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Incentive Conflicts, Bundling Claims, and the Interaction among Financial Claimants

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ABSTRACT

We show that for certain capital structures equity has an incentive to buy out another claim and alter the firm's investment strategy so as to maximize the combined value of equity and the acquired claim. This restructuring may reintroduce agency problems into capital structures which appear to avoid agency conflicts. By bundling claims, it is possible to avoid this agency problem. The agency problem is also eliminated by dispersed ownership of the claims.

IN THE PRESENCE OF other claims issued by the firm, the projects that maximize the value of equity need not maximize the value of the firm (e.g., Myers (1977)). An agency problem occurs because the equityholders select projects that are in the interest of equity rather than the firm as a whole. For example, if the firm comprises equity and bonds, then equity would prefer risky projects and may undertake projects that are undesirable from the firm's perspective but desirable from the equity perspective. The resulting difference in the claimants' preferences among projects could be limited by structuring the financial claims so that the preferences of equityholders among projects are similar to those of the firm as a whole. The issuance of separate claims that are individually both more risky and less risky than the equity can control incentive problems for a fixed capitalization without trading (see Jensen and Meckling (1976) and Green (1984)). Green (1984) examines the financing of a new project by a combination of warrants and bonds and shows that if the warrants and bonds correctly balance, then the incentives of equity still will be to maximize the value of the firm. For this fixed capital structure, the incentive conflicts are removed and the optimal strategy for equity coincides with the firm's optimal strategy.

A time inconsistency problem can arise when an earlier period is added to allow trading among the financial claimants. We initially suppose that the capital structure is designed so that without trading equity has an incentive

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to act in the interests of the firm. This can be illustrated by a firm composed of warrants, equity, and debt. The warrants benefit from increasing firm risk, while the debt benefits from decreasing firm risk. Equity's incentives lie between those of warrants and those of debt. For the proper mixture of debt and warrants, the equity has an incentive to act optimally for the firm and the agency problem is eliminated. However, if trading is allowed, the agency problem may be reintroduced. For example, if equity were to acquire the warrants, then the resulting optimal investment strategy for equity would no longer be the optimal strategy for the firm, because equity would then act in the combined interest of equity and warrants rather than in just the interest of equity. Therefore, the value of the combined position of warrants and equity will be greater than the sum of their values when separately held. This synergy between equity and warrants provides an incentive for equity to buy out the warrants and alter the firm's investment strategy. The new strategy would no longer be in the interest of the firm. This combination of equity and warrants and the resulting change in investment policy is detrimental to debtholders.

If equity were to buy the warrants, then the resulting project selection by equity would be riskier than if there was no trading, thereby reducing the value of the debt. Therefore, debt might be willing to sell its position for less than its value in a model without trading. Alternatively, if equity could acquire debt, then the warrantholders would be worse off. We show that, in equilibrium, trading makes both warrants and debt worse off. The pricing of the debt reflects the fear that equity will acquire warrants, and the pricing of the warrants reflects the fear that equity will acquire debt. The result is that equity can achieve a double squeeze and obtain both the debt and warrants at low prices.[1]

The time inconsistency problem can apply to various capital structures composed of such claims as public debt, bank loans, strip financing, equity, warrants, executive stock options, and share appreciation rights. Also, pension plans that have some risk of default would fit into this framework. In situations where a fixed set of claims exist that resolve incentive problems, there is still the danger that agency problems arise due to the possibility of trading.[2]

[1] The same type of double-sided squeeze also would emerge as an equilibrium in a game in which equity makes tender offers simultaneously to both claimants (however, unlike the sequential game, this solution is not unique in the simultaneous offer game).

[2] Bergman and Callen (1991) apply the Rubinstein (1982) bargaining approach to analyze the effect of equity's selection of projects over time in reducing the value of the debt and forcing debt renegotiation. As long as the initial value of the firm is low enough equity can credibly force renegotiation of the debt because of equity's continuing selection of projects. Our analysis is driven by rivalry between the claimants rather than the assumed direct form of dissipation of value through project selection. We also do not require a discrete precommitment to player strategies (e.g., compare to Bergman and Callen (1991)). The implications of the observed bankruptcy law for the division of value across financial claims is explored in Brown (1989) using the Rubinstein (1982) solution.

Interestingly, if trading were not allowed, then a convertible bond would be similar to debt with detachable warrants.[3] For a fixed capital structure, debt with warrants and convertible debt would have a similar impact upon equity's incentives. However, once trading is allowed a clear distinction emerges because convertibles still can prevent agency problems, whereas warrants with straight debt cannot.[4,5] If bonds and warrants are bundled together to form convertible debt, then equity cannot purchase one claim without acquiring the other. Although warrants may be issued in a package with debt, once an investor holds the package of warrants and debt it is possible to split the claims, selling debt to some investors and warrants to others. Because it is possible to separate these claims we regard the debt and warrants as unbundled. This contrasts with convertible bonds, where the warrant and debt are permanently tied or "bundled" to each other. This bundling of the claims in the form of convertible debt prevents equity from forming a coalition with either just the warrants or just the debt.

An illustration of the double squeeze is the tactic of railroad companies in the western United States during the 19th century of purchasing large tracts of land along alternative routes for a proposed rail line (even today railroads still have relatively large land holdings in the West). The value of the land would be substantially higher if the railroad were built nearby. Hence, by playing routes off against one another the railroad could acquire valuable land at a low price. Further, the railroad's selection of route would be influenced by its relative landholdings so that availability of land along one route reduced the cost of the land purchases along the other route.[6] The route selection problem is analogous to the capital structure problem in which the purchase of debt by equity results in a less-desirable selection of projects from the perspective of the warrants and reduces the warrantholder's bargaining power in the tender game (see condition (6) below).

An alternative to bundling claims to prevent incentive problems caused by trading is widely dispersed ownership. Widely held claims are more difficult to buy out than concentrated claims. Free rider problems, such as those

[3]Of course, equilibrium tax clientele and tax option effects can also create advantages to separate claims that either appeal to diverse tax clienteles or possess valuable tax options, but our analysis abstracts from such tax effects.

[4]The influence of coalitions among the firm's claims upon valuation and project selection is also studied by Aivazian and Callen (1980). They use a cooperative game theoretic approach and emphasize the nonexistence of the core in the presence of at least three claimants. Stepping outside of game theory, they interpret the nonexistence of the core as suggesting high negotiation costs, providing a "rationalization" for the bundled convertible debt contract.

[5]The difference between convertible debt and a combination of debt and warrants also is highlighted in Dunn, Heston, and Spatt (1992), who study the allocation of joint value in the pricing of "usable" debt and warrants. That study focuses upon pricing in a setting without incentive conflicts.

[6]The ability to select which claim to reward also was illustrated in the marketplace in September 1991 when Dow Chemical had to decide whether to redeem its contingent value right (a type of put) on Marion Merrell Dow common or to eliminate the value of the right by merging with the firm at a high price.

addressed by Grossman and Hart (1980), suggest that a holder of a small portion of a claim may prefer to hold out and not tender in order to free ride on the effort of the offerer. Interestingly, in our model such free riding is desirable because it may prevent the agency problem from reappearing.

We explore the effects of bundling and unbundling claims in Section I. In Section II we analyze the effect of wide ownership on trading, and we conclude in Section III.

I. Concentrated Ownership

To explore the agency problem we model the rivalry among the nonequity claimants to the firm by assuming the existence of equity, risky (nonconvertible) debt, and warrants.[7] Our purpose is to show how an agency problem can arise due to restructuring of this capital structure. Fixing the outstanding claims issued by the firm we let a denote the vector of decisions by the firm about which investment projects to undertake (including investment levels). We define $a(x, y)$ as the project choice that maximizes the value of equity plus (the fractions) x of the debt and y of the warrants. We assume that $a(x, y)$ is unique.[8] For example, $a(0, 0)$ is the optimal choice for equity if equity owns neither debt nor warrants, while $a(1, 0.5)$ is the optimal strategy for equity if equity owns the debt and half the warrants. We let $a^* = a(1, 1)$ denote the project choice that maximizes the total value of the firm. The values of the equity, debt and warrant claims for a given action choice a are denoted respectively by $e[a]$, $d[a]$, and $w[a]$. The total value of the firm $v[a]$ satisfies the identity[9]

$$v[a] = e[a] + w[a] + d[a] \text{ for all } a. \tag{1}$$

[7] The following sharing rules illustrate the division of the payout among risky debt, equity, and warrants. The debt receives the first F dollars of payout (riskiness of the debt implies that with positive probability the total value of the project is strictly below F). Equity receives the next G dollars plus a share of $s/(r + s)$ beyond the first $F + G$, where s is the number of original equity shares and r is the number of new equity shares that would result from the warrant exercise. The warrantholder receives a share of $r/(r + s)$ of the total payout beyond $F + G$.

[8] Specifically, we exclude the possibility that equity holding a fraction x of debt and y of warrants is indifferent among investment strategies, though a pure debtholder prefers one alternative and a pure warrantholder prefers another.

[9] For simplicity our analysis abstracts from taxes and the potential tax advantage of debt due to the deductibility of interest under the corporate income tax. This tax advantage would reduce the attractiveness of the recapitalizations we have described and suggest our model is more directly applicable to the partnership sector or prior to the corporate income tax. In fact, in the railroad example in the introduction, there would be no tax bias toward either perspective coalition, even were the corporate income tax in effect. The tax cost would be mitigated if there is no free rider problem among the equityholders (e.g., a single equity claimant) and the debt were purchased directly by the equity claimant rather than by the firm (so that the interest deduction remains on the firm's books). Miller (1977) suggests that personal tax offsets would reduce the effect of the corporate tax deductibility of interest. We also do not consider the informational aspects of the claims, unlike such analyses as Brennan and Kraus (1987) and Williams (1988).

The definitions of $a(0, 1)$, $a(1, 0)$ and a^* imply

$$e[a(0,1)] + w[a(0,1)] > e[a] + w[a] \text{ for } a \neq a(0,1), \tag{2}$$

$$e[a(1,0)] + d[a(1,0)] > e[a] + d[a] \text{ for } a \neq a(1,0), \tag{3}$$

and

$$e[a^*] + w[a^*] + d[a^*] > e[a] + w[a] + d[a] \text{ for } a \neq a^*. \tag{4}$$

We suppose that the combination of existing equity, warrants, and debt resolves the incentive conflict (see Green (1984)), i.e., we assume

$$e[a^*] > e[a] \text{ for } a \neq a^*, \tag{5}$$

so that the optimal project choice is also preferred by equity. Therefore, for the initial capitalization the incentive conflict between equity and the firm is eliminated.[10] We further assume that $x_1 > x_2$ implies $d[a(x_1, z)] > d[a(x_2, z)]$ and that $y_1 > y_2$ implies $w[a(z, y_1)] > w[a(z, y_2)]$.[11,12] Yet if equity succeeds in convincing one of the other two types of claimants (warrant or bond) to accept a tender offer from equity, then the incentive conflict will nevertheless arise.[13] If equity can force all investors to accept the tender offers, then the value of the nonequity claims is reduced. However, if all claimants accept their respective tender offers, then because of the assumption of condition (5) an incentive conflict does not arise, i.e., the investment decisions are still optimal.

To model this we use a sequential game in which equity makes offers sequentially. We consider a simple four-move game. The first move is for equity to make an offer to debt, the second move is for debt to either accept or reject the offer. The third move is for equity to make an offer to the warrants.

[10] The essential aspect of this assumption is that the objective of equity is intermediate between those of warrantholders and debtholders. For example, if equity acquired the warrants, then the action $a(0, 1)$ which maximizes the combined value of warrants and equity would be worse than a^* from the debtholder's perspective. The restriction imposed by (5) limits the financial claims to which the model applies. For example, (5) would fail in the case of equity and two similar classes of nonconvertible risky debt.

[11] A simple example of such a technology is illustrated by the profit function $\theta k^{3/4} - k$, where θ is a random variable with mean $\bar{\theta}$ and variance σ^2. The firm picks k to maximize the expected payoff by setting $k^* = (0.75\bar{\theta})^4$. The variance of the realized profit is $k^{3/2}\sigma^2$, which is increasing in k. Following our treatment in the text we suppose that the capital structure is such that equity also prefers k^*. The warrantholders prefer higher variance and therefore a value k^w which exceeds k^*. The debtholders prefer a value k^d which is below k^* and has a lower variance. If the equityholder has all the equity and some debt, then the value chosen is between k^d and k^*.

[12] The definitions of d, w and a are sufficient to show that $d[a(x_1, z)] \geq d[a(x_2, z)]$ and that $w[a(z, y_1)] \geq w[a(z, y_2)]$. Therefore, the only content of the text assumption is to make the inequalities strict. This assumption rules out the possibility of the absence of an agency problem, such as when the choice a is independent of how much debt or warrants the equity held.

[13] In the context of bankruptcy Bulow and Shoven (1978) analyze interesting strategic interaction between banks and equity. They suggest that banks collude with equity to satisfy the short-term obligations of the firm (temporarily avoiding bankruptcy) by issuing additional bank debt and undercutting the long-run value of other debt claims.

The final move of the game is for the warrantholder to accept or reject the offer. In this section we assume that the warrantholder can only tender his entire position or none of his position rather than a fraction of his position. The strategies of the debtholder (or warrantholder) form a rule about which offers are accepted and which are rejected. A strategy for the equity is an offer to debt and a rule describing what offer to make to the warrants if the debt offer is accepted and what offer to make if the debt offer is rejected. Of course, we could reverse the order by going to the warrantholder first, but the results would be the same. We will focus attention on the subgame perfect equilibria.[14]

Based upon the claims it has purchased, equity selects its most profitable investment projects. The resulting project selection is less favorable to the warrantholder (debtholder) if equity has acquired the debt (warrants) but not the warrants (debt). The strict inequalities discussed in footnote 12 imply that

$$w[a^*] > w[a(1,0)]. \tag{6}$$

Thus, the payoff to the warrants is reduced if equity acquires the debt. Similarly,

$$d[a^*] > d[a(0,1)], \tag{7}$$

so that the payoff to the debt is reduced if equity acquires the warrants. Whether or not equity acquires the debt, it still would attempt to obtain the warrants (though the price offered would be different). If equity acquired the debt in the first stage (e.g., while squeezing the debtholder), then it could eliminate the resulting incentive costs while squeezing the warrantholder in the second stage (because it would be optimal to select $a(1, 0)$ if the warrants did not accept the tender offer). The warrantholder would have an incentive to accept offers below $w[a^*]$, because the equity that now holds the debt, would not choose a^* if the warrants decline the offer.

If debt did not accept the offer in the first stage, then equity could offer $w[a^*] + \varepsilon$ to the warrantholder, who will then accept the offer. The resulting project selection would be $a(0, 1)$ and the payoff to debt would be $d[a(0, 1)]$. Hence, the debtholder is indifferent to accepting an initial offer of $d[a(0, 1)]$, reflecting the threat of equity to purchase warrants. Since $d[(a(0, 1)] < d[a^*]$, debt can be convinced to accept offers below $d[a^*]$. The best strategy for equity is to make the lowest offer that debt will accept.

Then after acquiring the debt, equity can use the debtholder's acceptance to squeeze the warrantholder to accept an offer at $w[a(1, 0)]$, reflecting the selection of $a(1, 0)$ by equity in the event that the warrantholder rejects the offer. In fact, this solution is the unique equilibrium (with endogenous offers) to this sequential offer game. In the final round the equity makes an offer to the warrantholder. If the offer is equal to $w[a(1, 0)]$, then the warrantholder

[14]A solution is a subgame perfect equilibrium if it is a Nash equilibrium in each subgame of the overall game. The concept of subgame perfection is described in Selten (1975) and applied to sinking fund debt contracts in Dunn and Spatt (1984).

is indifferent between accepting or rejecting the offer. A subgame perfect equilibrium can be formed in which the equity offers $w[a(1,0)]$ and the warrantholder accepts the offer. In equilibrium the warrantholder must accept any offer greater than $w[a(1,0)]$. If the warrantholder's strategy is to reject an offer of $w[a(1,0)]$, then the equity is better off offering $w[a(1,0)] + \varepsilon$, where $\varepsilon > 0$ (which is accepted). However, if the equityholder does this, then a better strategy is to offer $w[a(1,0)] + \varepsilon/2$ (which also would be accepted). Since there is no smallest offer greater than $w[a(1,0)]$, the only subgame perfect equilibrium has warrants accepting an offer of $w[a(1,0)]$.[15] This entails the warrantholder using a weakly dominated strategy. If the offer were slightly higher, then acceptance by the warrantholder would not be weakly dominated. The unique subgame perfect equilibrium is for equity to offer the debtholder $D = d[a(0,1)]$, which it accepts, and then for equity to offer the warrantholder $W = w[a(1,0)]$, which it accepts.

However, note that the bundling of the warrant and debt claims would eliminate the possibility of a squeeze because the rivals could not be played off against each other. As a result, the bundling of the warrant and debt would protect their combined value. This provides a reason for issuing convertible debt. This is consistent with empirical evidence in Finnerty (1986) that most firms issue convertible debt rather than debt with warrants.[16]

In order to pursue these strategies the equity needs sufficient time to alter the firm's action choice and subsequent cash flows. The value of these strategies can be reduced greatly by restricting the maturity of the warrants. Issuing short-term claims may be an alternative to bundling as a means of protecting against such squeezes. Finnerty (1986) notes that when firms issue debt with warrants, the maturity of the warrants is often less than the debt.

It is possible to eliminate the benefits of trading if each asset prefers the same choice of a, so that each asset desires $a = a^*$. If two assets each independently prefer a^*, then when they are merged into one asset the combined asset will have the same optimum. This implies that equity cannot benefit by acquiring such an asset and then changing the choice of a. An example is a firm with equity and convertible debt. The convertible debt is similar to bundling warrants and debt. If the convertible debt is designed along the lines of Green (1984), then equity desires $a = a^*$. However, the bundled convertible debt also desires $a = a^*$. The result is that the combination of equity and convertible debt will choose $a = a^*$. Thus, if equity acquires the convertible debt it will not change its choice of a. The capital

[15] Similar reasoning explains why in equilibrium the equity makes a first offer to the debtholder of $d[a(0,1)]$, which is accepted. In the subgame perfect equilibria the debtholder realizing that the warrants will be acquired by equity must accept any offer strictly greater than $d[a(0,1)]$. There is no smallest offer strictly greater than $d[a(0,1)]$, so the result is that the only equilibrium is an offer of $d[a(0,1)]$, which is accepted.

[16] Finnerty (1986) states that between 1981 and 1985 there was \$28.5 billion of new convertible bonds issued, but only \$7.4 billion of debt with warrants issued in the United States.

structure is stable, since the benefit to equity of buying the other asset is eliminated.

If instead other claims have different choices of a then a^*, then there is a benefit to equity in trading. An example that we explore is debt, equity, and warrants. The original capital structure may be designed along the lines of Green (1984), giving equity an incentive to choose $a = a^*$. However, debt prefers choosing projects with less risk than a^* and warrants prefer choosing projects with greater risk than a^*. This illustrates the potential effect of the types of trading we explore. However, by bundling the warrant and debt into a convertible bond this trading is avoided.

One question that arises is whether debt and warrants can negotiate with each other. The structure of our game prevents such negotiations and instead we use the subgame perfect equilibrium solution concept. If debt and warrants agreed (outside our game) not to trade with equity, they might prevent equity from restructuring. However, there would always be the risk that one party or the other would violate the agreement. One way to prevent this restructuring is to issue a bundled security.

A second way to prevent trading is to have widely held assets. If the ownership of the nonequity assets is sufficiently dispersed, then it will be difficult or impossible for equity to accumulate the other assets.

II. Dispersed Ownership

In this section we consider the effect of more widely held ownership distributions and allowing warrantholders or debtholders to tender a fraction of their holdings. To study the competition among the holders of a given claim we focus in this section upon unconditional offers. Permitting offers conditional upon the decisions of other holders of the same claim would allow equity to transform the problem to the single owner case considered in the preceding section.

A conditional offer is one that is made contingent upon a specified fraction accepting the offer. This type of offer typically leads to multiple equilibria as shown in Shleifer and Vishny (1986) and Bagnoli and Lipman (1988). One equilibrium involves acceptance by the specified fraction and rejection by the remaining holders that free ride. However, rejection by all holders is also an equilibrium response provided no holder owns an amount greater than or equal to the specified fraction. In this equilibrium the offer is rejected and no holder can unilaterally change the outcome. Shleifer and Vishny (1986) and Bagnoli and Lipman (1988) rule out this equilibrium involving weakly dominated strategies.

If we focus on the acceptance equilibrium, then the use of conditional offers would cause the analysis to resemble Section I. Equity can make an offer to debt of $D = d[a(0, 1)]$ conditional on unanimous acceptance, which all debtholders then would accept. Then equity makes an offer to warrants of $W = w[a(1, 0)]$ conditional on unanimous acceptance. Then all warrants would accept this offer in equilibrium. If any warrantholder rejected the offer, his payoff would be unaltered. In a takeover game Bagnoli and Lipman (1988) suggest a similar strategy in their Proposition II.

The analysis of unconditional offers also is partially justified by the potential practical difficulty in making the tender decision conditional upon the tender choices of others. Implementation of optimal strategies using conditional offers requires exact knowledge of the distribution of ownership and is difficult to implement (see, for example, Bagnoli and Lipman (1988, p. 100)). For example, there may be warrantholders who also hold sufficient debt claims that their incentives are affected.[17] In addition, a small number of warrantholders may be uninformed about the existence of the offer. In the context of a takeover game Stulz (1988) notes that different claimants face different tax incentives, so that the offerer may not possess the precise information needed for an efficient offer. For the purpose of our analysis, we will focus on unconditional bids.

The analysis in this section suggests that concentration of either the warrants or debt increases the ability of equity to squeeze the other claim. Therefore, the concentration of holdings generates an externality across claimants. For example, if both the debt and warrants are owned by a large number of traders, the free rider problem (e.g., see the paper by Grossman and Hart (1980)) prevents equity from acquiring even a portion of the warrants or debt. The nature of the observed ownership distributions provides a potential rationale for the lack of widespread evidence of double-sided squeezes. In bankruptcy Gertner and Scharfstein (1991) point out that the free rider problem can be partially resolved. The court can impose a settlement in bankruptcy backed by a majority of the holders of a particular claim upon all the holders of that claim. However, as they indicate, the Trust Indenture Act prevents this resolution outside of bankruptcy.

In this model we allow warrantholders and debtholders to tender a fraction of their holdings.[18] The first offer D is made by equity to debtholders. Each debtholder decides what fraction of his holdings to tender. Then an offer W is made to warrantholders and each warrantholder decides what fraction of his holdings to tender. We first analyze the final round in which offers can be made to the warrantholders, but not the debtholders. Understanding the characteristics and properties of the final round is critical to understanding the properties of the subgame perfect equilibrium. We focus on what would occur if a fraction z of the debt was already tendered before this final round. We assume that there are N identical warrantholders ($N \geq 1$) each having a fraction $\frac{1}{N}$ of the total warrants outstanding. If equity offers W for the warrants, then any warrantholder i may tender a fraction α_i of his warrants

[17] Notice that such ownership of the diverse claims is particularly likely to occur in the case of warrants and debt, since detachable warrants are often issued with debt. Therefore, the warrants and debt might be held by the same holders, though they are separate claims.

[18] We do not consider mixed strategies. Since claimants are allowed to tender any fraction, the strategy space is continuous.

and receive $\alpha_i \frac{1}{N} W$. The warrantholders would maximize $(1 - \alpha_i)\frac{1}{N}w + \alpha_i \frac{1}{N} W$, where α_i denotes the fraction of warrants tendered. The value w depends upon α_i because the more warrants tendered, the more favorable equity's project choice for the warrantholder. The favorable effect on equity's project choice increases the benefit to tendering warrants.

For an offer W for the warrants each warrantholder must choose a fraction α_i of his warrantholding to tender. The total fraction of all outstanding warrants tendered α is $\sum_{i=1}^{N} \alpha_i \frac{1}{N}$. Denote by $\beta_i = \sum_{j \neq i} \alpha_j \frac{1}{N}$ the quantity tendered by players other than i. Thus the total fraction of warrants tendered can also be written as $\alpha_i \frac{1}{N} + \beta_i$ for any i. In a subgame perfect equilibrium the ith warrantholder regards β_i as fixed and chooses α_i to maximize

$$P_i = \alpha_i \frac{1}{N} W + (1 - \alpha_i) \frac{1}{N} w\left[a\left(z, \alpha_i \frac{1}{N} + \beta_i\right)\right]. \tag{8}$$

The optimum is found by setting the first derivative equal to zero, assuming the function w is differentiable:

$$\frac{dP_i}{d\alpha_i} = \frac{1}{N} W - \frac{1}{N} w\left[a\left(z, \alpha_i \frac{1}{N} + \beta_i\right)\right] + \frac{1}{N^2}(1 - \alpha_i)\frac{\partial w}{\partial y} = 0, \tag{9}$$

where $\frac{\partial w}{\partial y}$ is the derivative of $w[a(z, \alpha_i \frac{1}{N} + \beta_i)]$ with respect to the second argument, $\alpha_i \frac{1}{N} + \beta_i$. Thus at the solution

$$W - w\left[a\left(z, \alpha_i \frac{1}{N} + \beta_i\right)\right] + \frac{1}{N}(1 - \alpha_i)\frac{\partial w}{\partial y} = 0. \tag{10}$$

The second-order condition requires that

$$\frac{d^2 P_i}{d\alpha_i^2} = -\frac{1}{N^2}\frac{\partial w}{\partial y} - \frac{1}{N^2}\frac{\partial w}{\partial y} + \frac{1}{N^2}(1 - \alpha_i)\frac{\partial^2 w}{\partial y^2}\left(\frac{1}{N}\right) \leq 0, \tag{11}$$

which is equivalent to

$$-2\frac{\partial w}{\partial y} + (1 - \alpha_i)\frac{1}{N}\frac{\partial^2 w}{\partial y^2} \leq 0. \tag{12}$$

The third term of equation (10) is positive. Hence, the sum of the first two terms must be negative to satisfy (10). Therefore, $w[a(z, \alpha_i \frac{1}{N} + \beta_i)]$ exceeds W so that the direct payoff to the warrants held is larger than to the warrants tendered. Increased tendering of warrants causes equity to act more in the interests of warrants and provides a positive externality to warrantholders. This externality is reflected in the third term of equation (10). Because of the public goods aspect of tendering warrants there would be partial acceptance (by each of the warrantholders) of an offer $W = w[a(z,0)]$ even in this final period. Even if $W < w[a(z,0)]$, there may be partial acceptance of the tender offer to the warrantholders.

Since the first two terms of the first-order condition are identical for all warrantholders (noting that $\beta_i + \alpha_i/N = \alpha$ is the total fraction of warrants tendered, which does not depend upon i), the third term also must be identical. This implies that the equilibrium must be symmetric with all warrantholders choosing identical values of α_i. Although in any subgame perfect equilibrium each investor chooses α_i assuming β_i is fixed, in equilibrium $\beta_i = \dfrac{N-1}{N}\alpha_i$. The first-order condition also allows us to establish that decreasing the concentration of the ownership of warrants decreases the fraction of warrants tendered in response to a fixed offer. This is a consequence of the increase in free riding resulting from the smaller holdings in more dispersed ownership.

PROPOSITION 1: *For a given offer W, the total fraction of warrants accepting the offer declines as N rises.*

Proof: Consider two values $\hat{\hat{N}}$ and $\hat{N}$, where $\hat{\hat{N}} > \hat{N}$. The solution for $\hat{N}$ is denoted $\hat{\alpha}_i$ and must satisfy the first-order condition given in equation (10). We evaluate the left side of equation (10) using $\hat{\alpha}_i$ (the solution for $\hat{N}$) but using the value $\hat{\hat{N}}$ instead of $\hat{N}$. Therefore, the corresponding solutions α are identical for $\hat{N}$ and $\hat{\hat{N}}$ as $\alpha = \sum_{i=1}^{\hat{\hat{N}}} \frac{1}{\hat{\hat{N}}}\hat{\alpha}_i = \sum_{i=1}^{\hat{N}} \frac{1}{\hat{N}}\hat{\alpha}_i$. The first term of equation (10) does not involve N and will be the same for $\hat{\hat{N}}$ as $\hat{N}$. The second term can be expressed as $-w[a(z, \alpha)]$, which does not depend upon N. The third term is positive, but decreasing in N. If the solution $\hat{\alpha}_i$ is used for $\hat{\hat{N}}$, then the left side of equation (10) is negative. Since both $\dfrac{\partial^2 P_i}{\partial \alpha_i^2}$ and $\dfrac{\partial^2 P_i}{\partial \alpha_i\, \partial \beta_i}$ are negative, a lower value of α_i satisfies equation (10). To satisfy the first-order condition warrantholders must use a lower value of α for $\hat{\hat{N}}$ than for $\hat{N}$. Q.E.D.

We next observe that as $N \to \infty$ the third term of the first-order condition approaches zero, provided that the derivative $\dfrac{\partial w}{\partial y}$ is bounded. The following proposition follows from equation (10).

PROPOSITION 2: *For a given offer, the payoff for untendered warrants converges to that for tendered warrants as* $N \to \infty$. *Similarly, if* α^* *is the value which equates the payoffs from tendered and untendered warrants, then as* $N \to \infty$, $\alpha \to \alpha^*$.[19]

[19] Similarly, if α^* is the value such that $W = w[a(z, \alpha^*)]$, then $\alpha \to \alpha^*$ as $N \to \infty$.

Proof: The payoff to untendered warrants is given by the second term in equation (10), while the payoff to tendered warrants is given by the first term in equation (10) and the third term of equation (10). Since the third term approaches zero as $N \to \infty$, the payoff to a tendered warrant must equal that of an untendered warrant as $N \to \infty$. Q.E.D.

The intuition behind this proposition is simply that for large N the public goods aspect of tendering warrants is ignored and warrantholders free ride by equating the first two terms of equation (10), i.e., $W = w[a(z, \alpha^*)]$. An interesting implication of Proposition 2 is that for an offer of $w[a(z, 0)]$, then $\alpha \to 0$ as $N \to \infty$. Clearly Proposition 2 indicates that the limiting value of α as N goes to infinity will be higher if the offer W is higher.[20] We also can establish that for a given concentration of ownership, increasing the offer will cause a larger fraction α of the warrants to be tendered. This is summarized in the following proposition.

PROPOSITION 3: *For a given N, if equity offers W, then increasing W increases the fraction of warrants tendered.*

Proof: Again consider $\hat{\hat{W}} > \hat{W}$. For $\hat{W}$ there is a solution $\hat{\alpha}_i$ to the first-order condition given in equation (10). Try the same solution $\hat{\alpha}_i$ for $\hat{\hat{W}}$ in the left side of equation (10). The result is positive since only the first term is changed. Since $\dfrac{\partial^2 P_i}{\partial \alpha_i^2}$ and $\dfrac{\partial^2 P_i}{\partial \alpha_i \, \partial \beta_i}$ are negative, increasing the value of α_i and hence of β_i will lower the left side of equation (10). Hence the solution $\hat{\hat{\alpha}}_i$ for $\hat{\hat{W}}$ must be higher than for $\hat{W}$. Q.E.D.

This shows that by increasing the offer equity can induce the warrantholders to tender a larger fraction of the warrants.

In a subgame perfect equilibrium the equity must choose an offer W that is optimal given the acceptance strategy of the warrantholders. We now explore an optimal offer strategy for equity assuming a fraction z of debt has been tendered. The payoff to equity Q is then given as

$$Q = zd[a(z, \alpha)] + e[a(z, \alpha)] + \alpha w[a(z, \alpha)] - \alpha W, \tag{13}$$

where the value of α depends upon the level of the offer W.[21] Raising the offer implies that α increases (see Proposition 3), so that the sum of the first three terms of (13) increases. However, a higher offer will also increase the absolute value of the final term of (13), which is negative.

[20] With *conditional* offers there is an additional incentive to tender in that a player might be pivotal so that without his acceptance the offer would fail. In the Stulz (1988) approach there is uncertainty about the reservation prices of other agents (due to taxes). The result is that there is uncertainty about whether a conditional offer will succeed. In this type of framework increasing N reduces the probability of a player being pivotal. This suggests that there would be more free riding as the number of warrantholders is increased.

[21] Alternatively, we could subtract from Q the amount zD, which equity paid for the debt. However, this is a sunk cost and will not affect equity's offer to the warrantholders.

One offer equity could always make is $W = w[a(z,0)]$. In this case $\alpha \geq 0$ and the payoff to equity is at least $zd[a(z,0)] + e[a(z,0)]$. Thus in a subgame perfect equilibrium equity must receive at least $zd[a(z,0)] + e[a(z,0)]$. The condition $Q \geq zd[a(z,0)] + e[a(z,0)]$ is necessary, but not sufficient for a subgame perfect equilibrium. This necessary condition is useful in proving the following proposition.

PROPOSITION 4: *Provided that* $\dfrac{\partial w}{\partial y}$ *is bounded, then for any* $\bar{\alpha} > 0$ *there exists an* N^* *such that any subgame perfect equilibrium with* $N > N^*$ *must have* $\alpha < \bar{\alpha}$.

Proof: Define $A = zd[a(z,0)] + e[a(z,0)]$ and $B = zd[a(z,\alpha)] + e[a(z,\alpha)]$. By offering $W = w[a(z,0)]$ to warrantholders, equity always will have a value of Q at least equal to A. Thus a necessary condition for a subgame perfect equilibrium is

$$A \leq B + \alpha w[a(z,\alpha)] - \alpha W. \tag{14}$$

Rearranging we have

$$A - B \leq \alpha w[a(z,\alpha)] - \alpha W. \tag{15}$$

By the definition of $a(x,y)$ and the strictness assumption discussed in footnote 12, $A - B$ is greater than zero for $\alpha > 0$ and increasing in α.[22] If $\alpha \geq \bar{\alpha}$, then $A - B$ must be greater than or equal to $zd[a(z,0)] + e[a(z,0)] - zd[a(z,\bar{\alpha})] - e[a(z,\bar{\alpha})]$, which is positive. However, Proposition 2 implies that by making N large enough $\alpha w[a(z,\alpha)] - \alpha W$ can be made arbitrarily close to zero. This implies that making N large enough will make the left side of (15) arbitrarily close to zero. Thus, for $\bar{\alpha} > 0$, there must exist N^* such that $N > N^*$ guarantees that a subgame perfect equilibrium cannot be found with $\alpha \geq \bar{\alpha}$. Q.E.D.

Propositions 1 through 3 explore the outcome for a specified offer, whereas Proposition 4 examines the equilibrium when the offer is endogenous. Propo-

[22] Start with $\alpha > 0$, which implies there exists α' such that $0 < \alpha' < \alpha$. Then by the definition of $a(z,\alpha')$ it is clear that

$$zd[a(z,\alpha)] + e[a(z,\alpha)] + \alpha' w[a(z,\alpha)] \leq zd[a(z,\alpha')] + e[a(z,\alpha')] + \alpha' w[a(z,\alpha')].$$

Then, by the strict inequalities discussed in footnote 12, $w[a(z,\alpha)] > w[a(z,\alpha')]$, so that

$$zd[a(z,\alpha)] + e[a(z,\alpha)] < zd[a(z,\alpha')] + e[a(z,\alpha')].$$

Therefore, $-B$ and $A - B$ (defined in the text) are strictly increasing in α. By the definition of $a(z,0)$,

$$zd[a(z,\alpha')] + e[a(z,\alpha')] \leq zd[a(z,0)] + e[a(z,0)].$$

Combining the last two inequalities yields

$$zd[a(z,\alpha)] + e[a(z,\alpha)] < zd[a(z,0)] + e[a(z,0)].$$

Therefore, $A - B > 0$ for $\alpha > 0$.

sition 4 implies that if the warrants are widely held, then the amount tendered in the final round can be made arbitrarily close to zero.

Propositions 1 through 4 have been developed assuming that the final round of the game involved the warrantholders. These propositions explore and demonstrate the effect that the distribution of ownership of warrants has on the results of this final round. If instead the final round involved the debtholders, then a similar set of propositions obtains for the debtholders. For these similar propositions the analogous results would apply to the debtholders. Of course, other capital structures also could be used.

Next we consider a case where both debt and warrants are each competitively held by a continuum of holders, invoking the subgame perfect equilibrium solution. An offer is made to one type of holder and then after it is accepted or rejected an offer is made to the other type of claimant. The previous analysis demonstrates that the warrantholders will not accept an offer in the last stage. Then applying the logic of the analysis to the debtholders in the penultimate stage implies that the offer to debtholders will not be accepted. Therefore, with an infinite number of debtholders and an infinite number of warrantholders, there would be no successful offers.

These results can be applied to the case in which the warrants are "competitively" held by a continuum of warrantholders. Debt is held by an oligopoly of debtholders. Using Proposition 4 we see that the warrantholders will not be tendering warrants in the final round. Thus, when the offer is made to the debtholders it is as if this were the final offer of the game. Since there are a finite number of debtholders, a positive fraction of the debt is tendered for any offer $D \geq d[a(0,0)]$. Equity is better off making an offer of $d[a(0,0)]$ to debt than making an offer which no debtholders accept. The optimal offer by equity need not be $d[a(0,0)]$, although the optimal offer must be one that is accepted by some of the debt. In the subgame perfect equilibrium some of the debt is tendered, but none of the warrants are tendered. Interestingly, the debtholders will receive more than $d[a(0,0)]$ and the warrantholders will receive less than $w[a(0,0)]$. The consolidation of the debt is slightly advantageous to debtholders and detrimental to warrantholders. We also can analyze the reverse situation in which there are a continuum of debtholders and a small number of warrantholders. The debtholders will not tender their debt, but a fraction of the warrants will be tendered.

III. Concluding Comments

The possibility of trading between financial claimants allows equity to alter the capital structure of the firm. We have shown that even if the capital structure is designed so that equity has the same incentives as the firm the agency problem may still occur. The agency conflict arises because equity may buy out claims so as to restructure the firm to its own advantage. This restructuring and the related agency problem may be eliminated by certain capital structure designs. By bundling claims together in a way that links claims preferring low-risk projects with claims preferring high-risk projects this incentive to restructure may be eliminated as shown in Section I. This issue applies to a variety of capital structures and potential financial claims. A specific example is the bundling of debt and warrants in the form of convertible debt, suggesting an explanation of why convertible bonds are more common than debt with warrants.

With widely dispersed ownership of warrants and debt, it is more difficult for equity to squeeze the other financial claimants. If the equity offers a certain amount per warrant to the warrantholder, then the warrantholder might tender only a fraction of his holdings. Then equity would acquire only a fraction of the warrants and debt. However, the fraction acquired is decreasing in the number of holders of a given class. In fact, we show that in the limit as the number of holders approaches infinity, the fraction of warrants acquired approaches zero. The free rider problem of Grossman and Hart (1980) has the beneficial effect of preventing an agency problem from occurring. Therefore, widely held claims prevents equity from restructuring the firm.

It is interesting that the claimants do worse when the debt and warrants are separately held by single owners rather than when both these claims are widely held. This is not because the warrants are less valuable when a single claimant holds them, but because the potential value of the debt is reduced when the warrants are held in concentrated form. Likewise, concentrating the ownership of the debt is detrimental to the warrantholders. This result is somewhat analogous to an example in Spatt and Sterbenz (1988) in which two duopoly warrantholders do worse than when their claims are widely held.

In recent years considerable attention has focused upon leveraged buyouts and other recapitalizations of the firm. In many of these transactions the corporate restructuring has severely reduced the value of outstanding debt (e.g., as illustrated by such recent examples as RJR Nabisco and Marriott). As we have shown, bundling claims is a way to eliminate the ability of equity to engage in opportunistic restructuring. One observed form of financing described in Copeland and Weston (1988) is strip financing in which the diverse claims of the firm are packaged in fixed proportion (making the package equivalent to unlevered equity). Jensen (1988) suggests that strip financing is a method of eliminating the conflicts between different claimants in the reorganization of a firm, especially if the securities are stapled together. Our analysis of the trading among claimants provides a further justification for stapling the securities together.

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