Crowding Out and the Informativeness of Security Prices
Author(s): Jonathan M. Paul
Source: *The Journal of Finance*, Vol. 48, No. 4 (Sep., 1993), pp. 1475-1496
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2329047
Accessed: 25/11/2009 11:34

THE JOURNAL OF FINANCE • VOL. XLVIII, NO. 4 • SEPTEMBER 1993

Crowding Out and the Informativeness of Security Prices

JONATHAN M. PAUL*

ABSTRACT

Individual investors trade less agressively on any particular piece of information as more investors observe it. The trades of the new investors observing a piece of information "crowd out" some of the trades of the old investors who observe that same piece of information. This paper shows that when traders are risk averse, these crowding out effects lead the proportions of traders who choose to observe one signal versus another to differ from the proportions that maximize the informativeness of prices.

ONE OF THE PRINCIPAL functions of security markets is to provide incentives for individuals to collect economically relevant information and then to incorporate this information into pricing. This paper shows that the presence of multiple types of information can have a major impact on investors' incentives to acquire information and the informativeness of the resulting security price. Specifically, the model studies the case in which there are two factors a and b whose sum is the asset's payoff and a fixed pool of investors each of which is about to choose whether to learn a or to learn b. The paper addresses the question, "Will the proportion of investors who choose to learn a in equilibrium maximize the security price's ability to predict terminal value?" The paper finds that, when traders are risk averse, the answer is no and that there is a clear economic interpretation for the forces which push the equilibrium allocation of information away from the allocation which maximizes price informativeness.

The paper also finds strikingly different results for risk-neutral and risk-averse investors. This suggests that intuitions gained from models with risk-neutral, strategic traders[1] may not be robust to situations with risk-averse investors.

In a recent paper Froot, Scharfstein, and Stein (1992) investigate investors' incentives to choose between different types of information in a model in which investors have short trading horizons.[2] Froot *et al.* show that with short horizons, as one investor increases the sensitivity of his trades to a

*School of Business Administration, University of Michigan, Ann Arbor. I am particularly indebted to David Scharfstein for comments which greatly improved this paper. I would also like to thank Anat Admati, Aaron Edlin, Bruce Grundy, Barbara Mace, Paul Milgrom, Paul Pfleiderer, Paul Seguin, Matt Spiegel, and Steve Slezak. All errors, of course, remain my own.

[1] See, for instance, the literature arising out of Kyle (1985).

[2] By short horizons, Froot *et al.* mean that investors cannot leave their money in the market for an indefinite period of time but instead must close out their positions quickly.

given piece of information, the equilibrium response of other traders with the same information is to increase the sensitivity of their trades to that same piece of information. That is, short horizons cause the sensitivity of the demands of different traders to the same piece of information to be strategic complements in the sense of Bulow, Geanakoplos, and Klemperer (1985).[3] Froot *et al.* then show that this strategic complementarity can lead to herding equilibria in which all investors choose to study the same information.

In this paper I examine a long-horizon version of the Froot, Scharfstein, and Stein model in which investors can leave their wealth in securities indefinitely. With long horizons, the sensitivity of the demands of different traders to the same piece of information are strategic substitutes[4] as demonstrated by Froot *et al.* In this paper I show that this strategic substitutability causes a crowding out effect which leads the proportions of risk-averse investors that study a versus b to differ from the proportions that would maximize the informativeness of prices.

This paper emphasizes the role played by the "crowding out effect" in causing investors' equilibrium research choices to differ from the research choices that would maximize the security price's ability to predict terminal asset value. The trades coming from new investors observing a piece of information only increase price informativeness to the extent that they do not merely replace (or crowd out) the trades of other traders already observing that information. The crowding out effect occurs to the extent that individual traders who observe a given piece of information trade less aggressively on that information in equilibrium as the number of traders who observe that piece of information increases. The trades of the additional investors "crowd out" some of the trades of the old investors who have the same information.

Suppose there are two different pieces of information, a and b. In equilibrium, the magnitude of the crowding out effect on investors who observe a of an additional type-a investor may not be the same as the magnitude of the crowding out effect on investors who observe b of an additional type-b investor. When these two crowding out effects have different magnitudes in equilibrium, the equilibrium proportion of investors studying one piece of information versus another will differ from the proportions that maximize price informativeness. An excessive proportion of the investors will study the type of information which has the greater crowding out effect.

This paper is related to work by Subrahmanyam (1991b) in that both papers compare results about price informativeness when traders are risk neutral to results when traders are risk averse. Subrahmanyam (1991b)

[3]The term "strategic complements" is a bit of a misnomer since the effect need not have anything to do with strategic behavior. Two variables are strategic complements if the equilibrium response to an increase in one variable is for the other variable to increase and vice versa. The equilibrium need not involve strategic behavior. It could be a competitive equilibrium or a game-theoretic equilibrium.

[4]Two variables are strategic substitutes if the equilibrium response to an increase in one variable is for the other variable to decrease and vice versa.

studies how many traders will choose to observe costly information while my paper looks at which types of information investors will choose to observe. Another important paper on this topic is by Lundholm (1991), who examines the effects of public information (such as accounting reports) on investors' incentives to collect information and the set of viable allocations of information.

The paper is organized as follows. Section I describes the model. Section II solves for the equilibrium allocations of information in the model when informed traders are risk neutral. Section III solves for the allocations of information that maximize the price's ability to predict terminal asset value. Section IV describes the crowding out effect and its role in driving the equilibrium allocation of information away from the allocation which maximizes price informativeness. Section V solves for equilibrium with risk-averse informed traders. Section VI compares results for the risk-neutral and risk-averse cases. Section VII shows that the paper's results hold for a large class of informativeness metrics. Section VIII discusses empirical implications and extensions of the paper's results. The paper concludes with Section IX.

I. The Model

The model studied in this paper is based on the model introduced by Froot *et al.* (1992). The terminal payoff of the asset is v which is the sum of two factors.

$$v = a + b$$

where $a \sim N(0, \sigma_a^2)$ and $b \sim N(0, \sigma_b^2)$ and a and b are independent and jointly normal.

A. Trading Behavior

Following Kyle (1985), security trading is modeled with three types of traders: informed traders, liquidity traders, and market makers. The informed traders become informed by observing either a or b. Before trading starts, each informed trader chooses whether to costlessly observe a or costlessly observe b.[5] There are n informed traders. Let αn be the number of the traders that choose to observe a and $(1 - \alpha)n$ be the number of traders who choose to observe b. I will ignore integer constraints.

[5]As is standard in this literature, I have assumed that deciding to observe a precludes one from observing b and vice versa. Hayek (1945) argued that the equilibrium price arises out of a complex interaction of many pieces of information and that the difficulty of calculating a price (or value) was so great that "to assume all knowledge to be given to a single mind is... to disregard everything that is important and significant in the real world." Hayek concluded that "the economic problem of society is... the utilization of knowledge not given to anyone in its totality." The assumption that no investor can simultaneously observe a and b is in the spirit of Hayek's view that one of the most important functions of the price system is the decentralized aggregation of information and that no one person or institution can process all information relevant to pricing.

After observing their information, the informed traders (without knowing price) submit market orders to the market maker. In this paper, I contrast results from two different versions of the model. The first version follows Kyle (1984), Admati and Pfleiderer (1988), and Froot *et al.* (1992) and studies a Cournot-Nash trading equilibrium with risk-neutral informed traders. The second version assumes that informed traders are risk averse (as in Holden and Subrahmanyam (1991) and Subrahmanyam (1991b)) and act competitively when trading.

The liquidity traders' aggregate demand for the risky asset is given by the random variable ε where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ and is independent of a and b. The market makers buy or sell as necessary to clear the market. The only information that the market makers can observe is the total number of orders to buy or sell at any given time. The market makers cannot tell whether the orders are coming from informed traders or liquidity traders. Market makers are assumed to be risk neutral and competitive. They earn zero expected profits by pegging the security price at the expected terminal value of the asset given the total order flow.

B. Pricing

Let q_a be the demand of a typical investor who knows a, and let q_b be the demand of a typical investor who knows b. Let F be the total order flow. $F = \alpha n q_a + (1 - \alpha) n q_b + \varepsilon$. The security price p will be the expected terminal value of the asset given the total order flow. As in Kyle (1985), I will only consider equilibria in which the demand of type-a traders q_a is linear in a and in which q_b is linear in b. Therefore,

$$p = \lambda F \quad \text{where} \quad \lambda = \frac{\operatorname{Cov}(v, F)}{\operatorname{Var}(F)}. \tag{1}$$

λ is analogous to the coefficient on F when one regresses v on F using OLS. Later in the paper, I confirm that such a linear equilibrium does indeed exist.

II. Equilibrium with Risk-neutral, Strategic Informed Traders

A. Equilibrium at the Trading Stage

In this section, I assume that informed traders are risk neutral and play a Cournot-Nash equilibrium at the trading stage of the model. When informed traders are risk neutral, an informed trader who observes a expects to earn profits of $\mathbf{E}[v - p|a]$ per unit of the security he purchases. Thus the expected utility of an informed trader who observes the realized value a and demands q_a^i is

$$U_a^i = q_a^i \mathbf{E}[v - p|a] = q_a^i\big[a - \lambda\big[q_a^i + (\alpha n - 1)q_a\big]\big]$$

which is concave in q_a^ι. The optimal value of q_a^ι is given by the first-order condition.[6]

$$a - 2\lambda q_a^\iota - \lambda(\alpha n - 1)q_a = 0.$$

In equilibrium, q_a^ι will equal q_a,[7] so we have $q_a = a/[\lambda(\alpha n + 1)]$. Thus we can write

$$q_a = \delta_a a \quad \text{where} \quad \delta_a = \frac{1}{\lambda(\alpha n + 1)}. \tag{2}$$

Similarly,

$$q_b = \delta_b b \quad \text{where} \quad \delta_b = \frac{1}{\lambda((1 - \alpha)n + 1)}. \tag{3}$$

The expected utility of an investor who studies a is

$$\mathbf{E}U_a = \mathbf{E}\{q_a \mathbf{E}[v - p|a]\} = \mathbf{E}\{\delta_a a[a - \lambda \alpha n \delta_a a]\} = [1 - \lambda \alpha n \delta_a]\delta_a \sigma_a^2.$$

Let $\beta_a = 1 - \lambda \alpha n \delta_a$. The coefficient β_a is analogous to the regression coefficient from regressing $v - p$ (the pricing error) on a. Similarly, let $\beta_b = 1 - \lambda(1 - \alpha)n\delta_b$. Therefore, the expected utility of investors who study a is

$$\mathbf{E}U_a = \beta_a \delta_a \sigma_a^2 = \beta_a \mathrm{Cov}(q_a, a).$$

Similarly, the expected utility of investors who study b is

$$\mathbf{E}U_b = \beta_b \delta_b \sigma_b^2 = \beta_b \mathrm{Cov}(q_b, b).$$

B. Equilibrium at the Information Collection Stage

The proportion α (the proportion that study a rather than b) is unobservable to both informed investors and market makers in all stages of the model, and informed investors play a Nash equilibrium at the research stage of the model. For extreme parameters,[8] the unique equilibrium is the corner solution in which all investors observe the more informative (and more profitable) signal. For less-extreme parameters,[9] $\mathbf{E}U_a < \mathbf{E}U_b$ at $\alpha = 1$ and $\mathbf{E}U_b < \mathbf{E}U_a$ at $\alpha = 0$. Here, the unique equilibrium occurs at the interior α where $\mathbf{E}U_a = \mathbf{E}U_b$. To see this, consider what would happen in an equilibrium in which $\mathbf{E}U_a = \mathbf{E}U_b$ if an investor, who in equilibrium studies a, deviates and studies b instead. The behavior of all other traders would be unchanged (since they cannot observe the deviation), but the deviation would increase the number of type-b traders (decreasing $\mathbf{E}U_b$) and would decrease the

[6] λ is determined by the market makers' beliefs about the joint distribution of F and v—not its actual distribution if a player deviates. Thus λ is a constant in the individual trader's optimization problem.

[7] Since the best response functions are strictly monotone, there is a unique Nash equilibrium and in this case it is symmetric.

[8] $\sigma_a > (n + 1)\sigma_b$ or $\sigma_b > (n + 1)\sigma_a$.

[9] $\sigma_b/(n + 1) < \sigma_a < (n + 1)\sigma_b$.

number of type-a traders (increasing $\mathbf{E}U_a$). Thus the deviation would be strictly unprofitable.[10] Thus, in any interior equilibrium, $\mathbf{E}U_a = \mathbf{E}U_b$, or equivalently,

$$\beta_a \mathrm{Cov}(q_a, a) = \beta_b \mathrm{Cov}(q_b, b). \tag{4}$$

Equation (4) also describes equilibrium in the research stage of the model if we assume that investors are small and act competitively when they decide whether to study a or b.

The goal of this paper is to look at whether security markets give investors the incentives to collect the right type of information. This question is trivial unless there is more than one valuable piece of information. In the corner equilibria, which only occur for extreme parameters, there is only one important piece of information and so we cannot address this question. Therefore, I will only examine the interior equilibria (described by equation (4)) in the remainder of the paper.

III. The Allocation of Information That Maximizes Price Informativeness

As is discussed in Section IV, the source of the inefficiency[11] found in this paper is the crowding out effect which will occur regardless of one's criteria for the informativeness of prices. However, I will start off by following the literature on price informativeness and adopting $\mathrm{Var}(v - p)$ as my criteria for the informativeness of prices. In Section VII, I show that the paper's results hold for a wide class of informativeness metrics.

In this section, I solve for the allocation of information that minimizes $\mathrm{Var}(v - p)$. Since a, b, and ε are independent, we can write

$$v - p = \frac{\mathrm{Cov}(v - p, a)}{\sigma_a^2} a + \frac{\mathrm{Cov}(v - p, b)}{\sigma_b^2} b + \frac{\mathrm{Cov}(v - p, \varepsilon)}{\sigma_\varepsilon^2} \varepsilon$$

This is analogous to regressing $v - p$ on the three independent sources of (normally distributed) uncertainty in the model.[12] Since a, b, and ε are

[10] I thank Matt Spiegel for pointing this out.

[11] In the literature on price informativeness and efficient markets, the term "efficiency" refers to statistical efficiency. In this literature, price efficiency is measured by $\mathrm{Var}(v - p)$ which measures the extent to which security prices reflect terminal value. See Grossman and Stiglitz (1980), Kyle (1984, 1985, 1989), Admati and Pfleiderer (1988), Foster and Viswanathan (1990), Froot, Scharfstein, and Stein (1992), Holden and Subrahmanyam (1991, 1992), Subrahmanyam (1991b), and Spiegel and Subrahmanyam (1992). This paper follows this literature and uses the term "efficiency" in this statistical sense. The paper makes no direct claims concerning social welfare though there are plausible models in which minimizing $\mathrm{Var}(v - p)$ is equivalent to maximizing social welfare. An example is available upon request from the author. Also see Section VII for a discussion of alternative metrics for price efficiency.

[12] Note that $a = 0$, $b = 0$, and $\varepsilon = 0$ implies that $v = p = 0$. So the constant in the "regression" is zero on the right-hand side.

independent, we get

$$\mathrm{Var}(v-p) = \frac{\mathrm{Cov}^2(v-p,a)}{\sigma_a^2} + \frac{\mathrm{Cov}^2(v-p,b)}{\sigma_b^2} + \frac{\mathrm{Cov}^2(v-p,\varepsilon)}{\sigma_\varepsilon^2}.$$

$$\mathrm{Var}(v-p) = [1-\lambda\alpha n\delta_a]^2\sigma_a^2 + [1-\lambda(1-\alpha)n\delta_b]^2\sigma_b^2 + \lambda^2\sigma_\varepsilon^2. \quad (5)$$

Assuming we do not have a corner solution, we can minimize $\mathrm{Var}(v-p)$ with respect to α and get the following first-order condition:

$$\lambda\sigma_\varepsilon^2\frac{d\lambda}{d\alpha} = [1-\lambda\alpha n\delta_a]\frac{d(\lambda\alpha n\delta_a\sigma_a^2)}{d\alpha} + [1-\lambda(1-\alpha)n\delta_b]\frac{d(\lambda(1-\alpha)n\delta_b\sigma_b^2)}{d\alpha}. \quad (6)$$

From the formula for λ in equation (1), we get

$$\lambda\sigma_\varepsilon^2 = [1-\lambda\alpha n\delta_a]\alpha n\delta_a\sigma_a^2 + [1-\lambda(1-\alpha)n\delta_b](1-\alpha)n\delta_b\sigma_b^2.$$

After substituting in this expression for $\lambda\sigma_\varepsilon^2$, equation (6) reduces to

$$[1-\lambda\alpha n\delta_a]\left(\alpha n\sigma_a^2\frac{d\delta_a}{d\alpha} + n\delta_a\sigma_a^2\right) = [1-\lambda(1-\alpha)n\delta_b]\left(n\delta_b\sigma_b^2 - (1-\alpha)n\sigma_b^2\frac{d\delta_b}{d\alpha}\right).$$

Recall that $\beta_a = 1-\lambda\alpha n\delta_a$ and $\beta_b = 1-\lambda(1-\alpha)n\delta_b$. Thus equation (6) further reduces to

$$\beta_a\frac{d(\alpha n\delta_a\sigma_a^2)}{d\alpha} = \beta_b\frac{d((1-\alpha)n\delta_b\sigma_b^2)}{d(1-\alpha)} \quad (7)$$

or, equivalently,

$$\beta_a\frac{d\,\mathrm{Cov}(F,a)}{d\alpha} = \beta_b\frac{d\,\mathrm{Cov}(F,b)}{d(1-\alpha)}. \quad (8)$$

Equation (8) is the first order condition for α to minimize $\mathrm{Var}(v-p)$. In the next section, I compare this condition to the condition that characterizes the equilibrium value of α.

IV. Crowding Out and the Informativeness of Prices

The trades coming from new investors observing a piece of information only increase price informativeness to the extent that they do not merely replace the trades of other traders already observing that information. The crowding out effect occurs because individual investors who observe a given piece of information trade less aggressively on that information as the number of investors who observe that piece of information increases. The variable δ_a

indicates the aggressiveness with which each individual type-a trader trades on his information. Recall that δ_a equals $1/[\lambda(\alpha n + 1)]$. Note that δ_a is decreasing in αn, and, similarly, δ_b is decreasing in $(1 - \alpha)n$. The trades of additional investors with the same information "crowd out" some of the trades of the old investors.

Consider the price informativeness problem of trying to determine the proportion of the informed investors that must study a rather than b in order to minimize $\mathrm{Var}(v - p)$. From equation (8), the first-order condition from the price informativeness problem is

$$\beta_a \frac{d\,\mathrm{Cov}(F, a)}{d\alpha} = \beta_b \frac{d\,\mathrm{Cov}(F, b)}{d(1 - \alpha)}.$$

Compare this to equation (4) which characterizes equilibrium:

$$\beta_a \mathrm{Cov}(q_a, a) = \beta_b \mathrm{Cov}(q_b, b).$$

The first-order condition for the price informativeness problem only differs from the equation that characterizes equilibrium in that the first-order condition for price informativeness has the change in the covariance of *aggregate* demand to the various pieces of information where the equilibrium conditions only have the covariance of the individual trader's demand to his information. The key point is that, to maximize price informativeness, we would care how adding one more investor of a given type affects the *total* sensitivity of *all* the investors' demands to that type of information, but the individual trader in equilibrium is only motivated by the covariance of *his* demand to his information. The crowding out effect drives a wedge between these two things and can cause the equilibrium research decisions of individual investors to differ from those that would maximize price informativeness.

In equilibrium, the strength of the crowding out effect on investors who observe a of an additional type-a investor may not be the same as the strength of the crowding out effect on investors who observe b of an additional type-b investor (see Section VI). When these two crowding out effects have different magnitudes in equilibrium, the equilibrium proportion of investors studying one piece of information versus another will be different from the proportions that maximize price informativeness. An excessive proportion of the investors will study the type of information which has the greater crowding out effect.

The strength of the crowding out effect for each type of information can be measured by calculating an elasticity. From equation (7), the first-order condition for the proportion of informed investors to minimize $\mathrm{Var}(v - p)$ can be written

$$\beta_a \left[\alpha \frac{d\delta_a}{d\alpha} + \delta_a \right] n \sigma_a^2 = \beta_b \left[(1 - \alpha) \frac{d\delta_b}{d(1 - \alpha)} + \delta_b \right] n \sigma_b^2$$

$$\beta_a \delta_a \sigma_a^2 [1 - \eta_a] = \beta_b \delta_b \sigma_b^2 [1 - \eta_b]$$

where $\eta_a = -\dfrac{d\delta_a}{d\alpha}\dfrac{\alpha}{\delta_a}$ which is the elasticity of δ_a with respect to α and $\eta_b = -\dfrac{d\delta_b}{d(1-\alpha)}\dfrac{1-\alpha}{\delta_b}$. Therefore the necessary conditions for the proportion of informed traders to minimize $\mathrm{Var}(v-p)$ are

$$\beta_a \mathrm{Cov}(q_a, a)[1-\eta_a] = \beta_b \mathrm{Cov}(q_b, b)[1-\eta_b]. \tag{9}$$

This condition differs from the condition that characterizes equilibrium (equation (4)) only by the terms $[1-\eta_a]$ and $[1-\eta_b]$. The elasticities η_a and η_b measure the size of the crowding out effect with respect to a and b respectively. Empirically, one could measure the impact of crowding out on investors' research decisions by estimating these elasticities.

V. Competitive, Risk-averse Informed Traders

The literature arising out of Kyle (1984, 1985) studies the polar case in which informed investors' incentive to trade on information is only mitigated by the effects of trading on price. In this literature, the informed traders are assumed to be risk neutral and play a Cournot-Nash equilibrium. In this section, I will examine the opposite polar case in which informed traders act competitively but are risk averse.

Consider the case in which informed traders have imprecise information about the terminal payoff of the security and are trading in a highly liquid financial market (low value of λ). In this case the informed trader would have to take a large position in the security exposing himself to a high degree of risk in order to significantly move price. Thus for liquid financial markets, assuming that informed traders ignore their effect on price and are only concerned about risk aversion is the more reasonable of the two polar cases. This is especially true if the informed trader's superior information still leaves him with a lot of residual uncertainty about the security's terminal payoff.

Intuitively, one would expect that price effects will only be important for traders who take large positions relative to the size of the market, but risk aversion will be an important factor whenever the investor's position is large relative to his own wealth.

When informed traders have the exponential utility function $U(\mathrm{w}) = -e^{-r\mathrm{w}}$, the certainty equivalent of a trader who observes a and demands q_a^i is

$$q_a^i \mathbf{E}(v-p|a) - \tfrac{1}{2} r (q_a^i)^2 \mathrm{Var}(v-p|a).$$

We will solve for the symmetric linear equilibrium. We first assume that demands are linear in equilibrium. Once we have found an equilibrium, we will verify this assumption. Now the objective function of an informed trader who has observed the value a can be written:

$$q_a^i[a - \lambda\alpha n q_a] - \tfrac{1}{2}r(q_a^i)^2\left\{[1 - \lambda(1-\alpha)n\delta_b]^2\sigma_b^2 + \lambda^2\sigma_\varepsilon\right\} \tag{10}$$

which is concave in q_a^i. The utility-maximizing value of q_a^i is given by the first-order condition:

$$[a - \lambda\alpha n q_a] = rq_a^i\left\{[1 - \lambda(1-\alpha)n\delta_b]^2\sigma_b^2 + \lambda^2\sigma_\varepsilon{}^2\right\}.$$

In any symmetric equilibrium, q_a^i will equal q_a, so we have

$$q_a = \frac{a}{r\left\{[1 - \lambda(1-\alpha)n\delta_b]^2\sigma_b^2 + \lambda^2\sigma_\varepsilon{}^2\right\} + \lambda\alpha n}.$$

Therefore in equilibrium, we can write $q_a = \delta_a a$ where

$$\delta_a = \frac{1}{r\left\{[1 - \lambda(1-\alpha)n\delta_b]^2\sigma_b^2 + \lambda^2\sigma_\varepsilon{}^2\right\} + \lambda\alpha n} \tag{11}$$

and $q_b = \delta_a b$ with

$$\delta_b = \frac{1}{r\left\{[1 - \lambda\alpha n\delta_a]^2\sigma_a^2 + \lambda^2\sigma_\varepsilon{}^2\right\} + \lambda(1-\alpha)n}. \tag{12}$$

The assumption that q_a is linear in a and q_b is linear in b holds true in equilibrium.

A. Equilibrium Allocations of Information

The expected utility of a trader who studies a is $\mathbf{E}U_a = \mathbf{E}\{q_a\mathbf{E}(v - p|a) - \frac{1}{2}rq_a^2\text{Var}(v - p|a)\}$. Recall that $\beta_a = 1 - \lambda\alpha n\delta_a$ and $\beta_b = 1 - \lambda(1-\alpha)n\delta_b$. From equations (10), (11), and (12),

$$\begin{aligned}
\mathbf{E}U_a &= \beta_a\delta_a\sigma_a^2 - \tfrac{1}{2}r\delta_a^2\sigma_a{}^2\{\beta_b^2\sigma_b^2 + \lambda^2\sigma_\varepsilon{}^2\} \\
&= \beta_a\text{Cov}(q_a, a) - \tfrac{1}{2}r\delta_a^2\sigma_a{}^2\left\{\beta_b^2\sigma_b^2 + \lambda^2\sigma_\varepsilon{}^2 + \frac{1}{r}\lambda\alpha n - \frac{1}{r}\lambda\alpha n\right\} \\
&= \beta_a\text{Cov}(q_a, a) - \tfrac{1}{2}\text{Cov}(q_a, a) + \tfrac{1}{2}\lambda\alpha n\delta_a^2\sigma_a^2 \\
&= \beta_a\text{Cov}(q_a, a) - \tfrac{1}{2}\beta_a\text{Cov}(q_a, a).
\end{aligned} \tag{13}$$

Therefore,

$$\mathbf{E}U_a = \tfrac{1}{2}\beta_a\text{Cov}(q_a, a) \quad \text{and} \quad \mathbf{E}U_b = \tfrac{1}{2}\text{Cov}(q_b, b). \tag{14}$$

Therefore in any interior equilibrium,

$$\beta_a\text{Cov}(q_a, a) = \beta_b\text{Cov}(q_b, b)$$

which is exactly the same condition that was derived for the case with risk-neutral, strategic informed traders. Thus equation (4) describes equilibrium in the case in which informed traders have exponential utility and act competitively.

Let α^{eq} be the proportion of the informed traders that choose to study a in equilibrium. Proposition 1 further characterizes equilibrium.

PROPOSITION 1: *If informed traders have exponential utility and act competitively in the trading stage of the model, then, in any interior equilibrium,*

$$(i) \qquad \frac{\delta_a}{\delta_b} = \frac{\sigma_b}{\sigma_a} \qquad \text{at } \alpha = \alpha^{eq}$$

$$(ii) \qquad \frac{\beta_a}{\beta_b} = \frac{\sigma_b}{\sigma_a} \qquad \text{at } \alpha = \alpha^{eq}$$

$$(iii) \qquad \alpha^{eq} > \frac{\sigma_a}{\sigma_a + \sigma_b} \qquad \text{for } \sigma_a > \sigma_b.$$

Proof: See Appendix B.

Proposition 1 shows that in equilibrium the variance of the demands of a type-a trader, $\delta_a^2\sigma_a^2$, equals the variance of the demands of a type-b trader, $\delta_b^2\sigma_b^2$. The expression $\beta_a^2\sigma_a^2$ is analogous to the explained sum of squares from regressing the pricing error $(v - p)$ on a. Similarly $\beta_b^2\sigma_b^2$ is analogous to the explained sum of squares from regressing the pricing error on b. Proposition 1 shows that in equilibrium these explained sums of squares are equal. Equivalently, $\mathrm{Var}(v - p|a)$ and $\mathrm{Var}(v - p|b)$ are equal in equilibrium.

B. The Allocation of Information That Maximizes Price Informativeness

Section III, which derived the necessary condition for the allocation of information that minimizes $\mathrm{Var}(v - p)$, never used any features specific to the risk-neutral strategic informed trader version of the model that do not hold equally true for the version of the model in which informed traders have exponential utility and act competitively. Therefore equation (8) applies to the case in which informed traders have exponential utility and act competitively. The same equation that describes the allocation of information that minimizes $\mathrm{Var}(v - p)$ for the case in which informed traders are risk neutral and play Cournot-Nash strategies also describes the allocation of information that minimizes $\mathrm{Var}(v - p)$ for the case in which informed traders have exponential utility and act competitively. The elasticities η_a and η_b continue to measure the crowding out effects with respect to a and b respectively.

VI. The Determinants of the Crowding Out Effect

As we have seen from Sections IV and V, the size of the crowding out effect for each type of information determines which signal is overemphasized in equilibrium. Thus in order to predict which type of information will be overemphasized, we must first understand the factors that cause the crowding out effect.

There are two factors that limit the aggressiveness with which individual investors trade on their information: (1) the magnitude of the marginal increase in total payoff risk arising from more aggressive trading and (2) the extent to which the trading on the information by those observing the signal diminishes the expected profits per unit traded. The crowding out effect is the marginal effect on these two factors of the trades coming from new investors who observe a given piece of information.

A. Risk-neutral, Strategic Informed Traders

In the risk-neutral case, investors only care about the informativeness of trading volume about terminal value. The riskiness of trading on information (point 1 above) does not enter into their calculations. With risk-neutral traders, the gains from trading on information are affected by the proportion of other investors also observing that information to the extent that α affects the total leakage of information into trading volume. For type-a traders, this is measured by the elasticity η_a. For type-b traders, it is measured by η_b. These elasticities are the same in equilibrium. This is confirmed in Proposition 2.

PROPOSITION 2: *Let $\sigma = \sigma_a/\sigma_b$. If informed traders are risk neutral and play a Cournot-Nash equilibrium in the trading stage of the model, then*

$$\eta_a = \eta_b = \frac{[(n+1)-\sigma][(n+1)\sigma - 1]}{n(n+2)\sigma} \quad \text{at} \quad \alpha = \alpha^{eq}.$$

Proof: See Appendix B.

As was shown in Section IV, these elasticities measure the size of the crowding out effect for each type of information. With risk-neutral traders, the crowding out effect for a and for b exactly cancel each other out in equilibrium. Therefore, with risk-neutral traders, the proportion of traders observing a and the proportion of traders observing b are exactly the proportions that minimize $\mathrm{Var}(v-p)$. This helps explain Scharfstein's result[13] that, for α^{eq}, $\alpha^* < 1$,

$$\alpha^{eq} = \alpha^* = \frac{(n+1)\sigma_a - \sigma_b}{n(\sigma_a + \sigma_b)}$$

where α^* is the proportion of informed traders that observe a in the allocation of information that minimizes $\mathrm{Var}(v-p)$ (see Appendix B).

[13] D. Scharfstein (1990), personal communication.

B. Risk-averse, Competitive Informed Traders

In the case in which investors are risk averse, the marginal impact of more aggressive trading on the investor's payoff risk (point 1 from the top of this section) does affect investors' decisions. In this case, the economy gains the benefit of the investor's trades in reducing $\mathrm{Var}(v - p)$, but the individual risk-averse investor privately bears the full risk of his trading. At the proportions that maximize informativeness, the investors that observe the less-informative signal about v (terminal value) bear more risk than investors that observe the more informative signal about v. At the α that maximizes informativeness, the investors that observe the less-informative signal are not adequately compensated in the trading stage of the model for their risk exposure. This intuition suggests that, with risk-averse investors, an excessive proportion of investors will observe whichever signal is more informative about terminal value.

Unfortunately, this version of the model quickly becomes intractable and cannot be solved for in closed form. Using numerical methods, I solved the model for many sets of parameter values. In each case I found that $\alpha^{eq} > \alpha^*$ whenever $\sigma_a^2 > \sigma_b^2$. The only exceptions to this were corner solutions where $\alpha^* = \alpha^{eq} = 1$. In equilibrium, an excessive proportion of the informed traders chose to study whichever variable explains more of the ex ante variability of the asset's payoff. Appendix B lists all of the more than 1000 sets of parameter values for which I have solved the model using numerical methods.

Crowding out can have a large effect on the resulting equilibrium. Table I shows that the informativeness-maximizing ratio of traders studying a versus b was 3.83 to 1 for $n = 25$, $r = 0.25$, $\sigma_\varepsilon^2 = 100$, $\sigma_a^2 = 80$, and $\sigma_b^2 = 20$, but the equilibrium ratio was 18.6 to 1. For the case in which $\sigma_a^2 = 83$ and $\sigma_b^2 = 17$, price informativeness is maximized at a ratio of 5 to 1 but in

Table I

Herding on the More Informative Signal: Some Computed Values

This table reports the equilibrium proportion of traders observing the more informative signal (α^{eq}) and the proportion required to maximize price informativeness (α^*). The parameters σ_a^2 and σ_b^2 are the variances of a and b respectively. In these simulations, there are 25 informed traders. Each trader has risk aversion coefficient 0.25, and the variance of liquidity trading is 100 which is also the variance of the security's terminal value.

σ_a^2	σ_b^2	α^{eq}	α^*	$\frac{\alpha^{eq}}{(1-\alpha^{eq})}$	$\frac{\alpha^*}{(1-\alpha^*)}$
67	33	0.744	0.650	2.91	1.86
75	25	0.867	0.733	6.52	2.75
80	20	0.949	0.793	18.6	3.83
83	17	1.00	0.833	∞	4.99
93	7	1.00	1.00	∞	∞

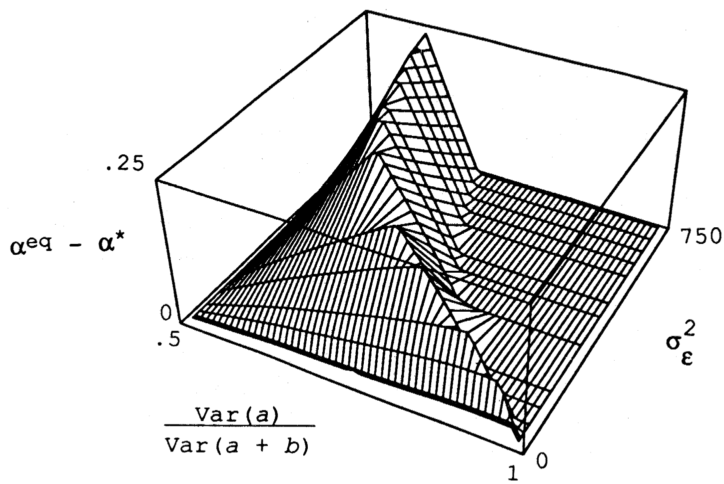

Figure 1. The effect of liquidity trading on crowding out. $\alpha^{eq} - \alpha^*$, which is the difference between the equilibrium and optimal proportions of traders observing a, is plotted on the vertical axis. The horizontal axis indicates the proportion of fundamental uncertainty comprised by a. Values on the horizontal axis range from 0.5 to 1. Depth indicates the variance of liquidity trading. It varies from 1 to 750. In this figure there are 25 informed traders, each trader has a risk aversion parameter of 0.25, and the variance of the security's terminal payoff is 100.

equilibrium all traders researched a. For the case in which $\sigma_a^2 = 93$ and $\sigma_b^2 = 7$, crowding out has no effect on the resulting equilibrium since both α^* and α^{eq} are cornered out at 1.

Figure 1 shows the effect of liquidity trading on the crowding out effect. The vertical axis records the value of $\alpha^{eq} - \alpha^*$ while depth indicates the amount of liquidity trading. Figure 1 illustrates a number of regularities. For any given level on the horizontal axis, the graph is hump-shaped as σ_ε^2 increases. That is, for any given ratio of $\text{Var}(a)/\text{Var}(a + b)$, the difference $\alpha^{eq} - \alpha^*$ first increases and then decreases as the amount of liquidity trading increases. This is what one would expect intuitively. Consider the limit as the amount of liquidity trading goes to zero. In this case, price always equals fundamental value and is unaffected by the number of traders studying each signal. The crowding out effect is 100 percent for each signal and so α^{eq} equals α^*. Similarly, for very low amounts of liquidity trading, the difference between terminal value and price is approximately zero regardless of the number of traders that study each signal. Here the crowding out effect is approximately 100 percent for each signal. Since the crowding out effect is roughly equal for each signal, α^{eq} and α^* are approximately equal.

On the other hand, when the variance of liquidity trading is extremely high, each trader's demands has a negligible effect on price and so has a negligible effect on the demands of other traders. In this case the crowding out effect is approximately zero for each signal and so α^{eq} and α^* are approximately equal. For intermediate levels of liquidity trading, crowding out can have a substantial effect on $\alpha^{eq} - \alpha^*$ as shown in Figure 1.[14]

Another striking feature of Figure 1 is that, in liquid markets, all traders will choose to observe the same variable even when the neglected variable determines a significant portion of terminal value. In the flat section to the northeast of the ridge in Figure 1, both α^{eq} and α^* equal 1. In the case in which the variance of a is 70, the variance of b is 30 and the variance of liquidity trading is 750, all investors choose to observe a even though b accounts for 30 percent of terminal value.[15] The intuition behind this is as follows. Consider the limit as the amount of liquidity trading goes to infinity. In this case, price is uninformative. All traders will choose to study the more informative signal no matter how slight the signal's edge in informativeness is over the other signal and no matter how many others also choose to study that signal. For sufficiently liquid markets, the variance of liquidity trading is high and all investors will choose to observe the same signal even if it only explains slightly more of fundamental value than the other signal. This clustering of investors observing the same signal in highly liquid markets maximizes price informativeness since, in the flat area to the northeast of the ridge in Figure 1, both α^{eq} and α^* equal one.

The final feature of Figure 1 is that the maximal values of $\alpha^{eq} - \alpha^*$ form a ridge which increases as the variance of liquidity trading and the similarity of the informativeness of the two signals increase.

VII. Alternative Metrics for the Informativenes of Prices

Up to this point, I have followed the rest of the literature on efficient markets and used $\mathrm{Var}(v - p)$ as my metric for the informativeness of prices. In this section, I will show that the paper's results hold for a wide class of informativeness metrics.

The zero expected profit condition for market makers implies that any pricing rule in the trading stage of the model must satisfy

$$v = p + (v - p) \quad \text{with} \quad \mathrm{Cov}(p, v - p) = 0.$$

Thus since v, p and $p|v$ are normally distributed,

$$p|v \sim N(v, \mathrm{Var}(v - p)) \quad \text{and} \quad p \sim N(0, \mathrm{Var}(v) - \mathrm{Var}(v - p)).$$

Note that $\mathrm{Var}(v)$ is a constant and v is unaffected by the pricing rule. Thus each possible pricing rule is uniquely parameterized by $\mathrm{Var}(v - p)$. Without

[14] The parameters for Table I were chosen to be reasonable and conservative. Figure 1 shows that, for other parameters, one can get more extreme examples of crowding out.

[15] This is not due to integer problems since each trader is modeled as a point on a continuum.

loss of generality, any metric for the informativeness of pricing can be written

$$\psi(\mathrm{Var}(v - p)).$$[16]

Assuming ψ is differentiable, the first-order condition that maximizes price informativeness under the ψ metric with respect to α is

$$\psi'\beta_a \frac{d\,\mathrm{Cov}(F, a)}{d\alpha} = \psi'\beta_b \frac{d\,\mathrm{Cov}(F, b)}{d(1 - \alpha)}. \tag{15}$$

Thus when $\psi' \neq 0$ at the optimum, we have

$$\beta_a \frac{d\,\mathrm{Cov}(F, a)}{d\alpha} = \beta_b \frac{d\,\mathrm{Cov}(F, b)}{d(1 - \alpha)}$$

which is our original result from equation (8). Note that equation (4) which describes the equilibrium allocation of information is not affected by the price informativeness metric. Thus η_a and η_b still describe the crowding out effect exactly as before, and the crowding out effect still describes the wedge between investors' equilibrium incentives to collect information and the incentives that would maximize price informativeness. As long as equation (15) has a unique solution, it will still be the case that an excessive proportion of the informed investors will choose to study the signal with the greater crowding out effect. If ψ measures the social welfare generated by the pricing rule, then the paper's results apply to the social efficiency as well as the statistical efficiency of pricing.

VIII. Extensions and Empirical Implications

A. Multiple Signals

All of the paper's results continue to hold in the case in which there are multiple signals $a, b, c, d, \ldots$ with terminal value v being the sum of these signals. This is true for both of the following cases:

1. The additional signals are added to the investors' choice set so that now investors have the choice of observing a or b or c or d and so on rather than just a or b.
2. The additional signals are residual risk to terminal value which cannot be observed by any of the informed investors.

B. Empirical Implications

Let $\{a, b, \ldots, z\}$ be the set of signals (ex post) observable to the econometrician. The elasticity η_i measures the crowding out effect with respect to signal

[16]Alternatively, $v|p$ completely describes informativeness regardless of metric. $v|p \sim N(p, \mathrm{Var}(v - p))$. Before p is observed, the unconditional distribution of $v|p$ is $N(0, \mathrm{Var}(v - p))$ which, once again, is completely parameterized by $\mathrm{Var}(v - p)$. Thus any metric for the informativeness of pricing can be written $\psi(\mathrm{Var}(v - p))$.

i. The model predicts that the informativeness of price about value ($1/\mathrm{Var}(v|p)$ or, equivalently, $1/\mathrm{Var}(v - p)$) will be decreasing in the η_i elasticities. In a regression of $\mathrm{Var}(v|\{a, b, \ldots, z\})/\mathrm{Var}(v - p)$ on $\eta_a, \eta_b, \ldots, \eta_z$ and $\sigma_\varepsilon{}^2$, the model predicts negative coefficients on the η elasticities.

IX. Conclusion

One of the central topics in financial economics is "how well do security prices reflect information about asset value?" Or, more succinctly, "do security prices accurately reflect fundamentals?" This paper studied whether the incentives individual investors have to collect information lead to allocations of information such that price reflects terminal value as accurately as possible.

Individual investors trade less aggressively on any particular piece of information as more investors observe it. The trades of the new investors observing a piece of information "crowd out" some of the trades of the old investors who observe that same piece of information. This paper shows that, when traders are risk averse, these crowding out effects drive a wedge between individual investors' incentives to collect information and the incentives that would maximize the informativeness of prices. An excessive number of traders will choose to observe the signal for which the crowding out effect is the greatest.

The paper models two cases. In the first, traders are risk neutral and play a Nash equilibrium. In the second, traders are risk averse with exponential utility and act competitively. In both cases:

1. The same equation (equation (4)) describes equilibrium.
2. The same equation (equation (8)) describes the allocation of information that maximizes the informativeness of prices.
3. The elasticities η_a and η_b measure the strength of the crowding out effect with respect to a and b respectively.

When traders are risk neutral, the crowding out effect for the first signal exactly offsets the crowding out effect for the second signal. This causes the proportions of traders studying each signal to maximize the informativeness of prices.

The result that equilibrium allocations of information maximize price informativeness is an artifact of the assumption of risk neutrality and does not hold for risk-averse investors. The case in which investors have exponential utility cannot be solved analytically. However, the results of more than 1000 numerical simulations indicate that an excessive proportion of investors will choose to study the signal that contains the most information about the security's terminal value. Crowding out can have a significant impact on equilibrium when traders are risk averse. The paper discusses an example in which price informativeness would be maximized if the ratio of traders studying one signal versus another was 3.83 to 1, but the equilibrium ratio was 18.6 to 1, and another example in which informativeness would be maximized at a ratio of 5 to 1 but in equilibrium all traders researched the same signal.

For the case in which investors have exponential utility, Proposition 1 shows that:

1. The variance of the demands of individual traders will be the same regardless of which type of information they choose to study.
2. Each piece of information will explain an equal amount of the pricing error $(v - p)$. Equivalently, $\mathrm{Var}(v - p|a)$ will equal $\mathrm{Var}(v - p|b)$.

The profoundly different results found for risk-neutral versus risk-averse investors suggests that intuitions gained from models with risk-neutral, strategic investors about the efficiency of investors' incentives to collect information may not be robust to situations with risk-averse investors.

There have been a large number of papers studying information aggregation and the informativeness of security prices using single-factor models in which all informed traders observe the same signal or the same signal with independent measurement error.[17] With the exception of Froot, Scharfstein, and Stein (1992), relatively little work has been done on investors' incentives to study the different factors that comprise terminal value. This paper shows that the presence of multiple factors can significantly alter the nature of information aggregation in financial markets.

Appendix A

Table II reports all parameters for which α^* and α^{eq} were solved for using numerical methods. In each case α^{eq} was greater then α^* whenever $\sigma_a^2 > \sigma_b^2$, except for cases in which both α^* and α^{eq} had corner solutions $\alpha^* = \alpha^{eq} = 1$.

Each row lists a set of simulations where α^* and α^{eq} were initially calculated for σ_a^2 = "Initial σ_a^2" and σ_b^2 = "Initial σ_b^2." Then σ_a^2 was increased and σ_b^2 was decreased by "step size," and α^* and α^{eq} were recalculated. This continued for each row until σ_a^2 = "Final σ_a^2" and σ_b^2 = "Final σ_b^2." Table II lists a total of 1286 simulations.

Appendix B

Proof of Proposition 1: From equations (13) and (14), we have at $\alpha = \alpha^{eq}$,

$$\beta_b^2\sigma_b^2(\beta_a\delta_a\sigma_a^2)^2\{\beta_b^2\sigma_b^2 + \lambda^2\sigma_\varepsilon^2\} = \beta_a^2\sigma_a^2(\beta_b\delta_b\sigma_b^2)^2\{\beta_a^2\sigma_a^2 + \lambda^2\sigma_\varepsilon^2\}. \quad \text{(A1)}$$

[17] See, for instance, Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), Kyle (1984, 1985, 1989), Admati (1988), Admati and Pfleiderer (1988), Foster and Viswanathan (1990), and Subrahmanyam (1991b). In his work on stock index futures, Subrahmanyam (1991a) models security payoffs as being the sum of a systematic factor and an idiosyncratic factor. Subrahmanyam (1991a) is not concerned with traders' incentives to collect information, however.

Since $\beta_a \delta_a \sigma_a^2 = \beta_b \delta_b \sigma_b^2$ in equilibrium, $\beta_b^2 \sigma_b^2 \{ \beta_b^2 \sigma_b^2 + \lambda^2 \sigma_\varepsilon^2 \} = \beta_a^2 \sigma_a^2 \{ \beta_a^2 \sigma_a^2 + \lambda^2 \sigma_\varepsilon^2 \}$.

Assume that $\beta_a \sigma_a \neq \beta_b \sigma_b$. We can write

$$\lambda^2 \sigma_\varepsilon^2 = \frac{(\beta_a^2 \sigma_a^2)^2 - (\beta_b^2 \sigma_b^2)^2}{\beta_b^2 \sigma_b^2 - \beta_a^2 \sigma_a^2} = -\left[\beta_a^2 \sigma_a^2 + \beta_b^2 \sigma_b^2 \right] < 0.$$

Contradiction, $\lambda^2 \sigma_\varepsilon^2$ cannot be negative. Therefore $\beta_a \sigma_a = \beta_b \sigma_b$. This gives us (ii). Substituting this result into equation (A1) yields (i). From (i) and equations (11) and (12),

$$\alpha^{eq} = \frac{r\{ \beta_a^2 \sigma_a^2 + \lambda^2 \sigma_\varepsilon^2 \}}{\lambda n} \frac{\sigma_a - \sigma_b}{\sigma_a + \sigma_b} + \frac{\sigma_a}{\sigma_a + \sigma_b}$$

$$> \frac{\sigma_a}{\sigma_a + \sigma_b} \quad \text{for} \quad \sigma_a > \sigma_b. \qquad \square$$

Scharfstein uses the following arguments to show that

$$\alpha^{eq} = \frac{(n+1)\sigma_a - \sigma_b}{n(\sigma_a + \sigma_b)} = \alpha^*.$$

The first equality is derived by substituting equations (2) and (3) into equation (4). To derive the second equality, substitute equations (2) and (3) into equation (1) this yields

$$\lambda^2 \sigma_\varepsilon^2 = \frac{\alpha n}{(\alpha n + 1)^2} \sigma_a^2 + \frac{(1-\alpha) n}{((1-\alpha)n + 1)^2} \sigma_b^2. \tag{A2}$$

Substituting this equation and equations (2) and (3) into equation (5), one gets

$$\mathbf{E}(v - p)^2 = \frac{\sigma_a^2}{\alpha n + 1} + \frac{\sigma_b^2}{(1-\alpha)n + 1}.$$

The first-order condition from differentiating the right-hand side of this equation with respect to α can be simplified to yield

$$\alpha^* = \frac{(n+1)\sigma_a - \sigma_b}{n(\sigma_a + \sigma_b)}, \qquad \square$$

Proof of Proposition 2: From equations (2) and (A2),

$$\delta_a = \frac{\sigma_\varepsilon}{\sigma_b \sqrt{n}} \left(\alpha \sigma^2 + \frac{(1-\alpha)(\alpha n + 1)^2}{[(1-\alpha)n + 1]^2} \right)^{-1/2} \quad \text{where} \quad \sigma = \frac{\sigma_a}{\sigma_b}.$$

Table II

Complete List of Simulations Conducted

This table reports the parameters for all simulations conducted during this research project, α^{eq} is the equilibrium proportion of traders observing the more informative signal, and α^* is the proportion required to maximize price informativeness. In each case α^{eq} was greater than α^*, except for cases in which both α^* and α^{eq} had corner solutions $\alpha^* = \alpha^{eq} = 1$. Each row lists a set of simulations where α^* and α^{eq} were initially calculated for σ_a^2 = "Initial σ_a^2" and σ_b^2 = "Initial σ_b^2." Then σ_a^2 was increased and σ_b^2 was decreased by "step size," and α^* and α^{eq} were recalculated. This continued for each row until σ_a^2 = "Final σ_a^2" and σ_b^2 = "Final σ_b^2." Table II lists a total of 1286 simulations. The parameter n is the number of informed traders and r is the risk aversion coefficient of each trader. The superscript a indicates that simulations stopped because $\alpha^* = \alpha^{eq} = 1$. The superscript b indicates that simulations stopped due to nonconvergence of numerical methods. σ_ε^2 is the variance of liquidity trading.

n	r	σ_ε^2	$\sigma_a^2 + \sigma_b^2$	Initial σ_a^2	Initial σ_b^2	Final σ_a^2	Final σ_b^2	Step Size
5	0.25	100	100	50	50	57 [a]	43	1
10	0.25	100	100	50	50	86 [a]	14	1
25	0.25	0.5	100	50	50	67 [b]	33	1
25	0.25	1	100	50	50	99	1	1
25	0.25	5	100	50	50	99	1	1
25	0.25	10	100	50	50	99	1	1
25	0.25	25	100	50	50	99	1	1
25	0.25	50	100	50	50	97 [a]	3	1
25	0.05	100	100	50	50	93 [a]	7	1
25	0.1	100	100	50	50	99	1	1
25	0.15	100	100	50	50	98 [a]	2	1
25	0.2	100	100	50	50	96 [a]	4	1
25	0.25	100	100	50	50	93 [a]	7	1
25	0.35	100	100	50	50	87 [a]	13	1
25	0.45	100	100	50	50	81 [a]	19	1
25	0.5	100	100	50	50	78 [a]	22	1
25	0.25	150	100	50	50	89 [a]	11	1
25	0.25	200	100	50	50	96 [a]	4	1
25	0.25	300	100	50	50	82 [a]	18	1
25	0.25	400	100	50	50	78 [a]	22	1
25	0.25	450	100	50	50	77 [a]	23	1
25	0.25	500	100	50	50	89 [a]	11	1
25	0.25	550	100	50	50	74 [a]	26	1
25	0.25	600	100	50	50	73 [a]	27	1
25	0.25	650	100	50	50	72 [a]	28	1
25	0.25	700	100	50	50	71 [a]	29	1
25	0.25	750	100	50	50	70 [a]	30	1
50	0.25	100	100	50	50	99	1	1
75	0.25	100	100	50	50	99	1	1
100	0.25	100	100	50	50	99	1	1
200	0.25	100	100	50	50	67 [b]	33	1
350	0.25	100	100	50	50	78 [b]	22	1
500	0.25	100	100	50	50	60 [b]	40	1

Table II—*Continued*

n	r	σ_ε^2	$\sigma_a^2 + \sigma_b^2$	Initial σ_a^2	Initial σ_b^2	Final σ_a^2	Final σ_b^2	Step Size
0.01	1	1	1	0.5	0.5	0.51[a]	0.49	0.01
2	1	1	1	0.5	0.5	0.99	0.01	0.01
5	1	1	1	0.5	0.5	0.72[a]	0.28	0.01
10	1	0.5	1	0.5	0.5	0.70[b]	0.30	0.01
10	1	1	1	0.5	0.5	0.74[b]	0.26	0.01
20	0.075	200	1	0.5	0.5	0.66[b]	0.37	0.01
30	1	1	1	0.5	0.5	0.51[b]	0.49	0.01
50	1	1	1	0.5	0.5	0.53[b]	0.47	0.01

[a]Simulations stopped because $\alpha^* = \alpha^{eq} = 1$.
[b]Simulations stopped due to nonconvergence of numerical methods.

Differentiating this expression with respect to α and multiplying by $-[\alpha/\delta_a]$ yields

$$\eta_a = -\frac{\alpha}{2}\,\frac{[(1-\alpha)n+1]^2\sigma^2+2n(1-\alpha)(\alpha n+1)-(\alpha n+1)^2+\dfrac{2n(1-\alpha)(\alpha n+1)^2}{(1-\alpha)n+1}}{\alpha\sigma^2[(1-\alpha)n+1]^2+(1-\alpha)(\alpha n+1)^2}.$$

Evaluating the right-hand side of this equation at $\alpha = \alpha^{eq} = \dfrac{(n+1)\sigma_a - \sigma_b}{n(\sigma_a+\sigma_b)}$ simplifies to

$$\eta_a = \frac{[(n+1)-\sigma][(n+1)\sigma-1]}{n(n+2)\sigma}. \tag{A3}$$

Repeating these steps for δ_b yields the second half of the proposition. □

REFERENCES

Admati, A. R., 1988, A noisy rational expectations equilibrium for multi-asset securities markets, *Econometrica* 53, 629–657.

——— and P. Pfleiderer, 1988. A theory of intraday patterns: Volume and price variability, *Review of Financial Studies* 1, 3–40.

Bulow, J., J. Geanakoplos, and P. Klemperer, 1985, Multimarket oligopoly: Strategic substitutes and complements, *Journal of Political Economy* 93, 488–511.

Diamond, D. W., and R. E. Verrecchia, 1981, Information aggregation in a noisy rational expectations economy, *Journal of Financial Economics* 9, 221–235.

Foster, F. D., and S. Viswanathan, 1990, A theory of interday variations in volume, variance, and trading costs in securities markets, *Review of Financial Studies* 3, 593–624.

Froot, K. A., D. S. Scharfstein, and J. C. Stein, 1992, Herd on the street: Informational inefficiencies in a market with short-term speculation, *Journal of Finance* 47, 1461–1484.

Grossman, S. J., and J. E. Stiglitz, 1980, On the impossibility of informationally efficient markets, *American Economic Review* 70, 393–408.

Hayek, F. A., 1945, The use of knowledge in society, *American Economic Review* 35, 519–530.

Hellwig, M. F., 1980, On the aggregation of information in competitive markets, *Journal of Economic Theory* 22, 477–498.

Holden, C. W., and A. Subrahmanyam, 1991, Risk aversion, imperfect competition, and long-lived private information, Discussion Paper No. 493, Graduate School of Business, Indiana University.

———, 1992, Long-lived private information and imperfect competition, *Journal of Finance* 47, 247–270.

Kyle, A. S., 1984, Market structure, information, futures markets, and price formation, in G. G. Storey, A. Schmitz, and A. H. Sarris, eds: *International Agricultural Trade: Advanced Readings in Price Formation, Market Structure, and Price Instability* (Westview Press, Boulder, Colo.), pp. 45–64.

———, 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315–1335.

———, 1989, Informed speculation with imperfect competition, *Review of Economic Studies* 56, 317–356.

Lundholm, R. J., 1991, Public signals and equilibrium allocation of private information, *Journal of Accounting Research* 29, 322–349.

Spiegel, M., and A. Subrahmanyam, 1992, Informed speculation and hedging in a noncompetitive securities market, *Review of Financial Studies* 5, 307–330.

Subrahmanyam, A., 1991a, A theory of trading in stock index futures, *Review of Financial Studies* 4, 17–51.

———, 1991b, Risk aversion, market liquidity, and price efficiency, *Review of Financial Studies* 4, 417–441.