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Futures Manipulation with "Cash Settlement"

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ABSTRACT

This paper investigates the susceptibility of futures markets to price manipulation in a two-period model with asymmetric information and "cash settlement" futures contracts. Without "physical delivery," strategies based on "corners" or "squeezes" are infeasible. However, uninformed investors still earn positive expected profits by establishing a futures position and then trading in the spot market to manipulate the spot price used to compute the cash settlement at delivery. We also show that as the number of manipulators grows, profits from manipulation fall to zero. However, even in the limit, manipulation still has a nontrivial impact on market liquidity. More broadly, we interpret manipulation as a form of endogenous "noise trading" which can arise in multiperiod security markets.

CAN UNINFORMED INVESTORS PROFITABLY manipulate security prices by strategic trading? This question has long intrigued both economists and the general public. For economists, since Keynes (1936) and Friedman (1953) the issue of interest has been whether such manipulation can arise as an equilibrium phenomenon in the presence of rational market makers and other traders.[1] Prima facie, it might appear that manipulation requires a divergence between a security's price and its "value" which other market participants ought to recognize and profitably counteract, thereby offsetting any incipient manipulation.

We show here, however, that this intuition is incorrect. Even if investors are risk-neutral (and hence risk-capacity limits as in De Long, Shleifer, Summers, and Waldmann (1990) are absent), profitable manipulation occurs in our model under a surprisingly weak set of assumptions.

Our analysis is conducted in the context of a two-date model in which trade occurs first in a futures market followed by a spot market. The idea is simple. Suppose that an "informed trader" privately learns the value of the underlying asset before delivery, but that this value becomes public only after the

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[1] See Hart (1977) and Jarrow (1989) for sufficient conditions on exogenous price processes which permit manipulation.

futures delivery date. The presence of an informed trader means that prices are affected by pre-delivery trades—including those of uninformed "manipulators." If futures accounts are closed through "cash settlement" rather than "physical delivery," then manipulative spot trading can improve the settlement price. For example, after establishing a "long" futures position, the settlement price can be artificially bid up by buying in the spot market. This is called "punching the settlement price." If the futures position is larger than the spot position, the net expected gain (i.e., profit on futures less loss on the spot) is positive.

"Cash settlement" allows the manipulator to convert "paper" capital gains into cash directly without taking delivery of the misvalued underlying asset. In effect, "cash settlement" acts as an infinitely liquid market in which *pre-existing* futures positions are closed out.[2] It is the resulting pattern of implicit liquidity at delivery preceded by a comparatively less liquid spot market (in which trading affects the price) which makes manipulation feasible.

"Cash settlement" is in practice mandatory for "index" futures (e.g., the CME S & P 500 and the CBOT municipal bond contracts) and a few other contracts (e.g., Eurodollars). The extent to which the analysis also applies to futures with a "physical delivery" option (e.g., T-bonds, wheat) is an issue we take up in the conclusion.

Our main results are the following:

1. We construct explicit examples of equilibrium manipulation in the context of a modified Kyle (1985) model.
2. Existence of manipulation per se is shown to be robust to a strikingly wide range of distributional, timing, and preference assumptions.
3. Exogenous spot "noise" trading is unnecessary for the spot market to open, although futures "noise" trading is needed.
4. Profits from manipulation disappear in the limit as manipulators are added, but welfare and price liquidity effects persist.
5. Imperfect informational linkage of futures and cash markets leads to "price pressure" (i.e., subsequently reversed price changes) in the futures market without market maker risk aversion or inventory control problems.

Previous papers on futures manipulation include Kyle (1984) and Vila (1986). These papers show that "squeezes" (or "corners") are possible in a two-period market given that (i) the option is available to force "physical delivery" of the underlying asset and (ii) the manipulator has ex ante superior information about the "noise" traders' net order. In contrast, we assume mandatory "cash settlement," treat a manipulator as being fully

[2] The term "cash settlement," as used in this paper, refers exclusively to the settlement mechanism at delivery. It differs from daily "marking to market" in that the position itself is closed out.

uninformed, allow for multiple manipulators, and introduce a separate strategic informed trader.

Manipulation succeeds because manipulators' trades are confused with those of the informed trader[3]—a point also raised in Black (1990). Froot, Scharfstein, and Stein (1992) use this idea to show that if investors have sufficiently short investment horizons, then "herding equilibria" are possible in which some investors optimally trade on "noise." Our paper differs from theirs in that our manipulators follow optimal dynamic strategies (i.e., exogenous execution lags are avoided) and manipulation is uncoordinated (i.e., manipulators do not coordinate on a common "sunspot" signal). We also show that in this setting the unique linear equilibrium always includes manipulation. A limitation of our model, however, is that it only applies around delivery dates for futures and, by extension, options and other derivative securities.[4]

This paper is organized as follows. Section I presents the basic model of futures manipulation with one manipulator. Section II establishes that adding manipulators drives manipulators' profits to zero. Section III gives an example of how manipulation can induce "price pressure" when futures and spot markets are less than fully informationally integrated. Section IV contains a brief summary.

I. Futures Manipulation with One Manipulator

The basic market environment used in this paper is a modification of the familiar Kyle (1985) model. Working in this stylized setting allows us to solve explicitly for prices and trading strategies. However, after illustrating the basic idea, we show that the existence of manipulation per se is much more general than this particular example.

There are three traded assets: first, a riskless bond which acts as the numeraire and which pays an interest rate of zero; second, a stock with a terminal value v distributed $N(\mu, \sigma_v^2)$ which is modeled as a "liquidating dividend"; and third, a futures contract on the stock. In contrast to Kyle (1984) and Vila (1986) who assume "physical delivery" on the delivery date, we assume "cash settlement" by a payment from the "shorts" to the "longs" equal to the prevailing spot price minus the futures price.

The sequence of events is illustrated in Figure 1. A futures market at date 1 is followed at date 2 by a spot market for the underlying stock. Date 3 is the

[3] A similar confusion arises concerning public tender offers in Bagnoli and Lipman (1989) and insider trading disclosures in Fishman and Hagerty (1991). Here, however, all manipulation occurs via trading rather than through deceptive public announcements. Allen and Gale (1992) also show that manipulative trading occurs in the context of a take-over game where pre-announcement trading signals that a potential target is "in play." However, unlike here, Allen and Gale assume that the manipulator is also the only potentially informed trader.

[4] A sell version of this strategy using options is described in "Are 'Slam Dunks' on Troubled Stocks a Foul?," *The Wall Street Journal*, February 1, 1991, Section C, pp. 1 and 13.

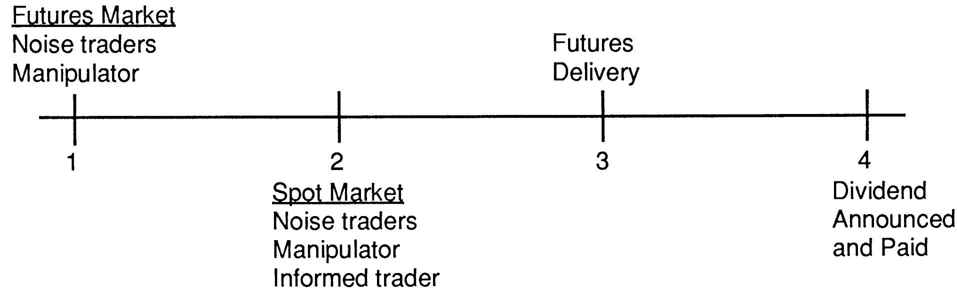

Figure 1. Timing of events.

futures delivery date. The liquidating dividend v is announced[5] and paid on date 4.

Four types of investors participate in these markets: first, a strategic risk-neutral "informed trader" who learns the value of v on date 2 and submits an order x in the spot market (where $x > 0$ if buying); second, a group of uninformed "noise" traders who trade $e \sim N(0, \sigma_e^2)$ in the futures market on date 1 and (independently) trade $u \sim N(0, \sigma_u^2)$ in the spot market on date 2; and third, groups of risk-neutral floor traders and specialists who set competitive futures and spot prices F and S and then clear their respective markets. The fourth trader is a risk-neutral uninformed "manipulator" who, *only if it is profitable*, strategically submits an order Δ in the futures market at date 1 and an order z in the spot market at date 2.

As will be shown, not only is the manipulator willing to trade here, but he will want to take an unbounded position. To obtain an equilibrium, we endow the manipulator with a random amount of initial wealth $|W|$ where W is distributed $N(0, \sigma_W^2)$ and then require that $|\Delta|$ be deposited in a margin account when taking a futures position Δ.[6] This margin assumption, although unnecessary for our general existence result, is also analytically useful here since it induces a normally distributed futures position, which in turn allows us to use linear projections in calculating spot prices.

The trading protocol is that at each date investors submit market orders which are then batched and sent to the appropriate market for execution. At date 1 floor traders observe the net futures order flow to be cleared

$$y_{1f} = e + \Delta \tag{1}$$

[5] Allowing for possible post-spot but *pre-delivery* announcement of v does not qualitatively change our results. This suggests that in multiperiod markets (where manipulation might occur over a series of costly spot and/or futures trades) the possibility of v becoming known before delivery (i.e., before the manipulated futures profit is realized) need not preclude manipulation.

[6] A more realistic device is simply to impose position limits. However, while this is tractable in a discrete space version of this model, it is not tractable in the normally distributed market considered here. Note also that spot positions are unconstrained.

and then set the futures price $F(y_{1f}) = E(S{:}y_{1f})$ to break even. Similarly, at date 2 specialists observe both the previous futures order flow y_{1f} and the current spot order flow

$$y_{2s} = x + u + z \tag{2}$$

and set a zero-profit spot price $S(y_{2s}, y_{1f}) = E(v{:}y_{2s}, y_{1f})$. The specialist conditions on the futures order flow y_{1f} not because y_{1f} is informative about fundamentals (i.e., the value of v), but because it is informative about the manipulator's position Δ and hence about his spot order z.

Given this protocol, the informed trader, who is assumed to learn the liquidating dividend v only at date 2, trades to maximize her expected profit given v and the previous futures order flow y_{1f}[7]

$$\text{Max}_x\ E_{u,z}\left\{x\left[v - S(x + u + z, y_{1f})\right]{:}v, y_{1f}\right\}. \tag{3}$$

The assumption that traders are uninformed at date 1, but not at date 2, is made for tractability.[8] A much weaker condition is used below to demonstrate the existence of manipulation in an extended market.

The manipulator establishes a futures position Δ at date 1 and then trades z at date 2 to maximize expected profits given both Δ and the date 1 noise traders' order e (which he can infer from y_{1f})

$$\begin{aligned}\text{Max}_\Delta\ E_e\{\text{Max}_z\ E_{v,u,x}\{\Delta[S(x + u + z, e + \Delta) - F(e + \Delta)]&\\ +z[v - S(x + u + z, e + \Delta)]{:}\,e\}\}&\end{aligned} \tag{4}$$

subject to $|\Delta| \leq |W|$.

The first-period order flow y_{1f} (or equivalently e together with Δ as written here) is relevant at date 2, because the manipulator uses it to calculate the conditional distribution over z which the specialist uses in setting S.

Given this market structure, there is a manipulative equilibrium in which an ex ante uninformed trader earns positive expected profits. (All proofs are given in the Appendix.)

[7] Sections I and II are unchanged if the informed trader does not know y_{1f}. This is not true of Section III where the specialist does not see y_{1f}. Also, note that the informed trader is implicitly constrained from trading futures at date 1. If in the multimanipulator context of Section II the informed trader is instead allowed to trade at date 1 *before learning* v, it can be shown that she trades futures like an ordinary manipulator and then submits a spot order at date 2 equal to the sum of the optimal informed and manipulator spot orders.

[8] Holden and Subrahmanyam (1992) and Foster and Viswanathan (1991) show that solving the system of difference equations which implicitly characterizes multiperiod Kyle markets requires calculating roots to a cubic equation. Note also that if the underlying asset is an index, then an extreme version of Subrahmanyam (1991) can be used to motivate the assumption that traders are uninformed in the futures market. Assume then that at date 1 a different informed trader has information about each component asset's idiosyncratic return. If this information is diversified away at the portfolio level (i.e., the sum of the idiosyncratic returns is identically 0), no one is informed about the index futures. There is, however, still informed trading in the individual spot markets at date 2. See also footnotes 12 and 13.

Proposition 1: *There is an equilibrium in which (i) the manipulator randomizes his futures order* Δ *on date 1 between* $|W|$ *and* $-|W|$ *with equal probability and then trades* $z = Q\Delta + Re$ *in the spot market where*

$$Q = (1 + \kappa)/2 \tag{5a}$$

$$R = \kappa/2 \tag{5b}$$

$$\kappa = \frac{\sigma_W^2}{\sigma_W^2 + \sigma_e^2}, \tag{5c}$$

(ii) the informed trader's spot order on date 2 is $x = \gamma(v - \mu)$ *where*

$$\gamma = \frac{\left[\sigma_u^2 + \sigma^2(z:y_{1f})\right]^{1/2}}{\sigma_v} \tag{5d}$$

$$\sigma^2(z:y_{1f}) = \kappa\sigma_e^2/4, \tag{5e}$$

(iii) the futures price is $F = \mu$ *and (iv) the spot price is set using the rule* $S = \mu + \lambda[y_{2s} - E(y_{2s}:y_{1f})]$ *where*

$$\lambda = \frac{\sigma_v}{2\left[\sigma_u^2 + \sigma^2(z:y_{1f})\right]^{1/2}} \tag{5f}$$

$$E(y_{2s}:y_{1f}) = E(z:y_{1f}) = \kappa y_{1f}. \tag{5g}$$

The manipulator goes long (short) in the futures market expecting to buy (sell) in the spot market so as to raise (depress) the spot price. Since his futures position Δ is larger than his expected spot position $E_e(z:\Delta)$, the profit on Δ allows the manipulator to recoup on average his loss from overpaying (being underpaid) on z.

The manipulator's ex ante "aggressiveness" in the spot market $Q = E_e(z:\Delta)/\Delta$ is increasing in the signal-to-noise ratio σ_W^2/σ_e^2 since this improves the ability of the specialist to predict the manipulator's futures position and hence to predict and then filter out the manipulator's uninformative trade z from the spot order flow. The "responsiveness" R of the actual spot order z to futures trading noise e also increases in the ratio σ_W^2/σ_e^2 since the specialist's expectation $E(z:y_{1f})$ is then more sensitive to large e realizations. Thus, if, for example, $\Delta > 0$ but e is large and negative, the manipulator can then buy fewer than $Q\Delta$ shares and still raise prices sufficiently given the specialist's depressed volume expectations.

One curious feature of the equilibrium is the asymmetry in the reactions of the futures price F and spot price S to the futures order flow y_{1f}. Even though floor traders know the spot price will be manipulated, they set the futures price equal to the unconditional mean μ regardless of y_{1f}. In contrast, holding the spot order flow y_{2s} fixed, the spot price is decreasing in y_{1f}. This asymmetry arises because the predictable part of the manipulator's spot order (given y_{1f}) is filtered out of the spot order flow by the *specialists* and thus has no effect on the spot price. Given "efficient" spot prices, iterated expectations implies that the futures price (i.e., the expected spot price given y_{1f}) is simply the unconditional mean μ and thus does not need to be adjusted given y_{1f}.

In this model the manipulator loses (on average) in the spot market, but gains in the futures market. If market makers break even, then not only do spot noise traders lose, but so do futures noise traders. The reason futures traders lose—even though the futures price μ is unbiased—is that contracts are settled using the manipulated spot price (from (5))

$$S = \mu/2 + v/2 + \lambda u + (\lambda/2)(1 - \kappa)\Delta - (\lambda\kappa/2)e \tag{6}$$

which is negatively correlated with futures noise trading e (i.e., through e's impact on expected spot trading).

To what extent then does the manipulator's profit depend on our assumption that futures noise traders must trade futures? For example, would "discretionary" noise traders (see Admati and Pfleiderer (1988)), if given a choice between futures at date 1 and the spot market at date 2, choose to trade futures?[9] The answer is yes, "discretionary" noise trading will concentrate in futures since from Proposition 1 the expected loss from trading e in the futures market, $E(S - F{:}e)e = (\lambda\kappa/2)e^2$, is less than the spot loss, λe^2.

Continuing in this vein, is nondiscretionary spot noise trading necessary for the date 2 spot market to open? Interestingly, the answer is no. Although futures noise trading e is necessary[10] (otherwise the manipulator's spot order z is perfectly predictable given y_{1f}), *exogenous* spot noise trading u is unnecessary either for manipulation to be profitable or for the spot market to be viable. The unexpected portion of the manipulator's order $z - E(z{:}y_{1f})$ is itself a form of *endogenous* trading noise which allows the spot market to open. Furthermore, the manipulator's profit increases in "thin" spot markets (i.e., where σ_u^2 is small or zero) since this increases the degree to which his order is confused with that of the informed trader.

Thus one way to view manipulation is as a mechanism transferring liquidity from futures to cash markets.[11] This increased spot liquidity (i.e., relative to what it would be if manipulative orders z were banned) clearly benefits both the informed trader and spot noise traders (whose expected losses are reduced). Hence, given the "zero sum" nature of Kyle-type trading games, the

[9] If "discretionary" noise traders can trade in *both* the futures and spot markets, they will do so. Under these conditions the futures and spot noise order flows will be correlated, so that the price rules and other trading strategies will differ somewhat from those here.

[10] For example, if we were to add a spot market *at date 1*, the expected loss from noise trading there would be 0, and nondiscretionary futures noise trading would be necessary for the futures market to open.

[11] However, unless the increased spot market liquidity leads to the entry of additional informed traders, manipulation has no impact on the market's informational efficiency as measured by the variance of the price forecast error $\sigma^2(v - S)$. Otherwise, as is well-known of Kyle-type models, increased spot "noise" trading is offset by a proportional increase in the trading aggressiveness γ of the informed trader.

manipulator, informed trader, and spot noise traders effectively split the futures noise traders' expected loss.

Turning next to robustness issues, we assert that our manipulative equilibrium is also the unique linear equilibrium. To see why, consider the hypothesis that the manipulator never trades. That is equivalent to setting $\sigma_W^2 = 0$ in the spot price rule. However, contrary to this hypothesis, the manipulator would still optimally trade $|W|$ or $-|W|$ at date 1 and then use $z = \Delta/2$ at date 2. The only other linear possibility is for the informed investor not to trade, but that clearly cannot be an equilibrium.

In solving for an explicit example of manipulation, we have made a series of strong assumptions. Before continuing with this example, it is important to be clear that these assumptions are not in any sense necessary for futures manipulation. Proposition 1 is simply an example of the following general result for two-period markets.

Theorem 1: *Given arbitrary informed trader preferences, information and noise trading distributions, margin constraints, and arrival probabilities for private information at date* 2, *any equilibrium in which the expected spot price is a nonconstant function of the spot order flow includes manipulation* (*i.e., non-zero futures and spot orders* Δ *and* z) *if*,

(*i*) *the manipulator and market makers are risk-neutral and*
(*ii*) *all traders are uninformed at date* 1.

An immediate implication of Theorem 1 is that if (as in this paper) the informed trader is also risk-neutral, then manipulation is generic. This follows, because if a trading rule x is optimal for the informed trader, then for each order $x(v, y_{1f})$ in the range of x the expected spot price must be increasing in the spot order flow. However, if the expected price is increasing conditional on $x(v, y_{1f})$, then it is also increasing unconditionally and hence is nonconstant as required by the theorem.

The two conditions identified in Theorem 1, although surprisingly weak, are themselves only sufficient and not necessary. Market maker risk neutrality and the absence of adverse selection at date 1 simply insure that the manipulator's futures order Δ does not move the futures price adversely. This latter property is not, however, necessary since manipulation still occurs in Section III where Δ does affect the futures price. This suggests that small price effects induced by "moderate" market maker risk aversion and "low grade" adverse selection at date 1 (e.g., low probability of informed traders) are unlikely to preclude manipulation.[12]

[12] What is critical is that, given a long (short) futures position, the manipulator can further raise (depress) the expected settlement price with a comparatively small spot trade. A simple way to construct examples of manipulation with adverse selection *in the futures market* is with a two-period Kyle market in which the informed trader's information is initially "noisy" at date 1, but becomes perfect at date 2. Increasing the variance of informational noise at date 1 increases futures liquidity (relative to spot liquidity). For a sufficiently high variance, it can then be shown that manipulation occurs despite the adverse selection problem at date 1.

Theorem 1 can be generalized further to allow for long-lived private information and for multiple rounds of concurrent spot and futures trading before and (in the spot market) after delivery. As long as the last pre-delivery expected spot price is a nonconstant function of the order flow, the settlement price can be manipulated. With more than two rounds of pre-delivery trading, the same arguments apply provided again no one is informed in the *first* round of futures trading. In this extended market, however, such an assumption is less strained since it implies only that at *some* time during the life of the futures contract adverse selection is not a factor in pricing.[13] *Post-delivery* trading simply provides an opportunity for the manipulator to unwind his spot position before the announcement of v (thereby potentially reducing his spot losses).

II. Manipulation with Many Manipulators

Since the manipulator in Section I is simply an uninformed, unconstrained investor of whom there are presumably many, other such investors may try to manipulate. In this section we investigate what happens as the number of manipulators grows.

To treat the case in which the number of potential manipulators n is common knowledge, we modify our initial model as follows: Manipulators, who are indexed by j, submit futures orders Δ_j (subject again to a margin requirement) and spot orders z_j. For simplicity, we take the case of symmetric ex ante wealth $|W_j|$ and assume that $W_j \sim$ i.i.d. $N(0, \sigma_W^2)$. With this change, the futures order flow is

$$y_{1f} = e + \Sigma_{j=1}^{n} \Delta_j \tag{7}$$

and the spot order flow is

$$y_{2s} = x + u + \Sigma_{j=1}^{n} z_j. \tag{8}$$

Otherwise the trading protocol is unchanged. In particular, the specialist still uses the futures order flow y_{1f} to predict the manipulators' aggregate spot order.

While the informed trader's problem (3) is straightforwardly adapted to this enlarged market, there is a new complication in the problem of a representative manipulator j. After date 1 each manipulator has private information not only about his own and the noise traders' futures positions, but also about the net positions of the other manipulators $\Sigma_{i \neq j} \Delta_i$ and hence about their net spot order $\Sigma_{i \neq j} z_i$. In particular, j is the only player to see $y_{1f} - \Delta_j$. Thus j must now try to manipulate both the specialist and the other manipulators (who are likewise privately informed and, unlike the coordinated manipulators in Froot et al. (1992), may try to manipulate the spot

[13] As an example of predictable variation in the adverse selection problem, Seppi (1992) finds evidence of private information about quarterly earnings during the week before earnings announcements, but not during the penultimate week.

price in different directions). In particular, each j takes a futures position Δ_j at date 1 and then trades z_j at date 2 to maximize his expected profit given his position Δ_j and information $y_{1f} - \Delta_j$,

$$\mathrm{Max}_{\Delta_j} E_{e,\{\Delta_i\}_{i \neq j}} \left\{ \mathrm{Max}_{z_j} E_{\substack{v,u,x,\\ \{z_i\}_{i\neq j}}} \{ \Delta_j [S(x + u + \Sigma_i z_i, e + \Sigma_i \Delta_i) \right.$$
$$\left. - F(e + \Sigma_i \Delta_i)] + z_j [v - S(x + u + \Sigma_i z_i, e + \Sigma_i \Delta_i)] : y_{1f} - \Delta_j \} \right\} \quad (9)$$

subject to $|\Delta_j| \leq |W_j|$.

With a known number of manipulators, a linear manipulative equilibrium still exists.

Proposition 2: *With n potential manipulators there is a symmetric equilibrium in which (i) each manipulator* $j = 1, \cdots, n$ *randomizes his futures order* Δ_j *on date* 1 *between* $|W_j|$ *and* $-|W_j|$ *with equal probability and then submits a spot order* $z_j = Q_n \Delta_j + R_n (y_{1f} - \Delta_j)$ *where*

$$Q_n = \left(\frac{1}{2}\right) \left\{ 1 + \kappa_n + (1 - \kappa_n) \frac{(n^2 - 1)\sigma_W^2}{(n^2 - 1)\sigma_W^2 + (2n + 2)\sigma_e^2} \right\} \quad (10a)$$

$$R_n = \left(\frac{1}{2}\right) \frac{\kappa_n (2n + 2)\sigma_e^2}{(n^2 - 1)\sigma_W^2 + (2n + 2)\sigma_e^2} \quad (10b)$$

$$\kappa_n = \frac{\sigma_W^2}{n\sigma_W^2 + \sigma_e^2}, \quad (10c)$$

(ii) the informed trader's spot order on date 2 *is* $x = \gamma_n (v - \mu)$ *where*

$$\gamma_n = \frac{\left[\sigma_u^2 + \sigma^2(\Sigma_{j=1}^n z_j : y_{1f})\right]^{1/2}}{\sigma_v} \quad (10d)$$

$$\sigma^2(\Sigma_{j=1}^n z_j : y_{1f})$$
$$= [Q_n + (n-1)R_n - n\kappa_n]^2 n \sigma_W^2 + n^2 (R_n - \kappa_n)^2 \sigma_e^2, \quad (10e)$$

(iii) the futures price is $F = \mu$ *and (iv) the spot price is set using the rule* $S = \mu + \lambda_n (y_{2s} - n\kappa_n y_{1f})$ *where*

$$\lambda_n = \frac{\sigma_v}{2\left[\sigma_u^2 + \sigma^2(\Sigma_{j=1}^n z_j : y_{1f})\right]^{1/2}}. \quad (10f)$$

The aggressiveness Q_n is again increasing in the signal-to-noise ratio σ_W^2/σ_e^2. However, the responsiveness R_n is now nonmonotone in σ_W^2/σ_e^2 taking a maximum at $[(2n + 2)/(n^3 - n)]^{1/2}$.

Letting the number of manipulators n grow, manipulators trade more aggressively (with $Q_n \to 1$ in the limit) and react less to their information $y_{1f} - \Delta_j$ (with $R_n \to 0$ in the limit). In addition, the conditional variance of the total manipulator order $\sigma^2(\Sigma_{j=1}^n z_j : y_{1f})$ grows to σ_e^2 in the limit. As this happens, spot prices become less responsive to the date 2 order flow, thereby increasing *informed* trader profits, but diminishing (along with the fact that $z_j \to \Delta_j$, the "break even" spot order) each manipulator's profit. Summing over the n manipulators, we have

Proposition 3: *The aggregate ex ante expected profit from manipulation falls to zero in the limit as the number of manipulators* $n \to \infty$.

In the limit, manipulation becomes a frictionless conduit transferring futures trading "noise" σ_e^2 to the spot market. However, although manipulators themselves only break even, manipulation continues—via its impact on market liquidity—to benefit informed and spot noise traders at the (indirect) expense of futures noise traders even in the limit.

III. Imperfect Informational Linkage and "Price Pressure"

Thus far, the futures price is unaffected by the futures order flow (i.e., $F = \mu$) because, with efficient spot prices, floor traders rely on the specialist to filter out manipulation which is predictable given y_{1f}. In practice, however, spot markets are unlikely to observe all order-related futures information (or infer it from futures prices). Under these conditions, the futures order flow affects the futures price even if (as in our model) y_{1f} is informative *only* about prospective spot manipulative trading at date 2, but is completely uninformative about fundamentals (i.e., v). The result is temporary "price pressure" in the futures price F. To our knowledge, this is a new type of "price pressure" since previously identified factors such as market maker risk aversion or inventory constraints[14] are absent here.

An illustrative, but extreme, case has the spot market completely "informationally decoupled" from the futures market (i.e., the specialist sees neither F nor y_{1f}) so that the spot price depends only on the spot order flow $S(y_{2s}) = E(v\!:\!y_{2s})$. In this setting the futures order flow y_{1f} is again informative about Δ which is only informative about z. However, with "decoupled" markets, floor traders can no longer rely on specialists to filter out the predictable manipulative noise $E(z\!:\!y_{1f})$ from the spot order flow y_{2s} (e.g., as in (5g)). Thus, the futures price $F(y_{1f}) = E(S\!:\!y_{1f})$ is set to reflect predicted spot price manipulation and therefore depends (nontrivially) on the futures order flow y_{1f}.

If the informed trader also does not see F or y_{1f}, her problem reduces to

$$\mathrm{Max}_x\, E_{u,z}\{x[v - S(x + u + z)]\!:\!v\}. \tag{11}$$

With one manipulator, the manipulator's problem becomes

$$\begin{aligned}\mathrm{Max}_\Delta\, E_e\{&\mathrm{Max}_z\, E_{v,u,x}\{\Delta[S(x + u + z) - F(e + \Delta)]\\ &+z[v - S(x + u + z)]\!:\!e\}\}\end{aligned} \tag{12}$$

subject to $|\Delta| \leq |W|$.

[14] See Stoll (1978), Amihud and Mendelson (1980), and Ho and Stoll (1983).

In this setting, futures trades (being uninformative about v) lead to temporary "price pressure" in the sense that the futures price F is a biased predictor of the stock's terminal value $E(F - v{:}F) = F - \mu \neq 0$.

Proposition 4: *If $\sigma_e^2 > \sigma_W^2$, there is an equilibrium with a "decoupled" spot market in which (i) the manipulator randomizes his futures order Δ on date 1 between $|W|$ and $-|W|$ with equal probability and then trades $z = \Delta/2$ in the spot market, (ii) the informed trader's spot order on date 2 is $x = \gamma(v - \mu)$ where*

$$\gamma = \frac{\left[\sigma_u^2 + \left(\frac{1}{4}\right)\sigma_W^2\right]^{1/2}}{\sigma_v}, \tag{13a}$$

(iii) the futures price is $F = \mu + (\lambda\kappa/2)y_{1f}$ and (iv) the spot price is $S = \mu + \lambda y_{2s}$ where

$$\kappa = \frac{\sigma_W^2}{\sigma_W^2 + \sigma_e^2} \tag{13b}$$

$$\lambda = \frac{\sigma_v}{2\left[\sigma_u^2 + \left(\frac{1}{4}\right)\sigma_W^2\right]^{1/2}}. \tag{13c}$$

In this "informationally decoupled" market the manipulator again loses in the spot market and gains in the futures market. However, unlike Proposition 1, futures noise traders lose, not because the spot price is manipulated (the spot price S is now uncorrelated with the futures noise trade e), but because futures "price pressure" $F - \mu$ is positively correlated with their order e. Insufficient futures trading "noise" σ_e^2 precludes a linear equilibrium[15] because futures prices are then "too" responsive to y_{1f} in the sense that the manipulator's futures profits no longer offset his spot losses.

IV. Summary

We argue here that with asymmetric information, futures contracts (and by extension other derivative securities) are inherently susceptible to manipulation—even if "cash settlement" is used in place of "physical delivery" to protect against "corners" or "squeezes." Since in equilibrium market participants understand this, this observation by itself has no clear-cut regulatory implications. Manipulation does, however, have nontrivial welfare implications since manipulation hurts futures noise traders, but benefits spot noise traders and informed traders through increased spot liquidity.

[15] However, using the same logic as in Theorem 1, if a *nonlinear* equilibrium exists with nonconstant spot prices, then it includes manipulation.

Intermarket manipulation also allows us to overcome a "local" version of the Milgrom and Stokey (1982) "no trade" theorem. By transferring liquidity from market A (with noise trading) to another market B (without indigenous noise trading), manipulation may allow market B to open even if, in isolation, the "no trade" theorem would apply. In this sense, manipulation acts, as Black (1990) suggests, as an *endogenous* form of noise trading. Thus to sustain trading across multiple markets, there need not be exogenous "noise" in *every* market, just in *some* markets.

Last, the model predicts interesting empirical differences in the way futures and spot prices react to their own and each other's order flows which, without such a model, might appear "anomalous." In particular, in our two-period example, "efficient" spot prices are decreasing in prior futures order flow, while futures prices are constant.

We conclude with a brief discussion of what factors might inhibit manipulation and whether assets other than "cash settlement" contracts are similarly susceptible to manipulation.

A. Inhibiting Factors

Given the robustness of our results (Theorem 1), what factors might inhibit manipulation in practice? Transaction costs are clearly one. Another is price discreteness combined with either (i) position limits, (ii) margin requirements with tightly constrained investor wealth, or (iii) manipulator risk aversion. Given price discreteness, a would-be manipulator's spot position must be large enough to move the price at least a "tick." However, with position limits (or given insufficient wealth to maintain large margin accounts) his maximal futures position may be inadequate to recover the expected losses on this "threshold" spot position. Alternatively, since manipulation is risky, risk aversion may dampen his willingness to undertake such large positions. Whether such market imperfections curtail manipulation in practice is an interesting empirical question.

B. Physical Delivery

Are futures contracts with a "physical delivery" option also susceptible to liquidity-driven manipulation? The answer depends on whether "offsetting" trades can be used to unwind a futures position with little price impact (i.e., relative to the impact of prior spot and futures trading). If they can, then such "offsetting" transactions may function effectively as a form of de facto "cash settlement."[16]

One scenario in which this occurs is if delivery is sufficiently costly (e.g., due to transportation costs, brokerage fees, inelastic "deliverable" supply, or

[16] These liquid "offsetting" trades (which manipulators would use to unwind their futures position) need not occur immediately prior to delivery. A more likely time in practice is before the "date of first notice" (i.e., the first day involuntary delivery obligations can be assigned) when the "open interest" of so-called "noncommercial participants" appears to shrink.

subsequent spot market illiquidity) so that *all* investors prefer avoiding delivery. Since an "offsetting" trade can always then be anticipated each time a futures position is initiated, the trade itself should have no price impact. Delivery must, however, be costly; otherwise futures "noise" traders would want to separate from manipulators by taking (or making) delivery. Similarly, informed traders, without costly delivery, could use pre-delivery trades not to unwind futures positions, but to enlarge them and then exploit the delivery option to change their spot position. Either case reduces market liquidity for "offsetting" trades.

C. Spot Market Manipulation

It is likely that spot markets can be manipulated directly given sufficient intertemporal variation in market liquidity (due perhaps to changes in the variance of exogenous "noise" trading). The main analytic difficulty in a more canonical single-market model is that the variation in liquidity must be endogenously determined rather than contractually mandated by "cash settlement" provisions.

Appendix

Proof of Proposition 1: The proof is divided into three steps. First, backwards induction is used to solve for the manipulator's optimal strategy given linear price rules and informed trader strategy. Starting at date 2, taking as given a futures position Δ and the order flow y_{1f}, the first-order condition for the manipulator's problem (4) is

$$z = \left[\Delta + E(z : y_{1f})\right]/2. \tag{A1}$$

The second-order condition for a maximum is satisfied if $\lambda > 0$.

Taking expectations of both sides of (A1), given y_{1f}, gives the specialist's conditional expectation of the manipulator's spot order to be $E(z:y_{1f}) = E(\Delta:y_{1f}) = \kappa y_{1f}$. Substituting this into (A1) and using $y_{1f} = \Delta + e$ confirms that $z = [(1 + \kappa)\Delta + \kappa e]/2$ is optimal given a futures position of Δ.

Turning to date 1, after substituting in the optimal spot strategy z, the manipulator's problem (4) is

$$\mathrm{Max}_{\Delta}(\lambda/4)\left[(1 - \kappa)^2\Delta^2 + \kappa^2\sigma_e^{\,2}\right] \tag{A2}$$
$$\text{subject to } |\Delta| \le |W|.$$

Since the maximand is increasing in Δ^2, the manipulator is indifferent between the two corner solutions of going long $|W|$ or short $-|W|$ futures contracts at date 1. Thus, he is willing to randomize with equal probability over these orders so that $\Delta \sim N(0, \sigma_W^2)$.

The second step of the proof is to solve for the informed trader's strategy in the spot market. Taking the spot price and manipulator strategy to be linear, the first-order condition for the informed trader's problem (3) at date 2 is

$$x = (v - \mu)/2\lambda. \tag{A3}$$

The second-order condition is satisfied if $\lambda > 0$.

Third, given linear informed trader and manipulator strategies, the specialist's conditional expectation of v given the futures and spot order flows is $E(v:y_{1f}, y_{2s}) = \mu + \lambda[y_{2s} - E(y_{2s}:y_{1f})]$ where $E(y_{2s}:y_{1f}) = E(z:y_{1f})$ and $\lambda = \text{cov}(v - \mu, y_{2s}:y_{1f})/\text{var}(y_{2s}:y_{1f})$. This last equation is quadratic in λ. Solving for the positive root gives (5f). Substituting this in for λ in (A3) gives (5d). Last, iterated expectations implies that $F = Ey_{2s}\{E_v(v:y_{2s}, y_{1f}):y_{1f}\} = \mu$. Q.E.D.

Proof of Theorem 1: Suppose not, so that an equilibrium exists in which the manipulator did not trade. In such an equilibium, the futures price is $F = \mu$ and the expected spot price $E_{x,u}\{S(x + u + z, y_{1f}):z\}$ either increases or decreases in z (although the spot price S may be nonmonotone). However, under these conditions not trading (and receiving a certain profit of 0) is not optimal for the manipulator.

If the expected spot price is *increasing* in z, then from (4) any futures position $\Delta > 0$ and spot order $\Delta > z > 0$ earns positive expected profits

$$\left[E_{x,u}\{S(x + u + z, y_{1f}): z\} - \mu\right](\Delta - z) > 0 \tag{A4}$$

since $E_{x,u}\{S(x + u + z, y_{1f}):z\} > E_{x,u}\{S(x + u + z, y_{1f}):z = 0\} = E_{x,u}\{E_v(v:x + u)\} = \mu$ where the first inequality follows from expected prices being increasing in z, the next equality from competition and specialist risk neutrality, and the last equality from iterated expectations. If the expected spot price is *decreasing* in z, then the manipulator can go long Δ in futures and then *short* in the spot market (to raise the spot price) so that now both trades are profitable on average. Thus in either case the manipulator trades in equilibrium. Q.E.D.

Proof of Proposition 2: The same steps are used to show that these modified prices and informed trading strategies are optimal. Thus all that needs to be shown is that the manipulators' strategies are optimal given n manipulators. To show this, the market makers' and informed trader's strategies are again assumed to be linear and backward induction is used to solve the trading problem of a typical manipulator j. Starting at date 2, with j's futures position Δ_j and the order flow y_{1f} taken as given, the first-order condition from (9) is

$$z_j = \left\{\Delta_j + E(z_j: y_{1f}) - \Sigma_{i \neq j}\left[E(z_i: \Delta_j, y_{1f}) - E(z_i: y_{1f})\right]\right\}/2. \tag{A5}$$

The second-order condition for a maximum is also satisfied if $\lambda_n > 0$.

Taking expectations of each side of (A5) with respect to y_{1f}, using iterated expectations and assuming symmetry at date 1 gives the specialist's conditional expected spot order from manipulator j and each manipulator

$i \neq j$ to be

$$E(z_j: y_{1f}) = E(\Delta_j: y_{1f}) = \kappa_n y_{1f} = E(\Delta_i: y_{1f}) = E(z_i: y_{1f}). \tag{A6}$$

In addition, if Δ_j, z_j, e, and the Δ_i's and z_i's are jointly multivariate Normal, manipulator j's expectations of the z_i's are given by

$$E(z_i: \Delta_j, y_{1f}) = \beta\Delta_j + \theta(y_{1f} - \Delta_j). \tag{A7}$$

Since we are looking for a symmetric equilibrium, it must also be true that $E(z_j:\Delta_i, y_{1f}) = \beta\Delta_i + \theta(y_{1f} - \Delta_i)$. Using (A6) and (A7) to substitute out the conditional expectations in (A5) together with the fact that β and θ are regression coefficients we obtain two linear equations in β and θ. Their solution is given by

$$\beta = \frac{\left[n^2\kappa_n - n(1 + \kappa_n) + 1\right]\sigma^2(\Delta) + 2n\kappa_n\sigma_e^2}{(n^2 - 1)\sigma^2(\Delta) + (2n + 2)\sigma_e^2} \tag{A8}$$

$$\theta = \frac{(n^2\kappa_n - n\kappa_n + 2)\sigma^2(\Delta) + 2n\kappa_n\sigma_e^2}{(n^2 - 1)\sigma^2(\Delta) + (2n + 2)\sigma_e^2} \tag{A9}$$

which when substituted into (A5) gives (10a) and (10b).

Turning to date 1, if (A5) through (A7) are substituted into (9), j's problem reduces to

$$\mathrm{Max}_{\Delta_j}(\lambda_n/4)E_{e,\{\Delta_i\}_{i \neq j}}\left\{\Delta_j - \kappa_n y_{1f} + \Sigma_{i \neq j}\left(\beta\Delta_j + \theta(y_{1f} - \Delta_j) - \kappa_n y_{1f}\right)\right\}^2 \tag{A10}$$

subject to $|\Delta_j| \leq |W_j|$.

Using (A8) and (A9) to substitute out β and θ, the maximand in (A10) can be shown to be strictly increasing in Δ_j^2. Thus, given the constraint on Δ_j, manipulator j is indifferent between going long $|W_j|$ or short $-|W_j|$ contracts in the futures market.

The last step of the proof is to observe that using (A6) and the fact that each manipulator's spot order is given by $z_j = Q_n\Delta_j + R_n(y_{1f} - \Delta_j)$, the conditional variance of the aggregate manipulator spot order $\sigma^2(\Sigma_{j=1}^n z_j : y_{1f})$ is given by (10e). Q.E.D.

Proof of Proposition 3: From Proposition 2, the net expected profit for all manipulators is

$$(\lambda/4)\Big\{n[1 + (n-1)\beta - n\kappa_n]^2\sigma_W^2$$
$$+n[(n-1)\theta - n\kappa_n]^2\left[\sigma_e^2 + (n-1)\sigma_W^2\right]\Big\}. \tag{A11}$$

Substituting in for κ_n, β, and θ (from above), the limit of the bracketed term is 0, while the price parameter λ tends to a finite limit. Q.E.D.

Proof of Proposition 4: With a "decoupled" spot market, the manipulator's problem at date 2 reduces to

$$\mathrm{Max}_z\, \lambda\Big\{\Delta z - \Delta E(z : y_{1f}) - z^2\Big\}. \tag{A12}$$

The new first-order condition is $z = \Delta/2$ with the second-order condition again satisfied if $\lambda > 0$.

Turning to date 1, after substituting in for F, S, and z, the manipulator's problem (12) reduces to

$$\mathrm{Max}_\Delta\, (\lambda/4)[1 - 2\kappa]\Delta^2 \tag{A13}$$
$$\text{subject to } |\Delta| \leq |W|.$$

When $\sigma_e^2 > \sigma_W^2$ the maximand is increasing in Δ^2 so that the manipulator is again willing to randomize his futures position Δ with equal probability over $|W|$ and $-|W|$ contracts.

The optimality of the informed strategy and the spot price (given unobservable y_{1f} and F) is established using the same steps as the proof of Theorem 1 in Kyle (1985) together with the observation that (since the specialist knows only the unconditional distribution of z) the contribution of z to the spot order flow "noise" is its full unconditional variance $\sigma_W^2/4$.

Taking expectations of the spot price S, floor traders set the futures price

$$F(y_{1f}) = \mu + \lambda E(z : y_{1f}) \tag{A14}$$

using their conditional expectation of the manipulator's spot order $E(z{:}y_{1f}) = (\kappa/2)y_{1f}$. Iterated expectations does not apply because S is no longer conditioned on y_{1f}. Q.E.D.

REFERENCES

Admati, A. and P. Pfleiderer, 1988, A theory of intraday patterns: Volume and price variability, *Review of Financial Studies*, 1, 3–40.

Allen, F. and D. Gale, 1992, Stock price manipulation, *Review of Financial Studies*, Forthcoming.

Amihud, Y. and H. Mendelson, 1980, Dealership market: Market-making with inventory, *Journal of Financial Economics* 8, 31–53.

Bagnoli, M. and B. Lipman, 1989, Stock price manipulation through takeover bids, Working paper, Carnegie Mellon University.

Black, F., 1990, Bluffing, Working paper, Goldman Sachs.

De Long, J., A. Shleifer, L. Summers, and R. Waldmann, 1990, Noise trader risk in financial markets, *Journal of Political Economy* 98, 703–738.

Fishman, M. and K. Hagerty, 1991, The mandatory disclosure of trades and market liquidity, Working paper, Northwestern University.

Foster, F. and S. Viswanathan, 1991, Market microstructure models with elliptically contoured distributions, Working paper, Duke University.

Friedman, M., 1953, The case for flexible exchange rates, in *Essays in Positive Economics* (University of Chicago Press, Chicago).

Froot, K., D. Scharfstein, and J. Stein, 1992, Herd on the street: Informational inefficiencies in a market with short-term speculation, *Journal of Finance* 47, 1461–1484.

Hart, O., 1977, On the profitability of speculation, *Quarterly Journal of Economics* 91, 579–591.

Ho, T. and H. Stoll, 1983, The dynamics of dealer markets under competition, *Journal of Finance* 38, 1053–1074.

Holden, C. and A. Subrahmanyam, 1992, Long-lived private information and imperfect competition, *Journal of Finance* 47, 247–270.

Jarrow, R., 1989, Market manipulation, bubbles, corners and short squeezes, Working paper, Cornell University.

Keynes, J., 1936, *The General Theory of Employment, Interest and Money* (MacMillan, London).

Kyle, A., 1984, A theory of futures market manipulations, in R. W. Anderson, ed.: *The Industrial Organization of Futures Markets* (Lexington Books, Lexington, MA).

———, 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315–1335.

Milgrom, P. and N. Stokey, 1982, Information, trade and common knowledge, *Journal of Economic Theory* 26, 17–27.

Seppi, D., 1992, Block trading and information revelation around quarterly earnings announcements, *Review of Financial Studies* 5, 281–305.

Stoll, H., 1978, The supply of dealer services in securities markets, *Journal of Finance* 33, 1133–1151.

Subrahmanyam, A., 1991, A theory of trading in stock index futures, *Review of Financial Studies* 4, 17–51.

Vila, J., 1986, The role of information in futures market manipulations, Working paper, Princeton University.