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ABSTRACT

Postwar U.S. data are characterized by negative correlations between real equity returns and inflation and by positive correlations between real equity returns and money growth. These patterns are closely matched quantitatively by an equilibrium monetary asset pricing model. The model also implies negative correlations between expected asset returns and expected inflation, and it predicts that the inflation-asset return correlation will be more strongly negative when inflation is generated by fluctuations in real economic activity than when it is generated by monetary fluctuations.

THE CO-MOVEMENTS OF REAL asset returns, inflation, and money growth have been studied extensively over the past fifteen years. Table I displays some of the statistical properties of these series using quarterly U.S. data from 1959 to 1990. Note first the significantly negative correlation between real equity returns and inflation.[1] Note also the negative correlation between inflation and the ex post real return to nominally riskless bonds. While this fact is not particularly surprising, numerous studies conclude that the correlations between ex ante real asset returns and expected inflation are also negative.[2] In contrast to these patterns, the correlation between money growth and real equity returns is (weakly) positive in the data, while the correlation between money growth and the real bond return is virtually zero.

Initially, these empirical patterns were viewed as anomalies. Negative correlations between inflation and ex post real stock returns contradict the traditional view that equities ought to act as an inflation hedge.[3] Negative correlations between expected inflation and ex ante real returns violate the

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[1] The tendency of equity returns to covary negatively with inflation has been extensively documented using a variety of data sets. See Lintner (1975); Bodie (1976); Fama and Schwert (1977); Cohn and Lessard (1981); Fama (1981); Fama and Gibbons (1982); Gultekin (1983); and Lee (1989).

[2] For example, see Jaffe and Mandelker (1976); Nelson (1976); Summers (1983); and Huizinga and Mishkin (1984).

[3] See, for example, Graham, Dodd, and Cottle (1962, p. 61).

Table I

Statistical Properties of Data

The data are quarterly observations from 1959:1 to 1990:2. All price and quantity data are from the CITIBASE database (in the following, CITIBASE acronyms are in parentheses), and all returns data are from the Center for Research on Security Prices (CRSP) database. Consumption is real quarterly expenditures on nondurables (GCN82) and services (GCS82). The price level is the implicit deflator for consumption. "Inflation Rate" is the growth rate of the price level. "Real Equity Return" is the one-month value-weighted return on the New York Stock Exchange from the CRSP Stock File, compounded to a quarterly return and deflated by the inflation rate. "Real Bond Return" is the yield to maturity of the 90-day Treasury Bill (computed as the average of the bid yield and the ask yield) from the CRSP Bond File, deflated by the inflation rate. "Money Growth Rate" is the rate of growth of the money supply as measured by M1 (FM1), scaled by the ratio of consumption to the gross national product (GNP82) in each period. "Velocity" is the ratio of nominal consumption to the money supply. All quantity variables are divided by the resident U.S. population (POPRES). The detrended variables are the residuals after a geometric trend is removed. The reported estimates were obtained using Generalized Method of Moments assuming sixth-order serial correlation in the residuals. Standard errors are in parentheses. The variance of each series is reported along the diagonal of the cross-correlation matrix displayed in panel B.

Panel A: Means

Variable	Mean	Mean of Detrended Variable
Real equity return	0.01638	
	(0.00686)	
Real bond return	0.00294	
	(0.00129)	
Inflation rate	0.01223	0.00699
	(0.00150)	(0.00132)
Money growth rate	0.01063	0.00171
	(0.00188)	(0.00139)
Velocity	1.382	0.9479
	(0.067)	(0.0165)

Table I—*Continued*

Panel B: Cross-Correlation Matrix

Real Equity Variable	Real Bond Return	Detrended Return	Detrended Inflation	Detrended Money Growth	Velocity
Real equity return	0.00707	0.1933	−0.2825	0.1304	−0.0928
	(0.00131)	(0.1092)	(0.1047)	(0.0779)	(0.0575)
Real bond return		0.452×10^{-4}	−0.6935	−0.0081	−0.0422
		(0.112×10^{-4})	(0.1032)	(0.1074)	(0.1751)
Detrended inflation			0.426×10^{-4}	0.0975	0.5494
			(0.111×10^{-4})	(0.0911)	(0.1057)
Detrended money growth				0.192×10^{-3}	0.0720
				(0.345×10^{-4})	(0.1363)
Detrended velocity					0.00526
					(0.00150)

so-called Fisher hypothesis, which states that nominal rates should move one-for-one with the expected inflation rate. Negative correlations between ex ante real equity returns and expected inflation appear to contradict the accepted theory that expected returns are determined by the risk associated with future cash flows. These apparent inconsistencies between the data and the predictions of economic theory have been cited as evidence of irrationality and market inefficiency.[4] Finally, the difference between inflation-asset return correlations and money growth-asset return correlations is inconsistent with the view that inflation is a purely monetary phenomenon.

Some of these issues have been addressed in theoretical research by Day (1984), Danthine and Donaldson (1986), and Stulz (1986), among others. In these papers, money is an asset whose real value is determined simultaneously with other asset values. Danthine and Donaldson (1986) demonstrate that equities do not hedge against that portion of inflation caused by fluctuations in real economic activity. Stulz's (1986) model predicts negative correlations between expected inflation and ex ante real equity returns. These models are highly stylized; they provide no evidence whether the predicted negative correlations are large enough in magnitude to match the data. Furthermore, these models do not attempt to reconcile the negative correlations between inflation and asset returns with the positive correlations between asset returns and money growth.

In this paper I formulate, estimate, and simulate a dynamic equilibrium monetary model to answer the following questions: (1) Does the model predict negative co-movements between expected inflation and ex ante real asset returns and between realized inflation and ex post real asset returns? (2) Can the model match the observed magnitudes of these correlations? (3) Does the sign or magnitude of these correlations depend on whether the inflation process is driven by fluctuations in real economic activity or by fluctuations in the money growth rate? By distinguishing between these two sources of fluctuations in inflation, can we reconcile negative inflation-equity return correlations with positive money growth-equity return correlations? I also examine the limitations of this class of models. That is, along which dimensions does the model fail empirically, and how do these failures provide guidance for future research?

Labadie (1989), Lee (1989), and Hodrick, Kocherlakota, and Lucas (1991) also simulate dynamic monetary asset pricing models. Labadie (1989) and Hodrick, Kocherlakota, and Lucas (1991) do not directly study the correlations between inflation and asset returns. Lee (1989) finds negative correlations between inflation and real asset returns in a stochastic growth model with a binding cash-in-advance constraint. Because that model restricts velocity to unity, it is not well suited to fitting stochastic fluctuations in money demand.

Section I describes the model. Section II examines theoretical implications of the model for the signs of the correlations between inflation and real asset

[4] See Modigliani and Cohn (1979); Cohn and Lessard (1981); and Summers (1983).

returns and asks whether these correlations depend on the source of inflationary fluctuations. Section III describes the strategy for estimating the model without imposing stationarity on the money growth process and presents the parameter estimates. Section IV considers simulations of the model and compares the model's predictions to the observed data. Section V concludes the paper.

I. A Simple Monetary Intertemporal Asset Pricing Model

Among the assets available to agents are money, equities, and bonds. Money reduces the costs of consumption transactions (e.g., Feenstra (1986)). That is, when real balance holdings equal m, consumption of c units of the single composite good requires expending an additional $\varphi(c, m)$ units of consumption as transaction costs. The function $\varphi(\cdot,\cdot)$ is non-negative and differentiable, with

$$\varphi_1 \geq 0, \qquad \varphi_2 \leq 0, \quad \text{and} \quad \varphi(0, m) = 0.$$

As suggested by Feenstra (1986), I assume that

$$\varphi_{11} \geq 0, \qquad \varphi_{22} \geq 0, \quad \text{and} \quad \varphi_{12} \leq 0. \tag{1.1}$$

The money growth rate, $\mu_t \equiv M_t/M_{t-1}$, is an exogenous stochastic process. Agents receive money as a lump-sum transfer τ_t. Let $\hat{M}_t$ denote nominal balances acquired by the representative agent in period $t-1$ for use in period t, and let M_t denote the nominal balances available for transaction use in period t. I assume that the money transfer τ_t is made at the beginning of the period and that agents can use this money immediately for transactions. Thus,

$$M_t = \hat{M}_t + \tau_t \tag{1.2}$$

The consumption good enters the economy as an exogenous endowment stream Y_t. Following Lucas (1978), I model equities as claims to the endowment. The total supply of equity shares is normalized to unity. There also exist N financial assets, which are in zero net supply. These include a one-period nominally riskless bond and a risk-free asset. To facilitate derivation of real returns, I normalize the real prices of these financial assets to one unit of the consumption good.

The representative agent has time- and state-separable preferences defined over consumption. I use the following additional notation:

c_t = agent's consumption in period t;
z_t = number of equity shares held by the agent at the beginning of period t;
a_t^i = number of shares of the ith financial asset held by the agent at the beginning of period t;
p_t^c = money price of one unit of consumption;
p_t^z = money price of one equity share;
r_t^i = real return to the ith financial asset;

$m_t = M_t/p_t^c$;
$\pi_t = p_t^c/p_{t-1}^c$; and
$E_t(\cdot)$ is the expectation operator, conditional on date t information.

The representative agent's problem is

$$\text{MAX}_{\{c_t, \hat{M}_{t+1}, z_{t+1}, a_{t+1}^i\}_{t=0,i=1}^{\infty\;\;N}} E_0 \sum_{t=0}^{\infty} \delta^t U(c_t), \tag{1.3}$$

subject to (1.2), (1.4), and (1.5):

$$p_t^c(c_t + \varphi(c_t, m_t)) + p_t^z z_{t+1} + p_t^c \sum_{i=1}^{N} a_{t+1}^i + \hat{M}_{t+1} \leq z_t(p_t^c Y_t + p_t^z) + p_t^c \sum_{i=1}^{N} r_t^i a_t^i + \hat{M}_t + \tau_t, \qquad t = 0, 1, 2, \ldots, \tag{1.4}$$

and

$$c_t \geq 0; \qquad z_{t+1} \geq \underline{z}; \qquad \hat{M}_{t+1} \geq 0; \qquad a_{t+1}^i \geq \underline{a}^i, \qquad \forall\, i, \forall\, t, \tag{1.5}$$

where z_0, $\hat{M}_0$, and a_0^i are given, $0 < \delta < 1$, $\underline{z} > -\infty$, and $\underline{a}^i > -\infty$. Constants $\underline{z}$ and $\underline{a}^i$ are short sales constraints. Without such lower bounds on asset holdings, the agent's objective function is unbounded. In equilibrium, though, constraints (1.5) never bind.

The first-order necessary conditions for a solution to the problem posed by (1.3) to (1.5) are the budget constraints (1.4) holding with equality and the following Euler equations:

$$\frac{1}{p_t^c} = E_t\left(\rho_{t+1} \cdot (1 - \varphi_2(c_{t+1}, m_{t+1}))\frac{1}{p_{t+1}^c}\right), \tag{1.6}$$

$$\frac{p_t^z}{p_t^c} = E_t\left(\rho_{t+1} \cdot \left(Y_{t+1} + \frac{p_{t+1}^z}{p_{t+1}^c}\right)\right), \tag{1.7}$$

and

$$1 = E_t(\rho_{t+1} \cdot r_{t+1}^i), \qquad i = 1, \ldots, N, \tag{1.8}$$

where

$$\rho_{t+1} \equiv \frac{\delta U'(c_{t+1})}{U'(c_t)}\left(\frac{1 + \varphi_1(c_t, m_t)}{1 + \varphi_1(c_{t+1}, m_{t+1})}\right). \tag{1.9}$$

The random variable ρ_{t+1} is the intertemporal marginal rate of substitution in wealth. Equations (1.7) and (1.8) equate each asset's real price to its expected real payoff discounted by the marginal rate of substitution. If the real return to money, denoted r_{t+1}^m, is defined by

$$r_{t+1}^m \equiv [1 - \varphi_2(c_{t+1}, m_{t+1})]/\pi_{t+1}, \tag{1.10}$$

then equation (1.6) can be put into the same form as (1.8):

$$1 = E_t(\rho_{t+1} \cdot r_{t+1}^m). \tag{1.11}$$

The return r_t^m has both a pecuniary component (inflation or deflation) and a nonpecuniary component (marginal transaction cost savings, as measured by $-\varphi_2$).

Let the net nominal interest rate be R_t. The real return to a one-period nominally riskless bond in period $t+1$ is then $(R_t + 1)/\pi_{t+1}$, so equations (1.6) and (1.8) imply

$$R_t = \frac{-E_t\left(\rho_{t+1} \cdot \dfrac{1}{p_{t+1}^c}\varphi_2(c_{t+1}, m_{t+1})\right)}{E_t\left(\rho_{t+1} \cdot \dfrac{1}{p_{t+1}^c}\right)}. \tag{1.12}$$

Equation (1.12) is a generalized money demand function of the type described by Lucas (1984) and McCallum and Goodfriend (1987). It is the focus of the estimation procedure used in Section III.

An equilibrium must satisfy (1.2), (1.4) holding with equality, (1.6) to (1.8), the money supply constraint

$$\tau_t = [\mu_t - 1]M_{t-1},$$

and the following market clearing conditions:

$$\begin{aligned}
c_t &= Y_t - \varphi(c_t, m_t) && \text{(goods market);}\\
\hat{M}_t &= M_{t-1} && \text{(money market);}\\
z_{t+1} &= 1 && \text{(equity market); and}\\
a_{t+1}^i &= 0, \qquad i = 1, \ldots, N && \text{(financial asset markets).}
\end{aligned}$$

II. Theoretical Implications of the Model for Correlations between Inflation and Asset Returns

In this section I explore the implications of the model described in Section I for the signs and magnitudes of the correlations between inflation and real asset returns. It is likely that these correlations will be strongly negative when inflation is caused primarily by fluctuations in real economic activity. In contrast, when inflation is caused primarily by fluctuations in the money growth rate, these correlations are ambiguous in sign and small in magnitude.

A. Expected Inflation and Expected Excess Real Asset Returns

Equation (1.8) implies the following conditional single-beta representation for the excess return to asset i:

$$E_t r_{t+1}^i - r_t^f = -\beta_t^i\left[\frac{\mathrm{var}_t(\rho_{t+1})}{E_t\,\rho_{t+1}}\right], \tag{2.1}$$

where r_t^f is the real risk-free rate and $\beta_t^i \equiv \text{cov}_t(r_{t+1}^i, \rho_{t+1})/\text{var}_t(\rho_{t+1})$. Equation (1.11) implies an analogous expression for money:

$$E_t r_{t+1}^m - r_t^f = -\beta_t^m \left[\frac{\text{var}_t(\rho_{t+1})}{E_t \rho_{t+1}} \right], \tag{2.2}$$

where $\beta_t^m \equiv \text{cov}_t(r_{t+1}^m, \rho_{t+1})/\text{var}_t(\rho_{t+1})$. Equations (2.1) and (2.2) together imply

$$E_t r_{t+1}^i - r_t^f = \frac{\beta_t^i}{\beta_t^m} \left[E_t r_{t+1}^m - r_t^f \right]. \tag{2.3}$$

If if β_t^i/β_t^m displays little variation through time (compared to the variation in excess returns),[5] then (2.3) implies

$$\begin{aligned} &\text{sign}\left[\text{cov}\left[E_t r_{t+1}^i - r_t^f, E_t \pi_{t+1}\right]\right] \\ &\quad = \text{sign}\left[\beta_t^i/\beta_t^m\right] \cdot \text{sign}\left[\text{cov}\left[E_t r_{t+1}^m - r_t^f, E_t \pi_{t+1}\right]\right]. \end{aligned} \tag{2.4}$$

The second covariance on the right-hand side of (2.4) is negative in any reasonable parameterization of the model: it is implausible that shifts in the marginal transaction cost φ_2 will completely offset the effect of increased inflation on the real return to money (see (1.10)). It follows that expected excess asset returns will covary negatively with expected inflation if and only if β_t^i/β_t^m is positive. Intuitively, β_t^i/β_t^m measures the degree to which the ith asset is a substitute for money in agents' portfolios. Increased expected inflation directly reduces the expected return to money and therefore reduces the equilibrium expected return to any asset which substitutes for money.

Inflation can be decomposed into a component caused by output growth fluctuations and a component caused by money growth fluctuations. The sign of β_t^i/β_t^m depends, in part, on which of these components is quantitatively more important. Consider the case where the ith asset has a positive excess return, so β_t^i is negative (see (2.1)). The ith asset will substitute for money, and therefore will covary negatively with expected inflation, if β_t^m is also negative. Negative values of β_t^m are likely if inflation innovations are caused primarily by shocks to output growth, since the conditional covariance between the real return to money and the intertemporal marginal rate of substitution is likely to be negative in this case. An unexpected increase in output decreases the marginal rate of substitution (since utility is concave) and decreases inflation (via the quantity relation), increasing the real return to money. When inflation innovations are driven primarily by money growth shocks, however, the sign of β_t^m, and therefore of the covariance between expected excess real asset returns and expected inflation, is indeterminate,

[5] If r_{t+1}^i, r_t^f, ρ_{t+1} and all conditioning variables are jointly normal, then β_t^i and β_t^m are constant. However, it is not clear whether this joint normality assumption is consistent with other features of the model.

depending in a complex way on the relative magnitudes of the first and second derivatives of φ.

If the variance of β_t^i/β_t^m is small, equation (2.3) implies that the component of expected inflation covarying more strongly with $E_t r_{t+1}^m - r_t^f$ will also covary more strongly with expected excess real asset returns. Consider first the component of expected inflation generated by real economic fluctuations. Hold μ_t constant, and consider an increase in expected inflation caused by a decrease in expected output. Both expected consumption and expected real balance holdings fall since reduced consumption requires a lower stock of real balances to mediate consumption transactions. With the second derivatives of φ given by equation (1.1), the simultaneous increase in the expected values of consumption and real balances has offsetting effects on $E_t \varphi_2(c_{t+1}, m_{t+1})$. The net effect on the expected value of φ_2 is likely to be small. Therefore, by (1.10), $E_t r_{t+1}^m$ changes by virtually the entire change in $E_t[1/\pi_{t+1}]$.

Now consider the component of expected inflation generated by monetary fluctuations. Hold Y_{t+1} constant, and consider an increase in expected inflation caused by an increase in expected money growth. Agents respond to the higher inflation by reducing their demand for real balances, so $E_t m_{t+1}$ falls. Unlike the preceding case, expected consumption is virtually unchanged, since expected output is held constant. According to (1.1), this reduction in $E_t m_{t+1}$ without a concurrent change in expected consumption decreases $E_t \varphi_2(c_{t+1}, m_{t+1})$. Thus, an increase in expected inflation from a purely monetary impulse simultaneously increases the numerator and denominator of the right-hand side of (1.10), attenuating the effect of the change in $E_t[1/\pi_{t+1}]$ on $E_t r_{t+1}^m$. Intuitively, the increase in expected inflation reduces the expected pecuniary return to money. To keep the total expected return to money competitive with other assets, the agent must increase the expected nonpecuniary return, $-\varphi_2$, by reducing expected real balance holdings. To summarize, $\operatorname{cov}[{}_t r_{t+1}^m - r_t^f, E_t \pi_{t+1}]$, and therefore the covariances between all expected excess returns and expected inflation, is likely to be stronger when inflation fluctuations are generated primarily by output fluctuations rather than by monetary fluctuations.

B. Real Asset Return Innovations and Inflation Innovations

Let r_t^z and r_t^b denote the real returns to equity and nominal bonds:

$$r_t^z \equiv [Y_t p_t^c + p_t^z]/(p_{t-1}^z \pi_t) \quad \text{and} \quad r_t^b \equiv (R_{t-1} + 1)/\pi_t. \tag{2.5}$$

Let ε_{t+1}^b, ε_{t+1}^z, and ε_{t+1}^π denote the innovations to r_{t+1}^b, r_{t+1}^z, and π_{t+1}, respectively. Because the nominal interest rate is predetermined, $\operatorname{cov}(\varepsilon_t^b, \varepsilon_t^\pi) < 0$. The sign of $\operatorname{cov}(\varepsilon_t^z, \varepsilon_t^\pi)$ depends on whether the innovation to inflation is driven primarily by real impulses or by monetary impulses. To take an extreme case in which inflation is driven by real impulses, hold the money growth rate constant. Unless the output process displays extremely negative serial correlation, a positive shock to the growth rate of output causes both a positive innovation to equity returns and a positive innovation to the growth rate in the demand for real balances. Since the growth rate in nominal balances is constant, the inflation rate must fall. Hence, the inflation innovation covaries negatively with the equity return innovation. This, in essence, is the argument put forth in Fama (1981) and Fama and Gibbons (1982).

In contrast, if innovations to the inflation process come primarily from monetary shocks, the sign of $\mathrm{cov}(\varepsilon_t^z, \varepsilon_t^\pi)$ is indeterminate. Suppose output were a constant $\overline{Y}$. Solving (1.7) forward, one obtains

$$\frac{p_{t+1}^z}{p_{t+1}^c} = \overline{Y} \cdot E_{t+1} \sum_{j=1}^{\infty}\Big[\prod_{i=1}^{j} \rho_{t+i+1}\Big];$$

ε_{t+1}^z is then given by

$$\varepsilon_{t+1}^z = \frac{\overline{Y}}{(p_t^z/p_t^c)}[E_{t+1} - E_t]\sum_{j=1}^{\infty}\Big[\prod_{i=1}^{j} \rho_{t+i+1}\Big].$$

The innovation to the real equity return is determined by the way new information shifts the conditional joint distribution of future values of the intertemporal marginal rate of substitution. The sign of $\mathrm{cov}(\varepsilon_t^z, \varepsilon_t^\pi)$ therefore depends, in a highly nonlinear way, on the serial correlation properties of money growth. If the money growth process induces positive serial correlation in inflation, $\mathrm{cov}(\varepsilon_t^z, \varepsilon_t^\pi)$ is likely to be positive: The positive inflation shock increases expected future inflation, which increases current demand for equities as individuals substitute away from money. This in turn could imply a positive correlation between the ex post equity return and the component of inflation generated by monetary fluctuations.

III. Estimation of the Model with Nonstationary Money Growth

A. A Tractable Model Parameterization with Nonstationary Money Growth

The preceding arguments demonstrate some intuitive properties of the model. I now turn to estimation of the model. Most previous empirical studies of monetary equilibria assume that the money growth rate is stationary. Table II presents evidence that the money growth rate displays a small but significant trend. The first row of Table II gives the Generalized Method of Moments (GMM) estimate (with standard errors) of the coefficient on time in a regression of $\log(\mu_t)$ on a constant and time trend.[6] The coefficient on the time trend is significant at the 0.7% level.

[6] The data set is described in Section III.*B*. I follow Stock and Watson's (1989) procedure to test the number of unit roots in the logarithm of the money stock, after linear and quadratic time trends are removed. I find no evidence against the hypothesis that $\log(M_t)$ is integrated of order 1. The presence of a second unit root is strongly rejected. These results imply that $\log(\mu_t)$ is stationary around a linear time trend.

Table II

Estimates of Exponential Time Trends

The data are quarterly observations from 1959:1 to 1990:2. All price and quantity data are from the CITIBASE database (in the following, CITIBASE acronyms are in parentheses), and all returns data are from the Center for Research on Security Prices (CRSP) database. "Consumption Growth Rate" is the growth rate of real quarterly expenditures on nondurables (GCN82) and services (GCS82). The price level is the implicit deflator for consumption. "Inflation" is the growth rate of the price level. "Real Equity Return" is the one-month value-weighted return on the New York Stock Exchange from the CRSP Stock File, compounded to a quarterly return and deflated by the inflation rate. "Nominal Interest Rate" is the yield to maturity of the 90-day Treasury Bill (computed as the average of the bid yield and the ask yield) from the CRSP Bond File. "Real Bond Return" is the nominal interest rate deflated by the inflation rate. "Money Growth Rate" is the rate of growth of the money supply as measured by M1 (FM1), scaled by the ratio of consumption to the gross national product (GNP82) in each period. "Velocity" is the ratio of nominal consumption to the money supply. All quantity variables are divided by the resident U.S. population (POPRES). "Trend" is the coefficient on time in the regression of the log of the indicated variable on time and a constant. Estimates and standard errors were obtained using the GMM procedure of Eichenbaum and Hansen (1990) assuming sixth-order serial correlation in the residuals.

Variable	Trend	Standard Error
Money growth rate	0.000142*	(0.000052)
Consumption growth rate	−0.000021	(0.000015)
Velocity	0.005705*	(0.000581)
Nominal interest rate	0.000127*	(0.000027)
Inflation	0.000083*	(0.000031)
Real bond return	0.000046	(0.000026)
Real equity return	0.000125	(0.000152)

* Trend is significantly different from zero at 1% significance level.

Stock and Watson (1989) argue that errors in economic inference can result if this trend in μ_t is ignored. Furthermore, Hansen's (1982) conditions for consistency and asymptotic normality of the GMM estimator will be violated if money growth is misspecified as a stationary process. Unfortunately, estimation of structural models with nonstationary exogenous processes usually presents insoluble technical problems because no internally consistent method can be derived for detrending the endogenous variables. One approach is to assume that the endogenous processes inherit a geometric trend from the exogenous process, and then to geometrically detrend both exogenous and endogenous variables. This approach can be justified rigorously only for a narrow class of models. (See Eichenbaum and Hansen (1990).) While the model of this paper does not fall into that class, it can be shown that this approach is valid asymptotically under the following four assumptions.

Assumption 3.1: $U(c) = log(c)$.

Assumption 3.2: *The process* Y_t/Y_{t-1} *is stationary and ergodic.*

I treat observed consumption as the empirical analogue to the endowment process Y_t.[7] Table II displays evidence that observed consumption growth does not exhibit a significant time trend.

Assumption 3.3: $\mu_t = \mu_t^* \theta^t$, *where* μ_t^* *is a stationary and ergodic stochastic process and* $\theta \geq 1$ *is a geometric time trend.*

Assumptions 3.2 and 3.3 imply that the inflation rate trends upward without bound. This would induce individuals to hold ever smaller stocks of real balances, implying that transaction costs would command an ever-increasing proportion of the endowment. To avoid this undesirable implication I assume that the transaction technology improves over time, allowing agents to economize on their money holdings. The transaction cost function is assumed to be a time-dependent Cobb-Douglas specification, as follows[8]:

Assumption 3.4: $\varphi(c_t, m_t, t) = (\gamma/\alpha) c_t^{\alpha} (\theta^t m_t)^{1-\alpha}$, $\gamma > 0$, $\alpha > 1, \theta \geq 1$.

In Assumption 3.4, the technical rate of growth in the efficiency with which real balances mediate transactions is assumed to equal θ, the trend in the money growth rate. This insures that the trend in money growth has no real effects.

In estimating the model, I do not impose all of the overidentifying restrictions. Rather, I examine how the model performs when the parameters of the transaction cost function are chosen to imply plausible money demand behavior. I therefore estimate the model using only the restrictions implied by the generalized money demand relation (1.12). Denoting velocity by $V_t \equiv c_t/m_t$, assumptions 3.1 to 3.4 and equation (1.12) imply:

$$E_t \left\{ \frac{1}{\mu_{t+1}} \left(\frac{R_t - \theta^{(1-\alpha)(t+1)}(\gamma/\alpha)(\alpha - 1)V_{t+1}^{\alpha}}{V_{t+1} + \theta^{(1-\alpha)(t+1)} \gamma V_{t+1}^{\alpha}} \right) \right\} = 0. \tag{3.1}$$

Let $V_t^* \equiv V_t/\theta^t$ and $R_t^* \equiv R_t/\theta^t$. Equation (3.1) can be rewritten

$$E_t \{ u_{t+1}^* / \theta^{t+1} \} = 0, \tag{3.2}$$

where the process u_t^* is defined by

$$u_t^* \equiv \frac{1}{\mu_t^*} \left(\frac{(R_{t-1}^*/\theta) - (\gamma/\alpha)(\alpha - 1)V_t^{*\alpha}}{V_t^* + \gamma V_t^{*\alpha}} \right). \tag{3.3}$$

It can be proved that, under assumptions 3.1–3.4 and two additional technical conditions, V_t^*, R_t^*, and π_t/θ^t all converge to stationary stochastic processes.[9] In other words, as $t \to \infty$, V_t, R_t, and π_t (and trivially μ_t) all

[7] In doing so, I assume that measured consumption equals consumption net of transaction costs plus the costs of transaction services used in purchasing these consumption goods. These services would include, for example, credit processing, banking services, and a portion of transportation expenditures.

[8] Note that with this transaction cost function the economy converges to a cash-in-advance economy as $\alpha \to \infty$.

[9] The proof is in a technical appendix available from me upon request.

converge to a limiting stochastic growth path along which the variables grow at the same geometric rate θ. I estimate the model under the assumption that the observed data are taken from an economy which has already converged to that limiting growth path. Under this assumption, u_t^* is a stationary stochastic process.

I obtain a set of unconditional moment restrictions by multiplying equation (3.2) by a vector of instrumental variables known at date t and then applying the law of iterated expectations. The instruments must grow at geometric rate θ to insure that the resulting moment conditions are stationary. I use the vector $\{\theta^t, R_t, \mu_t\}$, which implies the three moment conditions

$$E\left\{\frac{u_{t+1}^*}{\theta} \otimes \begin{pmatrix} 1 \\ R_t^* \\ \mu_t^* \end{pmatrix}\right\} = 0. \tag{3.4}$$

The maintained assumptions also imply that V_t, R_t, and μ_t share the same geometric time trend θ. This implies the moment conditions[10]

$$E\left(w_t^* \otimes \begin{pmatrix} 1 \\ t/T \end{pmatrix}\right) = 0. \tag{3.5}$$

where

$$w_t^* \equiv \begin{pmatrix} \log V_t - \bar{V}^* - t \cdot \log\theta \\ \log R_t - \bar{R}^* - t \cdot \log\theta \\ \log \mu_t - \bar{\mu}^* - t \cdot \log\theta \end{pmatrix}, \qquad T \equiv \text{sample size},$$

and

$$\bar{V}^* \equiv E(\log V_t^*), \qquad \bar{R}^* \equiv E(\log R_t^*), \qquad \bar{\mu}^* \equiv E(\log \mu_t^*).$$

By stacking equations (3.4) and (3.5), I obtain a system of nine moment conditions which is formally equivalent to the system studied in Eichenbaum and Hansen (1990). I estimate this system using their GMM procedure and refer the reader to their paper for a discussion of the consistency and asymptotic normality of this estimator.[11]

[10] Moment conditions (3.5) are equivalent to the normal equations from an ordinary least squares regression of $[\log(V_t)\ \log(R_t)\ \log(\mu_t)]'$ on a constant and a time trend, subject to the restriction that the coefficient on time is equal across the three equations.

[11] The model implies that the residual from condition (3.4) is serially uncorrelated. The lag structure of the residual from condition (3.5) is not specified by the model. In constructing the GMM weighting matrix, I assume that this residual follows a sixth-order moving average process. The weighting matrix used is the inverse of a Newey-West (1987) estimate of the covariance matrix of the residuals of the complete system of nine moment conditions, which incorporates the assumed moving average structure. See Eichenbaum and Hansen (1990, equation [4.19] and following) for a fuller discussion of the construction of the GMM criterion function.

B. Data

I use quarterly data from 1959:1 to 1990:2. The money supply is measured by M1; a quarterly series is constructed by sampling the first month of each quarter. The total money stock does not correspond to the money stock in the model because, in the model, money is held only to facilitate consumption transactions. Therefore, the measure of money appropriate for this empirical exercise is that portion of the total observed money stock used for consumption purchases. Since this object is not observed, I construct a proxy, which is used throughout the paper, by multiplying the total money stock by the ratio of consumption to the gross national product in each period.

To measure consumption, I use quarterly expenditures on nondurables and services from the National Income and Product Accounts. I assume that this measure equals the sum of consumption net of transaction services ("c" in the notation of the model) plus the transaction services used in purchasing consumption goods ("φ" in the model). Under this assumption, observed consumption is the observable analogue to the endowment process of the model.

To measure asset returns I use the Center for Research on Security Prices (CRSP) database. The gross nominal bond return is the inverse of the price of a Treasury bill that pays $1 with certainty three months later. The nominal equity return is the one-month CRSP value-weighted return series, compounded to a quarterly return.[12] The price level is the implicit deflator for consumption. All quantity variables are expressed in per capita terms by dividing by a measure of the resident U.S. population from the CITIBASE database.[12]

C. Estimation of Model Parameters

I set the quarterly discount rate δ equal to 0.99.[13] Table III displays estimates of the remaining three parameters $\{\gamma, \alpha, \theta\}$. The estimate of γ, the parameter which scales the transaction cost function, is rather small, implying that, on average, only 0.8% of total output is expended in transaction costs. This is a desirable property: the model would be suspect if transaction costs accounted for a sizable fraction of economic activity. The estimate for α, however, is close to 3, implying enough curvature in φ for marginal transaction costs to be nonnegligible.

The estimate of θ, the transaction technology growth parameter, is 1.000156, over three standard errors above unity. This confirms the rejection of money-growth stationarity reported in Table II. The estimate of θ is within one standard error of the estimated trend in μ_t and R_t (see Table II), but is significantly lower than the estimated trend in V_t. This discrepancy causes

[12] The actual stock and bond returns in the CRSP tapes are measured on the last business day of each month. I treat them as if they were measured on the first day of the subsequent month.

[13] The discount parameter is similarly fixed in Sargent (1978); Kydland and Prescott (1982); and Christiano and Eichenbaum (1987).

Table III

Estimates of Parameters

The parameters of the transaction cost function, $\varphi(c_t, m_t, t) = (\gamma/\alpha)c_t^{\alpha}(\theta^t m_t)^{1-\alpha}$, (where c_t denotes consumption and m_t denotes real balances) are estimated using the Generalized Method of Moments (GMM) procedure of Eichenbaum and Hansen (1990). The data are quarterly observations from 1959:1 to 1990:2. All quantity data are from the CITIBASE database (in the following, CITIBASE acronyms are in parentheses): Consumption is real quarterly expenditures on nondurables (GCN82) and services (GCS82); the money supply is M1 (FM1), scaled by the ratio of consumption to the gross national product (GNP82) in each period; real balances are the money supply divided by the implicit deflator for consumption. All quantity variables are divided by the resident U.S. population (POPRES). The nominal interest rate, denoted R_t, is the yield to maturity of the 90-day Treasury Bill (computed as the average of the bid yield and the ask yield) from the Center for Research on Security Prices Bond File. Asymptotic standard errors are in parentheses below the point estimates. J_T is the minimized GMM criterion function, which is asymptotically distributed χ^2 with three degrees of freedom. The marginal significance level of J_T is in parentheses below the point value.

Parameter	Point Estimate
γ	9.23×10^{-3}
	(8.61×10^{-4})
α	2.79
	(0.152)
θ	1.000156
	(0.0000428)
J_T	8.42
	(0.038)

the overidentifying restrictions incorporated in (3.4) and (3.5) to be rejected at the 5% significance level. (When the equations involving V_t are eliminated from system (3.5), the model is not rejected at conventional significance levels.). While this evidence is not overwhelming, it suggests that the model cannot explain the disparity between the trends in money growth and velocity.

In the next section, I examine the behavior of the model when it is evaluated at the estimated parameter values. I do so by simulating the model and comparing the behavior of the simulated time series with the behavior of observed time series.

IV. Comparison of the Model with the Data

A. Simulation Methodology

I assume that $(\log(Y_t/Y_{t-1}), \log(\mu_t))$ follows a first-order vector autoregression (VAR(1)), with a constant and a linear time trend.[14] I test for the presence of autoregressive conditional heteroskedasticity (ARCH) in the

[14] The Akaike Information Criterion, the Schwartz Criterion, and the sequential likelihood ratio tests (which test VAR(n) against VAR($n - 1$) for $n = 2, \ldots, 5$) all support the VAR(1) specification.

residuals from this VAR(1) by constructing a Ljung-Box statistic using correlations in squared residuals. There is no evidence of ARCH in the residuals of the Y_t/Y_{t-1} equation. There is some evidence of ARCH in the money growth equation. Specifically, when fewer than ten correlations are used to construct the test statistic, conditional homoskedasticity is rejected at the 5% marginal significance level. (When ten or more correlations are used, homoskedasticity is not rejected.) Since the focus of this paper is not on the effects of time-varying money growth volatility, I simulate the model under the assumption of conditional homoskedasticity. To incorporate ARCH effects would substantially increase the complexity of the model. However, the implications of ARCH in the money growth process ought to be a focus for future research.

I simulate the estimated process for $(Y_t/Y_{t-1}, \mu_t)$ 100 times by sampling randomly from the estimated residuals. For each simulation, I solve the model at the estimated parameter values using Marcet's (1991) method of parameterized expectations, modified to accommodate nonstationarity. (A brief description of the solution algorithm can be found in the Appendix.) Taylor and Uhlig (1990) provide evidence that this method is more accurate than many other commonly used approximation methods, such as linear-quadratic approximations or discrete state-space methods. Table IV provides evidence that the parameterized expectations algorithm generates highly accurate solutions for the model of this paper. According to the tests reported in that table, the Euler equation residuals implied by my numerical solutions are statistically indistinguishable from martingale difference sequences, as required by the theory.

Table IV

DHM(k, j) Statistics Testing the Predictability of Euler Equation Residuals in the Simulated Data

DHM(k, j) denotes the den Haan-Marcet (1989) statistic, testing whether the vector of Euler-equation residuals in the simulated data is uncorrelated with the jth lagged vector of exogenous variables raised to the kth power. The statistic is constructed as follows: Let η_t denote a 2×1 vector whose elements are the residuals from equations (1.6) and (1.7). Let $h_t^{k,j} \equiv [(Y_{t-j}/Y_{t-j-1})^k, \mu_{t-j}^k]'$. DHM($k, j$) $\equiv TB_T A_T^{-1} B_T'$, where $B_T \equiv 1/T\Sigma_t \eta_t \otimes h_t^{k,j}$, $A_T \equiv 1/T\Sigma_t[\eta_t \otimes h_t^{k,j}] \cdot [\eta_t \otimes h_t^{k,j}]'$, and $T \equiv$ sample size. DHM(k, j) is asymptotically $\chi^2(4)$ under the hypothesis that η_t is a martingale difference sequence. The test was performed for each of the 100 simulations. The reported statistics are averages across the 100 simulations. The average marginal significance levels (over the 100 simulations) are in parentheses.

	Test Statistic When jth-Lagged Exogenous Vector is Raised to Power (k)			
Order of Lag (j)	1	2	3	4
1	2.31	1.47	6.84	2.03
	(0.68)	(0.83)	(0.16)	(0.73)
2	3.80	3.52	3.77	4.02
	(0.45)	(0.49)	(0.45)	(0.42)
3	3.08	4.97	3.83	5.81
	(0.55)	(0.30)	(0.44)	(0.23)

B. Comparison of Asset Return-Inflation Co-Movements in the Model and the Data

In this section I address the questions posed in the introduction: Can the model replicate, in sign and magnitude, the observed patterns of co-movements between inflation and real asset returns? Do these patterns depend on whether inflation fluctuations are driven primarily by fluctuations in real economic impulses or by fluctuations in the money supply? I investigate these issues by comparing unconditional moments and impulse response patterns in the simulated time series with those in the observed data. Table V displays means, variances, and correlation coefficients of the time series generated by simulating the model. To induce stationarity, a geometric time trend is removed from the inflation rate, the money growth rate, and velocity.[15] Figure 1 displays the response of the real equity return, r_t^z, to a one-standard-deviation shock to the inflation rate, π_t, as implied by a bivariate vector autoregression including $[\log(\pi_t), \log(r_t^z)]$. Figure 2 displays the response of r_t^z to a one-standard-deviation shock in μ_t, as implied by a trivariate vector autoregression including $[\log(\mu_t), \log(Y_t/Y_{t-1}), \log(r_t^z)]$.[16]

B.1. Equity Return-Inflation Co-Movements

The model generates negative correlations between inflation and real equity returns of approximately the magnitude observed in the data. In Table I, the estimated correlation between ex post real equity returns and detrended inflation in the data is -0.28, almost three standard errors below zero. In Table V, the corresponding correlation in the model simulation is -0.26, within one standard error of the estimated value.

Consistent with the discussion in Section II.*A*, the correlation in the simulated data between the expected real equity return and expected inflation is negative. Panel C of Table V reports a value close to -1. The impulse response patterns in Figure 1 give a similar picture. For both the observed data and the data simulated from the model, a positive impulse to the inflation rate implies a negative response in the real equity return. In the observed data, this negative response is fairly persistent; in the model, the response dissipates in about one year. This discrepancy is not surprising since there are no endogenous sources of persistence (such as adjustment costs or temporally dependent preferences) in the model.

According to the analysis in Section II.*A*, expected real asset returns and expected inflation should covary more strongly when inflation fluctuations are caused by real economic fluctuations than when they are caused by monetary fluctuations. According to the analysis in Section II.*B*, the correla-

[15] Each nonstationary data series is detrended using a geometric trend individually estimated from that series; so, in computing the stationary component, I do not impose the condition that all trends in these endogenous variables equal the parameter θ.

[16] Figures 1 and 2 use first-order VARs. The order of orthogonalization for Figure 1 is $\{\pi_t, r_t^z\}$, and for Figure 2 is $\{\mu_t, Y_t/Y_{t-1}, r_t^z\}$. Reversing the order of the first two variables did not affect the qualitative results.

Table V

Statistical Properties of Data Simulated from the Model

The equilibrium asset pricing model is simulated 100 times. Logarithmic utility is assumed. The transaction cost function is

$$\varphi(c_t, m_t, t) = (0.00923/2.79)(c_t^{2.79})(1.000156^t m_t)^{(1-2.79)},$$

where c_t denotes consumption and m_t denotes real balances. The point values reported are the sample averages over the 100 simulations. Sample standard errors, reported in parentheses, are computed as the square root of the sample variance of the point values over the 100 simulations. In the model, the variables are defined as follows: "Real Equity Return" is the real quarterly return to holding a claim to the aggregate endowment; "Real Bond Return" is the real return to a nominally riskless three-month bond; "Inflation" is the growth rate of the price of consumption; "Money Growth" is the growth rate of the exogenously determined money supply; "Velocity" is the ratio of consumption to real balances. "Trend" denotes the coefficient on time in the regression of the log of the series on time and a constant. Detrended variables are residuals after a geometric trend is removed. The variance of each series is reported along the diagonal of the cross-correlation matrix displayed in panel B. In panel C the expected variables are the conditional expectations of date t variables given date $t-1$ information.

Panel A: Means and Trends

Variable	Mean	Trend	Mean of Detrended Variable
Real equity return	0.01391	−0.000021	
	(0.00019)	(0.000000)	
Real bond return	0.01407	−0.000029	
	(0.00024)	(0.000008)	
Inflation		0.000155	−0.00035
		(0.000007)	(0.00037)
Velocity		0.002150	1.438
		(0.000154)	(0.014)

Table V—*Continued*

Panel B: Cross-Correlation Matrix

Variable	Real Equity Return	Real Bond Return	Detrended Inflation	Detrended Money Growth	Detrended Velocity
Real equity return	0.282×10^{-4}	0.2683	−0.2634	0.1936	−0.3580
	(0.572×10^{-7})	(0.0150)	(0.0145)	(0.0012)	(0.1470)
Real bond return		0.204×10^{-3}	−0.9970	−0.8825	−0.2930
		(0.879×10^{-5})	(0.0013)	(0.0092)	(0.0779)
Detrended inflation			0.199×10^{-3}	0.8843	0.3018
			(0.812×10^{-5})	(0.0089)	(0.0684)
Detrended money growth				0.192×10^{-3}	0.1468
				(0.345×10^{-4})	(0.0683)
Detrended velocity					0.169×10^{-3}
					(0.126×10^{-3})

Panel C: Cross-Correlation Coefficients between Expected Inflation and Expected Real Returns in the Model Simulations

Correlation Between	Correlation Coefficient
Expected real equity return and expected detrended inflation	−0.8641
	(0.0345)
Expected real bond return and expected detrended inflation	−0.8634
	(0.0405)

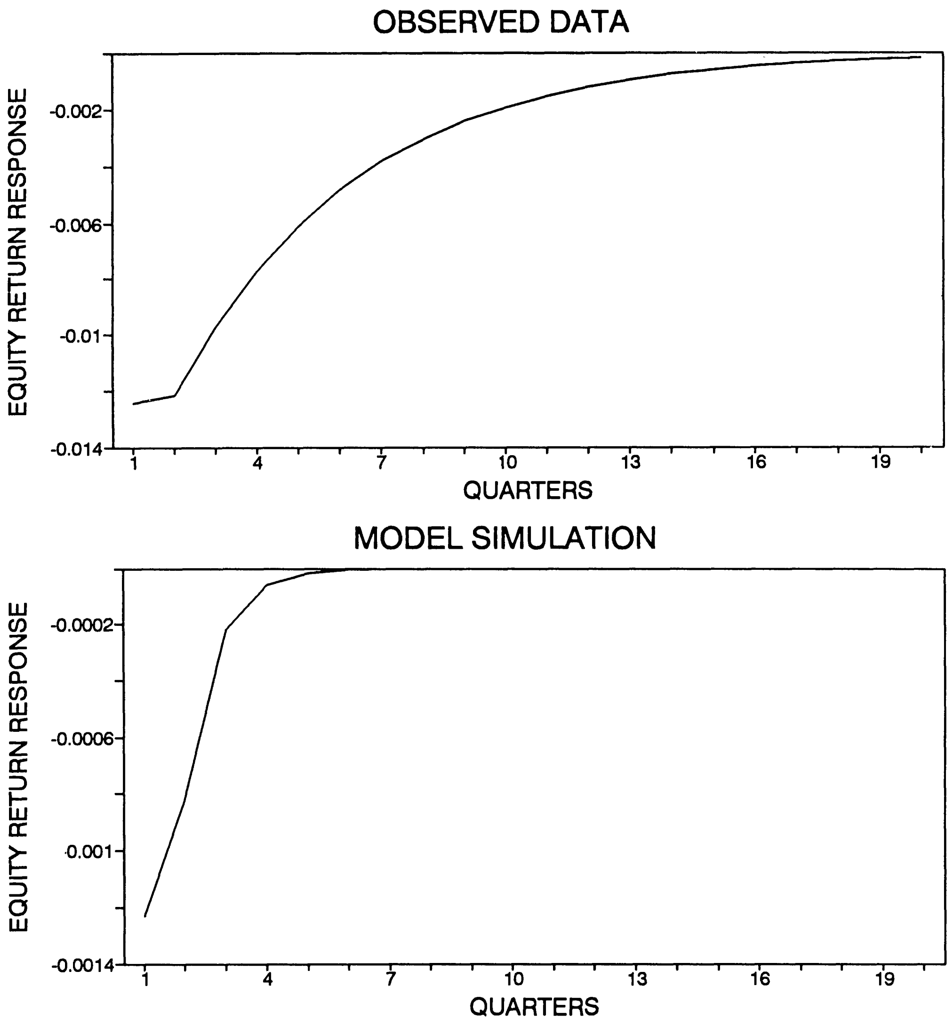

Figure 1. Response of real equity return to one-standard-deviation inflation impulse. The upper panel plots the impulse response over a 20-quarter horizon in observed data, as implied by a logarithmic VAR(1) including inflation, the real equity return, a constant, and a time trend. The lower panel plots the corresponding impulse response in data simulated from the model.

tion between ex post real stock returns and inflation should be negative when inflation is driven by real fluctuations but has an indeterminate sign in the case of monetary fluctuations. To examine these issues, I re-solve the model (at the estimated parameters) for the case of real fluctuations by fixing μ_t at its trend value. I then repeat the exercise for monetary fluctuations by holding endowment growth constant at its mean value. I report the average correlations across 50 simulations in Table VI.

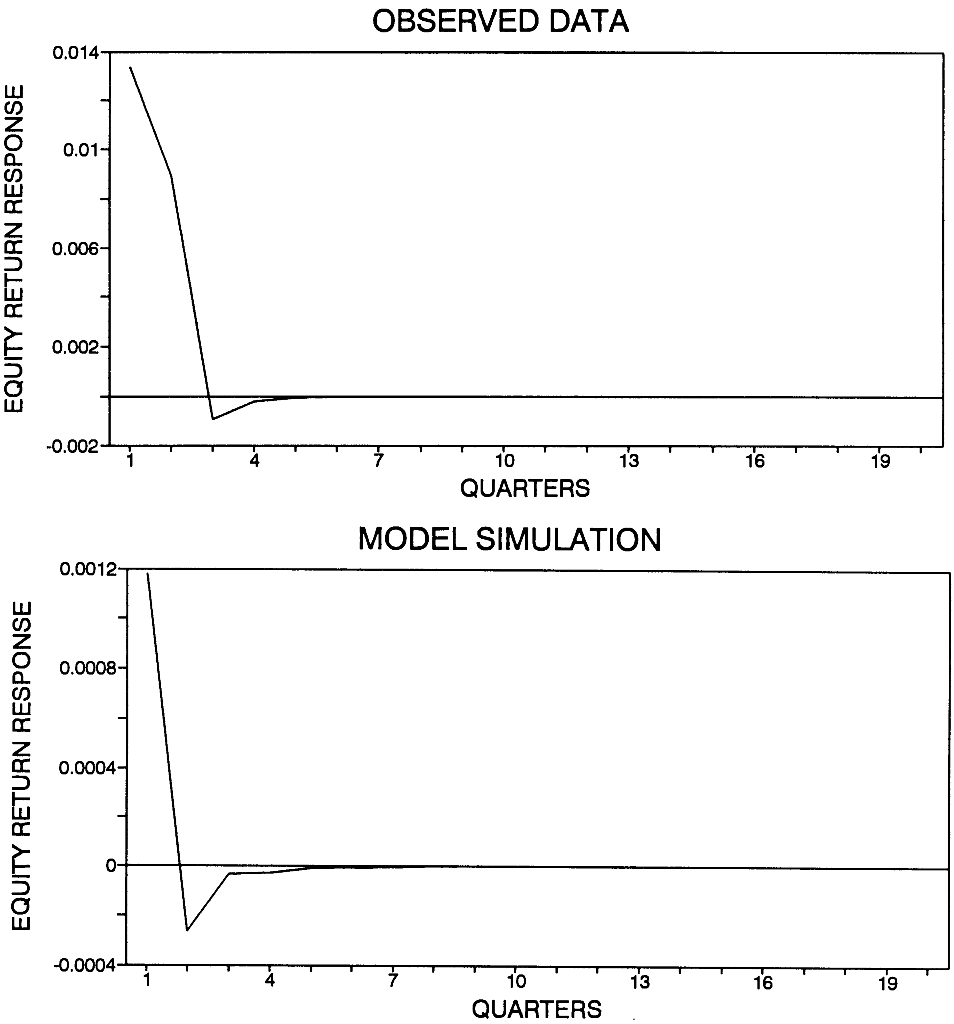

Figure 2. Response of real equity return to one-standard-deviation money-growth impulse. The upper panel plots the impulse response in observed data over a 20-quarter horizon, as implied by a logarithmic VAR(1) including money growth, endowment growth, the real equity return, a constant, and a time trend. The lower panel plots the corresponding impulse response in data simulated from the model.

These results correspond qualitatively to the intuition developed in Section II. When inflation fluctuations are generated by real impulses (column (A) of Table VI), the correlation between the ex ante real equity return and expected inflation is close to -1. When inflation fluctuations are generated by monetary impulses (column (B) of Table VI), the correlation between the ex ante real equity return and expected inflation, while negative, is small in magnitude with a fairly large sample standard error. The correlation between

Table VI

Correlation between Real Asset Returns and Inflation Implied by the Model When One Exogenous Variable Is Nonstochastic

The equilibrium asset pricing model is simulated 50 times. Logarithmic utility is assumed. The transaction cost function is

$$\varphi(c_t, m_t, t) = (0.00923/2.79)(c_t^{2.79})(1.000156^t m_t)^{(1-2.79)},$$

where c_t denotes consumption and m_t denotes real balances. The point values reported are the sample averages over the 50 simulations. Sample standard errors, reported in parentheses, are computed as the square root of the sample variance of the point values over the 50 simulations. In column (A), the money growth rate is fixed at its trend path $E(\mu^*)\eta^t$, where $\eta = 1.000142$ (the estimated geometric trend in the growth rate of the stock of money used for consumption transactions, defined as M1 scaled by the ratio of consumption expenditures on nondurables and services to gross national product) and $E(\mu^*) = 1.00171$ (the sample mean of the growth rate of the stock of money used for consumption transactions after a geometric trend is removed). In column (B), the endowment growth rate is held constant at 1.00369, the sample mean of the growth rate of quarterly consumption expenditures on nondurables and services. In the model, the variables are defined as follows: "Real Equity Return" is the real quarterly return to holding a claim to the aggregate endowment; "Real Bond Return" is the real return to a nominally riskless three-month bond; "Inflation" is the growth rate of the price of consumption. "Detrended Inflation" is the residual after a geometric trend is removed from inflation. The expected variables are the conditional expectations of date t variables given date $t-1$ information.

Correlation Between	(A) Money Growth Equals Its Trend Value	(B) Endowment Growth Equals Its Mean Value
Real equity return and detrended inflation	−0.9814	+0.7669
	(0.000008)	(0.0631)
Expected real equity return and expected detrended inflation	−0.9725	−0.1984
	(0.0921)	(0.0932)
Real bond return and detrended inflation	−0.9820	−0.9936
	(0.000029)	(0.0020)
Expected real bond return and expected detrended inflation	−0.9737	−0.4021
	(0.0088)	(0.1327)

the ex post equity return and realized inflation is negative when inflation is generated by real impulses. However, in the case of purely monetary inflation impulses, this correlation is highly positive.

These results suggest that the negative correlation observed between ex post equity returns and inflation are due to the component of inflation generated by real economic shocks. This intuition reconciles negative correlations between real equity returns and inflation with the observed positive correlation between real equity returns and money growth. Notice that the co-movement of the real equity return and money growth in the simulated data is similar to that observed in the data. Table V reports that the correlation between these two variables in the model is 0.19, within one standard deviation of the estimated value of 0.13 reported in Table I. The response in the model of the real equity return to a monetary shock (displayed in Figure 2) matches the qualitative pattern found in the data. The initial positive response is followed by an abrupt drop, with a return to zero in about one year.

B.2. Real Bond Return-Inflation Co-Movements

Table V, panel C, reports that the correlation between expected inflation and the ex ante real bond return in the simulated data is highly negative. Clearly, the model is non-Fisherian. As with equity returns, this correlation is smaller in magnitude with monetary inflation impulses than with real inflation impulses (see Table VI).

The model predicts that the correlation between the ex post real bond return, r_t^b, and detrended inflation is -1, a much stronger correlation than the estimated value of -0.69. To investigate this discrepancy between the model and the data, it is convenient to express the discrepancy in terms of covariances: The model predicts that $\operatorname{cov}(r_t^b, \pi_t) = -0.201 \times 10^{-3}$, while this covariance in the data is only -0.307×10^{-4}. The covariance can be decomposed as follows:

$$\operatorname{cov}(r_t^b, \pi_t) \approx \operatorname{cov}(E_{t-1} r_t^b, E_{t-1}\pi_t) - \operatorname{var}(\pi_t - E_{t-1}\pi_t). \tag{4.1}$$

According to (4.1), there are only two possible explanations for the excessively negative value of $\operatorname{cov}(r_t^b, \pi_t)$ in the model: either the first term on the right-hand side of (4.1) is excessively negative (i.e., the model is too non-Fisherian), or the variance of the inflation innovation is too high.

I argue that the first of these explanations is not credible. While $\operatorname{cov}(E_{t-1} r_t^b, E_{t-1}\pi_t)$ cannot be observed directly, an upper bound on this covariance can be estimated. Let $\mathscr{R}^2$ denote the population R^2 of some arbitrary regression of π_t on variables dated $t-1$ and earlier. $\mathscr{R}^2$ is bounded above by

$$\mathscr{R}^2 \leq \frac{\operatorname{var}(E_{t-1}\pi_t)}{\operatorname{var}(\pi_t)} = 1 - \frac{\operatorname{var}(\pi_t - E_{t-1}\pi_t)}{\operatorname{var}(\pi_t)}. \tag{4.2}$$

Substituting (4.1) into (4.2) and rearranging, one obtains

$$\operatorname{cov}(E_{t-1} r_t^b, E_{t-1}\pi_t) \leq \operatorname{var}(\pi_t)[1 - \mathscr{R}^2] + \operatorname{cov}(r_t^b, \pi_t). \tag{4.3}$$

The regression of π_t on a constant and three own lags has an R^2 of 0.66. Substituting this value for $\mathscr{R}^2$ in the right-hand side of (4.3), and using the values of $\operatorname{var}(\pi)$ and $\operatorname{cov}(r^b, \pi)$ implied by Table I, one obtains an upper bound for $\operatorname{cov}(E_{t-1} r_t^b, E_{t-1}\pi_t)$ of -0.161×10^{-4}. Since the value implied by the model for this covariance is only -0.208×10^{-5}, I conclude that the model underpredicts the degree of negative covariation between $E_{t-1}\pi_t$ and $E_{t-1} r_t^b$.

It follows that the excessively negative covariation between inflation and the real bond return predicted by the model is due to the second term on the right-hand side of (4.1). That is, the model implies a counterfactually strong negative correlation between r_t^b and π_t because it implies a counterfactually high variance for the inflation prediction error. I directly test the predictability of inflation. I compute the one-step-ahead prediction errors for the last 60 values of inflation using a regression which includes a constant, a time trend, and three lags each of inflation, consumption growth, and money growth. (Each prediction used the fitted value of a regression which only include past observations of the regressors.) When this exercise is performed using the actual data, the root-mean-squared prediction error is 4.6×10^{-3}. When the exercise is repeated using the 100 simulated data sets, the average root-mean-squared prediction error increases to 1.7×10^{-2} (with a sample standard error of 4.9×10^{-4}). According to these results, the model cannot replicate the predictability of observed inflation.

C. Other Discrepancies Between the Model and the Data

The model is unable to match the observed variance of real asset returns. While the mean of the real equity return in the simulated data is quite close to that in observed data, the variance is over two orders of magnitude smaller. Similarly, the response of the real equity return to inflation or monetary disturbances, reported in Figures 1 and 2, is about ten times smaller than the corresponding response in the observed data. In contrast, the variance of the real bond return is about five times higher in the model than in the data.

The counterfactually low variance of the real equity return is endemic to nonmonetary models with time- and state-separable preferences (see Hansen and Jagannathan (1991)), so it is not surprising that a monetary model with these preferences shares the same difficulty. It remains to be seen whether monetary models incorporating nonstandard preferences (such as those explored by Giovannini and Weil (1989); Constantinides (1990); Weil (1990); and Epstein and Zin (1991)) have more realistic implications for equity-return volatility. The counterfactually high variance of the ex post bond return in the model is due to the counterfactually high variance in the inflation innovation, discussed above in Section IV.*B*.2.

The contemporaneous correlation between detrended money growth and detrended inflation in the model is around 0.8. In the data, however, it is only 0.097 (with a standard error of 0.091). Evidently, the model is not equipped to capture the "long and variable lags" with which changes in money growth affect inflation. Finally, while this model avoids the counterfactual implication of cash-in-advance models that velocity is (virtually) constant, the variance of velocity implied by the model is well below that found in the data. Thus, while the model gives a better account of money demand variability than the cash-in-advance models commonly used, it provides at best an incomplete explanation for money demand fluctuations.

D. The Importance of the Trend in Money Growth for Economic Inference

In spite of the statistical evidence supporting a time trend in μ_t, the money growth rate is generally modeled in the literature as stationary. To see whether the results of this paper change when μ_t is assumed to be stationary, I estimated and simulated the model under the restriction that $\theta \equiv 1$.[17] The correlations between inflation and asset returns in this restricted model are similar to those in the unrestricted model. However, the impulse responses in the restricted model appear much less smooth than the corresponding response patterns in either the actual data or in the data simulated from the unrestricted model. This suggests that future work with equilibrium monetary models which look at dynamic patterns in greater detail may find results sensitive to the way the trend in μ_t is modeled.

V. Conclusion

This paper offers evidence that substantial negative correlations between real asset returns and inflation do not constitute evidence of money illusion or market inefficiency since correlations of at least the observed magnitude are implied by this model, which includes no disequilibrium elements, irrationality, or market frictions (beyond the implicit frictions needed to generate a demand for money in the first place). In particular, the Fisher hypothesis does not generally describe the implications of dynamic economic equilibria when the transaction role of money is explicitly taken into consideration.

The model is consistent with both a negative response of equity returns to inflation shocks and a positive response of equity returns to monetary shocks because the main source of fluctuations in inflation is fluctuations in real economic activity. Negative correlations between ex ante equity returns and expected inflation do not represent a puzzle in this model: expected inflation covaries negatively with the ex ante real return to money and, therefore, with the ex ante return to any asset which is a partial substitute for money. This effect, however, is small in the case of monetary inflation fluctuations.

The following issues for future research into monetary equilibria are suggested: (1) the observed persistence in inflation; (2) the lags with which monetary fluctuations affect inflation; (3) reconciling the low variance of consumption growth with the high variance of real equity returns; and (4) the high variability of money demand, as measured by the variance of velocity.

Appendix: Description of Solution Procedure

This appendix describes the method of parameterized expectations, which I use to simulate the model in Section IV. This approach was first proposed in Marcet (1991).

[17] This restricted model was estimated using only system (3.4) since, under the restriction $\theta = 1$, the instrument (t/T) renders (3.5) nonstationary. The restricted parameter estimates were $\gamma = 0.01085$ and $\alpha = 2.50$. The minimized GMM criterion function, which is asymptotically distributed $\chi^2(1)$, was 7.86, rejecting the restriction $\theta = 1$ at the 0.5% marginal significance level. The exogenous process $(Y_t/Y_{t-1}, \mu_t)$ was assumed to be a VAR(3) in logs, without time trend. (The Akaike Information Criterion and the sequential likelihood ratio tests support this specification.) The Ljung-Box statistic revealed no evidence of conditional heteroskedasticity in the residuals.

If the law of motion for $(Y_t/Y_{t-1}, \mu_t)$ has L lags, equilibrium conditions (1.6) and (1.7) can be written as

$$\Psi(x_t, y_t) = E(\Gamma(x_{t+1}, y_{t+1}) | x_t, t), \tag{A1}$$

where $x_t = (Y_{t-s}/Y_{t-s-1}, \mu_{t-s}, s = 0, 1, \cdots, L-1), y_t = (c_t, p_t^c, p_t^z)$, and where Ψ and Γ are known functions. I include the time-period index, t, in the conditioning set on the right-hand side of (A1) because, under assumption 3.3, the conditional expectation is time-dependent. Let $X \subset \mathbb{R}^{2L}$ denote the support for x_t. A solution for the model is defined as a function y: $X \to \mathbb{R}^3$ such that $y_t = y(x_t)$ solves (A1) for all $x_t \in X$.

The method of parameterized expectations approximates $E(\Gamma(x_{t+1}, y_{t+1}) | x_t, t)$ with a function $g(x_t, t, \theta)$, where θ denotes a vector of parameters. The function g must be chosen from a family of functions that is dense in the space of functions containing $E[\Gamma(x_{t+1}, y_{t+1}) | x_t, t]$ in the sense that, for some choice of θ, the unknown conditional expectation function can be approximated arbitrarily well by $g(\cdot, \cdot, \theta)$. In this model, the unknown conditional expectation is a strictly positive and continuous function of x_t, so I choose $g(x_t, t, \theta)$ to be an exponential polynomial in t and in the logs of the elements of x_t. That is,

$$g(x_t, t, \theta) \equiv \exp\left\{\theta'[\log(x_t), t]^{(n)}\right\},$$

where $\log(x_t)$ is the vector whose elements are the logs of the corresponding elements of x_t, and $[\log(x_t), t]^{(n)}$ is the vector of polynomial combinations of elements of $(\log(x_t), t)'$ up to order n.

An approximate rational expectations equilibrium is defined as an expectational rule $g(\cdot, \cdot, \theta^*)$ where, if the policy function $\hat{y}(\cdot, \cdot, \theta^*)$ is implicitly defined by

$$\Psi(x_t, \hat{y}(x_t, t, \theta^*)) \equiv g(x_t, t, \theta^*), \tag{A2}$$

then $g(x_t, t, \theta^*)$ is a nonlinear least squares (NLLS) estimator of $E[\Gamma(x_{t+1}, \hat{y}(x_{t+1}, t+1, \theta^*)) | x_t, t]$. Since the x_t process is time dependent, this NLLS estimation cannot be done using a single time series, so I simulate M independent paths for $\{x_t\}_{t=1}^T$ and estimate $g(\cdot, t, \theta)$ by integrating cross-sectionally for each t. Specifically, the algorithm is as follows:

(i) Fix n (the order of the approximating polynomial), M (the number of independent time series simulated), and T (the length of each simulated time series). Use the known law of motion of x_t to simulate M independent series $\{x_{m,t}\}_{t=1,}^{T} {}_{m=1}^{M}$, make an initial guess for the coefficient vector, θ_0, and compute the series $\psi_{m,t}^0 \equiv g(x_{m,t}, t, \theta_0)$, $t = 1, \ldots, T$, $m = 1, \ldots, M$.

(ii) Compute $y_{m,t}^0$ from the implicit relation $\psi_{m,t}^0 = \Psi(x_{m,t}, y_{m,t}^0)$, for $t = 1, \ldots, T$, $m = 1, \ldots, M$.

(iii) Solve the following least-squares problem:

$$\min_\theta \sum_{m=1}^{M} \sum_{t=1}^{T} \left(\Gamma(x_{m,t+1}, y_{m,t+1}^0) - g(x_{m,t}, t, \theta)\right)^2,$$

denoting the minimizing parameter vector θ_1.

(iv) Repeat steps (i) through (iii) with θ_0 replaced by θ_1.

(v) Iterate until the sum of squared residuals using θ_j (when $y_{m,t} = y^j_{m,t}$) approximates that obtained using θ_{j+1}. This converged value of the θ vector corresponds to the approximate equilibrium.

In Section IV, n is set equal to 2, and M is set to 25. (The approximate solution was virtually unchanged when n was increased to 3.) The 25 independent paths for $(Y_t/Y_{t-1}, \mu_t)$ are simulated according to the estimated vector autoregressions, using the first observation as the starting value. The sample size T for each simulated path in the nonstationary model is 123, matching the 124 observations in the data set less the starting value, which is discarded.

Once the value of θ^* corresponding to the approximate equilibrium has been computed, the model is simulated by computing $\{\hat{y}(x_t, t, \theta^*)\}_{t=1}^T$ using the x_t vector actually observed in the data. The conditional expectation of inflation is computed by simulating $g(x_t, t, \theta_\pi)$, where

$$\theta_\pi = \text{argmin}_\theta \sum_{m=1}^{M} \sum_{t=1}^{T} (\pi_{m,t+1} - g(x_{m,t}, t, \theta))^2.$$

and where $\pi_{m,t+1}$ is the value of inflation implied by $\hat{y}(x_{m,t+1}, t, \theta^*)$. The conditional expectations of real interest rates and real equity returns were computed analogously.

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