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Value of Latent Information: Alternative Event Study Methods

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ABSTRACT

This paper presents an econometric model to value latent information underlying corporate events. This model computes the market's inference of the value of latent information from the probability of an event, conditional on firm-specific, preevent information. It provides a convenient framework for testing significance of preevent information variables, such as accounting attributes and lagged stock return. Simulations show that this model, when applied to *both* event and preevent period data, can decrease the incidence of bias in event studies. If restricted to only event period data, this model reduces to a truncated regression and does not perform as well as standard procedures.

THIS PAPER PRESENTS AN econometric approach to measure the value of latent information underlying corporate events and conducts simulation experiments to analyze the performance of alternative event study methodologies. The original study by Fama, Fisher, Jensen, and Roll (1969) interprets the mean abnormal common stock return over the event period as the value of information released. This and numerous other event studies that followed recognize that events are not complete surprises and estimate the value of information by implicitly or explicitly accounting for the partially anticipated nature of events.[1] But, the (standard) methodology used in these studies does not account for the market's inference of the value of information released by a probable event, given publicly available information.

Most corporate events result from some decision process although the standard methodology presumes that events are exogenous. Decisions triggering events usually involve latent information not completely available to the market. For example, when a firm announces its bid to acquire a target

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[1] Brennan (1990, p. 709) emphasizes that events may not be complete surprises. It is easy to show that the Fama-Fisher-Jensen-Roll procedure implicitly models partially anticipated events. Malatesta and Thompson (1985) explicitly model such events.

firm, the information privately acquired by the bidding firm about the target can only be partially extracted by the market. Likewise, when a jury announces its verdict in a lawsuit, the information used by the jury in making the decision is not completely observed by the market. Jumps in stock prices are likely to depend on market inference of the latent information underlying the decision process (Acharya (1986)).

Inference of latent information depends on the market's prior information. This prior information very likely differs across firms making the same type of announcement. For example, consider two types of firms announcing an increase in dividend payout by the same amount. Suppose that these firms are similar except that, in the immediate past, the first type borrowed heavily while the second type invested heavily. Investors' evaluation of a firm's stock value when the dividend payout is increased is likely to be different across these firms. Not controlling for information observable by investors prior to a probable announcement can yield a biased, and certainly an incomplete, measure of the news the management signals through an event.

Many event studies cross-sectionally regress residual returns on firm-specific attributes.[2] Leaving aside the statistical issues involved in such a procedure, the main point here is that this cross-section regression is limited to only event periods for those firms that announce the event. This procedure ignores preevent periods when firms faced choice problems, but made no announcement. The decision to skip a probable announcement will carry valuable information if the event is presumed to impact stock value. Omitting preevent period data is likely to induce selectivity bias since the disturbance term in the chosen sample will have a nonzero mean. (See, e.g., Maddala (1984).)

While a dummy variable model can use stock return data over both event and nonevent periods, it does not account for potentially valuable market inference of the latent information underlying an event. It is well known in the econometrics literature that coefficient estimators in a dummy variable model are inconsistent when data on the dependent variable (stock return) are limited by choices of agents (decision rules that trigger events).

The latent variable model suggested in Acharya (1986) accounts for market inference and computes the inferred value of information underlying an event announcement process. This model measures the expected abnormal return to an event, conditional on firm-specific preevent information. Further, it uses data over both event and nonevent periods.[3] Thus, problems of inconsistency and selectivity bias do not arise. In this model, the inferred value of information underlying an event can be computed from the probability of the event, given preevent information. The latent variable model thus provides a convenient framework to test for significance of the effects of firm-specific variables on the expected abnormal return.

[2] See, e.g., Brown and Warner (1980, 1985) for ways of computing abnormal returns.

[3] See Acharya (1988) for stock price effects of discrete signals.

Preevent information could comprise firm-specific accounting variables and past stock return. Lagged stock return (in the preevent information set) of a firm could capture many details of relevant information not contained in accounting data. Accounting attributes can explain the probability of many corporate events, but they may not vary much over a relatively short period. If only time-invariant accounting data are used as preevent information, the latent variable model applied to an *individual* firm (with time series return data) will indeed be equivalent to a dummy variable model. In event studies involving many firms, the expected abnormal return to an event is likely to depend on cross-sectionally and intertemporally varying accounting attributes and lagged returns. While the latent variable model measures such a variable expected abnormal return, standard procedures estimate only a constant abnormal return. This imputes a large-sample bias in the expected abnormal return estimator within a standard procedure. This bias approaches zero as the cross-sectional and intertemporal variation in the firm-specific attributes vanishes.

Restricting the latent variable model to only event periods results in a truncated regression procedure.[4] A truncated regression model cannot exploit intertemporal variation in preevent information to measure expected abnormal returns. But, this approach promises to correct for potential selectivity bias, using data only over the event period. Indeed, simulation experiments in the econometrics literature show that this procedure can precisely estimate the basic model parameters (market model coefficients). Unfortunately, however, parameter estimates in the truncation (market inference) criterion are unreliable. (See, e.g., Maddala (1984, p. 267).)[5] Since the stock price effect of an event is a function of parameters in the truncation criterion, this procedure can result in imprecise estimates of the value of information released by corporate events. This raises an important question: Can parameters in the market inference criterion of the *untruncated* latent variable model be estimated reliably? To the best of my knowledge, no past simulation study has documented performance of parameter estimators based on this model.

We simulate alternative event study methodologies and measure their performance by the absolute deviation of parameter estimates from their true values. The untruncated latent variable model performs at least as well as other models. The truncated regression procedure estimates market model coefficients as accurately as other procedures, but imprecisely estimates parameters in the market inference criterion, providing a poor measure of the

[4] See, e.g., Eckbo, Maksimovic, and Williams (1990) for an application of this approach to corporate mergers. In this study, investors observe whether or not a bidder has identified a target, but not the latent information the bidder used to identify the target.

[5] Maddala writes: "It is usually the case that even though we are able to correct for the selectivity bias [in the basic model parameter estimates]..., the estimates of parameters in the selectivity criterion are not reliable." This problem has received limited attention in this literature perhaps because most applications were concerned with precise estimation of the basic model parameters.

stock price effect of events. Truncation does not pose a theoretical problem for parameter identification, but it nevertheless results in omission of critical information on firms that skipped announcement of a probable event.

The standard event study procedure and the dummy variable model perform better than the truncated regression procedure. For high probability events, the performance of the mean abnormal return estimator in the standard and the dummy variable procedures is close to that in the untruncated latent variable procedure; but, in the former procedures, the precision of estimation as indicated by the inverse of standard errors falls significantly for low probability events. Further, simulations reported here and applications to actual data (available from the author on request) indicate that effects of firm-specific attributes on the expected abnormal return are better captured in the untruncated latent variable approach than in other procedures. Latent variable models contribute nothing, when firm-specific attributes are unavailable or are unreliable predictors of the probability of an event.

Section I presents alternative event study models which are applied to simulated data in Section II. In Section III, we conclude.

I. Alternative Event Study Models

In this section we present four different models for measuring the value of information released by events.

A. Standard Event Study Model

The first model is a two-step procedure that corresponds to a standard event study. In the first step, parameters of a benchmark model are estimated using realized stock return data over a suitable period presumed to be unrelated to the event:

$$R_{it} = \beta_i' W_{it} + \varepsilon_{it}, \tag{1}$$

where $E(\varepsilon_{it}|W_{it}) = 0$, $t \in$ {period in which the event is improbable}, W_{it} is a vector of firm-specific attributes as of time $t-1$ and market-wide factors as of time t, including the return to a market index, and β_i is an associated vector of coefficients.[6] Model (1) conveniently expresses the realized return as a sum of two components: $\beta_i' W_{it}$ which is the expected return (given W_{it}) and

[6] Note that $0 = E(\varepsilon_{it}|W_{it}) = p_{eit} A_{eit} + (1 - p_{eit}) A_{nit}$, where $p_{eit} \in (0, 1)$ is the probability of occurrence of the event; A_{eit} is the expected abnormal return, conditional on announcement of an event; and A_{nit} is the expected abnormal return, conditional on skipping the probable announcement. Clearly whenever A_{eit} is nonzero, A_{nit} is nonzero. This implies that $E(\varepsilon_{it}|W_{it}, t \in \{\text{nonevent}\}) \neq 0$. Thus, the estimators of coefficients obtained from stock return data over a period when an event was probable but not announced will be inconsistent. Note that A_{eit} and A_{nit} can be interpreted as state-contingent valuations of the rate of change in the equity value of the firm and can therefore be predetermined before the event is announced, where the states are "event" and "nonevent."

the abnormal return ε_{it}. While most event studies use $\beta_i' W_{it} = \alpha_i + \beta_i R_{mt}$, where R_{mt} is the return to a market index, we use form (1) for notational simplicity and to maintain generality.

Using parameter estimates from the first step, event period residual returns, $\hat{\varepsilon}_{it}$, are computed by

$$\hat{\varepsilon}_{it} = R_{it} - \hat{\beta}_i' W_{it}, \tag{2}$$

for $t \in$ {event}. Notationally, $t \in$ {event} means that an event announcement was made during $(t-1, t)$, whereas $t \in$ {nonevent} means that an event announcement was probable during $(t-1, t)$ but not made.

In the second step, the following cross-sectional regression is performed:

$$\hat{\varepsilon}_{it} = \theta_0 + \theta_1' z_{i1t-1} + \cdots + \theta_J' z_{iJt-1} + \eta_{it}, \quad t \in \{\text{event}\}, \tag{3}$$

where $(z_{i1t-1}, \ldots, z_{iJt-1}) \in W_{it}$ is a J-vector of attributes of firm i in the time period just preceding the event period, and $\theta_0, \ldots, \theta_J$ is a conforming set of coefficients. Typical event studies use $\Sigma_{t \in \{\text{event}\}} \hat{\varepsilon}_{it}$ as the left-hand side variable in (3).

B. Dummy Variable Model of Event Study

The dummy variable approach corresponds to the event study procedure advocated by Schipper and Thompson (1983) and Binder (1985).[7] This procedure involves just one step and estimates the following model for a sample of firms:

$$R_{it} = \beta_i' W_{it} + \delta_e I_{eit} + \delta_n I_{nit} + \eta_{it}, \tag{4}$$

where $t \in$ {event} $\cup$ {nonevent}, I_{eit} is a dummy variable which takes a value 1 whenever $t \in$ {event} for firm i, and a value 0 otherwise; and $I_{nit} = 1 - I_{eit}$. In this model, δ_e measures the expected return, conditional on the event, and δ_n measures the expected return, conditional on the nonevent. The coefficients δ_e and δ_n do not have to be held constant across firms, and model (4) can be estimated for an individual firm experiencing the same type of event more than once in a sample period. Observe that if β_i's are known, (3) will be equivalent to (4), restricted to event periods $[I_{eit} = 1]$ with $\delta_e = \theta_0$; in this case (3) is nested within (4).

C. Latent Variable Models for Endogenous Events

The event study models (3) and (4) are statistical representations of events experienced by corporations. Implicit in these representations is an assumption that events are exogenous to corporations. In most circumstances, this assumption is unrealistic. Corporate events usually are the results of decisions based on information available to managements of corporations, although some events (e.g., the outcome of a lawsuit against a firm) are not controlled by managements.

[7] See also Malatesta and Thompson (1985). Binder attributes the dummy variable approach to Gibbons (1980).

In most, if not all, cases the decision process leading to an event is based on information not generally available to the market. It is, however, reasonable to suppose that the market can anticipate the nature of such a decision process and make at least an inference about the unobservable (latent) information. In this section, we obtain a simple statistical representation of this inference process. The basic idea is to model the unobserved information as a latent random variable whose realization at a point in time determines whether or not the event occurs. We first derive a latent information model that uses both the event period and preevent period data. In the next subsection, we show that the truncated regression model is nested within the latent information model.

First consider corporate events controlled by managers. Let $\tilde{x}_{it}$ denote firm i's evaluation of the net present value of announcing an event minus the net present value of not announcing the event at time t. It is reasonable to suppose that the manager announces the event whenever $\tilde{x}_{it}$ crosses a particular threshold. In case of corporate events not controlled by managers, for example, the outcome of a lawsuit, $\tilde{x}_{it}$ represents the amount of favorable evidence available to the jury. In this case too it is plausible to suppose that $\tilde{x}_{it}$ crosses a threshold for the jury to announce a favorable verdict. We can normalize the problem so that for any particular firm, this threshold may be assumed to be zero. From the point of view of investors in the market, $\tilde{x}_{it}$ is an unobservable latent random variable whose realization determines the corporate event.[8]

While the market may not observe the information, $\tilde{x}_{it}$, that determines an event, it can observe firm-specific characteristics

$$z_{it-1} \equiv (z_{i0t-1} \quad z_{i1t-1} \cdots z_{iJt-1})'$$

at $t-1$, where $z_{i0t-1} = 1$ may provide some information about $\tilde{x}_{it}$. To capture this, we specify:

$$\tilde{x}_{it} = \tilde{\theta}' z_{it-1} + \tilde{\xi}_{it},$$

where $\tilde{\theta}' z_{it-1}$ is the investors' expectation of $\tilde{x}_{it}$, conditional on prior publicly available information z_{it-1}, $\tilde{\theta}$ is a vector of coefficients, and $\tilde{\xi}_{it}$ is the component of $\tilde{x}_{it}$ that is unobservable to investors. Then, defining

$$x_{it} \equiv \frac{\tilde{x}_{it}}{\sigma_{\tilde{\xi}}}, \qquad \theta \equiv \frac{\tilde{\theta}}{\sigma_{\tilde{\xi}}}, \qquad \xi_{it} \equiv \frac{\tilde{\xi}_{it}}{\sigma_{\tilde{\xi}}},$$

where $\sigma_{\tilde{\xi}}$ is the standard deviation of $\tilde{\xi}_{it}$, it follows that[9]

$$x_{it} = \theta' z_{it-1} + \xi_{it}. \tag{5}$$

[8] Observe that in some cases, outcomes may not be simply binomial. See Acharya (1986) for binomial, multinomial, and continuous corporate announcements based on latent information; Lanen and Thompson (1988) and Eckbo, Maksimovic, and Williams (1990) for binomial actions.

[9] No generality is lost by the normalization in (5). Note, however, that $\tilde{\theta}$ and $\sigma_{\tilde{\xi}}$ are not separately identifiable, i.e., a probit model used for estimation later can identify only the ratio $\tilde{\theta}/\sigma_{\tilde{\xi}} = \theta$.

Henceforth we call ξ_{it} the latent information. The market's inference of the latent information is limited by the observability of the event indicator I_{eit} which takes a value 1 whenever $t \in \{\text{event}\}$, and a value 0 otherwise. Clearly, given (5), the market can infer that

$$I_{eit} = \begin{cases} 1 & \text{whenever } \theta' z_{it-1} + \xi_{it} > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Under a null hypothesis of nonzero information effect, x_{it} is related to the rate of return on the stock of firm i over period $(t-1, t)$. Given W_{it} (which includes z_{it-1}), this relationship can be represented by the relationship between ξ_{it} and the abnormal return to the stock of firm i, $\varepsilon_{it} \equiv R_{it} - \beta_i' W_{it}$. In particular, we specify the following cross-sectional relationship:

$$\varepsilon_{it} = q\xi_{it} + \eta_{it}, \tag{7}$$

where $q \equiv \text{Cov}(\varepsilon_{it}, \xi_{it})$ since $\text{var}(\xi_{it}) = 1$. Given the inference process (6), the market's reevaluation of the rate of change in value of equity of the firm when the event is announced is:

$$E(\varepsilon_{it} | I_{eit} = 1, W_{it}) = qE[\xi_{it} | z_{it-1}, \xi_{it} > -\theta' z_{it-1}] = q\frac{\phi(\psi_{it})}{\Phi(\psi_{it})}, \tag{8}$$

where

$$\psi_{it} \equiv \theta' z_{it-1}, \tag{9}$$

$$\phi(u) \equiv \frac{1}{\sqrt{2\pi}} e^{-(1/2)u^2}, \tag{10}$$

$$\Phi(s) \equiv \int_{-\infty}^{s} \phi(u)\, du, \tag{11}$$

i.e., $\phi(\cdot)$ and $\Phi(\cdot)$ are, respectively, the standard Normal density and cumulative density functions. (See, e.g., Johnson and Kotz (1970).) Similarly, the rate of change in value of equity of a firm when a probable announcement is skipped is

$$E(\varepsilon_{it} | I_{nit} = 1, W_{it}) = -q\frac{\phi(\psi_{it})}{1 - \Phi(\psi_{it})}, \quad I_{nit} = 1 - I_{eit}. \tag{12}$$

Although the market does not observe the latent information ξ_{it} which the firm uses to announce the event, the inference rule (6) allows the market to measure means of ξ_{it}, contingent on the event/nonevent indicator I_{eit} and preevent information z_{it-1}. These conditional means, $E(\xi_{it} | I_{eit} = 1, z_{it-1})$ and $E(\xi_{it} | I_{nit} = 1, z_{it-1})$, used in (8) and (12) to evaluate stock price effects, are indeed functions of the probability of occurrence of the event, $\Phi(\psi_{it})$. The market can measure this probability, given z_{it-1}. If the conditional means of the latent information are interpreted as the quantity of relevant information released by the event or nonevent, then the covariance q will be the quality of this information. Coefficient q translates the conditional means of latent information, ξ_{it}, into expected rates of return to the stock, contingent on the event or the nonevent. The quantities in (8) and (12) are thus the values of information released by the event and the nonevent.

Given specification (1),

$$E(R_{it}|W_{it}, I_{eit}) = \beta_i' W_{it} + \begin{cases} E(\varepsilon_{it}|I_{eit} = 1, W_{it}) & \text{iff } I_{eit} = 1 \\ E(\varepsilon_{it}|I_{eit} = 0, W_{it}) & \text{otherwise} \end{cases}$$

$$= \beta_i' W_{it} + \sum\nolimits_{a=e,n} E(\varepsilon_{it}|I_{ait} = 1, W_{it}) I_{ait}. \tag{13}$$

Substituting the conditional means (8) and (12) in (13),

$$R_{it} = \beta_i' W_{it} + q\left[\frac{\phi(\psi_{it})}{\Phi(\psi_{it})} I_{eit}\right] - q\left[\frac{\phi(\psi_{it})}{1 - \Phi(\psi_{it})} I_{nit}\right] + \nu_{it}, \tag{14}$$

since we can write $R_{it} = E(R_{it}|W_{it}, I_{eit}) + \nu_{it}$, where $E(\nu_{it}|W_{it}, I_{eit}) = 0$.

The latent variable model allows the market to update the probability of the event and estimates the expected abnormal return as a function of firm-specific information, z_{it-1}. In the standard procedures, the expected abnormal return and the probability of the event are both constants. (See, e.g., Malatesta and Thompson (1985) who make this point explicit.) To the extent variables z_{it-1} are useful for updating the probability, results from (14) are likely to be different from those obtained within standard procedures. Indeed, (14) provides a convenient framework to test for the significance of z_{it-1}, as discussed later.

Model (14) can be used for panel data sets in which each firm has data over a sample period. For example, to study the event of bidding for acquisition of firms, data over prebidding and bid announcement periods for bidders could be used. Since data over both event and preevent periods are used, the issue of selectivity bias does not arise in this model and hence no correction for bias is needed. Further, the coefficient estimators are consistent since $E(\nu_{it}|W_{it}, I_{eit}) = 0$. Model (14) could be used for a cross-section of firms that experienced the event and firms that had possibilities but did not experience the event if data on the latter group of firms were available. If the latter group of firms could not be identified, they might be proxied by nonevent observations on firms that experienced the event. Omitting nonevent period data will result in a truncated regression addressed in the next subsection.

It is possible that the expected abnormal return to a nonevent is very small compared to the expected abnormal return to the event (or vice versa). Then a pertinent question is whether (14) estimated over both event and nonevent periods can make a difference to the results. This question appears interesting in the case of a low probability event like bidding for acquisition of firms. There would be a few bidding firms compared to nonbidders at any time t in a given sample period. Even if we were to consider only those firms that made at least one bid in the sample period, the nonbidders are likely to far outnumber the bidders at any time. Observe, however, that

$$0 = E(\varepsilon_{it}|W_{it}) = E(\varepsilon_{it}|W_{it}, I_{eit} = 1)p_{eit} + E(\varepsilon_{it}|W_{it}, I_{nit} = 1)p_{nit}, \quad (15)$$

where $p_{ait} = \text{Prob}(I_{ait} = 1)$, $a = e, n$. Thus, even if $E(\varepsilon_{it}|W_{it}, I_{nit} = 1)$ is small, it is the product of this quantity with the probability p_{nit}, $E(\varepsilon_{it}|W_{it}, I_{nit} = 1)p_{nit}$, which is weighted against $E(\varepsilon_{it}|W_{it}, I_{eit} = 1)p_{eit}$ in the estimation of (14). In (13)–(14), the nonevent probability, p_{nit}, plays an important role via the indicator counter I_{nit} which accumulates the expected abnormal return to the nonevent, $E(\varepsilon_{it}|W_{it}, I_{nit} = 1)$, as many times as the number of nonevent observations in the sample are. In the bidding example, the first product could be comparable to the second product, although the expected abnormal return to the nonevent might be small.[10]

C.1. Estimation and Testing

Model (14) can be estimated by nonlinear least squares methods. But, a simple consistent two-stage procedure is feasible (Acharya (1986)). In the *first stage*, the probability of the event, $\Phi(\psi_{it})$, can be estimated using a probit model with I_{eit} as the dependent variable and z_{it-1} as right-hand side variables. This probit model assumes that ξ_{it}, $\forall i$, and t, are independent. This assumption seems plausible when ξ_{it} are forecast errors in the market's expectation of the value of x_{it}. Independence of ξ_{it} is, however, not crucial but simplifies the estimation.

In the *second stage*, model (14) is estimated by OLS, with the coefficients θ in ψ_{it} replaced by their maximum likelihood estimates, $\hat{\theta}$, obtained from the first-stage probit model.[11] Most econometrics packages have options to directly generate estimates of conditional means of a standard Normal random variable (ξ_{it}) within a probit estimation. Thus the estimation of (14) involves two simple regressions, a probit and an OLS.

Model (14) allows for two types of tests. First, effects (θ) of information variables z_{it-1} on the expected abnormal returns can be tested. This is fairly direct since for the j'th attribute, z_{ijt-1} [where $z_{i0t-1} = 1$],

$$\frac{\partial}{\partial z_{ijt-1}}\left[\frac{q\phi(\theta' z_{it-1})}{\Phi(\theta' z_{it-1})}\right] > 0 \Leftrightarrow q\theta_j < 0, \quad j = 0, 1, \ldots, J,$$

given the well known fact that $\phi(v)/\Phi(v)$ is a monotonically decreasing function of v. The second test, namely, the statistical significance of the

[10] I am thankful to René Stulz for pointing this out.

[11] Although the two-stage procedure is simpler, the co-variance matrix of the second stage OLS estimator should be estimated correctly. This estimation is fairly straightforward in a program like *RATS* by VAR Econometrics, 1800 Sherman Avenue, Evanston, Illinois 60201. In the two-stage procedures suggested by Heckman (1976), Lee (1979), and Hausman and Wise (1976), terms like $\phi(\psi_{it})/\Phi(\psi_{it})$ correct for selectivity biases in truncated and censored samples. Such truncated models are unnecessary here since data over the event period and the nonevent are used.

expected abnormal return itself can be performed by simply testing for q since

$$q\frac{\phi(\theta' z_{it-1})}{\Phi(\theta' z_{it-1})} > 0 \Leftrightarrow q > 0,$$

for all finite $\theta' z_{it-1}$. Further, since

$$E(\varepsilon_{it}|I_{eit}, W_{it}) - E(\varepsilon_{it}|I_{nit}, W_{it}) = q\left[\frac{\phi(\theta' z_{it-1})}{\Phi(\theta' z_{it-1})} + \frac{\phi(\theta' z_{it-1})}{1-\Phi(\theta' z_{it-1})}\right],$$

the expected abnormal return to the event is greater than the expected abnormal return to the nonevent if and only if $q > 0$. These tests do not need the means of cross-sectionally and intertemporally varying expected abnormal returns ((8) and (12)), but such means can be estimated for comparison with the constant abnormal returns traditionally estimated by the standard procedures.

D. Truncated Regression Model

As stated in the previous subsection, the latent variable model (14) may be estimated separately for two truncated groups of data, one over the event period (i.e., whenever $I_{eit} = 1$) and the other over the nonevent period (i.e., whenever $I_{nit} = 1$). Such estimation procedures can, theoretically, correct for the bias resulting from truncation of the data set. For example, as in Eckbo, Maksimovic, and Williams (1990), one may estimate the expected abnormal return conditional on the event using only the event period data:[12]

$$R_{it} = \beta_i' W_{it} + q\left[\frac{\phi(\psi_{it})}{\Phi(\psi_{it})}\right] + \nu_{it}, \tag{16}$$

where $t \in$ {event} and $i \in$ {sample of firms which experienced the event}. Observe that in the truncated regression model (16), $E(\nu_{it}|W_{it}, I_{eit}) = 0$,

[12] Eckbo, Maksimovic, Williams (1990) study an event in which the manager of a firm proposes a merger (with another firm) which can be challenged by government regulators and then be decided by a court. In this case (using the Eckbo, Maksimovic, and Williams's notation) the *event* of {proposing a merger} $\equiv \{d_{jt} = 1\} \equiv \{\eta_j \geq -x_j\gamma\}$, and has a *nonevent* {not proposing a merger}. In their equation (4), either the nonevent probability $[1 - \text{Prob}(\eta_j \geq -x_j\gamma) \equiv 1 - N(z_j)]$ or the nonevent stock price effect is zero, but this is incorrect when η_j has a nondegenerate distribution, e.g., Normal as they assume. Similarly, their model (3) incorrectly presumes that the nonevent effect is zero while it is not, given their specifications, as can be easily seen by evaluating a conditional mean similar to their (2). It is correct, however, to apply their model (3) [for $d_{jt} = 1$] to only event period data (proposed mergers). For $d_{jt} = 1$, their (3) is identical to our (16) if one specifies $\beta_i' W_{it} = a_i + b_i R_{mt} + \gamma z_{it-1}$—their model uses notation x for our z, and constrains $\gamma = \theta$. They do not use time subscripts for z_j (x_j in their model (3)), but, these attributes seem to be in the preevent information set. It is important to note that, in the truncated regression model, neither the nonevent probability nor the nonevent effect is zero. This fact should be accounted for in evaluating equations like Eckbo, Maksimovic, and Williams's (4) and (5), while applying the truncated regression model to truncated data.

which ensures consistency of estimators of basic model parameters β_i.[13] Correcting for selectivity bias in the estimator of β_i by using the term $q[\phi(\psi_{it})/\Phi(\psi_{it})]$ in (16) is well known in the econometrics literature. (See, e.g., Heckman (1976), Lee (1979), and Hausman and Wise (1976).) Here, this term represents the expected abnormal return to the event. As discussed earlier, this term is of primary relevance to an event study. We are thus concerned about accurately estimating parameters θ and q that define this quantity of interest. Recall that the parameter θ gets into this quantity due to its role in the truncation (market inference) criterion (6).

E. Consistency of Parameter Estimators

As discussed earlier, the standard procedure (with known β_i) is nested within the dummy variable model (4), restricted to the event period data [for all $I_{eit} = 1$]. It is well known in the econometrics literature that using the dummy variable model can result in inconsistent estimators of the information effect.[14] For example, when η_{it} are homoskedastic, the large-sample bias in the estimator $\hat{\delta}_e$ is given by:

$$p \lim_{s \to \infty} \hat{\delta}_e - \delta_e = q \frac{1}{\bar{p}_e} \mathrm{Cov}\left(\frac{\phi(\psi_{it})}{\Phi(\psi_{it})}, \Phi(\psi_{it}) \right), \tag{17}$$

where $s = nT$ for n firms and T time periods in the sample, and $\bar{p}_e$ is the mean probability of event, as defined in the Appendix. Clearly, this bias is zero if either (i) $q = 0$, or (ii) $\mathrm{Cov}(\phi(\psi_{it})/\Phi(\psi_{it}), \Phi(\psi_{it})) = 0$. Condition (i) holds if the true announcement effect is zero, and condition (ii) holds if the probability of occurrence of the event is constant, i.e., if investors in the capital market do not update the probability of the event, conditional on preevent information, z_{it-1}. Thus, the large-sample bias in the expected abnormal return estimator within a standard procedure approaches zero as the cross-sectional and intertemporal variation in the firm-specific attributes vanishes. These conditions can be tested using (14) and (16).

II. Simulation Experiments

As discussed earlier, β_i can be estimated precisely in the truncated regression model (16), but concerns have been raised about the reliability of estimates of parameters θ and q in the truncation (market inference) criterion (Muthen and Joreskog (1981) and Maddala (1984)). This raises a similar question about estimates of these parameters in the untruncated latent

[13] The truncated estimation procedures cannot, however, guarantee the basic specification that $E(\varepsilon_{it}|W_{it}) = 0$, which can cause problems (Acharya (1989)).

[14] Observe that existence of a large-sample bias is not as obvious as in the classical missing variable problem since the dummy variable model uses δ_e and δ_n, respectively, as proxies for $q(\phi(\psi_{it})/\Phi(\psi_{it}))$ and $-q\phi(\psi_{it})/(1 - \Phi(\psi_{it}))$.

variable model (14) too. In this section, we perform simulation experiments to study the performance of parameter estimators in all event study models.

For simplicity, we specify $W_{it} = R_{mt}$.[15] This reduces (1) to the familiar market model, with R_{it} and R_{mt} as excess returns to the stock and the market, respectively. Excess returns are realized returns minus the risk-free interest rate. We use Monte Carlo simulation methods to generate samples of data consisting of monthly excess stock returns, $\{R_{it}\}$, market excess returns, $\{R_{mt}\}$, firm attributes, $\{z_{it-1}\}$, and event indicators, $\{I_{eit}\}$. We use these simulated data to estimate parameters using 6 different approaches based on four models of event study and the estimation procedure adopted:

a. The standard two-step procedure.
b. The dummy variable model by OLS.
c. The latent variable model in two stages.
d. The latent variable model by one-step nonlinear least squares (NLLS).
e. The truncated regression model by NLLS.
f. The truncated regression by NLLS with known market model coefficients.

Using the above six approaches, we study the properties of expected abnormal return and other parameter estimators with regard to their true values.

A. Specification of Parameters

Each event study sample has 800 observations of data on $\{I_{eit}, R_{it}, R_{mt}, z_{it-1}\}$. Each sample could be interpreted to consist of 40 firms over 20 months around the event. To keep the simulations simple, we define the market expectation process (5) by a scalar attribute z_{it-1}:

$$\psi_{it} \equiv \theta_0 + \theta_1 z_{it-1}, \quad \forall i, t. \tag{18}$$

We generate four random variables from a joint Normal distribution with the mean vector and the variance-covariance matrix as follows:

$$\begin{bmatrix} \varepsilon_{it} \\ \xi_{it} \\ z_{it-1} \\ R_{mt} \end{bmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.0086 \end{pmatrix}, \begin{pmatrix} 0.005 & q & 0 & 0 \\ q & 1 & 0 & 0 \\ 0 & 0 & 1.44 & 0 \\ 0 & 0 & 0 & 0.002 \end{pmatrix}\right], \quad \forall i, t. \tag{19}$$

By (19), $z_{it-1} \sim N[0, 1.44]$. Thus, ψ_{it} is distributed Normally with mean θ_0 and variance $1.44\theta_1^2$, given (18). Further, the variance-covariance matrix in (19) suggests that the latent information ξ_{it} and the abnormal return ε_{it} are joint Normal with $\mathrm{Cov}(\xi_{it}, \varepsilon_{it}) = q$. The abnormal excess stock returns correspond to an actual sample of 8000 stock returns for 300 firms, with an average $\beta_i = 1.02$. The market return parameters correspond to an index of

[15] See Fama *et al.* (1969) for an excellent discussion on this specification.

equally weighted NYSE and AMEX stocks over the same period as for the stock returns. Using $(\varepsilon_{it}, R_{mt})$, we generate excess stock returns by:

$$R_{it} = \beta_i R_{mt} + \varepsilon_{it}. \tag{20}$$

Finally we use latent information ξ_{it} and specification (18) to generate the event indicator based on decision (market inference) criterion (6).

To use specifications (18)–(20) for simulating $\{R_{it}, R_{mt}, z_{it-1}, I_{eit}\}$, we need true values of parameters $(\theta_0, \theta_1, q, \beta_i)$. We specify these values to generate 2 sets of event study samples, Set A and Set B:

$$\begin{aligned} &q = 0.015, \quad \theta_0 = -1.0, \quad \theta_1 = 1.0, \quad \beta_i = 1.02 \quad (\text{in Set A}),\\ &q = 0.015, \quad \theta_0 = -3.0, \quad \theta_1 = 1.0, \quad \beta_i = 1.02 \quad (\text{in Set B}). \end{aligned} \tag{21}$$

Both Set A and Set B have 50 samples, each for an event study. The only different parameter underlying the samples of data in Sets A and B is θ_0. Since I_{eit} is generated by inference rule (6), using $\psi_{it} = \theta' z_{it-1} = \theta_0 + \theta_1 z_{it-1}$, the true value of θ_0 determines the underlying probability (degree of anticipation) of the event. The underlying probability of the event is calculated to be 0.26 for each sample in Set A and 0.03 for each sample in Set B.

Changing θ_1 is equivalent to changing the variance of market expectation ψ_{it} of the firm's evaluation of the event, x_{it}, specified in (5). Altering the variance of ψ_{it} did not change the nature of our results. Also, increasing the number of attributes in the preevent information set did not change the nature of our results. We conducted simulations by changing the value of q to 0.03, while retaining the values of $(\theta_0, \theta_1, \beta_i)$ as in (21), and obtained results similar to those reported for $q = 0.015$. Varying parameters underlying simulated stock return data did not alter the nature of our results. Including more variables than just R_{mt} in the set of variables W_{it} did not appear to alter the nature of these results since the basic model parameter β_i was very accurately estimated in all cases.

Given a joint Normal distribution for $(\varepsilon_{it}, \xi_{it})$, the true expected abnormal returns in each event study are:

$$A_{eit} \equiv q\left[\frac{\phi(\theta_0 + \theta_1 z_{it-1})}{\Phi(\theta_0 + \theta_1 z_{it-1})}\right], \qquad A_{nit} \equiv -q\left[\frac{\phi(\theta_0 + \theta_1 z_{it-1})}{1 - \Phi(\theta_0 + \theta_1 z_{it-1})}\right], \tag{22}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are defined in (10)–(11). We compute the means of these variable expected abnormal returns for every event study by:

$$\begin{aligned} A_a &= \frac{1}{N_a} \textstyle\sum_{I_{ait}=1} A_{ait},\\ N_a &= \textstyle\sum_{i,t} I_{ait}, \qquad a = e\ (\text{event}),\ n\ (\text{nonevent}). \end{aligned} \tag{23}$$

Thus, A_e $[A_n]$ is the true mean abnormal return to an event [nonevent] in each event study. The specifications of the previous subsection result in true mean abnormal returns to the event equal to 1.20% in Set A, and 2.21% in Set B; the true mean abnormal returns to the nonevent are -0.42% and

-0.06%, respectively. Observe that the stock price effect of the event, $q(\phi(v)/\Phi(v))$ for $v \equiv \theta_0 + \theta_1 z_{it-1}$, decreases with the event probability, $\Phi(v)$, since $q(\phi(v)/\Phi(v))$ is a decreasing function of $\Phi(v)$. That is, as the degree of anticipation increases, the surprise due to the event (stock price effect) reduces.

B. Parameter Estimators and Their Performance

We estimate some or all of the elements of the set $(\theta_0, \theta_1, q, A_e, A_n, \beta_i)$, depending on the event study approach used. We measure the performance (bias) of each estimator by

$$\text{Bias}(\hat{p}) \equiv \left| \frac{\hat{p} - p}{p} \right|, \tag{24}$$

where p is the true value and $\hat{p}$ the estimator of a parameter. Parameter estimators and estimates are denoted by hats. Observe that the expected abnormal returns are variables (not parameters) in the latent variable procedures (c, d, e, f). Latent variable procedures do not need these means for testing whether or not the expected abnormal returns are significant. These tests are based on parameters θ_0, θ_1, and q that define the expected abnormal returns, as discussed in the previous section. Technically, performance of latent variable procedures can be measured only by the reliability of parameter estimators $\hat{q}$, $\hat{\theta}_0$, $\hat{\theta}_1$, and $\hat{\beta}_i$, although we compute the means of variable expected abnormal returns for comparison across event study procedures.

We use the six different approaches to estimate $(\theta_0, \theta_1, q, A_e, A_n, \beta_i)$ as follows:

a. In the standard model:

$$\hat{A}_e = \frac{1}{N_e} \sum_{I_{eit}=1} \left(R_{it} - \hat{\beta}_i R_{mt} \right),$$

for $\hat{\beta}_i$ estimated from the nonevent period and N_e defined in (23). Further, we perform the second step (regression (3)) of a standard event study procedure to estimate the linear effect of attribute z_{it-1} on the abnormal residual return. Estimates of this effect are obviously not comparable to the true value of θ_1. This second-step regression merely helps us in gauging the power of the standard two-step procedure in capturing this effect, as indicated by the t-statistic for significance.

b. In the dummy variable model:

$$\hat{A}_a = \hat{\delta}_a, \quad (a = e, n),$$

are estimated along with $\hat{\beta}_i$ by OLS applied to (4) with $W_{it} = R_{mt}$.

c. In the latent variable model (estimated in two stages):

$$\hat{A}_e = \frac{1}{N_e}\hat{q}\sum_{I_{eit}=1}\frac{\phi(\hat{\psi}_{it})}{\Phi(\hat{\psi}_{it})}, \quad \text{and} \quad \hat{A}_n = -\frac{1}{N_n}\hat{q}\sum_{I_{nit}=1}\frac{\phi(\hat{\psi}_{it})}{1-\Phi(\hat{\psi}_{it})},$$

where $\hat{\psi}_{it} = \hat{\theta}_0 + \hat{\theta}_1 z_{it-1}$. $(\hat{\theta}_0, \hat{\theta}_1)$ are estimated from the probit model and $\hat{q}$ is estimated along with $\hat{\beta}_i$ in the second stage by OLS applied to (14) with $\hat{\psi}_{it}$ in place of ψ_{it} and $W_{it} = R_{mt}$.

d. In the latent variable model estimated by one-step nonlinear least squares (NLLS), the same formulas for mean abnormal return estimates as in (c) are used. But, $\hat{\beta}_i$, $\hat{q}$, $\hat{\theta}_0$, and $\hat{\theta}_1$ are all estimated by NLLS applied to (14) with $W_{it} = R_{mt}$.

e. In the truncated regression model,

$$\hat{A}_e = \hat{q}\frac{1}{N_e}\sum_{I_{eit}=1}\frac{\phi\left(\hat{\theta}_0 + \hat{\theta}_1 z_{it-1}\right)}{\Phi\left(\hat{\theta}_0 + \hat{\theta}_1 z_{it-1}\right)},$$

where $\hat{q}$, $\hat{\theta}_0$ and $\hat{\theta}_1$ are estimated along with $\hat{\beta}_i$ by NLLS applied to:

$$R_{it} = \beta_i R_{mt} + q\left[\frac{\phi(\theta_0 + \theta_1 z_{it-1})}{\Phi(\theta_0 + \theta_1 z_{it-1})}\right] + \nu_{it}.$$

f. In the truncated regression model (with known $\beta_i = 1.02$), the formula for mean abnormal return as in (e) is used, but $(\hat{q}, \hat{\theta}_0, \hat{\theta}_1)$ are estimated by NLLS applied to:

$$R_{it} - \beta_i R_{mt} = \varepsilon_{it} = q\left[\frac{\phi(\theta_0 + \theta_1 z_{it-1})}{\Phi(\theta_0 + \theta_1 z_{it-1})}\right] + \nu_{it}.$$

There is no practical difference between a case in which β_i is known, and a case in which β_i can be estimated from a period in which the event is improbable. We thus do not distinguish between these two cases in either the truncated regression procedure or the standard procedure. We, however, estimate the truncated regression procedure (approach (f)) by using $\varepsilon_{it} = R_{it} - \beta_i R_{mt}$ as the dependent variable, presuming that β_i is known. This helps us in claiming that our results for this procedure do not depend on the specification of W_{it}. For example, any other specification for W_{it}, as described in footnote 12, leaves the simulation results identical in approach (f) since simulated data on ε_{it} are generated directly.

C. Results

In Tables I and II, we report the mean performance of parameter estimators over 50 event studies in Sets A and B, respectively. In Tables III and IV, we report mean parameter estimates and their standard errors over 50 event studies. The true values of parameters used in generating data for each event

Table I

Performance of Alternative Approaches for Set A Event Studies by Simulation

The performance measure is $|(\hat{p} - p)/p|$, for each parameter with true value p and estimated value $\hat{p}$. Parameters are: θ_0 and θ_1 in the market inference criterion; the co-variance of abnormal return and latent information, q; the mean abnormal return to the event, A_e; the mean abnormal return to the nonevent, A_n; and the market beta, β_i. In the latent variable and the truncated regression procedures, A_e and A_n are functions of θ_0, θ_1, and q.

	Mean Performance Over 50 Event Studies					
Event Study Approach*	$\hat{\theta}_0$	$\hat{\theta}_1$	$\hat{q}$	$\hat{A}_e$	$\hat{A}_n$	$\hat{\beta}_i$
a. Standard (OLS)	na	na	na	0.45	na	0.06
b. Dummy Variable (OLS)	na	na	na	0.28	0.30	0.05
c. Latent Variable (Two-stage)	0.05	0.05	0.21	0.17	0.18	0.05
d. Latent Variable (NLLS)	0.29	0.44	0.21	0.35	0.35	0.04
e. Truncated (NLLS)	4.00	1.25	2.05	0.80	na	0.06
f. Truncated (NLLS, known β)	4.47	1.20	1.21	0.80	na	na
True parameters	−1.00	1.00	0.015	0.0120	−0.0042	1.02

"na" means not applicable since these parameters are not estimated.

* The estimation procedures followed in these approaches are as follows:

a. In the standard model, β_i is estimated from the nonevent period data in the first step. In the second step, the mean event period abnormal return is estimated by OLS.
b. The dummy variable model is estimated by OLS.
c. The latent variable model is estimated in two stages. In the first stage the probability of the event is estimated and in the second stage the expected abnormal returns are estimated.
d. The latent variable model is estimated by one-step nonlinear least squares (NLLS).
e. The truncated regression model is estimated by NLLS.
f. The truncated regression model is estimated by NLLS, using known market model beta ($\beta_i = 1.02$).

study in a set are reported in the last panel (row) of each table. In this row, $\theta_0 = -1.00$ for Set A events (Tables I and III) and $\theta_0 = -3.00$ for Set B events (Tables II and IV). The corresponding mean abnormal return to the event (A_e) is equal to 0.0120 in Tables I and III; and 0.0221 in Tables II and IV. The mean abnormal returns to the nonevent (A_n) are -0.0042 (in Tables I and III) and -0.0006 (in Tables II and IV). The true values of other parameters (θ_1, q, β_i) are equal for all event studies.

In either Table I or Table II, the dummy variable model performs at least as well as the standard procedure. The mean abnormal returns from these procedures reported in rows (a) and (b) of Tables III and IV are indeed quite close to their true values. In estimating mean abnormal returns to an event, these procedures appear to perform close to the untruncated latent variable procedure. But, rows (a)–(b) of Table IV show that the standard errors of parameter estimators ($\hat{A}_e$, $\hat{A}_n$) are quite high, resulting in statistically insignificant mean abnormal returns. Recall that the probability of the event is 0.03 for Set B events (Tables II and IV), compared to 0.26 for Set A events (Tables I and III). Thus, a decrease in the degree of anticipation of an event

Table II

Performance of Alternative Approaches for Set B Event Studies by Simulation

The performance measure is $|(\hat{p} - p)/p|$, for each parameter with true value p and estimated value $\hat{p}$. Parameters are: θ_0 and θ_1 in the market inference criterion; the co-variance of abnormal return and latent information, q; the mean abnormal return to the event, A_e; the mean abnormal return to the nonevent, A_n; and the market beta, β_ι. In the latent variable and the truncated regression procedures, A_e and A_n are functions of θ_0, θ_1, and q.

	Mean Performance Over 50 Event Studies					
Event Study Approach*	$\hat{\theta}_0$	$\hat{\theta}_1$	$\hat{q}$	$\hat{A}_e$	$\hat{A}_n$	$\hat{\beta}_\iota$
a. Standard (OLS)	na	na	na	0.49	na	0.05
b. Dummy Variable (OLS)	na	na	na	0.49	0.92	0.05
c. Latent Variable (Two-stage)	0.07	0.10	0.22	0.25	0.26	0.05
d. Latent Variable (NLLS)	0.23	0.48	0.30	0.33	0.70	0.03
e. Truncated (NLLS)	1.70	1.81	6.68	0.87	na	0.09
f. Truncated (NLLS, known β_ι)	1.47	1.50	2.25	0.73	na	na
True parameters	−3.00	1.00	0.015	0.0221	−0.0006	1.02

"na" means not applicable since these parameters are not estimated.

* The estimation procedures followed in these approaches are as follows:

a. In the standard model, β_ι is estimated from the nonevent period data in the first step. In the second step, the mean event period abnormal return is estimated by OLS.
b. The dummy variable model is estimated by OLS.
c. The latent variable model is estimated in two stages. In the first stage the probability of the event is estimated and in the second stage the expected abnormal returns are estimated.
d. The latent variable model is estimated by one-step nonlinear least squares (NLLS).
e. The truncated regression model is estimated by NLLS.
f. The truncated regression model is estimated by NLLS, using known market model beta ($\beta_\iota = 1.02$).

(the probability) leads to a deterioration in the precision of estimation (defined by the inverse of standard errors) within the standard and the dummy variable procedures. If the probability of the event is further lowered, the precision of estimation further decreases (results for event probability less than 0.03 are not reported).

In row (a) of Tables III and IV (columns for $\hat{\theta}_0$ and $\hat{\theta}_1$), we report results from linear regression of residual returns on a constant and the firm attribute z_{it-1} as done in a standard two-step procedure. The coefficient estimates are obviously not comparable with θ_0 and θ_1. But, the tests for significance should indicate the power of this two-step procedure in capturing the effect of firm-specific attributes on the abnormal return to an event. In either set of event studies, the two-step procedure does not capture the effect of z_{it-1}, although it estimates the signs correctly. Recall that the effect of z_{it-1} on the abnormal return has an opposite sign to the effect (θ_1) on the probability of the event. The t-statistic of this effect is -1.5 in case of the high probability event (Table III). For the low probability event (Table IV), the t-statistic is -0.7.

Table III

Estimated Parameters for Set A Event Studies

Parameters are: θ_0 and θ_1 in the market inference criterion; the co-variance of abnormal return and latent information, q; the mean abnormal return to the event, A_e; the mean abnormal return to the nonevent, A_n; and the market beta, β_i. In the latent variable and the truncated regression procedures, A_e and A_n are functions of θ_0, θ_1, and q.

	Mean of Estimates and Their Standard Errors in Parentheses Across 50 Simulated Event Studies					
Event Study Approach*	$\hat{\theta}_0$	$\hat{\theta}_1$	$\hat{q}$	$\hat{A}_e$	$\hat{A}_n$	$\hat{\beta}_i$
a. Standard (OLS)	0.027	−0.009	na	0.0160	na	1.108
	(0.008)	(0.006)		(0.005)		(0.074)
b. Dummy Variable (OLS)	na	na	na	0.01189	−0.0041	1.101
				(0.005)	(0.147)	(0.054)
c. Latent Variable (Two-stage)	−1.002	1.014	0.0149	0.0119	−0.0042	1.011
	(0.071)	(0.072)	(0.004)			(0.054)
d. Latent Variable (NLLS)	−1.077	1.077	0.0144	0.0119	−0.0042	1.006
	(0.668)	(0.705)	(0.005)			(0.006)
e. Truncated (NLLS)	−4.626	2.002	0.0260	0.0067	na	0.982
	(0.510)	(0.016)	(0.011)			(0.109)
f. Truncated (NLLS, known β)	−5.346	1.740	0.0117	0.0058	na	na
	(0.273)	(0.177)	(0.004)			
True parameters	−1.000	1.000	0.015	0.0120	−0.0042	1.02

"na" means not applicable since these parameters are not estimated.
* The estimation procedures followed in these approaches are as follows:

a. In the standard model, β_i is estimated from the nonevent period data in the first step. In the second step, the mean event period abnormal return is estimated by OLS. Also, linear effects of firm attributes on residual returns are estimated and reported in the columns for $\hat{\theta}_0$ and $\hat{\theta}_1$.
b. The dummy variable model is estimated by OLS.
c. The latent variable model is estimated in two stages. In the first stage the probability of the event is estimated and in the second stage the expected abnormal returns are estimated.
d. The latent variable model is estimated by one-step nonlinear least squares (NLLS).
e. The truncated regression model is estimated by NLLS.
f. The truncated regression model is estimated by NLLS, using known market model beta (β_i = 1.02).

Row (c) of Tables I and II indicates that parameter estimators in the two-stage latent variable methodology perform well, even better than when the same model is estimated by NLLS (row (d) of Tables I and II). Row (c) of Tables III and IV show that the two-stage latent variable procedure very accurately estimates the coefficients of interest to an event study (θ_0, θ_1, and q).

Row (e) of Tables I and II indicates that the truncated regression procedure estimates the basic model parameter (β_i) almost as accurately as the other procedures, as indicated by the bias reported in the last column of these tables. But, this procedure performs very poorly with regard to estimators

Table IV

Estimated Parameters for Set B Event Studies

Parameters are: θ_0 and θ_1 in the market inference criterion; the co-variance of abnormal return and latent information, q; the mean abnormal return to the event, A_e; the mean abnormal return to the nonevent, A_n; and the market beta, β_ι. In the latent variable and the truncated regression procedures, A_e and A_n are functions of θ_0, θ_1, and q.

	Mean of Estimates and Their Standard Errors in Parentheses Across 50 Simulated Event Studies					
Event Study Approach*	$\hat{\theta}_0$	$\hat{\theta}_1$	$\hat{q}$	$\hat{A}_e$	$\hat{A}_n$	$\hat{\beta}_\iota$
a. Standard (OLS)	0.048	−0.013	na	0.0188	na	0.999
	(0.044)	(0.019)		(0.0152)		(0.055)
b. Dummy Variable (OLS)	na	na	na	0.0188	−0.0004	1.001
				(0.0150)	(0.1279)	(0.054)
c. Latent Variable (Two-stage)	−3.085	1.054	0.0144	0.0204	−0.0006	1.001
	(0.301)	(0.164)	(0.0051)			(0.054)
d. Latent Variable (NLLS)	−2.841	0.792	0.0130	0.0185	−0.0007	1.013
	(2.100)	(0.669)	(0.0186)			(0.055)
e. Truncated (NLLS)	−4.040	1.794	0.5604	0.0192	na	1.028
	(15.05)	(9.054)	(0.877)			(0.407)
f. Truncated (NLLS, known β)	5.084	1.835	0.0364	0.0154	na	na
	(4.620)	(3.090)	(1.80)			
True parameters	−3.000	1.000	0.015	0.0221	−0.0006	1.02

"na" means not applicable since these parameters are not estimated.
* The estimation procedure followed in these approaches are as follows:

a. In the standard model, β_ι is estimated from the nonevent period data in the first step. In the second step, the mean event period abnormal return is estimated by OLS. Also, linear effects of firm attributes on residual returns are estimated and reported in the columns for $\hat{\theta}_0$ and $\hat{\theta}_1$.
b. The dummy variable model is estimated by OLS.
c. The latent variable model is estimated in two stages. In the first stage the probability of the event is estimated and in the second stage the expected abnormal returns are estimated.
d. The latent variable model is estimated by one-step nonlinear least squares (NLLS).
e. The truncated regression model is estimated by NLLS.
f. The truncated regression model is estimated by NLLS, using known market model beta ($\beta_\iota = 1.02$).

($\hat{q}$, $\hat{\theta}_0$, $\hat{\theta}_1$) that define the expected abnormal return to the event. Both sets of event studies (Tables I and II) indicate that $\hat{q}$, $\hat{\theta}_0$, and $\hat{\theta}_1$ deviate from their true values quite substantially. For example, row (e) in Table I shows that $\hat{\theta}_0$ deviates from its true value by 400%. Performance of estimators in Table II indicate that $\hat{\theta}_0$ and $\hat{\theta}_1$ (not $\hat{q}$) are closer to their true values, relative to estimates in Table I. But, the standard errors of $\hat{\theta}_0$, $\hat{\theta}_1$, and $\hat{q}$ reported in the corresponding row (e) of Table IV indicate that the precision of estimation declines substantially when the probability of event reduces to 0.03 (Table IV) from 0.26 (Table III). Either by the criterion of precision of estimation or by bias in estimates, estimators of parameters determining the mean abnormal return in a truncated regression procedure are unreliable. Results in this procedure do not improve even when β_i is known (rows (f) in Tables I, II, III, and IV).

At a first blush, the performance of the mean abnormal return estimator $(\hat{A}_e)$ is not as bad as the other parameter estimators in the truncated regression procedure. But, the performance of an estimator $(\hat{A}_e)$ which is a function of unreliable parameter estimators is not very meaningful. Theoretically truncation does not change the identifiability of basic model parameter, β_i, as long as the right correction is used. But, there is a loss of critical information due to omission of data for firms that skipped announcement of the event. This loss of information seems to be the main problem in accurately estimating the parameters in the truncation criterion, as argued by Maddala (1984).

III. Conclusion

Simulation experiments show that using *both* event and nonevent period data in a latent variable model that accounts for the market inference criterion reduce the incidence of bias. The same model restricted to event periods results in a truncated regression. Simulation experiments demonstrate significant biases in the estimators of parameters defining the expected abnormal return in truncated regressions.

The standard procedure and the dummy variable model perform better than the truncated regression procedure. For high-probability events, these procedures perform close to the untruncated latent variable model. But, for low-probability events, the precision of estimation (inverse of standard errors of estimators) in the former procedures declines substantially leading to statistical insignificance of abnormal returns.

Simulations reported here and applications to actual data (available from the author on request)[16] indicate that effects of firm-specific attributes on the expected abnormal return are better captured in the untruncated latent variable approach than in other procedures. Latent variable models contribute nothing, when firm-specific attributes are unavailable or are unreliable predictors of the probability of an event.

Appendix

To simplify notations, let $m_{eit} = \phi(\psi_{it})/\Phi(\psi_{it})$, and $m_{nit} = \phi(\psi_{it})/(1 - \Phi(\psi_{it}))$. We define $p_{ait} = \text{Prob}\{I_{ait} = 1\}$, $a = e, n$. We first show that $E(\eta_{it} I_{ait}) \neq 0$, almost surely, which will establish that estimators of informa-

[16] Comparative studies of alternative procedures applied to actual data—on merger attempts and on successful mergers of firms—could be obtained from the author on request. In Acharya (1986), comparisons have been done using data on convertible debt call events. In these applications, the standard two-step procedure does not capture significance of effects of most information variables. The latent variable approach, however, captures significant effects of many interesting information variables on the abnormal return and the probability of events.

tion effect in the dummy variable model (4) will be inconsistent. (See, e.g., White (1984).) We then derive an expression for the large-sample bias when $\mathrm{Var}(\eta_{it})$ is scalar. By comparing (4) and (14), it follows that $\eta_{it} = \nu_{it} + \sum_{a=e,n}(qm_{ait} - \delta_a)I_{ait}$. Then

$$\begin{aligned} E(\eta_{it}I_{eit}) &= E(\eta_{it}I_{eit} | I_{eit} = 1)p_{eit} + E(\eta_{it}I_{eit} | I_{eit} = 0)p_{nit} \\ &= E(\eta_{it} | I_{eit} = 1)p_{eit} \\ &= E\Big[\nu_{it} + \sum\nolimits_{a=e,n}(qm_{ait} - \delta_a)I_{ait}\Big| I_{eit} = 1\Big]p_{eit} \\ &= [qm_{eit} - \delta_e]p_{eit} \neq 0 \end{aligned}$$

almost surely, since m_{eit} is a random variable whereas δ_e is a constant. Similarly, $E(\eta_{it}I_{nit}) = [qm_{nit} - \delta_n]p_{nit} \neq 0$ almost surely. We rewrite the dummy variable model (4) as:

$$R_{it} - \mu_{it} = \delta' J_{it} + \eta_{it},$$

where $\mu_{it} = \beta_i' W_{it}$, $J_{it} \equiv (I_{eit} \;\; I_{nit})'$ and $\delta \equiv (\delta_e \;\; \delta_n)'$. Further, let $R \equiv (R_{11} - \mu_{11} \cdots R_{1T} - \mu_{11} \cdots R_{n1} - \mu_{n1} \cdots R_{nT} - \mu_{nT})'$, $J \equiv (J_{11} \cdots J_{1T} \cdots J_{n1} \cdots J_{nT})'$, $\eta \equiv (\eta_{11} \ldots \eta_{1T} \ldots \eta_{n1} \ldots \eta_{nT})'$, and $s \equiv nT$. Then the large-sample bias, B, in the estimator of δ is given by the following limit in probability, $p\lim_{s\to\infty}$:

$$B = p\lim_{s\to\infty} \hat{\delta}_s - \delta = \big[p\lim_{s\to\infty} s^{-1}J'J\big]^{-1}\big[p\lim_{s\to\infty} s^{-1}J'\eta\big]. \quad \text{(A1)}$$

Observe that the first bracketed expression on the right of (A1) is:

$$p\lim_{s\to\infty} s^{-1}J'J = p\lim_{s\to\infty} s^{-1}\begin{bmatrix} \sum_{it} I_{eit}^2 & \sum_{it} I_{nit}I_{eit} \\ \sum_{it} I_{eit}I_{nit} & \sum_{it} I_{nit}^2 \end{bmatrix} = \begin{bmatrix} \bar{p}_e & 0 \\ 0 & \bar{p}_n \end{bmatrix}, \quad \text{(A2)}$$

where $\bar{p}_a = p\lim_{s\to\infty} s^{-1}\sum_{it} E(I_{ait})$, $a = e, n$ and where we have used the law of large numbers. Observe that since $E(I_{ait}) = p_{ait}$, $\bar{p}_a$ represents the mean probability of announcement a, $(a = e, n)$. Further, the law of large numbers implies that $p\lim_{s\to\infty}[J'\eta - E(s^{-1}J'\eta)] = 0$. Thus, the second bracketed expression on the right of (A1) is:

$$p\lim_{s\to\infty}(s^{-1}J'\eta) = p\lim_{s\to\infty} E(J'\eta) = p\lim_{s\to\infty} s^{-1}\sum\nolimits_{it} E(J_{it}\eta_{it}). \quad \text{(A3)}$$

Since $E(I_{ait}\eta_{it}) = (qm_{ait} - \delta_a)p_{ait}$, $(a = e, n)$, (A3) simplifies to:

$$\begin{aligned} p\lim_{s\to\infty} s^{-1} J'\eta &= p\lim_{s\to\infty} s^{-1}\begin{bmatrix} \sum_{it} qm_{eit}p_{eit} - \sum_{it}\delta_e p_{eit} \\ \sum_{it} qm_{nit}p_{nit} - \sum_{it}\delta_n p_{nit} \end{bmatrix} \\ &= \begin{bmatrix} \sum_{it} qE(m_{eit}p_{eit}) - \sum_{it}\delta_e E(p_{eit}) \\ \sum_{it} qE(m_{nit}p_{nit}) - \sum_{it}\delta_n E(p_{nit}) \end{bmatrix}, \end{aligned}$$

where we can use $\mathrm{Cov}(m_{ait}, p_{ait}) = E(m_{ait} p_{ait}) - E(m_{ait}) E(p_{ait})$, $a = e, n$, after substituting for $\delta_a = qE(m_{ait})$ from (4) and (14). Thus,

$$p \lim_{s \to \infty} s^{-1} J'\eta = \begin{bmatrix} q \,\mathrm{Cov}(m_{eit}, p_{eit}) \\ q \,\mathrm{Cov}(m_{nit}, p_{nit}) \end{bmatrix}. \tag{A4}$$

Using (A-2) and (A-4) in (A-1), the bias is:

$$B = \begin{bmatrix} \dfrac{q \,\mathrm{Cov}(m_{eit}, p_{eit})}{\bar{p}_e} \\ \dfrac{q \,\mathrm{Cov}(m_{nit}, p_{nit})}{\bar{p}_n} \end{bmatrix}.$$

Observe that the bias is zero if $q = 0$, or $\mathrm{Cov}(m_{ait}, p_{ait}) = 0$, $a = e, n$. The first condition will hold if and only if the true information effect is zero. The second condition will hold whenever p_{ait} is constant, i.e., whenever the investors do not update their probability beliefs about the occurrence of the event, given their information.

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