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## Dual Trading in Futures Markets

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### ABSTRACT

With dual trading, brokers trade both for their customers and for their own account. We study dual trading and find that customers who are less likely to be informed have higher expected profits with dual trading while customers who are more likely to be informed have higher expected profits without dual trading. We also examine the effects of frontrunning. We test the major empirical implications of our model. Consistent with the model, dual traders earn higher profits than non-dual traders, and customers of dual-trading brokers do better than customers of non-dual-trading brokers.

ONE OF THE MOST controversial issues facing futures traders, exchanges, and regulators is whether floor traders who act as brokers in bringing customers' orders to the market should also be allowed to trade for their own account. This practice—known as dual trading—traditionally has been permitted at futures exchanges and has received widespread attention since the January 1989 announcement of a Federal investigation into alleged trading abuses at Chicago futures exchanges. Critics of dual trading argue that it causes a conflict of interest between brokers and their customers. Advocates of dual trading claim that customers benefit because it reduces the cost of trading.<sup>1</sup>

We develop a model of dual trading for the purpose of studying the effects of dual trading on futures prices and customer trading profits. In particular, we seek to identify who gains and who loses from dual trading. Our model is in the spirit of Glosten and Milgrom (1985) and addresses the role of the

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<sup>1</sup> Grossman (1989) provides a general discussion of dual trading and some of the likely effects of restricting it. Grossman also describes the forms of dual trading present in various stock and futures markets throughout the world.

broker in the market. Traders submit orders to a broker, who in turn, executes these orders in the market.

We assume that the broker knows more about his customers' motives for trading than do other floor traders. Specifically, we assume that the broker has better, although imperfect, information about whether a customer is making an informed or an uninformed trade. With this informational advantage, the broker can profit by trading after a customer's order is executed. The broker's optimal trading strategy is to mimic the trades of customers who are likely to be trading on the basis of fundamental information. This leads to two important empirical implications. First, the informational advantage conveyed by serving as a broker implies that a floor trader's personal trading profits should be higher at times when he is also observed to be filling customers' orders. Second, since the broker trades when his customers are likely to be informed, customers' trading profits should be higher at times when the broker is also observed to be trading for his own account. These implications form the basis for our empirical tests.

We also examine how a ban on dual trading affects the market. In doing this, we incorporate its effects on the trading commission charged by the broker. Specifically, we assume that the brokerage business is competitive in the sense that the commission is set at a level which offers the broker zero expected profits. Thus, prohibiting a broker from profiting by trading for his own account leads the broker to charge a higher commission to cover his costs. We find that before commissions, customers' expected trading profits increase if dual trading is banned. Net of commissions, however, a ban on dual trading leads to lower expected trading profits for uninformed customers and higher expected trading profits for informed customers.

We also study how the possibility that brokers trade ahead of their customers—frontrunning—affects the markets. As is the case with dual trading but no frontrunning, the broker's optimal strategy is to mimic the trades of customers who are likely to be informed. But since the broker fills his own order before filling his customer's order, the broker's order is filled at a more favorable price. This leads the broker to trade more aggressively in the sense that he mimics the trades of some customers who he would otherwise not mimic. Compared to a market with dual trading but no frontrunning, frontrunning leads to higher before-commission expected trading profits for customers that the broker does not frontrun and lower before-commission expected trading profits for customers that the broker does frontrun. Further, frontrunning may also have the same effects when compared to a market with no dual trading at all. Finally, with frontrunning, the broker's expected trading profit is higher and thus, he charges customers a lower commission.

We examine the empirical implications of our dual-trading model using transactions data for the actively traded soybean futures contract on the Chicago Board of Trade. A novel feature of the data set is that it identifies the trades of each individual trader present on the floor of the exchange. This

allows us to compute the trading profit for each floor trader and for his customers. We find that the personal trading profits of dual-trading brokers are significantly higher than those of other floor traders. Moreover, the personal trading profit of a given floor trader is higher on days he is dual trading than on other days. This suggests that the higher trading profits of dual traders are due to the information conveyed by customers' orders rather than to superior trading skills. We also find that the trading profits of customers of dual-trading brokers are higher than the trading profits of customers of non-dual-trading brokers. These findings are consistent with our model.

Related studies of dual trading include Roell (1990) in which a broker observes the trades of some uninformed traders, and Sarkar (1991) in which a broker has fundamental information of his own and observes the trades of another informed trader. Both studies find that the informed trader's expected profit is higher if dual trading is banned. Roell finds that uninformed traders whose trades are observed (not observed) by the broker have lower (higher) expected profits if dual trading is banned. Sarkar finds that if the broker's own fundamental information is precise (not precise) relative to the other informed trader's information, then uninformed traders' expected profits are lower (higher) if dual trading is banned. Neither study incorporates the effect of dual trading in reducing the brokerage commission.<sup>2</sup>

Smith and Whaley (1991) study the "top-step rule" imposed in June 1987 in the S&P 500 futures market at the Chicago Mercantile Exchange. This rule prohibits dual trading by floor traders who stand on the top step of the S&P 500 futures trading pit; they can trade only for customers. Prior to this time, any floor trader in the pit could dual trade. They find evidence that this rule, which substantially curtailed the amount of dual trading, led to a widening of the effective bid-ask spread. By contrast, our empirical work studies a market in which dual trading is allowed. We focus on floor traders' profits conditional on whether they dual traded and on customers' profits conditional on whether their brokers chose to dual trade. A study by the Commodity Futures Trading Commission (1989) focuses on bid-ask spreads in futures markets where dual trading is allowed. They find evidence that the effective bid-ask spreads faced by customers of dual-trading brokers are not significantly different from those faced by customers of non-dual-trading brokers.

The remainder of the paper is organized as follows. Section I presents the dual-trading model and characterizes the equilibrium of the model. Section II studies the effects of a ban on dual trading. Section III studies the effects of frontrunning. Section IV describes the data and presents the empirical findings. Section V contains concluding remarks.

<sup>2</sup> In a study of futures market manipulations, Kyle (1984) also considers a model in which a trader can profit by observing the trades of uninformed traders.

## I. The Dual-Trading Model

In this section, we develop the dual-trading model. First, we describe the underlying structure and assumptions of the model. We then characterize the equilibrium.

### A. The Model

Futures contracts are traded on a commodity or asset whose underlying value is  $\theta$ .<sup>3</sup> Assume that with equal probability,  $\theta$  equals  $\theta_1$  or  $\theta_2$ , ( $\theta_1 < \theta_2$ ) and let  $\bar{\theta} = (\theta_1 + \theta_2)/2$ . The value of  $\theta$  is publicly observed at the conclusion of trading and contracts are settled on the basis of  $\theta$ . For convenience, assume that there are no carrying costs.

Two customers may want to trade this futures contract, call them  $A$  and  $B$ . These customers either buy one futures contract, sell one futures contract, or do not trade. We want to consider a case in which customers sometimes trade because they have fundamental information about the value of the commodity, and sometimes trade for other reasons (for example, to hedge). Specifically, assume that with probability  $q$ ,  $q \in (0, 1)$ , customer  $i$ ,  $i = A, B$ , is informed; he observes the realization of  $\theta$ . With probability  $1 - q$ , customer  $i$  is uninformed. Assume that an informed customer always trades on his information; he buys (sells) if he observes  $\theta = \theta_2$  ( $\theta = \theta_1$ ). In addition, assume that if customer  $i$  is uninformed, then he buys with probability  $h_i/2$ , sells with probability  $h_i/2$ , and does not trade with probability  $1 - h_i$ . Let  $y_i$  denote customer  $i$ 's trade, where  $y_i = 1$  for a buy,  $-1$  for a sell, and  $0$  for no trade.

Assume that  $h_A$  and  $h_B$  are independently and identically distributed random variables, continuously distributed on the interval  $[0, 1]$ . Let  $f(\cdot)$  denote the probability density function of  $h_i$ , let  $F(\cdot)$  denote the cumulative distribution function of  $h_i$ , and let  $\bar{h} = E[h_i]$ . Assume that the random variables  $h_A$ ,  $h_B$ , whether customers are informed, and  $\theta$  are independent of one another.

Customers simultaneously submit their orders to a risk-neutral broker who brings their orders to the market. If both customers submit orders, then the broker fills them sequentially; each customer has a 50% chance of having his order filled first. Consistent with the actual practice of dual trading, the broker can trade for his own account, provided he does not trade ahead of his customers. That is, brokers do not frontrun their customers. In particular, once the broker has filled his customers' orders, he may buy one futures contract, sell one futures contract, or not trade (frontrunning is considered in Section III).

The broker faces a fixed cost (prorated over the time considered by the model) of maintaining a brokerage business of  $k_0 \geq 0$  and a variable cost of filling a customer's order of  $k_1 \geq 0$ . To cover these expenses, the broker

<sup>3</sup> Although we focus on the futures market, the model can be applied to any market in which dual trading can occur.

charges a commission of  $c$  for filling orders. A customer pays the broker  $c$  if he trades, and he pays the broker nothing otherwise. Assume that the brokerage business is competitive in the sense that  $c$  is chosen so that the broker earns a zero expected profit.

There is a competitive risk-neutral market maker who takes the other side of each trade at a price equal to the expected settlement price, conditional on the order flow. The market maker's actions are meant to represent the outcome of competition among a large number of floor traders in the trading pit.<sup>4</sup> The market maker cannot distinguish between orders that the broker submits for his own account and those that he submits for customers.

The broker knows  $h_A$  and  $h_B$ , but does not know whether customers are informed. Knowing  $h_A$  and  $h_B$  is useful for determining the likelihood that a given trade is information based. Given that customer  $i$  trades, the broker's beliefs are that customer  $i$  is informed with probability  $q/(q + h_i(1 - q))$ . The lower  $h_i$  is, the more likely the broker believes it to be that customer  $i$  is informed. The market maker's beliefs, based on the distribution of  $h_i$ , are that if customer  $i$  trades, then he is informed with probability  $q/(q + \bar{h}(1 - q))$ . That the broker can condition his beliefs on the actual value of  $h_i$  is his informational advantage over the market maker. The broker learns more than the market maker from his customers' orders and can use this information to earn trading profits. By allowing the broker to have better information about his customers, the model is consistent with actual practice in futures markets. For example, in futures trading, only the broker knows the identity of the customer whose order is being filled. Furthermore, brokers are likely to have information about customers' trading records—in particular, about the profitability of past trades.

Three trades could be submitted to the market maker, one for trader  $A$ , one for trader  $B$ , and one for the broker. Nevertheless, there will be at most two trades submitted to the market maker. This is because, by assumption, the broker fills his customers' orders first, and so if there is a third trade, the market maker knows that it must be the broker's trade. Since the broker is better informed than the market maker and since the broker only trades if he faces a positive expected profit, the market maker must, on average, lose when buying from the broker at a price above  $\theta_1$  or when selling to the broker at a price below  $\theta_2$ . Thus, on a third trade, the market maker is only willing to buy at a price of  $\theta_1$  and to sell at a price of  $\theta_2$ . At such prices the broker would not trade (the broker has better information than the market maker but not perfect information).

<sup>4</sup> By increasing the number of floor traders who can trade for their own account, dual trading may promote greater competition among floor traders and provide for greater risk-sharing opportunities. If so, then the market will be more liquid in that prices will not move as much in response to temporary order imbalances. See Grossman (1989) for a discussion. Addressing these issues would be of interest in developing a model of exchange membership prices. In this paper, however, we abstract from these roles of dual trading. This simplifies the pricing of futures contracts.

Let  $X(y_A, y_B, h_A, h_B)$  denote the broker's trading strategy;  $X$  specifies the broker's trade as a function of the trades of customers  $A$  and  $B$  and as a function of  $h_A$  and  $h_B$ . Assume that if the broker is indifferent between trading and not trading, then he does not trade. Let  $\omega_j$ ,  $j = 1, 2$ , denote the  $j$ th order submitted to the market maker, and let  $P_1(\omega_1)$  and  $P_2(\omega_1, \omega_2)$  denote the market maker's price functions; each specifies the transaction price as a function of the order flow up to and including that trade. In effect, the price functions specify bid and ask prices. For instance, for the second trade,  $P_2(\omega_1, 1)$  is the ask price and  $P_2(\omega_1, -1)$  is the bid price, each as a function of the first trade.

An equilibrium consists of a trading strategy for the broker,  $X$ , and price functions  $P_1$  and  $P_2$ , such that (i) taking  $P_1$  and  $P_2$  as given, the broker's strategy maximizes his expected trading profit; and (ii) taking  $X$  as given, the price functions satisfy  $P_1(\omega_1) = E[\theta | \omega_1]$  and  $P_2(\omega_1, \omega_2) = E[\theta | \omega_1, \omega_2]$ . That the broker does not trade if both  $A$  and  $B$  trade, as discussed above, means that if  $y_A \neq 0$  and  $y_B \neq 0$ , then  $X(y_A, y_B, h_A, h_B) = 0$ .

Finally, define  $\alpha(h)$  by the expression

$$\alpha(h) = \frac{\frac{1}{2}(q + (1-q)\bar{h}/2)^2 + (1-\bar{h})(1-q)F(h)(q + (1-q)E[h_i | h_i < h]/2)}{\frac{1}{4}(q^2 + (q + (1-q)\bar{h})^2) + (1-\bar{h})(1-q)F(h)(q + (1-q)E[h_i | h_i < h])}. \quad (1)$$

The function  $\alpha(h)$  will be used to express the market maker's updated beliefs conditional on observing either two buys or two sells. To explain, we anticipate the nature of the broker's equilibrium trading strategy. Suppose that if only customer  $i$  trades, then the broker mimics customer  $i$ 's trade if  $h_i < h$  and does not trade otherwise. Then two buys (or two sells) can be from either (i) two informed customers; (ii) two uninformed customers; (iii) one informed customer and one uninformed customer; (iv) one informed customer with  $h_i < h$  and the broker; or (v) one uninformed customer with  $h_i < h$  and the broker. Given these possibilities, Bayes rule can be used to compute the market maker's updated beliefs about  $\theta$ . It can be shown that  $\alpha(h)$  is the updated probability that  $\theta = \theta_2$  if there are two buys and  $\alpha(h)$  is the updated probability that  $\theta = \theta_1$  if there are two sells.

### B. The Dual-Trading Equilibrium

Proposition 1 characterizes the unique equilibrium of the dual-trading market.

**PROPOSITION 1:** *There is a unique equilibrium. In this equilibrium, the broker's strategy is as follows:*

- (i) Suppose  $y_A = 0$  and  $y_B = 0$ . Then  $X(y_A, y_B, h_A, h_B) = 0$ .

(ii) Suppose  $y_A \neq 0$  and  $y_B = 0$ . Then

$$X(y_A, y_B, h_A, h_B) = \begin{cases} y_A, & \text{if } h_A < h^*, \\ 0, & \text{if } h_A \geq h^*. \end{cases}$$

(iii) Suppose  $y_A = 0$  and  $y_B \neq 0$ . Then

$$X(y_A, y_B, h_A, h_B) = \begin{cases} y_B, & \text{if } h_B < h^*, \\ 0, & \text{if } h_B \geq h^*. \end{cases}$$

In this equilibrium, the price functions are

$$P_1(1) = \frac{\theta_2(q + (1 - q)\bar{h}/2) + \theta_1(1 - q)\bar{h}/2}{q + (1 - q)\bar{h}}, \quad (2)$$

$$P_1(-1) = \frac{\theta_2(1 - q)\bar{h}/2 + \theta_1(q + (1 - q)\bar{h}/2)}{q + (1 - q)\bar{h}}, \quad (3)$$

$$P_2(1, -1) = \bar{\theta}, \quad (4)$$

$$P_2(-1, 1) = \bar{\theta}, \quad (5)$$

$$P_2(1, 1) = \alpha(h^*)\theta_2 + (1 - \alpha(h^*))\theta_1, \quad (6)$$

$$P_2(-1, -1) = (1 - \alpha(h^*))\theta_2 + \alpha(h^*)\theta_1, \quad (7)$$

where  $h^*$  satisfies

$$\alpha(h^*) = \frac{q + (1 - q)h^*/2}{q + (1 - q)h^*}. \quad (8)$$

*Proof:* See the Appendix.

If neither customer  $A$  nor  $B$  trades, then the broker learns nothing about  $\theta$  and does not trade. If just customer  $i$  trades, then the broker may find it profitable to mimic this trade. In particular, if the broker knows that  $h_i$  is low, then the broker earns a positive expected profit by mimicking customer  $i$ 's trade. This is because if  $h_i$  is low, the broker knows that the market maker underestimates the probability that the trade is information based. The critical value of  $h_i$  is determined as follows. If the broker follows such a strategy, and the market maker conjectures that the critical value is  $h^c$ , then the market maker's updated beliefs in response to two buys (similarly for two sells) are that the asset is worth  $\alpha(h^c)\theta_2 + (1 - \alpha(h^c))\theta_1$ . Now, suppose customer  $A$  buys and customer  $B$  does not trade. The broker profits by buying if  $E[\theta | y_A = 1, y_B = 0, h_A, h_B] > \alpha(h^c)\theta_2 + (1 - \alpha(h^c))\theta_1$ . For any conjecture,  $h^c$ , by the market maker, there is a value  $h'$ , such that  $E[\theta | y_A = 1, y_B = 0, h_A, h_B] \geq \alpha(h^c)\theta_2 + (1 - \alpha(h^c))\theta_1$  for  $h_A < h'$ , with equality at  $h_A = h'$ . In equilibrium, the market maker's conjecture must be correct,  $h^c = h'$ . Thus, the equilibrium critical value,  $h^*$ , is the solution to  $E[\theta | y_A = 1, y_B = 0, h_A = h^*, h_B] = \alpha(h^*)\theta_2 + (1 - \alpha(h^*))\theta_1$ , which is equivalent to (8).

The bid-ask spread on the first trade is  $P_1(1) - P_1(-1) = (\theta_2 - \theta_1)q/(q + (1 - q)\bar{h})$ . For  $\omega_1 = 1$  or  $\omega_1 = -1$ , the bid-ask spread on the second trade is  $P_2(\omega_1, 1) - P_2(\omega_1, -1) = (\theta_2 - \theta_1)(\alpha(h^*) - 1/2) = (\theta_2 - \theta_1)q/(2(q + (1 - q)h^*))$ , where the second equality follows from (8). Other things equal, as the market maker's information improves, the spread will narrow since the adverse-selection problem is mitigated. Typically in models such as these, this implies a narrowing of the spread over time. Here, however, the spread on the second trade may exceed that of the first trade. Though the first trade conveys information to the market maker, it also conveys information to the broker; the former mitigates the adverse-selection problem while the latter aggravates it. The net effect may be a wider spread for the second trade.<sup>5</sup>

The broker's unconditional expected trading profit (prior to observing  $y_A$ ,  $y_B$ ,  $h_A$ , or  $h_B$ ) equals

$$\begin{aligned} \pi_B = & 2(1 - \bar{h})(1 - q)F(h^*)(q + (1 - q)E[h_i | h_i < h^*]) \\ & \times (E[\theta | y_i = 1, y_j = 0, h_i < h^*, h_j] - P_2(1, 1)). \end{aligned} \quad (9)$$

This can be explained as follows. With probability  $2(1 - \bar{h})(1 - q)F(h^*)$ , one customer does not trade and the other customer has  $h_i < h^*$ . Conditional on  $h_i < h^*$ , the probability that customer  $i$  trades is  $q + (1 - q)E[h_i | h_i < h^*]$ . These are the conditions under which the broker trades, and thus, the probability that the broker trades is the product of these two terms. Given that  $h_i < h^*$ , the broker's expected profit from mimicking customer  $i$ 's trade is  $E[\theta | y_i = 1, y_j = 0, h_i < h^*, h_j] - P_2(1, 1) > 0$ . Thus, the broker's unconditional expected profit is given by (9), and  $\pi_B > 0$ .

Given how prices are set, the market maker does not bear the (expected) cost of the broker's trading profits. The broker's trading profits come at the expense of his customer's trading profits. Given a competitive brokerage business, though, the broker's expected trading profit will be returned to customers via lower trading commissions. That is, with dual trading, the broker's overall expected profit equals

$$\Pi_B(c) = \pi_B + (c - k_1)E[|y_A| + |y_B|] - k_0, \quad (10)$$

where  $c$  is the commission that he charges. This is the broker's expected trading profit plus his expected profit from commissions less his fixed cost. Using the fact that  $E[|y_i|] = q + (1 - q)\bar{h}$ , the zero-expected-profit commission for the broker,  $c_D$  ( $D$  for dual trading), satisfies  $\Pi_B(c_D) = 0$  and is given by

$$c_D = k_1 + \frac{k_0 - \pi_B}{2(q + (1 - q)\bar{h})}. \quad (11)$$

<sup>5</sup> For example, if  $h_i$  is uniformly distributed on the interval  $[0, 1]$ , then  $h^* = (-1 - q + (2 + 2q^2)^{1/2})/(2 - 2q)$ , and for  $q \leq 0.142$ , the bid-ask spread is higher for the second trade.

The greater is the broker's expected trading profit, the lower is his commission.<sup>6</sup>

To illustrate the equilibrium, consider the following example:

*Example:* Let  $\theta_2 = 2$ ,  $\theta_1 = 1$ ,  $q = 0.1$ ,  $k_0 = 0$ ,  $k_1 = 0.01$ , and suppose that  $h_i$  is uniformly distributed on the interval  $[0, 1]$ . Then  $h^* = 0.178$ ; the broker mimics customer  $i$ 's trade only if  $h_i < 0.178$ . On the first trade, prices are  $P_1(1) = 1.591$  and  $P_1(-1) = 1.409$ . On the second trade, prices are  $P_2(1, 1) = 1.692$ ,  $P_2(-1, -1) = 1.308$  and  $P_2(1, -1) = P_2(-1, 1) = 1.500$ . The broker's expected trading profit is  $\pi_B = 0.0025$  and the broker's commission is  $c_D = 0.0077$ .

The basic premise of this dual-trading model is that brokers are better informed concerning the identities of their customers. This implies that brokers learn more than other floor traders from their customers' orders and can use this information to earn trading profits. Brokers only profit from such information, though, if their customers trade. This follows from Proposition 1; if the broker's customers do not trade, then he does not trade either. Thus, a broker's expected trading profit should be higher at times when he also fills orders for customers. This implication is examined empirically in Section IV.<sup>7</sup> A second implication that is examined empirically concerns customers' trading profits conditional on whether the broker also traded. By Proposition 1, the broker only mimics the trades of customers who are relatively likely to be informed—that is, customers for whom  $h_i < h^*$ . Thus, a customer's expected trading profit should be higher if his broker is also observed to trade for his own account.<sup>8</sup>

## II. The Effects of Banning Dual Trading

In this section, we compare the market with dual trading to one in which the broker is not allowed to trade for his own account. He serves exclusively as a broker for his customers. With a ban on dual trading, customers will trade at more favorable prices but will pay higher commissions. We consider

<sup>6</sup> Note that the commission is set before the broker observes  $h_A$  and  $h_B$ .

<sup>7</sup> A broker's expected trading profit is zero if his customers do not trade. This is due to the assumption that prices are set by a competitive risk-neutral market maker. If we had modeled the trading process such that order imbalances were absorbed by competitive risk-averse market makers, then a broker would trade, and earn a positive expected profit, even if his customers did not trade. He would take a portion of any order imbalance for his own account and be compensated for sharing the risk. Grossman and Miller (1988) study such a model.

<sup>8</sup> A model with more customers and trading rounds could lead to a situation in which the broker makes the opposite trade from his customers. That is, suppose several customers with high values of  $h_i$  all buy and drive up the price. Then the broker, knowing that the market is overestimating the likelihood that these buys are informed, may profit by selling. In general, though, we expect that a broker mimicking his customers is more likely. This is because it only takes one customer with a low  $h_i$  to provide the broker with a profitable mimicking trade. It takes multiple customers with high values of  $h_i$ , and who trade in the same direction, to provide the broker with a profitable opposing trade.

the effects of a ban on dual trading on customer's expected trading profits net of commissions.

### A. The No-Dual-Trading Equilibrium

Proposition 2 characterizes the equilibrium prices for the case in which there is no dual trading. We state this proposition without proof. It is simply the computation of prices as  $P_1(\omega_1) = E[\theta \mid y_i = \omega_1]$  and  $P_2(\omega_1, \omega_2) = E[\theta \mid y_i = \omega_1, y_j = \omega_2]$ .

**PROPOSITION 2:** *If the broker is prohibited from trading for his own account, then the equilibrium price functions satisfy (2), (3), (4), (5), and*

$$P_{2N}(1, 1) = \alpha(0)\theta_2 + (1 - \alpha(0))\theta_1, \quad (12)$$

$$P_{2N}(-1, -1) = (1 - \alpha(0))\theta_2 + \alpha(0)\theta_1. \quad (13)$$

The price of a first trade, whether a buy or sell, is the same with or without dual trading. This is because even with dual trading, the broker never trades if his customers do not. Thus, with or without dual trading, the first trade conveys the same information to the market maker. The price of a second trade is also unchanged if the second trade is the opposite of the first trade. This is because, with dual trading, the broker never makes the opposite trade of his customer. Thus, with or without dual trading, a buy followed by a sell, or the reverse, conveys the same information to the market maker. The price of a second trade is changed, though, if the second trade is the same as the first trade. Specifically, without dual trading, a customer who trades second and who makes the same trade as the previous customer pays a lower price if he is buying and receives a higher price if he is selling. That is,  $P_{2N}(1, 1) < P_2(1, 1)$  and  $P_{2N}(-1, -1) > P_2(-1, -1)$ . Equivalently, the bid-ask spread is narrower without dual trading. Customers trade at more favorable prices because the adverse-selection problem facing the market maker is diminished when the broker cannot trade. Since the broker only trades to capitalize on mispricings, the market maker widens the bid-ask spread to compensate for losses to the broker.

### B. The Effect of a Dual-Trading Ban on Customers' Expected Profits

As noted above, the price at which a customer trades depends on whether there is dual trading only if he makes the same trade as the other customer and if he trades second. In this case, he either sells at a higher price without dual trading or buys at a lower price without dual trading. Therefore, a customer's expected trading profit increases if dual trading is banned. This increase in expected trading profit is offset, however, by a higher brokerage commission.

The increase in expected trading profit equals the joint probability that a customer makes the same trade as the other customer and trades second, multiplied by  $P_2(1, 1) - P_{2N}(1, 1) = P_{2N}(-1, -1) - P_2(-1, -1)$ , the reduc-

tion in the price paid or the increase in the price received. For an uninformed customer, this equals

$$\Delta\pi_U = \frac{q/2 + (1-q)\bar{h}/2}{2} (P_2(1,1) - P_{2N}(1,1)). \quad (14)$$

The joint probability reflects the fact that if the other customer is informed, he makes the same trade with probability  $1/2$  and if the other customer is uninformed, he makes the same trade with probability  $\bar{h}/2$ . It also reflects the fact that if both customers trade, each faces a probability  $1/2$  of trading second. For an informed customer, the increase in expected trading profit equals

$$\Delta\pi_I = \frac{q + (1-q)\bar{h}/2}{2} (P_2(1,1) - P_{2N}(1,1)). \quad (15)$$

The difference between (14) and (15) reflects the fact that an informed customer's trade is positively correlated with the other customer's trade; if the other customer is also informed, then they make the same trade. In contrast, an uninformed customer's trade is uncorrelated with the other customer's trade. This implies that an informed customer is more likely to trade at a price of  $P_2(1,1)$  or  $P_{2N}(1,1)$  and thus, is more likely to benefit from a more favorable price due to a ban on dual trading. Therefore, an informed customer's expected trading profit increases by more than that of an uninformed customer if dual trading is banned;  $\Delta\pi_I > \Delta\pi_U$ . Note that with dual trading, the broker's expected trading profit can be expressed as

$$\pi_B = 2(q\Delta\pi_I + (1-q)\bar{h}\Delta\pi_U). \quad (16)$$

This is because, if dual trading is banned, the average change in a customer's expected trading profit equals  $q\Delta\pi_I + (1-q)\bar{h}\Delta\pi_U$ . With two customers, the total change is twice this amount. Further, if dual trading is banned, the change in the broker's expected trading profit is  $-\pi_B$ . Since the total change across the customers and the broker must sum to zero, (16) holds.

Although customers' expected trading profits are higher without dual trading, they now face higher trading commissions. This is because the broker can no longer profit by trading for his own account and must increase the commission to cover his costs. The broker's zero-expected-profit commission without dual trading is given by

$$c_N = k_1 + \frac{k_0}{2(q + (1-q)\bar{h})}. \quad (17)$$

This is the same as (11) with  $\pi_B$  replaced by zero. Of course,  $c_N > c_D$ .<sup>9</sup>

<sup>9</sup> See Stanley (1981) for a further discussion of how a ban on dual trading might affect the cost of brokerage services.

We can compute the net change in a customer's expected profit, including the effect of the increase in the brokerage commission,  $c_N - c_D$ . For an uninformed customer, the net change in expected profit equals

$$\begin{aligned}
 \Delta \Pi_U &= \Delta \pi_U - (c_N - c_D) \\
 &= \Delta \pi_U - \frac{\pi_B}{2(q + (1 - q)\bar{h})} \\
 &= \Delta \pi_U - \frac{q\Delta \pi_I + (1 - q)\bar{h}\Delta \pi_U}{q + (1 - q)\bar{h}} \\
 &= -\frac{q^2}{4(q + (1 - q)\bar{h})} (P_2(1, 1) - P_{2N}(1, 1)) < 0, \quad (18)
 \end{aligned}$$

where the second equality uses (11) and (17), the third equality uses (16), and the fourth equality uses (14) and (15). An uninformed customer's expected trading profit, net of commission, is lower if dual trading is banned. The increase in the commission more than offsets the increase in expected trading profit for an uninformed customer. The net change in an informed customer's expected profit equals

$$\begin{aligned}
 \Delta \Pi_I &= \Delta \pi_I - (c_N - c_D) \\
 &= \Delta \pi_I - \frac{\pi_B}{2(q + (1 - q)\bar{h})} \\
 &= \Delta \pi_I - \frac{q\Delta \pi_I + (1 - q)\bar{h}\Delta \pi_U}{q + (1 - q)\bar{h}} \\
 &= \frac{q(1 - q)\bar{h}}{4(q + (1 - q)\bar{h})} (P_2(1, 1) - P_{2N}(1, 1)) > 0. \quad (19)
 \end{aligned}$$

An informed customer's expected trading profit, net of commission, is higher if dual trading is banned. The increase in expected trading profit for an informed customer more than offsets the increase in the commission.

The effect of a ban on dual trading on a customer's unconditional expected profit net of commission (computed before knowing whether he is informed) is

$$\begin{aligned}
 q\Delta \Pi_I + (1 - q)h_i\Delta \Pi_U &= \frac{q^2(1 - q)}{4(q + (1 - q)\bar{h})} (\bar{h} - h_i)(P_2(1, 1) - P_{2N}(1, 1)) \\
 &\geq 0 \text{ as } h_i \leq \bar{h}. \quad (20)
 \end{aligned}$$

Customers who are relatively more likely to make uninformed trades have higher expected profits with dual trading and customers who are relatively more likely to make informed trades have higher expected profits without dual trading.

To illustrate the effect of a dual-trading ban, reconsider the example from Section I.B.

*Example (continued):* A ban on dual trading narrows the bid-ask spread on the second trade;  $P_{2N}(1, 1) = 1.676$  which is lower than with dual trading and  $P_{2N}(-1, -1) = 1.324$  which is higher than with dual trading. The broker now charges a higher commission;  $c_N = 0.01$ . The net change in an uninformed customer's expected profit is  $\Delta\Pi_U = -0.000072$  and the net change in an informed customer's expected profit is  $\Delta\Pi_I = 0.000324$ .

The above discussion focuses on customers' expected trading profits. The results are suggestive but it should be noted that since we have not specified customers' objectives in trading futures, we cannot make conclusive statements about customer welfare. For instance, suppose customers are risk averse and that their uninformed trades are motivated by hedging reasons. Dual trading has conflicting effects on the risk facing customers. The beneficial effect is to smooth customers' expected income by transferring expected trading profits from the times when a customer is informed to the times when he is uninformed. The detrimental effect is that, conditional on being either informed or uninformed, the volatility of customers' trading profits is higher (the commission is decreased, but the prices at which customers trade may be worse). It should also be noted that a ban on dual trading may also affect customers' incentives to collect information. For instance, suppose customers choose  $q$ , the probability that they are informed, with higher values of  $q$  being more costly to attain. Then, since a ban on dual trading increases the expected profit of informed customers, it may also induce customers to expend more resources collecting information.

### III. Frontrunning

Critics of dual trading argue that dual trading makes it easier for brokers to cheat their customers. One method for doing so is by frontrunning, in which case the broker trades for his own account prior to filling a customer order. A broker who frontruns a customer forces the customer to trade at a worse price. In this section, we analyze the effects of frontrunning.

We allow the broker to trade for his own account after he learns his customers' trades but before he fills his customers' trades. To maintain comparability with the previous analysis, we assume that the broker can trade only if zero customers trade or if one customer trades. Thus, the difference between this analysis and the analysis of Section I is that if the broker and one customer trade, the broker's trade is filled first. We use the same notation and definition of equilibrium as in Section I.<sup>10</sup>

Proposition 3 characterizes the unique equilibrium of the dual-trading model with frontrunning.

<sup>10</sup> Proposition 1 shows that there is no equilibrium in which the broker follows a customer's trade with the opposite trade for himself. Such a strategy is ruled out here by assuming that the broker's order is filled first. This assumption allows us to use the same notation as before and simplifies the proof of Proposition 3. It is not restrictive, though, since the logic behind the proof of Proposition 1 applies here as well.

PROPOSITION 3: *There is a unique equilibrium with frontrunning. In this equilibrium, the broker's strategy is as follows:*

- (i) *Suppose  $y_A = 0$  and  $y_B = 0$ . Then  $X(y_A, y_B, h_A, h_B) = 0$ .*
- (ii) *Suppose  $y_A \neq 0$  and  $y_B = 0$ . Then*

$$X(y_A, y_B, h_A, h_B) = \begin{cases} y_A, & \text{if } h_A < \bar{h}, \\ 0, & \text{if } h_A \geq \bar{h}. \end{cases}$$

- (iii) *Suppose  $y_A = 0$  and  $y_B \neq 0$ . Then*

$$X(y_A, y_B, h_A, h_B) = \begin{cases} y_B, & \text{if } h_B < \bar{h}, \\ 0, & \text{if } h_B \geq \bar{h}. \end{cases}$$

*In this equilibrium, the price functions satisfy (2), (3), (4), (5), and*

$$P_{2F}(1, 1) = \alpha(\bar{h})\theta_2 + (1 - \alpha(\bar{h}))\theta_1, \quad (21)$$

$$P_{2F}(-1, -1) = (1 - \alpha(\bar{h}))\theta_2 + \alpha(\bar{h})\theta_1. \quad (22)$$

*Proof:* See the Appendix.

Compare this equilibrium to the one with dual trading but no frontrunning, characterized in Proposition 1. The price of a first trade, whether a buy or sell, is the same with or without frontrunning. This is because if only customer  $i$  trades, then the first trade made is  $y_i$ . Without frontrunning, this trade is for customer  $i$ . With frontrunning, this trade is for customer  $i$  if  $h_i \geq \bar{h}$  and for the broker if  $h_i < \bar{h}$ . In either case, the trade is  $y_i$  and thus, there is no difference in the information conveyed to the market maker. Similarly, if both customers trade, then the order flow is unaffected by frontrunning. The price of a second trade, if the opposite of the first trade, is also unaffected by frontrunning.

Frontrunning does change the price of a second trade if it is the same as the first trade. With frontrunning, a trader who trades second, and makes the same trade as the first trade, trades at a more favorable price. He pays a lower price if buying and receives a higher price if selling. That is,  $P_{2F}(1, 1) < P_2(1, 1)$  and  $P_{2F}(-1, -1) > P_2(-1, -1)$ . Alternatively put, the bid-ask spread is narrower with frontrunning. This is because with frontrunning, the broker trades more aggressively. Since his own trade is filled first, he trades at a more favorable price; he buys at a lower price or sells at a higher price relative to trading second. Therefore, in equilibrium, he mimics a customer's trade for values of  $h_i$  for which he would not do so if there were no frontrunning. He mimics customer  $i$ 's trade if  $h_i < \bar{h}$ , whereas without frontrunning, he only mimics customer  $i$ 's trade if  $h_i < h^*$  (and  $h^* < \bar{h}$ ). This change in the critical value for which the broker mimics a customer's trade leads to a decrease (increase) in the expectation of  $\theta$  conditional on observing two consecutive buys (sells), and thus leads to a narrowing of the spread.<sup>11</sup>

<sup>11</sup> Formally,  $\alpha(h)$  is maximized at  $h = h^*$ . Thus,  $\alpha(h^*) > \alpha(\bar{h})$ , which implies that  $P_{2F}(1, 1) < P_2(1, 1)$  and  $P_{2F}(-1, -1) > P_2(-1, -1)$ .

The broker's expected trading profit is higher with frontrunning. If only customer  $i$  trades and  $h_i < h^*$ , then the broker trades with or without frontrunning, but his expected profit is higher with frontrunning since his trade is filled at a better price. If only customer  $i$  trades and  $h^* \leq h_i < \bar{h}$ , then the broker trades and has a positive expected profit with frontrunning, but does not trade without frontrunning. Since the broker's expected trading profit is higher, the commission he charges will be lower with frontrunning.

Frontrunning increases the before-commission expected trading profit of a customer with  $h_i \geq \bar{h}$ . Suppose this customer buys. Since the broker will not frontrun this customer, his chance of trading first vs. second is the same as with no frontrunning. If he trades first, he trades at  $P_1(1)$  with or without frontrunning; if he trades second and the other customer sells, he trades at  $P_2(-1, 1)$ , with or without frontrunning; and if he trades second and the other customer buys, he trades at  $P_{2F}(1, 1)$  with frontrunning and  $P_2(1, 1)$  without frontrunning. Since  $P_{2F}(1, 1) < P_2(1, 1)$ , this customer's trading profit is at least as high or higher with frontrunning. Moreover, given that the brokerage commission is lower with frontrunning, this customer clearly prefers to trade in a market with frontrunning. The gains of customers with  $h_i \geq \bar{h}$  come at the expense of customers with  $h_i < \bar{h}$ . These are the customers who the broker frontruns and their expected trading profits, before and after commissions, are lower with frontrunning. In summary, frontrunning leads to a transfer from the customers who are more likely to be informed to those who are less likely to be informed.

The above discussion compares a market with frontrunning to a market with dual trading and no frontrunning. Interestingly, the same results may obtain in a comparison between a market with frontrunning and a market with no dual trading at all. Suppose  $h_i$  is uniformly distributed on the interval  $[0, 1]$ . Then,  $\bar{h} = 1/2$ ,

$$\alpha(0) = \frac{q^2 + q(1 - q)/2 + (1 - q)^2/16}{q^2 + q(1 - q)/2 + (1 - q)^2/8}, \quad (23)$$

and

$$\alpha(1/2) = \frac{q^2 + q(1 - q) + (1 - q)^2/8}{q^2 + q(1 - q) + (1 - q)^2/4}. \quad (24)$$

It is straightforward to show that for this example  $\alpha(0) > \alpha(1/2)$ , and thus,  $P_{2F}(1, 1) < P_{2N}(1, 1)$  and  $P_{2F}(-1, -1) > P_{2N}(-1, -1)$ —the bid-ask spread on the second trade is narrower in a market with frontrunning as compared to a market in which dual trading is banned.<sup>12</sup>

The analysis of this section illustrates two general points. First, even before consideration of issues like risk sharing, customers' preferences regarding a market's trading rules cannot always be measured by the resulting bid-ask spread. Even though frontrunning can lead to a narrower bid-ask

<sup>12</sup> This is not true in general. For example, if the density of  $h_i$  is  $f(h_i) = 2 - 2h_i$ , then  $\bar{h} = 1/3$  and  $\alpha(0) \geq \alpha(1/3)$  as  $q \leq 0.5163$ .

spread, this will not be preferred by all customers. Second, viewing a practice such as frontrunning as harmful to all customers is too simple a view. This is because a broker will not frontrun all customers, and customers who are not frontrun by the broker are better off with frontrunning.<sup>13</sup>

#### IV. Empirical Results

In this section, we present the empirical results. We begin by describing the data used in the study and present summary statistics. Our data set has the important advantage of identifying the trades of each trader present on the floor of the futures exchange. Thus, we can compute the daily trading profits for each floor trader and for his customers. This allows us to examine directly the relation between dual trading and trading profitability. We discuss the implications of the empirical results for the dual-trading model.

##### *A. The Data*

The data for the study consist of records for every transaction executed in the soybean futures pit at the Chicago Board of Trade (CBOT) for 15 randomly selected trading days during the last quarter of 1988.<sup>14</sup> Soybean futures are one of the most actively traded futures contracts at the Chicago Board of Trade, with daily trading volume often exceeding 50,000 contracts. The size of the contract is 5,000 bushels and contracts are listed with expiration months of January, March, May, July, August, September, and November. The data set was provided by the Commodity Futures Trading Commission (CFTC) and includes nearly 400,000 transaction records. Each transaction record includes the date of the trade, whether the trade was a buy or a sell, the number of contracts traded, the price at which the trade was executed, an identification number for the individual floor trader who executed the trade, and the customer type indicator code (CTI code) for each trade.

The General Regulations under the Commodity Exchange Act require that the long and short parties to every futures trade be identified by a CTI code. These CTI codes are defined in CFTC rule 1.35(e) as

CTI 1: Was trading for his own account.

CTI 2: Was trading for his clearing member's house account.

<sup>13</sup> For another analysis of frontrunning, see Pagano and Roell (1990). Frontrunning is not the only possible abuse of dual trading. For example, suppose a broker receives a buy order from an outside customer and executes the order immediately. If, during the next few minutes, the futures price rises, the broker has the incentive to claim the previous trade for his own account and then execute another buy (at the higher price) for the customer. If, however, the futures price falls after the initial trade, the broker has no such incentive. Of course, such abuses are possible without dual trading—the broker just needs a partner.

<sup>14</sup> The fifteen days are October 7, 14, 24, and 25, November 17, 23, and 29, and December 6, 9, 13, 16, 19, 21, 28, and 30.

CTI 3: Was trading for another member present on the exchange floor, or an account controlled by such other member.

CTI 4: Was trading for any other type of customers.

The CTI 1 trades are transactions executed by floor traders for their own accounts. In the context of our model, this category represents the trades of market makers as well as the personal trades of dual-trading brokers. The CTI 1 trades represent nearly 60% of the total trading volume during the sample period. The majority of the remaining trading volume consists of CTI 4 trades. These trades are executed by CBOT floor traders for outside customers. For example, this category would include retail trades for brokerage house customers. In the context of our model, this category represents the trades of the customers who place their orders with a broker. The other two CTI categories represent trades where a trader was trading either for another member of the exchange or for another member's customer. Since we cannot differentiate between these two cases, we do not include these trades in the analysis. Trades designated CTI 2 or CTI 3 represent about 5.8% and 9.6% of the total trading volume, respectively.

Having the trading record for each floor trader for each day in the sample period allows us to compute the intraday trading profits for each trader. This is done by computing the difference between the trade and settlement prices, multiplying by the signed trading quantity, and then cumulating over all trades for the trader. This procedure gives the exact trading profit for futures positions that are initiated and closed out on the same day. Profits for the remaining positions, which represent a small proportion of the total, are computed as if closed out at the settlement price. Since intraday trading profits are based on actual transaction prices, they incorporate both the bid-ask-spread-related costs of trading as well as any gains or losses from holding intraday futures positions. In addition to computing the trading profit for each floor trader, we can also compute the aggregate profits for each broker's customers, although not the trading profits for individual customers. The reason for this is that the data set identifies the broker executing the trade, not the customer initiating the trade. Consequently, we cannot compute individual customers' trading profits—only the aggregate trading profits for all of an individual broker's customers on a given trading day.

### *B. Trading Activity*

By examining the transaction history for each floor trader, we can identify which floor traders traded for their own account only, for their customers only, or for both their customers and their own account. Once the various types of traders are identified, we can then examine the composition of the trading pit, the extent to which dual trading occurs, and compare profitability measures across different types of traders. Table I presents summary statistics about the distribution and activity levels of the various types of traders during the sample period.

**Table I**  
**Summary Statistics for the Activity of Floor Traders in the**  
**Soybean Futures Pit at the Chicago Board of Trade**

Own account denotes floor traders who traded exclusively for their own account during the sample period. Customers only denotes floor traders who traded exclusively for their customers during the sample period. Both denotes floor traders who traded both for their own account and for their customers at some time during the sample period. The sample period consists of fifteen randomly selected trading days during the fourth quarter of 1988.

|                                       | Own Account | Customers Only | Both   | All    |
|---------------------------------------|-------------|----------------|--------|--------|
| Number who traded                     | 336         | 433            | 385    | 1,154  |
| Average number trading on a given day | 42.33       | 39.73          | 219.13 | 301.20 |
| For own account only                  | 42.33       | —              | 77.33  | 119.66 |
| For customers only                    | —           | 39.73          | 26.13  | 65.86  |
| For both                              | —           | —              | 115.67 | 115.67 |
| Total number of trading days          | 635         | 596            | 3,287  | 4,518  |
| For own account only                  | 635         | —              | 1,160  | 1,795  |
| For customers only                    | —           | 596            | 392    | 988    |
| For both                              | —           | —              | 1,735  | 1,735  |
| Average number of days per trader     | 1.889       | 1.376          | 8.537  | 3.915  |
| For own account only                  | 1.889       | —              | 3.013  | 1.555  |
| For customers only                    | —           | 1.376          | 1.018  | .856   |
| For both                              | —           | —              | 4.506  | 1.504  |

Table I shows that the composition of the trading pit is fluid. During the 15-day sample period, 1,154 different floor traders executed trades. On an average day, however, only about 301 of these 1,154 traders actually traded. Thus, the representative trader averaged about 4 trading days out of the 15 days during the sample period. Of the 1,154 traders, 336 traded only for their own account during the sample period and 433 traded only for their customers. The remaining 385 traders traded both for their own account and for their customers at some point during the sample period, although not necessarily on the same day.

The activity level of the various types of traders varies widely. For example, the 336 traders who traded exclusively for their own account traded an average of only 1.889 days of the 15 days in the sample, resulting in  $336 \times 1.889 = 635$  observations of intraday trading profits and trading volume (635 trading days). The traders trading only for customers traded an average of 1.376 days, resulting in 596 observations. The most-active traders were those that traded both for their customers and for their own account. On average, these traders traded on 8.537 days, resulting in 3,287 observations. Of these 8.537 days, these traders traded for their own account exclusively 3.013 days (1160 observations), exclusively for their customers 1.018 days (392 observations), and dual traded the remaining 4.506 days (1735 observations).

**Table II**  
**Trading Activity Statistics for Floor Traders in the Soybean**  
**Futures Pit at the Chicago Board of Trade**

Trading for own account refers to the trading days on which a floor trader traded exclusively for his own account. Trading for customers refers to the trading days on which a floor trader traded exclusively for customers. Dual trading refers to trading days on which a floor trader traded both for his own account and for his customers. The sample period consists of fifteen randomly selected trading days during the fourth quarter of 1988.

|                       | Trading For<br>Own Account | Trading For<br>Customers | Dual<br>Trading | All<br>Days |
|-----------------------|----------------------------|--------------------------|-----------------|-------------|
| Number of trader days | 1,795                      | 988                      | 1,735           | 4,518       |
| Total trades          | 63,550                     | 5,750                    | 236,682         | 305,982     |
| For own account       | 63,550                     | —                        | 130,586         | 194,136     |
| For customers         | —                          | 5,750                    | 106,096         | 111,846     |
| Total volume          | 274,721                    | 23,038                   | 999,445         | 1,297,204   |
| For own account       | 274,721                    | —                        | 615,541         | 890,262     |
| For customers         | —                          | 23,038                   | 383,904         | 406,942     |
| Average daily trades  | 35.40                      | 5.82                     | 136.42          | 67.73       |
| For own account       | 35.40                      | —                        | 75.27           | 42.97       |
| For customers         | —                          | 5.82                     | 61.15           | 24.76       |
| Average daily volume  | 153.05                     | 23.32                    | 576.05          | 287.12      |
| For own account       | 153.05                     | —                        | 354.78          | 197.05      |
| For customers         | —                          | 23.32                    | 221.27          | 90.07       |
| Average trade size    | 4.32                       | 4.01                     | 4.22            | 4.24        |
| For own account       | 4.32                       | —                        | 4.71            | 4.59        |
| For customers         | —                          | 4.01                     | 3.62            | 3.64        |

Table II presents trading activity statistics for all 4,518 observations in the sample. In terms of trading volume, dual traders are the most-active class of traders on the trading floor. About 69% of CTI 1 trading volume is executed by traders for their own account on days when they also trade for customers. Similarly, about 95% of all customer trading volume is executed by brokers who trade for their own account on the same day the customer's order is executed. On a typical trading day, dual traders execute about 75 trades for their own account and about 61 trades for their customers. In contrast, traders trading only for their own account average about 35 trades per day and brokers trading only for customers average about 6 trades per day.

### *C. Trading Profits*

The dual-trading model implies that floor traders who also serve as brokers for customers have an informational advantage over floor traders who trade only for their own account. If these brokers take advantage of their information by trading for their own account, then their trading profits should be

higher than those earned by non-dual-trading floor traders. Our data set allows us to test this implication directly by comparing the intraday trading profits for the two types of traders.

Table III reports summary statistics for the trading profits of floor traders who traded only for their own account during the trading day, and for the trading profits of floor traders who traded both for their own account and a customer during the trading day. There are 1795 observations for the non-dual-trading traders and 1735 observations for the dual-trading traders. The average trading profits for both types of traders are positive, with non-dual traders earning \$367.49 per day and dual traders earning \$1,234.02 per day from trading. Neither of these means, however, is significantly different from zero because of the large standard deviation of the distribution of trading profits. One reason for this large standard deviation is that the distribution of trading profits includes a number of extreme positive and negative observations. For example, the range of trading profits is more than 100 times the interquartile range for both dual and non-dual traders.

Because of these extreme values, it is more appropriate to test for differences between the two distributions using nonparametric methods. Table III shows that 54.2% of the profits for non-dual traders are greater than zero, while 58.7% of the profits for dual traders are greater than zero. Both of these percentages are significantly greater than 50% based on a standard binomial test—we reject the hypothesis that floor traders earn zero or negative trading profits on average.

The hypothesis that the trading profits of dual traders are greater than those of non-dual traders can be tested using a simple test for the difference in medians. The median trading profit for non-dual traders is \$125.00 and for dual traders is \$475.00. Thus, the median profit for dual traders is nearly four times that of non-dual traders. The normally distributed test statistic for the difference in medians is 4.14 standard deviations above zero and is highly statistically significant.<sup>15</sup> These results provide direct evidence that dual traders earn greater trading profits than non-dual traders.<sup>16</sup>

While these results are consistent with the implications of the dual-trading model developed in this paper, it is important to acknowledge that there may be alternative explanations for these results. For example, Grossman (1989) argues that dual traders may be more skilled at trading than non-dual traders. Further, this may be what brings them customers in the first place. If so, this greater skill would translate into higher trading profits.

<sup>15</sup> The nonparametric test for the difference in medians is described in Mood, Graybill, and Boes (1974), chapter 11.

<sup>16</sup> We also computed trading profits on a per-contract basis by dividing total trading profits by the total volume of contracts traded. Non-dual traders earn a median profit of 56.3 cents per contract while dual traders earn a median profit of 106.1 cents per contract. Per-contract trading profits, however, may understate the difference between the two types of traders. For example, a trader with special information may earn the same per-contract profit as a trader without special information, yet may earn higher total profits by choosing to trade more contracts because of his information.

**Table III**  
**Summary Statistics for the Distribution of Daily Trading**  
**Profits in Dollars for Floor Traders in the Soybean Futures Pit**  
**at the Chicago Board of Trade**

Trading for own account refers to the trading profits for days on which the floor trader traded only for his own account. Dual trading refers to the trading profits for days on which the floor trader traded both for his own account and for his customers. Both the test statistics for the test that the proportion of observations that are positive exceeds 0.50 and the test that the medians are the same are distributed as a standard unit normal under the null hypothesis. The sample period consists of fifteen randomly selected trading days during the fourth quarter of 1988.

|                                    | Trading For Own Account | Dual Trading |
|------------------------------------|-------------------------|--------------|
| Number of trader days              | 1795                    | 1735         |
| Mean profit                        | 367.49                  | 1,234.02     |
| Standard deviation                 | 14,338.12               | 29,127.26    |
| Minimum                            | -203,750.00             | -427,562.50  |
| 1st quartile                       | -1,200.00               | -2,550.00    |
| Median                             | 125.00                  | 475.00       |
| 3rd quartile                       | 1,600.00                | 5,025.00     |
| Maximum                            | 164,875.00              | 380,000.00   |
| Proposition > zero                 | 0.542                   | 0.587        |
| Test $z$ -statistic for $p = 0.50$ | 3.54                    | 7.22         |
| Difference in medians              | 350.00                  |              |
| Test $z$ -statistic                | 4.14                    |              |

To address this issue, we focus on the subset of 205 floor traders who traded exclusively for their own account on at least one trading day and who also dual traded on at least one trading day. For each of these traders, we compare the average daily trading profit on dual-trading days to the average daily trading profit on non-dual-trading days. Note that in doing this, we hold the traders fixed. If the superior performance of these dual traders is due to trading skill, then there should be no difference between their average daily trading profits. On the other hand, if the superior performance of these dual traders is due to their informational advantage, their trading profits on dual-trading days should exceed their trading profits on non-dual-trading days.

Table IV presents summary statistics for the differences between the average daily trading profits on dual-trading and non-dual-trading days. As shown, the median difference is \$763.02. The difference in average trading profits is positive for 56.6% of the dual traders. Based on a binomial test, this percentage is 1.89 standard deviations above 50%, which provides evidence that the daily trading profit on dual-trading days exceeds the daily trading profit on non-dual-trading days.<sup>17</sup> While these results do not rule out the

<sup>17</sup> As before, the difference in means is not statistically different.

Table IV

**Summary Statistics for the Distribution of Differences Between  
the Average Daily Trading Profits in Dollars for Dual Traders  
on Days that They Dual Traded and Their Average Daily  
Trading Profits on Days that They Did Not Dual Trade**

The test statistic for the test that the proportion of observations that are positive exceeds 0.50 is distributed as a standard unit normal under the null hypothesis. The sample period consists of fifteen randomly selected trading days during the fourth quarter of 1988.

|                                    | Difference   |
|------------------------------------|--------------|
| Number of dual traders             | 205          |
| Mean difference                    | 3.53         |
| Standard deviation                 | 18,680.45    |
| Minimum                            | - 182,000.00 |
| 1st quartile                       | - 2,630.90   |
| Median                             | 763.02       |
| 3rd quartile                       | 4,563.54     |
| Maximum                            | 62,475.00    |
| Proportion > zero                  | 0.566        |
| Test $z$ -statistic for $p = 0.50$ | 1.89         |

possibility that some portion of the superior performance of dual traders is related to trading skill, they do suggest that the major source is related to their informational advantage.

The distribution of trading profits provides a number of additional insights into the behavior of floor traders. For example, the relatively tight interquartile range for trading profits indicates that most traders employ conservative trading strategies. This is consistent with traditional models of floor-trader behavior such as Silber (1984) in which floor traders provide liquidity to the market but avoid taking large speculative positions. The extreme observations, however, indicate that some floor traders do take large speculative positions. For example, the largest single daily trading loss of \$427,562.50 occurred for a trader (call him trader X) who went long 2641 contracts and short 2136 contracts on October 7, 1988, when the nearby soybean futures price dropped 22.75 cents per bushel or \$1,137.50 per contract. Interestingly, trader X also experienced the second largest daily trading loss in the sample of \$282,287.50 several weeks later. During the 15-day sample period, trader X traded on 14 days, went long 24,031 contracts, went short 22,511 contracts, and lost a total of \$384,625.00.

The largest single daily trading gain of \$380,000.00 also occurred on October 7, 1988. On this day, a trader (call him trader Y) made only two trades for his own account, each time going short 200 contracts. Surprisingly, trader Y traded for his own account only 2 days during the sample period although he traded for customers on 12 different days. Including the 2 trades on October 7, 1988, trader Y made only 4 trades during the sample period.

Furthermore, the additional 2 trades involved a total of only 10 contracts. Thus, the highly profitable trades on October 7, 1988, were virtually the only trades trader Y made for his own account. Trader Y's total trading profits during the sample period were \$389,375.00. Interestingly, the trader on the other side of trader Y's trades on October 7, 1988 was trader X.

In addition to providing information about the profits of floor traders, the data set allows us to determine the aggregate trading profits of outside customers. A second major implication of the model is that dual-trading brokers trade when they believe that their customers are likely to be trading for informational reasons. This means that trading profits for outside customers should be greater on days when their broker chooses to dual trade.

Table V presents summary statistics for the trading profits of traders' customers. These trading profits are classified into two categories, profits for customers whose broker traded for his own account on the same day the order was executed and profits for customers whose broker did not trade for his own account on the day the order was executed. Since the number of customers varies across brokers, we standardize the results by dividing the aggregate customer profits for each day by the number of contracts traded by the customers. Thus, all profits are on a per-contract basis.

**Table V**  
**Summary Statistics for the Distribution of Daily Per-Contract**  
**Trading Profits in Dollars for Customers of Floor Traders in**  
**the Soybean Futures Pit at the Chicago Board of Trade**

Trading for customers refers to the per-contract profits of customers for days on which their broker did not trade for his own account. Dual trading refers to the per-contract trading profits of customers for days on which their broker did trade for his own account. Both the test statistics for the test that the proportion of observations that are positive exceeds 0.50 and the test that the medians are the same are distributed as a standard unit normal under the null hypothesis. The sample period consists of fifteen randomly selected trading days during the fourth quarter of 1988.

|                                    | Trading For Customers | Dual Trading |
|------------------------------------|-----------------------|--------------|
| Number of trader days              | 988                   | 1735         |
| Mean profit per contract           | -45.13                | -22.79       |
| Standard deviation                 | 368.57                | 301.95       |
| Minimum                            | -1,125.00             | -1,100.00    |
| 1st quartile                       | -250.00               | -175.00      |
| Median                             | -25.00                | -14.10       |
| 3rd quartile                       | 129.34                | 109.20       |
| Maximum                            | 1,175.00              | 1,150.00     |
| Percent > zero                     | 0.413                 | 0.453        |
| Test $z$ -statistic for $p = 0.50$ | -5.47                 | -3.89        |
| Difference in medians              | 10.90                 |              |
| Test $z$ -statistic                | 1.79                  |              |

As shown, the average per-contract profits for outside customers are negative,  $-\$45.13$  for customers of non-dual traders and  $-\$22.79$  for customers of dual traders. The median per-contract loss is  $-\$25.00$  for customers of non-dual traders and  $-\$14.10$  for customers of dual traders. The percentages of per-contract profits greater than zero are 41.3 and 45.3, respectively, for the two types of customers. Both of these percentages are significantly less than 50% based on a binomial test.

The hypothesis that the per-contract trading profits differ across broker type can be examined by a test of the difference in medians. The test statistic for equality of medians is 1.79 standard deviations above zero. Hence, the evidence provides support for the hypothesis that customers do better on days when their broker also trades for his own account. This is consistent with the implications of the dual-trading model.

## V. Conclusion

We have developed a model of dual trading in order to study its effects on the expected trading profits of futures market participants. Our model incorporates the idea that a broker knows more about his customers and their motives for trading than do other floor traders. This gives the broker an informational advantage that can be exploited by mimicking the trades of customers who are likely to be informed. This strategy implies that the expected trading profits of floor traders should be higher when they dual trade than when they trade exclusively for their own account. Conversely, customers should earn higher expected trading profits when their broker dual trades than when he trades exclusively for customers. We test these two empirical implications using intraday transactions data from the Chicago Board of Trade. A key feature of our data set is that we can identify the trades of each floor trader and his customers. We find that both of these implications are supported by the data.

We consider the effects of a ban on dual trading on expected trading profits. We find that net of commissions, the expected profits of customers who are likely to be informed increase while the expected profits of customers who are less likely to be informed decrease. We also study the effects of frontrunning on traders' expected profits. We show that brokers have incentives to trade more aggressively when they can frontrun their customers. Our results imply that customers who are likely to be informed are worse off with frontrunning while customers who are less likely to be informed prefer that their broker frontrun customers. These results are particularly relevant given recent moves by the CFTC and some futures exchanges toward banning dual trading. For example, in May 1991, the Chicago Mercantile Exchange prohibited dual trading in mature liquid contracts, i.e., contract months that have a daily trading volume of approximately 10,000 contracts for six months.

There are number of interesting directions for future research. For example, we have shown that traders may have divergent views on whether dual

trading should be banned. Given this divergence, which type of exchange is most likely to survive—an exchange that permits dual trading or one that bans it? What are the implications for current attempts by futures exchanges to move toward computerized trading? Finally, is there any role for regulation of dual trading?

## Appendix

*Proof of Proposition 1:* (i) If the first trade is a buy (sell), then there is a positive probability that it is for an informed customer who observed  $\theta = \theta_2$  ( $\theta = \theta_1$ ), and a zero probability that it is for an informed customer who observed  $\theta = \theta_1$  ( $\theta = \theta_2$ ). Therefore,  $P_1(\omega_1) = E[\theta | \omega_1]$  implies that  $P_1(1) > \bar{\theta} > P_1(-1)$ . If  $y_A = y_B = 0$ , the broker believes that the commodity is worth  $E[\theta | y_A = y_B = 0, h_A, h_B] = \bar{\theta}$ . Therefore, his expected profit is maximized by not trading. Since the broker does not trade when  $y_A = y_B = 0$ , the first trade must be for a customer. Thus,  $P_1(\omega_1) = E[\theta | \omega_1] = E[\theta | y_i]$  for  $i = A$  or  $B$ , and is given by (2) and (3).

(ii) Suppose  $y_A = 1$  and  $y_B = 0$  (by symmetry, the case of  $y_A = 0$  and  $y_B = 1$  is identical). The broker's expected profit is 0 if he does not trade,  $E[\theta | y_A = 1, y_B = 0, h_A, h_B] - P_2(1, 1)$  if he buys, and  $P_2(1, -1) - E[\theta | y_A = 1, y_B = 0, h_A, h_B]$  if he sells, where

$$E[\theta | y_A = 1, y_B = 0, h_A, h_B] = \frac{\theta_2(q + (1 - q)h_A/2) + \theta_1(1 - q)h_A/2}{q + (1 - q)h_A}. \quad (A1)$$

We first show that there is no equilibrium in which the broker sells. The broker's expected profit from buying is decreasing in  $h_A$ , and his expected profit from selling is increasing in  $h_A$ . This is because  $E[\theta | y_A = 1, y_B = 0, h_A, h_B]$  is decreasing in  $h_A$  and  $P_2(1, \omega_2)$  does not depend on  $h_A$  (the market maker does not observe  $h_A$ ). Therefore, if the broker's expected profit is maximized by selling when  $y_A = 1$  and  $y_B = 0$  for some value of  $h_A$ , say  $h'$ , then the broker's expected profit is maximized by selling for all  $h_A \geq h'$ . Let  $\hat{h}$  denote the minimum value of  $h_A$  for which the broker's expected profit is maximized by selling when  $y_A = 1$  and  $y_B = 0$ . The value  $\hat{h}$  would satisfy

$$P_2(1, -1) - E[\theta | y_A = 1, y_B = 0, h_A = \hat{h}, h_B] \geq 0. \quad (A2)$$

In such an equilibrium, it can be shown that  $P_2(1, -1) = E[\theta | \omega_1 = 1, \omega_2 = -1] = \beta(\hat{h})\theta_2 + (1 - \beta(\hat{h}))\theta_1$ , where

$$\beta(\hat{h}) = \frac{q\bar{h}/2 + (1 - q)\bar{h}^2/4 + 2(1 - \bar{h})(q + (1 - q)E[h_i | h_i > \hat{h}]/2)(1 - F(\hat{h}))}{q\bar{h} + (1 - q)\bar{h}^2/2 + 2(1 - \bar{h})(q + (1 - q)E[h_i | h_i > \hat{h}]/2)(1 - F(\hat{h}))}. \quad (A3)$$

Thus, (A2) is equivalent to

$$\beta(\hat{h}) \geq \frac{q + (1 - q)\hat{h}/2}{q + (1 - q)\hat{h}}. \quad (\text{A4})$$

Inequality (A4) cannot be satisfied though, since

$$\begin{aligned} \beta(\hat{h}) &\leq \frac{q\bar{h}/2 + (1 - q)\bar{h}^2/4 + 2(1 - \bar{h})(q + (1 - q)E[h_i | h_i > \hat{h}]/2)}{q\bar{h} + (1 - q)\bar{h}^2/2 + 2(1 - \bar{h})(q + (1 - q)E[h_i | h_i > \hat{h}])} \\ &\leq \frac{q\bar{h}/2 + (1 - q)\bar{h}^2/4 + 2(1 - \bar{h})(q + (1 - q)\hat{h}/2)}{q\bar{h} + (1 - q)\bar{h}^2/2 + 2(1 - \bar{h})(q + (1 - q)\hat{h})} \\ &= \frac{q\bar{h}/(4(1 - \bar{h})) + (1 - q)\bar{h}^2/(8(1 - \bar{h})) + q + (1 - q)\hat{h}/2}{2(q\bar{h}/(4(1 - \bar{h})) + (1 - q)\bar{h}^2/(8(1 - \bar{h}))) + q + (1 - q)\hat{h}} \\ &< \frac{q + (1 - q)\hat{h}/2}{q + (1 - q)\hat{h}}. \end{aligned} \quad (\text{A5})$$

Thus, there is no equilibrium in which the broker sells when  $y_A = 1$  and  $y_B = 0$ . An analogous argument implies that there is no equilibrium in which the broker buys when  $y_i = -1$  and  $y_j = 0$ . Since the broker never sells after a customer buy or buys after a customer sell,  $P_2(1, -1) = E[\theta | \omega_1 = 1, \omega_2 = -1] = E[\theta | y_i = 1, y_j = -1] = E[\theta | \omega_1 = -1, \omega_2 = 1] = P_2(-1, 1)$ , and is given by (4) and (5).

We now show that in equilibrium, the broker buys if  $h_A < h^*$  and does not trade if  $h_A \geq h^*$ , where  $h^*$  satisfies (8). There is a positive probability that two consecutive buys are for uninformed customers. Therefore,  $P_2(1, 1) = E[\theta | \omega_1 = 1, \omega_2 = 1] < \theta_2$ . Since the broker's expected profit from buying equals  $\theta_2 - P_2(1, 1) > 0$  at  $h_A = 0$  and is decreasing in  $h_A$ , in equilibrium, there is a critical value, call it  $h^*$ , such that for  $h_A < h^*$  ( $h_A \geq h^*$ ), the broker's expected profit is maximized by buying (not trading). The critical value,  $h^*$ , satisfies

$$E[\theta | y_A = 1, y_B = 0, h_A = h^*, h_B] - P_2(1, 1) = 0. \quad (\text{A6})$$

By the definition of  $\alpha(\cdot)$ , in equilibrium,  $P_2(1, 1) = E[\theta | \omega_1 = 1, \omega_2 = 1]$  implies a price given by (6). Thus, using (A1), it follows that (A6) is equivalent to (8). Rewriting (8) using (1), multiplying through, and rearranging, yields

$$\begin{aligned} &(1 - q)(1 - \bar{h})(h^* - E[h_i | h_i < h^*])F(h^*) \\ &= \frac{(1 - q)\bar{h}^2}{4} - \frac{(q + (1 - q)\bar{h})}{2}h^*. \end{aligned} \quad (\text{A7})$$

The left side of (A7) is less (greater) than the right side of (A7) at  $h^* = 0$  ( $h^* = 1$ ). Further, the left (right) side of (A7) is increasing (decreasing) in  $h^*$

for  $0 \leq h^* \leq 1$ . Thus, there is a unique  $0 < h^* < 1$  for which (8) is satisfied. In equilibrium, the broker buys if  $h_A < h^*$  and does not trade if  $h_A \geq h^*$ . An analogous argument implies that in equilibrium,  $P_2(-1, -1)$  is given by (7) and if  $y_i = -1$  and  $y_j = 0$ , then the broker sells if  $h_i < h^*$  and does not trade if  $h_i \geq h^*$ . Q.E.D.

*Proof of Proposition 3:* We show first that there is no equilibrium in which the broker sells (buys) before a customer buy (sell). Suppose  $y_A = 1$  and  $y_B = 0$  (by symmetry the case of  $y_A = 0$  and  $y_B = 1$  is identical). The broker's expected profit is 0 if he does not trade,  $E[\theta | y_A = 1, y_B = 0, h_A, h_B] - P_1(1)$  if he buys, and  $P_1(-1) - E[\theta | y_A = 1, y_B = 0, h_A, h_B]$  if he sells, where  $E[\theta | y_A = 1, y_B = 0, h_A, h_B]$  is given by (A1). The broker's expected profit from buying is decreasing in  $h_A$  and his expected profit from selling is increasing in  $h_A$ . This is because  $E[\theta | y_A = 1, y_B = 0, h_A, h_B]$  is decreasing in  $h_A$  and  $P_1(\omega_1)$  does not depend on  $h_A$  (the market maker does not observe  $h_A$ ). Therefore, if the broker's expected profit is maximized by selling when  $y_A = 1$  and  $y_B = 0$  for some value of  $h_A$ , say  $h'$ , then the broker's expected profit is maximized by selling for all  $h_A \geq h'$ . Let  $\hat{h}$  denote the minimum value of  $h_A$  for which the broker would sell when  $y_A = 1$  and  $y_B = 0$ . The value  $\hat{h}$  would satisfy

$$P_1(-1) - E[\theta | y_A = 1, y_B = 0, h_A = \hat{h}, h_B] \geq 0. \quad (\text{A8})$$

Besides the broker selling before a customer buy when  $h_i \geq \hat{h}$ , the customer and broker trades that give rise to  $\omega_1 = -1$  are  $y_i = y_j = -1$  and  $x = 0$ ,  $y_i = x = -1$  and  $y_j = 0$ , or  $y_i = y_j = 0$  and  $x = -1$ , where  $x$  is the broker's trade. Conditional on each of these three possibilities, the expectation of  $\theta$  is less than or equal to  $\bar{\theta}$ . Therefore, since  $E[\theta | y_A = 1, y_B = 0, h_A \geq \hat{h}, h_B] > \bar{\theta}$ , it follows that  $P_1(-1) = E[\theta | \omega_1 = -1] < E[\theta | y_A = 1, y_B = 0, h_A \geq \hat{h}, h_B] < E[\theta | y_A = 1, y_B = 0, h_A = \hat{h}, h_B]$ , for all  $\hat{h}$ . This implies that (A8) cannot be satisfied. Thus, there is no equilibrium in which the broker sells before a customer buy. An analogous argument implies that there is no equilibrium in which the broker buys if  $y_i = -1$  and  $y_j = 0$ . Since the broker never sells (buys) before a customer buy (sell),  $P_2(1, -1) = E[\theta | \omega_1 = 1, \omega_2 = -1] = E[\theta | y_i = 1, y_j = -1] = E[\theta | \omega_1 = -1, \omega_2 = 1] = P_2(-1, 1)$ , and is given by (4) and (5).

We now show that there is no equilibrium in which the broker trades when neither customer trades. Suppose the first trade is a buy. Since the broker never buys before a customer sell, the market maker knows that there is a positive probability that the trade is for an informed customer who observed  $\theta = \theta_2$ , and a zero probability that it is for either an informed customer who observed  $\theta = \theta_1$  or a broker who is trading before an informed customer who observed  $\theta = \theta_1$ . Therefore,  $P_1(1) = E[\theta | \omega_1 = 1] > \bar{\theta}$ . An analogous argument implies that  $P_1(-1) < \bar{\theta}$ . If neither customer trades, then the broker believes that the commodity is worth  $E[\theta | y_A = y_B = 0, h_A, h_B] = \bar{\theta}$  and his expected profit is maximized if he does not trade.

The above results imply that the joint density of the random variables  $\omega_1$ ,  $y_A$ ,  $y_B$ ,  $h_A$ , and  $h_B$  is not changed by frontrunning. If  $y_A = y_B = 0$ , then

$\omega_1 = 0$  with or without frontrunning. If  $y_A \neq 0$  and  $y_B \neq 0$ , then the broker does not trade, with or without frontrunning, and with probability  $1/2$  ( $1/2$ ),  $\omega_1 = y_A$  ( $\omega_1 = y_B$ ). If  $y_i \neq 0$  and  $y_j = 0$ , then whether or not the broker frontruns customer  $i$  with a trade of  $y_i$  (the broker never frontruns a customer with the opposite trade),  $\omega_1 = y_i$ . Thus, just as in the case without frontrunning,  $P_1(\omega_1) = E[\theta | \omega_1] = E[\theta | y_i]$  for  $i = A$  or  $B$  and is given by (2) and (3).

We now show that in equilibrium, if  $y_A = 1$  and  $y_B = 0$ , then the broker frontruns customer  $A$  and buys if  $h_A < \bar{h}$  and does not trade if  $h_A \geq \bar{h}$  (by symmetry, the case of  $y_A = 0$  and  $y_B = 1$  is identical). It was shown above that the broker will not sell in this case. The broker's expected profit from not trading is 0 and his expected profit from buying is

$$\begin{aligned} & E[\theta | y_A = 1, y_B = 0, h_A, h_B] - P_1(1) \\ &= \frac{\theta_2(q + (1 - q)h_A/2) + \theta_1(1 - q)h_A/2}{q + (1 - q)h_A} \\ &\quad - \frac{\theta_2(q + (1 - q)\bar{h}/2) + \theta_1(1 - q)\bar{h}/2}{q + (1 - q)\bar{h}} \\ &\geq 0 \text{ as } h_A \leq \bar{h}. \end{aligned} \tag{A9}$$

Therefore, the broker's expected profit is maximized by buying if  $h_A < \bar{h}$  and not trading if  $h_A \geq \bar{h}$ . An analogous argument implies that if  $y_i = -1$  and  $y_j = 0$ , then the broker's expected profit is maximized by selling if  $h_i < \bar{h}$  and not trading if  $h_i \geq \bar{h}$ . By the definition of  $\alpha(\cdot)$ , in equilibrium,  $P_2(1, 1) = E[\theta | \omega_1 = 1, \omega_2 = 1]$  and  $P_2(-1, -1) = E[\theta | \omega_1 = -1, \omega_2 = -1]$  imply prices given by (21) and (22). Q.E.D.

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