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Security Design

ARNOUD W. A. BOOT and ANJAN V. THAKOR*

ABSTRACT

We explain why an issuer may wish to raise external capital by selling multiple financial claims that partition its total asset cash flows, rather than a single claim. We show that, in an asymmetric information environment, the issuer's expected revenue is enhanced by such cash flow partitioning because it makes informed trade more profitable. This approach seems capable of shedding light on corporate incentives to issue debt and equity, as well as on financial intermediaries' incentives to issue multiple classes of claims against portfolios of securitized assets.

THERE HAS RECENTLY BEEN substantial interest in the economic underpinnings of financial security design. There are perhaps many reasons for this, not the least of which may be the spectacular financial innovation witnessed recently. The key issues in security design are to explain why it may be important to partition the cash flows from an asset across financial claims with different risk characteristics, and to determine the optimal partition.

The main goal of this paper is to address two questions. First, why would a firm raising external capital wish to issue *multiple* types of financial claims against its cash flows? Second, why do firms pool individual assets into a portfolio and *then* partition the portfolio cash flows? These questions cut to the heart of security design. For example, why do firms simultaneously issue debt and equity, with high seniority for debt? Why do financial intermediaries issue multiple classes of claims, rank ordered by seniority, when they securitize loans and mortgages? Why does securitization involve the pooling of individual assets?

We develop a noisy rational expectations model which predicts that it is a revenue-maximizing strategy for an issuer to partition its asset cash flows across different financial claims when the value of these cash flows is a priori unknown to investors but can be discovered at a cost by some. We show that splitting a security into two components—one “informationally insensitive” and the other more “informationally sensitive” than the composite

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security—makes informed trading more profitable. The reason is that informed traders with constrained wealth endowments earn a higher return on their investment in information by allocating their wealth to the information-sensitive security. The consequent stimulation of informed trading moves the equilibrium price of the intrinsically more valuable security closer to its fundamental value, and increases the higher-valued issuer's total expected revenue.

Formalizing this intuition requires taking into account four key issues. First, trading on information should be profitable. This requires in turn that the equilibrium price should not be a sufficient statistic for the information processed by informed traders (Grossman and Stiglitz (1980)). Thus, there should be some noise in trading. Second, since our intuition is driven by the impact of security design on informed trading, the demand from informed traders should be endogenous. Third, we need a market-clearing mechanism such that the equilibrium price reflects at least *some* of the information of the informed traders. Finally, while our intuition suggests that the higher-valued firm desires to split its security, we need to verify that the lower-valued firm adopts the same strategy in a pooling (sequential) equilibrium. This is necessary because informed traders earn no profits in a separating equilibrium, and would therefore not acquire information.

Our model captures these issues in a simple way. We consider three types of traders: pure liquidity traders who are uninformed and whose demand is exogenous, traders who become informed at a cost about the firm's intrinsic value, and uninformed discretionary traders (UDTs) who could become informed but choose to remain uninformed; the demand from the last two groups of traders is endogenous. The privately informed issuing firm asks an intermediary to sell the security by soliciting bids from traders. The UDTs are the marginal investors and they submit bids to clear the market after observing the total demand from the liquidity and informed traders. The equilibrium price is set such that the UDTs earn zero expected profits, conditional on the information contained in the aggregate order flow. In this setting, the issuer determines security design to stimulate information production which leads to higher informed demand and greater information revealed on average by the aggregate order flow.

Our research is related to two strands of the literature, one on optimal security design and the other on the creation of "liquid securities." In their elegant analysis of security design under symmetric information, Allen and Gale (1988) consider a securities market rendered incomplete by short sales restrictions¹ and show the optimality of splitting the total cash flow across as many securities as there are states of nature. With investors possessing smooth preferences, the optimality of such "extremal" security design leaves no room for debt and equity. Madan and Soubra (1991) introduce marketing

¹ In a subsequent paper, Allen and Gale (1991) show that when unlimited short sales are allowed, financial innovation is not necessarily efficient, and markets may not be complete even when the cost of issuing new securities is negligible.

costs in the Allen and Gale framework to show that nonextremal securities may be optimal.

Our approach differs significantly. Rather than assuming risk-averse investors, as Allen and Gale and Madan and Soubra do, we assume pervasive risk neutrality. Thus, our security design is not aimed at improving the allocation of risky wealth in an incomplete market. Rather, it maximizes the informativeness of equilibrium prices and hence the issuers' expected revenue. Moreover, the earlier papers do not explain why securitization—a special form of security design—involves first pooling individual assets and then partitioning the portfolio cash flows; we do.

The other related literature is on liquid securities and consists of recent contributions by Gorton and Pennacchi (1990) and Subrahmanyam (1991). Gorton and Pennacchi argue that the trading losses suffered by uninformed investors can be diminished if trading is confined to relatively information-insensitive securities such as insured bank deposits. Thus, the issuer can make such investors better off by splitting the total asset cash flow so as to create a liquid asset whose payoff does not embody private information. Gorton and Pennacchi show that if the issuer does not provide such a split, the uninformed investors will avail of the desired split by issuing claims against the total cash flow themselves. Similarly, Subrahmanyam (1991) shows that strategic liquidity traders may prefer baskets of securities to individual securities because security-specific components of adverse selection get diversified away in the baskets.

One significant distinction between these papers and ours is that the impetus for splitting up the security is supply-driven in our analysis, whereas it is demand-driven in the earlier work. We consider security design from the issuing firm's perspective. Therefore, gains from improving security design accrue primarily to the issuing firm, and the incentive to optimize this design is supply induced. Another difference is that the driving force behind security splitting in the earlier research is the desire to create a liquid (informationally insensitive) security, whereas in our analysis it is the desire to create a relatively illiquid (informationally sensitive) security. Also, while these papers have explained why we observe diversified portfolios of securities, they do not explain why such portfolios are then partitioned. Indeed, such partitioning seems antithetical to the desire to create liquidity, which is the driving force behind the analysis in these papers.

This paper is organized as follows. The basic model and an analysis of the "composite security" equilibrium appear in Section I. Optimal security design is taken up in Section II where the "split securities" equilibrium is derived. In these two sections we allow the marginal uninformed traders to take negative as well as positive demand positions to clear the market. In Section III we examine the robustness of our model and discuss extensions. We first preclude negative demand from the UDTs, so that rationing may be needed to clear the market with a fixed security supply. Next, we allow limited short sales by all agents. We then relax the assumption of the basic model that all potentially informed traders face the same information acquisition cost. We

also take up some other robustness issues. The desirability of splitting the composite security is sustained in all of these extensions. The analysis thus far is based on a single initial security. In Section IV we permit multiple securities and explicate the issuer's gains from forming a (diversified) portfolio of these securities that is then partitioned. Section V contains a discussion of applications of our analysis. We claim that our model explains debt and equity as well as the stratified financial claims commonly encountered with securitization. Section VI concludes.

I. The Basic Model and Market Equilibrium with a Single Security

A. The Model

A.1. Information Structure and Preferences

We consider a two-date model. At date $t = 0$, a firm offers for sale a fixed supply of a "composite security," which represents a claim against all of the firm's assets at date 1. We normalize this supply to be 1 unit. The firm can be one of two types: high quality (good) and low quality (bad). The date-1 values of the good (G) and bad (B) firms are $\bar{x}$ and $\underline{x}$ respectively, with $0 < \underline{x} < \bar{x} < \infty$. At date 0, the firm knows its own type, but no one else knows a priori the firm's "true" type. The commonly known prior probability is $q \in (0, 1)$ that the firm is of high quality and $1 - q$ that it is of low quality. There is no discounting between dates 0 and 1, and there is universal risk neutrality. At date 1, each firm's "true" value becomes common knowledge.

A.2. Types of Traders

There are three types of investors/traders in the market: "pure" liquidity traders, UDTs, and informed traders. The aggregate asset demand, l , of the "pure" liquidity traders is random and exogenously specified by the continuously differentiable probability density function $f(l)$ which has a support of $(0, \infty)$.² We specify l in terms of the number of dollars the liquidity traders wish to invest in the security.³ Like the "pure" liquidity traders, the UDTs are a priori unaware of the precise date-1 value of the asset. However, they condition their aggregate demand on their observation of the sum of the demands of the other two types of traders. Each UDT can choose to either

² We have not bounded the support for $f(l)$ from above because we want to preclude states in which the aggregate demand perfectly reveals the presence of the informed traders in the market. In Section III, when we permit limited short sales, we will introduce a bounded support for $f(l)$, and analyze a version of the model in which the aggregate demand is sometimes perfectly revealing.

³ We can think of the liquidity traders as "naive" investors who have investible wealth but do not consider price-relevant data in making their investment decisions. For example, one can imagine an individual who has a fixed fraction of his salary automatically invested every period in predesignated securities; many mutual funds have automatic salary withdrawal plans for such investments. The precise number of such individuals in any period will be a random variable, so that their aggregate demand will be random.

remain uninformed or acquire information at a cost; adopting the latter strategy transforms him into an informed trader. For now, we assume that the information acquisition cost, $M \in (0, \infty)$, is the same for all prospective informed traders. We will later permit M to be heterogeneous cross-sectionally. In either case, the total number of informed traders is endogenously determined in this model by a “marginal investor” condition. An informed trader knows precisely whether the security is high quality or low quality. We define D_I as the aggregate demand (in dollar terms) from the informed traders.

A.3. Market Structure and Clearing

The securities market is competitive in the sense that any UDT's *expected* net gain (or net present value) from buying a security is zero in equilibrium. All of the UDTs who choose not to become informed end up being the marginal holders of the security.⁴ The issuing firm delegates the sale of the securities to an investment banker. The liquidity traders and the informed traders submit their orders to this investment banker; note that the demand of neither of these two groups of traders depends on the price. After observing the total demand, the investment banker communicates this information to the UDTs who absorb the net trade in the security, denoted by ξ . The market-clearing price of the security is one that produces zero expected profit for the UDTs. In the present version of the model, we permit short sales by the UDTs so that their demand can be negative. We will later limit their demand to be nonnegative. Although in this analysis all traders other than the UDTs are constrained to submit nonnegative demands, we later permit limited short sales for all.

Let ξ be the UDTs' demand (in terms of number of units) for the security, and define D as the aggregate demand (in dollar terms) from the liquidity traders and the informed traders. let P be the price of the security, so that we have

$$\xi = 1 - [D/P] \quad (1)$$

since the supply of the security from the firm is fixed at unity. The end-of-period value of the security is random for the UDTs since they do not know its precise value. Let $\tilde{x} \in \{\underline{x}, \bar{x}\}$ represent this random variable. Then, the market-clearing price of the security is

$$P^e = E(\tilde{x} \mid \xi). \quad (2)$$

That is, the UDTs observe only the total order flow from outsiders, D , and then set the (equilibrium) price to equate demand and supply such that they earn zero expected profit. We assume that each individual trader who sub-

⁴One way to think about this is to view the uninformed discretionary liquidity traders as forming a coalition called a “market maker”. One can imagine there being a sufficient number of other “professional” market makers, so that the market is competitive. The market maker is then the recipient of all the orders, and an agent who takes the position in the security required to clear the market at a price that yields him zero expected profit.

mits an order appears to the UDTs observationally identical and is also atomistic. That is, there is a continuum of traders of each type so that the demand-relevant measure of each individual trader is zero. Demand is positive only in the aggregate when integrated over a set of traders with positive measure.

A.4. Informational and Wealth Considerations for Informed Traders

Each potentially informed trader has $M + 1$ units of wealth, so that he has \$1 to invest in the security after investing M in acquiring information. The alternative to investment at date 0 is consumption which is valued the same as date-1 consumption. For now, an informed trader can neither borrow nor short sell.

Upon investing M in information acquisition, the trader receives a signal ϕ ; we assume for now that this signal reveals the firm's precise value to him, and later permit the signal to be noisy.⁵ Each informed agent receives the same signal, as in the "photocopy" information models of Grossman and Stiglitz (1980) and Admati and Pfleiderer (1987). Since the informed agent is risk neutral and his signal is perfectly revealing, his individual demand for the security will be at one of the two corners as long as he anticipates that the equilibrium price will reflect his information only noisily. Assuming for the moment that this property of the equilibrium price holds,⁶ we can write an individual informed trader's demand, d_I , as

$$d_I = d_I(\phi) = \begin{cases} 1 & \text{if } \phi = G \\ 0 & \text{if } \phi = B \end{cases} \quad (3)$$

where we have stated the demand in dollar terms rather than the number of securities demanded. The reason is that the equilibrium price is a random variable at the time demand is submitted, so that a trader can only specify how much he wishes to invest.

We assume that the number of traders who become informed takes values in a (possibly unbounded) continuum. Let θ be the (Lebesgue) measure of the set of informed traders.⁷ Hence, the aggregate dollar demand from informed traders is

$$D_I = D_I(\theta, \phi) = \theta d_I(\phi).$$

A.5. Decision Problem of the Informed Traders

Each trader who becomes informed ends up investing M in information acquisition and 1 in purchasing the security if his signal reveals $\phi = G$. The condition which determines θ says that, for the marginal informed investors,

⁵ Clearly, prior to investing M , ϕ is unknown to the trader, although its probability distribution is common knowledge.

⁶ We will verify that this is indeed true in a noisy rational expectations equilibrium. If the price perfectly reveals his information, the informed trader will be indifferent, but this possibility may dissuade him from investing in information in the first place.

⁷ We have in mind a situation in which exogenous parameter values are such that $\theta \in (0, 1)$.

the *expected* net gain from becoming informed is zero. Let V represent an investor's expected net gain to being informed, P^e the equilibrium price of the security as set by the market maker, and $P(\phi)$ the value of the security privately known to the informed agent who receives signal ϕ . Note that D will be a function of ϕ and l , so that (1) and (2) imply that $P^e = P^e(D(\phi, l))$. Since an informed trader will submit a buy order (choose $d_I = 1$) only when his signal reveals $\phi = G$, we can write

$$V = -M + q \int_0^\infty \{[\bar{x} - P^e(\theta + l)]/P^e(\theta + l)\} f(l) dl, \quad (4)$$

where we have substituted $P(\phi = G) = \bar{x}$ and $D(\phi, l) = \theta + l$ for $\phi = G$, and $[P^e(\theta + l)]^{-1}$ is the number of the units demanded. The equilibrium value of θ , call it θ^* , is determined by the following marginal condition

$$V(\theta^* | q, \underline{x}, \bar{x}, M, f(l)) = 0. \quad (5)$$

With M cross-sectionally constant, (5) holds for marginal and inframarginal informed investors.

A.6. Definition of Equilibrium

A (noisy) rational expectations Nash equilibrium is:

1. a measure of informed traders, θ^* , satisfying (5), in which each informed trader takes as given the equilibrium strategies of the other informed traders and the UDTs, but assumes that the impact of his own trade on the price is negligible;
2. an aggregate security demand from informed and uninformed liquidity traders equal to $D^*(\phi, l) = \theta^* d_I(\phi) + l$, with $d_I(\phi)$ given by (3);
3. a market-clearing price P^e given by (2), which is determined by the UDTs in such a way that supply and demand for the security are equated and the expected net gain to a priori uninformed security purchasers, conditional on the information contained in the order flow, $D^*(\phi, l)$, is zero; and
4. a security design by the issuing firm which, taking as given the above behavior by traders and the UDTs, maximizes the issuer's total expected revenue.

This Nash equilibrium is a strategic game in which the informed issuer moves first with its security design, and the UDTs respond with a price after observing total demand.

B. The Analysis

We first compute the equilibrium price of the security, taking as given that the issuer can issue only the composite security. For a realization $D(\phi, l)$ of

aggregate security demand $l + D_I$, the UDTs will (given the definition of equilibrium) set

$$\begin{aligned} P^e(D(\phi, l)) &= \Pr(\phi = G | D(\phi, l)) \bar{x} + [1 - \Pr(\phi = G | D(\phi, l))] \underline{x} \\ &= \Pr(\phi = G | D(\phi, l)) [\bar{x} - \underline{x}] + \underline{x} \end{aligned} \quad (6)$$

where $\Pr(\cdot | \cdot)$ is the conditional probability measure. Using Bayes' rule, we have

$$\Pr(\phi = G | D(\phi, l)) = \frac{f(D - \theta)q}{f(D - \theta)q + f(D)[1 - q]} \quad (7)$$

where we have taken into account that an aggregate demand of $D = D(\phi, l)$ implies a liquidity demand of $D - \theta$ if $\phi = G$, and D if $\phi = B$.⁸ We can now establish a result that will be useful in our subsequent analysis (primes denote partial derivatives).

LEMMA 1: $f'(l + \theta) < 0 \ \forall l \in (0, \infty)$ is sufficient for $\partial V / \partial \theta < 0$.

Note that this condition on the density function $f(\cdot)$ is sufficient (but not necessary) for our subsequent results; weaker, albeit more complicated, conditions can be invoked for these results. What we need is that the expected net gain to becoming informed be decreasing in the number (measure) of traders who become informed in equilibrium. This intuitive property holds under fairly general conditions.⁹ With this result in hand, we can examine the effect of informed trading on the type- G issuing firm's expected revenue. Note that $D(\phi, l) = \theta + l$ for a type- G issuing firm. Using (6), this expected revenue, R , can be written as

$$R = \int_0^\infty P^e(\theta + l) f(l) dl = \int_0^\infty \Pr(\phi = G | \theta + l) [\bar{x} - \underline{x}] f(l) dl + \underline{x}. \quad (8)$$

We now have the following result.

PROPOSITION 1: *The type- G issuing firm's equilibrium expected revenue, R , is increasing in the measure of the set of informed traders, θ . The type- B firm's R is decreasing in θ .*

This analysis of the market equilibrium with the composite security provides a benchmark for our subsequent examination of the gains from decomposing the security. Since the type- G issuer's expected revenue is increasing in the total demand from informed traders, our analysis suggests that this issuer should seek to design securities that induce more traders to become

⁸ Note that $D \geq \theta$ is needed to prevent perfect revelation. When $\phi = G$, this condition is obviously satisfied. Hence, whenever informed traders are in the market (i.e., $\phi = G$), they never face perfect revelation.

⁹ Unfortunately, we are unable to find a weaker sufficient condition in the general case. For the special case of $q = 0.5$, the *likelihood ratio property* is sufficient (we thank Peter DeMarzo for this observation). Note that all that is needed for our result is that the *marginal* value of being informed is *decreasing* in the number of informed traders.

informed. As it turns out, the issuer can accomplish this by decomposing securities. It will enhance the “information sensitivity” of some, and induce informed trading in these securities. On the other hand, the type-*B* issuer would prefer to offer securities that decrease the dollar investment of informed trades. But this would reveal the issuer’s type. Hence, it mimics the type-*G* issuer. This intuition forms the basis of our analysis of optimal security decomposition in the next section.

II. Decomposing the Composite Security

Our goal in this section is to show that selling a composite security is not revenue-maximizing for the type-*G* issuer. A simple decomposition of the composite security can increase the issuer’s total revenue. Consider the composite security being split up into two securities: a senior security *A* which is not “information sensitive,” and a junior security *S* which is more information sensitive than the composite security.¹⁰ We will shortly explain why this is the optimal partition. *A* promises a sure date-1 payoff of $\underline{x}$. *S* promises a claim against all of the issuing firm’s residual value after *A* is paid off. Since either type of issuer can pay off $\underline{x}$, the date-1 payoff to *A* is $\underline{x}$ with probability one. Security *S*, however, will pay off $\bar{x} - \underline{x}$ if issued by the type-*G* issuer (probability q), and zero if issued by the type-*B* issuer (probability $1 - q$).

Gorton and Pennacchi (1990) suggest that the creation of a less-information-sensitive security steers liquidity demand toward that security. We now account for this possibility by assuming that a fraction $\alpha \in [0, 1]$ of liquidity demand will be diverted from *S* to *A*.¹¹ Thus, liquidity demand for security *S* is now given by the density $f_s(\cdot)$, and a realization l^0 of liquidity demand for the composite security (when it is the only security offered, as in the previous analysis) “corresponds” to a realization $[1 - \alpha]l^0$ of liquidity demand for security *S* (when *S* and *A* are offered instead of the composite security), i.e., $f_s([1 - \alpha]l^0) = f(l^0) \forall l^0$. We wish to reiterate that liquidity demand is exogenous in our model, as is the anticipated migration α of liquidity demand to security *A*. We could therefore assume the same exogenous liquidity demand for each of the split securities *A* and *S* as for the composite security. It would then follow immediately that *A* and *S* represent the optimal split for the issuer. More generally, however, it seems intuitive to assume that α is weakly monotonically increasing in the difference in the information sensitivities of securities *A* and *S*. But even in this case, we will argue that *A* and *S* will be the optimal split as long as “sufficient” liquidity demand remains for *S*. We now have the following result.

¹⁰ Information sensitivity here refers to the percentage divergence between the “true” value of the security and its value based on the *prior* assessment of the uninformed.

¹¹ Although the UDTs in our model break even on average, the liquidity traders lose on average because they take “unbalanced positions,” buying too much of the security when the price is too high and too little when it is too low.

PROPOSITION 2: *The total equilibrium expected revenue that the type-G issuer obtains by issuing securities A and S is higher than that obtained by issuing the composite security. Thus, in equilibrium the type-G firm splits its composite security into A and S. Although the total expected revenue of the type-B firm is lower in the equilibrium involving securities A and S than in the equilibrium involving only the composite security, the type-B firm also splits its composite security into A and S. The Nash equilibrium involving securities A and S, when augmented by the UDTs' belief that a firm issuing the composite security is type-B with probability one, is sequential and survives the universal divinity refinement of Banks and Sobel (1987).*

This proposition asserts that splitting the composite security is wealth enhancing for the more valuable firm as long as some liquidity trade remains in S .¹² This result holds despite security splitting having an ambiguous effect on the equilibrium informed demand. On the one hand, splitting increases the value of information acquisition for investors in S and thus encourages informed demand. On the other hand, the migration of liquidity traders to A implies less noise in the price of S , which makes informed trading easier to “detect” and discourages informed demand. It is interesting that, despite this, the wealth enhancement from splitting for the type-G firm is qualitatively unaffected by the manner in which splitting causes liquidity demand to be allocated to the split securities, as long as sufficient liquidity trade remains in S to ensure positive informed demand in equilibrium (i.e., α stays below an upper bound).

The intuition for these results is as follows. By stripping away A from the composite security, the issuer separates out that component of firm value about which, loosely speaking, there is no informational asymmetry. Clearly, informed traders can hope to gain nothing from buying this security, even in an ex post sense.¹³ By thus separating out A , the issuer permits an informed trader to invest *all* of his wealth in the residual security S , rather than being implicitly forced to invest some of it in A as is the case when he purchases the composite security. That is, at the margin, an informed trader has more to gain by being informed when he has the option to purchase S than when he could only purchase the composite security. This “informational leveraging up” of his wealth position means that an informed trader can be compensated for his information acquisition cost with a *smaller* divergence between the “true” value and the equilibrium price for S than for the composite security. Since this divergence is what a type-G firm seeks to minimize, and A is priced at its true value because information about it is symmetric, splitting

¹² It is important that enough liquidity trade remains in security S , such that becoming informed is optimal for a *strictly positive* proportion of traders. If “not enough” liquidity trade remains in security S , the equilibrium price may reflect “too much” of the information of informed traders, and the expected return on information acquisition may become nonpositive.

¹³ That is, *after* they have expended M to become informed. In our model, in an ex ante sense the gain from being informed is zero in equilibrium, once M is accounted for. However, informed traders do expect to gain in an ex post sense, i.e., treating M as a sunk cost.

the composite security makes the type-*G* firm better off. And as long as there is a positive informed demand for *S*, the impact of splitting on liquidity demand does not jeopardize this wealth enhancement because it only affects the *level* of informed trading.

The situation is different for the type-*B* firm. In the “split securities” equilibrium, it is strictly worse off than in the “composite security” equilibrium because informed demand is more informative. The key is that aggregate demand is more informative not only when the informed traders place buy orders, but also when they don’t. However, despite this the type-*B* firm splits because it would otherwise be unambiguously identified as a type-*B* firm.

While our decomposition of the composite security into securities *A* and *S* was apparently arbitrary, it is indeed the *optimal* way to split the security. *A* is made *senior* to *S* because the type-*G* issuer wishes to have both securities priced as close to their “true” values as possible. By assigning *A* seniority over *S*, *A* is made *information insensitive* and hence priced at its true value. Moreover, since this makes *S* more information sensitive than the composite security, it makes informed trading more profitable, which moves its price closer to its true value.

As the preceding discussion indicates, the result that the type-*G* firm benefits from greater informed trading depends crucially on the assumed wealth constraints faced by investors. These constraints ensure that the risk-neutral informed investors do not take infinitely large positions when they discover a type-*G* firm. We could relax wealth constraints by allowing investors to borrow. Note, however, that borrowing is subject to informational distortions because lenders will be unable to distinguish between informed and uninformed traders. Moreover, borrowing by informed *and* uninformed traders will distort the traders’ incentives to become informed.

It is worth noting that mechanisms other than security splitting could be used by the issuer to stimulate informed trading and increase expected revenue.¹⁴ For example, firms could maximize the impact of information acquisition by selling futures contracts that require no net investment, or they could issue warrants or optionlike derivatives. These could create high “information leverage.” If despite doing this, sufficient noise remains in the prices of the underlying security and the derivative instruments to preserve information production incentives, then it will be optimal for the issuer to maximize this information leverage. However, if such actions eliminate noise in prices by driving away liquidity traders, then maximizing information leverage will not be optimal for the issuer. Maintaining some noise trading is important because of the Grossman and Stiglitz (1980) paradox with perfectly revealing prices.

The general implication of our analysis is not that it will always be optimal for a firm to create a risk-insensitive security. Indeed, for some payoff distributions, this may not even be feasible. Rather, our theory of security

¹⁴ We are grateful to René Stulz and David Hirshleifer for pointing this out.

design suggests that the firm will wish to issue a security that is as *information sensitive* as possible because this will make information production more profitable and augment the issuer's expected revenue. As a consequence, the firm's remaining security will automatically be less information sensitive than the composite security.¹⁵

III. Extensions and Generalizations

A. Market Clearing with Possible Rationing

We have assumed thus far that the UDTs absorb all the net trade in the security, even if that implies a negative demand for the security on their part. An alternative market-clearing process is one in which traders are rationed when demand exceeds supply. Such rationing is commonly observed for oversubscribed initial offerings. Note that whether there is rationing in security allocation depends on the market microstructure, which we take as exogenous. It is not our objective to explain how securities are sold, just how they are packaged. Thus, our purpose in examining the equilibrium with possible rationing is to see how robust our results are with respect to alternative market-clearing mechanisms, particularly one that is commonly observed.

Consider an investment banker who takes a fixed unit supply of the security to the market, commits to supply at least a predetermined fraction τ of it to the UDTs, and then receives orders from observationally indistinguishable informed and liquidity traders. If the total demand from the informed and liquidity traders exceeds $1 - \tau$, then the informed and liquidity traders are randomly rationed, i.e., each stands an equal chance of being rationed. If their total demand falls short of $1 - \tau$, then the UDTs are allowed to bid for a sufficient portion of the supply to enable the market to clear. The minimum allocation τ to the UDTs is to ensure these traders' participation.¹⁶

This kind of rationing will potentially affect the endogenously determined measure of those who become informed. For an individual trader, the expected profit from becoming informed is:

$$\hat{V} = -M + q \int_0^{1-\theta-\tau} \{[\bar{x} - P^e(\theta + \tau + l)]/P^e(\theta + \tau + l)\} f(l) dl \\ + q \int_{1-\theta-\tau}^{\infty} \left[\frac{1}{l + \theta + \tau} \right] \{[\bar{x} - P^e(\theta + \tau + l)]/P^e(\theta + \tau + l)\} f(l) dl \quad (9)$$

¹⁵ From a social welfare point of view, splitting securities has ambiguous implications. If migration is small, it will increase (dissipative) information production, and the associated costs are borne in equilibrium by the liquidity traders. However, if sufficient migration of liquidity demand to A occurs, splitting will reduce aggregate information production, and this enhances social wealth.

¹⁶ If the UDTs are not allocated a fixed fraction of the supply, then when the demand from the liquidity and informed traders exceeds the supply, the UDTs, in their role as "residual claimants," would not get any portion of the supply. With no bidding by the UDTs, the price becomes indeterminate. We are grateful to the referee for alerting us to this.

Note that the total demand from the informed and liquidity traders is $l + \theta$ when $\phi = G$, whereas the supply available to them is $1 - \tau$. As long as the total demand from the liquidity traders is less than or equal to $1 - \theta - \tau$, every trader's order can be filled. But when l exceeds $1 - \theta - \tau$ (so that $l + \theta + \tau > 1$), each trader's order is filled only with probability $[1/(l + \tau + \theta)] \in (0, 1)$. We will assume throughout that exogenous parameter values are such that $\theta < 1$, although this assumption is innocuous for our results. The equilibrium measure of informed traders, $\hat{\theta}$, is now determined by the following condition on the expected profit of the marginal informed trader.

$$\hat{V}(\hat{\theta} | q, \underline{x}, \bar{x}, M, f(l)) = 0. \quad (10)$$

Similar to the analysis in the no-rationing case, it can now be shown that the type- G firm's expected revenue is increasing in the measure of the set of informed traders, and the type- G insurer can enhance its expected revenue by issuing securities A and S instead of the composite security. The intuition is identical to that underlying the corresponding results in the no-rationing case. The reason is that the possibility of rationing does not alter the fact that splitting up the security makes informed trade more profitable and hence moves the type- G issuer's total revenue closer to what it would be if its true type were commonly known. However, as the next result shows, the type- G firm's revenue is adversely impacted by the possibility of rationing.

PROPOSITION 3: *In the composite security equilibrium as well as the split securities equilibrium, the total expected revenue of the type- G (or type- B) issuer is lower (or higher) when there is rationing than when there is no rationing.*

The intuition is that the possibility of rationing reduces the expected allocation to each informed trader and thus the expected gain from becoming informed; there is thus less informed trading when rationing is possible. Since, holding fixed the distribution of liquidity demand, the type- G issuer's expected revenue is lower when there is less informed trading, such an issuer is made worse off by rationing. This effect is reversed for the type- B issuer.

B. Heterogeneous Information Production Costs

We have assumed thus far that all potentially informed traders face the same information acquisition cost. This means that marginal and infra-marginal informed traders are identical. Suppose instead that trader i faces an information acquisition cost of M_i , and M_i varies cross-sectionally. We will continue to assume that trader i has a total initial endowment of $M_i + 1$. This means that each trader has exactly \$1 to invest in the security.¹⁷ The question is: how does heterogeneity in information acquisition costs affect the issuer's revenue and the economics of splitting up the composite security?

¹⁷ If we had held constant across traders the total wealth endowment of each trader rather than the amount he has to invest in the security, we would get perfect revelation of the trader's identity based on his investment demand.

We will examine this issue in the context of the no-rationing market-clearing scenario. For the composite security, the analog of (4) is

$$V_i = -M_i + q \int_0^\infty \{[\bar{x} - P^e(\theta + l)]/P^e(\theta + l)\} f(l) dl \quad (11)$$

Define I as the indicator set for all traders who have the *potential* to be informed, i.e., $i \in I$, and let M_i vary cross-sectionally over the compact interval $[M^-, M^+]$ which contains M (the constant information acquisition cost in our basic model). Let $h(M_i)$ be a density function representing the “weight” of an informed trader with information acquisition cost M_i in the population of traders who have the potential to be informed, and $H(\cdot)$ the associated cumulative distribution function. For any M_i , let $\theta(H(M_i))$ be the aggregate demand from (or the measure of) the set of informed traders with information acquisition costs not exceeding M_i . Now let θ^0 be the measure of the set of informed traders in equilibrium and I^0 the indicator set corresponding to the set of informed traders, i.e., each trader $i \in I^0$ becomes informed in equilibrium. Then, θ^0 is determined by:

$$V^0(\theta^0 | q, \underline{x}, \bar{x}, f(l), h(M_i)) = 0. \quad (12)$$

One noteworthy difference between (5) and (12) is that, unlike the former, the latter holds only for the marginal investor. That is, when M_i varies cross-sectionally, inframarginal informed traders make strictly positive expected profits. It is relatively straightforward to go through the steps we followed in the case in which M is cross-sectionally invariant to show that splitting up the security is beneficial for the type- G issuer even when information acquisition costs vary cross-sectionally. We skip the details here. What we wish to explore is whether heterogeneity in information acquisition costs benefits the issuing firm. It turns out that the answer depends on $h(M_i)$, as the following proposition asserts.

PROPOSITION 4: 1. Composite Security Equilibrium: *In this equilibrium, if*

- a. $\theta(H(M)) < \theta^*$, *then the type- G (or type- B) issuer's total expected revenue is lower (or higher) with heterogeneous information acquisition costs than with a constant information acquisition cost, with the reverse result holding if $\theta(H(M)) > \theta^*$;*
- b. $\theta(H(M)) = \theta^*$, *then the total expected revenue is identical with heterogeneous and constant information acquisition costs.*

2. The Split Securities Equilibrium: *In this equilibrium, if*

- a. *there is no migration of liquidity demand to security A, the type- G (or type- B) issuer's total expected revenue is lower (or higher) with heterogeneous information acquisition costs than with a constant information acquisition cost for a strictly larger set of $h(M_i)$ specifications than for the composite security equilibrium;*

- b. splitting causes a sufficiently large migration of liquidity demand to A (with constant and heterogeneous acquisition costs), then heterogeneous information acquisition costs enhance the value of splitting to type-G issuers.*

This proposition is intuitive. When $\theta(H(M))$ is relatively low, there is a low density of potentially informed traders with low information acquisition costs, as $h(M_i)$ puts relatively more weight on high-cost information acquisition traders. Consequently, the cumulative informed demand in equilibrium is low relative to the cumulative informed demand with a constant M , regardless of whether it is the composite security equilibrium or the split securities equilibrium. Our earlier analysis (Proposition 1) tells us that the type-G issuer's total expected revenue suffers due to this depressed informed demand. The latter half of the proposition addresses the effect of heterogeneity in information acquisition costs on the gains from splitting. Whether the gains from splitting are greater or smaller with heterogeneity depends on $h(M_i)$. With liquidity demand unchanged by splitting, there is a sense in which heterogeneity reduces the gains from splitting. The reason is that, loosely speaking, heterogeneity leads to a smaller increase in overall informed demand as one moves from a composite security to split securities. On the other hand, if splitting causes some liquidity demand to migrate to A , then the effect of heterogeneity on the gains from splitting depends on how much migration occurs. The lower liquidity demand in security S , relative to the case in which liquidity traders do not migrate to A , induces a lower informed demand for S in equilibrium. This means that the information acquisition cost of the marginal informed trader is lower, implying that prices can reflect greater information before rendering information acquisition unprofitable. Heterogeneity in this case can lead to a larger gain from splitting because it creates more room for participation by informed traders.

The upshot of the discussion of heterogeneous information acquisition costs is that our earlier conclusions are substantively unaffected. We turn next to another robustness issue.

C. Limited Short Sales

Thus far we have assumed that there is no short selling allowed for the liquidity and informed traders. The reason for doing this is that unlimited short selling by the informed agents would lead to a fully revealing equilibrium price. We will show now that *limited* short sales will not qualitatively affect our results.

Suppose liquidity demand is uniformly distributed, with density $f(l)$, and $l \in L \equiv (-\bar{l}, \bar{l})$, $\bar{l}, \bar{l} > 0$. We no longer need the assumption that $f'(l) < 0$. Each UDT demands $d \in [-\beta, 1]$ with $\beta > 0$. As before, the UDTs are the marginal holders of the security, and there are sufficiently many UDTs so that the market clears. The informed traders will demand $d_I = 1$ if $\phi = G$, and $d_I = -\beta$ if $\phi = B$.

There are now values of the aggregate demand such that the total demand perfectly reveals the informed traders' information. To see this, suppose $\phi = G$. In this case, the informed traders submit buy orders aggregating to θ , and the total demand from the informed and liquidity traders is completely revealing if $l \in (\bar{l} - [1 + \beta]\theta, \bar{l})$. The reason is as follows. If $\phi = B$, then the informed demand would be $-\beta\theta$. Even with the maximum liquidity demand of $\bar{l}$, the aggregate demand from informed and liquidity traders would not exceed $\bar{l} - \beta\theta$. Hence, if $l + \theta > \bar{l} - \beta\theta$, then regardless of the realized value of l , the informed agents could not have submitted a demand of $-\beta\theta$, i.e., when $l > \bar{l} - [1 + \beta]\theta$, the UDTs can infer that the informed agents know that $\phi = G$. Similarly, a realization $l \in (-\underline{l}, -\underline{l} + [1 + \beta]\theta)$ leads the UDTs to infer with probability 1 that the informed have observed $\phi = B$. Thus, if $\phi = B$, then $l \in L_B \equiv (-\underline{l}, -\underline{l} + [1 + \beta]\theta)$ leads to perfect revelation that $\phi = B$; if $\phi = G$, then $l \in L_G \equiv (\bar{l} - [1 + \beta]\theta, \bar{l})$ leads to perfect revelation that $\phi = G$.

Using (6), we have $P^e(D(\phi, l)) = \bar{x}$ if $l \in L_G$, and $P^e(D(\phi, l)) = \underline{x}$ if $l \in L_B$. If $l \in \{L \setminus L_B\}$ with $\phi = B$, or $l \in \{L \setminus L_G\}$ with $\phi = G$, then we have $P^e(D(\phi, l)) = q[\bar{x} - \underline{x}] + \underline{x}$ because $f(\cdot)$ is uniform. Note that the notation $A \setminus B$ means that set of elements that are in set A but not in set B . We can now write the revenue function of a type- G issuer as follows:

$$R = \Pr(l \in L \setminus L_G)\{q[\bar{x} - \underline{x}] + \underline{x}\} + \Pr(l \in L_G)\bar{x} \\ = \left\{ \frac{\bar{l} + \underline{l} - [1 + \beta]\theta}{\bar{l} + \underline{l}} \right\} \{q[\bar{x} - \underline{x}] + \underline{x}\} + \left\{ \frac{[1 + \beta]\theta}{[\bar{l} + \underline{l}]} \right\} \bar{x}. \quad (13)$$

Next, we can establish a result analogous to Proposition 2.

PROPOSITION 2': *With limited short sales, the total equilibrium expected revenue that the type- G issuer obtains by issuing securities A and S is higher than that obtained by issuing the composite security. Thus, in equilibrium the type- G firm splits its composite security into A and S . The type- B firm also splits its composite security.*

The reason why our basic result holds even with limited short sales is that constraints on short sales have the effect of masking the information of the informed in at least some states, thereby preserving these traders' profits from privileged information, albeit at a lower level. Security splitting is optimal because some of the information masking persists even with security S , and consequently informed traders still benefit from concentrating their wealth in S .

D. "Homemade" Splitting

In our supply-driven explanation for security splitting, the issuing firm determines whether the composite security should be split. Gorton and Pennacchi's (1990) demand-based explanation suggests that, if the issuing firm does not split the security, the uninformed traders will want to buy the composite security and split it themselves by issuing claims against it.

Although such “homemade” splitting is possible here, the type-*G* firm has no incentive to let traders do this because the additional rents from splitting, if any, are then captured by the uninformed traders rather than by the issuer. Moreover, the uninformed traders do not perceive any benefit from homemade splitting because they are unaware of the underlying true value. On average, across a cross-sectionally weighted sample of type-*G* and type-*B* securities, splitting will preserve the issuer’s total revenue. Hence, the issuer will always choose to split the security prior to sale.

E. More General Payoff Specifications and Richer Informational Asymmetries

We have assumed for simplicity that the end-of-period value of each security is nonrandom for an informed agent, and that for an uninformed agent it is a random variable that can take one of two values. A more realistic scenario would be one in which, even with just two types of firms, each firm has an end-of-period value that is a random variable that takes values in a continuum. If the lowest value that each random variable can take is zero or less, then it will not be possible to use splitting to create a positively valued riskless security. However, the optimal number of securities is likely to be finite in this case. The reason is as follows. As the issuer creates a greater number of securities while keeping the total asset cash flow unchanged, each security represents (on average) a smaller claim to the total cash flow. This may necessitate rationing, particularly for the most information-sensitive securities in which the potential profit for informed traders is the highest; the reason is that informed demand is nondecreasing in the security’s information sensitivity. Thus, if we limit the informed traders to order sizes that are identical across *all* traders—so as to preclude type revelation through the order size—then it is possible that the optimal number of securities is finite because creating more securities could reduce the measure of the set of informed traders. In the Appendix, we provide a numerical example in which this is the case.

Another possible extension of the model is to assume more than two possible types of issuers. This could introduce greater complexity into optimal security design. For instance, suppose there are three types of observationally indistinguishable firms, with intrinsic end-of-period values x_1 , x_2 , and x_3 , with $x_1 < x_2 < x_3$. Then, an issuer with value x_3 may split the composite security into *three* securities: the most senior security promising a date-1 payoff of x_1 , the security with the next level of seniority promising a date-1 payoff of $x_2 - x_1$, and the most junior security promising a date-1 payoff of $x_3 - x_2$. Note, however, that creating three securities may be suboptimal, even ignoring transactions costs. This is because, as mentioned earlier, increasing the number of securities beyond two may reduce total informed demand.¹⁸

¹⁸ If we allowed for richer informational asymmetries (e.g., multidimensional unobservable firm characteristics), and more general payoff distributions and also endogenized liquidity demand, we could envision a more complex interaction between security design and liquidity demand.

IV. Multiple Securities and Partitioning of Portfolio Cash Flows

Thus far we have assumed that there is only a single composite security and that the signal received by each informed agent is perfect. We now permit multiple securities and noisy signals. The basic idea in this section is that the noise in the informed agent's signal creates potential "information diversification" gains from pooling individual securities and issuing claims against the portfolio, rather than splitting individual securities themselves. Suppose there are N securities and that an informed agent can invest $\$M$ to obtain a *noisy* but informative signal, $\tilde{\phi}_i$, about security i ; we assume that M is invariant across investors. Each of the N securities can be either G or B , i.e., have a date-1 value of $\bar{x}$ or $\underline{x}$. The commonly known prior belief of each investor is that the probability is q that firm i has value $\bar{x}$.

The signal $\tilde{\phi}_i$, which can be thought of as being extracted from a random variable equal to the perfect signal ϕ (of the previous sections) plus white noise, is constructed as follows. Suppose $x_i \in \{\underline{x}, \bar{x}\}$ denotes the "true" value of security i . Then, the random variable $\tilde{\omega}_i$ is

$$\tilde{\omega}_i \equiv x_i + \tilde{\varepsilon}_i,$$

where $\tilde{\varepsilon}_i$ is a random variable with probability density function $k(\tilde{\varepsilon}_i)$, cumulative distribution function $K(\cdot)$, $E(\tilde{\varepsilon}_i) = 0 \forall i$, $\text{var}(\tilde{\varepsilon}_i) = \sigma \in (0, \infty) \forall i$, and $\text{cov}(\tilde{\varepsilon}_i, \tilde{\varepsilon}_j) = 0 \forall i \neq j$. Note that since σ is the variance of the mean-zero noise ε_i , $1/\sigma$ is the precision of an informed agent's signal. In general, $k(\tilde{\varepsilon}_i)$ is continuous and has unbounded support. The signal $\tilde{\phi}_i$ is now derived below:

$$\tilde{\phi}_i = \begin{cases} G & \text{if } \tilde{\omega}_i > \hat{x} \\ B & \text{if } \tilde{\omega}_i \leq \hat{x} \end{cases} \quad (14)$$

where $\hat{x}$ is a cutoff which equates the probabilities of type-I and type-II errors. Let this probability be $\delta(\sigma) \in (0, 1/2)$. Thus, $\Pr(\tilde{\phi}_i = G|G) = \Pr(\tilde{\phi}_i = B|B) \equiv 1 - \delta(\sigma)$, and $\Pr(\tilde{\phi}_i = B|G) = \Pr(\tilde{\phi}_i = G|B) \equiv \delta(\sigma)$, where the probabilities are conditioned on the issuer's true type (for details see the proof of Lemma 2). We now have the following result.

LEMMA 2: *Let k be unimodal and symmetric around zero. Then $\hat{x} = [\underline{x} + \bar{x}]/2$ and $\partial\delta/\partial\sigma > 0$.*

Assuming for now that k is unimodal and symmetric, the probability of error, $\delta(\sigma)$, embedded in the signal increases with the variance of $\tilde{\varepsilon}_i$, the idiosyncratic noise in the signal.

Our next step is to analyze the investor's incentive to acquire information and the type- G issuer's expected revenue, assuming no rationing. We assume that all informed investors receive the same noisy signal, ϕ_i , on security i , but that *across* different securities of a given type the signal realizations may be different. The aggregate demand for security i is given by $D_i = D_i(\phi_i, l)$.

The investor's net gain from being informed (dropping the subscript i for now) is

$$V = -M + q[1 - \delta(\sigma)] \int_0^\infty \{[\bar{x} - P^e(\theta + l)]/P^e(\theta + l)\} f(l) dl \\ - [1 - q] \delta(\sigma) \int_0^\infty \{[P^e(\theta + l) - \underline{x}]/P^e(\theta + l)\} f(l) dl \quad (15)$$

Note that (15) differs from (4) in that the informed investor's trading strategy is now driven also by recognition of the noise in the signal. Note that, even though the signal is noisy, the informed agent's information is a finer partition of the information available to the UDTs because the latter observe only the aggregate demand, $D(\tilde{\phi}_i, l)$, which imperfectly reveals the signal $\tilde{\phi}_i$. Hence, given our assumption that the signal is informative, the equilibrium will still be such that it pays for an informed agent to submit a buy order when $\tilde{\phi}_i = G$ and to eschew purchasing the security when $\tilde{\phi}_i = B$.¹⁹ That is, the investor submits a buy order when the security is truly type G (the probability of this is q) and the signal reveals type G (with conditional probability $1 - \delta(\sigma)$). The investor's expected gain is then $\bar{x} - P^e(\theta + l)$, and the joint probability of this event is $q[1 - \delta(\sigma)]$. The investor also submits a buy order when the security is truly type B (the probability of this is $1 - q$) and the signal reveals type G (with conditional probability $\delta(\sigma)$). In this case, the investor's loss is $P^e(\theta + l) - \underline{x}$, and the joint probability of this event is $[1 - q]\delta(\sigma)$.

The equilibrium value of θ , call it θ^{**} , is determined by the following marginal condition

$$V(\theta^{**} | q, \underline{x}, \bar{x}, M, f(l), \sigma) = 0. \quad (16)$$

Similar to (6), we can define the market price as

$$P^e(D(\tilde{\phi}, l)) = \Pr(G | D(\tilde{\phi}, l)) [\bar{x} - \underline{x}] + \underline{x}, \quad (17)$$

where $\Pr(G | D(\tilde{\phi}, l))$ is the probability that the true type is G , conditional on an aggregate demand of D . We have the following version of Lemma 1 in this setting.

LEMMA 3: $f'(l + \theta) < 0 \ \forall l \in (0, \infty)$ is sufficient for $\partial V / \partial \theta < 0$.

¹⁹ Note that the condition $\delta(\sigma) \in (0, 1/2)$ is necessary and sufficient for $\tilde{\phi}$ to be informative in a "nonperverse" sense, i.e., for the informed investor to increase his posterior belief in the direction recommended by the signal. To see this, note that the informed investor's posterior belief, conditional on $\tilde{\phi} = G$, is

$$\Pr(G | \tilde{\phi} = G) = \frac{[1 - \delta(\sigma)]q}{[1 - \delta(\sigma)]q + \delta(\sigma)[1 - q]}.$$

Simple algebra shows that $\Pr(G | \tilde{\phi} = G) > q$ when $\delta(\sigma) \in (0, 1/2)$. Similar logic holds for the other posteriors.

We will now present one of the two main results of this section.

PROPOSITION 5: *The equilibrium measure of the set of informed traders, θ^{**} , is strictly decreasing in the idiosyncratic variance σ .*

This result is proved by showing that V is decreasing in $\delta(\sigma)$ (and hence σ) for any θ . This is intuitive. Greater noise in the signal reduces the marginal benefit to becoming informed.

PROPOSITION 6: *The type- G issuing firm's expected revenue, R , is strictly decreasing in σ .*

The key implication of this proposition is that the type- G issuer would benefit if its security was sold as part of a portfolio of type- G securities. By combining securities with uncorrelated $\tilde{\varepsilon}_i$'s, the portfolio variance of the idiosyncratic noise can be reduced, and eliminated in the limit. A reduction in σ improves the precision of the information of the informed traders, increases their return on information and encourages higher informed demand. This moves the issuing firm's security price closer to its fundamental value and makes the type- G issuer better off as σ decreases.

This diversified portfolio can then be viewed as a "composite security" in our framework. Further revenue enhancement can be achieved by splitting the portfolio cash flows across multiple financial claims. We do not present formal details related to the gains from partitioning portfolio cash flows because the logic mirrors that developed earlier. Indeed, for $N \rightarrow \infty$, the portfolio variance of the idiosyncratic noise is zero, and we have the same payoff structure as in Section I.

V. Applications of Analysis

We now briefly discuss the implications of our analysis for "real world" security design.

A. Debt and Equity

Our analysis is transparently applicable to the issue of why firms may wish to partition total cash flows into flows accruing to debt- and equity holders. Security A in our model can be interpreted as riskless debt, and security revenue from selling claims against its assets are enhanced by issuing a debt claim that is relatively low in information sensitivity and is senior to the more information-sensitive equity claim.

Corporations issue a variety of other claims, some of which, like warrants, are more information-sensitive than equity. It may be possible that this variety reflects a certain amount of richness in informational asymmetry, but this needs careful analysis, since our earlier discussion indicates that greater informational asymmetries do not necessarily lead to more securities.

B. Securitization

Securitization of financial-intermediary-originated assets has grown significantly in recent years. Typically, securitization involves the pooling together

of many assets and then a partitioning of the portfolio cash flows into a number of securities rank ordered by seniority. For example, a collateralized mortgage obligation (CMO) usually has three or four tranches. The first tranche is fully paid off before the second tranche begins to be paid, and so on. This payoff structure is similar to the kind of security splitting shown to be optimal in Section IV.

As in our model, the marketing of securitized assets involves three primary parties: the issuer, the investment banker, and investors who buy the offering. The investment banker assists in the pricing and sale of the securitized assets. CMOs often contain individual mortgages that may be difficult for prospective buyers of CMO tranches to evaluate. However, when assembled in portfolios, these assets have payoff patterns that are easier to evaluate because some asset idiosyncrasies are eliminated through diversification.²⁰ Our analysis in Section IV produces implications consistent with these stylized facts since it implies that the issuer will wish to combine individual mortgages and sell them as a portfolio through an investment banker/market maker.²¹

VI. Conclusion

The perspective in our theory of security design is that a firm will partition its total asset cash flows into different claims because this maximizes its expected revenue. This supply side perspective is in contrast to the earlier contributions of Allen and Gale (1988), Gorton and Pennacchi (1990), and Subrahmanyam (1991) in which security design is dictated by demand considerations. Although our model is robust to many extensions, it ignores many complexities of real-world financial claims, such as covenants and convertibility features associated with debt contracts. These complexities suggest a fruitful agenda for future research.

Appendix

Proof of Lemma 1: We can write (4) as

$$V = -M + q \int_0^\infty \{\bar{x}/P^e(\theta + l)\} f(l) dl - q. \quad (\text{A1})$$

Using (A1) and (6) we have

$$V = -M + q \int_0^\infty \left\{ \frac{\bar{x}}{\Pr(\phi = G | \theta + l)[\bar{x} - \underline{x}] + \underline{x}} \right\} f(l) dl - q. \quad (\text{A2})$$

²⁰ This corresponds roughly to Subrahmanyam's (1991) intuition about the popularity of trading in indices. A different reason for forming portfolios is suggested by Millon and Thakor (1985) who point out that portfolio formation under asymmetric information may be worthwhile because of *cross-sectional information* reusability produced by systematic factors affecting the payoffs of all securities in the portfolio.

²¹ Our model requires that the securities assembled into a portfolio for sale by the investment banker all come from the same issuer, or if there are multiple issuers, the investment banker knows they are all of the same type. Investors would still be uncertain about the issuer's type.

Substituting (7) in (A2) and simplifying yields

$$V = -M + q \int_0^\infty \left\{ \frac{[f(l)q + f(l + \theta)[1 - q]]\bar{x}}{[f(l)q\bar{x} + f(l + \theta)[1 - q]\underline{x}} \right\} f(l) dl - q. \quad (\text{A3})$$

Differentiating with respect to θ gives us

$$\partial V / \partial \theta = \int_0^\infty \left\{ \frac{[1 - q][\bar{x} - \underline{x}]qf(l)f'(l + \theta)}{[f(l)q\bar{x} + f(l + \theta)[1 - q]\underline{x}]^2} \right\} f(l) dl.$$

It is transparent that $f'(l + \theta) < 0 \forall l$ is sufficient to ensure that $\partial V / \partial \theta < 0$. $\square$

Proof of Proposition 1: Note that $D(\phi, l) = l + \theta$ for the type- G issuer. Substituting (7) in (8) allows us to write the total expected revenue of the type- G issuer as

$$R = \underline{x} + \int_0^\infty \left\{ \frac{f(l)q}{f(l)q + f(l + \theta)[1 - q]} \right\} [\bar{x} - \underline{x}] f(l) dl \quad (\text{A4})$$

Differentiating with respect to θ yields

$$\partial R / \partial \theta = q[\bar{x} - \underline{x}] \int_0^\infty \left\{ \frac{-f(l)f'(l + \theta)[1 - q]}{[f(l)q + f(l + \theta)[1 - q]]^2} \right\} f(l) dl.$$

Clearly, given the sufficiency condition in Lemma 1, $\partial R / \partial \theta > 0$. Using similar steps, we can show that $dR / \partial \theta < 0$ for the type- B issuer. $\square$

Proof of Proposition 2: Define P_S^e as the equilibrium price of security S and $P_S(\phi)$ as the value of security S that is privately known to the informed trader who receives signal ϕ . The informed trader can gain nothing by purchasing security A , whereas there is a positive expected profit ex post from purchasing security S . Thus, an informed trader's optimal strategy is to invest his entire wealth endowment in security S . When security S is offered, the informed trader's expected net gain from being informed is (note that $f_s(\cdot)$ is the density function of liquidity demand in security S)

$$V_s = -M + q \int_0^\infty \left\{ \frac{[\bar{x} - \underline{x}] - P_S^e(l + \theta_s)}{P_S^e(l + \theta_s)} \right\} f_s(l) dl \quad (\text{A5})$$

where θ_s is the set of traders who become informed. In writing (A5), we have used the fact that the informed trader will submit an order for security S only when $\phi = G$, and in that case the privately known (intrinsic) value of that security is $\bar{x} - \underline{x}$. For a realization $D^S = D^S(\phi, l)$ of aggregate demand $l + D_l^S$ in security S , the market will set

$$P_S^e(D^S(\phi, l)) = \Pr(\phi = G | D^S(\phi, l)) [\bar{x} - \underline{x}] \quad (\text{A6})$$

Using Bayes' rule, we have

$$\Pr(\phi = G | D^S(\phi, l)) = \frac{f_S(D^S - \theta_S)q}{f_S(D^S - \theta_S)q + f_S(D^S)[1 - q]} \quad (\text{A7})$$

Substituting (A6) and (A7) in (A5) gives

$$V_S = -M + q \int_0^\infty \left\{ \frac{f_S(l)q + f_S(l + \theta_S)[1 - q]}{f_S(l)q} \right\} f_S(l) dl - q \quad (\text{A8})$$

Letting θ_S^* be the equilibrium value of θ_S , we have θ_S^* being determined by the marginal condition,

$$V_S(\theta_S^* | q, \bar{x}, \underline{x}, M, f_S(l)) = 0 \quad (\text{A9})$$

We now wish to compare θ_S^* to θ^* . To do this, compare (A9) to (5) by writing (A9) as

$$0 = -M + q \int_0^\infty \left\{ \frac{f_S(l)q + f_S(l + \theta_S^*)[1 - q]}{f_S(l)q} \right\} f_S(l) dl - q \quad (\text{A9}')$$

and (5) (using (A3)) as

$$0 = -M + q \int_0^\infty \left\{ \frac{f(l)q + f(l + \theta^*)[1 - q]}{f(l)q + f(l + \theta^*)[1 - q][\underline{x}/\bar{x}]} \right\} f(l) dl - q \quad (5')$$

As stated in the text, we have $f_S([1 - \alpha]l^0) = f(l^0) \forall l^0$. Now, ignoring $f(l + \theta^*)[1 - q][\underline{x}/\bar{x}]$ in the denominator of the integrand in (5'), the assumed relation between $f_S(\cdot)$ and $f(\cdot)$ would then imply that $\theta_S^* = [1 - \alpha]\theta^*$. Given $f(l + \theta^*)[1 - q][\underline{x}/\bar{x}]$, however, we now know that $\theta_S^* > [1 - \alpha]\theta^*$, using the result $\partial V / \partial \theta < 0$ proved in Lemma 1. Next we show that $\theta_S^* > [1 - \alpha]\theta^*$ implies the result stated in the proposition.

Define R_{AS} as the total expected revenue of the type- G issuer from selling securities A and S . Then, we have

$$\begin{aligned} R_{AS} &= \underline{x} + E(P_S^e) \\ &= \underline{x} + \int_0^\infty \left\{ \frac{f_S(l)q}{f_S(l)q + f_S(l + \theta_S)[1 - q]} \right\} [\bar{x} - \underline{x}] f_S(l) dl. \end{aligned} \quad (\text{A10})$$

In deriving (A10) we used $D^S(\phi, l) = l + \theta_S$ (i.e., the firm is type G), and the expression for P_S^e in (A6). It is straightforward to show that, given $f'_S(l + \theta_S) < 0$, we have $\partial R_{AS} / \partial \theta_S > 0$. Comparing (A4) and (A10), and given $f_S([1 - \alpha]l^0) = f(l^0) \forall l^0$, we observe that for $\theta_S = [1 - \alpha]\theta$, we have $R_{AS} = R$. However, we have established that $\theta_S^* > [1 - \alpha]\theta^*$. Hence, since $\partial R_{AS} / \partial \theta_S > 0$, splitting the composite security increases the type- G firm's expected revenue. In equilibrium then, the type- G firm will split the composite security.

Similar steps can be used to show that the type-*B* issuer's expected revenue declines when it splits the composite security. However, if it follows the conjectured equilibrium strategy of splitting the security, its total expected revenue (defined as $\bar{R}_{AS}$) is

$$\bar{R}_{AS} = \underline{x} + E_B(\bar{P}_S^e) \quad (\text{A11})$$

where $E_B(P_S^e)$ is the expected equilibrium price of security *S* for the type-*B* issuer. And if it chooses not to split the composite security, its expected equilibrium price is $\underline{x}$, since the UDTs believe with probability one that the defecting firm is of type *B*. It can be easily checked that the Kreps and Wilson (1982) requirement that the equilibrium strategies and beliefs represent a "consistent assessment" is satisfied here. Thus, with this out-of-equilibrium (o.o.e.) belief, the equilibrium in which both firms choose to split their securities is a sequential equilibrium. Note that this o.o.e. belief survives the universal divinity refinement of Banks and Sobel (1987). To see this, let p be the probability belief of the market maker that the defecting issuer is of type *G*. We will assume that the market maker prices the security to break even, conditional on his beliefs, even outside the equilibrium, i.e., his best response is fixed by his belief. Let p_G be the critical value of this probability such that $R_{AS} = R(p_G)$, where $R(p_G)$ is the type-*G* issuer's expected revenue if it defects from the equilibrium by issuing a composite security and the market maker believes with probability (w.p.) p_G that the defector is of type *G*. Clearly, $R_{AS} < R(p)$ for $p > p_G$ and $R_{AS} > R(p)$ for $p < p_G$, i.e., $R(p)$ is increasing in p . Similarly, define p_B through the equality $\bar{R}_{AS} = R(p_B)$. Since $\bar{R}_{AS} < R_{AS}$, it is clear that $R(p_G) > R(p_B)$. Hence, $(p_G, 1] \subset [p_B, 1]$, which means that, according to the universal divinity criterion, the market maker must attach zero probability to the defector being of type *G*. Since $E_B(\bar{P}_S^e) > 0 \forall l \in (0, \infty)$, it is privately optimal for the type-*B* issuer to split the security. $\square$

Proof of Proposition 3: When the market maker is permitted to ration, the expected profit of the informed trader is given by (9). Following familiar steps, it can be shown that $\partial \hat{V} / \partial \theta < 0$, given $f'(l + \theta) < 0 \forall l$. Moreover, recall that $V(\theta^* | q, \underline{x}, \bar{x}, f(l), M) = 0$. Evaluating $\hat{V}$ at θ^* , we see that $\hat{V}(\theta^* | q, \underline{x}, \bar{x}, f(l), M) < 0$. Hence, it must be true that $\hat{\theta} < \theta^*$, where $\hat{\theta}$ satisfies (10). Since the type-*G* issuer's expected revenue is unaffected by rationing, the type-*G* issuer's expected revenue is lowered by the possibility of rationing. Proofs of the remaining claims parallel earlier proofs. $\square$

Proof of Proposition 4: Consider first the case in which $\theta(H(M)) < \theta^*$ for the composite security equilibrium. This means that, at $M_i = M$, there is less informed demand with heterogenous information acquisition costs than with a cross-sectionally constant M . Since $V(\theta^* | q, \underline{x}, \bar{x}, f(l), M) = 0$, it must then be true that $V(\theta(H(M)) | q, \underline{x}, \bar{x}, f(l), h(M_i)) > 0$. This implies that the potentially informed trader with $M_i = M$ must be inframarginal and the M_i such that $\theta(H(M_i)) = \theta^0$ must exceed M . But this means that the marginal

trader with heterogeneous information acquisition costs has $M_i > M$. To ensure that

$$V(\theta^0 = \theta(H(M_i)) | q, \underline{x}, \bar{x}, f(l), h(M_i)) = V(\theta^* | q, \underline{x}, \bar{x}, f(l), M) = 0,$$

we must, therefore, have $\theta^0 < \theta^*$. To see that heterogeneous information acquisitions costs are more pernicious to the type- G issuer (i.e., there is a larger set of $h(M)$ specifications for which heterogeneous information acquisition costs lead to lower expected revenues than those attainable with constant information acquisition costs), consider the $\theta(H(M)) = \theta^*$ composite security equilibrium. While in that case heterogeneous information acquisition costs do not diminish this issuer's expected revenue, they will do so in the split securities equilibrium. Since $\theta_S(H(M_S)) > \theta(H(M))$, we have $M_S > M$. Therefore, $\theta_S(H(M_S)) < \theta_S^*$. The rest of the proof follows using similar logic. $\square$

Proof of Proposition 2': Note that using (13) and the logic used in proving Proposition 1, it follows that the type- G firm's equilibrium expected revenue is increasing in θ . The UDT's net gain to being informed is

$$\begin{aligned} V = & -M + q \int_{L \setminus L_G} \{[\bar{x} - P^e(\theta + l)]/P^e(\theta + l)\} f(l) dl \\ & + [1 - q] \int_{L \setminus L_B} \{[P^e(\theta + l) - \underline{x}]/P^e(\theta + l)\} f(l) dl. \end{aligned}$$

Substituting the values of $P^e(\theta + l)$ in the various intervals above and rearranging yields

$$V = -M + \left[\frac{\underline{l} + \bar{l} - [1 + \beta]\theta}{\underline{l} + \bar{l}} \right] \left[\frac{\bar{x} - \underline{x}}{q[\bar{x} - \underline{x}] + \underline{x}} \right] \{2q[1 - q]\} \quad (\text{A12})$$

We can derive similar expressions for R (for the type- G issuer) and V for the "split securities" equilibrium. Recalling that $f_S([1 - \alpha]l^0) = f(l^0) \forall l^0$, we have

$$\begin{aligned} R_{AS} = & \underline{x} + \left\{ \frac{[1 - \alpha][\underline{l} + \bar{l}] - [1 + \beta]\theta_S}{[1 - \alpha][\underline{l} + \bar{l}]} \right\} q[\bar{x} - \underline{x}] \\ & + \frac{[1 + \beta]\theta_S}{[1 + \alpha][\underline{l} + \bar{l}]} [\bar{x} - \underline{x}] \end{aligned} \quad (\text{A13})$$

and noting that $P_S^e(\theta + l) = q[\bar{x} - \underline{x}]$ when $l \in \{L \setminus L_G\}$, we have

$$V_S = -M + \left[\frac{[1 - \alpha][\underline{l} + \bar{l}] - [1 + \beta]\theta_S}{[1 - \alpha][\underline{l} + \bar{l}]} \right] \left[\frac{\bar{x} - \underline{x}}{q[\bar{x} - \underline{x}]} \right] \{2q[1 - q]\} \quad (\text{A14})$$

Comparing (13) and (A13) we see that, at $\theta_S = [1 - \alpha]\theta$, we have $R_{AS} = R$. However, using (A12) and (A14), we can show that $\theta_S^* > [1 - \alpha]\theta^*$ (i.e., at

$\theta_S^* = [1 - \alpha]\theta^*$, we have $V < V_S$, implying that $\theta_S^* > [1 - \alpha]\theta^*$ because $\partial V_S / \partial \theta_S < 0$). Since $\partial R_{AS} / \partial \theta_S^* > 0$, we have the desired result. $\square$

An Example With Two Types of Firms and a Continuum of Possible Terminal Values (Section III.E)

Suppose there are two types of firms, B and G . Firm G has a terminal value $\tilde{x}_G$ which is uniformly distributed over $[4.5, 7.5]$, and firm B has a terminal value $\tilde{x}_B$ which is uniformly distributed over $[1.5, 4.5]$. The cross-sectional proportion of each type of firm is 0.5. As before, an uninformed trader cannot distinguish B from G , so his prior probability that a randomly picked firm is G is 0.5.

We can create a "safe security" (call it A) that promises a payoff of 1.5. In addition, we can create two risky securities, S_L and S_H , each of which promises a payoff of 3, with S_L having priority over S_H . Thus, the promised and expected payoffs on A are both 1.5. The promised payoff on S_L is 3.0, whereas the expected payoff is

$$\begin{aligned} P_L &= 3 \times \Pr(\phi = G) + E(\tilde{x} - 1.5 | \phi = B) \times \Pr(\phi = B) \\ &= 0.5 \times 3 + 0.5 \times 1.5 = 2.25. \end{aligned}$$

The promised payoff on S_H is 3.0, whereas the expected payoff is

$$\begin{aligned} P_H &= \Pr(\phi = G) \times E(\tilde{x} - 4.5 | \phi = G) \\ &= 0.5 \times 1.5 = 0.75. \end{aligned}$$

Finally, for S (S_L and S_H combined), the promised payoff is 6, and the expected payoff is $P_L + P_H = P_S = 3.0$.

We wish to show that, from the issuer's standpoint, issuing S_L and S_H need not be better than issuing the combined security S . In this case, issuing two securities would be better than issuing three. Let the informed trade in S be 0 or 2, and let the liquidity trade be 2 with probability (w.p.) 0.5 and 4 w.p. 0.5. Note that the informed only gain if $\phi = G$ and $l = 2$, since $l = 4$ leads to a total demand of 6 and a fully revealing price. We suppress information acquisition costs for now. Thus,

$$V_S = 0.5 \times 0.5 [E(\tilde{x} - 1.5 | \phi = G) - P_S(D = 4)] [P_S(D = 4)]^{-1} \times 2 = 1/4.$$

Now suppose that S_L and S_H are separately traded. Assume for symmetry that informed trade is 0 or 1 in each, and liquidity trade is 1 w.p. 0.5 and 2 w.p. 0.5 in each. Then,

$$V_L = 0.25[3 - 2.25]/2.25 = 1/12.$$

$$V_H = 0.25[1.5 - 0.75]/0.75 = 1/4.$$

We will now account for rationing. For security S , the total demand is 4 while supply is 3. Thus, $V_S^{RAT} = (0.75) \times V_S = 3/16$, $V_L^{RAT} = V_L = 1/12$, (no rationing since demand and supply are both 2.25), and $V_H^{RAT} = (0.75/2) \times V_H = 3/32$. Since $V_L^{RAT} + V_H^{RAT} < V_S^{RAT}$, we see that security splitting has

reduced the value of becoming informed when rationing is accounted for. In this example, we assumed that total informed demand was unchanged when the security was split, but if we endogenize informed demand, this calculation shows that it is possible that security splitting will diminish informed demand.

Proof of Lemma 2: We first show that

$$K(\hat{x} - \bar{x}) + K(\hat{x} - \underline{x}) = K(\hat{x} - \underline{x}) + K(\hat{x} - \bar{x}) = 1 \quad (\text{A15})$$

is needed for the equality between type-I and type-II errors. Note that, given (14), we have

$$\Pr(\tilde{\phi}_i = G | G) = \Pr(\tilde{\varepsilon}_i > \hat{x} - \bar{x}) = 1 - K(\hat{x} - \bar{x}),$$

and

$$\Pr(\tilde{\phi}_i = B | B) = \Pr(\tilde{\varepsilon}_i \leq \hat{x} - \underline{x}) = K(\hat{x} - \underline{x}).$$

To ensure that $\Pr(\tilde{\phi}_i = G | G) = \Pr(\tilde{\phi}_i = B | B)$, we need

$$1 - K(\hat{x} - \bar{x}) = K(\hat{x} - \underline{x}). \quad (\text{A16})$$

Similarly,

$$\Pr(\tilde{\phi}_i = B | G) = \Pr(\tilde{\varepsilon}_i \leq \hat{x} - \bar{x}) = K(\hat{x} - \bar{x}),$$

and

$$\Pr(\tilde{\phi}_i = G | B) = \Pr(\tilde{\varepsilon}_i > \hat{x} - \underline{x}) = 1 - K(\hat{x} - \underline{x}).$$

To ensure that $\Pr(\tilde{\phi}_i = B | G) = \Pr(\tilde{\phi}_i = G | B)$, we need

$$K(\hat{x} - \bar{x}) = 1 - K(\hat{x} - \underline{x}). \quad (\text{A17})$$

By combining (A16) and (A17) we now obtain (A15).

To obtain the result stated in Lemma 2, note that if K is unimodal and symmetric around its mean of zero, (A15) can hold only if $\hat{x} - \bar{x}$ and $\hat{x} - \underline{x}$ are equidistant from zero and on either side of it. It is transparent now that

$$0 - [\hat{x} - \bar{x}] = [\hat{x} - \underline{x}] - 0,$$

or

$$\hat{x} = [\bar{x} + \underline{x}]/2.$$

Since $\delta(\sigma) = K(\hat{x} - \bar{x})$, it is clear that an increase in σ will increase $\delta(\sigma)$, i.e., $\partial\delta(\sigma)/\partial\sigma > 0$. $\square$

Proof of Lemma 3: we can rewrite (15) as

$$\begin{aligned} V = & -M - \{q[1 - \delta(\sigma)] + [1 - q]\delta(\sigma)\} \\ & + \{q[1 - \delta(\sigma)]\bar{x} + [1 - q]\delta(\sigma)\underline{x}\} \int_0^\infty [P^e(\theta + l)]^{-1} f(l) dl. \end{aligned} \quad (\text{A18})$$

Note now that

$$\Pr(G|D(\tilde{\phi}, l)) = \frac{\{[1 - \delta(\sigma)]f(D - \theta) + \delta(\sigma)f(D)\}q}{\{[1 - \delta(\sigma)]f(D - \theta) + \delta(\sigma)f(D)\}q + \{[1 - \delta(\sigma)]f(D) + \delta(\sigma)f(D - \theta)\}[1 - q]} \quad (\text{A19})$$

In writing (A19) we have accounted for the fact that an aggregate demand of $D = D(\tilde{\phi}, l)$ implies a liquidity demand of $D - \theta$ with probability (w.p.) $1 - \delta(\sigma)$ and D w.p. $\delta(\sigma)$ if the true type is G , and D w.p. $1 - \delta(\sigma)$ and $D - \theta$ w.p. $\delta(\sigma)$ if the true type is B . From (17) and (A19) we get

$$[P^e(\theta + l)]^{-1} = \frac{q\{[1 - \delta(\sigma)]f(l) + \delta(\sigma)f(l + \theta)\} + [1 - q]\{[1 - \delta(\sigma)]f(l + \theta) + \delta(\sigma)f(l)\}}{q\{[1 - \delta(\sigma)]f(l) + \delta(\sigma)f(l + \theta)\}\bar{x} + [1 - q]\{[1 - \delta(\sigma)]f(l + \theta) + \delta(\sigma)f(l)\}\underline{x}} \quad (\text{A20})$$

It can be shown that $f'(\theta + l) < 0 \forall l$ is sufficient to ensure that $\partial([P^e(\theta + l)]^{-1})/\partial\theta < 0$. Thus, from (A18) it now follows that $\partial V/\partial\theta < 0$. $\square$

Proof of Proposition 5: We first show that $\partial V/\partial\delta(\sigma) < 0 \forall \theta > 0$. It is sufficient to show that (A3) exceeds (A18), and that the differential is increasing in $\delta(\sigma)$. Substitute (A20) in (A18) and observe that (A3) exceeds (A18) if

$$\begin{aligned} q\bar{x} \left\{ \frac{qf(l) + [1 - q]f(l + \theta)}{qf(l)\bar{x} + [1 - q]f(l + \theta)\bar{x}} \right\} - q \\ > -q[1 - \delta(\sigma)] - [1 - q]\delta(\sigma) \\ + \{q[1 - \delta(\sigma)]\bar{x} + [1 - q]\delta(\sigma)\underline{x}\}\Gamma \forall l \end{aligned} \quad (\text{A21})$$

where

$$\Gamma \equiv \frac{q\{[1 - \delta(\sigma)]f(l) + \delta(\sigma)f(l + \theta)\} + [1 - q]\{[1 - \delta(\sigma)]f(l + \theta) + \delta(\sigma)f(l)\}}{q\{[1 - \delta(\sigma)]f(l) + \delta(\sigma)f(l + \theta)\}\bar{x} + [1 - q]\{[1 - \delta(\sigma)]f(l + \theta) + \delta(\sigma)f(l)\}\underline{x}}$$

Tedious algebra, involving premultiplication by

$$\begin{aligned} C \equiv q\{[1 - \delta(\sigma)]f(l) + \delta(\sigma)f(l + \theta)\}\bar{x} \\ + [1 - q]\{[1 - \delta(\sigma)]f(l + \theta) + \delta(\sigma)f(l)\}\underline{x} > 0, \end{aligned}$$

gives

$$\begin{aligned} \delta(\sigma)\{[qf(l) + [1 - q]f(l + \theta)][q\bar{x} + [1 - q]\underline{x}] \\ - \tau - [2q - 1][q\bar{x} + [1 - q]\underline{x}]\} > 0 \end{aligned} \quad (\text{A22})$$

where

$$\tau \equiv 1 - \frac{q\bar{x}[f(l) - f(l + \theta)]}{[1 - q]f(l + \theta)\underline{x} + qf(l)\bar{x}}$$

A little algebra shows that the left-hand side of (A22) is indeed strictly positive. Observe that the quantity on the left-hand side of (A22) is strictly increasing in $\delta(\sigma)$. Since the premultiplication involved C , with $\partial C / \partial \delta(\sigma) < 0$, it follows that $\partial V / \partial \delta(\sigma) < 0$. Since $\partial \delta(\sigma) / \partial \sigma > 0$, we have $\partial V / \partial \sigma < 0$.

Now, from (16) we know that θ^{**} solves

$$V(\theta^{**} | q, \underline{x}, \bar{x}, M, f(l), \sigma) = 0.$$

Given $\partial V / \partial \sigma < 0$ and $\partial V / \partial \theta < 0$ (from Lemma 3), it follows that $\partial \theta^{**} / \partial \sigma < 0$, i.e., a higher value of σ reduces the expected revenue of the informed investors, and hence discourages informed trading. $\square$

Proof of Proposition 6: The type- G issuing firm's expected revenue is

$$\begin{aligned} R = & [1 - \delta(\sigma)] \int_0^\infty \Pr(G | D = \theta + l) [\bar{x} - \underline{x}] f(l) dl \\ & + \delta(\sigma) \int_0^\theta \Pr(G | D = l) [\bar{x} - \underline{x}] f(l) dl \\ & + \delta(\sigma) \left\{ \frac{q\delta(\sigma)}{q\delta(\sigma) + [1 - q][1 - \delta(\sigma)]} \right\} [\bar{x} - \underline{x}] \int_0^\infty f(l) dl + \underline{x}. \quad (\text{A23}) \end{aligned}$$

The specification in (A23) takes into account the fact that there is a probability $\delta(\sigma)$ that a type- G issuer does not face informed demand. If the total demand $D(\tilde{\phi}, l)$ turns out to be less than θ , then the absence of informed demand is perfectly revealed to the market maker. The proof involves first showing that, holding θ fixed, R is decreasing in σ . Since R is increasing in θ , and θ is decreasing in σ , an increase in σ will impact θ in such a way as to further accentuate the decline in R . These details are not included here, but are available upon request. $\square$

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