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Seasonality and Consumption-Based Asset Pricing

WAYNE E. FERSON and CAMPBELL R. HARVEY*

ABSTRACT

Most of the evidence on consumption-based asset pricing is based on seasonally adjusted consumption data. The consumption-based models have not worked well for explaining asset returns, but with seasonally adjusted data there are reasons to expect spurious rejections of the models. This paper examines asset pricing models using not seasonally adjusted aggregate consumption data. We find evidence against models with time-separable preferences, even when the models incorporate seasonality and allow seasonal heteroskedasticity. A model that uses not seasonally adjusted consumption data and nonseparable preferences with seasonal effects works better according to several criteria. The parameter estimates imply a form of seasonal habit persistence in aggregate consumption expenditures.

STUDIES OF CONSUMPTION-BASED asset pricing have typically used data smoothed with the X-11 seasonal-adjustment program. The seasonally adjusted data are weighted averages of past and (in revised data) future expenditures. Such data smoothing creates potential problems. The future expenditures which X-11 includes in the current period data cannot provide utility in the current period. Obviously, one does not purchase goods at seasonally adjusted prices. Furthermore, seasonal adjustment can induce spurious correlation between the error terms of a model and past values of the variables, leading to bias and erroneous inferences (Wallis (1974)).

The effect of seasonal adjustment on the asset pricing evidence has received only limited attention in the literature. Miron (1986) extended the seminal work of Hansen and Singleton (1982) to incorporate seasonal “taste shocks.” English, Miron and Wilcox (1989) rejected Miron’s model using not seasonally adjusted data. However, they studied only a single asset, which provides no evidence about the effects of seasonality on excess returns or risk premiums. Furthermore, they mistakenly used a Treasury bill *discount rate* as their return.

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Previous studies have observed that consumption appears “too smooth” to fit average equity and bill returns in simple models, without resorting to implausibly large values of risk aversion. Mehra and Prescott (1985) described this “equity premium puzzle.” Dunn and Singleton (1986) found that the consumption covariances of Treasury bills differ too little relative to their average return differences. Such results, based on seasonally adjusted consumption, could potentially be an artifact of data smoothing.¹

We find that seasonal adjustment does not explain the poor fit of the simple consumption model to asset return data. For example, using not seasonally adjusted consumption, we examine the values of the concavity parameter for a time-separable utility function that are implied by a sample of average asset returns and covariances with consumption. The parameter values would be considered implausible by most observers. Furthermore, we find that the variation over time in conditional expected risk premiums is large relative to the variation over time in the conditional covariances of excess returns with consumption. Thus, a pattern similar to the equity premium puzzle exists at the level of *conditional* expectations.

We extend the investigation of seasonality in several ways. First, we generalize the model of seasonality proposed by Miron (1986). In Miron’s model the seasonal taste or technology parameters are multiplicative, and they do not enter the relation of expected risk premiums to consumption. When estimated using a cross-section of asset returns, such a model tends to “overadjust” for the seasonality in consumption. We study models with “subsistence levels,” in which the seasonal parameters are related to expected risk premiums. When the subsistence levels are deterministic, the time-separable models with seasonality still do not provide a good fit of the consumption and return data.

A model incorporating seasonal time-nonseparability fares better in the tests. In this model, the subsistence level is assumed to depend on a previous consumption level in the same season. If the coefficient on the lagged consumption is positive the model exhibits habit persistence, similar to Sundaresan (1989) and Constantinides (1990). If the coefficient is negative the model reflects the durability of seasonal consumption expenditures. We compare the results for this model with a nonseasonal, time-nonseparable model studied by Ferson and Constantinides (1991). The results indicate that seasonal habit persistence in aggregate consumption is empirically relevant.

This paper is organized as follows. Section I presents the models and Section II describes the empirical methodology. Section III presents the data, focusing on the differences between seasonally adjusted and not seasonally adjusted consumption. Section IV presents the empirical results and Section V is the conclusion.

¹ Mehra and Prescott used annual data. However, prior to 1929 the data are five-year moving averages from Kuznets. The Commerce Department constructs annual data from 1947 by aggregating the seasonally adjusted quarterly data. Dunn and Singleton (1986) used seasonally adjusted monthly consumption.

I. The Models

This section of the paper is organized in two subsections. In the first, we review the time-separable consumption models and motivate why we incorporate a subsistence level of consumption in the model. Part B describes the time-nonseparable models, reviewing the nonseasonal nonseparable model and presenting the seasonal nonseparable model that represents the main contribution of this paper.

A. Time-Separable, Consumption-Based Asset Pricing Models

Consumption-based models imply that in equilibrium the price of an asset is the expected discounted value of its future payoff, weighted by a marginal utility of consumption. Given a representative agent with a time-additive utility function $\sum_t \beta^t u(C_t)$, this condition can be written (Lucas (1978)) as:

$$E \left\{ \beta \left[\frac{u'(C_{t+1})}{u'(C_t)} \right] R_{t+1} \mid \Omega_t \right\} = 1, \quad (1)$$

where Ω_t is the public information at time t , R_{t+1} is one plus a real rate of return, $u'(C)$ is a marginal utility of consumption, and β is a time preference parameter. Assume a power utility function:

$$u(C) = \frac{C^{1-\alpha}}{1-\alpha} \quad (2)$$

where $\alpha > 0$ is the concavity parameter. Substituting equation (2) into equation (1) and applying the law of iterated expectations implies:

$$E \left\{ \beta \left[\frac{C_{t+1}}{C_t} \right]^{-\alpha} R_{t+1} \mid Z_t \right\} = 1 \quad (3)$$

where Z_t is a vector of instruments, i.e., any subset of the time- t public information Ω_t .

Equation (3) is the model studied by Hansen and Singleton (1982). If there is variation over time in the real returns that is predictable using Z_t , the model implies that the predictability is removed when R_{t+1} is multiplied by consumption growth raised to the correct power. This implies that a “seasonal” or other pattern in consumption growth that is predictable using Z_t should be reflected in the real rates of return of all assets. However, as we show below, any seasonal variation in expected asset returns is minuscule compared with the seasonal fluctuations in consumption.

Miron (1986) hypothesizes that there are seasonal “shocks” to preferences, replacing the utility function (2) with

$$u(C, s, t) = \left\{ [C_{\kappa_{s(t)}}]^{1-\alpha} \right\} / (1-\alpha),$$

where $\kappa_{s(t)}$ is a taste shift parameter indicating that the utility obtained from the level of consumption C at time t varies according to the prevailing season $s(t)$. $C_t \kappa_{s(t)}$ may be thought of as the “seasonally adjusted” consumption.² With this utility function, equation (1) implies:

$$E \left\{ \beta \left[\frac{C_{t+1}}{C_t} \right]^{-\alpha} \left[\frac{\kappa_{s(t+1)}}{\kappa_{s(t)}} \right]^{1-\alpha} R_{t+1} \mid Z_t \right\} = 1. \quad (4)$$

The Euler equation (4) implies that a multiplicative seasonal adjustment using $\kappa_{s(t)}$ should reduce the predictable seasonal fluctuations in consumption to a level which can be exactly mirrored in the real returns of all assets. Since the expectation of the product is the product of the expectations plus the covariance, expected returns differ across assets depending on their covariances with consumption, raised to the power $-\alpha$. Taking the differences in the equation (4), stated for two assets (subscripted i and f) implies:

$$E \left\{ \left[\frac{C_{t+1}}{C_t} \right]^{-\alpha} (R_{i,t+1} - R_{f,t+1}) \mid Z_t \right\} = 0. \quad (5)$$

The time preference and the seasonal taste shift parameters that appear in equation (4) are included in the consumer's information set and they cancel out of equation (5). These parameters are not identified using the expected risk premiums, because their effects are common to all assets in Miron's (1986) model of seasonality. We generalize the specification of the taste shift parameters, replacing the utility function with:

$$U(C, s, t) = (1 - \alpha)^{-1} [\kappa_{s(t)} C - \delta(t)]^{1-\alpha}.$$

The parameter $\delta(t)$ is a “subsistence level,” which represents the minimum level of seasonally adjusted consumption considered tolerable by the consumer. Consider a simple specification in which the subsistence level is a deterministic function of time: $\delta(t) = \delta e^{\mu t}$.³ This is similar to specifications examined by Eichenbaum and Hansen (1990) and Heaton (1990b), using seasonally-adjusted data. The Euler equation implies:

$$E \left[\left(\frac{\kappa_{s(t)} C_t - \delta e^{\mu t}}{\kappa_{s(t+1)} C_{t+1} - \delta e^{\mu(t+1)}} \right)^\alpha (R_{i,t+1} - R_{f,t+1}) \mid Z_t \right] = 0. \quad (6)$$

² Alternatively, the κ'_s can be interpreted as parameters of a household production function. Miron models $\kappa_{s(t)}$ in two ways. In his first model, $\ln \kappa_{s(t)} = \sum_{s=2,3,4} (G_s d_{s(t)} + G_5 t + G_6 t^2)$; where $d_{s(t)}$ is a seasonal dummy indicator, t is time and the G_i are parameters. In his second model, the taste shocks depend on a measure of temperature and a measure of precipitation. He found that the seasonal dummies largely subsume the weather variables.

³ In a previous version of this paper we examined a model in which the subsistence level depends on the season, but is not growing over time. In an economy with per capita growth in consumption, a fixed subsistence level would eventually become irrelevant. However, the results for this model with seasonal subsistence were similar to the results that we report for the growing subsistence level model.

When the parameter δ differs from zero, the $\kappa_{s(t)}$ parameters do not cancel out of the expression for risk premiums.

B. Time-Nonseparable Preferences

The models of equations (3)–(6) assume that the utility function is time-separable and that the consumption good is nondurable. Habit persistence in consumption preferences, following Sundaresan (1989) and Constantinides (1990), and durability of consumption goods following Dunn and Singleton (1986), both imply time nonseparability in the derived utility for consumption expenditures. Time nonseparability means that the marginal utility of the time- t expenditures C_t depends on the levels of expenditures at other dates as well. Ferson and Constantinides (1991) study a model in which lagged consumption expenditures determine a subsistence level and consumption expenditures may be durable. Their empirical model is based on the Euler equation (7):

$$u_{t+1} = -1 + \beta \left\{ \left[\frac{C_{t+1} - bC_t}{C_t - bC_{t-1}} \right]^{-\alpha} - b\beta \left[\frac{C_{t+2} - bC_{t+1}}{C_t - bC_{t-1}} \right]^{-\alpha} \right\} R_{t+1} \\ + b\beta \left[\frac{C_{t+1} - bC_t}{C_t - bC_{t-1}} \right]^{-\alpha} \\ E\{u_{t+1} | Z_t\} = 0 \quad (7)$$

Habit persistence implies that the lagged expenditures enter the indirect utility at time t as a negative effect ($b > 0$), while durability implies a positive effect ($b < 0$). Constantinides (1990) finds that habit persistence increases the implied variance of marginal utility and asset returns, thereby providing a partial resolution of the equity premium puzzle. Ferson and Constantinides (1991) find evidence for habit persistence in monthly, quarterly, and annual data. However, they use seasonally adjusted consumption data.

We consider a simple form of time nonseparability which emphasizes seasonality, and we use seasonally unadjusted consumption data. In our model the consumer is assumed to maximize the expected value of the utility function $\sum_t \beta^t (C_t - bC_{t-4})^{1-\alpha}$, where α is the concavity parameter. The parameter b measures seasonal habit persistence ($b > 0$) or durability ($b < 0$) of the consumption expenditures at time t . The timing convention assumes that the consumption expenditure decisions C_t are quarterly. The Euler equation is derived by a standard perturbation argument. The consumer considers the marginal effects of a prospective reduction in current expenditures C_t by a small amount, ε , which is invested in some asset with a quarterly gross real return, R_{t+1} . The proceeds εR_{t+1} are realized at time $t + 1$. This affects the subsistence levels that will be relevant at the future dates $t + 4$ and $t + 5$ in addition to the consumption at dates t and $t + 1$.

The net expected utility change is zero at an optimum. The following Euler equation is implied:⁴

$$u_{t+1} = -1 + \beta \left\{ \left[\frac{C_{t+1} - bC_{t-3}}{C_t - bC_{t-4}} \right]^{-\alpha} - b\beta^4 \left[\frac{C_{t+5} - bC_{t+1}}{C_t - bC_{t-4}} \right]^{-\alpha} \right\} R_{t+1} \\ + b\beta^4 \left[\frac{C_{t+4} - bC_t}{C_t - bC_{t-4}} \right]^{-\alpha} \\ E\{u_{t+1} | Z_t\} = 0 \quad (8)$$

The assumption that the subsistence level (in the case of habit persistence) or the flow of services (in the case of durability) depends only on the consumption expenditure in the same quarter of the previous year is obviously a strong one. Formally, this imposes zero restrictions on lagged values for the other quarters. A more general model would allow more recent and possibly more distant quarters to determine the subsistence level and/or the flow of consumption services, and would incorporate additional parameters representing the seasonal shocks. We experimented with some more complex models, in which more recent quarters entered the model and there were seasonal taste shift parameters. We were unable to reliably estimate models with the larger numbers of parameters. Therefore, we did not formally test the zero restrictions.

Imposing the zero restrictions implicit in equation (8) may be empirically reasonable. Ferson and Constantinides (1991) studied models with more than a single lagged consumption value and they were unable to separately estimate the individual coefficients for more than a single lag. This means that it is hard to tell which lags are the most important for determining the subsistence level. Given that consumption expenditures are highly autocorrelated in the levels, past values of expenditures at different lags could potentially proxy about as well for the subsistence level. Equation (8) has the advantage that only a single parameter is used to capture both time-nonseparability and seasonality.

The seasonal, nonseparable model, equation (8), is a parsimonious way to control seasonality for asset pricing purposes. We show below that a large fraction of consumption expenditures are seasonal. Seasonal fluctuations in consumption growth represent on the order of 90% of the variance of data. Therefore, focusing on nonseparability at the seasonal lag seems like a sensible approach to modelling the not seasonally adjusted data. Consider seasonal clothing (included in "nondurables") as a plausible example of seasonal durability. Seasonal habit persistence may be relevant for expenditures associated with vacations and other seasonal consumption.

⁴ Heaton (1990b) studies a similar model of seasonality. He assumes a single asset with a fixed real return which is a constant over time, and focuses on the properties of the endogenous consumption process under time aggregation. Winder and Palm (1990) studied a linearized approximation to the nonseparable Euler equation, in which they found support for seasonal habit persistence in the Netherlands.

II. Empirical Methods

A. Generalized Method of Moments Estimation

Consider any of the Euler equations (4–8). The value of the expression inside the conditional expectation operator at time t defines an error term u_{t+1} that should have conditional mean zero, given information at time t , if the model is correct. This implies $E(u_{t+1} | Z_t) = 0$ and therefore $E(u_{t+1} Z_t) = 0$, if Z_t is a set of instruments known by the market at time t . The parameters of the model are estimated and the model is tested by exploiting these orthogonality conditions using Hansen's (1982) generalized method of moments (GMM). Given N asset returns and L instruments Z_t , there are $N \times L$ orthogonality conditions. The GMM estimates are based on minimizing a quadratic form $(g' W g)$, where $g = \text{vec}(\sum_t u_{t+1} Z_t) / T$ is a vector of sample orthogonality conditions. The weighting matrix W is the inverse of the covariance matrix of the orthogonality conditions.

In the time-separable models u_t is a function of the variables R_t , c_{t-1} , and c_t , which are known at time t . Since u_t is in the consumer's information set at time t , the Euler equations imply that $E[u_{t+j} | u_t] = 0$, $j > 0$, and $E[u_{t+j} u_t] = 0$ for any j not equal to zero. The error term follows a moving average process of order zero (denoted $MA(0)$). In the seasonal time-nonseparable model of equation (8), u_t is a function of c_{t+j} , for $j = 1, 2, 3$, and 4, as well as R_t , c_{t-1} , and c_t . Since u_t is not known at time t , the Euler equation does not imply that $E[u_{t+1} | u_t] = 0$, but it does imply that $E[u_{t+j} | u_t] = 0$, $j > 4$. In this model, u_t is $MA(4)$. In the model of equation (7), the error term is $MA(1)$. The weighting matrix W is adjusted to account for the moving average terms, as in Hansen (1982).

In the time-nonseparable Euler equation (8), if we choose $\alpha = 0$ we obtain $u_{t+1} = (1 - b\beta^4) + \beta R_{t+1}(1 - b\beta^4)$. If we also choose $1 = b\beta^4$ we obtain a trivial solution to the Euler equation. Following Eichenbaum and Hansen (1990) and Ferson and Constantinides (1991), we note that the orthogonality condition $E(u_{t+1} | Z_t) = 0$ will still hold if u_{t+1} is divided by something in the information set at time t . We divide u_{t+1} by $(1 - b\beta)$ when we estimate equation (7) and by $(1 - b\beta^4)$ when we estimate equation (8), in order to avoid trivial solutions to the Euler equations.

Hansen (1982) provided sufficient conditions under which the GMM estimates are consistent and asymptotically normal and the minimized value of the quadratic form is asymptotically distributed as a chi-square. The minimized objective function provides a test statistic for the goodness-of-fit of the model. The number of degrees of freedom of the statistic is the difference between the number of orthogonality conditions and the number of parameters to be estimated.

Hansen (1982) showed that sufficient conditions for the asymptotic properties of the GMM include strict stationarity of the data. Strict stationarity may be violated for some kinds of seasonal variation. However, consistency and asymptotic normality of the estimators and the asymptotic distribution

of the test statistic can be demonstrated under weaker conditions.⁵ Stationarity may also be violated under some models of growth in consumption. We assume that the growth rates of real, per capita consumption expenditures are stationary in our empirical work. In the model of equation (6) we make the stronger assumption that the geometrically detrended levels of the expenditures are also stationary.⁶

The GMM methods are justified from asymptotic distribution theory, so there is a natural concern about their properties in small samples. Tauchen (1986) and Kocherlakota (1990) provided simulation evidence for the time-separable model, equation (3). Tauchen found that the test statistics perform well with as few as 50 annual observations, although he found a slight tendency to reject the model too infrequently. Kocherlakota, using a different set of parameter values, found cases where the model is rejected too often.⁷

Ferson and Foerster (1991) examined the finite sample properties of the GMM in a different conditional asset pricing context, using multiple asset returns. They found that a two-stage GMM approach, as described in Hansen and Singleton (1982), tends to reject the models too often in larger systems, while an iterated GMM approach provides more accurate test statistics. We use an iterated GMM approach in this study.⁸

If Z_t is chosen to be the null information set, the implications of equations (3)–(8) can be examined for unconditional, or “average” expected returns. We present evidence at the level of conditional and unconditional moments.

B. Restrictions on Intertemporal Marginal Rates of Substitution

Each consumption model implies that any real return R_{t+1} multiplied by an intertemporal marginal rate of substitution, m_{t+1} , has a constant conditional expectation:

$$E\{m_{t+1}R_{t+1} | Z_t\} = 1. \quad (9)$$

The form of m_{t+1} differs across the models. Hansen and Jagannathan (1991a) showed how the Euler equations place inequality restrictions on the mean and variance of the marginal rate of substitution. These bounds depend on the sample of assets. Using equation (9), iterated expectations implies that $E(m_{t+1}R'_{t+1}) = \underline{1}'$, where $E\{\cdot\}$ denotes the unconditional expectations and $\underline{1}'$ is a row vector of ones. A “benchmark” return (see Hansen and Richard

⁵ See Jagannathan (1983) and Lim (1985) for analyses of the asymptotic properties of the GMM under seasonality and nonstationarity.

⁶ Specifically, we assume that $C_t e^{-\mu t}$ is strictly stationary for the model of equation (6). This is stronger than assuming that the growth rates, (C_{t+1}/C_t) are stationary. See Eichenbaum and Hansen (1990) for technologies that are consistent with these assumptions about growth. See Stock and Watson (1988) for a review of variable trends in economic time series.

⁷ Kocherlakota (1990) uses $\alpha = 13.7$ and $\beta = 1.14$ as his parameter values.

⁸ Specifically, we construct the weighting matrix W using the parameter estimates from the n th stage, use this matrix to find parameters for stage $n + 1$ which minimize the quadratic form, and then use the new parameters to update the weighting matrix. The iterations continue until the objective function converges.

(1987)) can be formed as a linear combination of the assets: $m_{t+1}^* = w' R_{t+1}$ so that $E(m_{t+1}^* R_{t+1}') = \underline{1}'$. The regression weights $w' = \underline{1}' E(R_{t+1} R_{t+1}')^{-1}$ define a benchmark m_{t+1}^* that satisfies the regression relation $m_{t+1} = m_{t+1}^* + \varepsilon_{t+1}$, where ε_{t+1} is a projection error uncorrelated with R_{t+1} and m_{t+1}^* . Therefore (dropping the time subscripts):

$$\begin{aligned} \text{var}(m) &\geq \text{var}(m^*) \\ &= \underline{1}' [E(RR')^{-1}] V(R) [E(RR')^{-1}] \underline{1}, \end{aligned} \quad (10)$$

where $V(R)$ is the unconditional covariance matrix of returns. Using the definition of the covariance and the fact that $E(mR') = \underline{1}'$ we have:

$$\begin{aligned} E(m)E(R') &= \underline{1}' - \text{cov}(m, R) \\ &= \underline{1}' - \text{cov}(m^*, R) \\ &= \underline{1}' - \underline{1}' [E(RR')^{-1}] V(R) \end{aligned} \quad (11)$$

Substituting from equation (11) into the right hand side of equation (10) implies:

$$\begin{aligned} \text{var}(m) &\geq [\underline{1}' - E(m)E(R')] [E(RR')^{-1}] \underline{1} \\ &= [\underline{1}' - E(m)E(R')] \{V(R)^{-1}\} [\underline{1}' - E(m)E(R')]' . \end{aligned} \quad (12)$$

For a given sample of returns, equation (12) provides a lower bound for the variance of m_{t+1} as a function of the mean of m_{t+1} .⁹ The Hansen-Jagannathan bounds therefore provide an informal diagnostic for comparing the fit of the different models of m_{t+1} . For the purposes of this diagnostic the model of m_{t+1} includes the choice of the utility function parameters.

The time-nonseparable Euler equations (7) and (8) complicate the Hansen-Jagannathan analysis. These equations are of the form $E(m_{1,t+1} R_{t+1}' | Z_t) = E(m_{2,t+1} | Z_t)$, where $m_{1,t+1}$ and $m_{2,t+1}$ are functions of lagged and future consumption expenditures and the model parameters. Therefore, the nonseparable models imply that equation (9) holds if we define the m_{t+1} as $m_{1,t+1} / E(m_{2,t+1} | Z_t)$. When it is desired to exploit the information in the conditional moments the models are more complex than the time-separable models, because the conditional expectation in the denominator must be estimated (see Gallant, Hansen, and Tauchen (1990) for an example). However, using unconditional moments these complications can be avoided.

By iterated expectations the nonseparable Euler equations imply that $E(m_{1,t+1} R_{t+1}') = E(m_{2,t+1})$. Fix the choice of the other model parameters

⁹ The bound in (12) is not the sharpest lower bound that can be derived. Hansen and Jagannathan (1991a) show how imposing that m_{t+1} is a strictly positive random variable can sharpen the bound. Computing the bounds imposing positivity requires a numerical search procedure. However, the results of Hansen and Jagannathan show that the bounds imposing positivity are nearly coincident with the simpler bounds in the portion of the parabola where the standard deviation is low, and depart from the simpler bounds only when the standard deviation is relatively high. Imposing positivity is unlikely to materially affect our analysis.

and let $E(m_{2,t+1}) = \gamma$ be a parameter to be estimated. Define the relevant m_{t+1} as $m_{1,t+1}/\gamma$; then we have that $E(m_{t+1}R_{t+1}) = 1$ for all asset returns and therefore equation (12) must hold for this m_{t+1} . We are interested in estimating the unconditional mean, μ_m and the variance σ_m^2 of this m_{t+1} . The following moment conditions can be used to obtain the GMM estimators of the relevant parameters:¹⁰

$$\begin{aligned} u_{1,t+1} &= \gamma - m_{2,t+1} \\ u_{2,t+1} &= \mu_m - (m_{1,t+1}/\gamma) \\ u_{3,t+1} &= \sigma_m^2 - [(m_{1,t+1}/\gamma) - \mu_m]^2. \end{aligned} \quad (13)$$

The instruments consist of a vector of ones and the orthogonality conditions are given by $g = \sum_t (u_{1,t}, u_{2,t}, u_{3,t})/T$. The system is exactly identified, with three orthogonality conditions and three parameters, and the GMM estimates can be obtained in closed form by setting the sample orthogonality conditions to zero. The solution is simple. The GMM estimate of γ is the sample mean of the $m_{2,t+1}$. The estimate of μ_m is given by the sample mean of $\{m_{1,t+1}/(\sum_t m_{2,t+1}/T)\}$ and the estimate of σ_m^2 is given by the sample variance of $\{m_{1,t+1}/(\sum_t m_{2,t+1}/T)\}$. That is, we may simply divide $m_{1,t+1}$ by the sample mean of $m_{2,t+1}$ and proceed with the analysis as in the time-separable model.

Our approach may be contrasted with the approach of Gallant, Hansen, and Tauchen (1990) who estimate the conditional expectation of $m_{2,t+1}$ using semiparametric methods. By using the conditional moments they exploit more of the information in the data. For example, their marginal rate of substitution must price additional asset payoffs which are constructed by multiplying the returns by elements in the conditioning information.¹¹ However, as Hansen and Jagannathan (1991a) point out, their approach may be sensitive to errors in the specification of the conditional means. By using only the unconditional moments we are using less information, but our approach should be more robust.

III. The Data

We use the commerce department's not seasonally adjusted data on consumer expenditures for nondurables, services, and consumer durable goods for 1946 through 1987. These series measure *nominal* expenditures, whereas the models are stated in *real* terms. Price-deflators for personal consumption

¹⁰ We are grateful to Ravi Jagannathan for suggesting the analysis using the system (13). For this analysis, we assume that the relevant unconditional moments exist and that the data are sufficiently regular that the GMM can be applied.

¹¹ Another simple way to see the difference is suggested by a referee. We find the bounds on an m_{t+1} which satisfies $E(m_{t+1}R_{t+1}) = 1$. Using conditioning information we would have $E(M_{t+1}R_{t+1}|Z_t) = 1$. Therefore, $E[(m_{t+1} - M_{t+1})R_{t+1}] = 0$ and we can think of $e_{t+1} = m_{t+1} - M_{t+1}$ as "noise" which we add to the restrictions by not using the conditioning information Z_t .

expenditures (PCD) are available to us only in seasonally adjusted form, but unadjusted consumer price indices (CPI) are available. We use the components of the not seasonally adjusted CPI to deflate the nominal expenditures, constructing quarterly series of real, not seasonally adjusted per capita consumption. This is similar to the approach followed by Miron (1986), except that we focus on data at a higher level of aggregation.¹² We also examine the real, seasonally adjusted data for comparison purposes. The consumption and price indices are obtained from Data Resources, Inc.

We focus on seasonality and the effects of seasonal adjustment, but we recognize that there are other potential problems with consumption series used as measures of marginal utility for asset pricing. For example, because only a subset of the goods that provide utility is measured, a “missing variables” problem is created if preferences are not separable across the goods. This is analogous to the problem of identifying the “true” market portfolio in tests of the Capital Asset Pricing Model. Other potential problems include infrequent and nonsynchronous sampling, time-aggregation, publication lag, measurement errors, and the tax treatment of the returns to “investment” in consumer goods that differs from taxation of security returns.

Figures 1–3 illustrate the quarterly time series of the seasonally adjusted and unadjusted real consumption growth rates. The plots indicate no apparent trends in any of the growth rate series. The strong seasonality and higher volatility of the unadjusted data are readily apparent. The X-11 program removes a substantial fraction of the variability in consumption growth rates.

We study quarterly consumption and the rates of return to five portfolios. The portfolios include a value-weighted index of the smallest decile of common stocks on the NYSE. Size is based on the market value of equity outstanding at the end of the previous year. We also include a value-weighted index from the largest decile of stocks, a long-term government bond, a long-term corporate bond and a strategy of rolling over one-month Treasury bills. The real returns are the nominal returns deflated by the price index corresponding to the measure of consumption being examined. The security return data are from the Center for Research in Security Prices at the University of Chicago (CRSP). Table I presents sample means and standard deviations of the quarterly returns and growth rates for 1947:2–1987:4. To save space, we report statistics for nominal returns, in excess of the Treasury bill, and the real Treasury bill return deflated using the overall CPI.

The standard deviations of the unadjusted consumption are an order of magnitude larger than the seasonally adjusted ones. The overall means of

¹² As Miron (1986) points out, the expenditure categories used by the Commerce Department are difficult to match precisely with the CPI components. Miron constructs not seasonally adjusted data on 13 components for 1946–1982, using judgment to match the categories. We use the standard CPI components for nondurables, services and durables as provided by the Bureau of Labor Statistics.

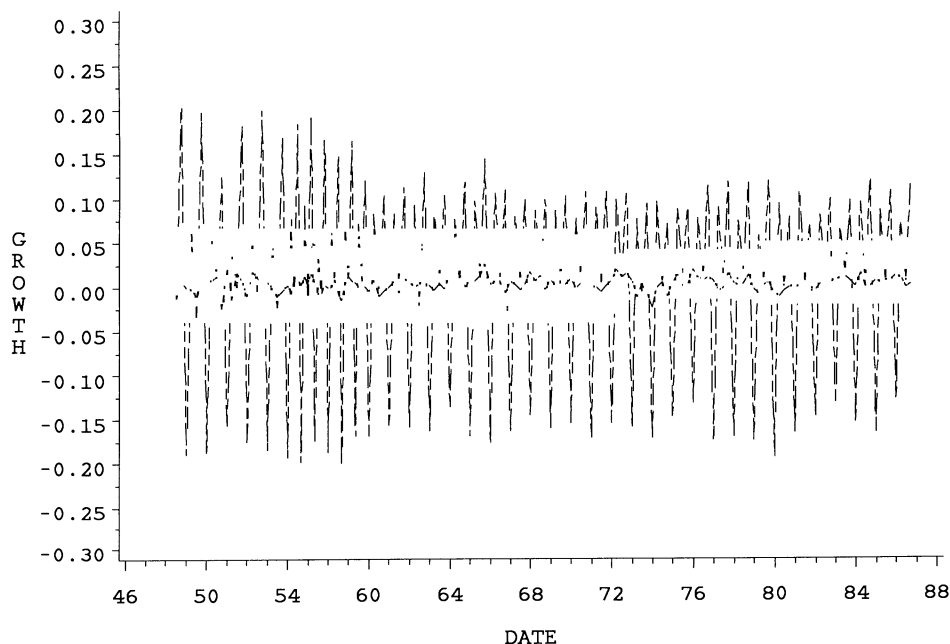


Figure 1. Quarterly real per capital growth of nondurables consumption. The growth rate is the ratio of period t consumption divided by period $t - 1$ consumption, minus 1.0. Seasonally adjusted data are denoted by the solid line and the not seasonally adjusted data are represented by the dashed line. The seasonally adjusted consumption is deflated by its own (seasonally adjusted) price deflator while the not seasonally adjusted consumption is deflated with the (not seasonally adjusted) Consumer Price Index for nondurables. All data are from the Department of Commerce.

the growth rates also differ, for two reasons. The X-11 program alters the means of the expenditure series and the price deflators are different.¹³ The autocorrelation structure of the unadjusted consumption data is, of course, very different from that of seasonally adjusted data. There is high autocorrelation at the seasonal lag. We compute the autocorrelations of the variables out to 60 lags. All of the autocorrelations appear to decay toward zero at longer lags. The autocorrelations of the not seasonally adjusted consumption growth rates display the systematic sign reversals and slow decay typical of a stationary, but strongly seasonal time series. These patterns and the lack of

¹³ Let g be the nominal growth rate of a consumption measure and i be the rate of price inflation. The expected real growth rate is: $E\{(1 + g)/(1 + i)\} = E(1 + g)E\{1/(1 + i)\} + \text{cov}\{g, 1/(1 + i)\}$. The most dramatic difference in mean real growth rates between unadjusted and adjusted data in Table I is the case of nondurables (0.010 versus 0.003, respectively). The values decompose according to the above expression as:

$$1.010 = 1.0195 \cdot 0.9911 - 0.00804 \text{ (Nondurables—not seasonally adjusted)}$$

$$1.003 = 1.0118 \cdot 0.9911 + 0.00001 \text{ (Nondurables—seasonally adjusted)}.$$

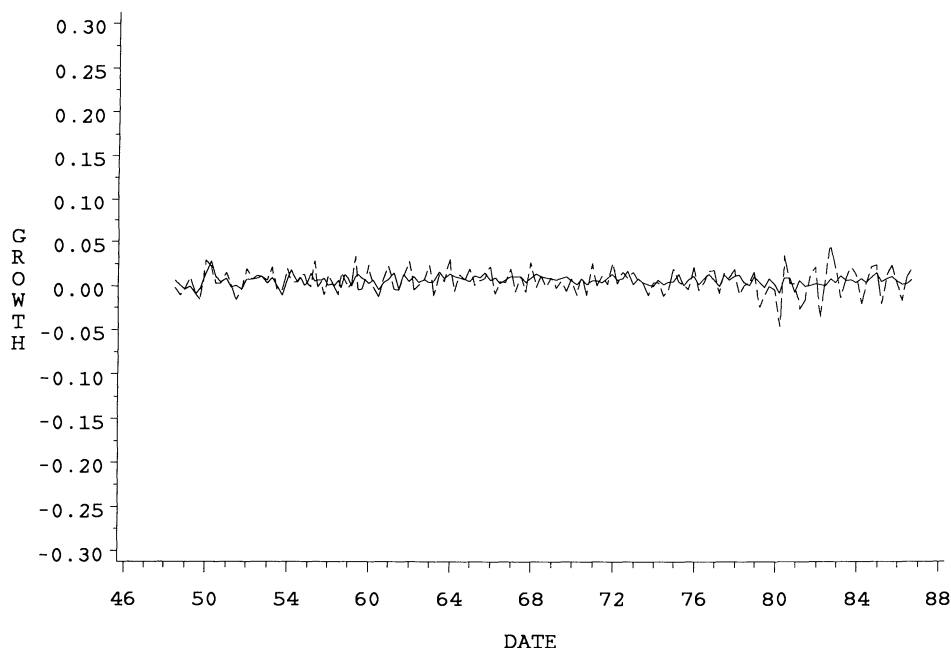


Figure 2. Quarterly real per capita growth of services consumption. The growth rate is the ratio of period t consumption divided by period $t - 1$ consumption, minus 1.0. Seasonally adjusted data are denoted by the solid line and the not seasonally adjusted data are represented by the dashed line. The seasonally adjusted consumption is deflated by its own (seasonally adjusted) price deflator while the not seasonally adjusted consumption is deflated with the (not seasonally adjusted) Consumer Price Index for services. All data are from the Department of Commerce.

trends in Figures 1–3 suggest that stationarity of the growth rates is a reasonable assumption.

Table I also displays the sample means of the variables for each quarter of the year. The mean unadjusted consumption growth rates differ markedly across the quarters. Expenditures for durable and nondurable goods are high in the fourth and the second quarters. The first and third quarter average growth rates are negative. Services display a different pattern, with the highest mean in the first quarter and the lowest in the second. An analysis of variance attributes slightly more than 30% of the variance of services growth to the differences in quarterly means; for nondurable goods the figure is 95%.

Security returns have seasonal mean differences, but to a lesser extent than the consumption measures. The well-known small-firm, turn of the year effect is evident in the smallest decile of common stocks. Most of the 13.5% annual excess return is earned in the first quarter (almost 11%). The other securities realize their largest average returns in the fourth quarter. This reflects in part a lower fourth quarter inflation rate. But the seasonal components of the price deflators and of the asset returns are minuscule compared with the consumption expenditure series.

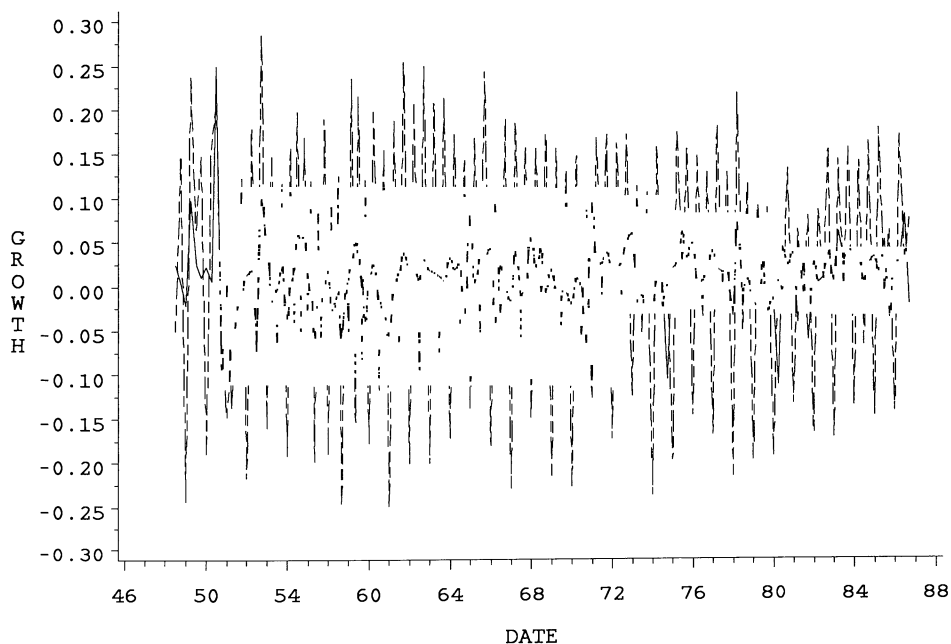


Figure 3. Quarterly real per capita growth of durables consumption. The growth rate is the ratio of period t consumption divided by period $t - 1$ consumption, minus 1.0. Seasonally adjusted data are denoted by the solid line and the not seasonally adjusted data are represented by the dashed line. The seasonally adjusted consumption is deflated by its own (seasonally adjusted) price deflator while the not seasonally adjusted consumption is deflated with the (not seasonally adjusted) Consumer Price Index for durables. All data are from the Department of Commerce.

The summary statistics in Table I suggest that an asset pricing model using not seasonally adjusted consumption needs to control for the strong seasonality in consumption expenditures that is not reflected in asset returns. In order to capture a cross-section of the expected returns, there should be seasonality in the conditional covariances with consumption that differs across the assets.

Recall that in Miron's (1986) model of seasonality, multiplicative dummy variables should reduce the seasonality in consumption expenditure levels to an amount that is mirrored in the real returns of all assets. Equivalently, additive dummy variables should control the seasonality in the continuously-compounded growth rates of consumption. We regressed the first differences of the logarithm of the per capita real consumption expenditures on dummy variables for the quarters and we examined the autocorrelations of the regression residuals. The residuals at lag j , $j = 1, 2, 3, 4, 8, 12$ were found to be $\{-0.32, -0.23, 0.10, 0.54, 0.41, 0.44\}$ for nondurable expenditures and $\{0.01, -0.35, 0.12, 0.44, 0.42, 0.30\}$ for services. The chi-square statistic for the joint significance of the first 12 residual autocorrelations was 171.8 (right-tail p -value = 0.000) for nondurables and 145.6 (p -value =

0.000) for services. There is a lot of seasonal behavior left in the consumption growth rates even after deterministic shocks are removed by the seasonal dummies. This suggests that Miron's (1986) model of seasonality may be inadequate to model the relation of not seasonally adjusted consumption to asset returns. However, looking at the autocorrelations is not sufficient to draw any firm conclusions, since both the means and the covariances of asset returns with consumption can combine to control the remaining seasonality.

Table II presents sample correlations of the variables. There are several interesting differences between the seasonally adjusted and the unadjusted data. When seasonally adjusted, the three components of personal consumption expenditures are weakly positively correlated. Seasonally unadjusted, the nondurable and durable goods expenditures are highly correlated (0.90). We study seasonally unadjusted nondurables but exclude durable goods expenditures from our analysis.¹⁴

The correlation of services with the other two components of expenditures is negative in the seasonally unadjusted data. This reflects the opposing seasonal patterns. We examine the services and the nondurables components separately to check the sensitivity of our results. Some studies use the sum of nondurables plus services as the consumption measure. In these data the sum behaves very much like the nondurables component, because nondurables are a large fraction of the total and they are much more volatile than services.

The correlations of the seasonally unadjusted consumption measures with the asset returns differ from seasonally adjusted data. The correlations of nondurables with the bond returns are higher, and the difference between the correlations of the large and the small stock returns are greater, when the data are not seasonally adjusted. This suggests that the "consumption betas" would differ more using these data.

To study conditional expectations we use instrumental variables Z_t . The instruments should be correlated with expected consumption growth and real returns. We use a constant plus four lagged values each of the growth rate of the consumption expenditure being examined and of the return on the Treasury bill. This is similar to a number of previous studies which used lagged consumption and returns. We use four lags of consumption in order to include the seasonal lag.¹⁵ Additional lagged values are excluded to avoid

¹⁴ Miron (1986) included durability in his theoretical model, but he assumed that utility is separable across goods, examined a single disaggregated durable expenditure, and did not attempt to estimate durable technology parameters. Ferson and Constantinides (1991) found that seasonally adjusted nondurable goods and durable goods expenditures produce similar results in their model.

¹⁵ This is a parsimonious alternative to including dummy variables for the quarters in the set of instruments. Regressions on the lagged values explain a larger fraction of the variance of consumption growth than regressions using dummy variables, and the residual autocorrelations are not large. The R^2 of regressions for the asset returns are slightly higher using the lagged, not seasonally adjusted consumption data than using the seasonally adjusted data. We have replicated some of our tests using dummy variables as instruments and we found that the results were similar.

Table I
Means, Standard Deviations and Autocorrelations of Real Consumption Growth, Asset Returns and Price Deflators

The real growth of per capita nondurables, durables, and services consumption expenditures are the arithmetic growth rates. Seasonally adjusted (SA) consumption data are deflated by seasonally adjusted personal consumption deflators (PCD). Not seasonally adjusted (NSA) consumption measures are deflated by the not seasonally adjusted consumer price indices (CPI) that correspond to the consumption measures. The consumption expenditures and the price deflators are from the Department of Commerce. The asset excess returns are computed net of the return to rolling over one-month Treasury bills for three months. The government and corporate bond and the Treasury bill data are from Ibbotson Associates. The smallest and largest decile returns are common stock portfolios formed by total equity market capitalization of the firms. These are from the Center for Research in Security Prices (CRSP). The real Treasury bill is the nominal quarterly return to rolling over one month bills each month deflated by the overall CPI. All units are decimal fractions per quarter. The sample is 1947:2-1987:4 (163 observations).

Variable	Overall		First Quarter		Second Quarter		Third Quarter		Fourth Quarter		Autocorrelations ^a					
	Mean	Standard Deviation	Mean	Mean	Mean	Mean	Mean	Mean	Mean	Mean	ρ_1	ρ_2	ρ_3	ρ_4	ρ_8	ρ_{12}
Panel A. Real consumption growth rates																
Nondurables (NSA)	0.01043	0.11337	-0.16621	0.07776	-0.00091	0.12679	-0.680	0.367	-0.669	0.983	0.979	0.982				
Nondurables (SA)	0.00279	0.00841	0.00320	0.00308	0.00260	0.00228	0.069	0.066	0.147	-0.052	-0.178	0.023				
Durables (NSA)	0.02056	0.14926	-0.18271	0.15080	-0.03598	0.14516	-0.779	0.655	-0.776	0.895	0.862	0.855				
Durables (SA)	0.00968	0.04248	0.01302	0.00631	0.01871	0.00079	-0.089	0.116	-0.124	0.005	-0.178	-0.113				
Services (NSA)	0.00531	0.01385	0.01758	-0.00378	0.00375	0.00399	-0.150	-0.263	-0.050	0.620	0.623	0.550				
Services (SA)	0.00614	0.00576	0.00635	0.00754	0.00483	0.00583	0.157	-0.036	0.027	-0.050	0.097	-0.023				

Table I—Continued

Variable	Overall Mean	Overall Standard Deviation	First Quarter Mean	Second Quarter Mean	Third Quarter Mean	Fourth Quarter Mean	Autocorrelations ^a					
							ρ_1	ρ_2	ρ_3	ρ_4	ρ_8	ρ_{12}
Panel B. Asset returns												
Excess government bond	-0.00038	0.04895	-0.00234	-0.00304	-0.00584	0.00966	-0.072	0.030	0.129	0.050	-0.085	-0.061
Excess corporate bond	0.00092	0.04806	-0.00071	-0.00563	-0.00272	0.01271	-0.023	0.025	0.118	0.039	-0.039	-0.061
Excess smallest decile	0.03384	0.13639	0.10988	-0.00606	0.02055	0.01284	0.008	-0.118	-0.105	0.139	-0.046	0.130
Excess largest decile	0.01721	0.07632	0.01742	0.01151	0.00457	0.03535	0.148	-0.054	-0.074	0.008	-0.086	0.053
Real Treasury bill	0.00157	0.00888	0.00264	-0.00083	0.00009	0.00442	0.532	0.357	0.493	0.402	0.168	0.313
Panel C. Growth rates of price deflators												
CPI nondurables (NSA)	0.00915	0.01358	0.00920	0.01241	0.01061	0.00436	0.427	0.330	0.420	0.322	0.057	0.181
PCD nondurables (SA)	0.00906	0.01084	0.00943	0.00873	0.00943	0.00864	0.687	0.543	0.481	0.310	0.088	0.175
CPI durables (NSA)	0.00783	0.01247	0.00298	0.00992	0.00750	0.01079	0.365	0.373	0.257	0.494	0.416	0.351
PCD durables (SA)	0.00694	0.01218	0.00951	0.00671	0.00693	0.00467	0.082	0.092	0.143	0.079	0.036	0.002
CPI services (NSA)	0.01320	0.00930	0.01379	0.01299	0.01443	0.01162	0.596	0.531	0.556	0.546	0.394	0.287
PCD services (SA)	0.01224	0.00655	0.01130	0.01114	0.01343	0.01307	0.734	0.706	0.723	0.714	0.583	0.501
CPI overall (NSA)	0.01029	0.01034	0.00913	0.01257	0.01176	0.00767	0.615	0.484	0.596	0.489	0.249	0.315

^a ρ_j is the sample autocorrelation at lag j .

multicollinearity.¹⁶ We use the overall CPI to deflate the lagged instruments, not the deflator corresponding to the measures of consumption and real returns. This is to reduce the risk of spurious correlation between the endogenous variables and the instruments that can arise if the deflator has autocorrelated measurement error.¹⁷

IV. Empirical Results

A. Seasonality and the Equity Premium Puzzle

Mehra and Prescott (1985) examined the equity premium, measured as the difference between NYSE stocks and Treasury bill returns. They found that the variance of aggregate consumption growth is too small to explain the unconditional mean of the equity and bill returns, using small values for the concavity parameter α in a time-separable consumption model. In equation (5), the equity premium depends on α and on the covariance of returns with consumption. In the more specialized model of Mehra and Prescott consumption is the payoff to “equity,” and the covariance of equity with consumption is determined by the variance of consumption.

The first panel of Table III shows the values of α in the time-separable model of equation (5), implied by the *unconditional* mean excess returns and covariances with the various measures of consumption. The estimates are obtained by the method of moments.¹⁸ With seasonally adjusted consumption data the estimates of α exceed 40.0 in absolute magnitude, in eight of the ten cases. This illustrates the equity premium puzzle, interpreted to mean that large values of the concavity parameter are implied by the unconditional moments.¹⁹ 18,19,Tbl III

Intuition suggests that not seasonally adjusted consumption data should imply estimates of α in equation (5) which are closer to zero, because of the higher volatility. The unconditional model chooses α to fit the total covaria-

¹⁶ Tauchen's (1986) simulations suggest the small sample properties of GMM estimators may suffer if excessive numbers of lagged instruments are used. We conduct experiments to check the sensitivity of some of our results to using fewer lagged values (e.g., 2 or 3 lags) and do not find that the results change qualitatively. See Hansen and Singleton (1987) for an analysis of optimal instrument selection in the context of linear models.

¹⁷ In the few cases we checked, there appeared to be stronger correlations between the instruments and the error terms of the models when the same deflator was used for both the endogenous variables and the lagged instruments.

¹⁸ We have also estimated α using maximum likelihood methods, assuming consumption is lognormally distributed, and obtained qualitatively similar results.

¹⁹ Pratt (1964) provides interpretations for values of α as a coefficient of risk aversion in timeless gambles. One case considers an agent who is indifferent between a gamble over h fraction of wealth, winning with probability p and losing with probability $1 - p$. Choosing $h = 1\%$, Pratt's analysis implies that (to a second order approximation) an agent with log utility ($\alpha = 1.0$) requires $p = 0.50025$. With $\alpha = 10$, $p = 0.5025$; $\alpha = 100$ implies $p = 0.75$. Under certainty, α is the inverse of the elasticity of intertemporal substitution. Unit elasticity ($\alpha = 1$) corresponds to offsetting income and substitution effects in some models (see Weil (1988)). Under this interpretation, $\alpha = 100$ implies that the elasticity is only 1% per quarter.

Table III
Estimates of the Concavity Parameter in a Representative Agent's Power Utility Function

The estimates are based on unconditional mean excess returns and four measures of consumption. Seasonally adjusted (SA) nondurables and services consumption data are deflated by seasonally adjusted personal consumption deflators. Not seasonally adjusted (NSA) nondurables and services consumption data are deflated by the not seasonally adjusted consumer price index (CPI) that corresponds to each consumption measure. The consumption expenditures and the price deflators are from the Department of Commerce. The government and corporate bond and the Treasury bill data are from Ibbotson Associates. The smallest and largest decile returns are common stock portfolios formed by total equity market capitalization of the firms. These are from the CRSP. Real returns are nominal returns deflated by the price index corresponding to the consumption growth measure used in the method of moments estimation. The estimation equations are:

$$E \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} (R_{i,t+1} - R_{f,t+1}) \right] = 0, \quad (5)$$

$$E \left\{ \left(\frac{C_{t+1} - bC_t}{C_t - bC_{t-1}} \right)^{-\alpha} - b\beta \left(\frac{C_{t+2} - bC_{t+1}}{C_t - bC_{t-1}} \right)^{-\alpha} \right\} (R_{i,t+1} - R_{f,t+1}) = 0, \quad (7')$$

$$E \left[\left\{ \left(\frac{C_{t+1} - bC_t}{C_t - bC_{t-1}} \right)^{-\alpha} - b\beta^4 \left(\frac{C_{t+5} - bC_{t+1}}{C_t - bC_{t-4}} \right)^{-\alpha} \right\} (R_{i,t+1} - R_{f,t+1}) \right] = 0, \quad (8')$$

where α is the concavity parameter, β is the time discount factor which is set equal to 1.0, b is the parameter representing seasonal habit persistence ($b > 0$ and set equal to 0.9) or durability ($b < 0$ and set equal to -0.3), $R_{i,t+1}$ is the quarterly real return on an asset and $R_{f,t+1}$ is the quarterly real return to rolling over one month Treasury bills each month. C_t is the real, per capita aggregate consumption. The sample is 1948:2-1987:4 (159 observations).

Table III—Continued

Excess return	Nondurables		Services	
	NSA	SA	NSA	SA
Panel A. Time-separable model (5)				
Government bond	-0.103 (10.64)	9.866 (329.99)	-5.800 (101.84)	7.359 (141.35)
Corporate bond	3.369 (9.46)	-92.81 (162.79)	8.602 (88.31)	-69.425 (342.76)
Smallest decile	-8.965 (15.38)	42.145 (23.46)	41.370 (29.95)	132.645 (47.66)
Largest decile	2.080 (2.50)	94.227 (46.39)	37.777 (20.33)	109.566 (39.87)
Four assets	-1.643 (1.69)	62.346 (57.69)	56.240 (20.93)	131.920 (92.94)
Panel B. Time nonseparable models (7') and (8') with habit persistence				
Government bond	-0.057 (1.25)	-0.065 (0.77)	-0.003 (0.15)	-0.055 (0.89)
Corporate bond	-0.251 (0.72)	-0.280 (0.68)	-0.049 (0.15)	0.291 (0.76)
Smallest decile	-0.670 (0.33)	-3.348 (3.02)	1.601 (0.91)	-4.627 (2.85)
Largest decile	-0.671 (0.37)	-3.841 (95.50)	-1.058 (0.50)	7.252 (9.84)
Four assets	-0.649 (0.30)	-0.760 (0.70)	0.278 (0.21)	0.351 (0.57)
Panel C. Time nonseparable models (7') and (8') with durability				
Four assets	-1.633 (1.72)	74.332 (51.38)	43.856 (17.32)	158.93 (86.27)

tion of returns with consumption, including the seasonal variation. Table III provides some confirmation of this intuition; using the not seasonally adjusted consumption the estimates of α are uniformly closer to zero. However, using services, three of the five estimates of α are still above 37.0 and using nondurables, three of the five estimates are below zero. Negative values of α are troublesome, because they indicate "risk-loving" behavior.²⁰

The evidence in Table III indicates that the equity premium puzzle is not explained as an artifact of seasonal adjustment in a time-separable model. The puzzle extends beyond the NYSE index, which is dominated by large stocks, to include corporate bonds and small stocks.

In further experiments we use observations for the individual quarters separately in the time-separable model. This allows all of the parameters of the model to vary according to the quarter. Seasonal differences can then be captured by differences in the α coefficients and in the consumption covariances.²¹ The estimates are often large in absolute magnitude and they are imprecise. We observe the largest values of α in the first quarter for small stocks, but no other systematic pattern across the quarters. About $\frac{2}{3}$ of the estimates are closer to zero using the unadjusted data than with the seasonally adjusted data. Therefore, even when seasonality is fully incorporated, the time-separable consumption models do not explain the equity premium puzzle.

The lower panels of Table III report a similar exercise using the nonseparable models of equations (7) and (8). We fix the values of the nonseparability parameter b and we obtain the values of the concavity parameter, α , implied by the unconditional expected risk premiums. We consider two values of the parameter b , which imply competing models of the nonseparability. The first, $b = 0.90$ is in the range of values estimated by Ferson and Constantinides (1991) and it represents strong habit persistence. The second value, $b = -0.30$ is in the range of values estimated by Dunn and Singleton (1986) and it represents durability of consumption expenditures.

The results for the nonseparable models are not directly comparable with the first panel of Table III because the concavity parameter α has a different interpretation under time-nonseparability. In a time-separable model, α is interpreted as the coefficient of relative risk aversion or as the inverse of the elasticity of consumption with respect to a fixed interest rate. In a nonseparable model α may differ substantially from the elasticity of substitution. However, Ferson and Constantinides (1991) show that α approximates closely the coefficient of relative risk aversion in a model with habit persistence.

The time nonseparable modes with habit persistence appear to fit the risk premiums better than the time-separable models, in the sense that the implied concavity parameter is dramatically reduced, along with the stan-

²⁰ Of course, if the seasonal behavior of consumption expenditures or of asset returns reflects factors outside the model (such as taxes, for example), the coefficient estimates can be biased.

²¹ Osborne (1988) adopted a similar approach for United Kingdom data, but assumed a constant real return.

dard errors. This is true for both the seasonal and the nonseasonal, time-nonseparable models. Using the value of the parameter which implies durability however, there is no apparent improvement. We examine the overidentifying restrictions in the four asset systems to obtain further information about the fit of the nonseparable models. With four assets and the one parameter α , the model provides three overidentifying restrictions. We find that the right-tail p -values of the chi-squared goodness-of-fit statistics for these restrictions are somewhat larger for the seasonal nonseparable model of equation (8), and provide little evidence against that model.²² However, the parameter estimates in Table III are imprecise. Only four of the twenty point estimates for the time-separable models exceed two standard errors and only three of the twenty point estimates for the nonseparable models are larger than two standard errors. Hence, it is difficult to distinguish between the models using these tests. The low information content of the unconditional moments motivates using conditioning information in the tests.

B. Tests of the Time-Separable Models

Table IV summarizes the results of estimating equations (3) and (4), providing a comparison of two methods for dealing with seasonality. In equation (4) preference shocks are estimated directly using not seasonally adjusted data. In equation (3), seasonality is controlled in a more ad hoc way by the X-11 program. We report results for the real Treasury bill return and for a system with the other four assets, excluding the Treasury bill. The estimates of the concavity parameter α are not unreasonably large, but they are occasionally negative. With seasonally adjusted data the goodness-of-fit tests reject the model for the Treasury bill return, but not for the other long-term assets, either individually or in a multiple asset system.²³

With not seasonally adjusted data, the first quarter of the year is chosen as a reference quarter ($\kappa_1 = 1$). The shift coefficients $\kappa_2, \kappa_3, \kappa_4$ indicate how the model seasonally adjusts consumption levels relative to the first quarter. Using only the Treasury bill, none of the shift coefficient estimates are larger than two standard errors. A test of their joint significance produces p -values of 0.11 (services) and 0.45 (nondurables).²⁴ In the systems with four assets,

²² The right-tail p -values for the seasonal, nonseparable model with habit persistence ($b = 0.9$) are 0.978 using not seasonally adjusted nondurables consumption and 0.077 for services. The p -values for the nonseasonal nonseparable model, using the seasonally adjusted data, are 0.093 for nondurables and 0.028 for services.

²³ We do not report the results for the individual long-term assets in order to save space. We also conducted tests of the model (3), which excludes the seasonal shift parameter, using not seasonally adjusted consumption data. The tests reject the model using Treasury bills or multiple-asset systems. The point estimates of the concavity parameter α are much smaller than with seasonally adjusted data. This reflects the high seasonal variation of consumption that is not matched by seasonal variation in the real returns. The Euler restrictions are fit more closely by raising the consumption growth rate to a power close to zero, when there are no seasonal shift parameters to control the seasonality.

²⁴ This is the standard, heteroskedasticity consistent Wald test.

Table IV
Tests of Consumption-Based Models Using Real Returns

The estimates are based on four measures of consumption. Seasonally adjusted (SA) nondurables and services consumption data are deflated by seasonally adjusted personal consumption deflators. Not seasonally adjusted (NSA) nondurables and services consumption data are deflated by the not seasonally adjusted consumer price index (CPI) that corresponds to each consumption measure. The consumption expenditures and the price deflators are from the Department of Commerce. The government and corporate bond and the Treasury bill data are from Ibbotson Associates. The quarterly Treasury bill is the return to rolling over one month bills each month. The smallest and largest decile returns are common stock portfolios formed by total equity market capitalization of the firms. These are from the CRSP. Real returns are nominal returns deflated by the price index corresponding to the consumption growth measure used. The four asset system includes the returns on government and corporate bonds as well as the smallest and largest equity decile returns. The Euler equations are:

$$E \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} R_{i,t+1} \mid Z_t \right] - 1 = 0 \quad (3)$$

$$E \left[\left(\frac{\kappa_{s(t+1)}}{\kappa_{s(t)}} \right)^{1-\alpha} \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} R_{i,t+1} \mid Z_t \right] - 1 = 0 \quad (4)$$

where α is the concavity parameter and β is the time discount factor. The κ_s are seasonal taste shifters for quarter s relative to the first quarter (κ_1 is set equal to 1.0). $R_{i,t+1}$ is the quarterly real return on an asset and C_t is the real, per capita consumption measure. The estimates are based on generalized method of moments using a constant, four lags of the real Treasury bill return and four lags of the real consumption growth as the instruments, Z_t . The instruments are deflated by the overall CPI. Standard errors are in parentheses. The sample is based on quarterly data from 1948:2–1987:4 (159 observations).

Table IV—Continued

Model	Consumption	α	β	Panel A. Treasury bill			$\ln(\kappa_j) = 0$ $j = 2, 3, 4$ χ^2_{2a}	Overidentifying Restrictions χ^2_{2b}
				$\ln(\kappa_2)$	$\ln(\kappa_3)$	$\ln(\kappa_4)$		
(3)	Nondurables SA	0.983 (0.235)	0.999 (0.001)	—	—	—	—	23.442 [0.001]
(3)	Services SA	-3.242 (0.729)	0.974 (0.005)	—	—	—	—	22.092 [0.002]
(4)	Nondurables NSA	0.606 (0.262)	0.998 (0.001)	0.125 (0.128)	0.128 (0.130)	0.293 (0.313)	2.665 [0.446]	12.424 [0.014]
(4)	Services NSA	0.533 (0.152)	1.005 (0.001)	-0.007 (0.008)	-0.009 (0.012)	-0.016 (0.013)	6.037 [0.110]	27.854 [< 0.001]
Panel B. Four asset system								
(3)	Nondurables SA	0.642 (0.853)	1.002 (0.004)	—	—	—	—	38.561 [0.271]
(3)	Services SA	4.253 (2.906)	1.021 (0.019)	—	—	—	—	43.785 [0.121]
(4)	Nondurables NSA	2.306 (1.117)	1.006 (0.005)	-0.134 (0.051)	-0.138 (0.055)	-0.323 (0.113)	15.439 [0.001]	56.333 [0.004]
(4)	Services NSA	1.894 (1.169)	1.016 (0.008)	0.028 (0.018)	0.034 (0.018)	0.057 (0.044)	4.972 [0.174]	48.646 [0.023]

^a The test of whether the seasonal taste shifters are jointly equal to zero has 3 degrees of freedom. The right-tail probability values are in brackets.

^b In the single asset model that uses seasonally adjusted data, there are 9 orthogonality conditions and 2 parameters leaving 7 degrees of freedom in the chi-square statistic. In the model that uses the not seasonally adjusted data there are 4 degrees of freedom. In the four asset system, there are 34 degrees of freedom in the model with seasonally adjusted data and 31 degrees of freedom in the model that uses not seasonally adjusted data. *p*-values are in brackets.

the point estimates are intuitively plausible and more precisely estimated. The values of κ_s are smaller than 1.0 for nondurables (the $\ln(\kappa_s)$ are less than zero), which reflects the low expenditures for nondurable goods in the first, or reference, quarter. The values of κ_s are larger than 1.0 for services. The smallest value of κ_s for nondurables and the largest value for services are in the fourth quarter. However, the goodness-of-fit tests reject the model with seasonality. p -values less than 0.025 are recorded, not only for the Treasury bill but also for the systems of four assets.

Table IV confirms that the model (4) with seasonality is misspecified. The evidence against the model found by English, Miron, and Wilcox (1989) does not depend on their use of the Treasury bill discount rate as a measure of return. Miron (1986) and English, Miron, and Wilcox (1989) assume homoskedasticity in their empirical work.²⁵ Hansen and Singleton (1983) and Ferson (1983) suggest that rejections of a linear version of equation (3) may be the result of changing conditional variances and covariances. Table IV allows general forms of conditional heteroskedasticity, which can be seasonally varying as a function of the seasonal instruments Z_t . Thus, the rejections of the model with seasonality are not explained by heteroskedasticity.

Table V focuses on expected excess returns. The upper panel examines equation (5) comparing results for seasonally adjusted and unadjusted data. Using seasonally adjusted consumption data, the Euler restrictions of model (5) are not rejected. The point estimates of the concavity parameter α are large in magnitude, compared with Table IV, but imprecise. Unlike Table IV, there is no attempt to fit the levels of the real asset returns, only the excess returns. The results in Table IV reflect the strong influence of the levels of real interest rates, which imply a lower concavity parameter (higher intertemporal substitution) than is implied by the risk premiums.

Tests of equation (5) using the not seasonally adjusted consumption produce rejections of the Euler restrictions. The seasonality in consumption is not controlled in this model, and this misspecification results in smaller point estimates of α and larger values of the test statistics, compared with the seasonally adjusted case.²⁶ The algorithm attempts to reduce the seasonal fluctuations in consumption by raising the growth rate to a smaller power ($-\alpha$), and it sacrifices the ability to fit the intertemporal fluctuations in expected risk premiums by doing so.²⁷

An additional experiment provides further information about where the time-separable models are failing. The four right hand columns of Table V report average pricing errors for each of the excess returns. A pricing error is defined as the difference between the excess return and the excess return predicted by the model. We estimate the mean pricing errors by

²⁵ These studies estimate their models using two-stage, nonlinear least squares.

²⁶ We examine tests for the individual excess returns, and find no evidence that particular excess returns are driving these rejections.

²⁷ In further experiments we replicate the tests in Table V for the not seasonally adjusted data, using dummy variables for the quarters as instruments in place of or in addition to the lagged consumption variables. The results are similar.

Table V
Tests of a Consumption-Based Model Using Excess Returns

The estimates are based on four measures of consumption. Seasonally adjusted (SA) nondurables and services consumption data are deflated by seasonally adjusted personal consumption deflators. Not seasonally adjusted (NSA) nondurables and services consumption data are deflated by the not seasonally adjusted consumer price index (CPI) that corresponds to each consumption measure. The consumption expenditures and the price deflators are from the Department of Commerce. The government and corporate bond and the Treasury bill data are from Ibbotson Associates. The smallest and largest decile returns are common stock portfolios formed by total equity market capitalization of the firms. These are from the CRSP. Real returns are nominal returns deflated by the price index corresponding to the consumption growth measure used. Four excess returns are used in the estimation: the government bond (GB), corporate bond (CB), smallest equity decile (D1) and largest equity decile (D10) returns. The Euler equation is:

$$E \left[\left(\frac{C_t}{C_{t+1}} \right)^\alpha (R_{i,t+1} - R_{f,t+1}) | Z_t \right] = 0, \quad (5)$$

where α is concavity parameter, $R_{i,t+1}$ is the real return on asset i , $R_{f,t+1}$ is real return to rolling over one month Treasury bills each month and C_t is the real, per capita consumption measure. The estimates are based on generalized method of moments using a constant, four lags of the real Treasury bill return and four lags of the real consumption growth as the instruments, Z_t . The instruments are deflated by the overall CPI. Standard errors are in parentheses. The sample is based on quarterly data from 1948:2–1987:4 (159 observations).

Consumption Measure	α	χ^2 ^a	p -value	Pricing ^b Error GB	Pricing Error CB	Pricing Error D1	Pricing Error D10
Nondurables	21.867	36.150	0.414	-0.0047 (0.0029)	-0.0033 (0.0029)	0.0214 (0.0100)	0.0079 (0.0059)
SA	(19.727)						
Services	323.585	38.590	0.311	-0.0117 (0.0014)	-0.0089 (0.0015)	-0.0006 (0.0056)	-0.0062 (0.0044)
SA	(38.078)						
Nondurables	-3.832	59.702	0.006	-0.0048 (0.0026)	-0.0041 (0.0025)	0.0241 (0.0087)	0.0140 (0.0052)
NSA	(1.104)						
Services	47.064	50.757	0.041	-0.0044 (0.0025)	-0.0022 (0.0024)	0.0225 (0.0092)	0.0133 (0.0052)
NSA	(9.861)						

^a In model (5), there are 35 degrees of freedom. p -value is the right-tail probability value.

^b The pricing error is the estimate of the λ_i when $(R_{i,t+1} - R_{f,t+1} - \lambda_i)$ replaces the excess return in the model. The standard error of the $\hat{\lambda}_i$ is in parentheses.

introducing asset specific parameters λ_i , replacing $(R_{i,t+1} - R_{f,t+1})$ with $(R_{i,t+1} - R_{f,t+1} - \lambda_i)$ in equation (5). The λ_i are analogous to "Jensen's alpha" coefficients in the excess return formulations of beta pricing models. The average pricing errors are similar for three of the four versions of model (5). They are on the order of $1/2\%$ per quarter for the bonds, and vary between 1 and $2\frac{1}{2}\%$ per quarter for the stock portfolios. Using seasonally adjusted services the mean pricing errors are smaller than this for the small stocks but larger for the bonds.

When the average pricing errors are introduced, the models place no restriction on the *unconditional* mean excess returns. This is in contrast with Table III, which focused attention on the unconditional means. Of course, the point estimates of the λ_i in Table V will be affected by both the conditional and the unconditional moment restrictions.²⁸ However, when the λ_i 's are included, then rejections of the models' restrictions are driven by the variation of the conditional expected values. Using unadjusted consumption, the test statistics for model (5) with the λ_i parameters imply *p*-values of 0.005 (nondurables) and 0.076 (services). Thus, the time-variation of the conditional expected values is sufficient to reject the time-separable model.

Table VI examines excess returns using the more general time-separable model (6) with a trending subsistence level. In this model the seasonal taste shift parameters do not cancel out of the expression for excess returns, as they do in equation (5). The point estimates of the κ_s parameters are comparable to those in Table IV. Unfortunately, there is no evidence that introducing a deterministic subsistence level improves the fit of the data. Few of the parameters are significantly different from zero, and the goodness-of-fit test statistic rejects the model at standard significance levels. We introduce parameters to estimate the mean pricing errors and to allow the unconditional mean premiums to be unrestricted. The results are similar to those reported in Table V.²⁹ We conclude that the time-separable models which incorporate general seasonal effects are unable to provide a good fit to consumption and asset returns data, as measured by the goodness-of-fit tests.

C. Time-Nonseparable Models: The Euler Equation Tests

Table VII presents the results of estimating the time-nonseparable models of equations (7) and (8) using the five-asset system. The first two rows are for the nonseasonal nonseparable model, equation (7). The estimates of the concavity parameter α and the time discount parameter β are reasonable in magnitude and the estimates of β are highly statistically significant. However, the estimates of α are not significantly different from zero in the nonseasonal model. The goodness-of-fit tests produce right-tail *p*-values of 0.33–0.34, which do not reject the model. The point estimates of the nonsepa-

²⁸ Unless the covariance matrix of the orthogonality conditions is block diagonal.

²⁹ We estimated mean pricing errors for a model with seasonally-varying subsistence levels and found similar results. The *p*-values of the chi-square goodness-of-fit statistic are 0.001 (nondurables) and 0.014 (services).

Table VI
Tests of a Seasonal Consumption-Based Model with a Trending Subsistence Level Using Excess Returns

Not seasonally adjusted (NSA) nondurables and services consumption data are deflated by the not seasonally adjusted consumer price index (CPI) that corresponds to each consumption measure. The consumption expenditures and the price deflators are from the Department of Commerce. Real returns are nominal returns deflated by the price index corresponding to the consumption growth measure used. Four excess returns are used in the estimation: the government bond, corporate bond, smallest equity decile and largest equity decile. The government and corporate bond data are from Ibbotson Associates. The smallest and largest decile returns are common stock portfolios formed by total equity market capitalization of the firms. These are from the CRSP. The returns are computed in excess of the return to rolling over one month Treasury bills each month. The model is:

$$E \left[\left(\frac{\kappa_{s(t)} C_t - e^{\mu t} \delta}{\kappa_{s(t+1)} C_{t+1} - e^{\mu(t+1)} \delta} \right)^\alpha (R_{t,t+1} - R_{f,t+1}) \mid Z_t \right] = 0 \quad (6)$$

where α is concavity parameter, $(\delta e^{\mu t})$ is the subsistence level, where δ and μ are parameters of the subsistence level, κ_s are seasonal taste shifters for quarter s relative to the first quarter, $R_{t,t+1}$ is the real return on asset i , $R_{f,t+1}$ is the real quarterly return to rolling over one month Treasury bills each month and C_t is the real, per capita consumption measure. The estimates are based on generalized method of moments using a constant, four lags of the real Treasury bill return and four lags of the real consumption growth as the instruments, Z_t . The instruments are deflated by the overall CPI. Standard errors are in parentheses. The sample is based on quarterly data from 1948:2–1987:4 (159 observations).

Consumption Measure	α	$\ln(\kappa_2)$	$\ln(\kappa_3)$	$\ln(\kappa_4)$	μ	δ	χ^2_a	p -value
Nondurables	1.749	-0.309	-0.415	-0.501	0.0039	0.256	53.71	0.005
NSA	(7.229)	(0.651)	(0.808)	(0.783)	(0.0049)	(0.421)		
Services	40.930	-0.001	0.003	0.007	-0.0111	0.565	40.87	0.089
NSA	(13.420)	(0.006)	(0.006)	(0.003)	(0.0040)	(0.082)		

^a There are 30 degrees of freedom. p -value is the right-tail probability value.

Table VII
Tests of Consumption-Based Models with Time Nonseparability
Using Real Returns

The estimates are based on four measures of consumption. Seasonally adjusted (SA) nondurables and services consumption data are deflated by seasonally adjusted personal consumption deflators. Not seasonally adjusted (NSA) nondurables and services consumption data are deflated by the not seasonally adjusted consumer price index (CPI) that corresponds to each consumption measure. The consumption expenditures and the price deflators are from the Department of Commerce. The models use five asset returns: the Treasury bill, government bond, corporate bond, smallest equity decile and largest equity decile. The government and corporate bond and the Treasury bill data are from Ibbotson Associates. The quarterly Treasury bill is the return to rolling over one month bills each month. The smallest and largest decile returns are common stock portfolios formed by total equity market capitalization of the firms. These are from the CRSP. Real returns are nominal returns deflated by the price index corresponding to the consumption growth measure used in the model. The models are:

$$E \left[-1 + R_{i,t+1} \beta \left\{ \left(\frac{C_{t+1} - bC_t}{C_t - bC_{t-1}} \right)^{-\alpha} - b\beta \left(\frac{C_{t+2} - bC_{t+1}}{C_t - bC_{t-1}} \right)^{-\alpha} \right\} + b\beta \left(\frac{C_{t+1} - bC_t}{C_t - bC_{t-1}} \right)^{-\alpha} \middle| Z_t \right] = 0, \quad (7)$$

$$E \left[-1 + R_{i,t+1} \beta \left\{ \left(\frac{C_{t+1} - bC_{t-3}}{C_t - bC_{t-4}} \right)^{-\alpha} - b\beta^4 \left(\frac{C_{t+5} - bC_{t+1}}{C_t - bC_{t-4}} \right)^{-\alpha} \right\} + b\beta^4 \left(\frac{C_{t+4} - bC_t}{C_t - bC_{t-4}} \right)^{-\alpha} \middle| Z_t \right] = 0, \quad (8)$$

where α is concavity parameter, β is the time discount factor, b is the parameter representing seasonal habit persistence ($b > 0$) or durability ($b < 0$), $R_{i,t+1}$ is one plus the real return on asset i , and C_t is the real, per capita not seasonally adjusted consumption measure. The estimates are based on generalized method of moments using a constant, four lags of the real Treasury bill return and four lags of the real consumption growth as the instruments, Z_t . The instruments are deflated by the overall CPI. Standard errors are in parentheses. The sample is based on quarterly data from 1948:4–1987:4 (153 observations).

Consumption Measure	α	β	b	χ^2 ^a	p -value ^b
Nondurables	0.840	0.997	0.371	45.38	0.333
SA equation (7)	(0.620)	(0.002)	(0.163)		
Services	0.253	0.999	0.399	45.17	0.341
SA equation (7)	(0.270)	(0.001)	(0.262)		
Nondurables	0.983	0.960	0.909	25.46	0.979
NSA equation (8)	(0.480)	(0.031)	(0.023)		
Services	1.111	0.967	0.938	24.93	0.983
NSA equation (8)	(0.319)	(0.013)	(0.002)		

^a χ^2 is the minimized value of the generalized method of moments criterion function. There are 42 degrees of freedom.

^b p -value is the right-tail probability value based on the asymptotic distribution.

rability parameter b are positive, at 0.37–0.40, which provides some evidence for habit persistence.

The last two rows of Table VII present the results for the seasonal nonseparable model of equation (8). The estimates of the concavity parameter α and the time discount parameter β are again reasonable in magnitude, and now they are statistically significant. The parameter estimates imply a strong form of seasonal habit persistence. The point estimates of the nonseparability parameter b are positive at 0.91–0.94, and they appear to be precisely estimated. These values are many standard errors away from zero; thus, we strongly reject the hypothesis that the utility is time-separable against the alternative of a seasonal nonseparable model.³⁰ In spite of this statistical power, the goodness-of-fit tests indicate virtually no evidence against the overidentifying restrictions of the seasonal nonseparable model. The p -values are 0.98 for both measures of consumption, a result strikingly different from the results for the previous models.³¹30,31

We conduct an alternative test for the significance of the nonseparability parameter b in the seasonal model, which is similar to a likelihood ratio test but is more general. The test is developed in Newey and West (1987) and Eichenbaum, Hansen, and Singleton (1988). It compares the time-separable model as the null hypothesis ($b = 0$) with the nonseparable model as the alternative hypothesis. We first estimate the model under the alternative hypothesis, minimizing the quadratic form which is the GMM criterion function while the parameter b is not constrained to equal zero. We use the GMM weighting matrix from this model, and holding it fixed we minimize the quadratic form while constraining b to equal zero. The difference between the two quadratic forms is distributed as a chi-square variable under the null hypothesis, with degrees of freedom equal to the number of restrictions implied by the null hypothesis, which is one in this example. We find that the right tail p -values of this test statistic are less than 0.001, which further confirms that the nonseparability parameter is significantly different from zero.

D. Controlling Seasonality

Since the seasonality in consumption is so strong compared with seasonality in asset returns, it is interesting to ask how well the alternative models control seasonality in consumption expenditures through the measure of the intertemporal marginal rate of substitution, m_{t+1} . Table VIII records summary statistics for the m_{t+1} implied by each of the models, evaluated at the parameter estimates from the combined asset systems. The first panel records

³⁰ Note that the t -statistic for the hypothesis that b is zero provides a rejection of the time-separable model against the nonseparable alternative, while allowing that the error terms follow an MA(4) process under the time-separable null hypothesis.

³¹ The seasonal habit model induces a longer moving average structure in the error terms than the nonseasonal habit model. This could affect the finite sample performance of the test statistics. Therefore, the p -values should be interpreted with some caution.

Table VIII

Quarterly Means and Autocorrelations of the Real Consumption Growth Rates and Measures of a Representative Agent's Intertemporal Marginal Rate of Substitution

Each model is estimated with not seasonally adjusted (NSA) nondurables and services consumption data deflated by the not seasonally adjusted consumer price index (CPI) that corresponds to each consumption measure. The consumption expenditures and the price deflators are from the Department of Commerce. Real returns are nominal returns deflated by the price index corresponding to the consumption growth measure. The returns include the Treasury bill, the government bond, corporate bond, smallest equity decile, and largest equity decile. The government bond, corporate bond and Treasury bill data are from Ibbotson Associates. The treasury bill is the return to rolling over one month bills each month. The smallest and largest decile returns are common stock portfolios formed by total equity market capitalization of the firms. These are from the CRSP. Panel A reports summary statistics for one "plus" the growth rate in the not seasonally adjusted consumption measures, C_t (from Table I). Panel B provides summary statistics for quarterly values of

$$\beta \left(\frac{\kappa_{s(t+1)}}{\kappa_{s(t)}} \right)^{1-\alpha} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha}$$

evaluated at the point estimates of β and the κ_s from the four asset system reported in Table IV (seasonal taste-shifter model). Panel C presents summary statistics for the quarterly values of

$$\left(\frac{\kappa_{s(t+1)} C_{t+1} - e^{\mu(t+1)} \delta}{\kappa_{s(t)} C_t - e^{\mu t} \delta} \right)^{-\alpha}$$

evaluated at the point estimates of the κ_s , δ and μ from the four excess return system reported in Table VI (seasonal taste-shifter model with trending subsistence). Finally, panel D shows the summary statistics for the quarterly values of

$$\beta \left\{ \left(\frac{C_{t+1} - bC_{t-3}}{C_t - bC_{t-4}} \right)^{-\alpha} - b\beta^4 \left(\frac{C_{t+5} - bC_{t+1}}{C_t - bC_{t-4}} \right)^{-\alpha} \right\}$$

evaluated at the point estimates of β , α , and b from the five asset system reported in Table VII (seasonal habit formation model). All estimates are based on generalized method of moments using a constant, four lags of the real Treasury bill return and four lags of the real consumption growth as the instruments, Z_t . The instruments are deflated by the overall CPI. The sample is based on quarterly data from 1948:2–1987:4 (159 observations).

Table VIII—Continued

Variable	First Quarter	Second Quarter	Third Quarter	Fourth Quarter	Autocorrelations ^a						
	Mean	Mean	Mean	Mean	ρ_1	ρ_2	ρ_3	ρ_4	ρ_8	ρ_{12}	
Panel A. Not-seasonally adjusted consumption growth rates plus 1.0 (from Table I)											
Nondurables	0.836	1.078	0.999	1.127	-0.68	0.37	-0.67	0.98	0.98	0.98	
Services	1.018	0.996	1.004	1.004	-0.15	-0.26	-0.05	0.62	0.62	0.55	
Panel B. Taste shifter model (from Table IV and equation (4))											
Nondurables	0.724	1.143	1.009	1.170	-0.64	0.30	-0.63	0.95	0.91	0.89	
Services	1.078	0.958	0.982	0.950	-0.47	0.05	-0.45	0.88	0.86	0.82	
Panel C. Taste shifter model with trending subsistence (Table VI and equation (6))											
Nondurables	0.245	2.667	1.833	0.860	-0.15	-0.67	0.14	0.96	0.93	0.90	
Services	0.467	1.438	0.559	0.582	-0.03	-0.06	-0.10	0.56	0.59	0.38	
Panel D. Seasonal habit formation model (Table VII and equation (8))											
Nondurables	0.213	0.209	0.187	0.170	0.20	0.17	0.08	-0.34	-0.17	0.23	
Services	0.122	-0.259	0.169	0.113	0.08	0.02	-0.01	-0.31	0.01	-0.04	

^a ρ_j is the sample autocorrelation at lag j .

one plus the quarterly means and the autocorrelations of the seasonally unadjusted consumption growth rates, for a comparison.

The second panel of Table VIII reports statistics for the multiplicative taste shifter model, equation (4). In this model, $m_{t+1} = \beta [\kappa_{s(t+1)} / \kappa_{s(t)}]^{1-\alpha} [C_{t+1} / C_t]^{-\alpha}$, and the seasonality should be controlled by the shift parameters, κ_2 , κ_3 , and κ_4 . The third panel shows the summary statistics for the taste shifter model with a trending subsistence level, equation (6). In this model, $m_{t+1} = [(\kappa_{s(t+1)} C_{t+1} - e^{\mu(t+1)} \delta) / (\kappa_{s(t)} C_t - e^{\mu t} \delta)]^{-\alpha}$. The table shows that the time-separable models with taste shifters do a poor job of controlling the seasonal fluctuations in the consumption expenditures. The autocorrelations of the m_{t+1} at the seasonal lags using nondurables are very high, similar to the autocorrelations of the nondurable growth rates. Using services, the autocorrelations of the m_{t+1} for equation (4) are actually *larger* than those of the expenditure series.³² The sample means show differences across the quarters that are as large as the differences in the seasonally unadjusted consumption growth rates.

The bottom panel of Table VIII presents the results for the nonseparable model. We use the implied value of $m_{1,t+1}$, which is equivalent to within a scale factor to the value of $m_{t+1} = m_{1,t+1} / E(m_{2,t+1})$, as described in Section II.B.³³ The summary statistics show that the model with seasonal habit persistence does a better job of controlling seasonality than the time-separable models. The seasonal autocorrelations are reduced and the means differ less across the seasons, at least for the more highly seasonal nondurable expenditures. We find that the seasonal mean differences for services in Table VIII are largely driven by two outlying observations.³⁴

E. Fitting the Hansen-Jagannathan Bounds

It is interesting to further compare the moments of the implied marginal rates of substitution for the different models, in relation to asset returns. We graph the mean-standard deviation boundary for m_{t+1} implied by our sample of returns as the solid curves in Figures 4–8. An important question to address initially is whether the Hansen-Jagannathan bounds have empirical content using our sample of assets. It may be that the lower bound on the standard deviation of m_{t+1} is indistinguishable from zero. Snow (1991) and Hansen and Jagannathan (1991b) propose a test of this hypothesis. They observe that if the variance of m_{t+1} is zero, then m_{t+1} may be chosen to be a constant. Let μ_o be the constant and $x_t = 1 - \mu_o R_{t+1}$, where we hypothesize that $E(x_t) = 0$. Hansen and Jagannathan (1991b) show that the statistic

³² The overadjustment is not driven by the seasonal pattern peculiar to the small stock portfolio. We repeat the experiment, excluding the small stocks from the asset system, and we obtain similar results.

³³ We also examined the term $[(C_{t+1} - bC_{t-3}) / (C_t - bC_{t-4})]^{-\alpha}$ and found similar results.

³⁴ When the observations for the months numbered 125 and 129 are deleted, the seasonal means in the bottom panel for services are $Q_1 = 0.150$, $Q_2 = 0.172$, $Q_3 = 0.197$ and $Q_4 = 0.141$. The autocorrelations are: $\{0.39, 0.24, 0.01, -0.26, -0.16, \text{ and } -0.27\}$.

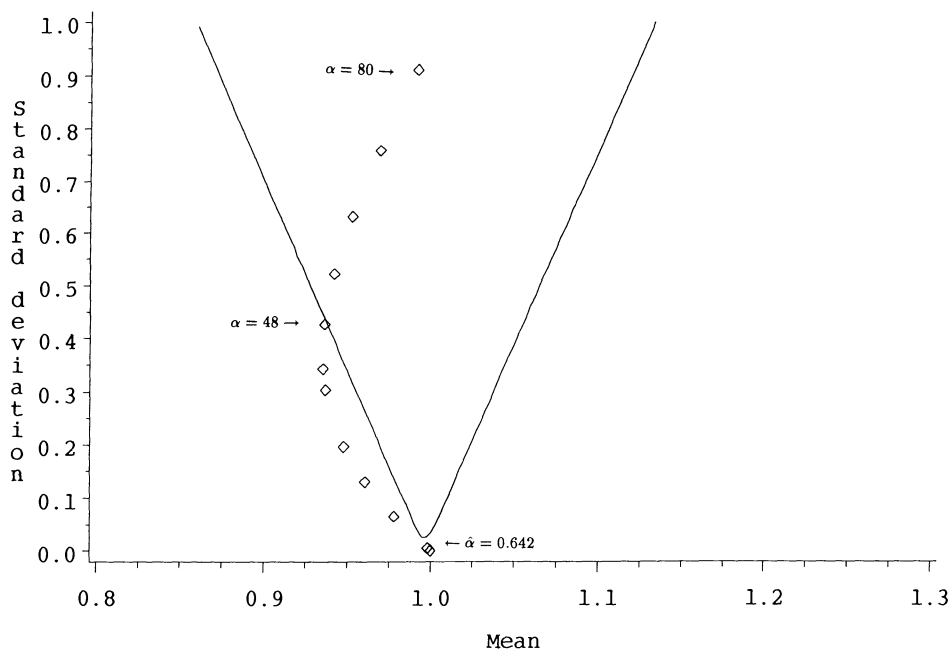


Figure 4. Intertemporal marginal rates of substitution—time-separable model with seasonally adjusted nondurables consumption. The mean-standard deviation boundary for the marginal rate of substitution implied by the our sample of asset returns is the solid curve. The diamonds depict the sample standard deviations and means of the implied marginal rate of substitution for a range of values for the concavity parameter, α , based on the model (3) presented in Table IV. The time-discount factor, β , is set equal to one. The point estimate of the concavity parameter from Table IV is denoted with a caret ($\hat{\alpha}$).

$K_T = \text{Min}_{\mu_0} \{(\sum x_t)' V_T^{-1} (\sum x_t)\} / T$ is asymptotically distributed as a chi-square random variable with $\dim(x_t) - 1$ degrees of freedom, where V_T is a consistent estimate of the covariance matrix of the x_t under the null. They suggest forming an empirical estimate of V_T by evaluating x_t at the value of the parameter μ_0 which minimizes the sample moment counterpart of the right hand side of equation (12). V_T is the sample moment counterpart to the asymptotic variance matrix: $\lim_{T \rightarrow \infty} \sum_{j=-\tau, \dots, \tau} \{(\tau - |j|)/\tau\} E(x_t x_{t-j}')$. Following Hansen and Jagannathan (1991b) we examine the K_T statistic using various lag lengths τ to see if the results are sensitive to the lag length. We find that the right tail p -values of the test statistics are between 0.020 and 0.066 for all lag lengths from $\tau = 0$ to $\tau = 8$. The results are not sensitive to the choice of the deflator which is used to define the real returns. The tests therefore can reject the hypothesis, at a 10% significance level, that the lower boundary of the regions in Figures 4–8 touch the horizontal axis at their lowest point.

Figure 4 shows the results of comparing the time-separable power utility model of equation (3) with the Hansen-Jagannathan bounds. The marginal rate of substitution implied by this model is $m_{t+1} = \beta[C_{t+1}/C_t]^{-\alpha}$. Season-

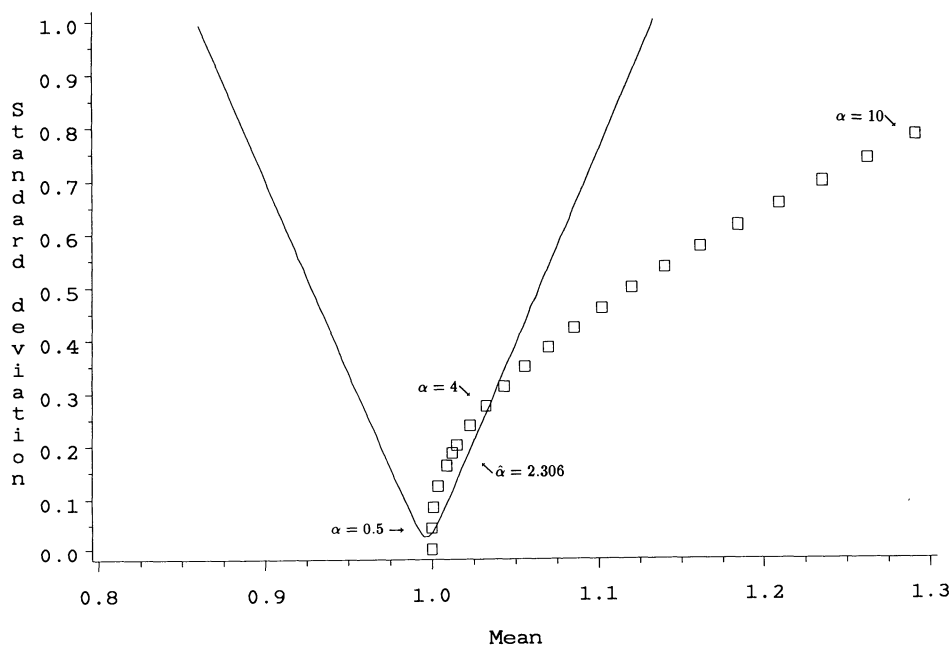


Figure 5. Intertemporal marginal rates of substitution—seasonal taste-shifter model with not seasonally adjusted nondurables consumption. The mean-standard deviation boundary for the marginal rate of substitution implied by the our sample of asset returns is the solid curve. The squares represent the sample standard deviations and means of the implied marginal rate of substitution for a range of values for the concavity parameter, α , based on the model (4) presented in Table IV. The time-discount factor, β , is set equal to one. The shifter parameters, κ_s , are set equal to the point estimates presented in Table IV. The point estimate for the concavity parameter from Table IV is denoted with a caret ($\hat{\cdot}$).

ally adjusted nondurables consumption data are used. Following Hansen and Jagannathan (1991a), we set the time preference parameter β equal to 1.0. The squares depict the sample standard deviations and means of the implied marginal rates of substitution for a range of values of the concavity parameter, α . The point estimate from Table IV is denoted with a $\hat{\cdot}$ (caret). Figure 4 shows that large values of α (in excess of 48) are required to satisfy the bounds. Similar results are obtained by Hansen and Jagannathan, for seasonally adjusted data and model (3).

Figure 5 depicts a similar analysis for not seasonally adjusted nondurables consumption and the model with multiplicative seasonal taste shifters, equation (4). The point estimates of the κ_s parameters from the four-asset system in Table IV are used. It is interesting that the mean and variance of the implied marginal rate of substitution satisfy the Hansen-Jagannathan bounds for a range of values for α between about $1/2$ and 4, including the point estimate of 2.3. This suggests that the rejections of the model (4) are not driven by the inability to fit the unconditional mean and variance. This result is consistent with our observation that the rejections are not “ex-

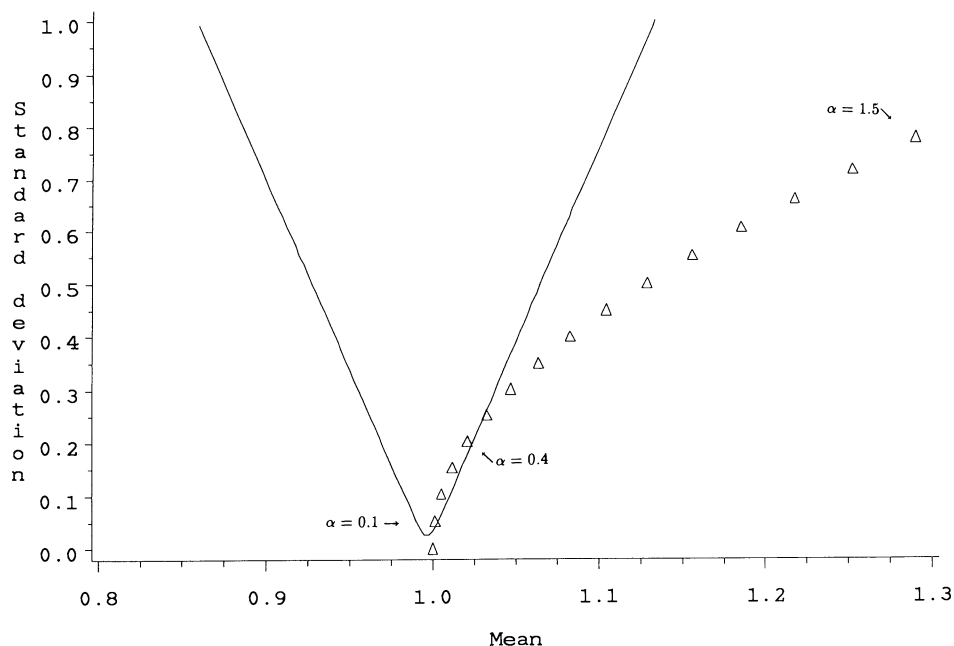


Figure 6. Intertemporal marginal rates of substitution—seasonal taste-shifter model and trending subsistence with not seasonally adjusted nondurables consumption. The mean-standard deviation boundary for the marginal rate of substitution implied by the our sample of asset returns is the solid curve. The triangles depict the sample standard deviations and means of the implied marginal rate of substitution for a range of values for the concavity parameter, α , based on the model presented in Table VI. The time-discount factor, β , is set equal to one. The shifter parameters, κ_s , and trend parameter, μ , are set equal to the point estimates presented in Table VI. The point estimate for the concavity parameter from Table VI is denoted with a caret ($\hat{\cdot}$).

plained” when the parameters λ_i are included to remove any restriction on the unconditional means in Table V. This reinforces our conclusion that the variation through time of the conditional expectations is sufficient to reject the time-separable model.

Figure 6 shows that the seasonal model with multiplicative taste shocks and a trending subsistence level, equation (6), fares worse than the model with no subsistence level in fitting the Hansen-Jagannathan bounds. Only for a narrow range of α values (0.10 to 0.40) are the bounds satisfied, and the point estimate of 1.75 implies a marginal rate of substitution far outside the parabola.

Figure 7 shows the results for the nonseasonal, time-nonseparable model, equation (7). The figure uses the point estimate from Table 7 (0.371) for the nonseparability parameter, which implies habit persistence. The sample mean and standard deviations of the implied m_{t+1} which we described in section II.B are plotted for different values of α . The points are inside the parabola for all values of α larger than about 3. However, the sample value

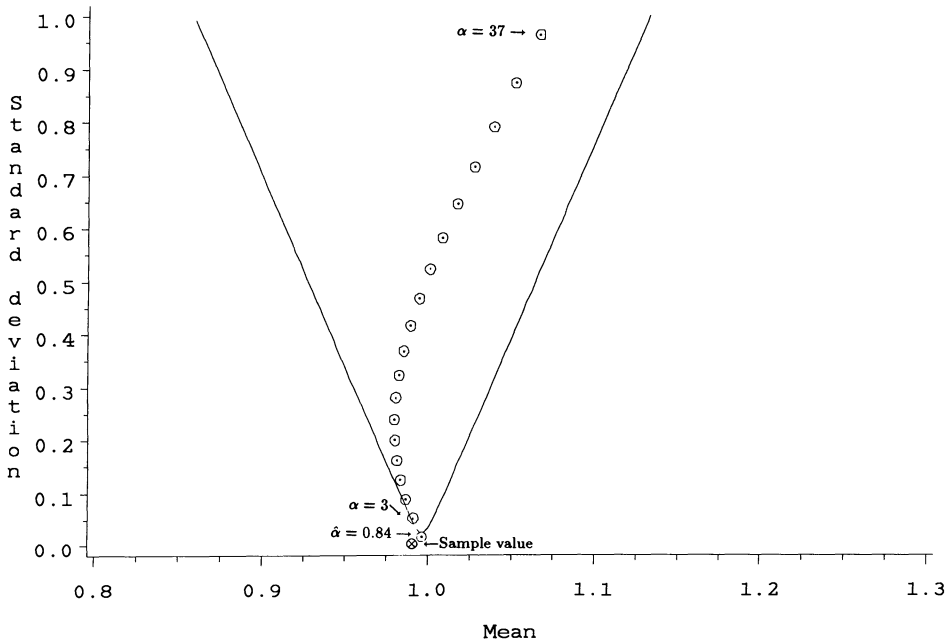


Figure 7. Intertemporal marginal rates of substitution—time-nonseparable model with seasonally adjusted nondurables consumption. The mean-standard deviation boundary for the marginal rate of substitution implied by our sample of asset returns is the solid curve. The circles represent the sample standard deviations and means of the implied marginal rate of substitution for a range of values for the concavity parameter, α , based on the model (7) presented in Table VII. The time-discount factor, β , is set equal to one. The nonseparability parameter, b , is set equal to 0.371 (from Table VII). The point estimate for the concavity parameter from Table VII is denoted with a caret ($\hat{\cdot}$). The point corresponding to $\beta = 0.997$ and $\alpha = 0.84$ (from Table VII) is denoted by the Kronecker product symbol ($\otimes$) and is labeled as the sample value.

of $\alpha = 0.84$ implies a point slightly outside the region. The point remains slightly outside the region when we replace the value of 1.0 for β with the sample estimate from Table VII. This point is denoted by a Kronecker product symbol ($\otimes$). Figure 7 shows that habit persistence allows the model with seasonally adjusted data to satisfy the bounds, and provides an improvement over the time-separable models.³⁵

Figure 8 provides the analysis for the seasonal time-nonseparable model, equation (8). The figure uses the point estimate of $b = 0.909$ for the nonseparability parameter, which implies strong seasonal habit persistence. The mean and standard deviation of the implied m_{t+1} are well inside the parabola for virtually all values of α , including the point estimate of $\alpha = 0.98$. This

³⁵ Hansen and Jagannathan (1991a), Gallant, Hansen, and Tauchen (1990) and Heaton (1990a) also find that habit persistence improves the fit to mean-variance bounds on the implied marginal rates of substitution using seasonally adjusted data.

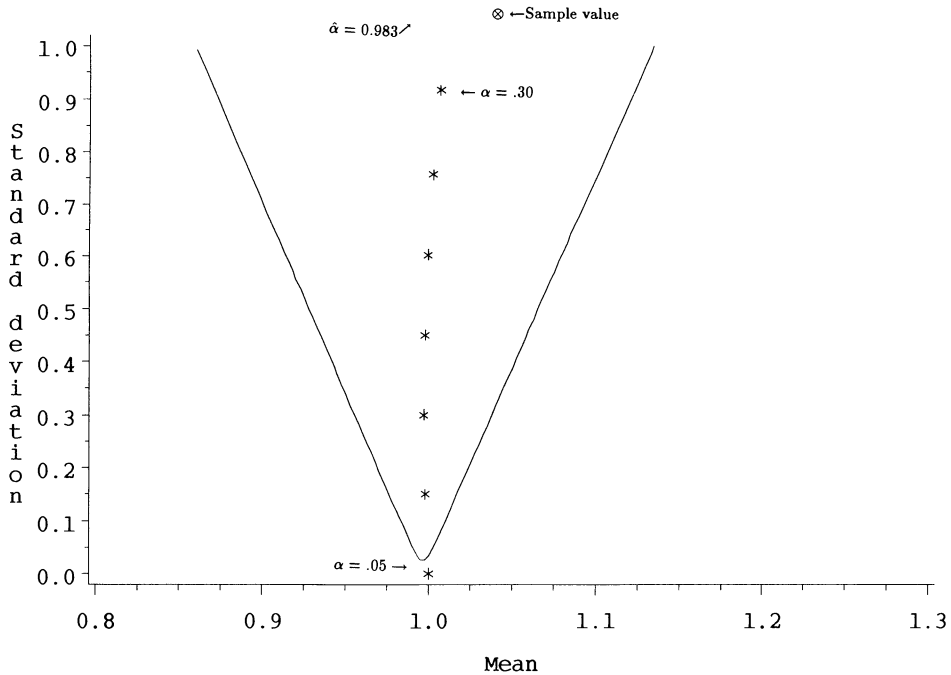


Figure 8. Intertemporal marginal rates of substitution—seasonal time-nonseparable model with not seasonally adjusted nondurables consumption. The mean-standard deviation boundary for the marginal rate of substitution implied by the our sample of asset returns is the solid curve. The stars depict the sample standard deviations and means of the implied marginal rate of substitution for a range of values for the concavity parameter, α , based on the model (8) presented in Table VII. The time-discount factor, β , is set equal to one. The non-separability parameter, b , is set equal to 0.909 (from Table VII). The point estimate for the concavity parameter from Table VII is denoted with a caret ($\hat{\cdot}$). The point corresponding to $\beta = 0.96$ and $\alpha = 0.98$ (from Table VII) is denoted by the Kronecker product symbol ($\otimes$) and is labeled as the sample value.

further confirms the improved fit of the model with habit persistence and not seasonally adjusted data.

IV. Conclusions

We examined consumption-based asset pricing models using not seasonally adjusted consumption data. The data suggest that such models should control for the strong seasonality in consumption that is not mirrored in asset returns. We incorporated seasonal taste shift parameters and estimated them jointly with the other model parameters. We examined several specifications. We found that in the time-separable models this approach does not control the seasonality in consumption. In fact, the models overadjusted for seasonality. A time-nonseparable model with seasonal nonseparability is more parsimonious and it does a better job of controlling the seasonality in consump-

tion. The parameter estimates imply a strong form of seasonal habit persistence.

Our tests of the Euler equation restrictions based on conditional moments emphasized the dynamic properties of consumption and asset returns. In time-separable models we found that the empirical results with not seasonally adjusted consumption are similar to the results found using X-11 seasonally adjusted data. For example, individual long-term bonds or stock portfolios do not produce striking evidence against the Euler restrictions, but Treasury bill returns and systems with multiple assets do. Examining multiple assets, the evidence against the time-separable models is even more striking using the not seasonally adjusted data, than when the X-11 seasonally adjusted data are used. The time-nonseparable models are not rejected by the Euler equation tests. The results for the model with seasonal nonseparability are especially striking: the right-tail p -value of the goodness-of-fit tests are greater than 0.9, while at the same time the parameter estimates are reasonable and statistically significant.

The equity premium puzzle of Mehra and Prescott (1985) implied that consumption data are too smooth to explain average equity and bill returns in a time-separable model, without resorting to implausible values of the concavity parameter. The not seasonally adjusted consumption data are much more volatile than the seasonally adjusted data, but the equity premium puzzle is not an artifact of the smoothing implied by seasonal adjustment. None of the models that we examined provides a precise explanation for the average excess returns.

We showed how the bounds on intertemporal marginal rates of substitution implied by asset returns, as developed by Hansen and Jagannathan (1991a), can be simply extended for nontime-separable models. It is not necessary to estimate conditional expectations using our approach. We find that the seasonal, time nonseparable model delivers an implied intertemporal marginal rate of substitution that easily satisfies the Hansen-Jagannathan bounds. We conclude that seasonal time-nonseparabilities are empirically relevant.

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