess return on a portfolio where restricted short selling is allowed but is handled at the portfolio level like Lintner. Such a return can be expressed as follows:

$$\begin{aligned} E(r_p) - r_{f1} = & \sum_{i=1}^{N} X_i\left[E(r_i) - r_{f1}\right] \\ & + \left\{\sum_{i \epsilon S} X_i\left[E(r_i)(1-m) + m r_{f1} - r_{f2}\right]\right\}/m \end{aligned} \tag{15}$$

[5] This description assumes that all the securities have positive betas. For an alternative procedure, see Kwan (1984) and Cheung and Kwan (1988).

where S denotes the set of securities that are sold short and X_i can be either positive (for a long position) or negative (for a short position). Solving equation (7) with the right-hand side of this expression in the numerator involves having to decide simultaneously which securities are in S as well as what the values of X_i are for $i = 1, \ldots, N$.[6]

Alternatively, the imposition of a margin constraint with restricted short selling can be handled at the individual security level, as it avoids having to decide which securities are in S. Hence, the Elton, Gruber, and Padberg (EGP) algorithm can still be used to find the solution, provided some modifications are made. All that needs to be done is to take the set of N risky securities and artificially create a second set of N securities that are equivalent to short selling the first set of N securities. Specifically, the estimates of $E(r_i)$, β_i, and $\sigma_{\epsilon i}^2$ for $i = 1, \ldots, N$ are modified for $i = N + 1, \ldots, 2N$ as follows:

$$E(r_{N+i}) = -E(r_i)/m + r_{f1} + r_{f2}/m \tag{16}$$

$$\beta_{N+i} = -\beta_i/m \tag{17}$$

$$\sigma_{\epsilon N+i}^2 = \sigma_{\epsilon i}^2/m^2. \tag{18}$$

The EGP no-short-selling algorithm is then applied to the resulting set of $2N$ securities. However, before doing so a potential problem must be overcome. Note that this algorithm makes use of, inter alia, the assumption that the unsystematic risks of securities i and j are uncorrelated. This assumption remains valid when short selling is introduced by creating a second set of N securities except for when i and j involve going long and short the same security, a situation that will occur when $j = N + i$. When this happens,

$$\sigma_{i,N+i} = \beta_i \beta_{N+i} \sigma_M^2 - \sigma_{\epsilon i}^2/m. \tag{19}$$

What this means is that the denominator σ_p of the objective function given in (9) must be modified to be:

$$\sigma_p = \left[\sum_{i=1}^{2N} X_i^2 \beta_i^2 \sigma_M^2 + \sum_{i=1}^{2N} \sum_{\substack{j=1 \\ i \neq j}}^{2N} X_i X_j \beta_i \beta_j \sigma_M^2 + \sum_{i=1}^{2N} X_i^2 \sigma_{\epsilon i}^2 - 2 \sum_{i=1}^{N} X_i X_{N+i} \sigma_{\epsilon i}^2/m \right]^{1/2} \tag{20}$$

which in turn means that equations (13) and (14) are no longer correct.

This problem can be overcome by first showing that the tangency portfolio would never involve both a long and short position in the same security. As mentioned in Elton, Gruber, and Padberg (1977, p. 334), the Kuhn-Tucker

[6] Additional complications arise when adjustments are made to the denominator of equation (7) in order to treat the margin constraint at the aggregate portfolio level.

conditions can be written as:

$$Z_i \sigma_i^2 + \sum_{\substack{j=1 \\ j \neq i}}^{2N} Z_j \sigma_{i,j} = E(r_i) - r_{f1} + \mu_i \qquad i = 1, \ldots, 2N. \tag{21}$$

This equation can be rewritten as:

$$Z_i \sigma_i^2 + \sum_{\substack{j=1 \\ j \neq i}}^{N} Z_j \sigma_{i,j} + \sum_{\substack{j=N+1 \\ j \neq N+i}}^{2N} Z_j \sigma_{i,j} + Z_{N+i} \sigma_{i,N+i} = E(r_i) - r_{f1} + \mu_i \tag{22}$$

where i is one of the first N securities, meaning it represents a long position in security i. Similarly, let $N + i$ be one of the second N securities representing a short position in security i. The Kuhn-Tucker condition given in equation (21) for this artificial security can be written as:

$$Z_{N+i} \sigma_{N+i}^2 + \sum_{\substack{j=1 \\ j \neq i}}^{N} Z_j \sigma_{N+i,j} + Z_i \sigma_{N+i,i} + \sum_{\substack{j=N+1 \\ j \neq N+i}}^{2N} Z_j \sigma_{N+i,j}$$
$$= E(r_{N+i}) - r_{f1} + \mu_{N+i}. \tag{23}$$

Rewriting equation (23) by substituting equations (2) through (6) in the appropriate places and then summing the resulting equation with equation (22) produces:

$$0 = r_{f2} - r_{f1} + \mu_i + \mu_{N+i} \tag{24}$$

or:

$$\mu_i + \mu_{N+i} = r_{f1} - r_{f2}. \tag{25}$$

Now if the tangency portfolio involves a long position in i, then $\mu_i = 0$. Hence, $\mu_{N+i} = r_{f1} - r_{f2} > 0$, indicating that $Z_{N+i} = 0$ since the optimal solution requires $Z_i u_i = 0$ for $i = 1, \ldots, 2N$. Thus, the tangency portfolio cannot involve both long and short positions in the same security.[7]

Next, consider deleting the term $-\sigma_{\epsilon i}^2/m$ in equations (19) and (20). This is equivalent to assuming that equation (8b) can be used to denote the covariance of returns between any two securities, even when they represent long and short positions in the same security. Deliberately misstating this relationship allows the EGP algorithm to still be used, provided it can be shown that it will never produce a solution involving long and short positions in the same security. This can be seen by examining equations (13) and (14).

Imagine a long position in security i is in the set of securities in the optimal portfolio. That is, assume $Z_i > 0$ (and hence $\mu_i = 0$). Given that

[7] Following Sharpe (1991), all the securities that are "in" the optimal portfolio have the same marginal utility while each one of those that are "out" have smaller marginal utility. Hence, an intuitive interpretation of why a security cannot be held both long and short in the investor's optimal portfolio is simply because the long and short positions cannot have the same level of marginal utility.

$\beta_i > 0$, the term $\beta_i/\sigma_{\epsilon i}^2$ is positive.[8] Hence the term $\{[E(r_i) - r_{f1}]/\beta_i - \phi_k\}$ must also be positive. In considering a short position in i, note that because $\beta_{N+i} < 0$, the term $\beta_{N+i}/\sigma_{\epsilon N+i}^2$ is negative. Simplifying the term $\{[E(r_{N+i}) - r_{f1}]/\beta_{N+i} - \phi_k\}$ results in the expression $\{[E(r_i) - r_{f2}]/\beta_i - \phi_k\}$, which is positive since it is larger than the positive quantity $\{[E(r_i) - r_{f1}]/\beta_i - \phi_k\}$. However, this means that $\mu_{N+i} > 0$, indicating that a short position in i cannot be in the optimal solution. Accordingly, security i cannot be held both long and short in the optimal solution.

In summary, the EGP algorithm can be applied to the set of $2N$ securities in routine fashion. Even though it involves a misspecification of the covariance of returns between long and short positions in the same security, this is of no consequence because the algorithm will never produce a solution where long and short positions are taken in the same security.

III. Riskfree Borrowing

So far it has been assumed that the riskfree rate r_{f1} was available for both lending and borrowing. This might not be the case in that either riskfree borrowing is available but at a higher rate than r_{f1} or is unavailable.[9] In the first case the efficient set consists of a linear segment from r_{f1} to a tangency portfolio T_L, then a concave segment from T_L to a second tangency portfolio T_B that corresponds with the riskfree borrowing rate r_{fB}, and then a linear segment extending outward from T_B. The second case is the same as the first except that the second linear segment does not exist; instead the concave segment extends from T_L to the maximum expected return security.

In the case of restricted short selling with $r_{fB} > r_{f1}$, the second tangency portfolio T_B can be identified by simply replacing r_{f1} in equations (13) and (14) with r_{fB}, the riskfree borrowing rate and then solving for $Z_i, i = 1, \ldots, 2N$. The other riskfree rates in existence—r_{f1} and r_{f2}—are utilized only in equation (2) where the expected returns for "securities" $N + 1$ through

[8] Since almost all securities have positive betas, the requirement that β_i be nonnegative is not an unrealistic one. See, for example, Blume (1971).

[9] Not considered here is the case where riskfree borrowing is allowed but is subject to a margin constraint at the individual security level instead of at the aggregate portfolio level. With margin constrained at the portfolio level (meaning an investor having a portfolio worth D dollars and facing a margin requirement of m can borrow $(D - mD)/m$ dollars regardless of what the portfolio consists of in terms of long and short positions), the efficient set will be a transformation of the efficient set when riskfree borrowing is unavailable (see Fama (1976), pp. 295–298). However, if margin for borrowing is constrained at the individual security level (meaning that the previously described investor would only be able to borrow $(D_L - mD_L)/m$ dollars where D_L denotes the market value of the long positions in the portfolio), then the efficient set where riskfree borrowing is allowed might bear no resemblance to the efficient set where riskfree borrowing is prohibited. Hence, one might not simply be a transformation of the other.

For example, imagine that all the efficient portfolios when riskfree borrowing is unavailable involve only short positions. By allowing riskfree borrowing with margin requirements based on long positions, the new efficient set might be a transformation of a set of previously inefficient portfolios.

$2N$ are determined. Portfolios between T_L and T_B can be identified by sequentially decreasing the riskfree rate in equations (13) and (14) from r_{fB} by arbitrarily larger and larger fractions of the amount $r_{fB} - r_{f1}$.[10] Similarly, if riskfree borrowing is not allowed, then the riskfree rate r_{f1} is used in equations (13) and (14) to determine T_L. Thereafter r_{f1} is sequentially increased by arbitrarily larger and larger fractions of the amount $E(r_{\max}) - r_{f1}$ (where $E(r_{\max})$ denotes the maximum expected return of any "security" from 1 through $2N$) in equations (13) and (14)—but is kept at its original level in equation (2).[11]

Interestingly, it is now possible for an investor to rationally borrow money while at the same time lend money risklessly at a lower rate. This is because the tangency portfolio T_B may involve short selling, which in turn requires the investor to purchase riskfree securities having a return of r_{f1}. Hence, any efficient portfolio that involves leveraging T_B will involve borrowing at r_{fB} and lending at r_{f1}.

IV. Conclusion

Short selling can be readily incorporated into mean variance analysis. The Lintnerian case involving initial margin requirements of 100 percent and the payment of riskfree interest on both the margin and short proceeds has previously been examined. Specifically, the EGP algorithm has been shown to be capable of identifying the composition and location of the tangency portfolio associated with a riskfree borrowing and lending rate in this situation.

The restricted case where the initial margin requirement is less than 100 percent and the riskfree interest that is paid on the short proceeds is less than the rate paid on the initial margin is considered here. The EGP algorithm is shown to be capable of identifying the tangency portfolio in this situation as well.[12] All that needs to be done is to create an artificial second set of N securities that correspond to short selling the first set of N securities, and then proceed to use the algorithm to examine the resulting set of $2N$ securities. Furthermore, this method is capable of tracing out the

[10] Alternatively, one could seek to identify the corner portfolios lying between T_L and T_B using the procedure providing by Elton, Gruber, and Padberg (1978, pp. 298–300).

[11] The procedure where the weights X_i are allowed to be negative when examining Lintnerian short sales cannot be used in either of these cases. Allowing negative weights to denote Lintnerian short sales and using r_{fB} in place of r_{f1} in equations (10) through (12) incorrectly treats r_{fB} as the riskfree lending rate, as can be seen by examining equation (2). Hence, the only way Lintnerian short sales can be correctly examined when either $r_{fB} > r_{f1}$ or when riskfree borrowing is disallowed is by creating the second set of N artificial securities corresponding to short positions in the first set of N securities and then using the no-short-selling algorithm, as is done here.

[12] Alexander (1993) shows that the EGP algorithm can also be used under restricted short selling when either the constant correlation model of Elton and Gruber (1973) or the Bayesian model of Brown (1979) are used to cope with estimation risk. For more on these two models, see Elton, Gruber, and Padberg (1976), Chen and Brown (1983), and Alexander and Resnick (1985).

efficient set if riskfree borrowing is either disallowed or is allowed but at a higher rate than the rate available for riskfree lending.

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