Information, Asset Prices, and the Volume of Trade

Author(s): Gregory W. Huffman

Source: *The Journal of Finance*, Vol. 47, No. 4 (Sep., 1992), pp. 1575-1590
Published by: Blackwell Publishing for the American Finance Association
Stable URL: http://www.jstor.org/stable/2328954
Accessed: 25/11/2009 11:57

THE JOURNAL OF FINANCE • VOL. XLVII, NO. 4 • SEPTEMBER 1992

Information, Asset Prices, and the Volume of Trade

GREGORY W. HUFFMAN*

ABSTRACT

A dynamic equilibrium model is constructed in which agents with access to different information sets participate in the capital market. Agents must use the equilibrium price of capital to make optimal forecasts of the return to holding capital. Examples show that the volume of trade, as well as the price of capital, can be highly correlated with a measure of the information content of prices. This measure of information is the difference between the unconditional entropy of the dividend and the entropy of the dividend conditional on observing the price of capital.

IN THIS PAPER AN equilibrium model is constructed in which optimizing agents, who possess differential information, choose portfolios based on expected future returns. Agents who receive less information must make inferences about the information possessed by other agents and therefore also about the beliefs of these latter agents, after observing the equilibrium level of financial market variables. The presence of differential information gives rise to different portfolios and consumption choices by distinct agents and consequently to an endogenous volume of trade in capital. It can be shown that in some instances the level of transaction volume and the level of the price of capital are highly correlated with a measure of the "information" content of observed prices.

It has long been recognized that to gain an understanding of the dynamic behavior of asset prices and rates of return, dynamic models must be constructed and analyzed in order to see what features of an agent's environment affect the equilibrium prices and rates of return to holding various capital assets. But while much progress has been made in analyzing asset prices in dynamic models, comparatively little progress has been made in analyzing the determinants of transaction volume in asset markets. This is especially puzzling when one considers that it is likely that the same fundamental forces impinge on *both* of these phenomena simultaneously.

It has not been easy to date to construct models in which agents' decision rules depend non-trivially on the expected future returns of assets, *and* in which agents *trade* assets in equilibrium. Milgrom and Stokey (1982) have attempted to analyze such a model, but the result they obtain is that for a

* Huffman is from the Southern Methodist University, and is a Research Associate at the Federal Reserve Bank of Dallas. The author wishes to thank an anonymous referee and the editors for their valuable comments. The financial support of the Bradley Foundation, and of the Social Sciences and Humanities Research Council of Canada, is gratefully acknowledged.

restricted set of economies, there will be *no trade* in equilibrium. One important reason why trade may not arise is related to ideas expressed by Grossman (1978), and Grossman and Stiglitz (1980). Consider an economy in which agents are initially endowed with diverse information sets, and in which this fact is common knowledge. Agents' portfolio decisions will be affected by their information sets, and consequently the equilibrium price (and possibly the volume of trade) will reflect all this information. But if all information is reflected in financial market variables, then agents will not have diverse information in equilibrium, and there may then be no reason for agents to trade.

In the simple model constructed in this paper, there will be two types of agent: those agents who are "informed" about the future rate of return on the single asset, and those agents who are not informed. Both types of agent maximize expected utility subject to their budget constraints and agents' decision rules will depend non-trivially on expected future returns of assets. The informed agents also receive another type of shock, which could be an endowment or preference shock. The equilibrium value of the capital asset will reflect all such shocks as well as expected future returns of the asset. The uninformed agents use the equilibrium price to make inferences about the underlying information set possessed by the informed agents, and thereby gain information from the equilibrium price. Because there are two types of exogenous shock and because the equilibrium price of capital does not reveal the exact nature of these disturbances, the uninformed agents are not able to acquire perfect information by observing the equilibrium price. Consequently, there may still be trade in assets in equilibrium due to the fact that agents have differential information about future returns of the asset.

The model is consistent with what many may believe to be important ingredients of observed asset markets. In some periods the price of capital reveals more about the underlying fundamentals of the asset (or of some other feature of the economy) than it does in other periods. Uninformed agents, or those agents with relatively little information about future rates of return, attempt to infer the information possessed by the informed agents through the implied actions of the latter agents. The differential information in the economy contributes to the purchase and sale of capital, and in equilibrium capital is actually traded. Furthermore, the volume of trade that is present is different from what would prevail if there were perfect information.

Two examples are presented that are illustrative of the behavior of the economy in question. These examples show that when the realized price of capital is near its average or expected level, the volume of trade is low, and little information is conveyed by financial market variables to the uninformed agents. On the other hand, when the price of capital deviates substantially from its expected level, the volume of trade will be high and more information will be conveyed by financial market variables. These latter features can be shown to be consistent with the observed behavior of the corresponding financial market variables described by Karpoff (1987). Furthermore, I show that the price of capital carries information about the underlying fundamental of the asset, which in this case is the dividend. A measure of the amount of this information is available and it is the difference between the unconditional entropy of the dividend, and the entropy of the dividend conditional on observing the price of capital.[1] I show that this measure of information can be highly correlated with the volume of capital traded, and that the information content of prices is high when the price of capital is unusually high or low.

The remainder of the paper is organized as follows. In Section I the economic environment is presented in detail with reference to a description of preferences, information sets, and the trading opportunities available to agents. The equilibrium of the economy is studied, and a proof of the existence of an equilibrium for the economy is presented in the Appendix. In Section II two examples are presented that illustrate the dynamic behavior of the economy. Some final remarks are presented in Section III.

I. Description of Economy

A. Environment

The environment under study is one in which there are three time periods ($t = 1, 2, 3$). At date $t = 1$, there is a population of size N that is present in the economy for three periods. Each agent has identical preferences such that they wish to consume at dates $t = 2$ and $t = 3$. An agent (j) then has preferences over the single consumption good in these periods described by the function

$$\left[\frac{(c_2^j)^{1-\rho}}{1-\rho}\right] + \left[\frac{\alpha_3^j (c_3^j)^{1-\rho}}{1-\rho}\right] \quad \rho > 0, \rho \neq 1.$$

At date $t = 1$, all agents merely hold an equal amount of capital, and do not consume. Here c_i^j denotes period i consumption by agent (j). The variable α_3^j is a random preference shock for agent j, that will be discussed in more detail below. There is productive capital in the economy that yields a random payment or dividend (r_t) to its holder in periods $t = 2, 3$, in units of the consumption good. The consumption good is nonstorable and the capital stock cannot be consumed or augmented. To simplify the analysis it will be assumed that the random variables $(\alpha_3^j,\ r_2,\ r_3) j = 1, \ldots N$, are generated from a probability distribution function $F(\alpha_3^j,\ r_2,\ r_3)$. Furthermore, this stochastic process is such that these variables are strictly positive and bounded from above with probability one, and the α_3^j satisfy certain other restrictions described below. The number of units of capital is normalized to N, so that the per capita quantity is unity.

[1] See Papoulis (1984), Ash (1965), or Feinstein (1958) for a more detailed analysis of the concept of entropy. Maasoumi (1988) addresses how this concept can be utilized from an economic point of view.

The behavior of agents is as follows. In period 1 all agents are endowed with an equal amount of capital. In the second period the agent will want to consume, and the capital that he holds will also pay a dividend. The agent may then wish to sell off some of his holdings of capital in order to increase consumption even further, or he may wish to use some of the dividend received in order to purchase more capital for the next period. In period 3, the agent receives a final dividend proportional to his capital holdings.[2]

B. An Agent's Optimization Problem

Let x_t^j denote the holdings of capital by an agent j chosen in period $t - 1$ taken into period t. Let P_2 represent the price of capital during period 2, and $x_2^j = 1$ be the amount of capital taken into period 2. During the next two periods the agent must then choose x_3^j so as to maximize

$$E^j\left(\left[\frac{(c_2^j)^{1-\rho}}{1-\rho}\right] + \left[\alpha_3^j\frac{(c_3^j)^{1-\rho}}{1-\rho}\right]\right) \tag{1}$$

subject to the constraints

$$c_2^j \leq (P_2 + r_2)(x_2^j) - (P_2 x_3^j) \tag{2}$$

$$c_3^j \leq r_3 x_3^j \tag{3}$$

$$c_2^j, c_3^j, x_3^j, \geq 0.$$

The subscript j of the expectation operator $E^j(\bullet)$ in equation (1) reflects the fact that some agents may have different information sets available to them at the same date.[3] This feature will be described in much detail in the next sections. The fact that the number of units of capital has been normalized to N implies that the equilibrium condition for capital is the following

$$\sum_{j=1}^{N} x_3^j = N. \tag{4}$$

[2] In Huffman (1991) it is shown that the main features of the present analysis can be incorporated into a model of 3-period-lived overlapping generations, and in which there is an infinite horizon to the economy. This model then would generate a long time series path for such variables as prices and transaction volume.

[3] The utility function that is employed is meant to be of a general nature, while excluding certain uninteresting cases. For ease of exposition it has been assumed that the "risk aversion" or intertemporal substitution parameter (ρ) is the same for period 2 and 3 consumption. For the utility of first-period consumption, (ρ) must be greater than zero, and otherwise no restrictions need apply. For the utility of second-period consumption (ρ) cannot equal unity, for this is the case of logarithmic utility, and in this instance the agent's saving–consumption decision rules are independent of the expected return on capital. This would take away any interesting possible avenue for agents to infer information from the contemporaneous price of capital since agents would never act on this information. Section II contains two examples in which the first period utility is of the form $\rho = 1$, while second period utility has $\rho = 0$. These parameterizations will easily fit into the present analysis.

C. Information Structure

One of the important features of this model is that some agents will have more information about certain random variables. In particular, all agents are identical in every respect in period 1. However, in period 2, a fraction λ of these agents will receive two signals. First, they will receive a signal about the value of *their own* preference shock for the next period α_3^j. As a benchmark, it will be assumed that this signal is perfectly informative and is common among agents that receive such signals, although these are certainly not necessary assumptions. The remainder of the agents who do not receive such a signal have preference disturbances (α_3^j) that are constant, and are known to be constant at the level equal to the unconditional mean of the preference shocks for the agents with random preference shocks. Secondly, the same fraction λ of the agents who received the random preference shocks also receive another signal. In period 2 these agents observe a signal of the level of dividends in the following period (r_3). Again, as a benchmark it is assumed that this signal is perfectly informative, and hence to these agents r_3 is predictable at date 2. The remainder of the agents receive no such signal, and consequently barring any other information their objective probability distribution over the level of dividends in the following period is the *unconditional* probability distribution for this random variable. It is important that these "signals" observed by agents about their preference shocks or the future level of dividends, are *private information*, and not observable by other agents. Therefore, agents have no way in which to insure against adverse realizations of these disturbances. Henceforth, the agents (i) who receive the "signals" will be referred to as the *informed* agents and the other agents (u) will be referred to as the *uninformed* agents.[4]

There is an avenue, however, through which uninformed agents may infer the information received by the informed agents. By observing the price of capital at which they can trade, the uninformed agents can *infer* the behavior of the informed agents. They can do this as follows. The decision rules of the informed agents will reflect the information they have obtained on the dividend and their own preference disturbance, and therefore the price of capital, being a function of the decision rules of all agents, will also reflect this information. Since the uninformed agents know the unconditional probability distributions of both the dividend and the preference shocks for the informed agents, and because they observe the price of capital at which trade can take place, they can then compute a *conditional* probability distribution

[4] The assumption that the same agents receive the two information signals is for convenience only. In this setup there are only two groups of agents to deal with and therefore the model is somewhat tractable. If the signals were randomly given to agents in the economy, then there could be four groups of agents to deal with: Those who observe only their preference shock, those who observe only the future value of the dividend, those who receive both pieces of information, and those who receive neither. Although this latter setup may perhaps be a more accurate pattern of how information is dispersed throughout actual economies, its implementation here would appear to only complicate the present analysis and would not produce greater insights.

over the future dividend for capital that reflects the information embodied in the price of capital. Furthermore, the uninformed agent's conditional and unconditional probability distributions over the future level of dividends must be consistent with Bayes's rule.[5]

D. A Stationary Competitive Equilibrium

In light of the description of the information structure, it is now possible to give a somewhat loose description of the competitive equilibrium. Let w denote the state vector $(r_3,\ \alpha_3^j)$ for this economy which encompasses all variables of importance in an agent's decisions.

Definition: A Competitive Equilibrium for this economy is characterized by a collection of functions $[P(w),\ x^j(w)]$ defined for all $w = (r_3,\ \alpha_3^j)$, $j = (i,\ u)$, such that:

i) Given the pricing functions $P_2 = P(w)$, where $w = (r_3,\ \alpha_3^j)$, the asset choice x_3^j solves the optimization problem for agent $j(j = i,\ u)$, given by equations (1)–(3).
ii) For all $t = 2$, 3, and all w, equation (4) holds.

It is shown in the Appendix that a stationary equilibrium exists for a wide class of economies of the sort described above.[6]

Consider now the optimization problem for an informed agent (i) who enters the economy at date $(t - 1)$. The optimization condition for the agent is

$$P_2\left[r_2 + P_2(1 - x_3^i)\right]^{-\rho} = E_2^i\left\{\alpha_3^i\left[r_3 x_3^i\right]^{-\rho} r_3\right\}. \tag{5}$$

However, the vector $(r_3,\ \alpha_3^i)$ is in the agent's information set at date t. Hence this expression can be rewritten as

$$\left[P_2 x_3^i\right]^{\rho}\left[r_2 + P_2(1 - x_3^i)\right]^{-\rho} = \alpha_3^i\left(\frac{r_3}{P_2}\right)^{1-\rho}. \tag{6}$$

This expression makes it clear that the value of the agent's holdings of capital after period 2 will be an increasing function of the rate of return on capital for $\rho \in (0,\ 1)$.

For an uninformed agent (u) on the other hand, the optimization problem is identical to that for agent (i) given by equation (5). However, r_3 is not known with certainty to such an agent, and so this optimization condition can

[5] This discussion is intended to give meaning to the expectation operator given in equation (1).

[6] It is implicit from the description of an equilibrium that in condition (i), the decision rules solve the optimization problem of maximizing the conditional expected utility of agent i, where the respective "conditional" expectations reflect all the information to which agents have access, including that embedded in the capital-pricing function itself.

be rewritten as

$$[P_2 x_3^u]^{\rho}[r_2 + P_2(1 - x_3^u)]^{-\rho} = E_2^u\left[\alpha_3^u\left(\frac{r_3}{P_2}\right)^{1-\rho}\right]. \tag{7}$$

Again, this equation displays a similar relationship to that of the corresponding equation (6) for informed agents. The quantity of capital purchased by informed agents will reflect the *actual* rate of return on capital, whereas the quantity of capital purchased by uninformed agents will reflect expectations of (a function of) the rate of return on capital. The capital market clearing condition can then be used to show that the *actual* price must be a function of the actual return to capital. Consequently the equilibrium price of capital must then reflect the actual return on capital. Therefore, uninformed agents will then use their observation on the contemporaneous price of capital to make inferences about its future return and this will thereby affect the manner in which the expected value in equation (7) is calculated. Letting $R_3 = (r_3/P_2)$, equation (6) can be written as

$$\left(\frac{P_2 x_3^i}{(P_2 + r_2) - P_2 x_3^i}\right) = \left[\alpha_3^i (R_3)^{1-\rho}\right]^{1/\rho} \equiv \Theta_3^i \tag{8}$$

or

$$(P_2 x_3^i) = \left(\frac{\Theta_3^i}{1 + \Theta_3^i}\right) x_2^i (P_2 + r_2).$$

Similarly, for uninformed agents equation (7) can be written as

$$(P_2 x_3^u) = \left(\frac{\Theta_3^u}{1 + \Theta_3^u}\right) x_2^u (P_2 + r_2)$$

where

$$\Theta_3^u = \left[\alpha_3^u E_2^u (R_3)^{1-\rho}\right]^{1/\rho}. \tag{9}$$

These conditions together with equation (4), and with the fact that for t even, $x_2^u = x_2^i = 1$ imply

$$P_2 = (P_2 + r_2)\left[\left(\frac{(1-\lambda)\Theta_3^u}{1 + \Theta_3^u}\right) + \left(\frac{\lambda \Theta_3^i}{1 + \Theta_3^i}\right)\right]. \tag{10}$$

Obviously this is an implicit equation for the price of capital since Θ_3^i and Θ_3^u are functions of P_2. From this expression it is clear that given an observation on P_2 and r_2, and given an uninformed agents expectation of $(R_3)^{1-\rho}$, this latter expectation must be consistent with the *conditional* probability distribution of r_3 (or R_3) implied by equations (8) and (9). Rather than further analyzing this somewhat general case, it is more illuminating to explore two specific examples. This is taken up in the next section.

II. Two Examples

In this section two examples are presented to illustrate the behavior of such an economy, and to further investigate a measure of the "information" implied by prices and the volume of trade changes as a function of this information. In both cases, to ease the exposition it will be assumed that the utility function for all agents is of the form

$$\ln(c_2^j) + \alpha_3^j(c_3^j).$$

In the first example the probability distribution for the exogenous random variables is discrete, while in the second case it is continuous.

A. Example 1

Let the probability distribution of the independent random variables be of the following form:

$$r_3 = \begin{cases} 1.25 & \text{with probability } 1/2 \\ 1.1111 & \text{with probability } 1/2 \end{cases}$$

$$\alpha_3^i = \begin{cases} 0.9 & \text{with probability } 1/2 \\ 0.8 & \text{with probability } 1/2 \end{cases}$$

Also $r_2 = 1$, $\alpha^u = 0.85$, and $\lambda = 0.2$. Now the optimization condition for an informed agent (equation (6)) can be written as

$$\left[\frac{P_2}{(P_2 + r_2) - x_3^i P_2}\right] = \alpha_3^i r_3.$$

The example is constructed so that the right side of this equation has the following distribution

$$\alpha_3^i r_3 = \begin{cases} 1.125 & \text{with probability } 1/4 \\ 1.0 & \text{with probability } 1/2\,. \\ 0.889 & \text{with probability } 1/4 \end{cases}$$

The decision rule for the informed agent will be of the form

$$(P_2 x_3^i) = (P_2 + r_2) - \left[\frac{P_2}{\alpha_3^i r_3}\right].$$

Similarly, for uninformed agents their optimization condition can be written as

$$(P_2 x_3^u) = (P_2 + r_2) - \left[\frac{P_2}{\alpha_3^u E_2^u(r_3)}\right].$$

Using these decision rules in equation (10) produces the following equilibrium capital-pricing equation

$$P_2 = (r_2)\left[\left(\frac{\lambda}{\alpha_3^i r_3}\right) + \left(\frac{(1-\lambda)}{\alpha_3^u E_2^u(r_3)}\right)\right]^{-1}.$$

It is then straightforward to show that there exists an equilibrium with the properties listed in Table I. When the preference shock for informed agents is high and the future dividend is high, the uninformed agents can infer from the price of capital that the future dividend will be high with certainty. Similarly, when the preference shock for the informed agents is low and the future dividend is low, the uninformed agents can infer that the future dividend will be low with certainty. In the other two cases, however the price of capital does not tell the uninformed agents "enough" information because the price of capital is identical in both states and consequently they are forced to compute a *conditional* expected value of the future dividend. It should also be noted that the volume of trade is highly correlated with the level of information conveyed by the price. Volume is highest when both types of agent are certain about the values of both stocks.

B. Example 2

In this example continuous probability distributions for the exogenous random variables will be employed. In particular, it is assumed that the independent random variables $\log(\alpha_3^i)$ and $\log(r_3)$, are both uniformly distributed on the interval (0, 1). Additionally, let $\lambda = 0.5$, $r_2 = 0.5$, $\alpha^u = E(\alpha_3^i) = (e-1)$, and $\beta = 0.8$. The utility function for both types of agent is as follows[7]

$$\ln(c_2^j) + \beta\alpha_3^j(c_3^j).$$

The uninformed agents can infer the value of the random variable $(\alpha_3^i r_3)$ from the price of capital. Hence these agents must compute the conditional probability distribution of r_3 implied by this observation. It can be shown that

$$E(r_3 | \alpha_3^i r_3 = y \le e) = \left[\frac{y-1}{\log(y)}\right],$$

[7] The form of the utility function used in both of the examples in this section was chosen for convenience only. The fact that it is linear in second-period consumption greatly simplifies the algebra involved in deriving the individual agent's decision rules. If second-period utility were nonlinear, then these decision rules would be more cumbersome since they would be a function of the marginal utility of second-period consumption and the expected rate of return on capital, as opposed to just being influenced by the expected rate of return to capital. Though complicating the analysis, however, this is not a change that would materially change the results.

Table I

Values of the Price of Capital and Volume for Example 1

There are four possible states for the economy, and these are characterized by the four possible realizations for the exogenous disturbances (α_3^i, r_3). "Probability" represents the probability of each event. "Price" is the equilibrium price of capital in each state, and "Volume" is the equilibrium transaction volume in each state.

State	(α_3^i, r_3)	Probability	Price	Volume
I	(0.9, 1.25)	1/4	1.074438202	0.004333
II	(0.9, 1.1111)	1/4	1.002775851	0.000277
III	(0.8, 1.25)	1/4	1.002775851	0.000277
IV	(0.8, 1.1111)	1/4	0.932784637	0.005110

and

$$E(r_3 | \alpha_3^i r_3 = y \geq e) = \frac{[e - (y/e)]}{[2 - \log(y)]}.$$

The resulting pattern of observations in this economy for transaction volume and the price of capital are shown in Figure 1. Transaction volume is highest when the price of capital is unusually high or low. The pattern of trade that takes place in this and the previous example can be described as follows. When the values of r_3 and α_3^i are unusually high, and hence the price of capital is high, the informed agents purchases capital from the uninformed because of the informed agents' increased desire for third-period consumption. Despite the fact that the high price tells the uninformed agents that the future dividend will be high, the high current price lowers the *rate of return* and these agents then save less. For the informed agents the high value of α_3^i increases desired future consumption and hence the demand for capital. The higher are the pair of variables r_3 and α_3^i, the more capital the informed agents will purchase. The converse takes place for simultaneous low realizations of r_3 and α_3^i, with the informed agents selling capital.

Considering the structure of the economy, it is not surprising that the price of capital conveys information about the future payoff of the asset. In particular, high (or low) values for the price of capital imply high (or low) values for the expected value of future dividends. Fortunately, there is a method available from the statistics literature for calculating the "information content" of prices in this economy. Consider a continuously distributed random variable x with probability density $f(\bullet)$. The entropy of x is defined to be

$$H(x) = E\{-\ln(f(x))\} = -\int_{-\infty}^{\infty} f(x)\ln(f(x))\,dx.$$

Now consider random variables x and y which are jointly continuously distributed with joint density $F(x, y)$, and with marginal probability densities $f(\bullet)$ and $g(\bullet)$, and the probability density of x *conditional* on observing y in denoted $m(x|y)$. The entropy of the random variable x conditional on

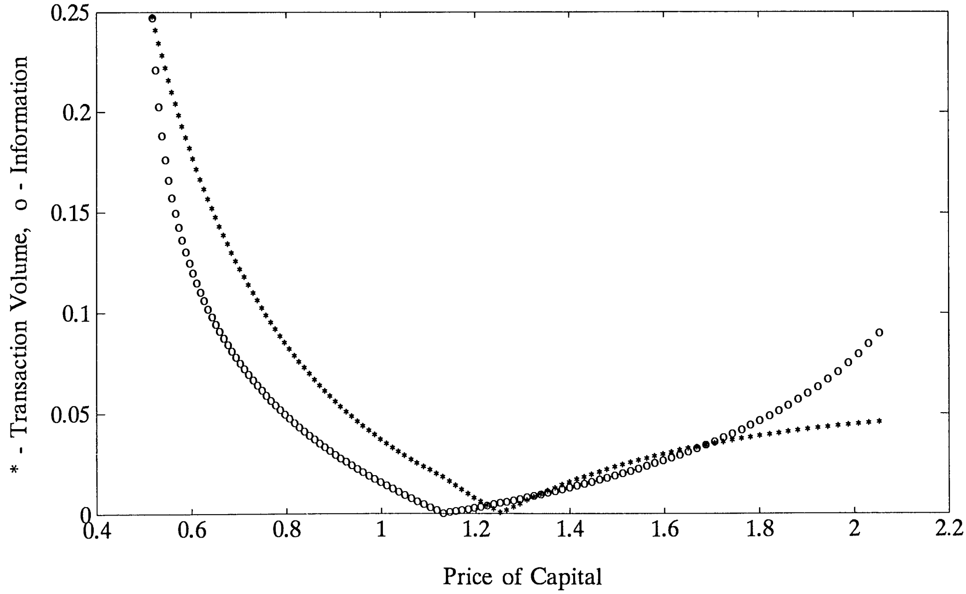

Figure 1. Pattern of transaction volume and information as a function of the price of capital for example 2. ($*$) represents the level of transaction volume shown for each realization of the price of capital. (o) represents the level of information conveyed by the price of capital for each realization of the price of capital. The measure of information is the difference between the unconditional entropy of the dividend and the entropy of the dividend conditional on the level of the price of capital. This measure is then divided by 16 to make it fit the vertical axis.

observing y is defined to be

$$H(x|y) = E\{-\ln(m(x|y))|y\} = -\int_{-\infty}^{\infty} m(x|y)\ln(m(x|y))\,dx.$$

The mutual information, or the information about x contained in an observation of y is then defined as

$$I(x,y) \equiv E\left\{\ln\left(\frac{F(x,y)}{f(x)g(y)}\right)\right\} = H(x) - H(x|y).$$

It can be shown that $I(x, y) \geq 0$, and that $I(x, y) = 0$ if and only if x and y are independent.[8]

[8] Given a probability density $f(x)$ for a random variable x, the entropy is then, loosely speaking, a measure of the uncertainty associated with this random variable. For example, the entropy of the probability distribution of a coin which has a probability p of landing heads equals $-(p)\log(p) - (1-p)\log(1-p)$. It is easy to see this entropy or uncertainty is maximized when $p = 1/2$ (i.e., when there is the greatest amount of uncertainty about the realization). If x and y are random variables drawn from the joint probability density $F(x, y)$, then observing y can provide information, or reduce the uncertainty associated with the distribution of x, and can therefore result in a lower entropy ($H(x) - H(x|y) > 0$). See Papoulis (1984) for a more detailed characterization.

Now for the economy under study, the uninformed agents know the unconditional distribution of the dividend and can calculate the distribution of the dividend conditional on observing the price of capital. Hence the information content of the price of capital can be computed as the difference between the unconditional entropy of the dividend, and the entropy of the dividend conditional on observing the price of capital.

The information content of prices, measured as described above as the information about the *dividend* contained in the current *price* of capital is also shown in Figure 1. It is clear that more information is conveyed by unusually high or low prices. Figure 1 reveals that the information content of the price of capital is high (or low) when the volume of trade is high (or low). This is further illustrated in Figure 2 where these variables alone are shown.[9] In this instance the correlation between the volume of trade and the measure of information is 0.9005.

To see the impact that the information structure alone has on these patterns, it may be enlightening to compare this economy with one which is exactly the same except that the uninformed agents also receive the same information about future dividends that is received by informed agents (i.e., all agents are informed about the dividend). Figure 3 shows the pattern of transaction volume and the price of capital that is now produced when there is perfect information. This is to be compared with Figure 1. Obviously the pattern of Figure 3 looks considerably different and appears to be a surface rather than a line as in Figure 1. The reason for this is that in Figure 1, for each realization of the multiplicative random variable ($\alpha_3^i r_3$), there exists an equilibrium price and level of volume as shown in Figure 1. Two different levels for the pair of random variables (α_3^i, r_3) which give rise to the same value for the multiplicative random variable ($\alpha_3^i r_3$) will produce the same price and level of volume since the uninformed agents cannot discern the separate value for the dividend r_3. However, under perfect information, two different values for the pair of random variables (α_3^i, r_3) will generally give rise to two different values for the price of capital and two different values for transaction volume, even if the different pairs (α_3^i, r_3) give rise to the same value for the multiplicative random variable ($\alpha_3^i r_3$). The presence of uninformed agents appears to impose considerably more structure on this pattern than would otherwise appear if all agents were informed. This example also calls into question whether the observed patterns between transaction volume and prices are in some important manner attributable to the presence of private information among the economic agents since the correlation between transaction volume and price seen in Figure 1 is greatly changed when agents are allowed to have complete information.

[9] In each of the diagrams the space of preference shocks (α_t^i) and dividends (r_t) was discretized so that there were 63 possible values for each random variables. Clearly more information is conveyed by the price when the preference shock and dividend are either both high, or both low.

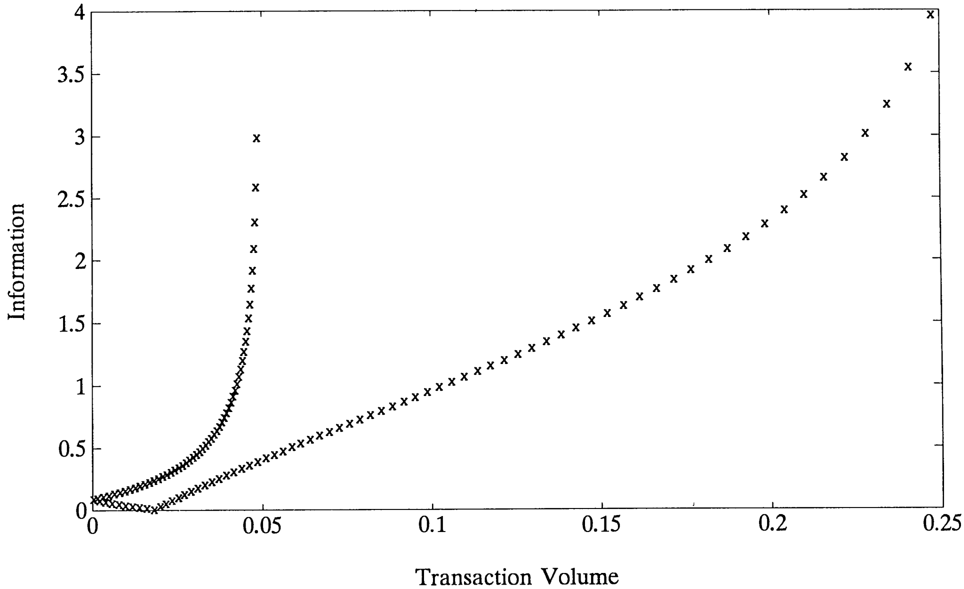

Figure 2. Pattern of information and the transaction volume for example 2. The measure of information is the difference between the unconditional entropy of the dividend and the entropy of the dividend conditional on the level of the price of capital.

III. Final Remarks

It is the purpose of this study to provide a somewhat specialized general equilibrium framework within which an analysis could be conducted of the relationship between a measure of the information content of asset prices on the one hand, and the statistical relationship between volume of trade and price of an asset on the other. The agents with the least information face a "signal-extraction" problem that is reminiscent of that employed in Lucas (1972). Examples illustrate that the pattern of transaction volume differs substantially from that which would exist if there were perfect information in the economy. Additionally, the examples reveal that the relationship between the transaction volume and the price of capital closely conforms with that observed in actual asset markets. That is, transaction volume is highly correlated with the change in the price of capital. This general equilibrium model then gives content to a long-prevailing view that there is a relationship between the volume of trade in financial markets and the behavior of the change in the price of capital. In Huffman (1991) it is shown that these patterns can be influenced by government policy variables. For example, a tax on dividends can influence the amount of information that is revealed by a change in the price of capital.

It would appear that there is plenty of latitude for introducing such "signal-extraction" issues into environments of this sort, as there appears to

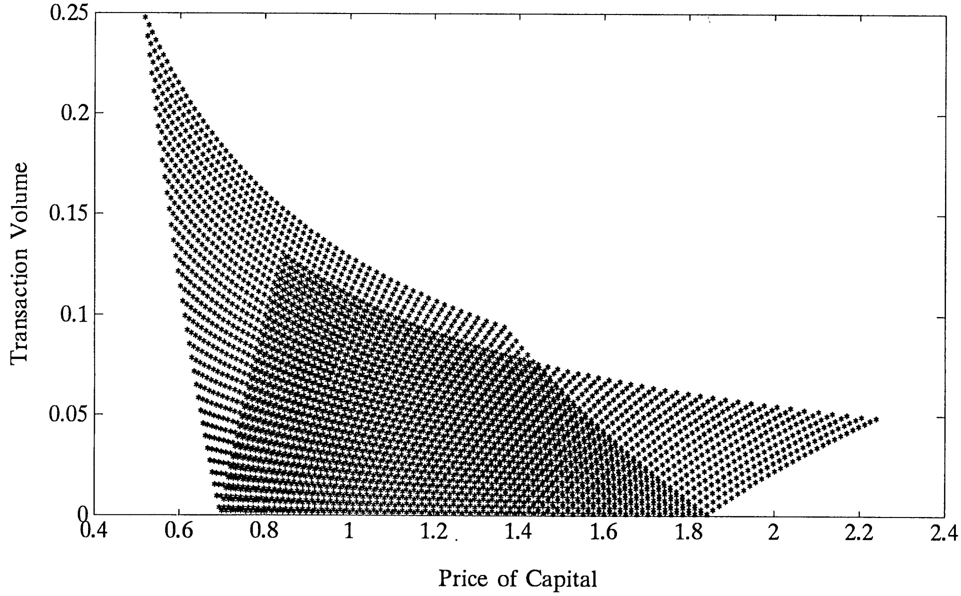

Figure 3. Pattern of transaction volume and the price of capital under complete information for example 2. All agents know the realizations of all exogenous random variables. This produces a different value for the price of capital and transaction volume for each value of the pair (α_3^i, r_3).

be room for a whole host of factors which are *fundamental* to the payoff on an asset that may be imbedded as information in the current price of an asset in a manner similar to the effect that other *non-fundamental* factors would have on the price. There would then be no need to resort to the use of preference shocks so that the price of capital does not convey all information.

Appendix

Since the equilibrium condition in the capital market as well as the individual optimization conditions are already imposed in the capital-pricing equation (10), all that remains to be verified is that there exists a pricing function for capital such that when the uninformed agents observe the price of capital they use the information conveyed by the price. Given that an uninformed agent knows the value of Θ^u, as well as λ and r_2, an observation of the price (P_2) for such an agent conveys the same information as an observation on $\alpha_3^i(r_3)^{1-\rho}$. Let $\Gamma(r_3|\alpha_3^i(r_3)^{1-\rho})$ denote the resulting conditional distribution of r_3, given this information as calculated from the unconditional probability distribution $F(\alpha_3^j, r_2, r_3)$. Then let

$$\Theta^u = \left[\alpha^u \int \left(\frac{r_3}{P_2}\right)\Gamma\left(dr_3|\alpha_3^i(r_3)^{1-\rho}\right)\right]^{1/\rho} \tag{A1}$$

as implied by equation (9). Equation (10) can be rewritten as

$$P_2\left(1-\left[\frac{(1-\lambda)\Theta^u}{(1+\Theta^u)}+\frac{\lambda\Theta^i}{(1+\Theta^i)}\right]\right)=r_2\left[\frac{(1-\lambda)\Theta^u}{(1+\Theta^u)}+\frac{\lambda\Theta^i}{(1+\Theta^i)}\right] \quad \text{(A2)}$$

where

$$\Theta^i=\left[\alpha_3^i\left(\frac{r_3}{P_2}\right)^{1-\rho}\right]^{1/\rho}$$

and Θ^u is given by equation (A1) above. Now there are two different cases to consider.

Case (i): $0<\rho<1$.

Note that the right side of (A2) is strictly increasing in Θ^u and Θ^i, while both of these latter expressions are strictly decreasing in P_2. Hence, the right side of (A2), viewed as a function of P_2 on $(0,\infty)$ is a strictly decreasing function taking values from $+\infty$ to 0. Similarly, the left side of (A2) is strictly decreasing in Θ^u and Θ^i and hence viewed as a function of P_2 on $(0,\infty)$, the left side of (A2) is strictly increasing taking values from 0 to $+\infty$. Therefore, because of the continuity of A2 in P_2, by the intermediate value theorem there is a unique fixed point solution to equation (A2) for P_2.

Case (ii): $\rho>1$

In this case both Θ^u and Θ^i are both increasing in P_2 and so the left side of (A2), viewed as a function of P_2 on $(0,\infty)$ takes values from 0 to $+\infty$. The right side of (A2) viewed similarly increases from 0 to r_2. Hence, to show there exists a solution to equation (A2) in P_2, it need only be shown that there exists a value of P_2 such that when both sides of (A2) are evaluated at this value, the right side is greater than the left side. To establish this note that for given $\varepsilon>0$, there exists a $\delta>0$ such that $|P_2|<\delta$ implies

$$\left|P_2-P_2\left(1-\left[\frac{(1-\lambda)\Theta^u}{(1+\Theta^u)}+\frac{\lambda\Theta^i}{(1+\Theta^i)}\right]\right)\right|<\varepsilon.$$

Hence for $|P_2|<\delta$, the left side of (A2) is less than $(P_2-\varepsilon)$ and hence is strictly less than P_2. Taking the derivative of the right side of (A2) with respect to P_2 reveals,

$$\frac{d}{dP_2}\left[\frac{(1-\lambda)\Theta^u}{(1+\Theta^u)}+\frac{\lambda\Theta^i}{(1+\Theta^i)}\right]\to\infty \text{ as } P_2\to 0$$

for $\rho>1$. Hence the right side of (A3), viewed as a function P_2 can be made larger than $P_2-\varepsilon$. Since (A2) is continuous in P_2, the intermediate value theorem again can be used to establish the existence of a fixed point to this equation. Employing equation (A1) in equation (10) produces an equilibrium pricing function for capital in which the price of capital conveys information to the uninformed agents, who use the same information in their decisions which are reflected in the price.

REFERENCES

Ash, R., 1965, *Information Theory* (Interscience Publishers, New York).

Feinstein, Abraham, 1958, *Foundations of Information Theory* (McGraw-Hill Book Company, New York).

Grossman, Sanford J., 1978, Further results on the informational efficiency of competitive stock markets, *Journal of Economic Theory* 18, 81–101.

——— and Joseph E. Stiglitz, 1980, On the impossibility of informationally efficient markets, *American Economic Review* 66, 393–408.

Huffman, Gregory W., 1991, The role of transaction volume in producing information about asset prices, Research Report No. 9106, Department of Economics, University of Western Ontario.

Karpoff, Jonathan M., 1987, The relation between price changes and trading volume: A survey, *Journal of Financial and Quantitative Analysis* 22, 109–126.

Lucas, Robert E., Jr., 1972, Expectations and the neutrality of money, *Journal of Economic Theory* 4, 103–124.

Maasoumi, Esfandiar, 1988, Information theory, in John Eatwell, Murray Milgate, and Peter Newman, eds., *The New Palgrave: A Dictionary of Economics,* Vol. 2 (Stockton Press, New York).

Milgrom, Paul, and Nancy Stokey, 1982, Information, trade and common knowledge, *Journal of Economic Theory* 26, 17–27.

Papoulis, Anthanasios, 1984, *Probability, Random Variables, and Stochastic Processes* (McGraw-Hill, New York).